# Anaxagoras and the Size of the Sun*

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Plutarch and others report that Anaxagoras compared the size of the sun with the Peloponnesus. It is the aim of this paper to show that this was a fair estimate, from his point of view, which is that of a flat earth. More precisely, I will show that, with the instruments and the geometrical knowledge available, Anaxagoras must have been able to use the procedures and perform the calculations needed to reach approximately his result.

Plutarch says that, according to Anaxagoras, "the sun is much bigger than the Peloponnesus"; Hippolytus, that "the sun surpasses the Peloponnesus in size"; and Diogenes Laërtius: "the sun is bigger than the Peloponnesus".[1] Gershenson and Greenberg classify these reports as "late traditions whose validity is uncertain".[2] The comparison between the sun and the Peloponnesus, however, is so unusual and surprising, that it is hardly believable that it was inserted by a doxographer. Moreover, according to Diels,[3] these reports go back to Theophrastus, who probably still had access to Anaxagoras' writings. We may conclude that the comparison of the sun with the Peloponnesus, in one way or another, was made by Anaxagoras himself.

When we look somewhat closer at these texts, another feature might strike us. Saying that the sun is bigger than the Peloponnesus, without any further addition, sounds a little bit odd. If one would say something that makes sense, it would be something like: "The sun is bigger than the Peloponnesus, *but smaller than Greece*", or: "The sun is *ten times* bigger than the Peloponnesus", or even "the sun is *a little bit* bigger than the Peloponnesus". It is noteworthy that this last is, explicitly or implicitly, how authors like Dreyer, West, Sider, and Fehling,[4] read it, without explaining,

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1 DK 59A72, 59A42(8), and 59A1(8).

2 Gershenson and Greenberg, 1964:352. They mention also Eusebius and Theodoretus, who repeat Plutarch's version.

3 Diels, 1879:138.

4 Dreyer, 1953:31. West, 1971:233. Sider 1973. Fehling 1985:209 explicitly points to a textual exaggeration: "vielfach so groβ nach Aëtius, ihm war das Richtige nicht groβ genug".

however, why we should read "*somewhat* bigger than", whereas the texts have "(much) bigger than". I tend to go further and doubt whether the qualifications like "bigger than" and "much bigger than", which the doxographers added to the word "Peloponnesus", reflect what Anaxagoras has really said. These additions rather seem to express the uneasiness the doxographers must have felt when they read that Anaxagoras compared the size of the sun with the Peloponnesus. We must not forget that Plutarch and the other doxographers lived at a time when it was known that the sun is very far away and very big. They lived a considerable time after Aristarchus, who was the first to try to measure the distance between sun and earth. His conclusions were that "the distance of the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon from the earth", and that "the diameter of the sun is greater than eighteen times, but less than twenty times, the diameter of the moon".[5] Qualifications like "bigger than" and "much bigger than" rather seem to express an attempt of the doxographers to make Anaxagoras' strange comparison more acceptable to the contemporary readers. For these reasons I think that Anaxagoras originally must have said something like: "The sun is about the size of the Peloponnesus". An additional argument for this claim will be put forward in the next section.

Another report of Plutarch states that, according to Anaxagoras, "the moon is as big as the Peloponnesus".[6] Two features of this text catch the eye. The first is, that it is the moon, and not the sun, that is compared here with the Peloponnesus. The second is, that the indication "(much) bigger than" is missing, or, to put it positively, the moon is said to be the size of the Peloponnesus. At first sight now everything seems to be clear: Anaxagoras had measured the absolute size of the moon (as big as the Peloponnesus), and inferred from that to the relative size of the sun (bigger than the moon, that is, bigger than the Peloponnesus).[7] We may wonder, however, whether there is any way in which Anaxagoras could have measured the absolute size of the moon. Therefore, I agree with Fehling, who holds that Plutarch simply transferred the report on the size of the sun to the moon.[8] When we read "sun" instead of "moon", then the sun is said to be as big as the Peloponnesus. However this may be, the least we could say is that Anaxagoras, thinking about the size of the sun, somehow choose the Peloponnesus as his point of reference.

Sider argues that Anaxagoras could have estimated the minimal size of the sun with the help of the solar eclipse of 30 April 463 BC. The width of the path of this eclipse, which passed through Greece, was 219 km.[9] Knowing the laws of perspective, Sider says, he may have concluded that the sun was bigger than 219 km. However, as

5 Propositions 7 and 9 from Aristarchus' treatise on the distances of the sun and the moon, quoted from Heath, 1915:377 and 383.

6 Plutarch, *De facie in orbe lunae* 19.9 (932a), not in DK. See Gershenson and Greenberg 1964:123 (no. 189).

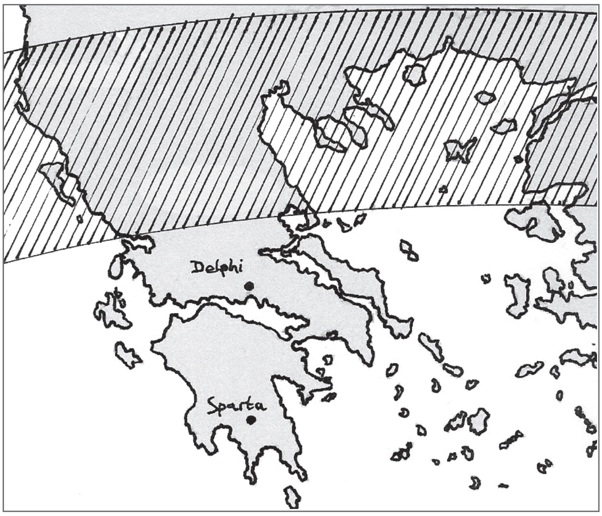
7 This is how Görgemanns, 1970:135(24) reads it. See also Panchenko, 2002[1]:333, n. 24.

8 Fehling, 1985:209, n.38.

9 Sider, 1973.

far as I know there is no evidence that the Greeks, or any ancient people whatsoever, bothered about the width of the path of a solar eclipse. In the second place, the path of the eclipse, as can be seen from recent calculations,[10] extended from Thessaly into Macedonia, and did not go across the Peloponnesus, as Sider hoped.[11] Accordingly, it was rather difficult for Anaxagoras to gather the information needed. The strongest argument against Sider's suggestion is that no conclusion whatsoever regarding the absolute size of the sun or the moon can be drawn from the path of a solar eclipse, unless one knows the distances of both the sun and the moon.

**Figure 1:** *The path of the solar eclipse of 30 April 463 BC*




As a first step, let us ask whether it is possible, when you think that the earth is flat, to estimate the distance of the sun. According to the doxography, Anaximander was the first to express the distance of the sun from the earth in a number. He thought of the sun as a kind of invisible ring or wheel around the earth, filled with fire, which we can see at only one point where there is an opening in the wheel, through which the fire shines. This sun-wheel, he says, is 27 times the earth. Diels takes this to mean that the diameter of the sun-wheel is 27 times the diameter of the earth.[12]

The greatest height the sun could reach at Miletus (lying at $37.5^0$ N), during the summer solstice, is $76^0$. An unexpected consequence of Anaximander's number is, as Figure 2 shows, that there is no place on Anaximander's earth where the sun could stand in the zenith. Either Anaximander was not aware of this consequence, or he did not know yet that there are places on earth where the sun can be in the zenith. Anaxagoras, on the other hand, probably knew that there are indeed places on earth,

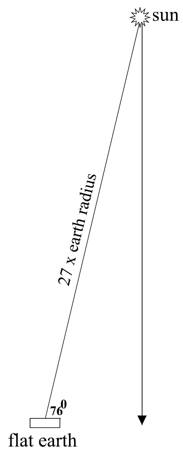
[10] See http://sunearth.gsfc.nasa.gov/eclipse/SEatlas/SEatlas-1/SEatlas-0479.GIF

[11] Sider, 1973:129, n.12.

[12] Diels, 1897:231.

where the sun is right above your head, and thus that the sun must be rather close to the flat earth, and. consequently, must be rather small.

**Figure 2:** *On Anaximander's earth the sun never stands in the zenith*




When we want to show that it is even possible to *calculate* the distance of the sun from a flat earth, we have to look at China, where astronomers, who also believed the earth to be flat, solved this problem in principle. In the third chapter of the *Huai nan tzu,*[13] about 120 BC, it is told how Chinese astronomers set up a gnomon (AB in Figure 3), the length of which was 10 *chi* (Chinese feet). The astronomers lived at Yangcheng ($33.3^0$ N, $111.7^0$ E), which is the place where the observations with the gnomon usually were made.[14] On the day of the summer solstice, at noon, they observed that their gnomon cast a shadow (BX) of 2 *chi*. This is the case when the angle at X, which indicates the height of the sun above the horizon, amounts to $78.7^0$.[15] They supposed that at the same moment a second gnomon (CD), put at a distance of 1000 *li* (Chinese miles) due south of the first one, cast a shadow (DY) of 1.9 *chi* (1 *li* = 415.8 meters;[16] 1 *chi* = 1/1500 *li* = 27.72 cm). They concluded that, when for every thousand *li* southward the shadow shortened by 1 *cun* (Chinese thumb; 1 *chi* = 10 *cun*, so 1 *cun* = 2.772 cm) there must be a point T, at a distance of 20,000 *li* to the south of the first gnomon, where a gnomon would cast no shadow at all. As the proportions of the triangle XAB are the same as those of the triangle XST, and AB : BX = 10 : 2 = 5 : 1, they could measure the length of ST, being 5 x 20,000 *li* = 100,000 *li*, which is 41,580 km.[17] In this way they managed to calculate the distance of the sun to the earth.

[13] Quoted by Needham, 1959:225.

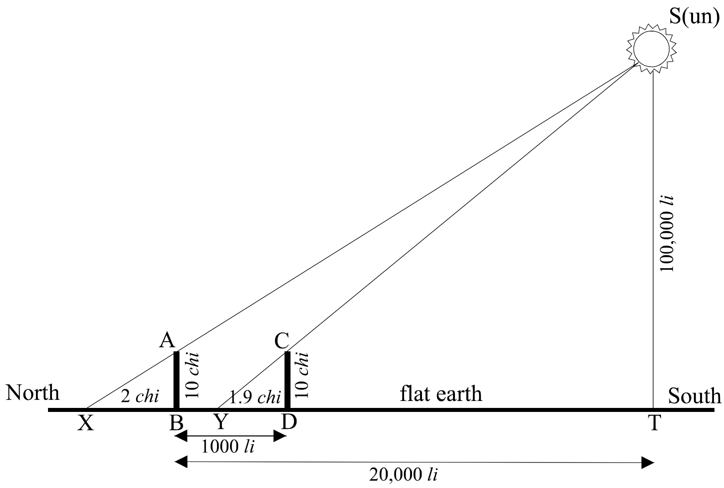
[14] See Cullen, 1996:222.

[15] tan-1(10 / 2) = 78.69.

[16] See Dubs, 1955:160, n.7.

[17] See Needham, 1959:225.

**Figure 3:** *How Chinese astronomers measured the distance of the sun (not to scale)*




Surprisingly, the Chinese astronomers did not *measure* with how many *cun* the shadow of a gnomon shortened for every 1000 *li* southward. They simply *took* that to be 1 *cun* per 1000 *li*. When we try to calculate the real difference between the two shadows, completely other figures result. It appears that 1000 *li* (415.8 km) to the south of $33.3^0$ N is approximately at $29.5^0$ N. On that latitude, at the time of the summer solstice, the angle at C is $6^0$ (viz. $29.5^0 - 23.5^0$, the inclination of the ecliptic) and accordingly the angle at Y = $84^0$. The length of the shadow (DY), then, is about 1.1 *chi*.[18] The discrepancy with the supposition of the Chinese astronomers, that the shadow shortens for 1 *cun* per 1000 *li*, is so significant that their number cannot be the result of observation. The same conclusion also follows from the strange consequence of these Chinese measurements, that at the summer solstice one has to go 20,000 *li* (8316 km) to the south, in order to find a place where the sun is in the zenith. In reality, however, the Tropic of Cancer runs through the south of China at a distance of about 1100 km from Yangcheng. Recently, Panchenko has suggested that the method, used by these astronomers "was established somewhere outside China and that, in the process of the transmission, the Chinese *li* was substituted for a foreign measure".[19] Panchenko argues that "somewhere outside China" must have been Greece.

Exactly speaking, the Chinese did not measure *the* distance of the sun from the earth, but only one of its many possible distances. Actually, what is measured in this way is the greatest distance. On a flat earth the distance of the sun differs according to time and place. Not only is (strangely enough) the sun in winter closer to the flat earth than in summer, but it is also closer in the morning and in the evening than at noon. The Chinese astronomers seemed to have been aware of this, as the *Huai nan tzu*,

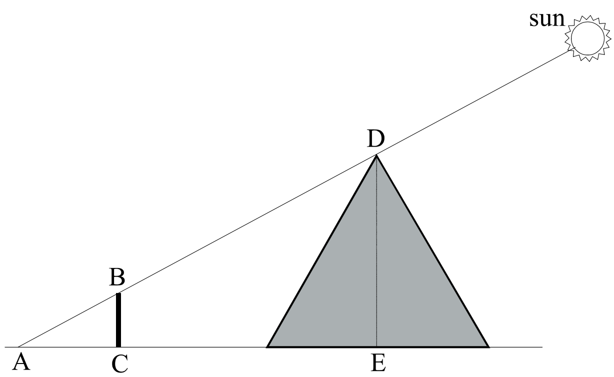
[18] 10 / tan 84 = 1,05.

[19] Panchenko, $2002^2$:252.

which is supposed to have measured at the summer solstice, gives another distance (100,000 *li*) than the *Zhou bi* (80,000 *li*), which probably measured about the time of an equinox.[20] And still another text says that at the winter solstice the sun is 20,000 *li* above the land.[21]

In principle, Anaxagoras could have used the procedure of the Chinese astronomers. The use of the gnomon was known since Anaximander had introduced the instrument into the Greek world. Moreover, the method, used by the Chinese in order to measure the distance of the sun was essentially the same as Thales is told by Plutarch to have used for measuring the height of a pyramid. Figure 4 illustrates his description: "Set up a stick (BC) at the extremity of the shadow cast by the pyramid and, having thus made two triangles by the touching of (the top of both pyramid and stick by) the sun's ray, show that the shadow (of the pyramid, AE) has the same ratio to the shadow (of the stick, AC) as which the (height of the) pyramid (DE) has to the stick (BC)". This picture almost invites you, as it were, to draw another line perpendicular from the sun to the earth in order in the same way to measure its distance.

**Figure 4:** *How Thales is said to have measured the height of a pyramid*




The procedure of the Chinese astronomers is also analogous to the famous experiment, by which Eratosthenes measured the circumference of the earth. The only difference is that Eratosthenes, knowing that the earth is spherical, managed to measure the circumference of the earth, whereas the Chinese astronomers, supposing that the earth is flat, measured the distance of the sun.[22] I would suggest the possibility that Eratosthenes used the setting of a procedure like that of the Chinese astronomers, set up by a now forgotten Greek astronomer, and replacing the supposition of a flat earth by the assumption of a spherical earth.

Before we enter into further calculations it is appropriate to underline that the calculations below will have to be looked upon as rough approximations, although the

20 See Cullen, 1996:78 and 178.

21 See Cullen, 1996:189.

22 See also Cullen, 1976.

figures used may give the impression that they are rather precise. The ancient Greek arithmetical operations were complex and laborious, especially when fractions were involved. In Anaxagoras' time, people will probably have rounded off broken numbers, in order not to make calculations too complicated. Moreover, "in the absence of all but the most basic trigonometry..., the measurement of angles was not the most obvious of ploys".[23] Accordingly, I will present calculations in which no angles will have to be measured. And, finally, the instruments used hardly made any exact measurement possible. The gnomon, e.g., was not so easy to put exactly perpendicular, and as the sun has a certain width, accurate measurements of the shadow-lengths were difficult to obtain in practice. The calculations given below, therefore, indicate an order of magnitude, no more and no less, but this will suffice fully for the aim of this article.

The Greeks considered Delphi ($38.5^0$ N, $22.5^0$ E) to be the navel of their circular flat earth. Let us suppose that Anaxagoras erected there one gnomon (AB) of 200 cm length, and another gnomon (CD) of equal length in the heart of the Peloponnesus, at Sparta ($37.1^0$ N, $22.5^0$ E), about 156 km due south of Delphi. We might even imagine that he made use of the gnomon that is said to have been erected there by Anaximander.[24] At the time of the summer solstice at noon, the shadow BX of the first gnomon was about 53.6 cm long, and the shadow DY of the second one 48.4 cm.[25] Subsequently, he could have extrapolated, that for every 156 km the shadow shortens by 5.2 cm, and he could have concluded that at about 1608 km to the south of the first gnomon the point (T) must be, where the sun stands right in the zenith. This number fits reasonably into the information he could have gathered from travelers to the south of Egypt.[26] Using the properties of the similar triangles XBA and XTS, the distance from the earth to the sun (TS in Figure 5) follows from the equation 53.6 : 200 = 1608 : x, and is 6000 km.

The next step he could have taken is, that he calculated the distance of the sun to Delphi, which is the hypotenuse XS of the triangle XTS in Figure 5. According to Pythagoras' theorem, the hypotenuse is $\sqrt{(1608^2 + 6000^2)} = 6212$ km. If the big figures involved in squaring and extracting the square root might have yielded a problem in order to learn the length of the hypotenuse XS, it is not necessary to make use of Pythagoras' theorem. Since in figure 5 AB and XB are known, XA can be measured with a measuring rope as about 207 cm. The length of XS, then, is the result of the equation XA : AB = XS : ST, thus 207 : 200 = XS : 6000, which makes XS = 6210 km.

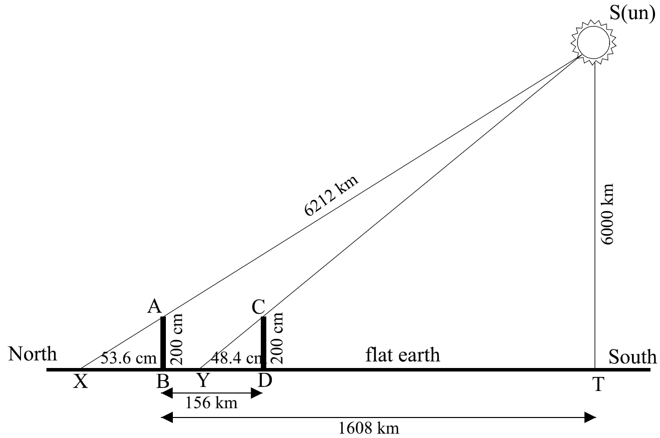
23 Lewis, 2001:41.

24 DK 12A1.

25 Anaxagoras could have observed this. We, however, can also calculate it. The angle at the top of the gnomon at the summer solstice at Delphi is 38.5 – 23.5 = 150, and hence the other opposite angle = 750. The length of the shadow is 200 : tan 75 = 53.6 cm, and at Sparta 200 : tan 76.4 = 48.4 cm.

26 The real distance between Delphi and the tropic of Cancer is 1670 km.

**Figure 5:** *How Anaxagoras could have measured the distance of the sun (not to scale)*




In order to understand the last step, we have to realise that Delphi, being the centre of the flat earth, could also be considered as the centre of the sun's orbit around the earth. The radius of this orbit is, as we saw, 6212 km, and thus the complete orbit of the sun around the earth 2π x 6212 km = 39,031 km.[27] Thales is said to have discovered that the angular (or apparent) diameter of the sun is 1/720 its orbit.[28] This attribution is certainly false, but it makes good sense to ascribe this discovery to Anaximander, since he was the first to describe the sun's orbit as a full circle around the earth. This means that Anaxagoras could have been acquainted with it. He could have calculated the sun's angular diameter with the help of a water clock or *clepsydra*, as explained by Cleomedes, in the second century AD. Cleomedes concluded that the angular diameter of the sun was 1/750th of its orbit.[29] As the angular diameter of the sun actually is about $0.5^0$, the result of 1/720th, ascribed to Thales, was more accurate. However, it is the method, which has a certain intrinsic inaccuracy, that counts here. As the angular diameter of the sun is about $0.5^0$, the real diameter of the sun must be 1/720 of 39,031 = about 54 km.

The last step in the procedure can be carried out in another way as well. In order to show this, let us return, for one last time, to the Chinese astronomers. The ultimate intention of the section of the *Zhou bi* was to measure the diameter of the sun. They took a hollow bamboo tube of 8 *chi* with an internal diameter of 1 *cun* (= 0.1 *chi*), and found that the sun exactly fitted into the bore. Then they "worked things out in proportion", as the text says. "Working things out in proportion" must mean that they calculated, again, with two similar triangles: OPQ and OYZ. The length of the tube is the perpendicular from O on PQ (the diameter of the tube), and is also the perpendicular on YZ (the diameter of the sun). Now the following equation holds:

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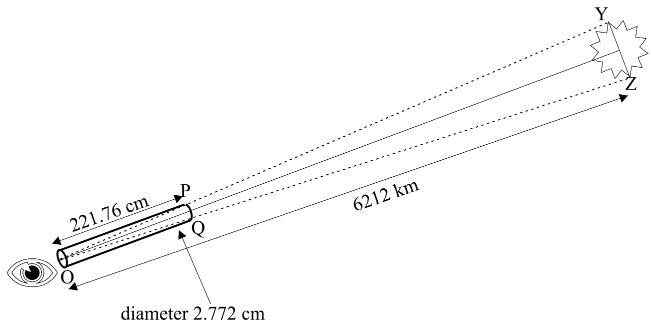
[27] In ancient civilizations the common practice was to take value of π simply as 3.

[28] DK 11A1(24) and 11A19.

[29] Cleomedes, *De motu circulari corporum celestium*, 2.75 (ed. Ziegler, Leipzig 1891:36).

80 : 1 = 100,000 : x. So x (the diameter of the sun) = 1250 *li* (520 km).[30] As explained above, the figure of 100,000 *li* (41,580 km) is much too big, according to the false supposition that the Chinese astronomers made. Accordingly, this measurement of the size of the sun as well went wrong. When we convert, however, the Chinese *chi* and *cun* into centimeters, and take 6212 km as the distance from the eye of the observer to the sun, as found above, instead of the wrong number of 41,580 km, the resulting diameter of the sun is: 221.76 : 2.772 = 6212 : x, so x = about 78 km. Given the inaccurateness of the methods and instruments used, this number may be taken as lying in the same range as the 54 km we found earlier. Again, this method could have been used by Anaxagoras as well, as the sighting tube was known to Aristotle,[31] but it must have been a much older instrument.

**Figure 6:** *Measuring the diameter of the sun with the help of a sighting tube (not to scale)*




The smallest east–west width of the Peloponnesus, measured through Sparta, is about 100 km. When the earth is supposed to be flat, the size of the sun can be calculated as about 54 km, or about 78 km, depending on the method used, as we have seen. We will have to realise, however, that all the calculations involved were necessarily rather rough and inaccurate in those ancient days, so that the measurements easily might have resulted in a (somewhat) bigger figure than we have found with either method. The calculations in this article, then, give no more than an indication of the range of magnitude, which nevertheless appears to be compatible with that of the Peloponnesus.

Supposing that Anaxagoras had his reasons to compare the size of the sun with the Peloponnesus, I have tried to bring forward circumstantial evidence in order to show that he had at his disposal the means to mathematically support his statement. However, whether he made an experiment like that of the Chinese astronomers or not, whether he measured the angular diameter of the sun with the help of a clepsydra or with any other method or not, or whether he simply made a reasonable guess, we may conclude, that Anaxagoras was quite right, from his point of view, when he compared the size of the sun with the Peloponnesus.

[30] Quoted in Cullen, 1996:78.

[31] See Aristotle, *De generatione animalium* 780b:19–22 and 781a:9–12.

However, Anaxagoras was fighting a lost battle. Plato, and definitely Aristotle argued that the earth is spherical, and this became the prevailing opinion. As a consequence of the sphericity of the earth, the sun was, as it were, catapulted into the heavens and became much bigger than the defenders of a flat earth could have ever imagined.

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