Closed-loop Frequency Tracking and Rejection *

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Abstract

This paper develops an adaptive controller for active vibration control. The method is based on the LQG approach via disturbance modelling given in De Nicolao [1]. This approach to the narrow band disturbance rejection problem is then applied to the problem of eliminating the effects of roll eccentricity in steel-strip rolling mills
Keywords: adaptive control, disturbance rejection, vibration control

1 Introduction

In many control applications the output of the process is subjected to a narrow band disturbance. In many cases this narrow band disturbance is sinusoidal, with unknown and possibly time-varying frequency. In this paper we examine one approach to designing a controller to reject such a disturbance.

2 Problem Statement

The problem of active vibration control consists of two parts: 1) estimating the frequency of the disturbance; and 2) given this frequency, designing a controller to reject the disturbance.

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This problem, with a fixed frequency disturbance, has been examined by Connolly *et al.* [2]. In this paper we consider the more difficult problem of vibration control when the frequency is time-varying.

2.1 Controller Design

There are two approaches to designing controllers to achieve the required disturbance rejection capabilities based on LQG. These are the disturbance modelling (DM) and frequency shaping (FS) approaches introduced by Gupta [3] and developed by several authors including Sievers [4, 5] and Anderson and Moore [6]. For narrow band disturbance rejection, the weighting functions used in these methods have poles on the imaginary axis to force the desired asymptotic rejection behaviour on the closed loop. However, the introduction of such weighting functions gives rise to uncontrollable modes on the imaginary axis in the DM case and to unobservable modes on the imaginary axis in the FS case, as noted in Sievers [7]. This makes the LQG synthesis difficult as the Riccati equations cannot be solved. In De Nicolao [8], these problems are elegantly overcome by the introduction of cross-coupling terms in the cost function for the DM case, and by correlated process and measurement noise in the FS case.

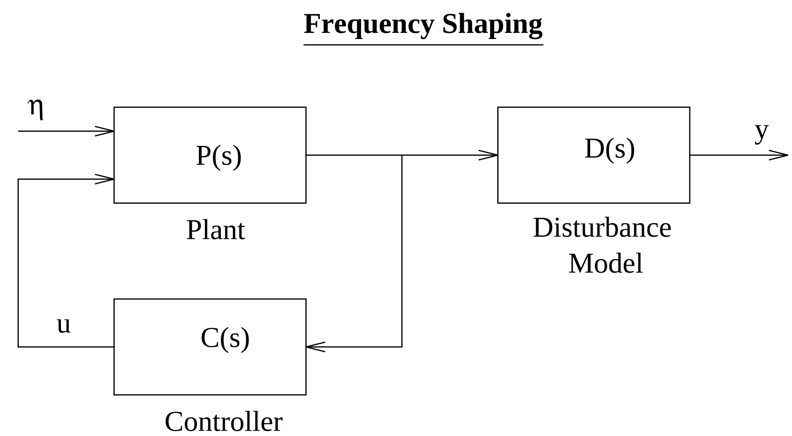
2.2 Frequency Estimation

There are a number of approaches to estimating the frequency of the sinusoid when that frequency is time-varying. One well-known technique for frequency tracking is adaptive notch filtering. Pei and Tseng [9] examine such filters for a variety of applications and discuss the appropriate formulations of such a filter in each case. Alternatively, frequency tracking can be performed using instantaneous frequency estimators recently described by Fertig and McClellan [10]. These estimators may be considered modifications and elaborations of Pisarenko [11]. Since these estimators use only (up to) 5 data points, they can track very rapid changes in signal frequency, but require

- a fairly high signal to noise ratio (greater than approximately 6dB)
- a reasonably high frequency (greater than approximately $f_s/5$) and
- that the signal is monocomponent (so that only a single frequency is present).

More complex estimators, such as those of Kay [12] and modifications of these by Lovell and Williamson [13], allow for much better noise performance over a greater range of frequencies. The trade-off using these estimators is that the signal frequency cannot change as quickly as with the shorter-term estimators of [10]. The signals must still be considered to be monocomponent.

Another common approach to frequency tracking is to employ an extended Kalman filter (EKF). In [14] the problem of designing an EKF frequency tracker for a single sinusoid at moderate signal-to-noise ratios was examined. The behaviour of the EKF when tracking multi-harmonic signals has been examined by James [15].



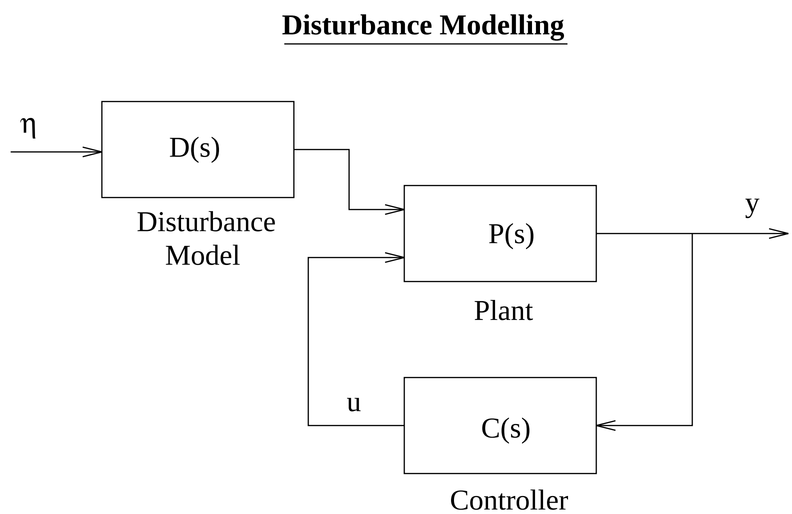


Figure 1: Block diagrams for Frequency Shaping and Disturbance Modelling

3 Problem Solution

3.1 Disturbance Rejection Using LQG Techniques

In LQG theory, the noise processes that drive the system are broadband, since they are white noise processes. Therefore, some modification is required to address the narrow band disturbance rejection problem. The modification considered here is that of disturbance modelling.

In the disturbance modelling (DM) approach, the process driving the plant is modelled as the output of a narrow band system driven by white noise. This technique can be viewed as conventional LQG synthesis problems with a suitably augmented plant model.

In the DM approach, the disturbance model is a filter consisting of a complimentary pair of poles at $\pm j\Omega$, where Ω is the frequency of the sinusoidal disturbance; let this disturbance model be

$$\begin{aligned}\dot{x}_d &= A_d x_d + B_d \eta \\ y_d &= C_d x_d.\end{aligned}$$

This is augmented with the plant model

$$\begin{aligned}\dot{x}_p &= A_p x_p + B_p(y_d + u) + \xi \\ y &= C_p x_p + v\end{aligned}$$

to give

$$\begin{aligned}\begin{bmatrix}\dot{x}_p \\ \dot{x}_d\end{bmatrix} &= \begin{bmatrix}A_p & B_p C_d \\ 0 & A_d\end{bmatrix}\begin{bmatrix}x_p \\ x_d\end{bmatrix} + \begin{bmatrix}B_p \\ 0\end{bmatrix} u \\ &\quad + \begin{bmatrix}I & 0 \\ 0 & B_d\end{bmatrix}\begin{bmatrix}\xi \\ \eta\end{bmatrix} \\ y &= [C_p \quad 0]\begin{bmatrix}x_p \\ x_d\end{bmatrix} + v.\end{aligned}$$

It can be seen that the dynamics of the disturbance model are uncontrollable, which is noted in Sievers [7].

This difficulty is overcome in De Nicolao [1] by using the cross-coupled objective function

$$J = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T \left(x_p' Q_c x_p + e' R_c e\right) dt\right\}, \tag{1}$$

in which $e = u + y_d$. Notice that the objective encourages $u + y_d$ to be small; this is the desired narrow band disturbance rejection property. Writing the dynamics in terms of the new variable e, we obtain

$$\begin{aligned}\begin{bmatrix}\dot{x}_p \\ \dot{x}_d\end{bmatrix} &= \begin{bmatrix}A_p & 0 \\ 0 & A_d\end{bmatrix}\begin{bmatrix}x_p \\ x_d\end{bmatrix} + \begin{bmatrix}B_p \\ 0\end{bmatrix} e \\ &\quad + \begin{bmatrix}I & 0 \\ 0 & B_d\end{bmatrix}\begin{bmatrix}\xi \\ \eta\end{bmatrix} \\ y &= [C_p \quad 0]\begin{bmatrix}x_p \\ x_d\end{bmatrix} + v.\end{aligned}$$

The dynamics of the disturbance model are decoupled from those of the plant; moreover, as they do not affect the cost function, we consider the reduced order control problem of optimising (1) subject to $\dot{x}_p = A_p x_p + B_p e$, which is a standard state feedback control problem with solution $e = -K_p x$. The state feedback control law is therefore given by

$$u = -[K_p \quad C_d]\begin{bmatrix}x_p \\ x_d\end{bmatrix} \tag{2}$$

Since there is no problem designing a Kalman filter (for the original augmented plant defined by (16)) to estimate the states x_p and x_d, this completes the DM approach to narrow-band disturbance rejection.

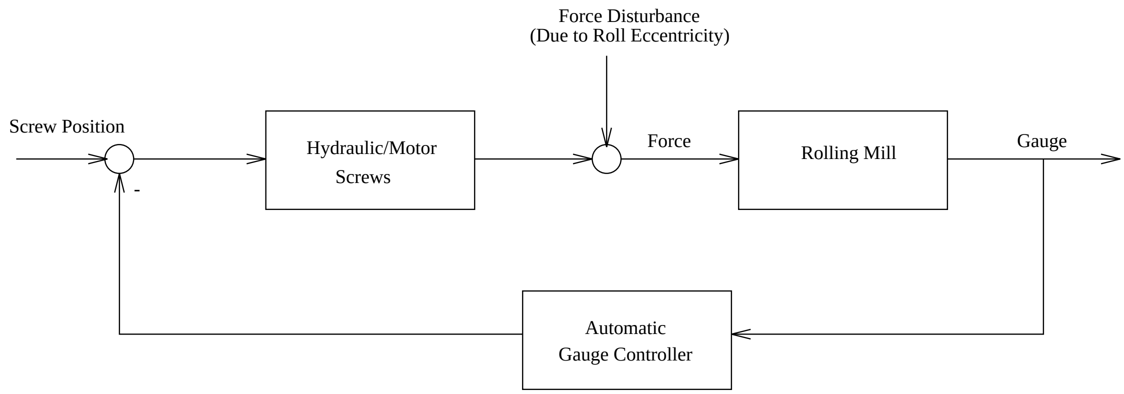


Figure 2: Force Disturbance in a Steel Mill

3.2 Frequency Tracking

The controller design above relies on an estimate of the disturbance frequency being available. In this paper we will examine the relative performances of two frequency tracking algorithms: 1) the adaptive notch filter; and 2) the instantaneous frequency estimators of Fertig and McClellan [10].

4 Application

One application for the controller given in Section 3.2 to the problem of eliminating roll eccentricity in a metal rolling mill. In a metal rolling mill, for example, steel rolls are forced together and the strip passes through them, thus reducing the thickness. The size of the reduction is controlled by the force exerted on the rolls, generally by hydraulic cylinders or motor driven screws. It is an impossible task to mount a roll in a metal strip rolling mill such that the roll's axis and the axis of rotation are perfectly concentric. This phenomenon, known as roll eccentricity, results in the metal strip experiencing a force variation as the roll rotates, which in turn causes a variation in the strip's thickness. This variation in strip thickness is generally a sinusoid, or at least can be well characterised by a single sinusoid. This sinusoidal variation in the gauge appears on the output of the gauge controller, as shown in Figure 2. The frequency of the disturbance is proportional to the roll speed, which is product dependent. In addition, as steel rolling is generally not a continuous process, the speed will vary during the rolling of a coil.

In this section we will describe the outcome of applying the controller design described above to a steel rolling mill.

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