Measuring the decoherence rate in a semiconductor charge qubit

S. D. Barrett*
Semiconductor Physics Group, Cavendish Laboratory,
University of Cambridge, Madingley Road, Cambridge CB3 0HE, U.K.†

G. J. Milburn
Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, U.K. and
Centre for Quantum Computer Technology, University of Queensland, St Lucia, Queensland 4072, Australia

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We describe a method by which the decoherence time of a solid state qubit may be measured. The qubit is coded in the orbital degree of freedom of a single electron bound to a pair of donor impurities in a semiconductor host. The qubit is manipulated by adiabatically varying an external electric field. We show that, by measuring the total probability of a successful qubit rotation as a function of the control field parameters, the decoherence rate may be determined. We estimate various system parameters, including the decoherence rates due to electromagnetic fluctuations and acoustic phonons. We find that, for reasonable physical parameters, the experiment is possible with existing technology. In particular, the use of adiabatic control fields implies that the experiment can be performed with control electronics with a time resolution of tens of nanoseconds.

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I. INTRODUCTION.

It is generally believed that scalable quantum computing devices will eventually be fabricated in solid state systems, and many ideas have been proposed.[1,2,3,4,5,6] Of these, only superconducting systems have, as yet, yielded single qubit devices capable of demonstrating a large number of single qubit rotations.[7,8] No multi qubit devices have been reported in the solid state. A single qubit device however is extremely useful as it enables an experimental measurement of the qubit decoherence time to be made. This number will ultimately determine if a particular solid state implementation is scalable (that is, capable of reaching the error threshold required for fault tolerant operation[9]). Decoherence refers to the uncontrollable coupling between the degree of freedom coding the qubit and other degrees of freedom in the qubit's environment. Such uncontrollable interactions lead to the qubit becoming entangled with these inaccessible degrees of freedom, with the result that the state of the qubit is not precisely defined by its preparation and subsequent control by unitary gates. Under such circumstances, the outcomes of direct measurements on the qubit are described by a mixed state, corresponding to an average over the inaccessible degrees of freedom.

In solid state systems the sources of decoherence are legion, and include phonons, nuclear spins, and electromagnetic fluctuations. Which sources of decoherence are relevant depend on what particular degrees of freedom are used to encode the qubit. A great deal of experimental and theoretical work remains to be done if we are to achieve understanding of the limitations of solid state implementations of qubits. In this paper we will focus on one particular qubit encoding based on the electron charge degree of freedom. (Decoherence of the charge degree of freedom is also of relevance to schemes in which *spin* qubits are coupled by an exchange interaction,[1,2,6] since charge decoherence can lead to leakage errors during exchange interaction gates.[10])

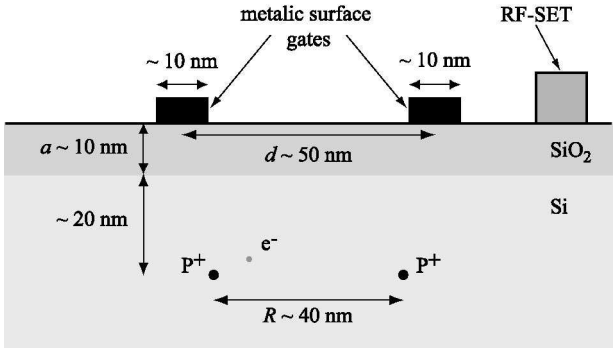

FIG. 1: A schematic representation of the double donor, single electron system. The qubit is encoded as the spatial localization of the electron charge, relative to the donor sites. The surface gates may be used to control the bias term, ε, in the qubit Hamiltonian. The radio frequency single electron transistor (RF-SET) may be used to readout the position of the electron.

To be specific, we will consider a system which consists of two phosphorus donors, embedded in a silicon substrate, which share a single excess electron.[11] The device is depicted schematically in Fig. 1. The qubit is coded in terms of the relative position of the electron. We denote localized single particle states by $\{|L\rangle, |R\rangle\}$ where $|L\rangle$ corresponds to an electron localized on the left donor site, while $|R\rangle$ denotes an electron localized on the right donor. These states are not eigenstates of the Hamiltonian when the potential is perfectly symmetrical. We may however represent localized states in terms of the

two lowest states of the potential; the symmetric ground state $|E_s\rangle$ and the antisymmetric first excited state $|E_{as}\rangle$, by $|L\,,R\rangle = (|E_s\rangle \pm |E_{as}\rangle)/\sqrt{2}$. The system prepared in one of these localized states will oscillate coherently between them at the tunnelling frequency $\Delta = E_{as} - E_s$. If the potential is biased sufficiently far from symmetry (by applying an external electric field) the localized states become good approximations to the energy eigenstates.

The single particle Hamiltonian for the double donor system may be approximated by the two level qubit Hamiltonian

$$H_{\rm qb} = -\hbar\frac{\varepsilon(t)}{2}\sigma_z - \hbar\frac{\Delta}{2}\sigma_x\,, \tag{1}$$

where $\sigma_x = |L\rangle\langle R| + |R\rangle\langle L|$ and $\sigma_z = |L\rangle\langle L| - |R\rangle\langle R|$ are the Pauli spin operators in the $\{|L\rangle, |R\rangle\}$ basis, and $\varepsilon(t)$ describes the bias of the potential away from symmetry, due to an external electric field. We have indicated that this may be a function of time, as discussed below. For a nonzero bias, the energy gap between the instantaneous ground state and first excited state is $\hbar E(t) = \hbar\sqrt{\Delta^2 + \varepsilon(t)^2}$.

This approximate Hamiltonian is valid if Δ, $|\varepsilon|$, $k_BT/\hbar \ll \omega_{01}$, where ω_{01} is the angular frequency corresponding to transitions between the ground and first excited states of an electron bound to a single isolated donor. For phosphorus donors in silicon, $\omega_{01} = 1.8\times10^{13}$ rad s^{-1}.[12] As we discuss in Sec. II, the tunnelling frequency, Δ, depends on the distance between the donors. For a donor separation of around 40 nm, the tunnelling frequency is approximately $\Delta \sim 10^{10}$ rad s^{-1}.

Decoherence in this system can be due to phonons that cause transitions between the energy eigenstates of the system. As we show in Appendix A, however, the corresponding timescale for such transitions can be made much longer than all other timescales in the problem by choosing an appropriate donor separation. Interactions with electromagnetic fluctuations in the environment (e.g. due to thermal voltage noise in nearby surface gates) however is more serious. In this paper we will model such processes using the spin boson model. This model has been extensively discussed in the literature (see for example Refs. 13 and 14).

Our objective is to find a way to experimentally determine the decoherence rate. It might be thought that this is easily done by monitoring the decay of the coherent tunnelling oscillation, by allowing the system to evolve for a time t and then determining the expected position of the electron relative to the double donor system, $\langle\sigma_z\rangle(t)$. Repeating for a number of different values of t, and observing the decay time of the oscillations in $\langle\sigma_z\rangle(t)$ would yield the decoherence rate. While this is possible in principle it is difficult in practice, because the coherent evolution must be turned on and off (for example, by rapidly changing the bias field $\varepsilon(t)$), on timescales much shorter than the reciprocal of the tunnelling frequency, $E(t)^{-1}$. Using this technique, the tunnelling frequency itself must be much larger than the decoherence time, which is expected to be of the order of nanoseconds (see Sec. IV B). Therefore, measuring the decay of coherent oscillations directly would require accurate switching of the qubit Hamiltonian on a timescale of tens of picoseconds. Despite these difficulties, a similar experiment has been achieved in a superconducting charge qubit.[7]

Other experiments have focused on *continuous* measurement of the charge degree of freedom of excess electrons in a coupled quantum dot system, using a nearby quantum point contact electrometer.[15,16] A signature of charge decoherence in the coupled dot system was observed by monitoring the average current through the electrometer, although a large contribution to the observed decoherence rate is thought to be due to the back action of the electrometer on the coupled dot system. This back action is due to the shot noise of the electrons tunnelling through the quantum point contact.

In a recent paper,[17] an alternative method was proposed to determine the decoherence rate for flux qubits implemented in a radio frequency SQUID system.[18] Rather than attempting to observe the decay of coherent oscillations of the flux, the authors proposed that the qubit polarization be reversed by *adiabatically* sweeping the qubit Hamiltonian parameters. They argued that the decoherence time can be determined by observing the probability of success of the adiabatic inversion process as a function of the parameter sweep time.

In this paper, we describe a scheme for determining the decoherence rate in the single electron, double donor system described above. Our scheme also makes use of adiabatic manipulation of the Hamiltonian parameters. We show that an experimental estimate of the decoherence rate can be obtained by preparing the system in the ground state under strong positive bias (a state localized on the left donor), adiabatically sweeping the bias to zero ($\varepsilon(t) = 0$) and then holding the bias at zero for a period $t_{\rm hold}$, before adiabatically sweeping to the opposite bias and then determining whether or not the system has changed its localized charge state. The final charge state of the system can be measured using a radio frequency single electron transistor (RF-SET).[19,20] An RF-SET can be kept in a quiescent state during the qubit evolution, and therefore the detector back action should not add a significant contribution to the observed decoherence rate. A plot of the probability of finding the electron on the right donor site versus $t_{\rm hold}$ will in general fall from a value close to unity, to substantially less than unity, over a timescale determined by the decoherence rate.

The advantages of this method over one in which coherent oscillations are directly observed are twofold. Firstly, substantially fewer measurements are required, since it is not necessary to plot out several coherent oscillations. Secondly, the timescales over which $\varepsilon(t)$ must be varied are determined by the decoherence timescale itself, rather than the (much shorter) timescale for coherent oscillations, $E(t)^{-1}$. Estimates of the relevant parameters, presented below, suggest that the experiment can be performed with electronics with a time resolution of tens of

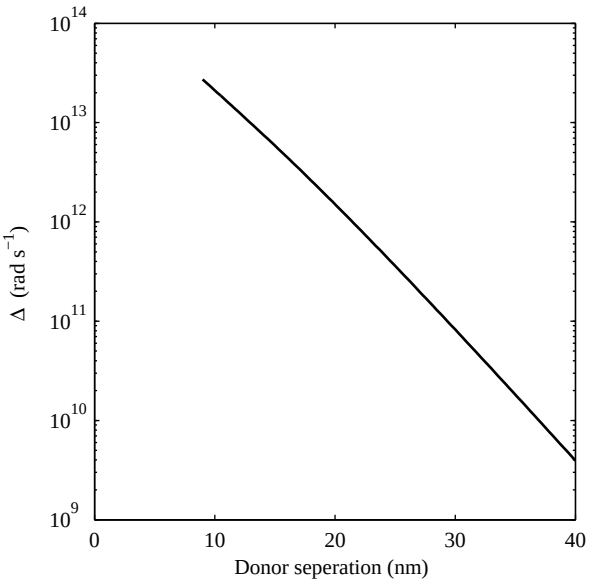

FIG. 2: Approximate energy gap, Δ, between the lowest symmetric and antisymmetric eigenstates of double donor, single electron system, under zero bias, as a function of donor separation.

nanoseconds, rather than tens of picoseconds.

This paper is organized as follows. In Sec. II we estimate the tunnelling frequency, Δ, for the double donor, single electron system, as a function of the donor separation. In Sec. III we describe the scheme for determining the decoherence rate in more detail. In Sec. IV A introduce the spin-boson model for the coupling of the qubit to the environment. In Sec. IV B we calculate an estimate for the strength of the system-environment coupling for the case of decoherence due to thermal voltage noise in nearby surface gates, and present the results of numerical calculations of the evolution of the qubit under such a coupling. In order for the experiment to be viable, a number of constraints must be satisfied. We quantitatively discuss these in Sec. V, and also find a set of experimentally achievable parameters that satisfy these constraints. We also discuss a number of other issues relating to the implementation of this scheme.

II. APPROXIMATE ENERGY LEVELS OF THE SINGLE ELECTRON, DOUBLE DONOR SYSTEM

The tunnelling frequency, Δ, may be estimated by determining approximate energy eigenvalues for the lowest energy symmetric and antisymmetric eigenstates for the double donor, single electron system. Finding exact values for these energy levels is complicated by the fact that the conduction band electron dispersion relation in silicon is anisotropic, and also by the valley-orbit interaction.[21] For the purposes of this work, however, it will be sufficient to gain an order of magnitude estimate for Δ. To this end, we ignore the conduction band anisotropy and assume that the localized states $|L\rangle$ and $|R\rangle$ may be represented by 1s-orbitals centered on the left and right donor sites respectively. We take the Bohr radius for an isolated donor state to be

$$a_B^* = \epsilon_{\rm Si} \left(\frac{m_e}{m_T} \right) a_B \,, \tag{2}$$

where $\epsilon_{\rm Si} = 11.7$ is the dielectric constant for silicon, m_e is the mass of a free electron, $m_T = 0.2 m_e$ is the transverse conduction band effective mass in silicon, and $a_B = 5.3 \times 10^{-11}$ m is the Bohr radius for the hydrogen atom.[22] We take the binding energy of a single electron to a single donor to be the experimentally observed value of -45.5 meV.[12]

With these assumptions, the energy levels of the double donor, single electron system can be determined by the same variational linear combination of atomic orbitals (LCAO) technique used to calculate the eigenvalues of an H_2^+ molecule.[23] In contrast to a real H_2^+ molecule, however, the position of the donors is fixed within the silicon lattice, and so it is not necessary to minimize the energy with respect to the donor separation. We plot the tunnelling frequency, Δ, as a function of the donor separation, R, in Fig. 2.

III. OUTLINE OF THE SCHEME

The scheme for measuring the decoherence rate for the charge qubit proceeds as follows. Initially, the electron is prepared in the $|L\rangle$ state by placing a large electric field across the double donor system, such that the bias term in the qubit Hamiltonian takes the value $\varepsilon_0 \gg \Delta$. ε_0 must be chosen such that the total energy gap for the two level system satisfies $\hbar E = \hbar\sqrt{\Delta^2 + \varepsilon_0^2} > k_B T$. The electron will then relax to the ground state, which is strongly localized on the left donor site.

The system is then placed in the symmetric superposition state, $(|L\rangle + |R\rangle)/\sqrt{2}$, by adiabatically sweeping the bias field to the symmetry point $\varepsilon(t) = 0$. The bias field sweep should be performed quickly, so that there is negligible decoherence during the sweep. However the sweep must not be made too quickly, or there will be coherent non-adiabatic transitions into the excited state. We discuss these requirements in more detail in Sec. V.

The bias field is held at zero for a time $t_{\rm hold}$. During this time, as a result of the interaction with the environment, the qubit will lose coherence. This loss of coherence will be manifest in the decay of the off diagonal elements of the qubit density matrix (written in the $|L\rangle$, $|R\rangle$ basis) at a rate Γ_0.

The bias field is then swept, rapidly but adiabatically, to a large negative value, $-\varepsilon_0$, and held at this value while the position of the electron is read out by a nearby electrometer. Thus $\varepsilon(t)$ has the following time dependence:

$$\varepsilon(t) = \begin{cases} \varepsilon_0 & t \leq 0 \\ \varepsilon_0 \frac{t_{\rm sw}-t}{t_{\rm sw}} & 0 < \ t \leq t_{\rm sw} \\ 0 & t_{\rm sw} < \ t \leq t_{\rm sw} + t_{\rm hold} \\ -\varepsilon_0 \frac{t-t_{\rm sw}-t_{\rm hold}}{t_{\rm sw}} & t_{\rm sw} + t_{\rm hold} < \ t \leq 2t_{\rm sw} + t_{\rm hold} \\ -\varepsilon_0 & 2t_{\rm sw} + t_{\rm hold} < \ t \,, \end{cases} \tag{3}$$

where $t_{\rm sw}$ is the time taken for each bias sweep.

Provided the electrometer can determine the position of the electron charge in a timescale shorter than the relaxation time for the qubit under the large bias ($-\epsilon_0$), the readout process will correspond to a strong quantum measurement in the $\{|L\rangle, |R\rangle\}$ basis. As we discuss further in Sec. V this measurement can be implemented by existing radio frequency single electron transistor (RF-SET) technology.[19,20,24]

By repeating the above preparation, bias sweep, and measurement steps a number of times, one can determine the probability, P_R, of finding the electron on the right donor site at the end of the sweep. If the decay of the off-diagonal elements of the density matrix (in the $|L\rangle$, $|R\rangle$ basis) during the time $t_{\rm hold}$ for which the bias is held at $\varepsilon = 0$ is negligible, then the electron will coherently tunnel through to the right donor site as the bias is swept through to $\varepsilon = -\varepsilon_0$. The final state will be, approximately, the pure state, $|R\rangle$. Thus the observed probability of finding the electron on the right donor will be close to unity. Conversely, if there has been substantial decay of the off-diagonal elements during the 'hold' part of the evolution, the final state will be mixed, and the observed P_R will be substantially less than unity. The off-diagonal density matrix elements are expected to decay over a timescale Γ_0^{-1}. Thus repeating the whole procedure for different values of $t_{\rm hold}$, and plotting P_R as a function of $t_{\rm hold}$ should allow one to determine Γ_0^{-1}.

IV. ESTIMATING THE DECOHERENCE RATE BY ADIABATIC TUNNELLING

A. The model of decoherence

In order to study the effects of the environment on the qubit, we model the environment as a bath of harmonic oscillator modes, linearly coupled to the $z-$component of the qubit, via the spin boson Hamiltonian,

$$H = H_{\rm qb} + \sigma_z \sum_i \hbar\lambda_i (a_i^\dagger + a_i) + \sum_i \hbar\omega_i a_i^\dagger a_i \,. \tag{4}$$

$H_{\rm qb}$ is the qubit Hamiltonian of Eq.(1). The second term describes the coupling between the position degree of freedom of the electron (σ_z), and the displacement operators for the bath modes ($a_i^\dagger + a_i$), where the λ_i's are coupling coefficients. The last term represents the free Hamiltonian of the oscillator bath, where the ω_i's are the angular frequencies of the bath modes.

The spin boson Hamiltonian has been studied extensively.[13,14] The behavior of the system depends crucially on the *spectral density* of the bath, defined as

$$J(\omega) = \sum_i \lambda_i^2 \delta(\omega - \omega_i) \,. \tag{5}$$

In general, the dynamics of the spin-boson model, for an arbitrary spectral density, is rather complicated. For the purposes of this work, however, a number of simplifying assumptions can be made. Firstly, we assume that the sweep of the bias field, $\varepsilon(t)$, is made sufficiently slowly that an adiabatic approximation can be employed. In particular, we require[25]

$$\frac{\pi}{2} \frac{\Delta^2 t_{\rm sw}}{\varepsilon_0} \gg 1 \,. \tag{6}$$

(We discuss this adiabaticity requirement further in Sec. V.) Secondly, we assume a weak system-bath coupling, such that $J(k_{\rm B}T/\hbar) \ll E(t)$, throughout the sweep. Finally, we take the initial state of the qubit to be the thermal state

$$\rho_0 = \frac{\exp(-H_{\rm qb,0}/k_B T)}{{\rm tr}[\exp(-H_{\rm qb,0}/k_B T)]} \,, \tag{7}$$

where $H_{\rm qb,0}$ is the initial qubit Hamiltonian, i.e. Eqn. (1) with $\varepsilon(t) = \varepsilon_0$. Note that ρ_0 is diagonal in the energy eigenbasis of the initial qubit Hamiltonian. Under these assumptions, the density matrix of the qubit is always diagonal in the instantaneous energy eigenbasis of the qubit Hamiltonian.[26] In this case, the Bloch vector $\vec{r}(t) = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$ always lies parallel to the vector $\vec{B} = (\Delta, 0, \varepsilon(t))$, and the dynamics can be understood by considering the evolution of $r(t) = |\vec{r}(t)|$, the length of the Bloch vector. The evolution of $r(t)$ under the above assumptions, is given by[26]

$$\dot{r}(t) = -\Gamma(t) \left(r(t) - r_{\rm eq}(t) \right) \,, \tag{8}$$

where the instantaneous relaxation rate $\Gamma(t)$ depends on the spin boson model parameters,[13,14]

$$\Gamma(t) = \frac{\pi}{2} \sin^2\theta J\left(E(t)\right) \coth\left(\frac{\hbar E(t)}{2k_{\rm B}T}\right) \,, \tag{9}$$

where $\theta = \tan^{-1}(\Delta/\varepsilon)$. $r_{\rm eq}(t)$ is the thermal equilibrium value of the Bloch vector, evaluated for the instantaneous energy gap of the system, $r_{\rm eq}(t) = \tanh\left(\hbar E(t)/2k_{\rm B}T\right)$.

At low frequencies, the spectral density of the bath typically has a power law behavior,[13,14] $J(\omega) \propto \omega^s$, where the exponent s depends on the nature of the environment. Two potentially serious sources of decoherence in this system are a deformation potential coupling between the qubit and acoustic phonons, and an electrostatic coupling to Nyquist-Johnson voltage fluctuations, which may originate in the surface electrodes used to control the qubit Hamiltonian parameters. The former is described by a superohmic spectral density ($s > 1$). However, as we show in Appendix A, with a judicious choice of donor configuration, the decoherence rate due to phonons can be made negligibly small, and therefore we neglect it in what follows.

B. Results for Ohmic damping

In this section, we concentrate on the case of decoherence due to Nyquist-Johnson voltage noise, which is characterized by a bath with an Ohmic spectral density ($s = 1$). At low frequencies, the spectral density may be written[13,14]

$$J(\omega) = 2\alpha\omega\,, \tag{10}$$

where α is a dimensionless parameter which characterizes the strength of the system-bath coupling.

In order to estimate α, we follow a similar procedure to that employed in Ref. 18. We first define the bath operator

$$X = \sum_i \lambda_i \left(a_i^\dagger + a_i\right)\,, \tag{11}$$

which couples to the σ_z operator of the qubit, via the second term in Eq.(4). To proceed, we calculate the spectrum of fluctuations in X in terms of the spectral density $J(\omega)$, and relate this to the spectrum of Nyquist-Johnson fluctuations in the surface gates. For a bath of harmonic oscillator modes in thermal equilibrium at temperature T, the Fourier transform of the symmetrized correlation function of this operator takes the form

$$\begin{aligned} S_X(\omega) &= \int_{-\infty}^{\infty} \frac{1}{2}\left\langle [X(t+\tau), X(t)]_+ \right\rangle e^{-i\omega\tau} d\tau \\ &= \pi J(\omega) \coth\left(\frac{\hbar\omega}{2k_BT}\right)\,, \end{aligned} \tag{12}$$

where $[A,B]_+ = AB + BA$ denotes an anti-commutator, $X(t) = e^{iHt/\hbar}Xe^{-iHt/\hbar}$ is the bath operator in the Heisenberg picture, and $\langle O\rangle = \mathrm{tr}[O\rho_{\rm env}]$ denotes the expectation of O for an environment in a thermal equilibrium state, $\rho_{\rm env}$.

For noise due to voltage fluctuations, X may be related to a perturbation δV_{LR} in the potential difference between the two donor sites by

$$X = \frac{e\delta V_{LR}}{2\hbar}\,. \tag{13}$$

δV_{LR} is related to the voltage fluctuations in the surface gates by

$$\delta V_{LR} = \beta \delta V_{\rm gate}\,. \tag{14}$$

where the dimensionless parameter β quantifies the electrostatic coupling between the surface gates and the donor sites, and is determined by the device geometry. For the geometry shown in Fig. 1, β may be approximated by elementary electrostatics as

$$\beta \approx \frac{2\ln\left(\frac{r_2}{r_1}\right)}{\left\{1+\frac{\epsilon_2}{\epsilon_1}\right\}\left\{\ln\left(\frac{d-r_0}{r_0}\right) + \frac{1}{2}\left(\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\right)\ln\left(\frac{(d-r_0)^2+4a^2}{r_0^2+4a^2}\right)\right\}}\,, \tag{15}$$

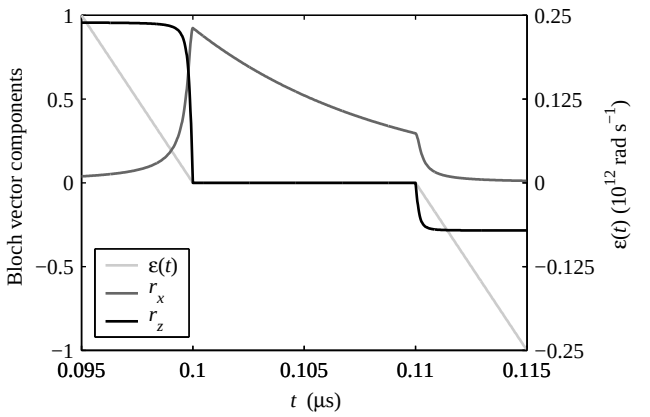

FIG. 3: Sample evolution of the Bloch vector components r_x and r_z, for part of the bias sweep. Also shown is the time profile of the bias sweep itself, $\varepsilon(t)$ (right axis).

where d is the distance between the two surface electrodes, a is the thickness of the oxide layer, r_1 is the distance between the left donor and the left electrode, r_2 is the distance between the left donor and the right electrode, r_0 is the effective radius of the electrode, and ϵ_1 and ϵ_2 are the dielectric constants of the oxide and silicon layers, respectively. In deriving this expression we have assumed that the gates may be represented by long, cylindrical conductors, and that $r_0 \ll a, d$. Using the values for r_1, r_2, d, and a given in Fig. 1, and taking $r_0 = 5\,\mathrm{nm}$, $\epsilon_1 = 4$, and $\epsilon_2 = 12$, we find $\beta = 0.17$.

Substituting Eq.(14) into Eq.(13), and calculating the corresponding power spectrum yields

$$S_X(\omega) = \frac{e^2\beta^2}{4\hbar^2} S_V(\omega)\,. \tag{16}$$

For Nyquist-Johnson noise, the voltage fluctuations are characterized by[27]

$$\begin{aligned} S_V(\omega) &= \int_{-\infty}^{\infty} \langle \delta V_{\rm gate}(t+\tau)\delta V_{\rm gate}(t)\rangle\, e^{-i\omega\tau} d\tau \\ &= R_{\rm gate}\hbar\omega \coth\left(\frac{\hbar\omega}{2k_BT}\right)\,, \end{aligned} \tag{17}$$

where $R_{\rm gate}$ is the impedance of the circuit which generates the gate voltages, and T is the corresponding noise temperature. Substituting this expression into Eq. (16) and comparing with Eqs. (10) and (12), we find that the system-bath coupling parameter is

$$\alpha = \frac{\beta^2 R_{\rm gate}}{4R_Q}\,, \tag{18}$$

where $R_Q = h/e^2 = 25.8\,\mathrm{k\Omega}$ is the quantum resistance. Taking $R_{\rm gate} = 50\,\Omega$ and $\beta = 0.17$, we have $\alpha = 1.4 \times 10^{-5}$.

We numerically solved Eqs. (8) and (9) for the Ohmic spectral density of Eq. (10), assuming a bias sweep $\varepsilon(t)$ of the form described in section III. Figure 3 shows the

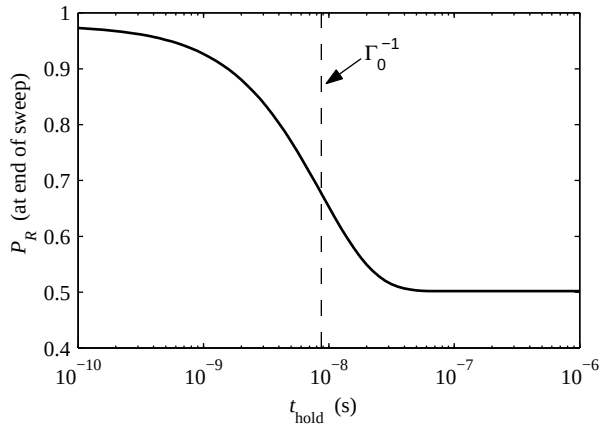

FIG. 4: Probability of finding electron on the right donor, P_R, at the end of the bias sweep, as a function of $t_{\rm hold}$. The broken line represents the reciprocal of the decoherence rate for zero bias, Γ_0^{-1}.

evolution of the x and z components of the Bloch vector, $\vec{r}(t)$, for a bias sweep with parameters $\varepsilon_0 = 5 \times 10^{12}\,{\rm s}^{-1}$, $t_{\rm sw} = 10^{-7}\,{\rm s}$, and $t_{\rm hold} = 10^{-8}\,{\rm s}$. We assume also that $\Delta = 10^{10}\,{\rm s}^{-1}$ and $T = 10\,{\rm K}$.

Figure 4 shows the resultant probability, $P_R = (1 - r_z)/2$, that the electron is found on the right donor at the end of the sweep, as a function of $t_{\rm hold}$. The other parameters used in this calculation are the same as those used in Fig. 3. For values of $t_{\rm hold} \ll \Gamma_0^{-1}$, P_R is close to unity, indicating that the electron has coherently tunnelled from the left donor site to the right donor site. Note that P_R saturates to a value slightly less than unity, as a result of a small amount of decoherence during the 'sweep' parts of the evolution. For values of $t_{\rm hold} \gtrsim \Gamma_0^{-1}$, the resultant probability is substantially less than unity, indicating a loss of coherence during the 'hold' part of the evolution, due to interaction with the environment. The transition between these regimes occurs at a value of $t_{\rm hold} \sim \Gamma_0^{-1}$. Thus measuring P_R at the end of the sweep provides a method for estimating the decoherence time Γ_0, and hence for estimating the strength of the system-environment coupling.

V. DISCUSSION

In order that the transition from coherent tunnelling ($P_R \approx 1$ at the end of the sweep) to incoherent behavior (P_R substantially less than 1) can be observed, and hence Γ_0 be determined, the parameters Δ, ε_0 and $t_{\rm sw}$ must satisfy a number of constraints. Firstly, we require that at time $t = 0$, the electron must be strongly localized at the left hand donor site. This can be achieved by placing a large bias ε_0 across the double donor system, and waiting for the donor to relax to its ground state. This implies that we require

$$\hbar\varepsilon_0 \gg k_B T\,. \tag{19}$$

Secondly, we require that the minimum energy gap between the ground and excited states satisfies

$$\hbar\Delta \ll k_B T\,, \tag{20}$$

otherwise the system will simply remain in its ground state, throughout the bias sweep, and it will not be possible to observe the effects of decoherence.

Thirdly, coherent, non-adiabatic transitions into the excited level should be minimized. The problem of non-adiabatic transitions in two level systems was considered by Landau[28] and Zener[25]. The results of Ref. 25 are directly applicable to the present work. For negligible nonadiabatic transitions, we require

$$\frac{\pi}{2}\frac{\Delta^2 t_{\rm sw}}{\varepsilon_0} \gg 1\,. \tag{21}$$

Finally, it is necessary to ensure that relaxation at the end of the bias sweep (when $\varepsilon(t) < -k_B T$) is negligible. If there is significant relaxation over the last part of the bias sweep or during the measurement process, the system will be found to be in its ground state regardless of $t_{\rm hold}$, and it will not be possible to observe the effects of decoherence. The probability that the electron will relax into the ground state, over the last part of the sweep, is approximately

$$P_{\rm relax} \approx \int_{t_*}^{t_f} \Gamma(t) dt\,. \tag{22}$$

where $\Gamma(t)$is the relaxation rate of Eq. (9), and t_* is the time for which $\varepsilon(t) = \varepsilon_* = -k_B T$, and $t_f = 2t_{\rm sw} + t_{\rm hold}$ is the time corresponding to the end of the sweep. Performing the integral, and requiring that $P_{\rm relax} \ll 1$, we have

$$\frac{\alpha\pi\Delta^2}{\varepsilon_0} \ln\left(\frac{\varepsilon_0}{\varepsilon_*}\right) \ll t_{\rm sw}^{-1}\,. \tag{23}$$

In arriving at this expression, we have made the approximation $\coth(\hbar E(t)/2k_B T) \approx 1$ for $t \geq t_*$, and that $\varepsilon_0 \gg \varepsilon_*$.

In order that there is no significant relaxation to the ground state during the measurement process, we require

$$\frac{\alpha\pi\Delta^2}{\varepsilon_0} \ll t_{\rm meas}^{-1}\,, \tag{24}$$

where $t_{\rm meas}$ is the characteristic time for the electrometer to detect the presence or absence of the electron on the right donor site. We take $t_{\rm meas} = 1\mu s$, which is readily achievable with existing radio frequency single electron transistor (RF-SET) technology.[19,20,24]

In the preceding analysis, T corresponds to the noise temperature of the electronics which generate the bias sweep. Taking $T = 10\,{\rm K}$, we find that $k_B T/\hbar = 1.3 \times 10^{12}\,{\rm s}^{-1}$. In order to satisfy the inequalities of Eq. (19) and Eq. (20), we choose $\Delta = 10^{10}\,{\rm s}^{-1}$ and $\varepsilon_0 = 5 \times$

$10^{12}\,\mathrm{s}^{-1}$. The inequality of Eq. (21) can then be satisfied if we choose $t_{\mathrm{sw}} = 10^{-7}\,\mathrm{s}$. With these parameters, Eq. (23) and Eq. (24) imply that unwanted relaxation is negligible provided $\alpha \leq 3 \times 10^{-3}$. Comparison with our earlier estimate, from Sec. IV B, of $\alpha \approx 10^{-5}$, suggests that the experiment is indeed feasible.

A central element of the scheme introduced in Sec. III is that the bias field, $\varepsilon(t)$, is held at zero for a time t_{hold}. This bias field will be related to V_{gate}, the voltage across the surface electrodes in Fig. 1. However, imperfections in the fabrication of a real device, and the existence of other surface electrodes (for instance, the plunger gate used to tune the RF-SET), may alter the potential landscape in the vicinity of the donors, leading to a small systematic error, $\delta\varepsilon$, in the bias field. This will lead to a systematic error in the observed value of the decoherence rate. According to Eq. (9), provided $\hbar\delta\varepsilon \ll k_B T$, the observed rate will be

$$\Gamma_0' = \frac{\Delta^2}{\Delta^2 + \delta\varepsilon^2}\Gamma_0\,, \tag{25}$$

where Γ_0 is the decoherence rate evaluated for $\delta\varepsilon = 0$. Thus the true decoherence rate can be inferred by determining Γ_0' for a range of different offset voltages and fitting the results to Eq. (25). Note that for sufficiently small offsets, $\delta\varepsilon < \Delta$, we have

$$\Gamma_0' \approx \left(1 - \frac{\delta\varepsilon^2}{\Delta^2}\right)\Gamma_0\,, \tag{26}$$

i.e. the error in the observed decoherence rate is only quadratic in the offset error.

In our discussion of decoherence mechanisms, we have not explicitly considered errors due to background charge fluctuations. These fluctuations vary from sample to sample, and typically have a $1/f$ spectrum with a shoulder at $100 - 1000$ Hz.[29] This timescale is longer than the time taken for each preparation, sweep, and measure cycle. Background charge fluctuations will therefore have the same effect as adding a small random offset bias, $\delta\varepsilon$, which may vary between cycles, but will be essentially constant over each bias sweep. As described above, the effect of such an offset becomes unimportant provided that, in a given sample, the charge fluctuations are sufficiently small that the corresponding offsets satisfy $\delta\varepsilon < \Delta$.

VI. CONCLUSION

In summary, we have proposed and analyzed, theoretically, an experimentally feasible scheme for directly determining the decoherence rate for a solid state charge qubit consisting of a single electron bound to a pair of donor impurities in a semiconductor host. The qubit is manipulated by adiabatically varying the bias term in the Hamiltonian. For a specific implementation using phosphorous donors embedded in a silicon host, we have theoretically obtained quantitative estimates for the Hamiltonian parameters, and for decoherence rates corresponding to interactions with both acoustic phonons and voltage fluctuations. We have analyzed various constraints which must be satisfied in order for the experiment to be feasible. We have found appropriate, experimentally achievable parameters which satisfy these constraints. Our results indicate that the control field needs to be manipulated with a time resolution of tens of nanoseconds, which is well within reach of current technology. Performing this experiment would be a vital step towards the implementation of a scalable solid state quantum computer.

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APPENDIX A: RELAXATION DUE TO PHONONS

In this appendix, we estimate the decoherence rate due to interaction with acoustic phonons. The problem of electron scattering by acoustic phonons in silicon was originally considered by Bardeen and Shockley.[30] More recently, electron relaxation, due to phonons, in low dimensional semiconductor systems has been considered.[31,32] Due to the confinement of the electrons in these systems, and the resulting discrete spectrum of the electronic energy levels, relaxation due to phonons is suppressed.

The rate for phonon emission in confined systems is[31,32]

$$\begin{aligned}\Gamma_{\mathrm{ph}} &= \frac{D^2 q_{if}^3}{8\pi^2\rho\hbar c_s^2}\left[n_B(E, T_{\mathrm{ph}}) + 1\right] \\ &\quad \times \int d\Omega_q |\langle\psi_f|e^{i\vec{q}\cdot\vec{r}}|\psi_i\rangle|^2\,,\end{aligned} \tag{A1}$$

where D is a deformation potential, ρ is the density of silicon, c_s is the speed of sound, $\hbar E$ is the energy difference between the initial and final electron states, $n_B(\omega, T_{\mathrm{ph}}) = (\exp(\hbar\omega/k_B T_{\mathrm{ph}}) - 1)^{-1}$ is the Bose occupation function for a bath of phonons at temperature T_{ph}, and q_{if} is the wave number of the emitted phonon. q_{if} is fixed by the energy gap between the ground and excited states and the phonon dispersion relation as $q_{if} = E/c_s$. The integral in Eq. (A1) is over all solid angles in q space, and is evaluated for $q = q_{if}$. In general, owing

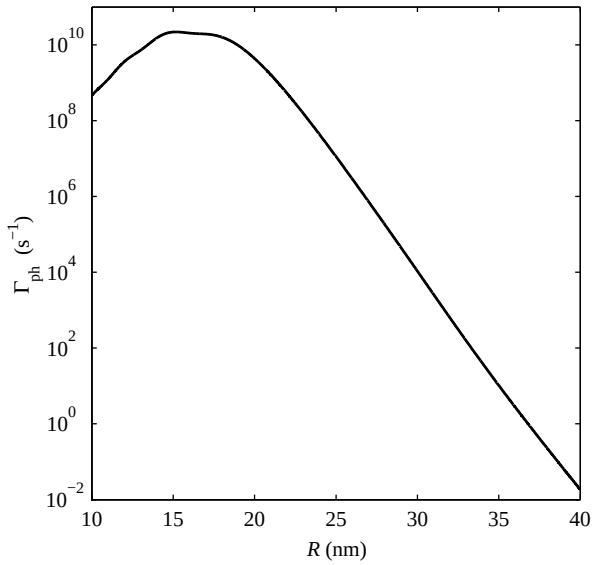

FIG. 5: Approximate relaxation rate due to LA phonons, $\Gamma_{\rm ph}$, as a function of donor separation, for zero bias ($\varepsilon = 0$). We assume $T_{\rm ph} = 0.1$ K, $D = 3.3$ eV, $\rho = 2.33$ kg m^{-3}, $c_s = 9.0 \times 10^3$m s $^{-1}$ and $a_0 = 3$ nm.

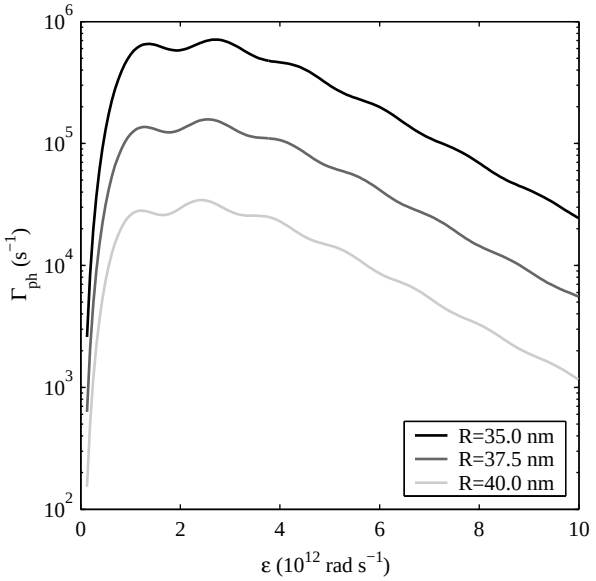

FIG. 6: Approximate relaxation rate due to LA phonons, $\Gamma_{\rm ph}$, as a function of bias energy, ε, for donor separations $R = 35.0$ nm, $R = 37.5$ nm, and $R = 40.0$ nm. We assume $T_{\rm ph} = 0.1$ K, $D = 3.3$ eV, $\rho = 2.33$ kg m^{-3}, $c_s = 9.0 \times 10^3$m s $^{-1}$ and $a_0 = 3$ nm.

to the anisotropy of the crystal, both D and c_s will be tensors. However, for the purpose of gaining an order of magnitude estimate of $\Gamma_{\rm ph}$, we will ignore these subtleties and treat these quantities as being isotropic.

The initial and final electron states are

$$|\psi_i\rangle = \cos\frac{\theta}{2}|L\rangle + \sin\frac{\theta}{2}|R\rangle\,, \tag{A2}$$

$$|\psi_f\rangle = \sin\frac{\theta}{2}|L\rangle - \cos\frac{\theta}{2}|R\rangle\,, \tag{A3}$$

where $\theta = \tan^{-1}(\Delta/\varepsilon)$, and $|L\rangle = (\pi a_B^3)^{-\frac{1}{2}}\exp(-r_a/a_B)$ and $|R\rangle = (\pi a_B^3)^{-\frac{1}{2}}\exp(-r_b/a_B)$ are 1s-orbitals, with Bohr radius a_B, localized on the left and right donor site respectively. Taking the origin to be the mid point of the line joining the two donors, we have $\vec{r}_a = \vec{r} + \frac{1}{2}R\vec{u}_x$ and $\vec{r}_b = \vec{r} - \frac{1}{2}R\vec{u}_x$, where $\vec{u}_x$ is the unit vector along the line joining the two donors, and R is the donor separation. In these coordinates, the matrix element in Eq. (A1) may be written

$$\begin{aligned}\langle\psi_f|e^{i\vec{q}\cdot\vec{r}}|\psi_i\rangle &= \frac{\sin\theta}{2\pi a_B^3}\int dV e^{i\vec{q}\cdot\vec{r}}\left(e^{-2r_a/a_B} - e^{-2r_b/a_B}\right)\\ &\quad + O\left((R/a_B)^3 e^{-R/a_B}\right)\,.\end{aligned} \tag{A4}$$

The last term in this expression may be neglected for donor separations $R \gg 3a_B$. Performing this integral, with the aid of the convolution theorem, we find that

$$\langle\psi_f|e^{i\vec{q}\cdot\vec{r}}|\psi_i\rangle = \frac{-16i\sin\theta\sin(q_xR/2)}{\left[(qa_B)^2+4\right]^2}\,, \tag{A5}$$

where q_x is the component of the phonon wavevector along the line joining the two donors. Substituting this expression into Eq. (A1), and performing the integral over all solid angles, gives

$$\Gamma_{\rm ph} = \frac{64D^2q_{if}^3\sin^2\theta\left[n_B(E,T_{\rm ph})+1\right]\left[1-{\rm sinc}(q_{if}R)\right]}{\pi\rho\hbar c_s\left[(q_{if}a_B)^2+4\right]^4}\,. \tag{A6}$$

Note that this rate is, in general, a function of the lattice temperature, $T_{\rm ph}$, the distance between the donors, R (which fixes Δ as shown in Fig. 2), and the bias between the donors, ε.

In Fig. 5 we plot $\Gamma_{\rm ph}$ as a function of donor separation for zero bias ($\varepsilon = 0$). In Fig. 6 we plot $\Gamma_{\rm ph}$ for a nonzero bias, for three different donor separations ($R = 35.0$ nm, $R = 37.5$ nm, and $R = 40.0$ nm). In these calculations we assume $T_{\rm ph} = 0.1$ K, $\rho = 2.33$ kg m^{-3}, $D = 3.3$ eV, $c_s = 9.0 \times 10^3$ m s^{-1} (LA phonons) and $a_0 = 3$ nm.[22] Note that the lattice temperature $T_{\rm ph}$ used here is much less than the effective noise temperature ($T = 10$ K) assumed in Sec. V; the latter is due to noise in the electronics used to generate the bias sweep, which is typically much greater than the sample base temperature.

These results indicate that $\Gamma_{\rm ph}$ is a strongly decreasing function of the donor separation (for $R \gtrsim 20$ nm). For donor separations greater than about 35 nm, $\Gamma_{\rm ph}^{-1}$ is significantly longer than the other relevant timescales in the problem, and therefore we are justified in neglecting phonons as a source of decoherence in this system.

* Electronic address: sean.barrett@hp.com
† Present address: Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol BS34 8QZ, U.K.

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