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Bifurcation Control Of A Flexible Beam Under Principal Parametric Excitation

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Abstract

A new approach is proposed in this paper for the nonlinear control of a flexible cantilever beam which is excited under a principal parametric vibration. The approach is based on a bifurcation control principle that combines linear and nonlinear feedback control strategies. A simplified nonlinear vibration model that accounts for the beam's flexibility and the control feedback, is first derived by using the method of multiple scales. The analysis of stability based on Krasovskii's theorem is then applied to the above system. Simulation results show that the proposed approach is effective for use in the case of parametric vibration control.

Key words: Bifurcation control, nonlinear model, parametric excitation, multiple scales.

1 Introduction

Nonlinear motions of parametrically-excited systems are frequently encountered in various structures where the excitation frequency is close to twice that of any of the structure's natural frequencies. Many researchers have investigated the non-linear response of parametrically-excited systems. A review of the literature can be found in the texts by Nayfeh and Mook[1], and Schmidt and Tondl[2]. In contrast, however, there is little literature on the subject of bifurcation control of parametrically-excited systems, because of the lack of a universal technique for the analysis of these kinds of nonlinear systems. Recently, it has been shown by Oueini and Nayfeh[3] that vibration amplitudes resulting from nonlinear resonance can be suppressed by a cubic-velocity and a cubic-position feedback. Based on their work, a new method which combines the bifurcation control and the nonlinear-feedback control, is proposed in this paper to suppress the principal resonance even when the excitation frequency is close to twice that of the first mode's frequency. In this manner, the onset of bifurcation can be avoided. Two feedback methods are employed; a linear-velocity feedback method is employed to suppress the bifurcation, and a nonlinear-cubic-velocity feedback method is employed to minimise the amplitude of vibration. Furthermore, Krasovskii's nonlinear system stability theorem is used to justify the stability of the control system. Simulation results show that the proposed control strategy leads to effective bifurcation control and vibration suppression at the principal resonance points.

2 System Description

Many structural elements can be modelled as a slender beam. When the support of a beam undergoes motion, the beam is subjected to vibration which is either external, or parametric, or both. A flexible cantilever beam mounted on a shaker and actuated by piezoelectric patches is chosen as the experimental model, because it is a convenient structure for exhibiting such complicated phenomena in a controlled setting. In accordance with the model proposed by Crespo da Silva and Glynn[4], the dynamics of the first mode of vibration for a long slender non-extendible beam can be expressed in the following non-dimensional form:

$$\ddot{v} + v + 2\varepsilon\mu_1\dot{v} + \varepsilon\hat{\mu}_2|\dot{v}|\dot{v} + \varepsilon\alpha_1 v^3 + \varepsilon\alpha_2 v^2\ddot{v} + \varepsilon\alpha_3 v\dot{v}^2 = \varepsilon v F\cos(\Omega t) + \varepsilon P, \quad (1)$$

where v is the generalised co-ordinate, F and Ω are the forcing amplitude and frequency, respectively, P is a control feedback, and ε is a dimensionless bookkeeping parameter. Consider the principal parametric excitation case where $\Omega = 2 + \varepsilon\sigma$, (2)
in which σ is a detuning parameter. The method of multiple scales is used for the solution of (1), in which the uniform expansion of v is expressed as $v(T_0,T_1,\varepsilon) = v_0(T_0,T_1) + \varepsilon v_1(T_0,T_1) + \cdots$, (3)
where $T_0 = t$ is a fast scale characterising the motion at the frequency Ω, and $T_1 = \varepsilon t$ is a slow scale characterising the time variations of the amplitude and phase. Substituting (3) into (1) yields $D_0^2 v_0 + v_0 = 0$, (4) and

$$D_0^2 v_1 + v_1 = -2D_0D_1v_0 - 2\mu_1 D_0 v_0 - \hat{\mu}_2|D_0v_0|D_0v_0 - \alpha_1 v_0^3 - \alpha_2 v_0^2 D_0^2 v_0 - \alpha_3 v_0 (D_0 v_0)^2 + v_0 F\cos(\Omega T_0). \quad (5)$$

The solution can be expressed as $v_0 = A(T_1)e^{iT_0} + \bar{A}(T_1)e^{-iT_0}$, where $A(T_1)$ is a complex-valued function to be determined, and $\bar{A}(T_1)$ is its complex conjugate. According to the solvability condition and by substituting $\alpha = 3\alpha_1 - 3\alpha_2 + \alpha_3, f = F/4$ in (5),

$$2i(D_1A + \mu_1 A) + \frac{\hat{\mu}_2}{2\pi}\int_0^{2\pi}|D_0v_0|D_0v_0e^{-iT_0}dT_0 + \alpha A^2\bar{A} - 2\bar{A}fe^{iT_1} = 0. \quad (6)$$

Expressing A in polar form gives $A = a(T_1)e^{i\beta(T_1)}/2$. The selected feedback control law is expressed as $P = -(K_c\dot{v}^3 + K_v\dot{v})$, where K_C is the gain of the cubic-velocity feedback and K_V is the gain of the velocity feedback; both of which are tuneable. Using the above expressions, yields the simplified nonlinear vibration model:

$$\begin{cases} a' = -\lambda_1 a - \lambda_2 a^2 - \lambda_3 a^3 + af\sin\gamma \\ a\gamma' = \sigma a - \alpha a^3 + 2af\cos\gamma \end{cases},$$

where $\gamma = \sigma T_1 - 2\beta$, $\lambda_2 = \dfrac{\hat{\mu}_2}{2}$, $\lambda_1 = \mu_1 + K_v$, $and\, \lambda_3 = \dfrac{3K_c}{8}$.

3 Nonlinear Stability Analysis

For stability analysis, the Cartesian form is used for $A = (p - iq)e^{\eta T_1}/2, \eta = \sigma/2$. Substituting it into (6) yields:

$$\begin{cases} p' = -\lambda_1 p - \lambda_2 p\sqrt{p^2+q^2} - \lambda_3 q(p^2+q^2) - \eta q + \alpha_e q(p^2+q^2) + qf \\ q' = -\lambda_1 q - \lambda_2 q\sqrt{p^2+q^2} - \lambda_3 p(p^2+q^2) + \eta p - \alpha_e p(p^2+q^2) + pf \end{cases}$$

According to Krasovskii's theorem[5], two symmetric positive definite matrices $\mathbf{Q}$ and $\mathbf{I}$ (unit matrix) are chosen such that, $\forall p \neq 0$ & $q \neq 0$, if the matrix $\mathbf{F}$ described by $\mathbf{F} = \mathbf{JQ} + \mathbf{QJ}^T + \mathbf{I}$, is negative semi-definite (where $\mathbf{J}$ is the Jacobian matrix), then the system is asymptotically stable. This leads to:

$|\sigma| \geq 2\sqrt{f^2 - (\mu_1 + K_v)^2}$, (7) or $f \leq \sqrt{(\mu_1 + K_v)^2 + \sigma^2/4}$. (8)

It is evident from (7) that the bifurcation can be avoided by using velocity feedback with large K_V. Furthermore, a large value of K_V will satisfy stability condition (8) even for large excitations.

4 Simulation Results

Figure 1 shows the frequency-response curves of the closed-loop system for different selections of K_C and K_V, compared with the frequency-response curve of the open-loop system. The figure indicates a parametric resonance of the softening type. There are three distinct regions bounded by σ_A&σ_D, σ_A&σ_B, and σ_B&σ_D, respectively. For the first region defined by $\sigma<\sigma_A$ or $\sigma>\sigma_D$, only the trivial solution is possible and stable. For the second region $\sigma_A<\sigma<\sigma_B$, under open-loop conditions, there are three possible solutions; two non-trivial solutions – the larger of which is stable and the smaller unstable, and a trivial solution which is stable. The initial conditions of the system will determine the response for the second region. For the third region $\sigma_B<\sigma<\sigma_D$, there is one stable non-trivial solution and one unstable trivial solution. If only the cubic-velocity feedback is applied, i.e., K_V=0, it can seen that the resonant amplitudes of the vibration are reduced (from amplitude E to F). When the cubic velocity and the velocity feedback are combined together, both the resonance range and the resonant amplitudes can be reduced from σ_B to σ_C and E to G, respectively.

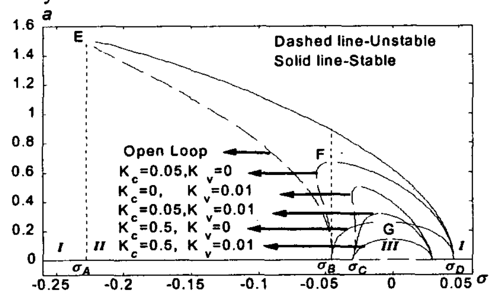

Figure 1. Frequency-response f = 0.025, μ_1 = 0.01, λ_2 = 0.01, α = -0.05.

Figure 2 shows the force-response curves of the closed-loop system for different selections of K_C and K_V, compared with the force-response curve of the open-loop system. Similar to Figure 1, there are also three distinct regions corresponding to the different amplitudes of the excitation force. The same results as shown in Figure 1 can be seen, i.e., the cubic-velocity feedback can suppress the amplitudes of vibration, and the velocity feedback can allow a larger range of excitation. The best result as illustrated in both figures is obtained when K_C=0.5 & K_V=0.01.

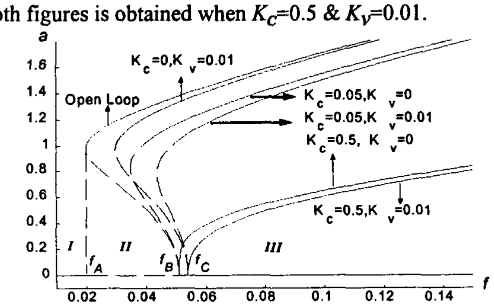

Figure 2. Force-response curves σ = -0.1, μ_1 = 0.01, λ_2 = 0.01, α = -0.05.

The numerical simulations of the system time response are shown in Figure 3 for the six cases described below:

(a) When $\Omega = 2$, the principal parametric resonance is inevitable. However, with cubic-velocity feedback alone, i.e., $K_C=0.05$ & $K_V=0$, the amplitude of resonance is *0.6* even when the excitatory amplitude is quite small (i.e., only *0.1*).

(b) When $\Omega = 1.93$, the resonance can be suppressed. However, with the same cubic-velocity feedback as (a), an ineffective vibration suppression with a large settling time is observed.

(c) When $\Omega = 1.93$, the resonance is effectively suppressed as a consequence of the combined control of the cubic-velocity and velocity feedback, i.e., $K_C=0.05$ & $K_V=0.01$.

(d) When $\Omega = 1.96$, the resonance can not be suppressed by cubic-velocity feedback alone, even when a large feedback gain is used ($K_C=0.5$ & $K_V=0$). This is because the resulting system has a high-amplitude limit-cycle.

(e) When $\Omega = 1.96$, the resonance is fully suppressed as a consequence of the combined control of the cubic-velocity and velocity feedback, i.e., $K_C=0.5$ & $K_V=0.02$.

(f) When $\Omega = 1.99$, the excitation frequency is even closer to the resonance point, the bifurcation is fully suppressed as a consequence of the combined control of the cubic-velocity and velocity feedback with large gains, i.e., $K_C=0.5$ & $K_V=0.04$.

5 Conclusion

From Figures 1 and 2, it can be seen that: (i) by increasing the velocity-feedback gain, the range of bifurcation can be eliminated, and (ii) by increasing the cubic-velocity-feedback gain, the amplitude of vibration can be suppressed. From the simulation results, it is evident that the combination of the cubic and linear velocity feedback is effective for suppressing the principal parametric resonance. The effectiveness of the combined method can be explained by the fact that the velocity feedback increases the viscous damping; whereas the cubic-velocity feedback compensates for the effects of nonlinear curvature and nonlinear inertia. Furthermore, if the gain of the velocity feedback is large enough, then the bifurcation can be fully controlled.

6 References

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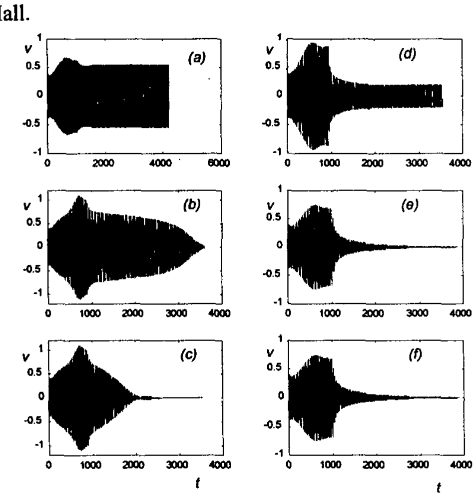

Figure 3. Numerical simulations of the time response, when f = 0.1, μ_1 = 0.01, λ_2 = 0.01, α = -0.05.