Enhanced adaptive array performance via DOA detection

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***Abstract*—In various communications-based adaptive array applications, the directions of arrival (DOAs) of the desired user signal are sparsely separated. As such, the desired beam-pattern has a sparse structure. We propose an NLMS based adaptive algorithm which exploits this sparse DOA structure and provides significantly improved convergence speeds.**

***Index Terms*—Sparse, least mean square, least squares, estimation.**

I. Introduction

Adaptive arrays find applications in many areas, and particularly within the communications field. Examples include base-station arrays in cellular communications, and microphone arrays in acoustic communication systems. The primary objective of the array is to spatially suppress noise and interference, which in turn allows for higher data rates, lower signal bandwidths, and/or provides improved quality of communications (*e.g.* lower bit error rates (BER)). Increasingly, arrays within these applications are being made adaptive. This enables the array to track time variations within the 'spatial channel' and allows for the array elements/weights to be initialised (essentially) arbitrarily.

The performance of an adaptive array is commonly measured by its steady state error (under time invariant conditions), convergence speed, tracking speed, computational cost, as well as its stability. The least mean square LMS, or its normalised equivelent NLMS algorithm, is the most commonly used algorithm for adaptation [1], [2]. This is due to its relatively low computational cost and very good stabilty properties. However, its main drawback is its relatively slow convergence and tracking speeds when the adaptive filter length is 'large' [2], [3]. In communications-based adaptive array applications this may occur, for example, with densely populated cellular communications cells, where long arrays are required to produce highly directional beam-patterns.

An approach to combat this 'parameter dimension' effect (within any adaptive application), is to incorporate dimension reduction techniques within the adaptive LMS/NLMS algorithm. This may be realised in a number of ways, depending on the characterstics of the system/channel being estimated or equalised. In the case of communications-based adaptive arrays, the spatial channel is often characterised by having a 'sparse' structure. That is, the desired user signal typically has only a small number of sparsely separated directions of arrival (desired-DOAs). Accordingly, a possible approach to dimension reduction involves transforming the adaptive system into the spatial (beampattern) domain and subsequently adaptively estimate only the dominant or 'active' desired-DOAs. In this paper we follow this approach. The key idea is the use of a criterion for accurately detecting the active desired-DOAs (DOAs of the desired user signal). Following the work of Homer *et. al.* [4], [5], we propose an activity criterion which is based on the minimisation of a structurally consistent least squares (SC-LS) cost function. Ultimately, we propose an NLMS based adaptive array algorithm which, for spatially sparse communications channels, demonstrates significantly higher convergence and tracking speeds than the standard NLMS algorithm. Furthermore, this is achieved with only a moderate increase in computational cost and without compromising the (time-invariant channel) steady state error.

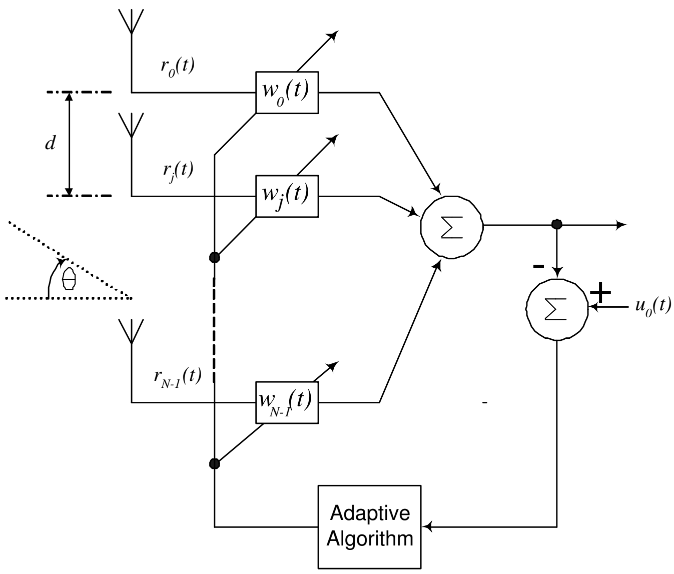

Fig. 1. $N-$ element adaptive array.

In this paper we assume the communication temporal channel is a zero delay non-time disersive (narrowband) channel. Accordingly, we consider only adaptive arrays with a (complex valued) scalar weight applied to each array element, as illustrated in Figure 1. Current research is investigating the extension of the proposed adaptive algorithm to nonzero delay and/or time dispersive (broadband) channels; that is to adaptive arrays with a temporal (adaptive) filter applied to each array element.

This paper is organised as follows. The next section introduces notation and describes the standard NLMS and spatial beampattern domain NLMS algotithms. Section III uses the

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notion of a structurally consistent least squares cost function to derive an activity measure in the beampattern domain. Section IV describes the implementation of the algorithm. Section V presents some simulation results and we conclude in Section VI.

II. Preliminaries

The configuration we consider throughout this paper is shown in Figure 1. We consider an N element uniformly spaced linear array and assume only a 2-dimensional spatial system. That is, all the received user signals (desired and interfering) lie in the same 2-dimensional plane, and that the linear adaptive array lies within this plane. We assume: the uniform antenna element spacing is $d \leq \lambda/2$ where λ is the wavelength of the narrowband transmitted user signals; and each antenna element is isotropic.

We consider an 'equivalent sampled complex baseband' system. That is, we assume all signals are sampled and complex basebanded. At sampling instant t: $u_0(t)$ is the transmitted desired user signal; $u_i(t),\ i = 1, 2, ..., n$ is the transmitted i^{th} interfering user signal; $r_j(t),\ j = 0, 1, ..., N-1$ is the signal received at the j^{th} antenna array element; $s_j(t)$ is the noise signal at the j^{th} antenna array element.

We assume: each user transmitted signal $u_i(t),\ i = 0, 1, 2, ..., n$ is described by a zero mean, bounded, wide sense stationary process of variance σ_u^2; the different user transmitted signals are uncorrelated with each other; the noise signal of each antenna element is a zero mean, bounded, wide sense stationary white process of variance σ_s^2; the noise signals are uncorrelated with each other and uncorrelated with the user signals.

We assume that the i^{th} $(i = 0, 1, 2, ..., n)$ user transmitted signal arrives from a 'small' number m_i of sparsely separated directions; and each of these directions is characterised by an angle of arrival $\theta_{i,k}, k = 1, 2, ..., m_i$ and a complex valued gain $g_{i,k}$. We choose the direction perpendicular to the linear array line as the zero angle ($\theta = 0$ radians) direction.

Note: for the sake of simplicity (notation and algorithm development), we have assumed the spatial characteristics of the communication channel are time-invariant. It needs to be emphasised that the proposed detection guided adaptive array algorithm is suitable for time-varying spatial channels, as well as for time-invariant spatial channels.

Accordingly, the complex baseband signal received at the j^{th} array element is:

$$r_j(t) = \sum_{i=0}^{n}\sum_{k=1}^{m_i} u_i(t) g_{i,k} \exp(-j2\pi dj \sin[\theta_{i,k}]/\lambda + \phi_{i,k}) + s_j(t) \quad (1)$$

where $\phi_{i,k}$ is the phase at the $j = 0$ element of the i^{th} user signal arriving from direction $\theta_{i,k}$.

The signal output by the adaptive array is:

$$\begin{aligned} v(t) &= \sum_{j=1}^{N} w_j(t) r_j(t) = W^T(t)R(t), \\ W(t) &= [w_1(t), w_2(t), ..., w_N(t)]^T, \\ R(t) &= [r_1(t), r_2(t), ..., r_N(t)] \end{aligned}$$

and where $w_j(t)$ is the scalar weight applied at the j^{th} array element at sample time t. The standard NLMS adaptation equation for the array weight vector is:

$$W(t+1) = W(t) + \frac{\mu}{R^T(t)R(t) + \epsilon} R^*(t)e(t).$$

where: * denotes complex conjugate, $e(t) = u_0(t) - v(t)$ and $\mu,\ \epsilon$ are small positive constants.

As discussed earlier, in order to exploit the sparse spatial characteristics of the desired user's signal DOAs, we carry out the NLMS adaptation in the spatial beampattern domain. The corresponding NLMS algorithm is:

$$W_f(t+1) = W_f(t) + \frac{\mu}{R_f^T(t)R_f(t) + \epsilon} R_f(t)e(t),$$

where $R_f(t) = [r_{f,1}(t), r_{f,2}(t), ..., r_{f,N}(t)]^T = FT[R^*(t)]$, $W_f(t) = [w_{f,1}(t), w_{f,2}(t), ..., w_{f,N}(t)]^T = FT[W(t)]$, $FT[X(t)]$ denotes the Fourier transform of the vector X at time t, and $e(t) = u_0(t) - v(t) = u_0(t) - R^T(t)W(t)$.

We define $W^{(0)} = [w_0^{(0)}, w_1^{(0)}, ..., w_{N-1}^{(0)}]$ as the desired weight-domain vector, that is the weight vector which minimises the cost function of (2) below. Similarly, we define $W_f^{(0)} = FT[W^{(0)}] = [w_{f,0}^{(0)}, w_{f,1}^{(0)}, ..., w_{f,N-1}^{(0)}]$ as the desired beampattern-domain vector. In many cases, $W_f^{(0)}$ will show a sparse structure. The same is not generally true for $W^{(0)}$. The desired vector $W^{(0)}$ or $W_f^{(0)}$ depends both on the DOAs of the desired user signal and the interfering user signals.

III. Activity Detection

As discussed earlier, our proposed NLMS based adaptive array algorithm incorporates an activity criterion for detecting the active (or existing) desired-DOAs. We begin by developing an activity criterion in the array weight domain. We then develop an equivalent activity criterion in the array beampattern domain via application of appropriate Fourier transformations.

A. Array Weight-Domain

Following on from the work of Homer *et. al* [4], [5], the activity criterion we employ is derived from the following structurally consistent least squares based cost function [4]:

$$V_{SCLS}(T) = V_{LS}(T) + m\sigma_u^2 \log T \quad (2)$$

where: $V_{LS}(T) = \sum_{t=1}^{T} |u_0(t) - W^T R(t)|^2$; σ_u^2 = variance of $u_0(t)$; W = estimated array weight-domain vector, which contains only m active/nonzero elements.

In general, minimisation of $V_{SCLS}(T)$ requires examination and comparison of a large number $\binom{N}{m}$ of weight index sets with $\binom{N}{m} = \sum_{m=1}^{N} \frac{N!}{(N-m)!m!}$. To circumvent this large comparison problem, we begin by introducing an assumption, which is not generally valid in the weight vector domain, but which greatly simplifies the cost function analysis. We then include a number of modifications to offset the effects of the simplifying assumption.

Assume the received signal vector $R(t)$ has uncorrelated elements. Then, for sufficiently large T, we may approximate $V_{SCLS}(T)$ of (2) by [4]:

$$\tilde{V}_{SCLS}(T) = \sum_{t=1}^{T} v^2(t) - \sum_{k=1}^{m} [X_{a_k}(T) - \sigma_u^2 \log T] \quad (3)$$

$$X_{a_k}(T) = \frac{|\sum_{t=1}^{T} u_0(t) r_{a_k}^*(t)|^2}{\sum_{t=1}^{T} |r_{a_k}(t)|^2} \quad (4)$$

where * denotes conjugate, $|.|$ denotes modulus, and $a_k(k = 1, 2, ...m)$ are the unknown indices of the active elements of the desired weight vector $W^{(0)}$.

It is apparent that $\tilde{V}_{SCLS}(T)$ is minimised by (and hence the indices of the desired active elements correspond to) those indices j which satisfy:

$$X_j(T) > L(T) \quad (5)$$

where

$$L(T) = \sigma_u^2 \log T \approx \frac{\log T}{T} \sum_{t=1}^{T} |u_0(t)|^2.$$

Equation 5 provides us with a suitable activity criterion for the (unrealistic) case in which the elements of $R(t)$ are uncorrelated. This activity criterion, however, is not suitable for the (more realistic) case in which the received signal vector elements are correlated. This is because the correlation causes neighbouring indices to contribute significantly to the numerator term of $X_j(T)$.

To reduce this coupling effect from neighbouring indices, we propose the following three modifications.

Modification 1: Replace $X_j(T)$ by:

$$XX_j(T) = \frac{|\sum_{t=1}^{T} \{e(t) + w_j(t) r_j(t)\} r_j^*(t)|^2}{\sum_{t=1}^{T} |r_j(t)|^2}. \quad (6)$$

Modification 2: Replace $L(T)$ by:

$$LL(T) = \frac{\log T}{T} \sum_{t=1}^{T} |e(t)|^2. \quad (7)$$

Modification 3: Include an exponentially forgetting factor: $(1-\gamma)$, $0 < \gamma \ll 1$ within the summation terms of $XX_j(T)$ and $LL(T)$.

The reasons for the three modification are described below. For the sake of clarity, we assume for the discussion below that all the interfering user signals are absent, so that:

$$r_j(t) = \sum_{k=1}^{m_0} g_{0,k} \exp(-j2\pi d_j \sin[\theta_{0,k}]/\lambda + \phi_{0,k}) + s_j(t). \quad (8)$$

In this case, the desired weight vector $W^{(0)}$ is given by:

$$W^{(0)} = [w_0^{(0)}, w_{,2}^{(0)}, ..., w_{N-1}^{(0)}]^T$$

$$w_j^{(0)} = \beta \sum_{k=1}^{m_0} g_{0,k} \exp(j2\pi d_j \sin[\theta_{0,k}]/\lambda - \phi_{0,k})$$

where β is a positive scaling constant, such that $u_0(t) = R^T(t) W^{(0)} + \tilde{s}(t)$, with $\tilde{s}(t)$ a residual noise signal which is uncorrelated with the elements of $R(t)$. It needs to be emphasised that the above assumption is not a requirement for the proposed DOA detection guided NLMS adaptive algorithm.

Modification 1 is based on the following. The cause of neighbour coupling in $X_k(T)$ arises from the following numerator term:

$$\begin{aligned} numX_j(T) &\triangleq \frac{1}{T} \sum_{t=1}^{T} u_0(t) r_j(t) \\ &= \frac{1}{T} [\sum_{t=1}^{T} \sum_{p \neq j} (w_p^{(0)} r_p(t) r_j(t) \\ &+ w_j^{(0)} r_j(t) r_j(t) + \tilde{s}(t) r_j(t)]. \end{aligned}$$

The first component in the summation is the cause of neighbour coupling. This becomes more significant with an increase in the correlation amongst the received signal elements.

The equivalent numerator term of $XX_j(T)$ is:

$$\begin{aligned} numXX_j(T) &\triangleq \frac{1}{T} [\sum_{t=1}^{T} \{e(t) + w_j(t) r_j(t)\} r_j(t)] \\ &= \frac{1}{T} [\sum_{t=1}^{T} \sum_{p \neq j} (w_p^{(0)} - w_p(t)) r_p(t) r_j(t) \\ &+ w_j^{(0)} r_j(t) r_j(t) + \tilde{s}(t) r_j(t)]. \end{aligned}$$

Here the coupling effect of the first term should be significantly weakened, assuming $w_p(t)$ converges towards $w_p^{(0)}$ (for $p = 0, ..., N-1$.)

Modification 2 stems from the realisation that for *inactive* elements (and assuming $w_{inactive} \approx 0$) the numerator term $numXX_j(T)$ is approximately:

$$numXX_j(T)) \approx \tfrac{1}{T} [\sum_{t=1}^{T} e(t) r_j(t)].$$

Consequently, combining this with the LS theory on which the original activity criterion (5) is based, then suggests this second proposed modification. This reasoning for Modification 2, however, is only relevant if the estimation error vector $\Delta W(t) \triangleq W^{(0)} - W(t)$ is non-time-varying.

Clearly, this is not the case. Modification 3 reduces the effect of the time-varying nature of ΔW.

Note: The inclusion of Modification 3 also improves the applicability of the detection guided NLMS adaptive array to spatially time-varying systems. This capability of the proposed adaptive array is not explored in this paper.

B. Array Beampattern-Domain

Transformation of the above weight-domain activity criterion to the beampattern domain involves replacing the numerator and denominator of $XX_j(T)$ with Fourier transform equivalents:

(i) Let $NumXX_f(T) = FT[NumXX(T)]$ where $NumXX(T) = [numXX_0(T), ..., numXX_{N-1}(T)]$ $NumXX_f(T) = [numXX_{f,0}(T), ..., numXX_{f,N-1}(T)]$. Replace the numerator term $numXX_j(T)$ with

$numXX_{f,j}(T)$.

(ii) Replace the denominator term $r_j(T)$ with $r_{f,j}(T)$.

IV. Detection Guided NLMS Adaptive Algorithm

The proposed DOA detection guided NLMS adaptive algorithm is as follows.

Initialisation:

(a) For each array element index j, initialise $b_j(0) = w_{f,j}(0) = 0$ and $d_j(0) = \epsilon_1,\ 0 < \epsilon_1 \ll \sigma_r^2$, where σ_r^2 is the variance of the received signals.

(b) Initialise: $q(0) = C(0) = 0$.

At each sample interval T:

(a) Standard signal operations:

$$\begin{aligned} R_f(T) &= FT[R(T)] \\ W(T) &= IFT[W_f(T)] \\ v(t) &= W^T(t)R(t) \\ e(t) &= u_0(t) - v(t). \end{aligned}$$

(b) Activity threshold calculation:

$$\begin{aligned} q(T) &= (1-\gamma)q(T-1) + |e(T)|^2 \\ C(T) &= (1-\gamma)C(T-1) + 1 \\ LL(T) &= q(T)\log\{C(T)\}/C(T) \end{aligned}$$

(c) Activity measure calculation, for element index j:

$$\begin{aligned} b_j(T) &= (1-\gamma)b_j(T-1) \\ &\quad + [e(T) + w_j(T)r_j(T)]]r_j(T) \\ d_j(T) &= (1-\gamma)d_j(T-1) + |r_{f,j}(T)|^2 \\ B(T) &= [b_0(T), b_1(T), ..., b_{N-1}(T)]^T, \\ B_f(T) &= FT[B(T)] \\ &= [b_{f,0}(T), b_{f,1}(T), ..., b_{f,N-1}(T)] \\ XX_{f,j}(T) &= \frac{|b_{f,j}(T)|^2}{d_j(T)}. \end{aligned}$$

(d) Application of activity criterion, for element j:
If $XX_{f,j}(T) > LL(T)$ then label j as an active element index a_k; otherwise label j as an inactive element index.

(e) NLMS adaptation, for element j:
If $j = a_k$ (that is, corresponds to a detected active index) then:

$$\begin{aligned} w_{f,j}(T) &= w_{f,j}(T-1) \\ &+ \frac{\mu}{\sum_{a_k} |r_{f,a_k}(T)|^2 + \epsilon} r_{f,j}(T)e(T) \end{aligned}$$

where $\sum_{a_k}$ =summation over all detected active indices.
If $j \neq a_k$ then

$$w_{f,j}(T) = 0.$$

TABLE I
Desired signal and interfering signal parameters

Desired Signal					
Angles, $\{\theta_{0,k}\}_{k=1}^4$	-75^o	-35^o	$+20^o$	$+55^o$	
Gains, $\{g_{0,k}\}_{k=1}^4$	$0.5e^{j\pi/3}$	$1.0e^{j\pi}$	$0.3e^{j\pi/9}$	0.5	
First Interfering signal DOAs:					
Angles, $\{\theta_{1,k}\}_{k=1}^3$	-55^o	-5^o	$+70^o$		
Gains, $\{g_{1,k}\}_{k=1}^3$	$1.0e^{j\pi/3}$	$1.0e^{j\pi/7}$	0.5		
Second Interfering signal DOAs:					
Angles, $\{\theta_{2,k}\}_{k=1}^5$	-50^o	-10^o	$+5^o$	$+35^o$	$+75^o$
Gains, $\{g_{2,k}\}_{k=1}^5$	$1.2e^{j\pi/3}$	$0.8e^{j\pi}$	1.6	0.9	$0.8e^{j\pi}$

A. *Choice of γ Value*

The parameter decoupling capabilities provided by the modifications depend largely on the value of γ: a larger value leads to a faster forgetting rate, and subsequently faster decoupling and a greater convergence rate. On the other hand, a larger γ value corresponds to each of the summations (within the active tap criterion) providing a poorer 'averaging' effect. As such, when a larger γ is used, there is a greater tendency for failure to detect the smaller active taps. Alternatively, γ could be periodically tuned, such that it is increased/decreased until an unacceptable/acceptable level of estimation error occurs. This would be indicated by, for example, a general increase in the power of the error signal $u_0(t) - v(t)$.

V. Simulations

Simulations were conducted to compare the performance of the standard NLMS adaptive array with that of the proposed DOA detection guided NLMS adaptive array.

The simulation conditions were as follows.

Signal wavelength $\lambda = 20$mm; Array element spacing $d = \lambda/2$; Number of array elements $N = 64$; Adaptation constants: $\mu = 0.02,\ \epsilon = 0.1,\ \epsilon_1 = 0.01$.

Desired signal u_0: random binary real valued signal (generated using Matlab: $u_0 = sign(randn(1, 25600))$).

Firts interfering signal u_1: random binary real valued signal with amplitude twice that of desired signal (generated using Matlab: $u_1 = 2 * sign(randn(1, 25600))$).

Second interfering signal u_2: random binary real valued signal with amplitude equal to that of desired signal (generated using Matlab: $u_2 = sign(randn(1, 25600))$).

Antenna element noise signal s_j: complex valued random Gaussian signal with variance $\sigma_s^2 = 2$ (generated using Matlab: $s_j = randn(1, 25600) + j * randn(1, 25600)$).

Table I shows the simulation multipath parameters of the desired and interfering signals.

Figure 2 shows the results of the simulation. [Shown are the average of ten similar simluations.) Figure 2(a) is a plot of the squared error $|e(t)|^2$ over time (sample number), for both the DOA detection guided NLMS adaptive array and the standard NLMS adaptive array. Figure 2(b) is a plot over time of the number of desired-DOA indices detected as being active ... via the proposed activity criterion.

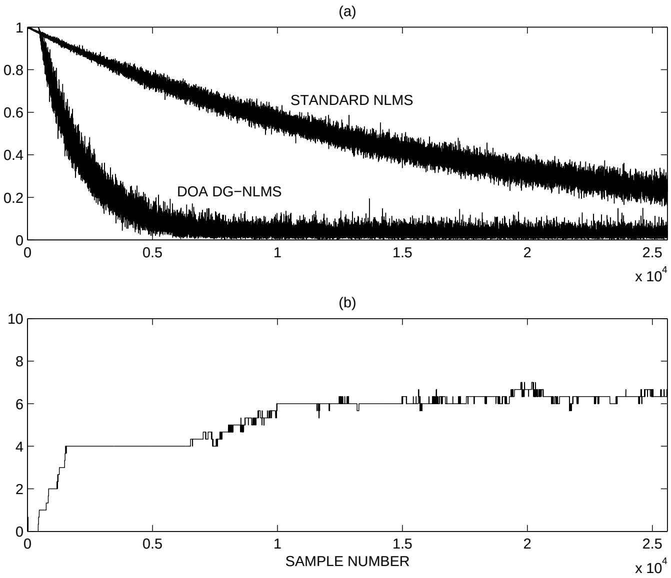

Fig. 2. Simulation results: (a) Plot of squared error $|e(t)|^2$ over time (sample number); (b) Plot of number of desired-DOA indices detected as being active over time.

VI. Conclusions

We have proposed a detection guided NLMS adaptive array algorithm that works in the spatial beampattern domain. Simulation results show that this algorithm converges considerably more quickly than the standard NLMS adaptive array algorithm. Many other simulations, not presented here, also indicate the same significantly superior performance.

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