

# Quantum measurement of coherence in coupled quantum dots

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We describe the conditional and unconditional dynamics of two coupled quantum dots when one dot is subjected to a measurement of its occupation number using a single electron transistor (SET). The measurement is made when the bare tunneling rate through the SET is changed by the occupation number of one of the dots. We show that there is a difference between the time scale for the measurement-induced decoherence between the localized states of the dots and the time scale on which the system becomes localized due to the measurement. A comparison between theory and current experiments is made.

## I. INTRODUCTION

There have recently been a number of suggestions for a quantum computer architecture that use quantum dots of varying kinds [1–3]. If these schemes are to be practical many important physical questions need to be answered, one of which is how to readout physical properties such as charge or spin at a single electron level [4,5]. In this paper we present a quantum trajectory analysis of a general scheme to readout a single electronic qubit using a single electron transistor (SET). We adopt a general phenomenological description of the SET in which the tunneling rate through the SET is conditioned on the occupation or otherwise of a nearby quantum dot.

We consider two spatially separated quantum dots which are strongly coupled so that delocalized states of their relevant degrees of freedom can form. To be specific, we imagine each dot to have a single electronic bound state that can be occupied. Thus the average occupation number of each dot must be less than unity. This restriction can easily be removed to account for spin, or multiple electron states. We label each dot with an index 1, 2 and let  $c_i, c_i^\dagger$  represent the Fermi annihilation and creation operators for each single electron state (see figure 1).

The two dots are strongly coupled via the tunnel coupling Hamiltonian

$$V = i\hbar \frac{\Omega}{2} (c_1^\dagger c_2 - c_2^\dagger c_1) \quad (1)$$

Thus the total Hamiltonian of the two-dot system is

$$H = \hbar \sum_{i=1}^2 \omega_i c_i^\dagger c_i + V \quad (2)$$

In what follows we will work in an interaction picture and assume that the energies of each bound state are equal (again this can be relaxed). Coulomb blockade effects have been ignored at this stage, but can easily be included without significantly changing the results of this paper.

The single particle eigenstates of this Hamiltonian are even and odd superpositions of the bare states of each well. Such states are thus delocalized over the two-dot system and are sometimes called 'molecular states' in the literature [6]. The localized states can then be represented as an even and odd superposition of the delocalized states. The localized states are not stationary; rather, the system will periodically oscillate between them. That is to say the system will tunnel coherently between the two dots.

To this coherent system we add a measurement device which determines the presence of an electron on one of the dots, say dot 1, which we shall refer to as the target (see figure 1). The model is based on a SET tunnel junction containing a single bound state on the island. The interaction between the target and the SET is via a Coulomb blockade. Thus the interaction Hamiltonian between the SET and the target must commute with the target electron number operator,  $c_1^\dagger c_1$ . This makes it a QND (quantum nondemolition measurement) of electron number [7]. The Coulomb blockade changes the current flowing through the SET. In simple terms if there is no electron on the target the island state is biased so as to allow little or no current to flow through the SET. This is the quiescent state of the

SET. However when there is an electron on the target, the coulomb blockade shifts the bound state on the island to allow a greater current to flow through the device (see figure 2)

We derive a master equation to describe the behaviour of the target system. This master equation describes the unconditional evolution of the measured system when the results of all measurement records (that is current records) are averaged over. This will tell us the rate at which coherence in the target system is destroyed by the measurement. However we also need to know how the system state depends on the actual current through the device in order to determine how quickly the conditional state of the electron becomes localized which is measure of the quality of the measurement.

One approach to this problem is to keep track of many different states of the system, corresponding to the different numbers of electrons which have tunneled through the SET. This is the approach used for example in Ref. [8]. Here we adopt an alternate method which gives a more intuitive picture for the conditional dynamics. We use a conditional *stochastic* master equation which gives the evolution of the measured system, conditioned on a particular realization of the measured current. The instantaneous state of the target conditions the measured current while the measured current itself conditions the future evolution of the measured system in a self consistent manner. This approach to measurements has been variously called the quantum trajectory method [9] or quantum monte carlo method [10].

## II. SET MODEL

Consider a two dot system with the coupling in Eq. (1). If the electron is in dot-2 the quiescent rate of current tunneling through the SET is a constant which we will denote  $D_0$ . However if there is an electron on dot-1, the rate of tunneling through the SET changes to  $D_0 + D_1$  with  $D_1 > 0$ . If  $D_0$  does not equal zero then a current spike (resulting from a tunnel event in the SET) does not necessarily imply that the electron in the measured system is in dot-1. In an ideal device the quiescent tunneling rate,  $D_0$  is zero. In reality Johnson noise on the circuit containing the SET will give a non-zero quiescent tunneling current.

Assuming that the SET island state can be adiabatically eliminated, it is possible to derive a master equation for the state of the coupled dot system. This is done in the appendix, and the result is

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma_{\text{dec}}\mathcal{D}[c_1^\dagger c_1]\rho = \mathcal{L}\rho \quad (3)$$

where the irreversible part is defined for arbitrary operators  $A$  and  $B$  by

$$\mathcal{D}[A]B = \mathcal{J}[A]B - \mathcal{A}[A]B, \quad (4)$$

where

$$\mathcal{J}[A]B = ABA^\dagger \quad (5)$$

$$\mathcal{A}[A]B = \frac{1}{2}(A^\dagger AB + BA^\dagger A). \quad (6)$$

The decoherence rate is given by

$$\gamma_{\text{dec}} = 2D_0 + D_1. \quad (7)$$

The fact that the irreversible term is a function of the number operator in the target qubit is an indication that this describes a QND measurement of the occupation of the dot-1. It is easy to verify that the stationary solution of this master equation is an equal mixture of the two accessible electronic states.

The stochastic record of measurement ideally comprises a sequence of times, being the times at which electrons tunneled through the SET. In practice of course these events are not seen due to a finite frequency response of the circuit (including the SET) which averages each event over some time. However for the purpose of this paper we will take the zero response time limit. In this limit the current consists of a sequence of  $\delta$  function spikes. Formally we can write  $i(t) = edN/dt$ , where  $dN(t)$  is a classical point process which represents the number (either zero or one) of tunneling events seen in an infinitesimal time  $dt$ , and  $e$  is the electronic charge. We can think of  $dN(t)$  as the increment in the number of electrons  $N(t)$  in the collector in time  $dt$ . It is this variable, the accumulated electron number transmitted by the SET, which is used in Ref. [5].

The point process  $dN(t)$  is formally defined by the conditions

$$[dN(t)]^2 = dN(t) \quad (8)$$

$$\begin{aligned} \text{E}[dN(t)]/dt &= D_0\text{Tr}[(1 - n_1)\rho_c(t)(1 - n_1)] + (D_0 + D_1)\text{Tr}[n_1\rho_c(t)n_1] \\ &= D_0 + D_1 \langle n_1 \rangle_c(t). \end{aligned} \quad (9)$$

Here  $E[x]$  denotes a classical average of a classical stochastic process  $x$ , and

$$n_1 = c_1^\dagger c_1 \quad (10)$$

is the occupation number operator for the first dot. The first of these equations simply expresses the fact that  $dN(t)$  equals zero or one. The second says that the rate of events is equal to a background rate  $D_0$  plus an additional rate  $D_1$  if and only if the electron is in the first dot.

In Eq. (9), the system state matrix  $\rho_c(t)$  is *not* the solution of the master equation (3). That is because if one has a record of the current  $dN/dt$  through the SET then one knows more about the system than the master equation indicates. That is to say,  $\rho_c(t)$  is actually conditioned by  $dN(t')$  for  $t' < t$ , hence the subscript  $c$ . The first way of writing Eq. (9) hints at how  $dN(t')$  conditions  $\rho_c(t)$ . From the appendix, the state at time  $t + dt$  given  $dN(t) = 1$  is

$$\tilde{\rho}_1(t + dt) = dt [ D_0(1 - n_1)\rho(t)(1 - n_1) + (D_0 + D_1)n_1\rho(t)n_1 ] \quad (11)$$

This is an unnormalized state whose norm is equal to the probability of that event ( $dN(t) = 1$ ) occurring, as seen above in Eq. (9).

The normalized state can be written more elegantly as

$$\rho_1(t + dt) = \frac{(D_0 + D_1\mathcal{J}[n_1] + 2D_0\mathcal{D}[n_1])\rho(t)}{\text{Tr}\{(D_0 + D_1\mathcal{J}[n_1] + 2D_0\mathcal{D}[n_1])\rho(t)\}} = \frac{(D_0\mathcal{J}[1 - n_1] + (D_0 + D_1)\mathcal{J}[n_1])\rho(t)}{E[dN(t)/dt]}, \quad (12)$$

where  $\mathcal{J}$  is as defined above in Eq. (5).

To write the full conditioned evolution we need to know the density operator  $\rho_0(t + dt)$  given that  $dN(t) = 0$ . This can be found from Eq. (12) plus the fact that when averaged over the observed classical point process  $dN$ ,

$$\tilde{\rho}_0(t + dt) + \tilde{\rho}_1(t + dt) = (1 + \mathcal{L}dt)\rho(t). \quad (13)$$

That is to say, *on average* the system still obeys the master equation (3). From this equation, we obtain

$$\tilde{\rho}_0(t + dt) = \rho(t) - dt\{D_0\mathcal{A}[1 - n_1]\rho(t) + (D_0 + D_1)\mathcal{A}[n_1]\rho(t) + i[H, \rho(t)]\}, \quad (14)$$

$$= \rho(t) - dt\{D_0\rho(t) + D_1\mathcal{A}[n_1]\rho(t) - i[H, \rho(t)]\}, \quad (15)$$

where  $\mathcal{A}$  is as defined above in Eq. (6). Once again this state is unnormalized and its norm gives the probability that  $dN(t) = 0$ , that is

$$\text{Tr}[\tilde{\rho}_0(t + dt)] = 1 - E[dN(t)] \quad (16)$$

Using the variable  $dN(t)$  explicitly, the conditioned state at time  $t + dt$  is

$$\rho_c(t + dt) = dN(t)\frac{\tilde{\rho}_1(t + dt)}{\text{Tr}[\tilde{\rho}_1(t + dt)]} + [1 - dN(t)]\frac{\tilde{\rho}_0(t + dt)}{\text{Tr}[\tilde{\rho}_0(t + dt)]}. \quad (17)$$

Since  $dN(t)$  is almost always zero we can set  $dN(t)dt = 0$  and expand this expression to finally obtain the stochastic master equation, conditioned on the observed event in time  $dt$

$$d\rho_c = dN(t) \left[ \frac{D_0 + D_1\mathcal{J}[n_1] + 2D_0\mathcal{D}[n_1]}{D_0 + D_1\text{Tr}[\rho_c n_1]} - 1 \right] \rho_c + dt \{-D_1\mathcal{A}[n_1]\rho_c + D_1\text{Tr}[\rho_c n_1]\rho - i[H, \rho_c]\} \quad (18)$$

Note that averaging this equation over the observed stochastic process (by setting  $dN$  equal to its expected value) gives the unconditional master equation (3).

### III. AVERAGE STEADY STATE PROPERTIES

We now analyze in some detail the ensemble averaged properties of the system based on the unconditional master equation. In particular we calculate the stationary noise power spectrum of the current through the SET when there is the possibility of coherent tunneling between dot-2 and the measured dot-1. The details of how the quantum stochastic processes in the SET determine the average current though the SET are given in reference [12]. The link with the stochastic formalism of the preceding section is that the current  $i(t)$  through the SET is given by

$$i(t) = e \frac{dN(t)}{dt}. \quad (19)$$

First we calculate the steady state current

$$\begin{aligned} i_\infty &= \text{E}[i(t)]_\infty \\ &= e(D_0 + D_1 \langle c_1^\dagger c_1 \rangle_\infty) \end{aligned} \quad (20)$$

$$= e(D_0 + \frac{D_1}{2}), \quad (21)$$

where the  $\infty$  subscript indicates that the system is at steady-state. The fluctuations in the observed current,  $i(t)$  are quantified by the two-time correlation function:

$$G(\tau) = \text{E}[i(t)i(t+\tau) - i_\infty^2]_\infty \quad (22)$$

$$= ei_\infty \delta(\tau) + \langle i(t), i(t+\tau) \rangle_\infty^{\tau \neq 0}. \quad (23)$$

Here  $\langle A, B \rangle \equiv \langle AB \rangle - \langle A \rangle \langle B \rangle$ . The fact that the multiplier of the shot noise is  $ei_\infty$  rather than the usual  $(e/2)i_\infty$  is because of the approximation we have made in treating the SET. Specifically, we have adiabatically eliminated the SET by taking the limit where any electron which tunnels onto the SET island from the emitter immediately tunnels off to the collector. This means that, on the time scales we are interested in, there is a perfect correlation between the emitter current and collector current. This leads to a doubling of the shot noise level. Of course, at very high frequencies, higher than we are interested in, the true shot noise level of  $(e/2)i_\infty$  could still be seen in principle.

To relate these classical averages to the fundamental quantum processes occurring in the well we apply the theory of open quantum system [9,11] to the present system. Specifically, we can relate the correlation function for the current to the following quantum averages

$$\langle i(t)i(t+\tau) \rangle_\infty^{\tau \neq 0} = e^2 \text{Tr} [(D_0 + D_1 \mathcal{J}[n_1] + 2D_0 \mathcal{D}[n_1]) e^{\mathcal{L}\tau} (D_0 + D_1 \mathcal{J}[n_1] + 2D_0 \mathcal{D}[n_1]) \rho_\infty]. \quad (24)$$

Because  $\rho_\infty$  is an equal mixture of the two electron states, it satisfies

$$\mathcal{D}[n_1] \rho_\infty = 0. \quad (25)$$

In addition, the following identities for arbitrary operators  $A$  and  $B$  are easy to prove:  $\text{Tr} [\mathcal{D}[A]B] \equiv 0$ ,  $\text{Tr} [\mathcal{J}[n_1]B] \equiv \text{Tr}[n_1 B]$ ,  $\text{Tr}[e^{\mathcal{L}\tau} B] = \text{Tr}[B]$ , and  $\text{Tr}[Ae^{\mathcal{L}\tau} \rho_\infty] = \text{Tr}[A\rho_\infty]$ . Using these simplifications we obtain

$$\langle i(t), i(t+\tau) \rangle_\infty^{\tau \neq 0} = D_1^2 e^2 \{ \text{Tr} [n_1 e^{\mathcal{L}\tau} \mathcal{J}[n_1] \rho_\infty] - \text{Tr}[n_1 \rho_\infty]^2 \}. \quad (26)$$

Evaluating this expression we find

$$G(\tau) = ei_\infty \delta(\tau) + \frac{e^2 D_1^2}{8} \left( \frac{\mu_+ e^{\mu_- \tau} - \mu_- e^{\mu_+ \tau}}{\sqrt{(\gamma_{\text{dec}}/4)^2 - \Omega^2}} \right), \quad (27)$$

where

$$\mu_\pm = -(\gamma_{\text{dec}}/4) \pm \sqrt{(\gamma_{\text{dec}}/4)^2 - \Omega^2}, \quad (28)$$

and where the first term represents the shot noise component as discussed above. The power spectrum of the noise is

$$S(\omega) = \int_0^\infty d\tau G(\tau) 2 \cos(\omega\tau), \quad (29)$$

which evaluates to

$$S(\omega) = ei_\infty + \frac{e^2 D_1^2 \Omega^2 / 2}{\sqrt{(\gamma_{\text{dec}}/4)^2 - \Omega^2}} \left\{ \frac{1}{\mu_+^2 + \omega^2} - \frac{1}{\mu_-^2 + \omega^2} \right\}. \quad (30)$$

In the case that  $\Omega > \gamma_{\text{dec}}/4$  the spectrum will have a double peak structure indicating that coherent tunneling is taking place between the two coupled dots. For smaller  $\Omega$  only a single peak appears in the spectrum. We can thus use the noise power spectrum of the current through the SET as a means to measure the tunnel coupling between dots if the tunnel coupling is high enough. We illustrate this in figure 3.

#### IV. ANALYTICAL RESULTS FOR CONDITIONAL DYNAMICS

We now return to the stochastic master equation for the conditioned state,

$$d\rho_c = dN \left[ \frac{D_0 + D_1 \mathcal{J}[c_1^\dagger c_1] + 2D_0 \mathcal{D}[c_1^\dagger c_1]}{D_0 + D_1 \text{Tr}[\rho_c c_1^\dagger c_1]} - 1 \right] \rho_c + dt \left\{ -D_1 \frac{1}{2} \{c_1^\dagger c_1, \rho_c\} + D_1 \text{Tr}[\rho_c c_1^\dagger c_1] \rho - i[H, \rho_c] \right\}. \quad (31)$$

Comparing this to the unconditional master equation

$$\dot{\rho} = -i[H, \rho] + \gamma_{\text{dec}} \mathcal{D}[c_1^\dagger c_1] \rho \quad (32)$$

we see that decoherence between the two coupled dots, 1 and 2, takes place at the rate  $\gamma_{\text{dec}} = 2D_0 + D_1$ , but that the system decides between the two possibilities (electron on dot-1 or on dot-2) on a time scale that depends on  $D_1$  and  $D_0$  in some more complicated way. Of course this measurement time scale is necessarily at least as large as the decoherence time scale because successfully distinguishing between the two dots would by definition destroy any coherence between them.

The different measurement time scales can be derived most easily by introducing the Bloch representation of the state matrix:

$$\rho = \frac{1}{2} (I + x\sigma_x + y\sigma_y + z\sigma_z) \quad (33)$$

where the Pauli matrices are defined using the Fermi operators for the two dots

$$\sigma_x = c_1^\dagger c_2 + c_2^\dagger c_1 \quad (34)$$

$$\sigma_y = -ic_1^\dagger c_2 + ic_2^\dagger c_1 \quad (35)$$

$$\sigma_z = c_2^\dagger c_2 - c_1^\dagger c_1 \quad (36)$$

In this representation the means of the Pauli matrices  $\sigma_\alpha$  are given by the respective coefficient  $\alpha$ , with  $\alpha = x, y, z$ .

The stochastic master equation can now be written as a set of coupled stochastic differential equations for the Bloch sphere variables as

$$dz_c = \Omega x_c dt + \frac{D_1}{2} (1 - z_c^2) dt - dN(t) \frac{D_1 (1 - z_c^2)/2}{D_0 + D_1 (1 - z_c)/2} \quad (37)$$

$$dx_c = -\Omega z_c dt - \frac{D_1}{2} z_c x_c dt - dN(t) x_c \quad (38)$$

$$dy_c = -\frac{D_1}{2} z_c y_c dt - dN(t) y_c. \quad (39)$$

Again the c-subscript is to emphasize that these variables refer to the conditional state. If we average over the noise, the ensemble dynamics is then seen to be given by

$$\frac{dz}{dt} = \Omega x \quad (40)$$

$$\frac{dx}{dt} = -\Omega z - \frac{\gamma_{\text{dec}}}{2} x \quad (41)$$

$$\frac{dy}{dt} = -\frac{\gamma_{\text{dec}}}{2} y \quad (42)$$

where  $\alpha = E[\alpha_c]$  denotes the averaging over the ensemble of conditional states. These equations are exactly what would be obtained directly from the ensemble averaged master equation Eq(3). In particular we note that the average population difference  $z$  between the dots is a constant of the motion in the absence of any free Hamiltonian. However the stochastic differential equations enable us to calculate important averages that are *not* obtainable from the master equation. For example, if the model does indeed describe a measurement of  $c_1^\dagger c_1 = (1 - \sigma_z)/2$ , then, in the absence of tunneling, we would expect to see the conditional state become localized at either  $z = 1$  or  $z = -1$ . Indeed for  $\Omega = 0$  we can see from the conditional equation for  $z_c$  that  $z_c = \pm 1$  is a fixed point.

We can take into account both fixed points by considering  $z_c^2$ . In the absence of tunneling this must approach 1 for all trajectories, since the system will eventually become localized due to the measurement in one dot or the other. Therefore it is sensible to take the ensemble average  $E[z_c^2]$  and find the rate at which this deterministic quantity approaches one. Noting that for a stochastic variable we have  $d(z^2) = 2zdz + dzdz$ , and that  $E[dN^2] = E[dN] = [D_0 + D_1(1 - z_c)/2]dt$ , we find that

$$\frac{dE[z_c^2]}{dt} = E \left[ \frac{D_1^2(1 - z_c^2)^2}{4D_0 + 2D_1(1 - z_c)} \right]. \quad (43)$$

If the system starts state which has an equal probability for single electron to be on each dot then  $z_c(0) = 0$  and in the ensemble average this would remain the case. However if we ensemble average  $z_c^2$  over many quantum trajectories then for short times we find

$$E[z_c^2(\delta t)] = \frac{D_1^2}{4D_0 + 2D_1} \delta t \quad (44)$$

That is to say, the system tends towards a definite state (with  $z_c = \pm 1$  so  $z_c^2 = 1$ ) at an initial rate of  $D_1^2/(4D_0 + 2D_1)$ . For vanishing  $D_0$ , this is the same as the decoherence rate,  $D_1/2$ , as expected. But for  $D_0 \gg D_1$ , the rate goes to  $(D_1/2D_0) \times D_1/2 \ll D_1/2$ . That is, the rate at which the system becomes localized at one or the other dot is much less than the decoherence rate. This result cannot be obtained from the ensemble averaged master equation alone. It is a direct reflection of the fact that for  $D_0 \neq 0$  a tunneling event cannot be unambiguously attributed to the location of an electron on the double dot system. As the rate of localization is a direct indication of the quality of the measurement, we can use the localization rate defined as

$$\gamma_{\text{loc}} = \frac{D_1^2}{4D_0 + 2D_1} \quad (45)$$

as an important parameter defining the quality of the measurement. This parameter is related to the signal-to-noise ratio for this measurement as we now show.

For a Poisson process at rate  $R$ , the probability for  $m$  events to occur in time  $T$  is

$$p(m; T) = \frac{(RT)^m}{m!} e^{-RT} \quad (46)$$

The mean and variance of this distribution are equal and given by  $E(m) = \text{Var}(m) = RT$ . Now consider an electron which is, with equal likelihood, in either dot, so that  $z = 0$ . If the electron is in dot 1 then the rate of electrons passing through the SET is  $D_0 + D_1$ ; if it is in dot two then it is just  $D_0$ . These two possibilities will begin to be distinguishable when the difference in the means of the two distributions  $p(m, T)$  is of order the square root of the sum of the variances. That is, when

$$D_1 T \sim \sqrt{D_0 T + (D_0 + D_1) T} \quad (47)$$

Solving this for  $T$  gives a characteristic rate

$$T^{-1} \sim \frac{D_1^2}{2D_0 + D_1}. \quad (48)$$

The right hand side of this expression is simply twice the  $\gamma_{\text{loc}}$  defined above. A similar conclusion is reached in reference [5].

In the ideal limit of no quiescent current in the SET  $D_0 = 0$ , the stochastic master equation can be replaced by a stochastic Schrödinger equation, and will collapse to a single possibility at a rate  $D_1$  which is the same as the decoherence rate. The effect of  $D_0$  is most clearly seen in the other limit,  $D_1 \ll D_0$ , as noted above. In this limit the single electron makes only a small relative change in the tunneling rate through the SET. As the rate of jumps also becomes large, the trajectories in this limit take on the appearance of diffusion rather than jumps. The rate at which the electron localizes into one well or the other scales as  $D_1^{-2} D_0$ , which is much longer than the decoherence time scale  $D_0^{-1}$ .

## V. NUMERICAL SIMULATIONS OF CONDITIONAL DYNAMICS

We now turn to numerical simulations of the conditional evolution and to estimate the conditions for a good measurement. Unlike traditional condensed matter measurements we wish to describe repeated measurements made on a single quantum system rather than a single measurement made upon an ensemble of systems. To do this we use the conditional dynamics of the measured system given a particular measurement record as described by the above stochastic evolution equation (31)

We return to the Bloch description defined in Eq. (33). In what follows we will assume that  $y(0) = 0$ . For the form of tunneling used here the value of  $y$  does not in fact change under either conditional or ensemble averaged dynamics. If the conditional state of the system remains in a pure state then  $x_c^2 + y_c^2 + z_c^2 = x_c^2 + z_c^2 = 1$ . As noted previously this can only occur if the bare tunneling rate ( $D_0$ ) is zero, when a tunneling event can unambiguously be attributed to the occupation of the target dot and no information is lost about the state of the system. In the more realistic case in which  $D_0 \neq 0$ , we can use the quantity  $x_c^2 + z_c^2$  as a measure of the purity of the state, or equivalently as a measure of how much information the conditional record of measurements gives about the actual state of the two coupled dots. If the conditional state is a maximally mixed state of a two state system then  $x_c^2 + z_c^2 = 0$ . We now describe in detail the numerical simulation of the conditional dynamics.

### A. No Background Current

First we consider the case  $D_0 = 0$ , so the system is always in a pure state. Typical trajectories are shown in Fig. 4 for various values of  $\Omega$ . For small  $\Omega \ll D_1/2$  we see little evidence for coherent tunneling. Most of the time the electron is localized almost entirely in one well or the other. However, there is an asymmetry between the wells. A transition from dot-2 into dot-1 (the target) is sudden, occurring whenever an electron tunnels through the SET. A transition the other way takes a time of order  $2/D_1$ . This time is still much smaller than the average time between state-changing transitions, which can be shown analytically to be  $D_1/\Omega^2$ . Thus over a long time, as shown in Fig. 4(a), the system still has the appearance of a random telegraph. This behaviour gives rise to a single-peaked noise spectrum, as shown in Fig. 3(a).

For moderate  $\Omega \sim D_1/2$  the system is no longer well-localized in one dot or the other. Rather, the dynamics is complicated with clearly non-sinusoidal oscillations from one dot to the other interspersed with jumps into the target dot. The fact that oscillations are present gives peaks in the current noise spectrum, as shown in Fig. 3(b). The position of these peaks is a frequency less than  $\Omega$ , as shown analytically in Sec. III.

For  $\Omega \gg D_1/2$  the dynamics once again becomes simple, with nearly sinusoidal oscillations interspersed with jumps which occur with an average rate of  $D_1/2$ . This corresponds to a noise spectrum having a very sharp feature at  $\omega \approx \pm\Omega$ , as shown in Fig. 3(c).

The change in behaviour as  $\Omega$  increases is summarized in Fig. 5. There we plot  $E[z_c^2]$  versus  $\Omega/D_1$ . The quantity  $E[z_c^2]$  measures how well localized the electron is at one well or the other, and would be 1 if the electron were always localized and 0 if it were never localized. It is actually possible to calculate this quantity numerically without using a stochastic ensemble, as follows.

With no background current, every time an electron tunnels through the SET the electron on the dots is known to be on dot 1. If there are no further SET tunneling events for a time  $t$  later then from Eq. (14), the system evolves up to that time by the equation

$$d\tilde{\rho}_0(t) = -dt\{D_1\{c_1^\dagger c_1, \tilde{\rho}_0(t)\}/2 + i[H, \tilde{\rho}_0(t)]\}. \quad (49)$$

Because there is no background current, and because the initial state is pure, it is possible to rewrite this in terms of a non-Hermitian Schrödinger equation

$$d|\tilde{\psi}_0(t)\rangle = -dt(iH + D_1 c_1^\dagger c_1/2)|\tilde{\psi}_0(t)\rangle. \quad (50)$$

Here it must be remembered that the norm of this state represents the probability for the event that no electron has passed through the SET since the last one a time  $t$  ago:

$$p_0(t) = \text{Tr}[\tilde{\rho}_0(t)] = \langle\tilde{\psi}_0(t)|\tilde{\psi}_0(t)\rangle. \quad (51)$$

It is not difficult to show that the solution to Eq. (50) satisfying the initial condition  $|\psi_0(0)\rangle = |1, 0\rangle$  is

$$|\tilde{\psi}_0(t)\rangle = \alpha(t)|1, 0\rangle + \beta(t)|0, 1\rangle, \quad (52)$$

where the occupation numbers refer to the dots one and two in order. Here  $\alpha$  and  $\beta$  are real numbers defined by

$$\alpha(t) = \frac{1}{\lambda_+ - \lambda_-} (\lambda_+ e^{\lambda_+ t} - \lambda_- e^{\lambda_- t}), \quad (53)$$

$$\beta(t) = \frac{-\Omega/2}{\lambda_+ - \lambda_-} (e^{\lambda_+ t} - e^{\lambda_- t}), \quad (54)$$

where

$$\lambda_{\pm} = \frac{1}{2} \left[ -\frac{D_1}{2} \pm \sqrt{\left(\frac{D_1}{2}\right)^2 - \Omega^2} \right] \quad (55)$$

The conditioned quantum expectation value for  $\sigma_z$  is

$$z_0(t) = \frac{\langle \tilde{\psi}_0(t) | \sigma_z | \tilde{\psi}_0(t) \rangle}{p_0(t)} = \frac{\beta^2 - \alpha^2}{\beta^2 + \alpha^2} \quad (56)$$

Now in steady state the probability  $p_0(t)$  that there is no increment in  $N(t)$  for a time  $t$  ago is related to the probability  $q_0(t)$  that the last increment was a time  $t$  ago by

$$q_0(t) = \frac{p_0(t)}{\int_0^{\infty} p_0(s) ds}. \quad (57)$$

Since at steady state all conditioned states are uniquely identified by how long it has been since the last SET event, the ensemble average for  $z_c^2$  is simply given by

$$E[z_c^2] = \int_0^{\infty} q_0(t) [z_0(t)]^2 dt \quad (58)$$

$$= \left[ \int_0^{\infty} (\beta^2 + \alpha^2) dt \right]^{-1} \int_0^{\infty} \frac{(\beta^2 - \alpha^2)^2}{\beta^2 + \alpha^2} dt. \quad (59)$$

Unfortunately it does not appear possible to evaluate the second integral here analytically. However a numerical integration is easy. The results, shown in Fig. 5, is in agreement with the ensemble averages obtained numerically using the stochastic master equation.

## B. A Finite Background Current

We next consider the case where  $D_0 \neq 0$ . We show two plots, both with  $\Omega = D_1$ , which is a regime in which coherent tunneling is clearly evident in the current noise spectrum. The first plot, in Fig. 6, is for  $D_0 = D_1$ . Here coherent oscillations are still evident in  $z$ , but  $z$  rarely attains its extreme values of  $\pm 1$ . The conditioned state is no longer pure, even immediately after a count. Also, the conditioned state following a count now depends on the state before the count. For this reason an exact solution by the method of the preceding section is impossible.

The second plot, in Fig. 7, is for  $D_0 = 10D_1$ . Here coherent oscillations are no longer obvious in the condition mean of  $\sigma_z$ , even though they are present in the spectrum (as small features above the shot noise), as calculated in Sec. III. In this regime  $D_0 \gg D_1$  so the diffusive limit discussed at the end of Sec. IV applies.

## VI. DISCUSSION AND CONCLUSION

The three parameters we need to compare our theoretical results with experiment are  $\Omega$ ,  $D_0$  and  $D_1$ . The two incoherent tunneling rates can be obtained by considering how they determine the steady state current through the device. This is given in Eq (21). Our model implicitly assumes that the quiescent noise in the SET is shot-noise limited, based as it is on elementary tunneling events. However from the point of view of the macroscopic circuit in which the SET is placed, the tunnel junctions appear as a capacitor in series with a resistor ( see note 14 in reference [13]). If the resistance of the junction is  $R$  and the capacitance is  $C$  the fundamental time constant for the junction is  $\frac{1}{RC}$ . This sets an upper bound for the tunneling rate  $\gamma \leq \frac{1}{RC}$ . For a typical Al/AlO<sub>x</sub>/Al junctions we have  $C \approx 0.2$ fF



and  $R \approx 50k\Omega$  and thus the time constant is  $\gamma \approx 10^{11} \text{ s}^{-1}$ . For a double tunnel junction device, the maximum conductance is achieved for a symmetric pair of tunnel junctions. The value of  $D_1$  in this case is given by  $D_1 = \gamma/2$  where  $\gamma$  is the tunneling rate of the SET under conditions of maximum conductance (see appendix). We thus estimate that  $D_1 \approx 5 \times 10^{10} \text{ s}^{-1}$ . The background tunneling rate through the SET depends on temperature as well as the bias conditions (see appendix). Typical maximum and minimum conductance for the SET at different temperatures have been measured by Joyez *et al.* [15]. At a temperature of 100mK the minimum conductance is approximately zero, and thus at this temperature we can safely take  $D_0 = 0$ , the zero temperature result. However at a temperature of 400mK the ratio of the maximum to minimum conductance is 2.2. This indicates that at 400 mK,  $D_0 = 4 \times 10^{10} \text{ s}^{-1}$ .

The value we choose for the tunneling rate depends strongly on the particular quantum dot system. We will consider the value appropriate for the single electron measurement scheme of Kane *et al.* [14]. In this model the two localized states correspond to an electron on a tellurium ion donor or at a nearby interface below the donor. The tunneling rate is then between the donor state and the interface state. Kane *et al.* estimate that for this system  $\Omega \approx 10^9 \text{ s}^{-1}$ . This value is too low to observe a peak in the noise power spectrum away from zero frequency. However the tunneling rate could be increased by changing the donor interface bias voltage.

In conclusion we have presented a simple model to describe single electron measurements on a coupled double quantum dot system using an SET. We have given both the stationary (ensemble averaged) properties of the current through the SET as well as the conditional dynamics of repeated measurements on a single system. This illustrates how quantum trajectory methods may be naturally adapted to single electronics, and aid in the interpretation of ensemble averaged properties. We believe these models will prove useful in current attempts to fabricate quantum logic gates in solid state devices.

## APPENDIX A: DERIVATION OF THE MASTER EQUATION.

It will suffice to consider a single quantum dot near the SET. This allows us to remove any reference to the electron field labeled by  $c_2$ ,  $c_2^\dagger$ . The SET is modeled as a single biased double barrier (single well) device with a single bound state on the well described by the Fermi operators  $b$ ,  $b^\dagger$ . The total Hamiltonian for the system including the reservoirs is

$$H = H_0 + H_{CB} + H_{RT} + H_{LT} \quad (\text{A1})$$

The term  $H_{CB}$  is the Coulomb blockade term and is given by [3]

$$H_{CB} = \hbar\chi c_1^\dagger c_1 b^\dagger b \quad (\text{A2})$$

where  $\hbar\chi$  is the Coulomb blockade energy gap (see figure 2). Note this term can only be nonzero if there is an electron on the island and on the dot, in which case the energy of the island electron is increased. The terms  $H_{RT}$ ,  $H_{LT}$  described the tunneling between the many modes in the left and right ohmic contacts and the bound state on the SET [12]

$$\begin{aligned} H_{LT} &= \sum_k T_{Lk} a_{Lk}^\dagger b + T_{Lk}^* a_{Lk} b^\dagger \\ H_{RT} &= \sum_k T_{Rk} a_{Rk}^\dagger b + T_{Rk}^* a_{Rk} b^\dagger \end{aligned}$$

where  $a_{Lk}$ ,  $a_{Rk}$  are respectively the Fermi field annihilation operators for the left and right reservoir states at momentum  $k$ . The tunneling matrix elements between respectively the left and right Ohmic contacts and the island are  $T_{Lk}$ ,  $T_{Rk}$ . The free Hamiltonian for the the system is

$$H_0 = \hbar \sum_k \omega_k^L a_{Lk}^\dagger a_{Lk} + \omega_k^R a_{Rk}^\dagger a_{Rk} + \hbar\omega_1 c_1^\dagger c_1 + \hbar\omega_0 b^\dagger b \quad (\text{A3})$$

We now transform to an interaction picture to remove the terms  $H_0 + H_{CB}$ . The dynamics in the Schrödinger picture is now described by the time dependent Hamiltonian

$$\begin{aligned} H_I(t) &= \sum_k T_{Lk} a_{Lk}^\dagger b e^{i\chi t c_1^\dagger c_1} e^{-i(\omega_k^L - \omega_0)t} \\ &\quad + T_{Lk}^* a_{Lk} b^\dagger e^{-i\chi t c_1^\dagger c_1} e^{i(\omega_k^L - \omega_0)t} \\ &\quad + T_{Rk} a_{Rk}^\dagger b e^{i\chi t c_1^\dagger c_1} e^{-i(\omega_k^R - \omega_0)t} \\ &\quad + T_{Rk}^* a_{Rk} b^\dagger e^{-i\chi t c_1^\dagger c_1} e^{i(\omega_k^R - \omega_0)t} \end{aligned}$$

Using the fact that  $(c_1^\dagger c_1)^n = c_1^\dagger c_1$  we find

$$H_I(t) = H_1(t) + H_2(t)$$

where

$$H_1(t) = (1 - c_1^\dagger c_1) \sum_k (T_{Lk} a_{Lk}^\dagger b e^{-i(\omega_k^L - \omega_0)t} + \text{H.c.}) + (T_{Rk} a_{Rk}^\dagger b e^{-i(\omega_k^R - \omega_0)t} + \text{H.c.})$$

$$H_2(t) = c_1^\dagger c_1 \sum_k (T_{Lk} a_{Lk}^\dagger b e^{-i(\omega_k^L - \omega_0 - \chi)t} + \text{H.c.}) + (T_{Rk} a_{Rk}^\dagger b e^{-i(\omega_k^R - \omega_0 - \chi)t} + \text{H.c.})$$

Notice that if there is no electron on the dot and  $c_1^\dagger c_1 \rightarrow 0$  then the second term is zero and the first term is a standard tunneling interaction onto a bound state with energy  $\hbar\omega_0$ . On the other hand if there is an electron on the dot  $c_1^\dagger c_1 \rightarrow 1$  and the first term is zero and the second term is a standard tunneling interaction onto a bound state with energy  $\hbar(\omega_0 + \chi)$  as expected.

The derivation of the master equation for the state matrix  $R$  for the system (SET and quantum dot) can now proceed using standard techniques which we will sketch. The objective is to obtain a semigroup evolution in Lindblad form (that is to say positivity-preserving irreversible dynamics) for the state of the SET island and the dot alone with no reference to the ohmic contacts. The ohmic contacts are treated as perfect Fermi thermal reservoirs with a very fast relaxation constants. Each ohmic contact (left and right) remains in thermal equilibrium with chemical potentials  $\mu_L$ ,  $\mu_R$ , but the total system is not in thermal equilibrium due to the external bias potential,  $V$  with  $eV = \mu_L - \mu_R$  (see references [16,17] for further discussion). We first define a time interval  $\delta t$  which is slow compared to the dynamics of the island and the dot but very long compared to the time scale in which the ohmic contacts relax back to their steady state. The change in the state matrix  $W$  of the system (SET and dot) and environment (Ohmic contacts) from time  $t$  to  $t + \delta t$ , to second order in the tunnel coupling energy, is given by

$$W(t + \delta t) = W(t) - i\delta t[H_I(t), W(t)] - \delta t \int_t^{t+\delta t} dt_1 [H_I(t), [H_I(t_1), W(t_1)]] \quad (\text{A4})$$

We now make the first Markov approximation and assume that at any time the state of the total system may be approximated by  $W(t) = R(t) \otimes \rho_L \otimes \rho_R$ , that is to say the left and right ohmic contacts instantaneously relax back to Fermi distributions. We now obtain an evolution equation for  $R(t)$ , the state of the island and the dot by tracing over the reservoirs. The result is

$$\begin{aligned} \frac{dR(t)}{dt} = & [\gamma_L(1 - f_L(\omega_0)) + \gamma_R(1 - f_R(\omega_0))] \mathcal{D}[b(1 - c_1^\dagger c_1)]R \\ & + [\gamma_L f_L(\omega_0) + \gamma_R f_R(\omega_0)] \mathcal{D}[b^\dagger(1 - c_1^\dagger c_1)]R \\ & + [\gamma'_L(1 - f_L(\omega_0 + \chi)) + \gamma'_R(1 - f_R(\omega_0 + \chi))] \mathcal{D}[bc_1^\dagger c_1]R \\ & + [\gamma'_L f_L(\omega_0 + \chi) + \gamma'_R f_R(\omega_0 + \chi)] \mathcal{D}[b^\dagger c_1^\dagger c_1]R \end{aligned}$$

where for arbitrary operators  $A$  and  $B$ ,  $\mathcal{D}[A]B = ABA^\dagger - \frac{1}{2}(A^\dagger AB - BA^\dagger A)$  and where  $f_{L,R}(\omega)$  is the Fermi filling probability for the left/right ohmic contact at the energy  $\hbar\omega$ . The rates  $\gamma_{L,R}$  and  $\gamma'_{L,R}$  determine the rate of injection from the left ohmic contact into the island or emission from the island into the right ohmic contact under the conditions of no electron on the dot (unprimed) and with an electron on the dot (primed). These are evaluated using the second markov approximation as

$$\gamma_L = |T_{Lk_0}|^2, \quad (\text{A5})$$

$$\gamma_R = |T_{Rk_0}|^2, \quad (\text{A6})$$

$$\gamma'_L = |T_{Lk'_0}|^2 \quad (\text{A7})$$

$$\gamma'_R = |T_{Rk'_0}|^2 \quad (\text{A8})$$

where  $k_0 = \sqrt{2m\omega_0/\hbar}$  and  $k'_0 = \sqrt{2m(\omega_0 + \chi)/\hbar}$ .

The ideal quiescent state of the SET is defined as  $f_L(\omega_0) = 1$ ,  $f_R(\omega_0) = 1$  while  $f_L(\omega_0 + \chi) = 1$ ,  $f_R(\omega_0 + \chi) = 0$ . Under these conditions the master equation reduces to

$$\frac{dR}{dt} = \gamma_R \mathcal{D}[b(1 - c_1^\dagger c_1)]R + \gamma_L \mathcal{D}[b^\dagger(1 - c_1^\dagger c_1)]R + \gamma'_R \mathcal{D}[bc_1^\dagger c_1]R + \gamma'_L \mathcal{D}[b^\dagger c_1^\dagger c_1]R \quad (\text{A9})$$

We now wish to derive a master equation for the state matrix  $\rho$  for the dot alone. This is easiest if we assume that  $\gamma_R, \gamma'_R$  are much larger than all other rates in the system. In this case it is possible to adiabatically eliminate the SET island using techniques similar to that in Ref. [18]. We expand the state matrix  $R$  in powers of  $1/\gamma_R$  or  $1/\gamma'_R$  as

$$R = \rho_0 \otimes |0\rangle\langle 0| + \rho_1 \otimes |0\rangle\langle 0|. \quad (\text{A10})$$

The equations of motion for  $\rho_1$  and  $\rho_0$  are

$$\dot{\rho}_1 = -\gamma_R \mathcal{A}[1 - n_1] \rho_1 + \gamma_L \mathcal{J}[1 - n_1] \rho_0 - \gamma'_R \mathcal{A}[n_1] \rho_1 + \gamma'_L \mathcal{J}[n_1] \rho_0 \quad (\text{A11})$$

$$\dot{\rho}_0 = \gamma_R \mathcal{J}[1 - n_1] \rho_1 - \gamma_L \mathcal{A}[1 - n_1] \rho_0 + \gamma'_R \mathcal{J}[n_1] \rho_1 - \gamma'_L \mathcal{A}[n_1] \rho_1 \quad (\text{A12})$$

Here  $n_1 = c_1^\dagger c_1$  and  $\mathcal{J}$  and  $\mathcal{A}$  are as defined in Eqs. (5), (6). Under the above conditions, we can slave  $\rho_1$  to  $\rho_0$  so that

$$(\gamma_R \mathcal{A}[1 - n_1] + \gamma'_R \mathcal{A}[n_1]) \rho_1 = (\gamma_L \mathcal{J}[1 - n_1] + \gamma'_L \mathcal{J}[n_1]) \rho_0 \quad (\text{A13})$$

Operating on both sides alternately by  $\mathcal{J}[n_1]$  and  $\mathcal{J}[1 - n_1]$  it is easy to show that

$$\gamma'_R \mathcal{J}[n_1] \rho_1 = \gamma'_L \mathcal{J}[n_1] \rho_0 \quad (\text{A14})$$

$$\gamma_R \mathcal{J}[1 - n_1] \rho_1 = \gamma_L \mathcal{J}[1 - n_1] \rho_0. \quad (\text{A15})$$

Substituting these into Eq. (A12) yields

$$\dot{\rho}_0 = (\gamma'_L + \gamma_L) \mathcal{D}[n_1] \rho_0 \quad (\text{A16})$$

Since the probability of their being an electron on the SET is very small we can say  $\rho \simeq \rho_0$ . Hence we have derived The master equation (3) (without the Hamiltonian term) for the dot alone

$$\dot{\rho} = (2D_0 + D_1) \mathcal{D}[c_1^\dagger c_1] \rho. \quad (\text{A17})$$

Here we have defined

$$D_1 = \gamma'_L - \gamma_L, \quad (\text{A18})$$

$$D_0 = \gamma_L. \quad (\text{A19})$$

Because the SET collector reservoir in the two cases (an electron on the dot and an electron not on the dot) are independent (due to the SET energy shift), the state conditioned on an electron entering the collector is an incoherent mixture of the two possible paths. From quantum trajectory theory [9,11], the unnormalized state conditioned on this event is

$$dt (\gamma_R \mathcal{J}[b(1 - n_1)] + \gamma'_R \mathcal{J}[bn_1]) R. \quad (\text{A20})$$

The norm of this state matrix gives the probability for this event, and is equal to the norm of

$$dt (\gamma_R \mathcal{J}[1 - n_1] + \gamma'_R \mathcal{J}[n_1]) \rho_1. \quad (\text{A21})$$

From the adiabatic elimination procedure above, this is equal to

$$dt (\gamma_L \mathcal{J}[1 - n_1] + \gamma'_L \mathcal{J}[n_1]) \rho. \quad (\text{A22})$$

This is the unnormalized state  $\tilde{\rho}_1(t + dt)$  of the dot alone conditioned on an electron tunneling through the SET. From this it is easy to derive the rate of such tunnelings as

$$\gamma_L \langle 1 - n_1 \rangle + \gamma'_L \langle n_1 \rangle = D_0 + D_1 \langle c_1^\dagger c_1 \rangle. \quad (\text{A23})$$

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FIG. 1. Schematic representation of a single electron measurement for two coupled quantum dots

FIG. 2. SET using coulomb blockade for a single electron measurement. The Coulomb blockade gap is labeled  $E_{CB}$  and the tunneling rates in the 'on' position are  $\gamma_L$  and  $\gamma_R$  through the left and right barriers respectively. In (a) the electron is localized on dot-2 and a background current  $eD_0$  flows. In (b) the electron is localized on dot-1 and the current  $e(D_0 + D_1)$  flows in the SET.

FIG. 3. A plot of the noise power spectrum normalized by the shot noise level for  $D_0 = 0$ . (a)  $\Omega = 0.1$ , (b)  $\Omega = 0.5$ , (c)  $\Omega = 5.0$ .

FIG. 4. A plot of the conditional population difference dynamics of  $z_c(t)$  versus scaled time for various values of the tunneling rate. In all cases  $D_0 = 0$  and time is measured in units of  $D_1^{-1}$ . (a)  $\Omega = 0.1$ , (b)  $\Omega = 0.5$ , (c)  $\Omega = 5.0$ .

FIG. 5. A plot of the average of the conditional quantity  $z_c^2$  versus  $\Omega/D_1$ , which measures the extent to which the measurement localises the state of the dot. We set  $D_0 = 0$ . The solid lines refer to the exact result Eq(59)

FIG. 6. The effect of finite background current with  $D_0 = 1.0$  and  $\Omega = 1.0$ . A plot of (a) the 'purity measure'  $x_c^2 + y_c^2 + z_c^2$  versus scaled time (units of  $D_1^{-1}$ ) and (b) the conditional population difference  $z_c(t)$  for a typical trajectory.

FIG. 7. The effect of finite background current with  $D_0 = 10.0$  and  $\Omega = 1.0$ . A plot of (a) the 'purity measure'  $x_c^2 + y_c^2 + z_c^2$  versus scaled time (units of  $D_1^{-1}$ ) and (b) the conditional population difference  $z_c(t)$  for a typical trajectory.

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