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RECURRENCE PLOT FEATURES: AN EXAMPLE USING ECG

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ABSTRACT

Electrocardiogram (ECG) signals are analysed using the nonlinear method of recurrence plots, which reveals subtle time correlations in time-domain signals. Large-scale features in the recurrence plots, which consist entirely of single dots, line segments of different orientations and white spaces, are directly related to time-domain features in the original signals. The relationship between recurrence plot features and time-domain features is easy to see for these ECG signals, and can be used to infer time-domain features of other signals (such as other bioelectric signals) that are more difficult to interpret due to their complexity.

1. INTRODUCTION

The aim of this study was to demonstrate how features that can be clearly observed in a time-domain signal give rise to particular features in recurrence plots of the data. Originally proposed in the physics literature, recurrence plots are a nonlinear dynamical analysis method describing subtle time correlation information about a signal [1]. They have since been applied to a diverse range of biomedical data, including bioelectric signals [2]–[5], interval analysis [4], [6] and motion analysis [7]. However, little attention has been paid to the way in which large scale features of recurrence plots relate to specific features of the analysed signals. In this study, recurrence plot analysis was applied to electrocardiogram (ECG) signals which have clear time-domain features.

2. RECURRENCE PLOTS

2.1 Formal Definition

Fundamental to the definition of recurrence plots is the idea that any scalar time series $s(n)$ can be considered as the projection of a multivariate signal $\mathbf{x}(n)$ onto the single dimension that we observe. Consider, for example, the ECG which originates in 3 dimensions, yet each lead is only a 1-dimensional signal. Not all variables represented by $\mathbf{x}(n)$ are necessarily observable. By use of Takens and Mañé's Embedding Theorem, we can create d-dimensional vectors $\mathbf{y}(n)$ from the original time series $s(n)$ so that the evolution in time of $\mathbf{y}(n)$ follows that of $\mathbf{x}(n)$, even though the dimensions of $\mathbf{x}(n)$ and $\mathbf{y}(n)$ may differ [8]. The form of these vectors is:

$$\mathbf{y}(n) = [s(n), s(n + T), \ldots, s(n + (d - 1)T)] \qquad \ldots(1)$$

Lag T is chosen as the value that gives the first minimum of the average mutual information (AMI) between $s(n)$ and $s(n + T)$ [9]. Embedding dimension d is the smallest dimension that gives no 'global false nearest neighbours'; that is, the distance between $\mathbf{y}(n)$ and its nearest neighbour in dimension d is not increased too much by extending both vectors into dimension $(d + 1)$ [8].

Two forms of recurrence plots, which we refer to as *direct* and *relative* forms, appear in the literature. Suppose that we have constructed N vectors $\mathbf{y}(n)$ from a given scalar time series. Then a *direct form* recurrence plot is a scatter plot of dots in an $N \times N$ square, where a dot at (i,j) indicates that $\mathbf{y}(i)$ is close to $\mathbf{y}(j)$ in d-dimensional space [1]. Direct form recurrence plots contain a line of identity, since $\mathbf{y}(i)$ must be close to $\mathbf{y}(j)$ if $i = j$. The test for whether two vectors are 'close' has been defined variously in terms of a required number of neighbours for each vector [1], or in terms of a threshold for the distance between vectors [3], [4] where the distance may be defined by different norms [6], [10]. If a fixed threshold is used, then a direct form recurrence plot is symmetrical about its line of identity. A *relative form* recurrence plot is a modified form where a dot at (i,j) indicates that $\mathbf{y}(i)$ is close to $\mathbf{y}(i + j)$ [3], [6]. In these plots, the vertical time index J is relative to the horizontal index I. Before interpreting any recurrence plot, care must be taken to ascertain whether it is in direct or relative form.

2.2 Alternative Interpretation

While recurrence plots are strictly defined as above, they can also be understood based on more traditional signal processing concepts. The axes of the plot can be thought of as the time indices of two sliding windows I and J, similar to the time index for short-time Fourier transforms. For direct form recurrence plots, each dot (i,j) indicates that windows I and J are similar, where the measure of similarity is whether vectors formed from the time-ordered windowed data are 'close'. If I is considered the current window, then above the line of identity, J represents future windows; below, it represents past windows. In relative form plots, J only represents future windows. The data may be oversampled for the purposes of generating a recurrence plot; that is, the mutual information between successive samples may be too high. This can be remedied by downsampling the data within each window by a factor T, where T is a lag value corresponding to the first minimum of the average mutual information between $s(n)$ and $s(n + T)$ [9]. A global false nearest neighbours test [8] is then used to

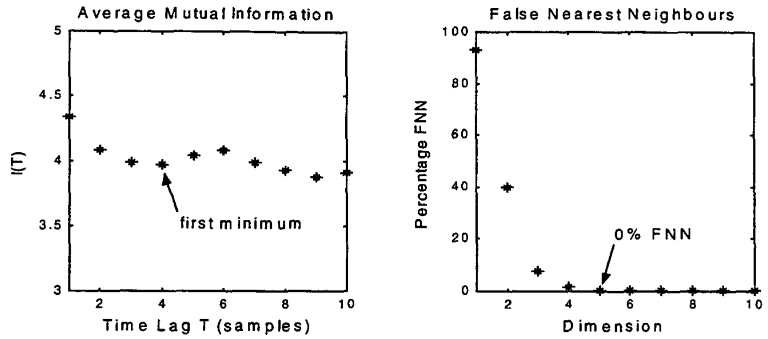

Figure 1. Plots used to determine appropriate parameters for calculating the recurrence plot for the normal ECG record in figure 2(a). The first minimum of the average mutual information between the original and lagged data occurs at a lag of 4 samples. Embedding the data using this lag, the percentage of false nearest neighbours becomes zero at an embedding dimension of 5. Similar results were found for the paced ECG.

determine how many samples d should be contained in each (downsampled) window. This number d is such that if windows I and J each contain d samples and are similar, then they will also be similar if they contain $(d+1)$ samples.

3. MATERIALS AND METHODS

ECG records of several beats were obtained from the MIT-BIH Arrhythmia Database (Massachusetts Institute of Technology, 1992). Three successive beats each from a noisy, normal ECG and a rhythm paced ECG are shown in figures 2(a) and 2(b) respectively. Programs were written using Matlab (version 5.2, The MathWorks, Inc., Natick, MA) to determine an appropriate time lag and embedding dimension for creating vectors from the data [8], and for generating direct form recurrence plots using these values and a threshold of 10% of the maximum Euclidean distance (L_2 norm) between vectors [3].

4. RESULTS

Plots of average mutual information and percentage of false nearest neighbours for the normal ECG are shown in figure 1. For both the normal ECG and the paced ECG, the lag corresponding to the first minimum of the average mutual information was found to be 4 samples and the embedding dimension was found to be 5. These values were used to generate the recurrence plots shown in figures 2(c) and 2(d) respectively.

5. DISCUSSION

The following is a discussion of the qualitative features of direct form recurrence plots. See [3] for features of relative form plots, and [4] for a discussion of possible quantitative measures that could also be obtained.

Each recurrence plot in figure 2 can be divided roughly into nine regions or sub-plots *a*–*i*:

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

With respect to the signals in figures 2(a) and 2(b), regions *g*, *e* and *c* correspond to the first, second and third beats respectively; regions *d* and *h* both compare the first and second beats; regions *a* and *i* both compare the first and third beats; and regions *b* and *f* both compare the second and third beats.

The plots consist of isolated dots, dots joined into line segments (two or more adjacent dots) and white spaces [4]. Care must be taken to analyse recurrence plots at an adequate screen or print resolution, otherwise some of these features may be lost. Little can be concluded from isolated dots, as they are due to chance recurrences such as in white noise. Line segments and white spaces, however, can be clearly related to features of the signal.

In region *e* of figure 2(c), horizontal line segments at $J = 415$, reflected as vertical line segments at $I = 415$, correspond to the baseline crossing between the R and S waves in beat 2 of the normal ECG. The horizontal line segments indicate that several successive overlapping windows $\{I_1, I_2, \ldots, I_n\}$ (baseline activity between beats 2 and 3) are all similar to a separate window J (baseline crossing between R and S), and so $\{I_1, I_2, \ldots, I_n\}$ are also similar to each other. This may indicate slow-changing regions of a signal. Since vertical line segments are reflections of horizontal line segments about the line of identity, they can be interpreted similarly; a vertical line segment indicates that several successive overlapping windows $\{J_1, J_2, \ldots, J_n\}$ are similar to a separate window I.

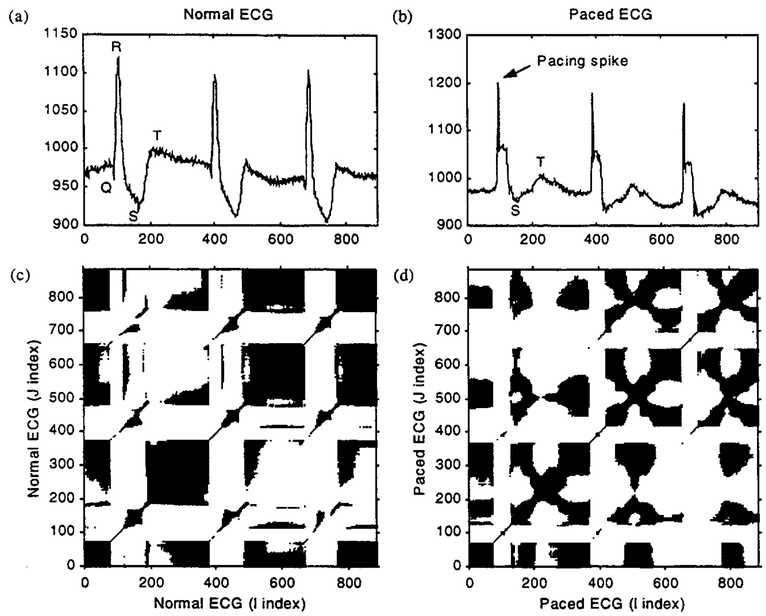

Figure 2. (a) Normal, noisy ECG signal. (b) Paced ECG signal. Abscissa units are sample numbers, ordinate units are arbitrary. (c) Recurrence plot of the normal ECG signal. (d) Recurrence plot of the paced ECG signal. Lag = 4 samples, embedding dimension = 5 and threshold = 39 units for both plots.

Each pair of reflected horizontal and vertical line segments corresponds to a pair of line segments that appear to split away from the line of identity, although these may be obscured by larger 'shapes'.

Upwards diagonal line segments with a slope of 1 can be seen clearly in regions *a*, *b*, *d*, *f*, *h* and *i* of figure 2(c), and in regions *b* and *f* of figure 2(d). These indicate that while successive windows may not be close, the closeness between two windows from different parts of the signal is maintained as the windows slide simultaneously along the signal. They therefore show where the same dynamics occur in different parts of the signal so are a particularly important feature of recurrence plots. The upwards diagonals identified above indicate the recurring waveform. Similar shapes located along regularly spaced, separate diagonals can indicate a periodic or, as in this case, a quasi-periodic component to the signal.

The larger shapes evident in both recurrence plots can be thought of as consisting of line segments of any orientation stacked on top of each other. However, the overall shape may suggest an orientation of line segment that is better than others for describing the signal. Squarish shapes suggest horizontal and vertical line segments which indicate slow-changing parts of the signal. Examples are baseline activity between beats and sections of the paced ECG immediately following the pacing spike; see *I*, *J* = 100, 400 and 680 in figure 2(c). The main parts of the arrowhead-like shapes along the line of identity in both plots suggest upwards diagonals, indicating that nearby windows are changing similarly, as is certainly the case during the corresponding S waves of both signals. However, the diagonals suggested here seem to have slopes other than 1. Where the slope is steeper, the part of the signal corresponding to the *I* index is changing faster than the section corresponding to the *J* index; where the slope is more gradual, the part of the signal corresponding to the *I* index is changing slower than the section corresponding to the *J* index. In the normal ECG for example, the descending part of the S wave has a steeper slope in beat 2 than in beat 1 and this is shown by the arrowhead shapes in regions *d* and *h* of figure 2(c). The 'barbs' of these shapes suggest downwards diagonal line segments (negative slope). These correspond to parts of the signal that have opposite (time-reversed) dynamics within the same amplitude range, such as either side of the minimum of the S waves. Similarly, the broad downwards diagonals in regions *c*, *e* and *g* of figure 2(d) indicate the rising and falling parts of the T waves of the paced ECG. The apparent curve of these diagonals is due to the T wave rising faster than it falls. Similar to upward diagonals, we observe for downwards diagonals that where the slope is steeper, the part of the signal corresponding to the *I* index is changing faster than the section corresponding to the J index; where the slope is more gradual, the part of the signal corresponding to the I index is changing slower than the section corresponding to the J index.

White spaces in the recurrence plots are also important. Bands due to the high amplitude transients (R wave and pacing spike) run horizontally and vertically through both plots. In figure 2(c), there is an additional band due to the S wave that mostly merges with the first band, although they are sometimes separated such as in region a at $I = 120$. In contrast, the lower magnitude S wave of the paced signal results in irregular-shaped white areas, such as that at $(I,J) = (140,50)$ in figure 2(d). The T wave is partly obscured by noise in the normal ECG but is more prominent in the paced ECG. The irregular white spaces due to the T wave are therefore more prominent in figure 2(d), contributing to the large X shapes.

White space can also indicate general nonstationarities such as baseline drift in the signal. Regions c, e and g appear similar within both figures 2(c) and (d), indicating that the waveforms for the different beats are similar within both signals. If the beats were identical then all 9 regions of the recurrence plot would also be identical. But slight differences and baseline drift cause windows of the signal that would otherwise appear similar to become further apart, resulting in 'paling' [1] or increased white space away from the line of identity. A negative drift is evident in both signals, but more so in the paced ECG. This shows as a complete loss of the distinctive shape away from the line of identity in regions a and i of figure 2(d).

Each of the large scale features of recurrence plots described above is related quite simply to features of the ECG signals. Similar features may be identified in the more complex recurrence plots of signals such as electroencephalograms [2] and electromyograms [5]. Since recurrence plot features relate directly to features of time-domain signals, recurrence plots have the potential to reveal time-domain features in complex signals.

6. SUMMARY

Features of direct form recurrence plots are:

- Isolated dots indicate chance recurrences.
- Horizontal and vertical line segments indicate regions of the signal that change little and are similar to another separate window of the signal.
- Upward line segments with a slope of 1 indicate that the same dynamic occurs in separate parts of the signal. A steeper or more gradual slope indicates that the part of the signal corresponding to the I index is changing faster or slower (respectively) than the section corresponding to the J index.
- Downward line segments with a slope of -1 indicate that opposite dynamics occur in separate parts of the signal. A steeper or more gradual slope indicates that the part of the signal corresponding to the I index is changing faster or slower (respectively) than the section corresponding to the J index.
- Larger shapes are stacks of the above line segments.
- White bands and irregular white areas indicate transients in the time-domain signal.

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