Optical alignment and spinning of laser-trapped microscopic particles

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Light-induced rotation of absorbing microscopic particles by transfer of angular momentum from light to the material raises the possibility of optically driven micromachines. The phenomenon has been observed using elliptically polarized laser beams [1] or beams with helical phase structure [2, 3]. But it is difficult to develop high power in such experiments because of overheating and unwanted axial forces, limiting the achievable rotation rates to a few hertz. This problem can in principle be overcome by using transparent particles, transferring angular momentum by a mechanism first observed by Beth in 1936 [4], when he reported a tiny torque developed in a quartz waveplate due to the change in polarization of transmitted light. Here we show that an optical torque can be induced on microscopic birefringent particles of calcite held by optical tweezers [5]. Depending on the polarization of the incident beam, the particles either become aligned with the plane of polarization (and thus can be rotated through specified angles) or spin with constant rotation frequency. Because these microscopic particles are transparent, they can be held in three-dimensional optical traps at very high power without heating. We have observed rotation rates in excess of 350 Hz.

A typical optical tweezers arrangement was used to trap microscopic calcite particles in three dimensions using between 30 and 300 mW of laser light at a wavelength of 1064 nm. The optical trap used a 100× oil-immersion, high numerical aperture (NA = 1.3) microscope objective. The trapping beam was initially linearly polarized, and the plane of polarization could be rotated using a half-wave plate. Alternatively, a quarter-wave plate allowed the ellipticity of polarization to be varied. The particles were fragments obtained by crushing a small calcite crystal, giving irregular particles 1–15 μm across. They were dispersed in distilled water in a trapping cell consisting of a well in a microscope slide with a coverslip.

Because of their birefringent nature, calcite particles can act as wave-plates; a calcite particle 3 μm thick is a $\lambda/2$ plate for 1064 nm light. On passage through a fragment of calcite, the ordinary and extraordinary components of the incident light will undergo different phase shifts. If this results in a change in the angular momentum carried by the light, there will be a corresponding torque on the material. Our results can be understood using a simple plane wave picture; the interaction between an incident plane wave and a waveplate is outlined below. We note that the calcite waveplate is trapped at the focal point of the beam, where the wavefronts are nearly plane.

An incident laser beam can, in general, have both cir-

culary polarized and plane polarized components; that is, it will be elliptically polarized. Elliptically polarized light can be described by

$$\mathbf{E} = E_0 \exp(-\mathrm{i}\omega t)\cos\phi\hat{\mathbf{x}} + \mathrm{i}E_0 \exp(-\mathrm{i}\omega t)\sin\phi\hat{\mathbf{y}}$$

where ϕ describes the degree of ellipticity of the light ($\phi = 0$ or $\pi/2$) indicates plane-polarized light, $\phi = \pi/4$ circularly polarized light). The angular momentum of a plane electromagnetic wave (the incident light) of angular frequency ω can be found from the electric field $\mathbf{E}$ and its complex conjugate $\mathbf{E}^\star$ by integrating over all spatial elements $\mathrm{d}^3 r$ giving

$$\mathbf{J} = \frac{\epsilon}{2\mathrm{i}\omega} \int \mathrm{d}^3 r \mathbf{E}^\star \times \mathbf{E},$$

where ϵ is the permittivity.

To calculate the change in angular momentum of the light after passage through a birefringent material, the incident elliptically polarized light is first expressed in terms of components parallel and perpendicular to the optic axis of the material by

$$\begin{aligned}
\mathbf{E} = {} & E_0 \exp(-\mathrm{i}\omega t)(\cos\phi\cos\theta - \mathrm{i}\sin\phi\sin\theta)\hat{\mathbf{i}} \\
& + E_0 \exp(-\mathrm{i}\omega t)(\cos\phi\sin\theta + \mathrm{i}\sin\phi\cos\theta)\hat{\mathbf{j}} \quad (1)
\end{aligned}$$

where θ is the angle between the fast axis of the quarter-wave plate producing the elliptically polarized light and the optic axis of the birefringent material. The phase shift due to passing through a thickness d with refractive

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index n is kdn, where k is the free-space wavenumber, so the emergent light field will be

$$\begin{aligned}
\mathbf{E} = {} & E_0 \exp(-\mathrm{i}\omega t) \exp(\mathrm{i}kdn_e) \\
& (\cos\phi\cos\theta - \mathrm{i}\sin\phi\sin\theta)\hat{\mathbf{i}} \\
& + E_0 \exp(-\mathrm{i}\omega t)\exp(\mathrm{i}kdn_o) \\
& (\cos\phi\sin\theta + \mathrm{i}\sin\phi\cos\theta)\hat{\mathbf{j}} \quad (2)
\end{aligned}$$

where n_e and n_o are the extraordinary and ordinary refractive indices of the birefringent material.

The changes in the angular momentum of the light cause a reaction torque per unit area on the thickness d of material of

$$\begin{aligned}
\tau = {} & -\frac{c\epsilon}{2\omega}E_0^2 \sin(kd(n_0 - n_e))\cos 2\phi \sin 2\theta \\
& + \frac{c\epsilon}{2\omega}E_0^2 \{1 - \cos(kd(n_0 - n_e))\}\sin 2\phi \quad (3)
\end{aligned}$$

In general, the first term of equation (3) is the torque due to the plane-polarized component of elliptically polarized light while the second term is due to the change in polarization caused by passage through the medium. For plane-polarized light, $\phi = 0$ or $\pi/2$, so the torque on the particle is proportional to $\sin 2\theta$, so that a particle will experience torque so long as θ is non-zero, and will be at a stable equilibrium when the fast axis of the crystal is aligned with the plane of polarization ($\theta = 0$). We found that calcite fragments trapped in plane-polarized light are aligned in a particular orientation, and a particular particle is always aligned in the same plane each time it is trapped. When the plane of polarization is rotated using a half-wave plate, a particle's alignment exactly follows the rotation of the plane of polarization. In figure 1, a calcite fragment is shown to rotate through 80° as a half-wave plate controlling the polarization of the trapping beam is rotated through 40°, illustrating the alignment of birefringent particles to the plane of polarization. To our knowledge, this is the first report of an optically trapped particle being rotated through a preset
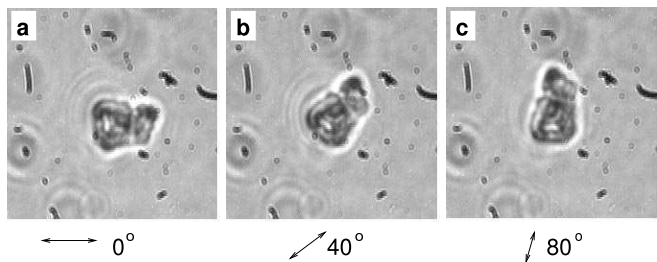

FIG. 1: Three sequential photographs (frames) of a trapped calcite crystal, showing alignment with the plane of polarization of the trapping beam. A $\lambda/2$ waveplate was rotated by 20°between successive photographs, rotating the plane of polarization by 40°, as shown by the arrows, and exerting an alignment torque on the crystal, causing it to rotate to a new position. This can be used to rotate the particle at a controlled speed, or to control its orientation.
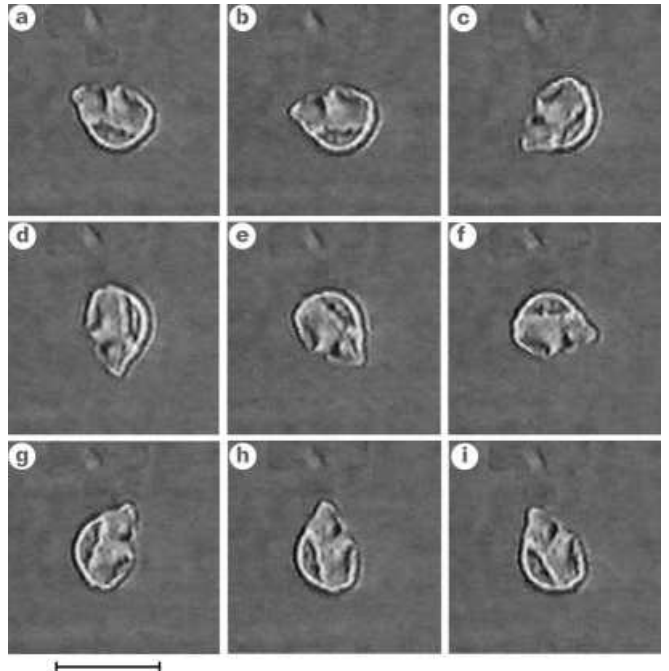

FIG. 2: Nine frames of a trapped calcite crystal, showing free rotation due to an elliptically polarized trapping beam. The speed of rotation is limited by the viscous drag on the particle. As the optical torque acting on the particle depends on its orientation, the rotation speed is not constant. The frames are 40 ms apart. Scale bar is 10 µm.

angle; a modification to the setup whereby the half-wave plate could be spun at a set rate would also allow the particle to rotate at a preset frequency.

The second term of equation (3) will be constant for a given laser power and ellipticity of polarization (characterized by ϕ), and will be maximum for circularly polarized light when the first term vanishes. Hence, when trapped in a circularly polarized beam, a birefringent particle will experience constant torque. In a viscous medium, this torque will be balanced by the drag torque, $t_D = D\Omega$, where D is the drag coefficient and Ω is the angular speed, so in this case a birefringent particle will rotate with constant frequency and angular speed. We measured the rotation frequencies of trapped calcite fragments by detection of back-scattered light [1]; the results show that calcite fragments rotate at constant frequency in circularly polarized light, and that this frequency is proportional to the laser power. A rotating particle is shown in figure 2. The fastest rotation frequency measured was 357 Hz, for a particle 1 µm thick trapped in a 300 mW laser beam.

For the general case of elliptically polarized light, both the alignment torque and the spinning torque will act, and the effect on the birefringent particle will depend on the thickness d and the ellipticity ϕ of the light. The particle will only rotate if the maximum alignment torque is less than the spinning torque. This is shown in figure 3. In figure 4 we plot the variation of rotation rate of
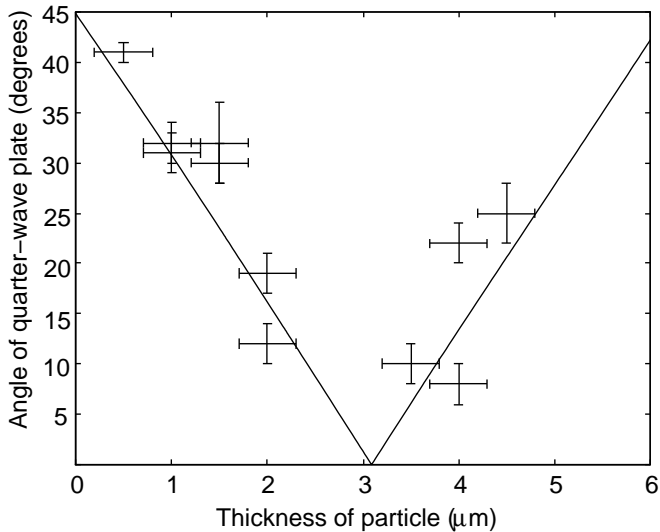
FIG. 3: The degree of circular polarization required to cause spinning of a trapped particle depends on its thickness. Here we compare measurements of the minimum angle required for rotation with the theoretical solution. Only if the particle is the exact thickness of a half-wave plate will it always spin in elliptical light. For all other particle thicknesses, there is some angle θ for which the maximum alignment torque will be greater than the spinning torque. The degree of ellipticity of polarization (measured by ϕ) required for the onset of rotation is found from the case where the alignment torque is maximum and the total torque is zero, which is when $\sin[kd(n_o - n_e)] \cos 2\phi = \{1 - \cos[kd(n_o - n_e)]\} \sin 2\phi$. The solution to this is $\phi_{\text{rotate}} = [\pi - kd(n_o - n_e)]/4$. In general a particle will be aligned to the plane of polarization of the trapping beam unless there is sufficient torque due to the circularly polarized component to set it into rotation, when it will experience a position-dependent torque.

a larger calcite crystal with degree of ellipticity of the trapping beam ϕ, showing the characteristic behaviour of a birefringent particle in elliptically polarized light.

The agreement between our results and the theory outlined above shows that the calcite particles act as microscopic wave-plates. The measurement of the rotation speed of spinning particles is a less accurate but simpler analogue of Beth's experiment [4]. In particular, assuming conventional viscous drag, the observed speeds are consistent with the accepted intrinsic spin angular momentum of $\hbar$ per photon (see figure 4). Our results also show how optical torques can be exerted on certain microscopic objects with high precision and efficiency, with minimal heating. Depending on the details of the arrangement, either a constant torque independent of orientation can be exerted, leading to rotation rates up to hundreds of hertz, or alternatively, the orientation of the object can be smoothly controlled.

The controllability of the motion of calcite particles and the minimal absorption involved suggests calcite as

an ideal material for optically driven rotary micromachines. Such micromachines could include pumps, stir-
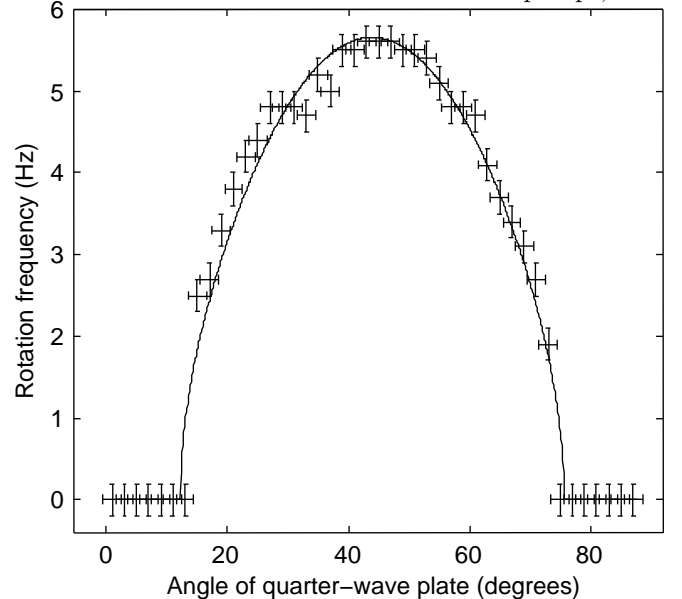

FIG. 4: The variation of rotation frequency with the polarization of the trapping beam. The sudden onset of rotation when the minimum angle is reached can be clearly seen. The theoretical response of the particle allows a particle of unknown size to be measured, or can be used to determine the viscosity of a fluid. In this case, the trapping beam has a power of 50 mW, and the particle is 6.15 µm in radius and 2.3 µm thick. The frequency of rotation for a trapping beam of power P and optical frequency ω is $f = [\text{Re}(P\{[1 - \cos kd(n_o - n_e)]^2 \sin^2 2\phi - \sin^2 kd(n_o - n_e) \cos^2 2\phi\}^{1/2})]/(2\pi\omega D)$. The drag torque coefficient D can be estimated by representing the particle as an ellipsoid [6]. The drag will lie between that of a sphere of radius a in a medium of viscosity μ ($D = 8\pi\mu a^3$) and a disk of the same radius ($D = (32/3)\mu a^3$). Under the very low Reynolds number flow conditions encountered here, the surface texture and fine structure of the particle are unimportant. The maximum rotation speed, $f = P/\pi\omega D$, will result when the incident light is circularly polarized and the particle is a half-wave plate. Small particles will generally rotate faster due to less drag, but as particles become too small, their thickness becomes much less than the ideal half-wave case, and they will not intercept all of the power available to spin larger particles.

rers, or optically powered cogwheels. The rotation could also be used to study the viscosity of small samples of fluids, or the alignment of calcite particles could be used to hold probe particles in particular orientations, which could be useful for atomic-force or other forms of microscopy. The birefringence of biological samples is usually much less than that of calcite, but may sometimes be large enough to allow the same alignment and free rotation to be achieved.

[1] Friese, M. E. J., Enger, J., Rubinsztein-Dunlop, H. and Heckenberg, N. R. "Optical angular-momentum transfer to trapped absorbing particles." *Phys. Rev. A* **54**, 1593–1596 (1996).

[2] He, H., Friese, M. E. J., Heckenberg, N. R. and Rubinsztein-Dunlop, H. "Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity." *Phys. Rev. Lett.* **75**, 826–829 (1995).

[3] Simpson, N. B., Dholakia, K., Allen, L. and Padgett, M. J. "Mechanicalequivalence ofthe spin and orbital angular momentum of light: an optical spanner." *Opt. Lett.* **22**, 52–54 (1997).

[4] Beth, R. A. "Mechanical detection and measurement of the angular momentum of light." *Phys. Rev.* **50**, 115–125 (1936).

[5] Ashkin, A., Dziedzic, J. M., Bjorkholm, J. E. and Chu, S. "Observation of a single-beam gradient force optical trap for dielectric particles." *Opt. Lett.* **11**, 288–290 (1986).

[6] Constantinescu, V. N. *Laminar Viscous Flow* (Springer, New York, 1995).