Wilsonian renormalization group equation for nuclear current operators

A. N. Kvinikhidze*

A. Razmadze Mathematical Institute, Georgian Academy of Sciences, Aleksidze Str.1, Tbilisi 0193, Georgia and Department of Physics, Flinders University, Bedford Park, South Australia 5042, Australia

B. Blankleider†

Department of Physics, Flinders University, Bedford Park, South Australia 5042, Australia

(Received 3 July 2007; revised manuscript received 7 November 2007; published 28 December 2007)

We present the solution to the recently derived Wilsonian renormalization group (RG) equation for nuclear current operators. To eliminate the present ambiguity in the RG equation itself, we introduce a new condition specifying the cutoff independence of the five-point Green function corresponding to the two-body propagator with current operator insertion. The resulting effective current operator is then shown to obey a modified Ward-Takahashi identity that differs from the usual one, but that nevertheless leads to current conservation.

DOI: 10.1103/PhysRevC.76.064003 PACS number(s): 25.10.+s, 05.10.Cc, 21.30.Fe, 21.60.−n

I. INTRODUCTION

The use of the Wilsonian renormalization group (RG) method [1–3] to impose a cutoff Λ on the momenta of virtual states is an important tool for studying various aspects of nuclear effective field theory (EFT) [4–9]. In this context it has mostly been used to study the strong interactions of nonrelativistic two-nucleon systems where the central starting point is the RG equation for the two-body effective potential V_Λ [4]. Recently, however, Nakamura and Ando (NA) [10] have extended the scope of such studies by deriving the RG equation for the two-body effective current operator O^μ_Λ. The main purpose of the present article is to present the unambiguous solution to this equation. As our solution differs from the one given by NA, we have endeavored to give a detailed account of both the solution and the RG equation itself. In particular, we present an *off-shell* cutoff-independence condition that leads *necessarily* to NA's RG equation. By contrast, NA derived their equation as only a *sufficient* condition for an *on-shell* cutoff-independence condition. In this way we eliminate the consequent ambiguity of NA's RG equation. Last, we examine the question of current conservation for the derived effective current operator O^μ_Λ. We find that even in the best case where O^μ_Λ is obtained (via the RG equation) from the full field-theoretic current operator O^μ_∞ that satisfies the usual Ward-Takahashi (WT) identity [11], O^μ_Λ will not obey this identity (it will instead satisfy a modified WT identity). Nevertheless, the same operator O^μ_Λ is shown to conserve current in matrix elements.

II. SOLUTION TO THE CURRENT OPERATOR RG EQUATION

We consider a nonrelativistic two-body system for which the RG method is used to introduce a momentum cutoff of Λ. For this purpose it is convenient to use the projection operators [10]

$$\eta = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\mathbf{k}\rangle\langle\mathbf{k}|\, \theta(\Lambda - k), \tag{1}$$

$$\lambda = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\mathbf{k}\rangle\langle\mathbf{k}|\, \theta(\bar\Lambda - k)\theta(k-\Lambda), \tag{2}$$

where $\bar\Lambda > \Lambda$. The RG equation for the reduced space effective potential V_Λ [4] can then be written as

$$\frac{\partial V_\Lambda}{\partial\Lambda} = V_\Lambda G_0 \frac{\partial\lambda}{\partial\Lambda} V_\Lambda, \tag{3}$$

where $G_0 = (E - H_0)^{-1}$ is the two-body free propagator and

$$\frac{\partial\lambda}{\partial\Lambda} = -\int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\mathbf{k}\rangle\langle\mathbf{k}|\, \delta(k-\Lambda). \tag{4}$$

Equation (3) can be derived from the reduced space Lippmann-Schwinger equation

$$T = V_\Lambda + V_\Lambda \eta G_0 T \tag{5}$$

by using the fact that the off-shell scattering amplitude, T, does not depend on Λ.

Although Eq. (3) has been used to study nuclear EFT [4–9], such investigations have been limited to the purely hadronic sector. However, in a recent work NA have extended the scope of such studies by deriving the corresponding RG equation for the reduced space effective current operator O^μ_Λ [10]. Writing this current operator as

$$O^\mu_\Lambda = \eta\Gamma^\mu_\Lambda\eta, \tag{6}$$

the RG equation derived by NA can be expressed as

$$\frac{\partial\Gamma^\mu_\Lambda}{\partial\Lambda} = V_\Lambda G_0 \frac{\partial\lambda}{\partial\Lambda}\Gamma^\mu_\Lambda + \Gamma^\mu_\Lambda \frac{\partial\lambda}{\partial\Lambda} G_0 V_\Lambda. \tag{7}$$

Here we provide the solution to Eq. (7), noting that the solution given in Refs. [10,12,13] differs from ours. We find, unambiguously, that

$$\begin{aligned} O^\mu_\Lambda &= \eta(1 - V_{\bar\Lambda}G_0\lambda)^{-1} O^\mu_{\bar\Lambda}(1 - \lambda G_0 V_{\bar\Lambda})^{-1}\eta \\ &= \eta[1 + V_{\bar\Lambda}\lambda G_0(1 - V_{\bar\Lambda}G_0\lambda)^{-1}] O^\mu_{\bar\Lambda} \end{aligned}$$

*sasha_kvinikhidze@hotmail.com
†boris.blankleider@flinders.edu.au

$$\times [1 + (1 - \lambda G_0 V_{\bar\Lambda})^{-1} G_0 \lambda V_{\bar\Lambda}]\eta$$
$$= \eta[1 + V_{\bar\Lambda}\lambda(E' - H_0 - V_{\bar\Lambda}\lambda)^{-1}]O^\mu_{\bar\Lambda}$$
$$\times [1 + (E - H_0 - \lambda V_{\bar\Lambda})^{-1}\lambda V_{\bar\Lambda}]\eta, \quad (8)$$

where O^μ_Λ is given at $\Lambda = \bar\Lambda$ by its starting value $O^\mu_{\bar\Lambda}$ and $V_{\bar\Lambda}$ is the two-body interaction defined in the model space with the cutoff $\bar\Lambda$.[1] The last line of Eq. (8) has been written in a form that is most easily compared with Refs. [10,12,13].

To prove Eq. (8), we show explicitly that the corresponding Γ^μ_Λ,

$$\Gamma^\mu_\Lambda = (1 - V_{\bar\Lambda} G_0 \lambda)^{-1} O^\mu_{\bar\Lambda} (1 - \lambda G_0 V_{\bar\Lambda})^{-1}, \quad (9)$$

satisfies Eq. (7).[2]

We first use Eqs. (A2) to write Eq. (9) as

$$\Gamma^\mu_\Lambda = (1 + V_\Lambda G_0 \lambda) O^\mu_{\bar\Lambda} (1 + \lambda G_0 V_\Lambda), \quad (10)$$

and therefore

$$\frac{\partial \Gamma^\mu_\Lambda}{\partial \Lambda} = \frac{\partial V_\Lambda G_0 \lambda}{\partial \Lambda} O^\mu_{\bar\Lambda}(1 + \lambda G_0 V_\Lambda) + (1 + V_\Lambda G_0 \lambda) \times O^\mu_{\bar\Lambda} \frac{\partial \lambda G_0 V_\Lambda}{\partial \Lambda}. \quad (11)$$

The use of the RG equation for V_Λ, Eq. (3), further gives

$$\frac{\partial V_\Lambda G_0 \lambda}{\partial \Lambda} = \frac{\partial V_\Lambda}{\partial \Lambda} G_0 \lambda + V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda}$$
$$= V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda} V_\Lambda G_0 \lambda + V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda}$$
$$= V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda}(V_\Lambda G_0 \lambda + 1). \quad (12)$$

Using this in Eq. (11) then gives the RG equation for the current operator, Eq. (7).

Our solution, Eq. (8), should be compared with the solution first given by NA in Ref. [10] and then used for RG analyses in Refs. [10,12,13]:

$$O^\mu_\Lambda = \eta[1 + V_{\bar\Lambda}\lambda G_0(1 - V_{\bar\Lambda} G_0)^{-1}\lambda] O^\mu_{\bar\Lambda}$$
$$\times [1 + \lambda(1 - G_0 V_{\bar\Lambda})^{-1} G_0 \lambda V_{\bar\Lambda}]\eta$$
$$= \eta[1 + V_{\bar\Lambda}\lambda(E' - H_0 - V_{\bar\Lambda})^{-1}\lambda] O^\mu_{\bar\Lambda}$$
$$\times [1 + \lambda(E - H_0 - V_{\bar\Lambda})^{-1}\lambda V_{\bar\Lambda}]\eta. \quad (13)$$

It is seen that our solution differs substantially from the one of Ref. [10]; in particular, the interaction operator $V_{\bar\Lambda}$ in the denominators $(E' - H_0 - V_{\bar\Lambda}\lambda)^{-1}$ and $(E - H_0 - \lambda V_{\bar\Lambda})^{-1}$ of Eq. (8) is projected by λ, so that each intermediate state in the perturbation series for $(E' - H_0 - V_{\bar\Lambda}\lambda)^{-1}$ and $(E - H_0 - \lambda V_{\bar\Lambda})^{-1}$ involves relative momenta restricted to the interval $\Lambda < k < \bar\Lambda$. By contrast, no such restriction on momenta appears in the corresponding intermediate states of Eq. (13).

[1]It is worth noting that Eq. (8) is just the expression for an effective operator in the Bloch-Horowitz approach [14] or Feshbach's projection formalism [15] (with $P = \eta$ and $Q = \lambda$).

[2]Equation (8) can also be derived directly from our definition of Γ^μ_Λ, Eq. (22), by making use of Eqs. (A7) and Eqs. (A8).

III. UNAMBIGUOUS DERIVATION OF THE CURRENT OPERATOR RG EQUATION

The RG equation for Γ^μ_Λ, Eq. (7), was derived in Ref. [10] as only a sufficient condition for the Λ invariance of the physical matrix element of O^μ_Λ:

$$\frac{\partial \langle O^\mu_\Lambda \rangle}{\partial \Lambda} = \frac{\partial}{\partial \Lambda} \bar\psi_\beta \eta \Gamma^\mu_\Lambda(E_\beta, E_\alpha)\eta\psi_\alpha = 0. \quad (14)$$

That is, the equation used to define Γ^μ_Λ was chosen to be[3]

$$\langle O^\mu_\Lambda \rangle \equiv \bar\psi_\beta \eta \Gamma^\mu_\Lambda(E_\beta, E_\alpha)\eta\psi_\alpha = \bar\psi_\beta \Gamma^\mu(E_\beta, E_\alpha)\psi_\alpha, \quad (15)$$

where

$$\Gamma^\mu(E_\beta, E_\alpha) \equiv O^\mu_{\bar\Lambda}(E_\beta, E_\alpha) = \eta\Gamma^\mu_\Lambda(E_\beta, E_\alpha)\eta\big|_{\Lambda=\bar\Lambda} \quad (16)$$

can be identified with the current vertex function of the full space [16] in the limit $\bar\Lambda \to \infty$. The sandwiching two-body wave functions $\bar\psi_\beta$ and ψ_α include bound states and scattering states whose relative momenta, p' and p, respectively, are smaller than the cutoff parameter: $p', p < \Lambda$. Although not emphasized in Ref. [10], this restriction is essential for the derivation of Eq. (7) in the case of scattering states, and it also prevents Eq. (15) from a possible mathematical inconsistency of having more equations than the number of unknown variables. This point is clarified under Eq. (22).

We stress that Eq. (7) is only a sufficient condition for the Λ independence of $\bar\psi\eta\Gamma^\mu_\Lambda\eta\psi$, as expressed by Eq. (14), even though this equation involves matrix elements between *all* states ψ (bound and scattering). The exact nature of this ambiguity can be illustrated by substituting either the scattering state equation,

$$\psi_{\mathbf{p}}(\mathbf{k}) = (2\pi)^3\delta^3(\mathbf{p} - \mathbf{k}) + G_0(E_p, k) \times \int V_\Lambda(E_p; \mathbf{k}, \mathbf{k}')\theta(\Lambda - k')\psi_{\mathbf{p}}(\mathbf{k}')\frac{d^3k'}{(2\pi)^3}, \quad (17)$$

or the corresponding bound state equation, into Eq. (14) (here $E_{p'}$ and E_p are the on-shell energies), thereby resulting in

$$\bar\psi_{\mathbf{p}'}\eta\frac{\partial \Gamma^\mu_\Lambda(E_{p'}, E_p)}{\partial \Lambda}\eta\psi_{\mathbf{p}}$$
$$= \bar\psi_{\mathbf{p}'}\eta V_\Lambda(E_{p'})G_0(E_{p'})\frac{\partial \lambda}{\partial \Lambda}\Gamma^\mu_\Lambda(E_{p'}, E_p)\eta\psi_{\mathbf{p}}$$
$$+ \bar\psi_{\mathbf{p}'}\eta\Gamma^\mu_\Lambda(E_{p'}, E_p)\frac{\partial \lambda}{\partial \Lambda}G_0(E_p)V_\Lambda(E_p)\eta\psi_{\mathbf{p}}. \quad (18)$$

Clearly Eq. (7) provides only a sufficient condition to guarantee Eq. (18).

Here we eliminate the ambiguity in the validity of Eq. (7) by showing that this RG equation is a sufficient and *necessary* condition for Λ independence of the five-point function $\eta G\eta\Gamma^\mu_\Lambda\eta G\eta$. In other words, rather than basing the RG approach to the current operator on the condition [Eq. (15)]

$$\bar\psi\eta\Gamma^\mu_\Lambda\eta\psi = \bar\psi\Gamma^\mu\psi \quad (19)$$

[3]To save on notation we suppress total momentum variables from the argument of Γ^μ_Λ.

for all $\Lambda < \bar{\Lambda}$, we suggest that it be based on the condition

$$\eta G\eta\Gamma^{\mu}_{\Lambda}\eta G\eta = \eta G^{\mu}\eta \tag{20}$$

for all $\Lambda < \bar{\Lambda}$, where G^{μ} is the five-point function defined as

$$G^{\mu} = G\eta\Gamma^{\mu}_{\Lambda}\eta G\big|_{\bar{\Lambda}=\Lambda} = G\Gamma^{\mu}G. \tag{21}$$

We note that G^{μ} corresponds to the two-body Green function G with all possible insertions of a current [16]. In the five-point function $\eta G^{\mu}\eta$, neither the incoming nor the outgoing two-body states are on the energy shell; by contrast, both these states are on the energy shell in $\bar{\psi}\Gamma^{\mu}\psi$. At the same time, such five-point Green functions are necessary ingredients for three-body currents where two-body subsystems are off-shell. In this sense the use of $\eta G^{\mu}\eta$ for the RG approach to the current operator is naturally related to the RG approach to the two-body interaction, where the cutoff independence of the fully off-shell two-body scattering amplitude is used [4]. Equation (20) thus defines the effective current vertex Γ^{μ}_{Λ} so that the five-point Green function $G\eta\Gamma^{\mu}_{\Lambda}\eta G$ coincides with the five-point Green function $G^{\mu} = G\Gamma^{\mu}G$ if the relative momenta of incoming and outgoing nucleons are below Λ. Showing the two-body energy arguments, Eq. (20) is

$$\begin{aligned}\eta G(E')\eta\, \Gamma^{\mu}_{\Lambda}(E',E)\eta G(E)\eta &= \eta G^{\mu}(E',E)\eta\\ &= \eta G(E')\Gamma^{\mu}(E',E)G(E)\eta,\end{aligned} \tag{22}$$

where the external η's ensure the above-mentioned restriction on the relative momenta of incoming and outgoing particles. Without this restriction, one would have the self-consistency constraint $\Gamma^{\mu} = \eta\Gamma^{\mu}_{\Lambda}\eta$, which would simply mean that the model is cutoff by Λ from the very beginning, leaving us with nothing further to be done.

The Λ independence specified by Eq. (22) leads to the RG equation

$$\eta G(E')\frac{\partial\eta\Gamma^{\mu}_{\Lambda}(E',E)\eta}{\partial\Lambda}G(E)\eta = 0. \tag{23}$$

The cutoff-independence condition of Eq. (15) differs from the one of Eq. (22). It is also a weaker condition as Eq. (15) follows from Eq. (22); i.e., Eq. (15) is a necessary but not a sufficient condition for Eq. (22) to be satisfied. The essential difference between these two conditions is that Eq. (22) involves off-shell scattering amplitudes through $\eta G(E)\eta$, whereas Eq. (15) involves half-on-shell amplitudes through the scattering states

$$\psi_{\mathbf{p}}(\mathbf{k}) = (2\pi)^3\delta(\mathbf{p}-\mathbf{k}) + G_0(E_p,k)T(E_p;\mathbf{k},\mathbf{p}). \tag{24}$$

In this sense Eq. (15) looks more like an extension of the RG approach discussed in Ref. [17] (which is based on the independence of the *half-on-shell* scattering amplitude) to the case of current operators. Moreover, Eq. (22) defines $\Gamma^{\mu}_{\Lambda}(E',E)$ *unambiguously* whereas there is an ambiguity in its definition by Eq. (15) (this ambiguity has already been pointed out in Ref. [10]).

To see the difference between these two conditions yet more precisely, let's write Eq. (23) in expanded form for the case of scattering states:

$$\begin{aligned}&\frac{\partial}{\partial\Lambda}\int\left[(2\pi)^3\delta(\mathbf{p}'-\mathbf{k}') + T(E';\mathbf{p}',\mathbf{k}')G_0(E',k')\right]\\ &\quad\times\frac{d^3k'}{(2\pi)^3}\theta(\Lambda-k')\Gamma^{\mu}_{\Lambda}(E',E,\mathbf{k}',\mathbf{k})\theta(\Lambda-k)\frac{d^3k}{(2\pi)^3}\\ &\quad\times\left[(2\pi)^3\delta(\mathbf{k}-\mathbf{p}) + G_0(E,k)T(E;\mathbf{k},\mathbf{p})\right] = 0,\end{aligned} \tag{25}$$

where no restriction is put on E' and E. Writing Eq. (14) in a similar way, it becomes clear that only that part of Eq. (25) corresponding to $E = E_p$, and $E' = E_{p'}$, reproduces Eq. (14). With no restriction being put on the external relative momenta $\mathbf{p}$ and $\mathbf{p}'$ of Eq. (25) (apart from $p', p < \Lambda$), one can invert the external Green functions to obtain the RG equation for Γ^{μ}_{Λ}. That is why the RG equation for Γ^{μ}_{Λ} is not only a sufficient but also a *necessary* condition for Eq. (23), whereas it is only a sufficient condition for Eq. (14).

To show explicitly how one obtains the RG equation unambiguously, we use Eqs. (A6) and the shorthand notation $\delta \equiv \partial\eta/\partial\Lambda = -\partial\lambda/\partial\Lambda$ in the following:

$$\begin{aligned}0 &= \eta G\frac{\partial\eta\Gamma^{\mu}_{\Lambda}\eta}{\partial\Lambda}G\eta = \eta G\left(\eta\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda}\eta + \delta\Gamma^{\mu}_{\Lambda}\eta + \eta\Gamma^{\mu}_{\Lambda}\delta\right)G\eta\\ &= \eta G\eta\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda}\eta G\eta + \eta G\delta\Gamma^{\mu}_{\Lambda}\eta G\eta + \eta G\eta\Gamma^{\mu}_{\Lambda}\delta G\eta\\ &= \eta G\eta\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda}\eta G\eta + (\eta G_0 + \eta G\eta V_{\Lambda}G_0)\delta\Gamma^{\mu}_{\Lambda}\eta G\eta\\ &\quad + \eta G\eta\Gamma^{\mu}_{\Lambda}\delta(\eta G_0 + G_0V_{\Lambda}\eta G\eta)\\ &= \eta G\eta\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda}\eta G\eta + \eta G\eta V_{\Lambda}G_0\delta\Gamma^{\mu}_{\Lambda}\eta G\eta\\ &\quad + \eta G\eta\Gamma^{\mu}_{\Lambda}\delta G_0V_{\Lambda}\eta G\eta + \eta G_0\delta\Gamma^{\mu}_{\Lambda}\eta G\eta + \eta G\eta\Gamma^{\mu}_{\Lambda}\delta\eta G_0\\ &= \eta G\eta\left(\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda} + V_{\Lambda}G_0\delta\Gamma^{\mu}_{\Lambda} + \Gamma^{\mu}_{\Lambda}\delta G_0V_{\Lambda}\right)\eta G\eta\\ &\quad + \eta G_0\delta\Gamma^{\mu}_{\Lambda}\eta G\eta + \eta G\eta\Gamma^{\mu}_{\Lambda}\delta\eta G_0.\end{aligned} \tag{26}$$

Furthermore, as we are interested in external relative momenta strictly below Λ, the last two terms of Eq. (26) are zero since $\delta\eta = 0$. One can then invert $\eta G\eta$ in the reduced subspace by acting on Eq. (26) with $1 - V_{\Lambda}G_0\eta$ from the right side and with $1 - \eta G_0V_{\Lambda}$ from the left:

$$\begin{aligned}0 &= (1-\eta G_0V_{\Lambda})\eta G\eta\left(\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda} + V_{\Lambda}G_0\delta\Gamma^{\mu}_{\Lambda} + \Gamma^{\mu}_{\Lambda}\delta G_0V_{\Lambda}\right)\\ &\quad\times\eta G\eta(1-V_{\Lambda}G_0\eta)\\ &= G_0\eta\left(\frac{\partial\Gamma^{\mu}_{\Lambda}}{\partial\Lambda} + V_{\Lambda}G_0\delta\Gamma^{\mu}_{\Lambda} + \Gamma^{\mu}_{\Lambda}\delta G_0V_{\Lambda}\right)\eta G_0,\end{aligned} \tag{27}$$

where Eqs. (A7) have been used. In this way we derive the RG equation for the current operator, Eq. (7), unambiguously.

IV. CURRENT CONSERVATION

The question of how to properly implement current conservation in EFT with a cutoff, so that gauge invariance is ensured in practical calculations, is a subtle one [18]. Here we show that the problem of current conservation in the RG approach is likewise not so simple (it is certainly not as simple as presented in Ref. [10]).

To avoid the well-known problems of current conservation in theories with a finite cutoff, we consider the simple case where the starting cutoff is taken to infinity, $\bar{\Lambda} = \infty$. Then in the best case we will have the usual two-body Ward-Takahashi (WT) identities [11]

$$q_\mu G^\mu(E', E) = \Gamma_0^0 G(E) - G(E')\Gamma_0^0, \tag{28a}$$
$$q_\mu \Gamma^\mu(E', E) = G^{-1}(E')\Gamma_0^0 - \Gamma_0^0 G^{-1}(E), \tag{28b}$$

where Γ_0^0 is the zero'th component of the current operator Γ_0^μ of two noninteracting particles and is specified for initial (final) total four-momentum $P = p_1 + p_2$ $(P' = p_1' + p_2')$ and relative momentm $\mathbf{p}$ $(\mathbf{p}')$ as

$$\begin{aligned}\langle \mathbf{p}'|\Gamma_0^0(P', P)|\mathbf{p}\rangle &= i(2\pi)^3[e_1\delta(\mathbf{p}_2' - \mathbf{p}_2) + e_2\delta(\mathbf{p}_1' - \mathbf{p}_1)]\\ &= i(2\pi)^3[e_1\delta(\mathbf{p}' - \mathbf{p} - \mathbf{q}/2) + e_2\delta(\mathbf{p}' - \mathbf{p} + \mathbf{q}/2)].\end{aligned} \tag{29}$$

It is important to realize that the WT identities of Eqs. (28) are damaged after the introduction of a finite momentum cutoff Λ. In particular, introducing the cutoff into Eq. (28a) gives

$$q_\mu \eta G^\mu(E', E)\eta = \eta\Gamma_0^0 G(E)\eta - \eta G(E')\Gamma_0^0\eta. \tag{30}$$

Because $\eta\Gamma_0^0 G(E)\eta \neq \eta\Gamma_0^0\eta G(E)\eta$ (the cutoff η does not commute with Γ_0^0), it is evident that Eq. (30) is not of the same form as Eq. (28a), so it is not a usual WT identity. Similarly, to see how the WT identity for current operator $O_\Lambda^\mu = \eta\Gamma_\Lambda^\mu\eta$ (which does depend on Λ) differs from the usual one, we use Eq. (8) and Eq. (28b) to write

$$\begin{aligned}q_\mu \eta\Gamma_\Lambda^\mu(E', E)\eta &= \eta[1 - V_{\bar{\Lambda}} G_0(E')\lambda]^{-1}\left[G^{-1}(E')\Gamma_0^0 - \Gamma_0^0 G^{-1}(E)\right]\\ &\quad\times[1 - \lambda G_0(E)V_{\bar{\Lambda}}]^{-1}\eta\\ &= \eta[1 + V_\Lambda G_0(E')\lambda]\left[G^{-1}(E')\Gamma_0^0 - \Gamma_0^0 G^{-1}(E)\right]\\ &\quad\times[1 + \lambda G_0(E)V_\Lambda]\eta.\end{aligned} \tag{31}$$

This expression can be simplified using

$$G^{-1}(1 + \lambda G_0 V_\Lambda) = G_0^{-1} - \eta V_\Lambda \tag{32a}$$
$$(1 + V_\Lambda G_0\lambda)G^{-1} = G_0^{-1} - V_\Lambda\eta, \tag{32b}$$

which follow from the equations for G, Eqs. (A5a) and (A5b). Thus,

$$\begin{aligned}q_\mu \eta\Gamma_\Lambda^\mu(E', E)\eta &= \eta[G_0^{-1}(E') - V_\Lambda(E')]\eta\Gamma_0^0[1 + \lambda G_0(E)V_\Lambda(E)]\eta\\ &\quad - \eta[1 + V_\Lambda(E')G_0(E')\lambda]\Gamma_0^0\eta[G_0^{-1}(E) - V_\Lambda(E)]\eta.\end{aligned} \tag{33}$$

Although Eq. (33) is not a usual WT identity, it still leads to a conserved current due to the operators in the curly brackets, $[G_0^{-1} - V_\Lambda]$:

$$q_\mu \bar{\psi}_{\mathbf{p}'}\eta\Gamma_\Lambda^\mu(E', E)\eta\psi_{\mathbf{p}} = 0. \tag{34}$$

Having derived the modified WT identity, Eq. (33), it is easy to realize that there was no obligation of pushing the starting cutoff to infinity. We could have started with a finite cutoff $\bar{\Lambda}$; however, our starting WT identities would then need to be Eqs. (30) and (33) (with Λ replaced by $\bar{\Lambda}$), instead of the usual ones, Eqs. (28). In this way we would come to the same result [Eqs. (30) and (33) for any $\Lambda < \bar{\Lambda}$].

It is important to note that the modified WT identity, Eq. (33), relates the reduced space effective current vertex Γ_Λ^μ, only to the corresponding effective potential V_Λ ($V_{\bar{\Lambda}}$ is not involved), and that it enters the WT identity only with relative momenta below Λ for all physically interesting low energy transitions. These properties are indispensable for constructing a self-contained EFT in the reduced momentum space [18].

ACKNOWLEDGMENTS

The research described in this publication was made possible in part by GNSF Grant GNSF/ST06/4-050.

APPENDIX: USEFUL EQUATIONS

Here we gather together some standard equations of the RG approach that are made use of in the main text. First, we note that the solution of the RG equation for the effective potential, Eq. (3), can be formally written in terms of the initial potential $V_{\bar{\Lambda}}$ as

$$V_\Lambda = (1 - V_{\bar{\Lambda}} G_0\lambda)^{-1} V_{\bar{\Lambda}}, \tag{A1a}$$
$$= V_{\bar{\Lambda}}(1 - \lambda G_0 V_{\bar{\Lambda}})^{-1}. \tag{A1b}$$

These equations then give the useful relations

$$(1 - V_{\bar{\Lambda}} G_0\lambda)^{-1} = 1 + V_\Lambda G_0\lambda, \tag{A2a}$$
$$(1 - \lambda G_0 V_{\bar{\Lambda}})^{-1} = 1 + \lambda G_0 V_\Lambda. \tag{A2b}$$

Second, we note that in this article we assume that a finite value of $\bar{\Lambda}$ defines the full model space. That is, all relative momenta are assumed to lie within a sphere of radius $\bar{\Lambda}$ so that

$$\bar{\eta} \equiv \eta|_{\Lambda=\bar{\Lambda}} = 1, \quad \bar{\lambda} \equiv \lambda|_{\Lambda=\bar{\Lambda}} = 0, \tag{A3}$$

and

$$\lambda = 1 - \eta. \tag{A4}$$

We also note that Eq. (5), and its reverse form $T = V_\Lambda + T G_0\eta V_\Lambda$, implies that the two-body Green function $G \equiv G_0 + G_0 T G_0$ satisfies the equations

$$G = G_0 + (\lambda G_0 + G\eta)V_\Lambda G_0, \tag{A5a}$$
$$G = G_0 + G_0 V_\Lambda(\lambda G_0 + \eta G), \tag{A5b}$$
$$G = G_0 + G V_{\bar{\Lambda}} G_0, \tag{A5c}$$
$$G = G_0 + G_0 V_{\bar{\Lambda}} G. \tag{A5d}$$

These then imply the following equations (together with their reversed forms):

$$\eta G = \eta G_0 + \eta G\eta V_\Lambda G_0, \tag{A6a}$$
$$\eta G = \eta G_0 + \eta G V_{\bar{\Lambda}} G_0, \tag{A6b}$$
$$\eta G\eta = \eta G_0 + \eta G V_{\bar{\Lambda}} G_0\eta. \tag{A6c}$$

The first of these equations, and its reversed form, then gives

$$\eta G\eta(1 - V_\Lambda G_0\eta) = \eta G_0, \tag{A7a}$$

$$(1 - \eta G_0 V_\Lambda)\eta G\eta = G_0\eta, \tag{A7b}$$

while the last two equations of Eqs. (A6), and their reversed forms, give

$$\eta G = \eta G\eta\,(1 - V_{\bar\Lambda} G_0\lambda)^{-1}\,, \tag{A8a}$$

$$G\eta = (1 - \lambda G_0 V_{\bar\Lambda})^{-1}\,\eta G\eta. \tag{A8b}$$

[1] K. G. Wilson and J. B. Kogut, Phys. Rep. **12**, 75 (1974).
[2] J. Polchinski, Nucl. Phys. **B231**, 269 (1984).
[3] T. R. Morris, Prog. Theor. Phys. Suppl. **131**, 395 (1998).
[4] M. C. Birse, J. A. McGovern, and K. G. Richardson, Phys. Lett. **B464**, 169 (1999).
[5] M. C. Birse, Phys. Rev. C **74**, 014003 (2006).
[6] K. Harada, K. Inoue, and H. Kubo, Phys. Lett. **B636**, 305 (2006).
[7] K. Harada and H. Kubo, Nucl. Phys. **B758**, 304 (2006).
[8] K. Harada, H. Kubo, and A. Ninomiya (2007), arXiv:nucl-th/0702074.
[9] S. X. Nakamura, Prog. Theor. Phys. **114**, 77 (2005).
[10] S. X. Nakamura and S.-I. Ando, Phys. Rev. C **74**, 034004 (2006).
[11] W. Bentz, Nucl. Phys. **A446**, 678 (1985).
[12] S. X. Nakamura and S.-i. Ando, Talk given at the 5th International Workshop on Chiral Dynamics, Theory and Experiment, 18–22 September 2006, Durham/Chapel Hill, NC, USA, arXiv:nucl-th/0611052.
[13] S. X. Nakamura and A. Gårdestig (2007), arXiv:0704.3757 [nucl-th].
[14] C. Bloch and J. Horowitz, Nucl. Phys. **8**, 91 (1958).
[15] H. Feshbach, Ann. Phys. **5**, 357 (1958); **19**, 287 (1962).
[16] A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60**, 044003 (1999).
[17] S. K. Bogner, T. T. S. Kuo, A. Schwenk, D. R. Entem, and R. Machleidt, Phys. Lett. **B576**, 265 (2003).
[18] A. N. Kvnikhidze *et al.* (in preparation).