Radio Direction Finding for Maritime Search and Rescue

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Abstract

Two Unmanned Aerial Vehicles (UAVs) are required to respond to a maritime emergency beacon, localise it, and then circle around it in an optimal configuration. Each UAV is able to measure bearing to the beacon but not its range. Novel guidance and control strategies based on cost function gradient search techniques are developed to achieve a continuous reduction in the estimate of the beacon location. Simulation studies reveal the localisation and circling behaviour achieved with the various cost functions, and a new minimum estimation error configuration is discovered. A direct cost function based on an 'area-of-uncertainty' metric achieved the best localisation considering flight and localisation time.

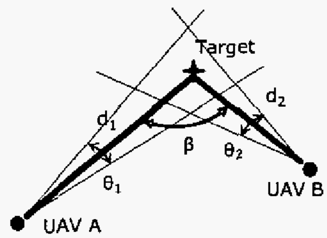
1 Introduction

A proposed Maritime Search and Rescue System comprises a low-cost radio beacon and two recoverable UAVs equipped with inexpensive radio direction finding equipment. This Honours student project involved the investigation, design, and simulation of guidance strategies for the UAVs which result in a continuous reduction in the estimate of the radio beacon location until they are within close range of the transmitter, at which point the UAVs will circle the transmitter in a minimal estimation error configuration.

The following assumptions are employed. The search area is large relative to the initial UAV separation. The UAV positions and velocities are known exactly and are communicated with no delay. Angle measurement noise is assumed to be small, Gaussian, and zero mean. For simplicity, the problem is limited to 2D, and the UAVs move with constant speed.

The angle noise contained within the UAVs' beacon bearing measurements causes an area of uncertainty [1] in the target localisation estimate. For small noise angles and large distances between the observers (i.e. the UAVs) and the target, the pairs of opposite lines bounding the area of uncertainty approach parallel. This area is derived as given in formula (1) and minimised by reducing the distances between the observers and target, and setting β to 90°. A stand-off distance, d_0, is required to allow a clear overhead space and permit a reasonable radius for circling around the beacon. A minimal estimation error configuration is circling d_0 from the target with $\beta = 90°$.

The aim of the work is to find a control strategy, which results in trajectories that are acceptable.



$$A = \frac{\theta_1 \theta_2 d_1 d_2}{\sin \beta} \quad (1)$$

Figure 1: Problem geometry

2 Literature Review

Although there is no directly related work reported in the literature, some studies relating to aspects of the problem are presented here for completeness.

Various aspects of bearings-only tracking and localisation have been thoroughly researched. Localisation using passive bearings-only information was pioneered in 1947 by Stansfield [2], for the purpose of cross-fixing a target through interception of radio transmissions by a distributed set of receivers.

Most work on bearings-only target localisation focus on a single observer, where the problem is to determine a target's position (or additionally, velocity) from a series of noisy angle measurements and employing various forms of optimal estimation [3-7]. The case of triangulating a single stationary target scenario from two or more stationary receivers is analysed in [8-9]. In [10], a technique is proposed for tracking multiple targets with multiple noisy bearings-only sensors. An inverse problem for estimating the position of hydrophone sensors in a towed submarine sonar array is estimated based on angular information [11].

Recent studies [12-16] also examine the issue of cooperative control between multiple agents, typically UAVs. To date, however, no literature has been found to discuss the cooperative control of two autonomous moving observers for locating and tracking a stationary or moving target with bearings only information.

3 Methodology

The problem is broken down into four phases: (i) before detection, (ii) one observer detects beacon, (iii) both observers detect beacon, and (iv) terminal phase when observers are in close proximity to the target. This scheme allows flexibility when measures of performance and potential strategies vary during flight, but demands careful definition of the handover conditions for robustness.

Initial phase strategies might include tail chase or grid search patterns. The development of cost-function-based guidance strategy employed in phases (iii) and (iv) forms the focus of this work.

It is assumed that the UAV direction can be controlled directly by setting the lateral acceleration, within the maximum constraint. Given the UAV heading, a simple feedback controller can be implemented.

The controller can use the current bearing, and desired bearing, and generate the required corrective lateral acceleration. We start with the most basic proportional control.

The approach chosen is to have both UAVs making independent decisions about the best direction to fly in. Each UAV makes a decision, implements that decision, and flies until the next decision time.

The optimal heading decision can be made by searching for the heading that would result in the maximum reduction in the value of the chosen cost function. For the purposes of this optimisation, trial headings are defined counter-clockwise from the line joining the observer to the target as α_1 and α_2 for the first and second observers, respectively. The change in cost function due to the selection of α_1 and α_2 is derived as follows:

$$\mathrm{dV} = \frac{\delta V}{\delta d_1}\Delta d_1 + \frac{\delta V}{\delta d_2}\Delta d_2 + \frac{\delta V}{\delta \beta}\Delta\beta \quad (2)$$

where V is the cost function,

$$\Delta d_1 = -v\Delta t\cos\alpha_1 \quad (3),$$

$$\Delta d_2 = -v\Delta t\cos\alpha_2 \quad (4), \text{ and}$$

$$\Delta\beta = \frac{v\Delta t\sin\alpha_1}{d_1} - \frac{v\Delta t\sin\alpha_2}{d_2} \quad (5)$$

We now need to find the values of α_1 and α_2 that minimise dV. A steepest-descent method should give a rapid reduction in cost function value, and in turn reduce the area of uncertainty. Because dV can be split into terms dependent on the decisions of the first observer (α_1) and second observer (α_2), the two UAVs can perform their respective optimisations independently.

The area of uncertainty depends on d_1, d_2 and β so it makes sense that if we wish to minimise the area of uncertainty, we should aim to minimise d_1, d_2 and drive the angle β to 90°. A cost function based on these principles is:

$$F = K_1 d_1^2 + K_2 d_2^2 + K_3(\beta - \frac{\pi}{2})^2 \quad (6)$$

To keep the UAVs away from the target, two penalty terms are added which penalise any radius greater or less than d_0 (formula (7)).

If the distances are expressed in meters, and angle β is measured in radians, the cost function is dominated by the relatively large distance terms. As a result, when angles α_1 and α_2 are selected through optimisation, the guidance will favour a reduction in radius over a correction in β (by a factor of $> 10^7$).

A further resulting problem is that the effect of varying the constants is hidden. Functions which normalise the relative weighting between terms were also investigated ('nondimensional functions', formulae (10-11, 13-14)).

Another approach is to use the area of uncertainty directly for the cost function (formulae (8, 12-14)).

The various functions examined are shown in Table 1. The simulation of these cost functions is discussed in the next section.

Table 1: Function definitions

Title	Function
Classic indirect (7)	$F_7 = K_1 d_1^2 + K_2 d_2^2 + K_3(\beta - \frac{\pi}{2})^2 + K_4(d_1 - d_0)^2 + K_5(d_2 - d_0)^2$
Classic direct (8)	$F_8 = \frac{K_1 d_1 K_2 d_2}{K_3 \sin\beta}$
Alternate penalty indirect (9)	$F_9 = K_1 d_1^2 + K_2 d_2^2 + K_3(\beta - \frac{\pi}{2})^2 + K_4(\frac{d_0}{d_1})^2 + K_5(\frac{d_0}{d_2})^2$
Nondimens. Indirect (10)	$F_{10} = K_1(\frac{d_1}{d_0})^2 + K_2(\frac{d_2}{d_0})^2 + K_3(\beta - \frac{\pi}{2})^2 + K_4(\frac{d_1}{d_0})^2 + K_5(\frac{d_2}{d_0})^2$
Angle range bias indirect (11)	$F_{11} = K_1(\frac{d_1}{d_0})^2 + K_2(\frac{d_2}{d_0})^2 + K_3(\frac{d_1 d_2}{d_0^2})^2(\beta - \frac{\pi}{2}) + K_4(\frac{d_1}{d_0} - 1)^2 + K_5(\frac{d_2}{d_0} - 1)^2$
Direct with penalty (12)	$F_{12} = \frac{K_1 d_1 K_2 d_2}{K_3 \sin\beta} + K_4(d_1 - d_0)^2 + K_5(d_2 - d_0)^2$
Direct with alternate penalty (13)	$F_{13} = \frac{K_1 d_1 K_2 d_2}{K_3 d_0^2 \sin\beta} + K_4(\frac{d_1}{d_0})^2 + K_5(\frac{d_2}{d_0})^2$
Direct with nondimens. Penalty (14)	$F_{14} = \frac{K_1 d_1 K_2 d_2}{K_3 d_0^2 \sin\beta} + K_4(\frac{d_1}{d_0} - 1)^2 + K_5(\frac{d_2}{d_0} - 1)^2$

4 Simulator Design

Design requirements for the software simulator were considered. The core requirement was to simulate two observers travelling towards a target. There must be the ability to apply guidance and control strategies, and measure the results in some reasonable way. Additional goals were identified.

Although the speed of each observer is fixed throughout the flight, it may be important to be able to change the speed before running the simulation. In fact, all quantities that might be varied for experimentation should be able to be modified easily. This is achieved through the use of software variables. Variables were defined together towards the start of the software code allowing them to be quickly located and changed.

Other potential inputs identified include a sampling period, experiment time, noise type / bias / standard deviation, and performance / cost weightings. The simulator terminates after a programmed number of samples are completed in phase (iv).

All warnings and errors produced were displayed to assist in identifying bugs. Algorithms for control and guidance were made flexible using functions and variables appropriately to allow parts of the code to be rewritten if required. The calculation identifying the ideal angle to travel in was separated from the application of that information to control each UAV. An allowance is made for noise to be added to the measured angle i.e. even though the target position is known to the simulator it is recalculated from angle data.

The simulation is designed to complete in a few minutes with the resources of a standard desktop PC. The control strategy itself should be one that can be implemented in a basic microcontroller and calculated at a reasonable control frequency.

MATLAB® was selected for the simulator. To produce trajectory plots, positions of the observers must be calculated over the period of the flight, taking into account the control applied and the equations of motion that the observers operate under. There are at least two approaches: model the system in MATLAB's system analysis tool, Simulink, and let it perform numerical integration, or use MATLAB code to directly calculate the aircraft position at a set of discrete points.

Each subsequent position and velocity was calculated by taking the previous position and velocity, and combining it with the lateral acceleration applied as a result of the control strategy.

The guidance frequency and 'sampling frequency' (how often new positions and velocities are calculated) were kept separate for flexibility.

At each control interval, the guidance algorithm must search for the current ideal angle to travel in, in order to achieve the maximum reduction in the cost function value. The Optimisation Toolbox function 'fminbnd' was selected as it returns the minimum of a function of one variable over a fixed interval (the solution for α_1, α_2 is limited between $0 <= \alpha_{1,2} <= 2\pi$).

Trajectory plots are useful to see the path taken by the two observers, but have disadvantages. For example, it is difficult to see the synchronisation between observer positions. When the observers circle around the target, it is difficult to tell the direction of flight and distinguish between circuits. An animation was developed to give an idea of what the UAVs were doing 'live'.

In addition to the animation a range of other outputs are produced: command window output for diagnostics/flight time/minimum stand-off distance etc, and figures. Figures produced include trajectory plot, cost function and area of uncertainty against time, angular acceleration and angular quantity diagnostic graphs.

A GUI was developed to make launching the simulator more 'user-friendly' and to reduce the time taken between simulation runs to change observer / target positions, cost functions, and weighting constants. A batch mode was also created so that the simulation could be run a number of times by computer, with variations. For example, the batch mode could be used to run the simulator with a hundred different constant values, or run the simulator automatically with four different cost functions to produce a graph comparing them.

Basic steps were taken to verify the operation of the simulator. The maximum lateral acceleration was set, and trajectories examined to make sure they corresponded with the theoretical turning radius. Left and right turns were set to ensure the conventions for error angle and lateral acceleration carried through to the trajectory. The simulator was executed with starting points rotated through all four quadrants, and results compared to check for angle calculation or angle range errors. Several random starting positions for each observer were checked for any unusual behaviour.

An integration of the uncertainty area is performed as a measure of uncertainty across the flight in. In general, a curve sitting completely above another will produce a higher integration sum. However, it is less useful when the two curves cross over one or more times: then we must refer to a graph comparing what happens to the uncertainty over time and decide which is more desirable (for example, decide between rapid reduction to start with followed by more gradual reduction).

5 Simulation Results

5.1 Inward Flight

As expected, it is found that travelling directly towards the target reduces the area of uncertainty continuously. By simulating the 'classic indirect' function with K_3, K_4 and K_5 set to zero (no importance placed on angle or stand-off) the observers fly directly to the target.

The effect of angle importance to the indirect functions is examined by varying K_3 (see Figure 2). A trade-off relationship emerges: travel straight to the target allows the quickest path, but sharper continuous declines in uncertainty are achieved with separation. In practice, a point would be selected at which both the area of uncertainty and time of flight are acceptable.

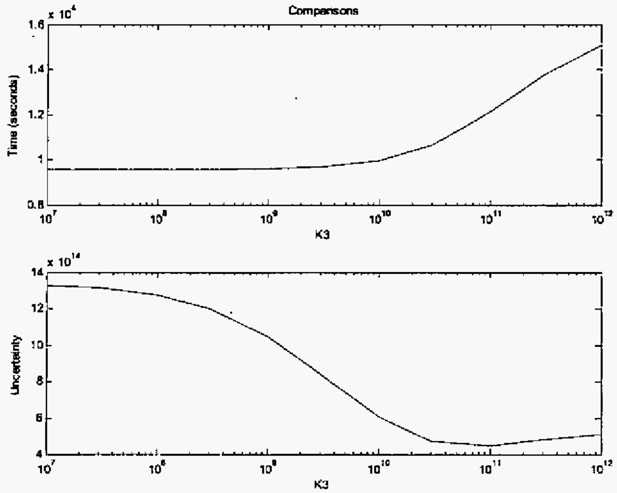


Figure 2: Flight time and uncertainty integral for varying K_3

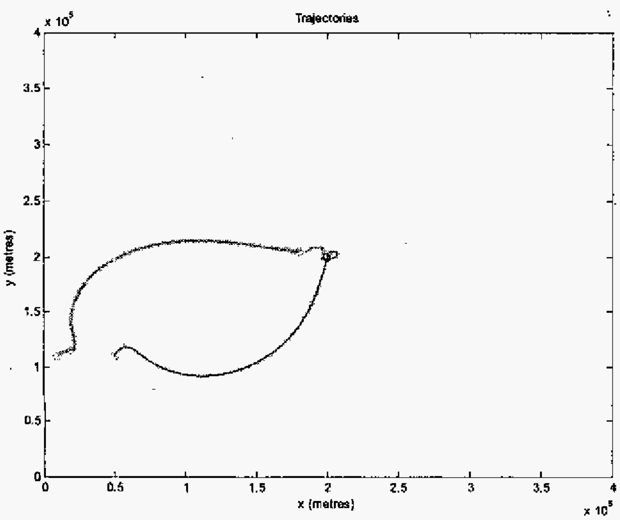


Figure 3: Classic Direct trajectories

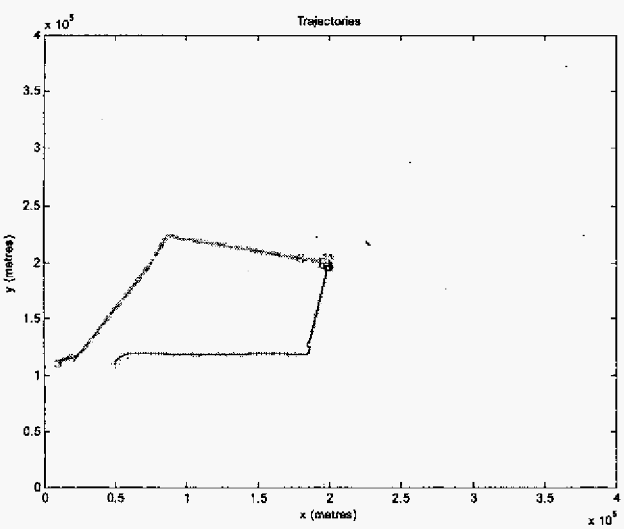


Figure 4: Angle Range Bias Indirect trajectories

The direct functions, as well as the angle range bias indirect function, place a greater importance on angle in their default configuration and the observers are seen to separate as part of their natural flight (Figures 3, 4).

Other functions were compared, with differing uncertainty integrations and flight times. The area of uncertainty variation for these functions is plotted in Figure 5.

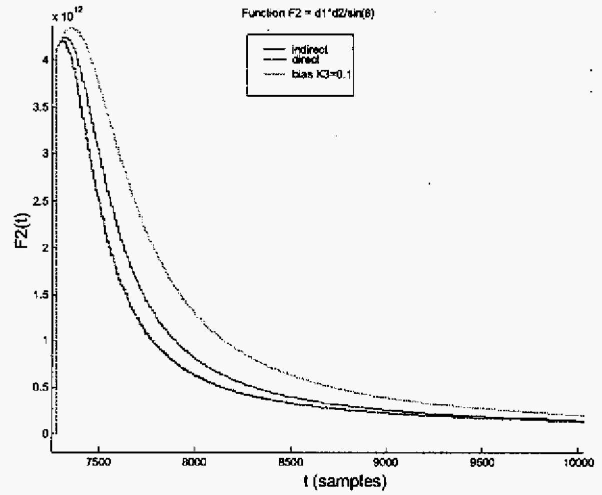


Figure 5: Area of Uncertainty for three cost functions

The optimum-in-time flight is trivial (i.e., fly directly to the target) when compared to the optimum-in-area trajectory. The optimum-in-area trajectory could be defined as the one with the lowest area of uncertainty integral.

It was thought that the direct cost function should produce this ideal trajectory, but when areas of uncertainty are plotted on the same graph, it does not generate the most rapid decline in area. It is theorised that this is because the controller does not achieve the flight requested of it by the guidance algorithm. The guidance strategy produces an ideal angle variation, and the controller turns in the direction of the variation. If the turn is insufficient to achieve that angle variation, the control is 'weak' and the trajectory will only approach that requested by the guidance algorithm.

To test this theory, the controller's proportional constant is varied temporarily from 0.1 to values of 0.2, 0.5, 1.0, 2.0 and 5.0. This implies that the UAVs will make sharper turns proportional to the guidance error.

As the constant is increased, the 'classic direct' function produces progressively smaller integrals for the area-of-uncertainty function thus putting it ahead of all tested functions.

While it is unlikely that this trajectory is ideal, it is at least the closest to it. A study of the proportional constant and resulting error angle variations during flight (plotted by the simulator) could further improve the results.

In summary, even the basic 'classic indirect' function produces continuous reductions in the area of uncertainty, thus meeting one of the project goals. Increasing the angle weighting (either by increasing the indirect K_3 constant, adding an angle range bias term, or using the 'direct' family of functions) causes the observers to separate, which while lengthening the flight time, decreases the uncertainty more rapidly.

During testing, various starting position were tried to determine whether the cost-functions performance comparisons is valid irrespective of starting locations. It is found that the cost-functions performance ranking for flight times and uncertainty integrals remained constant for corresponding starting positions

Based on empirical evidence alone (across a small test set), it therefore appears that varying the starting position does not affect the general behaviour of the observers during the flight in.

5.2 Unintentional Terminal Behaviour

It is not just the flight in behaviour that is important. To meet the project goals, the observers must circle around the target in a minimum error configuration. By testing various combinations of different starting positions, constant values and cost functions, a collection of behaviours was built up.

Regardless of starting position, most simulations across all cost functions with unity weighting do not result in the intended terminal behaviour. The 'classic indirect' function appears to produce circling observers in the final phase, but the observers circle the target in opposite directions (the angle β varies through its full range, and as a result the area of uncertainty varies significantly).

The 'opposite direction' circling is due to the order of the angle weighting term in the cost function. Other terms are measured in metres, resulting in distance being much more sensitive to angle choice optimisation than the separation term. The obvious solution of increasing K_3 (or equivalently nondimensionalising the function through division of distance terms) results in two observers obsessed with maintaining separation at the expense of circling as shown in Figure 6.

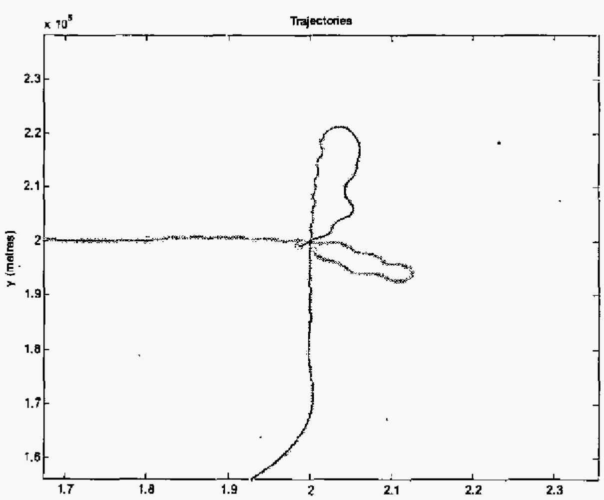


Figure 6: Undesirable behaviour from K_3 change

Other behaviours were less expected: for example, the discovery that localisation can be achieved through circling on the spot (Figure 7). This was an interesting unpredicted result, and was produced by several different functions with different circling radii.

In some cases, circling on the spot could be more desirable than circling around the target. It may be an alternative required behaviour, especially if a threat or obstruction existed on the opposite side of the target (e.g. a natural hazard in search and rescue). Further cost function terms could be added or alternate guidance implemented to choose the correct side to circle on.

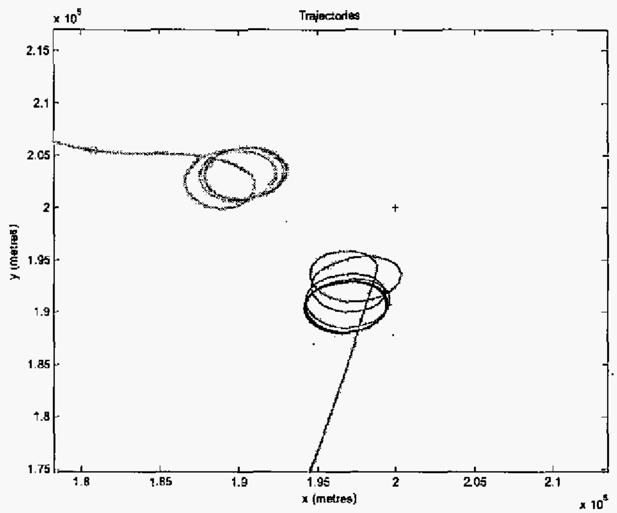


Figure 7: On-spot circling

Although circling on the spot still maintains localisation, it is harder to maintain a constant area of uncertainty in this pattern. Even if the circling between the two observers could be synchronised, the varying distance between the observers and target would produce a varying localisation error.

The undesirable terminal behaviours result from a combination of the cost function 'valleys' that the UAVs operate in (operating around local minima, not necessarily global minima) and the dynamics of the controller implemented.

5.3 Intentional Terminal Behaviour

It was found that by tuning the constant values, the UAVs could be induced to eventually fly in a circle, with a 90° separation relative to the target: the minimum estimation error configuration set by the problem statement. The constants were found using trial and error. It was not only possible to tune one of the functions to produce this 'terminal circling' – four nondimensionalised cost functions were tuned to produce desirable terminal behaviour. This set of functions is named the 'tuned functions'.

The lateral acceleration graph was analysed for a variety of cost functions and constant values. The maximum lateral acceleration constraint (1 ms^{-2}) was rarely reached. It could be that only gentle turns are required in the observer movement – or that the lateral acceleration constant is insufficient and that quicker system response could be attained.

Increasing the lateral acceleration results in different trajectory plots and terminal behaviour. Further study is required to investigate this behaviour, as well as to examine what happens when the lateral acceleration requested exceeds the maximum permitted thus resulting in actuator saturation.

The controller performance can be evaluated by analysing the cost functions. For each cost function, a theoretical minimum value can be calculated (the cost functions used in this project are always bounded below). We can look at whether the minimum is achieved, and how stable the cost function value is in the terminal phase. The following is a derivation of the minimum cost function value for the 'direct with nondimensionalised penalty' function (formula (14)) with tuned constants $K_4 = 100$, $K_5 = 10$, and $K_0=K_1K_2/K_3= 1$.

$$dF_{14} = \Delta d_1\Big[\frac{d_2 K_0}{d_0^2 \sin\beta} + \frac{2K_4}{d_0}\Big(\frac{d_1}{d_0}-1\Big)\Big] + \Delta d_2\Big[\frac{d_1 K_0}{d_0^2 \sin\beta} + \frac{2K_5}{d_0}\Big(\frac{d_2}{d_0}-1\Big)\Big] + \Delta\beta\Big[\frac{-d_1 d_2 \cos\beta}{d_0^2 \sin^2\beta}\Big] \quad (15)$$

For minimum dF_8, $\beta = 90°$ and

$$\frac{d_1}{d_0} = \frac{2K_0K_5 - 4K_4K_5}{K_0^2 - 4K_4K_5} \quad (16) \qquad \frac{d_2}{d_0} = \frac{2K_0K_4 - 4K_4K_5}{K_0^2 - 4K_4K_5} \quad (17)$$

Substituting into F_{14}, the theoretical minimum cost function value is 0.9727.

The graph of cost function value from the simulation is shown in Figure 8 and Figure 9. The cost function value during simulation decreases from 31,600 to reach 0.976 (0.3% above minimum) and varies up to 1.08 (11% above minimum).

Similar analysis can be performed for the other cost functions and constant values.

By comparing the functions, it can be seen that untuned functions, i.e., those with default constant values, achieved localisation, however, the circling on the spot behaviour during the terminal phase leads to large variations in the area of uncertainty. Tuned functions which circle the target are more likely to produce a stable area of uncertainty, and less variation in the angle β.

Tuning the functions allows the UAVs to reach the target quicker, due to the reduced relative weighting of β. There is however a corresponding small degradation of the initial localisation performance.

All four normalised functions ultimately produced good median localisations, regardless of whether or not they are tuned.

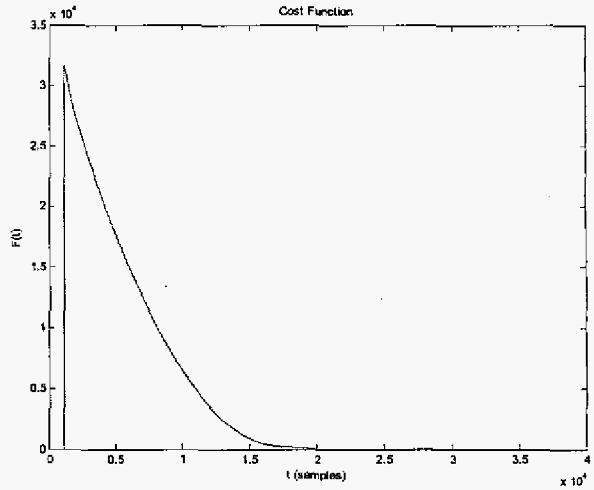


Figure 8: Cost Function for F_{14}

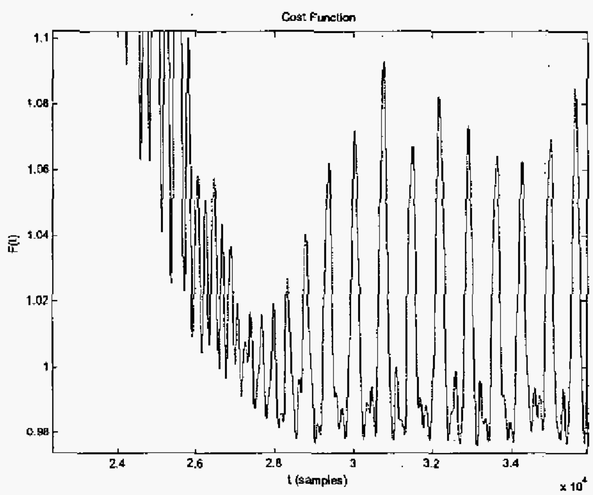


Figure 9: Magnified section of Figure 8

6 Conclusions

The aim of the project was to design and simulate guidance strategies for two UAVs. A simulator was implemented in MATLAB, and optimal control techniques used to fly the two UAVs towards a radio beacon. The guidance strategy is based on cost function gradient search techniques that adjust each vehicle's lateral acceleration. The simulator produces animation, trajectory plots and diagnostic graphs of function behaviour over time.

The guidance and control strategies are designed to achieve a continuous reduction in the estimate of the radio beacon location. Once it was demonstrated that localisation could be attained with the simplest cost function, comparisons were made with other cost functions and varied weightings within each cost function.

Successful localisation and terminal circling behaviour are achieved with a variety of cost functions and a new minimum estimation error configuration is identified. This unexpected 'circling on the spot' behaviour results in larger variations in the angle β between the UAVs and the target and larger variations in the area of uncertainty. There may be multiple desirable stable terminal behaviours depending on the situation at the target.

The direct cost functions provide the best localisation when graphs of the area over time are compared, but it depends on the objectives set: whether it is preferable to reach the target quickly, to conserve fuel, and so on.

Nondimensionalising (which normalises the weightings between cost function terms) should be applied to all cost functions. When the terms are around the same order, the effects of varying the weight are more prominent. It avoids the situation where angular quantities (radians) are almost ignored by the optimisation process because of their insignificance relative to the distance values (in the order of 10^5 m).

The approach of using a cost function allows flexibility in design (through weight selection and the ability to add further goals easily) but the selection of weights between terms is non-trivial. Weights were found through sensible selection, increasing the weighting of the standoff term and penalising any deviation away from the circular path around the target. Tuning the constants in this way has the effect of reducing time and increasing uncertainty over the inward flight: by using different cost functions in different phases this side-effect could be removed.

The objective was to develop an autonomous strategy that results in continuous reduction in uncertainty error and eventual circling around the target. This objective was achieved.

While there is some way to go before the techniques developed in the project could be safely applied to a real life search and rescue scenario, they make a small contribution to making UAVs 'smarter'. Autonomous UAVs that can search for and localise a target have many potential advantages.

7 Future Work

Following on from the advances made in this project, there is still substantial work to be done on this two-observer control problem. While most of the suggested work is in varying the simulation, a useful direction would be to look at the problem analytically (to find a way to describe the ideal trajectories and control input as continuous functions).

Currently, the lateral acceleration is controlled proportionally. Steady-state oscillations may be able to be reduced or removed by a controller with proportional, integral and derivative terms (PID). Bang-bang or 'on-off control' is another controller that could be implemented.

Without changing the controller's structure, its proportional constant could be varied. The maximum lateral acceleration is rarely reached, so increasing the lateral acceleration constant should give a quicker response at the expense of overshoot and oscillation.

Exactly the same control is applied to achieve different goals in Phases (iii) and (iv) – as a result, the performance of both is worse than their respective optimum. Guidance and control during the flight in could focus on reducing the area of uncertainty, and during the terminal phase focus on maintaining the area with a very accurate circling manoeuvre without oscillation. Handover between phases consequently becomes important.

More understanding could be gained of how varying the constants and starting positions affects the results: eg. How does the starting orientation affect the flight time to the target? The batch mode of the simulator tool could be useful to perform a Monte Carlo analysis: eg. Run 10,000 simulations with random starting positions.

The problem could be extended to a moving target. The goal would be not only to find the target location, but to discover its direction and speed of motion. The target could be moving with a constant velocity, performing some manoeuvre, or moving entirely randomly. The existing cost functions could be applied to test their performance with a moving target. One would expect that some knowledge of the target motion should be integrated into the functions, in order to 'intercept' [17] rather than 'chase' the target.

Other proposed work:

- Add communications latency.
- Add noise to the GPS readings.
- Extend to three dimensions.
- Model the system in Simulink software.
- Integrate a UAV simulator for aerodynamics.
- Allow the observers to vary their own speed.
- Implement a search routine in Phase (i).
- Add further penalties (control effort, fuel usage, etc.).
- Add noise to the received angle.
- Develop a position estimate in Phase (ii).

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