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A statistical analytical method for fatigue reliability containing very-high-cycle fatigue regime

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Abstract

Fatigue tests usually cost a lot of time and expenses especially for very-high-cycle fatigue (VHCF). Hence, a convenient and fast method is necessary for obtaining the S-N curve under different probabilities (i.e. P-S-N curve) s that to give an entire trend of the fatigue property of metallic materials under different stress levels. In this paper, a method is proposed to estimate the P-S-N curve containing VHCF regime. With the method, the fatigue life under different stress levels is first transferred to the fatigue life under an arbitrary stress level. Then, the P-S-N curve is obtained by studying the fatigue life in logarithm of base 10 under the chosen stress level. The method is used to study the reliability of fatigue life of steels and aluminium alloys, and the results are in agreement with the experimental data. The distribution form of fatigue life under the same stress level is also examined for VHCF regime. It is indicated that, for VHCF regime, the fatigue life under the same stress level approximately follows the normal distribution in logarithm of base 10.

Keywords: very-high-cycle fatigue; fatigue life; reliability; P-S-N curve

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1. Introduction

Many investigations have indicated that the fatigue life for the same group of specimens tested under an identical condition usually follows a certain distribution form, i.e. the probability associated. It is obvious that P-S-N curve (i.e. determining the fatigue life according to a certain survival probability P) is more objective to reflect the fatigue properties of materials from the perspective of statistics. Several methods have been developed to estimate P-S-N curve from a limited number of fatigue test data by statistical methods [1,2] for low cycle and high cycle fatigue. For example, Ling and Pan [3] proposed a maximum likelihood method for predicting the P-S-N curve by using a 3-parameter non-linear expression for S-N curve, in which a group of specimens were needed to be tested under a chosen reference stress.

It is known that the fatigue failures of metallic materials may still occur at the fatigue life up to 10^8~10^{10} cycles [4-6]. The development of modern industry often needs the metal components to endure exceeding 10^8 cycles of loading in service. For example, 10^8 corresponds to the loading cycles that a rotating component of a high speed train has been operated for about 5 months. Therefore, a convenient and fast method is necessary for obtaining the P-S-N curve especially for very-high-cycle fatigue (VHCF) regime in order to give a general view of the fatigue property of metallic materials under different stress levels. In this paper, a method is proposed to predict the P-S-N curve. The method is also compared with the tested results based on the conventional method [7].

2. Model and Analysis

2.1. Model construction

It is assumed that the fatigue life on a stress level is expressed as

$$N_i = A_i f(\sigma) \tag{1}$$

where the parameter A_i denotes the effect of other factors except the applied stress.

From Eq. (1), the fatigue life $N_{i_k,k}$ under stress levels σ_k $(k = 1, 2, \ldots, n)$ can be transferred to the fatigue life under an arbitrary stress level σ', i.e.

$$N'_{i_k,k} = \frac{f(\sigma')}{f(\sigma_k)} N_{i_k,k} \quad \text{or} \quad \log_{10} N'_{i_k,k} = \log_{10}\frac{f(\sigma')}{f(\sigma_k)} + \log_{10} N_{i_k,k} \quad (k = 1, 2, \ldots, n) \tag{2}$$

where the subscript i_k denotes the effect of other factors under the stress level σ_k, which together results in the fatigue life $N_{i_k,k}$.

Thus, the variance of fatigue life in logarithm of base 10 under stress level σ' is

$$\frac{1}{n}\sum_{j=1}^{n}\left(\log_{10} N'_{i_j,j} - \frac{1}{n}\sum_{k=1}^{n}\log_{10} N'_{i_k,k}\right)^2 = \frac{1}{n}\sum_{j=1}^{n}\left(\log_{10} N_{i_j,j} - \log_{10} f(\sigma_j) - \frac{1}{n}\sum_{k=1}^{n}(\log_{10} N_{i_k,k} - \log_{10} f(\sigma_k))\right)^2 \tag{3}$$

Eq. (3) indicates that the variance is independent of the referred stress level. So, the minimum value of Eq. (3) is reasonable to be used to determine the function $f(\sigma)$, which is used in the present paper. Then, one obtains the P-S-N curve by deriving the fatigue life in logarithm of base 10 under the chosen stress level σ'.

Further, if the scatter under different stress levels is considered, the tested stress levels can be divided into several parts and the corresponding stress levels (for example, $\sigma'_1, \sigma'_2, \sigma'_3$) is chosen for each part. Especially, for the fatigue life tested under several different stress levels, the analysis degenerates to the conventional method for determining P-S-N curve.

3. Results and Discussion

3.1. Distribution form of fatigue life

First, the distribution form of fatigue life in VHCF for high-strength steels under the same stress level is examined. For the fatigue life, the values of $\log_{10}N$ are ranked from the smallest, and labelled as $\log_{10}N_1 \le \log_{10}N_2 \le \cdots \log_{10}N_n$. The cumulative probability of fatigue life no larger than N_k ($k = 1, 2, \ldots, n$) is calculated by

$$F(\log_{10} N_k) = k/(n+1) \tag{4}$$

Figure 1 plots the statistical evaluations of fatigue life in logarithm of base 10 under the same stress level for a high carbon chromium steel. It is seen that, for the same stress level, the fatigue life in logarithm of base 10 in VHCF regime approximately follows the normal distribution.

3.2. P-S-N curve

Here, the form of traditional S-N curve $N = AS^a$ (i.e. $f(\sigma) = \sigma^a$) is attempted, and the normal distribution is considered for the fatigue life under the same stress level. Figure 2 shows the comparison of predicted P-S-N curve with experimental results. The comparison of the present method with the tested results based on the conventional method is shown in Fig. 2(d). The values of a and A_{i_p} obtained under different probability P are listed in Table 1, which are determined by using the experimental data by Shiozawa et al. [10] in Fig. 2(c) and by using the experimental data (solid sphere in Fig. 2(d)) for the maximum likelihood method by Ling and Pan [3] except for the test data under 372.78 MPa in Fig. 2(d). The population mean and variance of fatigue life in logarithm of base 10 are obtained by the unbiased estimation. It is seen that the predicted P-S-N curve obtained by the present method agrees with the experimental results. It is noted that the shape of S-N curve for high-strength steels often presents a duplex pattern corresponding to surface-initiated fracture mode and interior-initiated fracture mode [9,11]. So, only the interior-initiated fracture mode is considered here.

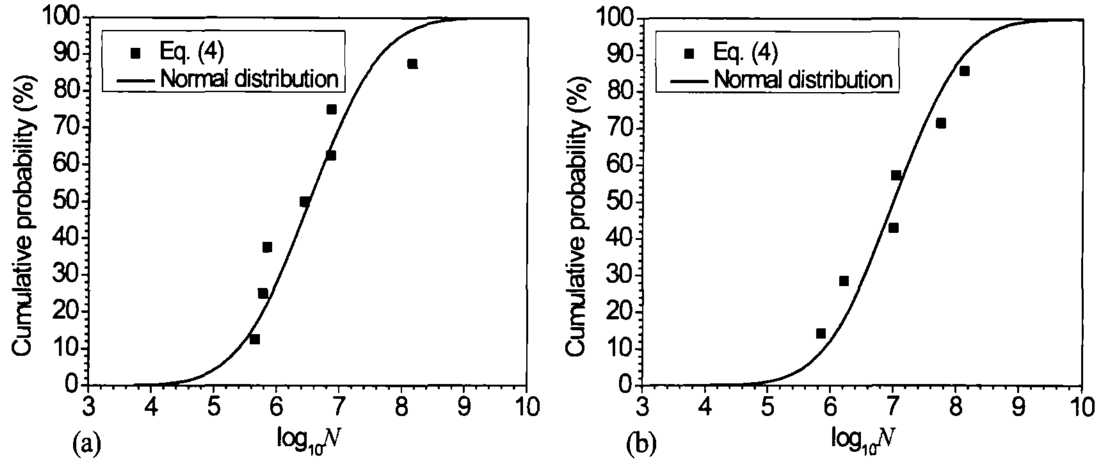

Fig. 1. Statistical evaluations of fatigue life in logarithm of base 10 under the same stress level for a high carbon chromium steel [8]. (a) σ_a =860 MPa; (b) σ_a =880 MPa

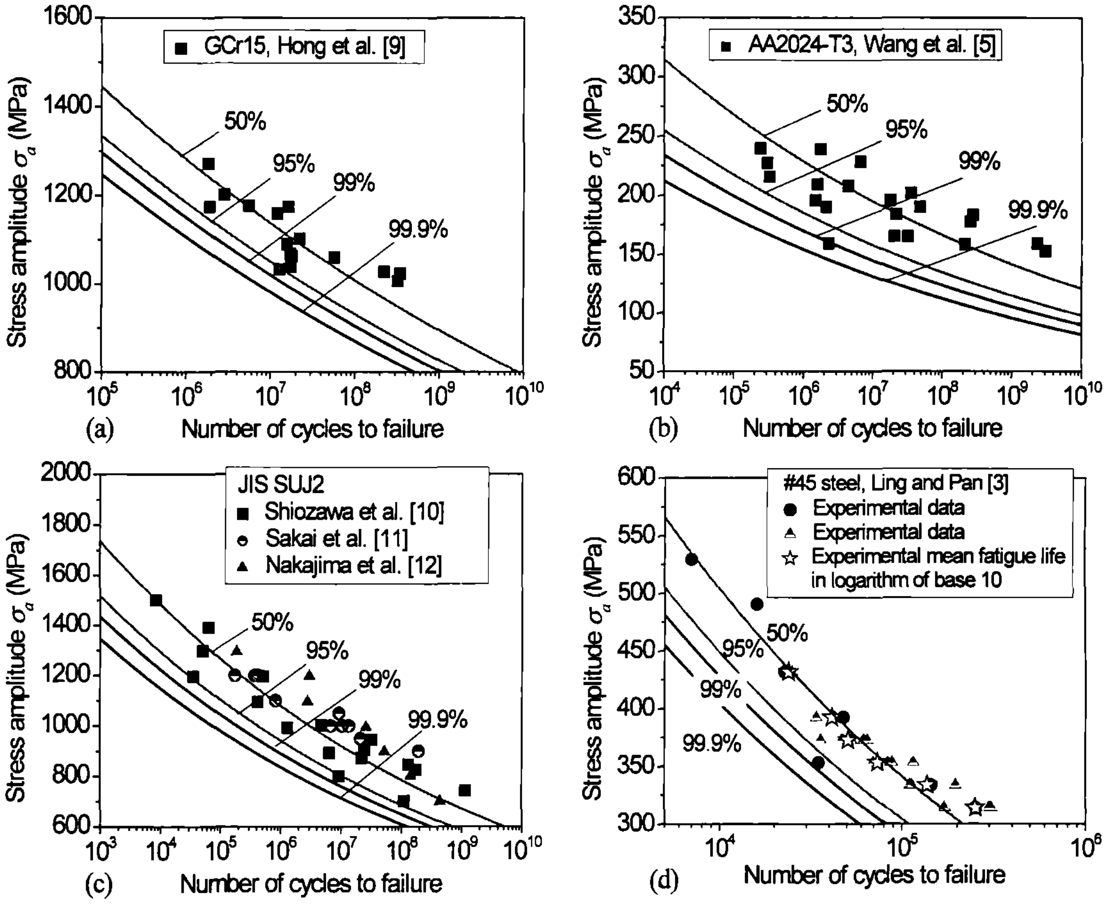

Fig. 2. Comparison of predicted P-S-N curves with experimental results at R=–1. (a) rotating bending test; (b) ultrasonic test (20 kHz); (c) axial loading test; (d) axial loading test

Table 1. Parameters a and A_{i_P} obtained under different probability P associated with Fig. 2

Materials	a	A_{i_P} at P=50%	A_{i_P} at P=95%	A_{i_P} at P=99%	A_{i_P} at P=99.9%
GCr15, Hong et al. [9]	−19.4	1.79×10^{182}	3.91×10^{181}	2.10×10^{181}	1.03×10^{181}
JIS SUJ2, Shiozawa et al. [10]	−14.5	1.81×10^{137}	2.50×10^{136}	1.12×10^{136}	4.45×10^{135}
AA2024–T3, Wang et al. [5]	−14.5	6.22×10^{126}	2.98×10^{125}	8.59×10^{124}	2.11×10^{124}
#45 steel, Ling and Pan [3]	−5.93	4.02×10^{55}	2.01×10^{55}	1.53×10^{55}	1.11×10^{55}

4. Conclusions

In this paper, a method is proposed for predicting the P-S-N curve containing VHCF regime, which is in agreement with our previous experimental data and the experimental data in literature for steels and aluminium alloys. The present method needs less experimental data for P-S-N curve prediction compared with the maximum likelihood method and the conventional method. The paper also indicates that, for VHCF regime, the distribution form of fatigue life under the same stress level can also be described by the normal distribution in logarithm of base 10.

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