Hydraulic conditions leading to exponential mine tailings delta profiles

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A recent theoretical model[1] of the subaerial deposition of mine tailings slurries predicts that the profile of a tailings delta formed with constant inputs will conform to an exponential curve.[1,2] Both the profiles of and the particle sorting on one platinum and two coal tailings deltas have been shown to conform to the theoretical model, as have the profiles of three laboratory coal tailings deltas.[1,2] The profiles of a further eight metalliferous and two coal tailings deltas, with a wide range of input tailings parameters, are shown here also to conform to the exponential curve predicted by the theoretical model.

The theoretical model was, however, based in part on an earlier analysis by de Vries[3] of the relative celerities of mobile bed flows, which enabled the governing equations to be simplified on the assumption that the flow was quasi-steady, but did not allow the maximum values of the volumetric solids concentration, c, compatible with the analysis to be evaluated. A more rigorous analysis of the relative celerities of mobile bed flows[4] has since shown that de Vries' analysis implies low c unrepresentative of mine tailings, for which values ≥ 0.25 are possible. Thus, both de Vries' analysis and the governing equations of mobile bed flows with low c that were used to derive the theoretical model[1] are inappropriate. In the present note the model is revised on the basis of an analysis of the governing equations of mobile bed flow with high c.

Delta profiles

According to the recent theoretical model of subaerial deposition of mine tailings slurries,[1] the profile of a tailings delta formed with the discharge and the initial volumetric solids concentration, c_0, specific gravity, G_s, and particle-size distribution of the input tailings all constant will conform to

$$S = S_0 \exp\left(-\omega \frac{x}{L}\right) \tag{1}$$

where x is longitudinal coordinate relative to the highest point on the delta, L is length of the delta, S is bed slope, $S_0 = S$ at $x = 0$ (Fig. 1) and ω is a non-dimensional positive constant given by[2]

$$\omega = \ln\left(\frac{S_0}{S_L}\right) \tag{2}$$

where $S_L = S$ at $x = L$.

Equation 1 was previously fitted with the use of least-squares regression methods to South African platinum tailings delta profile data obtained by Bentel[5] and to full-size

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Table 1 Parameters characterizing input tailings and rates of deposition on bed

Tailings	**Discharge, $m^3\ s^{-1}$**	c_0	G_s	D_{50}, **mm**	z', **m** s^{-1}
Full-size deltas					
1 Aberdare	–	7.9×10^{-2}	1.50	1.34×10^{-1}	–
2 Meandu	–	2.5×10^{-1}	1.92	1.78×10^{-2}	–
3 Oaky Creek 1	–	2.0×10^{-1}	1.93	6.40×10^{-3}	–
4 Oaky Creek 2	–	2.0×10^{-1}	1.93	6.40×10^{-3}	–
5 Platinum	–	2.5×10^{-1}	3.05	8.40×10^{-2}	–
6 Molybdenum 1	1.9×10^{-2}	8.5×10^{-2}	2.71	7.68×10^{-2}	5×10^{-8}
7 Molybdenum 2	2.8×10^{-2}	1.3×10^{-1}	2.71	1.36×10^{-1}	6×10^{-8}
8 Molybdenum 3	4.6×10^{-1}	8.5×10^{-2}	2.71	7.68×10^{-2}	2×10^{-6}
9 Copper	6.0×10^{-3}	2.0×10^{-1}	2.80	4.58×10^{-2}	1×10^{-8}
10 Gold sulphides	3.0×10^{-3}	9.6×10^{-2}	2.82	1.12×10^{-1}	2×10^{-6}
11 Silver	1.3×10^{-2}	7.3×10^{-2}	2.90	3.31×10^{-2}	2×10^{-6}
12 Uranium 1	5.0×10^{-2}	2.2×10^{-1}	2.69	2.62×10^{-1}	2×10^{-5}
13 Uranium 2	3.1×10^{-2}	2.0×10^{-1}	2.68	2.63×10^{-1}	3×10^{-6}
Laboratory model deltas					
14 Aberdare	–	3.7×10^{-2}	2.00	1.01×10^{-1}	–
15 Meandu 1	–	–*	–	–	–
16 Meandu 2	5.0×10^{-4}	2.4×10^{-1}	1.90	4.21×10^{-2}	3×10^{-4}

*Initial gravimetric solids concentration = 0.42.

and laboratory-model delta profile data for tailings from Aberdare and Meandu collieries in southeast Queensland, Australia.[1,2] Similar methods have since been used to analyse profile data obtained from Oaky Creek colliery in central Queensland and collected for U.S. metalliferous tailings by Smith.[6,7] The discharge, c_0, G_s and median particle diameter, D_{50}, of the input tailings for all 16 deltas are listed in Table 1. Their particle-size distributions are shown in Fig. 2.

The profiles of the Oaky Creek and U.S. metalliferous tailings deltas are shown in Fig. 3. The results of the statistical analyses of all the profiles, including the linear correlation coefficients, R, are presented in Table 2. The level of significance was less than 0.04% in all cases, indicating very strong correlations in support of the theoretical model.[1,2]

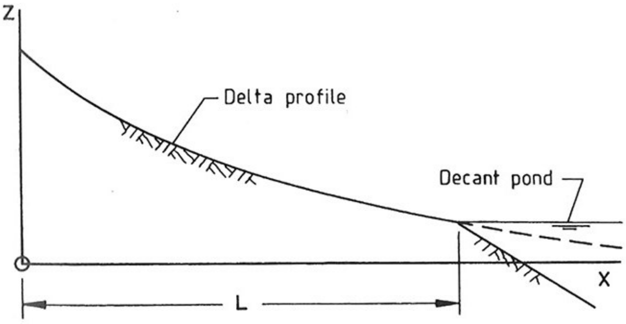

Fig. 1 Diagrammatic representation of tailings delta profile

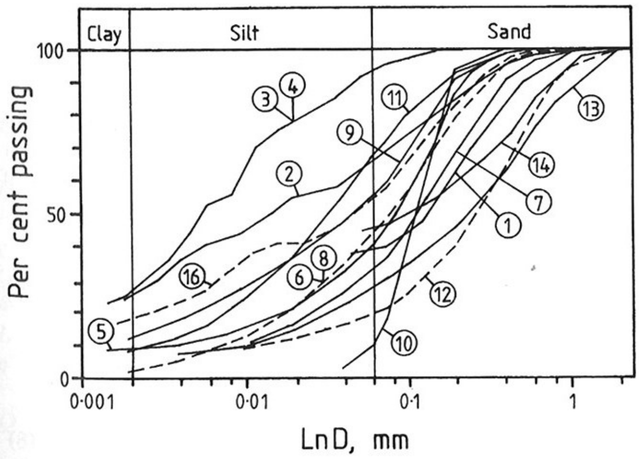

Fig. 2 Particle-size distribution curves of tailings listed in Table 1

The theoretical model incorporates, however, an assumption of quasi-steady flow conditions based on the relatively low flow Froude numbers, Fr, associated with tailings deltas.[1] The bed forms and (water) surface waveforms that occur on mine tailings deltas are, in general, characteristic of Fr less than 1, although transitory antidunes, which are characteristic of Fr greater than 1, occasionally appear.[8] For Fr of about 0.8—a reasonably representative value for many tailings deltas—a rigorous analysis[4] of the relative celerities of mobile bed flows with high c shows that c values of about 0.01 or less are necessary to permit the assumption of quasi-steady flow conditions. For $c \geq 0.1$ Fr of about 0.1 or less, which are unrealistically low values for many tailings deltas, are necessary.

The 15 c_0 values in Table 1 are all significantly greater than 0.01 and only six of them are less than 0.1. The assumption of quasi-steady flow on the basis of relatively low Fr is thus clearly untenable for mine tailings deltas, and the theoretical model of subaerial deposition[1] must be modified to account for the high c characteristic of mine tailings.

Governing equations

In general, flows on mine tailings deltas are essentially two-dimensional—that is, flow width/depth ≥10. Unsteady two-dimensional flow with finite c can be described by the following three equations.[4]

The continuity equations for water and sediment are, respectively

$$\frac{\partial uh\left(1-c\right)}{\partial x} + \frac{\partial h\left(1-c\right)}{\partial t} + \lambda\frac{\partial z}{\partial t} = 0 \tag{3}$$

and

$$\frac{\partial uhc}{\partial x} + \frac{\partial hc}{\partial t} + \left(1-\lambda\right)\frac{\partial z}{\partial t} = 0 \tag{4}$$

where u and h are velocity and depth of flow, respectively, t is elapsed time, λ is bed porosity (assumed constant[1,4]) and z is

Table 2 Parameters characterizing delta profiles

Tailings	***L*, m**	**S_0**	**ω**	***R***
Full-size deltas				
1 Aberdare	70	1.26×10^{-2}	1.614	−0.949
2 Meandu	180	1.59×10^{-3}	0.512	−0.999
3 Oaky Creek 1	66	2.12×10^{-2}	1.102	−0.982
4 Oaky Creek 2	480	1.37×10^{-2}	1.771	−0.996
5 Platinum	115	5.49×10^{-2}	1.982	−0.997
6 Molybdenum 1	344	3.27×10^{-2}	3.617	−0.982
7 Molybdenum 2	492	2.98×10^{-2}	2.225	−0.984
8 Molybdenum 3	295	1.35×10^{-2}	1.097	−0.997
9 Copper	636	8.08×10^{-3}	1.627	−0.997
10 Gold sulphides	23	7.61×10^{-2}	1.452	−0.995
11 Silver	49	3.36×10^{-2}	1.180	−0.976
12 Uranium 1	41	8.20×10^{-2}	0.741	−0.989
13 Uranium 2	82	4.42×10^{-2}	0.470	−0.992
Laboratory model deltas				
14 Aberdare	1.5	1.74×10^{-2}	1.686	−0.991
15 Meandu 1	1.6	8.18×10^{-3}	0.630	−0.996
16 Meandu 2	1.5	6.07×10^{-3}	0.092	−0.995

the bed elevation relative to an arbitrary horizontal datum (Fig. 1). The products $uh(1 - c)$ and uhc give the volumetric water, q, and sediment, Q, discharges per unit width of bed, respectively.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} + \frac{(\rho_s - \rho)gh}{2\rho_m}\frac{\partial c}{\partial x}$$

$$- \frac{((1-\lambda)\rho_s + \lambda\rho)u}{\rho_m h}\frac{\partial z}{\partial t} + g\frac{\partial z}{\partial x} + gS_f = 0 \qquad (5)$$

where g is acceleration due to gravity, ρ_s and ρ are densities of sediment and water, respectively, $\rho_m = c\rho_s + (1 - c)\rho$ and S_f is friction slope.

Simplification of governing equations

To obtain an analytical model of mine tailings delta formation equations 3–5 must be linearized using constraints related to the hydraulic conditions that arise on such deltas. These comprise the rate of deposition and the rates at which the flow varies in space and time.

As shown by the approximate values of the average rate of deposition over the entire delta, z', listed in Table 1, deposition on tailings deltas is slow. In the absence of better data the z' values were calculated on the assumption that all of the solids were deposited on the delta proper and none were carried in suspension to the decant pond. The plan areas of the full-size deltas were assumed to equal $0.5 L^2$. The λ values were estimated from the input D_{50} (Table 1) using the relationship for fine sediments in natural rivers, lakes and dams of Komura[9]

$$\lambda = 0.245 + 0.140 \times D_{50}^{-0.21} \qquad (6)$$

This gives λ values ranging from 0.43 (for uranium tailings) to 0.65 (for Oaky Creek coal tailings).

The z' for the full-size deltas are, thus, order-of-magnitude estimates, which may underestimate the true deposition rates in areas of diffuse flow. (Concentrated flows are more likely to erode than to deposit tailings.) The highest z' estimate (3×10^{-4} m s^{-1}), however, was obtained for the second Meandu laboratory model delta, for which the flow was uniform over the whole width and the plan area is accurately known.

Quasi-steady, slowly varying flow is characteristic of many natural streams,[10,11] where, even on steep slopes and during floods or other flows involving high accelerations, the accelerations and the depth slope, $\partial h/\partial x$, are of the order of 5–0.5% or less of S and S_f.[12,13,14] However, although some natural streams resemble flows on tailings deltas, the assumption that flows on tailings deltas vary slowly and are quasi-steady is justified primarily by the strong congruence of the resulting analytical solution and the available field data (Table 2 and Fig. 3).[1,2]

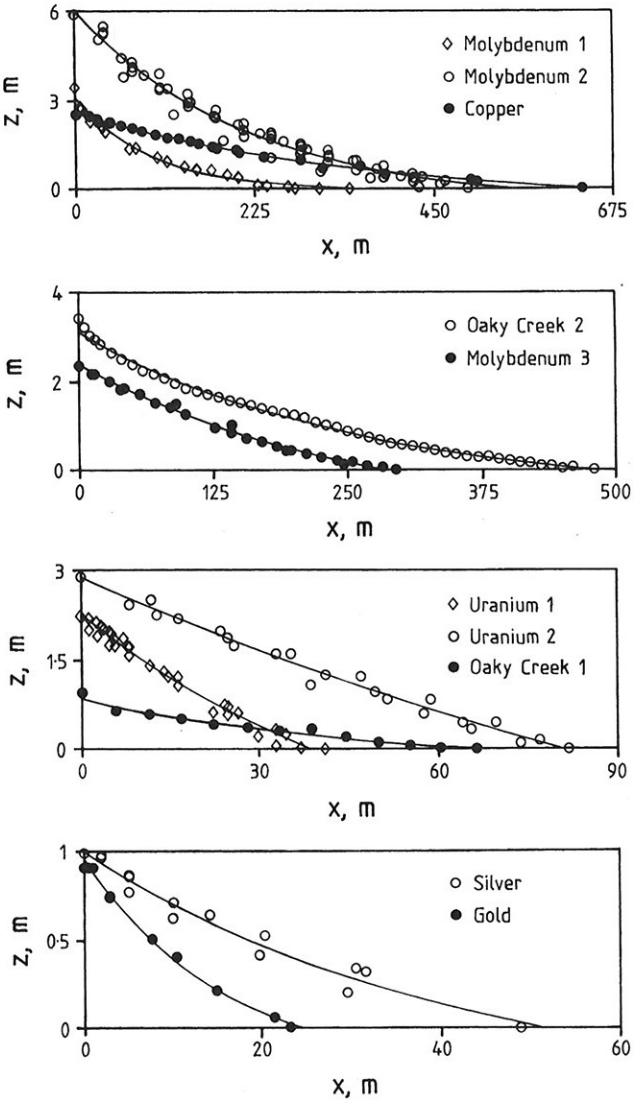

Fig. 3 Profiles of Oaky Creek coal and U.S. metalliferous tailings deltas

For quasi-steady flow with deposition the partial derivatives with respect to time of u, h and c approximate to zero and the partial derivative of z is a non-zero constant. For slowly varying flow the partial derivatives with respect to x of u, h and c approximate to zero. For slow deposition

$$\left|\lambda\frac{\partial z}{\partial t}\right| << \left|\frac{\partial uh\,(1-c)}{\partial x} + \frac{\partial h\,(1-c)}{\partial t}\right| \qquad (7)$$

and

$$\left|\frac{((1-\lambda)\rho_s + \lambda\rho)u}{\rho_m gh}\;\frac{\partial z}{\partial t}\right| << |S_f - S| \qquad (8)$$

where S equals $-\partial z/\partial x$.

Hence, for quasi-steady flow and slow deposition (inequality 7) equation 3 reduces to

$$q = \text{constant} \tag{9}$$

For quasi-steady flow equation 4 becomes

$$\frac{\partial z}{\partial t} + \frac{1}{1-\lambda}\frac{\partial Q}{\partial X} = 0 \tag{10}$$

and for quasi-steady, slowly varying flow and slow deposition (inequality 8) equation 5 reduces to

$$S = S_f \tag{11}$$

Equations 9–11 are identical to the equivalent equations for flows with low c[1] and lead to the same analytical solution with exponential delta profiles (equation 1) and exponential sorting, provided that the sorting-parameter distribution of the tailings is approximately log-uniform.[1]

This analysis has placed no explicit limits on either c or Fr. Inequalities 7 and 8 must be satisfied, but the data presented in Tables 1 and 2 imply that, on tailings deltas, this is possible for c of up to at least 0.25.

Conclusion

The exponential delta profile equation arising from a recent theoretical model of the subaerial deposition of mine tailings[1,2] matches the profiles of a wide range of coal and metalliferous tailings deltas well. The original model implies, however, low volumetric solids concentrations, c, that are unrepresentative of many tailings slurries.

A revision of the model based on an analysis of the governing equations of mobile bed flows with high c values has shown that quasi-steady, slowly varying flow and slow deposition (as well as tailings with a log-uniform sorting-parameter distribution[1]) are necessary conditions for exponential mine tailings delta profiles to be formed. Deposition on tailings deltas is slow; and the flow in many natural streams and, by analogy, on tailings deltas is quasi-steady and slowly varying. The assumption that such flow dominates on tailings deltas is, however, justified primarily by the strong congruence of the revised model and the available field and laboratory data.

The predictions of the theoretical model are essentially unchanged by this revision. However, there are now no explicit limits on either c or the flow Froude number.

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Symbols

c	Volumetric solids concentration
D_{50}	Median particle diameter, mm
Fr	Froude number of flow
g	Gravitational acceleration, m s^{-2}
G_s	Specific gravity of tailings
h	Depth of flow, m
L	Overall length of delta profile, m
q	Volumetric water discharge per unit width of bed, m^2 s^{-1}
Q	Volumetric sediment discharge per unit width of bed, m^2 s^{-1}
R	Linear correlation coefficient
S	Bed slope
S_f	Friction slope
t	Time, s
u	Velocity of flow, m s^{-1}
x	Distance down profile measured from its highest point, m
z	Height above datum of sediment deposited on bed, m
z'	Average rate of deposition on delta, m s^{-1}

Greek

λ	Porosity of bed
ρ	Density of water, kg m^{-3}
ρ_m	$c\rho_s + (1 - c)\rho$, kg m^{-3}
ρ_s	Density of sediment, kg m^{-3}
ω	Dimensionless positive constant defining profile for particular delta

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