

# Decoherence in ion traps due to laser intensity and phase fluctuations

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## Abstract

We consider one source of decoherence for a single trapped ion due to intensity and phase fluctuations in the exciting laser pulses. For simplicity we assume that the stochastic processes involved are white noise processes, which enables us to give a simple master equation description of this source of decoherence. This master equation is averaged over the noise, and is sufficient to describe the results of experiments that probe the oscillations in the electronic populations as energy is exchanged between the internal and electronic motion. Our results are in good qualitative agreement with recent experiments and predict that the decoherence rate will depend on vibrational quantum number in different ways depending on which vibrational excitation sideband is used.

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## I. INTRODUCTION

Recent advances in laser cooling now enable a single trapped ion to be prepared in a chosen quantum state of the center-of-mass vibrational motion [1]. In some cases, the state is highly nonclassical, such as the recent preparation of a superposition of two oscillator coherent states [2]. Quantum dynamical features, such as collapse and revival oscillations, have also been observed [3]. The key innovation in such experiments is the ability to tailor the effective potential experienced by the ion by coupling the center-of-mass motion to the electronic states by external laser pulses. If more than a single ion is trapped, individual ions may be addressed by different laser pulses leading to an entanglement of the collective vibrational motion of the ions and their electronic states. This is the principle behind the suggestion of Cirac and Zoller [4] for implementing quantum computation in ion traps. So far however only a single controlled-NOT gate, a key component in a quantum computer, has been implemented experimentally [5].

Despite these heroic experimental achievements, the quantum motion of a single trapped ion is obviously limited by sources of decoherence. Decoherence arises from random and unknown perturbations of the Hamiltonian. If these perturbations cannot be followed exactly, experiments must average over them. This leads to an effective irreversible evolution of the trapped ion and a suppression of coherent quantum features through the decay of off-diagonal matrix elements of the density operator in some basis. Complementary to the decay of off-diagonal matrix elements, noise is added to conjugate variables. This can appear as a heating of the ion if noise is added to the momentum variable.

In this paper we consider one source of decoherence for a single trapped ion due to intensity and phase fluctuations in the exciting laser pulses. For simplicity we assume that the stochastic processes involved are white noise processes, which enables us to give a simple master equation description of this source of decoherence. Section II contains a general overview of the kind of system we investigate. In the first subsection we concentrate on intensity fluctuation in the laser, whereas the second subsection is devoted to phase

fluctuations. We conclude with a discussion on the experimental relevance of our results.

## II. LASER FLUCTUATIONS.

A single two-level ion, with mass  $m$ , tightly bound in a harmonic trap, and laser cooled to the Lamb-Dicke limit, can be prepared in a variety of states by carefully controlling the effective detuning of external laser fields which couple the vibrational motion and the internal electronic states. For simplicity we will assume the ion is constrained to move in a single dimension at harmonic frequency  $\nu$ . A reference frequency is provided by the atomic transition frequency  $\omega_A$ . If the effective laser frequency  $\omega_L$  is tuned below or above this frequency by multiples of the harmonic trap frequency  $\nu$ , a variety of effective potentials may be obtained. In the NIST experiments [1], two laser fields are used to excite two-photon stimulated Raman transitions. However in this paper we will consider the simplest case of a single classical laser, with wave vector  $k_L$  and frequency  $\omega_L$ , where the field is propagating in the same direction in which the ion is constrained to vibrate.

In the interaction picture with the dipole and rotating wave approximation the interaction Hamiltonian is [1]

$$H_I = \hbar\Omega(t)(\sigma_+ + \sigma_-) \left( e^{i\eta(a^{-i\nu t} + a^\dagger e^{i\nu t}) - i\delta t + i\phi(t)} \right) + h.c. , \quad (1)$$

where  $\Omega(t)$  is the effective Rabi frequency for this transition, written as a function of time to account for fluctuations resulting from laser intensity fluctuations,  $\phi(t)$  represents fluctuations in the laser phase,  $\eta = k_L(\hbar/2m\nu)^{1/2}$  is the Lamb-Dicke parameter,  $\delta = \omega_A - \omega_L$  is the detuning between the laser and the electronic states, and  $\sigma_\pm$  are the usual two-level atom transition operators. In order to excite particular transitions,  $|g, n\rangle \leftrightarrow |e, n'\rangle$ , of the coupled electronic/vibrational spectrum we choose the detuning

$$\delta = \nu(n - n') , \quad (2)$$

where  $n, n'$  are integers. In this paper we will assume that the amplitude of the ions motion in the direction of the laser field is much less than a wavelength. In this limit we can expand

the interaction Hamiltonian to lowest order in  $\eta$  [1]. Furthermore, to illustrate the effect of laser fluctuations it will suffice to consider four cases: (i) carrier excitation,  $n = n'$ , (ii) first red sideband excitation  $n' = n - 1$ , (iii) first blue sideband,  $n' = n + 1$ , (iv) second red sideband,  $n' = n + 2$ . The interaction Hamiltonians for these four cases are:

$$H_I(t) = \hbar\Omega(t)(1 + \eta^2 a^\dagger a) [\sigma_+ e^{i\phi(t)} + h.c.] \quad \text{carrier} , \quad (3)$$

$$H_I(t) = \hbar\Omega(t)\eta [\sigma_+ a e^{i\phi(t)} + h.c.] \quad \text{red sideband} , \quad (4)$$

$$H_I(t) = \hbar\Omega(t)\eta [\sigma_+ a^\dagger e^{i\phi(t)} + h.c.] \quad \text{blue sideband} , \quad (5)$$

$$H_I(t) = \hbar\Omega(t)\eta [\sigma_+ a^2 e^{i\phi(t)} + h.c.] \quad \text{second red sideband} . \quad (6)$$

The red sideband Hamiltonian corresponds to the familiar Jaynes-Cummings Hamiltonian [7] of quantum optics.

We will specify the noise by defining a stochastic process for  $\Omega(t)$  and  $\phi(t)$ . In the case of laser amplitude fluctuations we take

$$\Omega(t)dt = \Omega_0(dt + \sqrt{\Gamma}dW(t)) , \quad (7)$$

where  $dW(t)$  is the increment of a real Wiener process [6], and  $\Omega_0$  is the non-fluctuating component of the Rabi frequency. The parameter  $\Gamma$  scales the noise. The interpretation of  $\Gamma$  is given by integrating Eq.(7) to obtain the pulse area,  $A(T)$ , which is also a stochastic variable,

$$A(T) = \Omega_0 T + \Delta A(T) , \quad (8)$$

where  $T$  is the pulse duration. The last term in this equation is a Gaussian random variable with mean zero and variance  $E(\Delta A(T)^2) = \Omega_0^2 \Gamma T$ . If we then consider the ratio of the r.m.s. fluctuations in the pulse area to the deterministic pulse area we find

$$\frac{E(\Delta A(T)^2)^{1/2}}{\Omega_0 T} = \sqrt{\frac{\Gamma}{T}} . \quad (9)$$

For phase fluctuations we take a simple diffusion,

$$\phi(t) = \sqrt{\gamma}W(t) , \quad (10)$$

where  $W(t)$  is the Wiener process.

### A. Intensity fluctuations

The Hamiltonians in Eqs. (6) are stochastic. We first consider the effect of laser intensity fluctuations and ignore phase fluctuations. We thus set  $\phi(t) = 0$  (constant). To obtain the corresponding Schrödinger equation requires some care, as the white noise process is quite singular. We can however define a stochastic Schrödinger equation in the Ito formalism [8], or more appropriately a stochastic Liouville-von Neumann equation,

$$d\rho(t) = -i[G, \rho]dt - i\sqrt{\Gamma}[G, \rho]dW(t) - \frac{\Gamma}{2}[G, [G, \rho]]dt , \quad (11)$$

where  $G$  takes one of the four forms,

$$G = \Omega_0\sigma_x(1 + \eta^2 a^\dagger a) \quad \text{carrier} , \quad (12)$$

$$G = \eta\Omega_0(a\sigma_+ + a^\dagger\sigma_-) \quad \text{red sideband} , \quad (13)$$

$$G = \eta\Omega_0(a^\dagger\sigma_+ + a\sigma_-) \quad \text{blue sideband} , \quad (14)$$

$$G = \eta\Omega_0(a^2\sigma_+ + (a^\dagger)^2\sigma_-) \quad \text{second red sideband} . \quad (15)$$

This equation gives the evolution of the system density operator conditioned on a particular noise history. In an experiment involving a number of pulses, with data from each pulse combined, the noise is effectively averaged and we obtain the following master equation describing the system

$$d\rho(t) = -i[G, \rho]dt - \frac{\Gamma}{2}[G, [G, \rho]]dt . \quad (16)$$

This equation has a similar form to that considered in reference [9] for a model of intrinsic decoherence. Indeed, that model of intrinsic decoherence has been explicitly solved for the case of the Jaynes-Cummings Hamiltonian (the red side-band case) by Moya-Cessa *et al.* [10] (see also [11]).

The last term in Eq.(16) is responsible for decoherence and, complementary to decoherence, it leads to diffusive growth in observables that do not commute with  $G$ , which is proportional to the interaction Hamiltonian of the system. In each case, the eigenstates of

$G$  are discrete and labeled by an index  $n$  corresponding to a vibrational quantum number, and a sign index  $\pm$  arising from the two dimensional Hilbert space of the electronic motion. We will designate these states as  $\{|e_n^\pm\rangle\}$ . Thus

$$\frac{\partial}{\partial t}\langle e_n^\pm|\rho(t)|e_m^\pm\rangle = \left[-i(e_n^\pm - e_m^\pm) - \frac{\Gamma}{2}(e_n^\pm - e_m^\pm)^2\right]\langle e_n^\pm|\rho(t)|e_m^\pm\rangle. \quad (17)$$

In this form the decay of off-diagonal coherence is quite explicit. Notice however that decoherence takes place in a joint basis of both the electronic and vibrational Hilbert spaces. This is in contrast to many similar proposals for decoherence, such as Brownian motion, which only involve a single Hilbert space. Furthermore, the form of the decoherence ensures that total energy is conserved even in the presence of noise, as expected for a stochastic Hamiltonian.

The eigenstates and eigenvalues for each of the operators in Eqs.(11-13) are as follows. For the case of carrier excitation:

$$|e_n^\pm\rangle = \frac{1}{\sqrt{2}}(|g, n\rangle \pm |e, n\rangle) \quad (18)$$

$$e_n^\pm = \pm \frac{\hbar\Omega_0}{2}(1 + n\eta^2). \quad (19)$$

For the case of red side-band excitation (Jaynes-Cummings):

$$|e_n^\pm\rangle = \frac{1}{\sqrt{2}}(|g, n+1\rangle \pm |e, n\rangle) \quad (20)$$

$$e_n^\pm = \pm\eta\Omega_0\sqrt{n+1}. \quad (21)$$

For the blue side-band case:

$$|e_n^\pm\rangle = \frac{1}{\sqrt{2}}(|g, n-1\rangle \pm |e, n\rangle) \quad (22)$$

$$e_n^\pm = \pm\eta\Omega_0\sqrt{n}. \quad (23)$$

For the second red sideband case:

$$|e_n^\pm\rangle = \frac{1}{\sqrt{2}}(|g, n+2\rangle \pm |e, n\rangle) \quad (24)$$

$$e_n^\pm = \pm\eta\Omega_0\sqrt{n(n+1)}. \quad (25)$$

The general solution to Eq. (16) may be written explicitly as

$$\rho(t) = e^{-iGt - \frac{\Gamma t}{2}G^2} \left( e^{\mathcal{J}t} \rho(0) \right) e^{iGt - \frac{\Gamma t}{2}G^2} , \quad (26)$$

where the superoperator in the middle of this expression is defined by the power series expansion for the exponential with

$$\mathcal{J}^m \rho = \Gamma^m G^m \rho G^m . \quad (27)$$

In the energy eigenstate basis we can solve Eq.(17) explicitly to give

$$\langle e_n^\pm | \rho(t) | e_m^\pm \rangle = \exp \left[ -\frac{i}{\hbar} t (e_n^\pm - e_m^\pm) - \frac{\Gamma t}{2\hbar^2} (e_n^\pm - e_m^\pm)^2 \right] \langle e_n^\pm | \rho(0) | e_m^\pm \rangle . \quad (28)$$

However it is of rather more use to exhibit the solution explicitly for particular initial conditions of relevance to the experiments. With this in mind we will assume that the initial state is prepared to be a particular vibrational energy eigenstate with the ion prepared in the electronic ground state,

$$|\psi(0)\rangle = |g, n\rangle . \quad (29)$$

In the experiments done so far, it is possible to probe directly which electronic state the ion occupies. In the work of Meekhof et al. [3] the internal state  $|g\rangle$  is the  $2s \ ^2S_{1/2}$  ( $F = 2, M_F = 2$ ) state of  $^9\text{Be}^+$ , and the state  $|e\rangle$  corresponds to the  $2s \ ^2S_{1/2}$  ( $F = 1, M_F = 1$ ) state as shown in figure 1. The state  $|g\rangle$  is detected by applying a nearly resonant  $\sigma^+$  polarized laser probe field to drive a strong transition between the state  $|g\rangle$  and another state,  $^2P_{3/2}$  ( $F = 3, M_F = 3$ ). As this other state can only decay back to  $|g\rangle$ , any fluorescence observed on this transition is evidence that the atom was in the state  $|g\rangle$  at the start of this probe pulse. As the intensity of the fluorescence on this transition is strong, it is almost certain to detect a photon and thus the quantum efficiency of this state determination is near unity. In other words, this measurement scheme realizes an almost perfect projection valued measurement onto the electronic state  $|g\rangle$ . A sequence of probe pulses delayed a time  $\tau$  after the initial state preparation can thus be used to build up the probability  $P_g(t)$  to find

the ion in the ground state. Of course to do this many repeats of the experiment must be performed and it is not possible to track laser fluctuations exactly one each run. The final result for  $P_g(t)$  must then represent an ensemble average over these fluctuations, and the ion dynamics is then described by Eq.(16). Given  $P_g(t)$ , one easily sees that  $P_e(t) = 1 - P_g(t)$ .

With  $\psi(0) = |g, n\rangle$ , the solution for  $P_g(t)$  in each case is as follows. For carrier excitation:

$$P_g(t) = \frac{1}{2} \left[ 1 + \exp \left\{ -\frac{\Gamma t \Omega_0^2}{2} (1 + 2n\eta^2) \right\} \cos \left( \Omega_0 t (1 + n\eta^2) \right) \right]. \quad (30)$$

For red side-band excitation:

$$P_g(t) = \frac{1}{2} \left[ 1 + \exp \left( -2\Gamma\eta^2\Omega_0^2 nt \right) \cos \left( 2\eta\Omega_0\sqrt{nt} \right) \right]. \quad (31)$$

For blue sideband excitation;

$$P_g(t) = \frac{1}{2} \left[ 1 + \exp \left( -2\Gamma\eta^2\Omega_0^2 (n+1)t \right) \cos \left( 2\eta\Omega_0\sqrt{n+1}t \right) \right]. \quad (32)$$

For second red sideband excitation:

$$P_g(t) = \frac{1}{2} \left[ 1 + \exp \left( -2\Gamma\eta^2\Omega_0^2 n(n-1)t \right) \cos \left( 2\eta\Omega_0\sqrt{n(n-1)}t \right) \right]. \quad (33)$$

So to test the dependence of decoherence on the excitation in the vibrational state experimentally, the second red sideband (or any higher order sideband) is a better choice than just the first order sidebands, since the dependence of the damping on  $n$  is quadratic (or of even higher order for higher sidebands). In figure 2 we illustrate a typical result using Eq. (32) and the parameters given in [3].

## B. Phase fluctuations

Now we investigate phase fluctuation in the laser instead of intensity fluctuations, i.e. we set  $\Omega(t)$  in Eqs. (3)–(6) to  $\Omega_0$  and introduce white phase noise  $\phi(t) = \sqrt{\lambda}W(t)$  instead. This means that the Hamiltonian is a nonlinear function of the noise which will lead to technical difficulties in deriving the corresponding Ito differential equation for the system state. However for our purposes a simple transformation can simplify matters considerably.



We follow [8] and use a random canonical transformation to get rid of the nonlinearity in the noise source. In effect this is an instantaneous rotation of the system through a fluctuating angle, analogous to the standard method of transformation to an interaction picture.

$$\hat{U} = \exp(i\phi(t)\hat{\sigma}_+\hat{\sigma}_-) . \quad (34)$$

Thus

$$\rho \longrightarrow \tilde{\rho} = \exp(-i\phi(t)\hat{\sigma}_+\hat{\sigma}_-) \rho \exp(i\phi(t)\hat{\sigma}_+\hat{\sigma}_-) \quad (35)$$

$$H \longrightarrow H_0 = G - \phi(t)\hat{\sigma}_+\hat{\sigma}_- , \quad (36)$$

where  $G$  is one of the operators defined by Eqs. (12)–(15), depending on which sideband the laser is tuned to. The reason we can use this stochastic transformation is that we are only interested in the population of the electronic levels. As the generator of the unitary transformation commutes with the population operator, the populations in the instantaneous transformed frame are the same as those in the original frame. However other moments will not be the same, and we could not easily use the transformed state,  $\tilde{\rho}$ , after averaging, to reconstruct moments in the original frame. This transformation can be described covering all four cases of  $G$  in one, since it only affects the internal state and the operators describing that ( $\hat{\sigma}_+$  and  $\hat{\sigma}_-$  appear in the same order and form in all the interactions we consider here). The corresponding master equation after averaging out the noise reads

$$\frac{d\tilde{\rho}}{dt} = -i [G, \tilde{\rho}] - \lambda [\hat{\sigma}_+\hat{\sigma}_-, [\hat{\sigma}_+\hat{\sigma}_-, \tilde{\rho}]] . \quad (37)$$

For the initial condition  $\hat{\rho} = |g\rangle\langle g| \otimes |n\rangle\langle n|$  we can solve this for all sidebands. However, here we restrict ourselves to the red sideband since that gives an idea on how the effects arising from phase fluctuations are different from intensity fluctuations. The probability for the atom to be in the upper state in this case is given by

$$\tilde{P}_g(t) = \frac{1}{2} \left[ 1 + \cos(\tilde{\omega}_n) \exp(-(\lambda/2)t) + \frac{\lambda}{2\tilde{\omega}_n} \sin(\tilde{\omega}_n) \exp(-(\lambda/2)t) \right] \text{ (red sideband),} \quad (38)$$

with

$$\tilde{\omega}_n = \sqrt{4\eta^2\Omega_0^2n - \lambda^2/4} . \quad (39)$$

So here the effective Rabi frequency depends on the coherence decay rate and not vice versa as in the case of intensity fluctuations.

### III. DISCUSSION

We have shown how to model fluctuations in the laser causing the interaction between center-of-mass motion and internal states in ion traps. Intensity fluctuations lead to decoherence processes which depend on the kind of interaction the laser is causing.

What is the value for  $\Gamma$  for fluctuations in the laser intensity? A rough estimate from the figures given in [3], using the quoted experimental values, leads to  $\Gamma \approx 1.4 \cdot 10^{-8}s$ . But this is really a very rough estimation. If we define the fractional error by the quotient of the r.m.s. and the pulse area,

$$\frac{\text{r.m.s.}}{A(T)} = \frac{\Omega_0\sqrt{\Gamma T}}{\Omega_0 T} = \sqrt{\frac{\Gamma}{T}} , \quad (40)$$

a fractional error of one percent leads to  $\Gamma \approx 10^{-10}$ . With a fractional error of ten percent, however, we get  $\Gamma \approx 10^{-8}$  and we are in the range of the roughly estimated value for  $\Gamma$ .

In [3] they experimentally estimate the  $n$ -dependence of their damping which they fit with

$$P_g(t) = \frac{1}{2} \left[ 1 + \sum_{n=0} P_n \cos(2\Omega_{n,n+1}t) e^{-\gamma_n t} \right] \quad (41)$$

to be of the form

$$\gamma_n = \gamma_o(n+1)^{0.7} . \quad (42)$$

We derive the coefficient to be 0.5 instead of 0.7 if the decoherence is just due to intensity fluctuations alone. However there are other sources of decoherence due, for example, to fluctuations in the trap potential itself. These fluctuations lead to, among other things, a fluctuating trap center point and will be addressed in a future publication.

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## FIGURES

Fig. 1

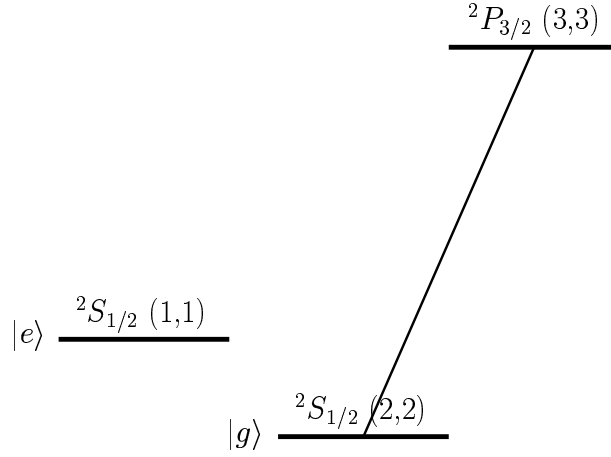


FIG. 1. Internal level scheme of  ${}^9\text{Be}^+$ . The ground state  $|g\rangle$  is the  $2s\ {}^2S_{1/2}$  ( $F = 2, M_F = 2$ ) state and the excited state  $|e\rangle$  is the  $2s\ {}^2S_{1/2}$  ( $F = 1, M_F = 1$ ) state. The state  $|g\rangle$  is detected by applying a nearly resonant  $\sigma^+$  polarized laser probe field to drive a strong transition between  $|g\rangle$  and another state  ${}^2P_{3/2}$  ( $F = 3, M_F = 3$ ) as indicated. As this other state can only decay back to  $|g\rangle$ , any fluorescence observed on this transition is evidence that the atom was in the state  $|g\rangle$  at the start of the probe pulse.

Fig. 2

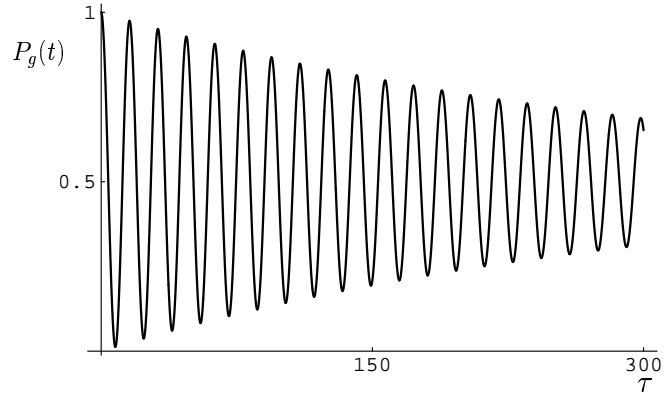


FIG. 2.  $P_g(t)$  for an initial  $|g, n = 0\rangle$  state of the atom driven by the first blue sideband, Eq. (32). The time parameter  $\tau = \Omega_0 t$  is scaled with the effective Rabi frequency  $\Omega_0 = 470$  kHz. The value for the scaled damping coefficient  $\Gamma' = \Gamma\Omega_0$  is 0.041 and the one for  $\eta$  is 0.2. All the values are rough estimates from the experimental values given in [3].