

A Model for Analyzing the Dynamic Behavior of a Bank Facing a Financial Crisis

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Abstract

This paper presents a model for analyzing the optimal dynamic behavior of a bank. The model considers the development of the bank's balance sheet in a situation involving the risk of a financial crisis which may or may not materialize, and the timing of which is uncertain. Depending on the parameter configurations, the crisis may involve defaulting of loans as well as a reduction in the availability of funding. The model considers the precautionary measures that the bank takes in preparation for the potential crisis as well as the bank's reactions to the crisis, in case it materializes. An optimization model is presented, in which the bank dynamically adjusts the size and composition of its balance sheet, which includes cash, loans, deposits and equity. The model is shown to exhibit behavior that appears to be intuitively logical as e.g. lending is reduced as a precautionary measure for the anticipated financial crisis and cash reserves are increased in order to maintain sufficient liquidity. The process of maturity transformation, being a concept very central to banking, is also an essential element of the model.

Key words: Banking, financial crisis, stochastic programming, ALM

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1 Introduction

One of the most central functions of a bank is that of maturity transformation, in which the bank transforms liabilities that have short maturities into assets, the maturities of which are longer. As a result, the turnover rate of the bank's liabilities is higher than that of its assets, i.e. the bank needs to renew its funding at a relatively high frequency. From the perspective of a bank, a financial crisis may involve on one hand the defaulting of some of its lending and on the other hand a difficulty of obtaining new funding as the market may have concerns regarding the solvency of the bank and since other financial institutions may want to hold on to their liquid assets in such a situation. Both of these issues reduce the bank's ability to pay back its maturing liabilities during a financial crisis. As a result, in order for a bank to remain liquid, i.e. being able to repay its maturing liabilities, it needs to adjust its balance sheet as a precautionary measure for the possibility of a financial crisis. This it needs to do even if it does not know if and when the anticipated crisis is going to materialize. In addition, it may need to take actions during the crisis.

This paper presents a model for analyzing the dynamic behavior of a bank and the development of the bank's balance sheet. The model is applied to analyzing the bank's actions before and during a financial crisis in a situation where it is uncertain when the crisis is going to start and whether it is going to materialize at all. The model is an optimization model, in which the bank's funding and lending, as well as their different maturities, are modeled explicitly, thereby exhibiting the process of maturity transformation.

Much of the literature in economics relating to crises in banking considers bank runs, such as the well-known paper by Diamond and Dybvig (1983) and contagious bank runs, i.e. bank panics, such as the papers e.g. by Allen and Gale (1998), Bougheas (1999), and Dasgupta (2004). Other models, concerning financial crises from the viewpoint of systemic risk and financial contagion, include e.g. the simulation-based models by Iori et al. (2006) and, more recently, by Ladley (2010).

Microeconomic models focusing on the management of a bank's balance sheet are rare. Some models of this kind may be found within the Industrial Organization approach to banking (see e.g. Freixas and Rochet (2008)). One model along these lines which considers the behavior of an individual bank is presented by Klein (1971). In contrast, in the field of management science there is a substantial amount of literature regarding the modeling of individual banks and their balance sheets. These models are often Asset-Liability Management (ALM) models based on the methodology of *stochastic programming* (see. e.g. Dupacova (1995)). Such models are presented e.g. by Kusy and Ziemba (1986), Booth et al. (1989), Oguzsoy and Guven (1997), and Mulvey and Shetty (2004).

This paper applies the methodology of stochastic programming, more commonly used in the field of management science, to the field of economics. The methodology and the dynamic management of the balance sheet of the modeled bank differentiate the model presented here from typical banking models in the field of economics. The methodology allows for a detailed analysis of the dynam-

ics relating to the bank’s optimal adjustments of its balance sheet. On the other hand, compared to existing management science literature involving stochastic programming ALM models, the main difference is that instead of presenting a model for the purpose of financial planning in a bank, the focus here is on developing a model for getting general-level, economic insight into the actions of a bank facing a financial crisis. Therefore, compared to similar models in the field of management science, the amount of parameters is relatively low and the model structure is relatively simple. The simplified model structure allows for increased transparency of the functioning of the model and makes it possible to draw economic conclusions from the outcomes. Another difference to typical stochastic programming models is the different structure of the scenario tree applied. In comparison to many stochastic programming models, the scenario tree structure applied here is very simple, as it involves a very small number of points where the scenario tree diverges into more than one subsequent scenarios. This allows for a large number of decision-making periods whereas the number of alternative economic scenarios is modest. As a result of the larger number of periods, shorter period lengths may be applied, which in turn allows for more accurate modeling of decisions relating to lending and funding happening on a relatively short time-scale.

The purpose of this paper is to present a model for analyzing the dynamic behavior of a bank, particularly in a situation potentially involving a financial crisis, and to confirm that the model behaves logically. The model has potential for being applied to the analysis of a wider range of crises, scenario structures and parameter configurations than the ones considered here. The model is presented in Section 2. The outcomes of the model are analyzed in the case of financial crises of three different types. First, Section 3.1.1 considers a crisis involving defaulting loans. Then, Section 3.1.2 considers a liquidity crisis, i.e. a crisis where the bank has difficulties obtaining new funding from the market. Third, Section 3.1.3 combines these two basic types of crises by considering a liquidity crisis involving defaulting loans. Finally, Section 3.2 considers the impact that different parameter configurations have on the bank’s behavior before a financial crisis starts, by varying the values of parameters defining the type, severity and duration of the crisis.

2 The Model

2.1 Assumptions Regarding the Model Structure

The methodology applied in the model is that of *scenario based multistage stochastic programming with recourse* (see. e.g. Dupacova (1995)). The model is formulated as a linear programming model. The model differs from typical stochastic programming models in that the total number of scenarios in the scenario tree is very small relative to the number of periods. The planning horizon considered is divided into a large number of periods in order to allow for a high resolution in the modeling of the bank’s dynamic behavior.

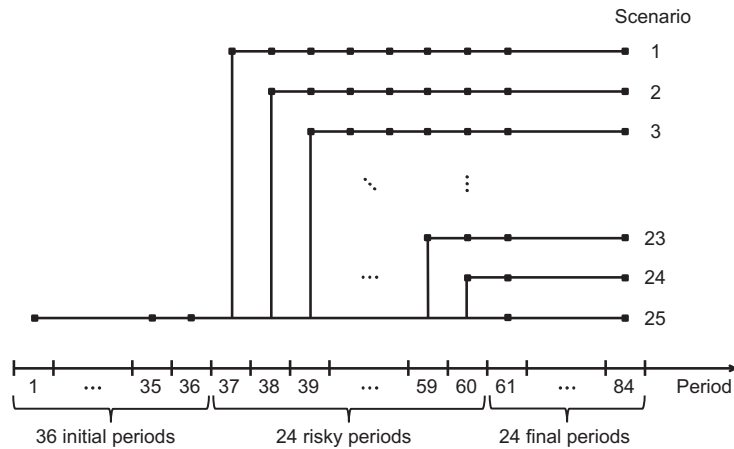


Figure 1: The scenario tree structure of the model.

Figure 1 presents the scenario tree structure applied in the model. The total number of periods in the model is 84, and each period represents the time span of one month. The first 36 periods, i.e. the *initial periods* are spent in preparation for the crisis, and it is assumed, that there is no stochasticity during these. The next 24 periods, i.e. Periods 37-60, are *risky periods*, during which the crisis may or may not start. Once the crisis has started, it is assumed that the bank knows how it is going to continue in that scenario. The 24 *final periods*, i.e. Periods 61-84, constitute a phase during which the crisis will not start anymore, but may continue, if it has started in one of the last risky periods. These last 24 periods do not involve stochasticity. The issues involving uncertainty in the model are thus whether the crisis will materialize at all and at what point in time it will start. There is a total of 25 scenarios in the model. Scenarios 1-24 represent the outcomes in which a crisis starts in Periods 37-60, respectively. Scenario 25 represents the outcome where the crisis does not materialize at all. In case that the crisis materializes, it is assumed to last for 6 consecutive periods. Figure 2 depicts the scenario-period combinations of the model.

The assumptions regarding the scenario tree structure are of course simplifications. In reality, e.g. a bank in a financial crisis would not know for sure how the crisis would be going to continue. In the context of analyzing the problem at hand, such simplifications may, however, be considered appropriate, as the focus is on getting an overview of the precautionary measures taken by a bank in preparation for an uncertain crisis, and on its reactions to a crisis of a specified kind. A simple scenario tree structure also improves the transparency of the model. Many of the simplifying assumptions of the model, such as those relating to the scenario tree structure, could be relaxed in future extensions of the model.

The asset side of the bank's balance sheet is assumed to consist of liquid

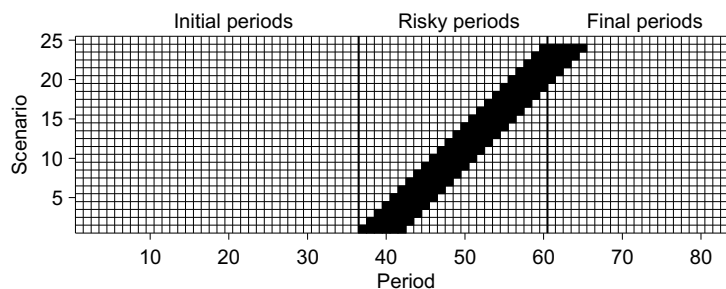


Figure 2: The scenarios and periods in the model. There are 84 time periods and 25 scenarios. The boxes painted in black represent those scenario-period combinations where there is a financial crisis going on. The financial crisis lasts for 6 consecutive periods. Scenario 25 represents the case that the financial crisis does not materialize at all.

Table 1: The initial balance sheet of the bank.

Cash	0	Deposits	90
Loans	100	Equity	10

assets represented by Cash, and non-liquid assets represented by Loans. The liability side is assumed to consist of Deposits, which are assumed to include, in addition to actual deposits, also a number of other forms of funding, such as borrowing from other financial institutions. In addition, the balance sheet contains Equity. While being a simplification, the balance sheet applied in the model is sufficient for capturing some of the most relevant aspects of the functioning of a bank, and for analyzing some of the most relevant decisions of the bank. These decisions include the allocation of the bank’s funds between liquid assets (Cash) and less liquid assets (Loans), as well as the amount of funding (Deposits) which is used to adjust the size of the bank’s balance sheet. The maturities of the bank’s Loans are longer than those of its Deposits, exhibiting the bank’s function of maturity transformation, which is a central element of the model. The amounts of each balance sheet item in the initial balance sheet are presented in Table 1.

2.2 The Equations

Tables 2 and 3 list the parameters and variables of the model, respectively. The indices $i \in 1 \dots I$ and $j \in 1 \dots J$ refer to the scenarios and periods of the model, respectively. The values of parameters $d_L^{i,j}$ and $m_D^{i,j}$ define the economic environment at a given point in time in a given scenario by determining the amounts of defaulting loans and the availability of new funding. During a crisis

Table 2: The parameters of the model. The values of parameters $d_L^{i,j}$ and $m_D^{i,j}$ presented in the table represent a situation, where there is no financial crisis going on, and they will obtain different values during financial crises.

Parameter	Value	Definition
I	25	Number of scenarios
J	84	Number of periods
J_1	36	Number of initial periods
$C^{i,0}$	0	Initial amount of Cash
$L^{i,0}$	100	Initial amount of Loans
$D^{i,0}$	90	Initial amount of Deposits
$E^{i,0}$	10	Initial amount of Equity
r_L	0.003274	Interest on lending per period (4 % per year)
r_D	0.001652	Interest on deposits per period (2 % per year)
τ_L	24	Maturity of loans (periods)
τ_D	12	Maturity of deposits (periods)
$d_L^{i,j}$	0	Fraction of maturing loans defaulting
$m_D^{i,j}$	0.5	Maximum amount of new deposits as a fraction of equity
$P^1 \dots P^{24}$	0.5/24	Probabilities of the scenarios involving a financial crisis
P^{25}	0.5	Probability of the scenario involving no financial crisis

Table 3: The variables of the model.

Variable	Definition
$C^{i,j}$	Cash
$L^{i,j}$	Loans
$D^{i,j}$	Deposits
$E^{i,j}$	Equity
$\Delta_L^{i,j}$	New Loans
$\Delta_D^{i,j}$	New Deposits
$M_L^{i,j}$	Loans maturing
$M_D^{i,j}$	Deposits maturing

involving defaulting loans, the parameter $d_L^{i,j}$ obtains positive values. This case is considered in Section 3.1.1. Similarly, during a liquidity crisis, the parameter $m_D^{i,j}$ obtains values that are small or zero. This case is considered in Section 3.1.2. Both of these aspects are present in the case of a liquidity crisis involving defaulting loans, which is considered in Section 3.1.3.

The bank is assumed to maximize its expected final-period utility. The bank's utility is assumed to equal its scenario-specific amount of Equity and thus the bank is assumed to be risk-neutral. Therefore, the corresponding objective function, presented in Equation 1 is defined as the expected amount of Equity in the final period.

Equations 2 and 3 define how the amounts of Loans and Deposits develop. They are decreased by the amounts maturing and increased by the amounts of new Loans or new Deposits. Equation 4 defines the amount of Equity based on Equity in the previous period and the profits obtained in the current period. Profits depend on the interest income on Loans, the interest costs of holding Deposits and the losses from defaulting loans.

Equation 5 ensures that the balance sheet identity holds, i.e. that assets and their funding are equal. The balance sheet identity makes it unnecessary to define a separate equation for the amount of Cash in the balance sheet, since it is implied by the amounts of Loans, Deposits and Equity.

It is assumed that the principals relating to Loans and Deposits are paid back at maturity. The maturities of Loans and Deposits are assumed to differ from each other, which reflects the bank's function of maturity transformation. It is assumed that the Loans initially in the balance sheet are composed of equal amounts of securities of each maturity, thus determining the amounts maturing in Periods $j \in 1 \dots \tau_L$. Similarly, the amounts of Deposits maturing in Periods $j \in 1 \dots \tau_D$ are determined by the Deposits initially in the balance sheet. These assumptions are reflected by Equations 6 and 8.

Equations 7 and 9 determine the amounts of Loans and Deposits maturing in periods in which all the securities held initially have already matured. Therefore, in these periods, the amounts of Loans and Deposits maturing are determined by the amounts of new Loans and new Deposits τ_L and τ_D periods earlier, respectively.

The bank's ability to obtain new funding in the form of Deposits is defined by Equation 10. It is assumed that Deposits are available in a linear relation to the amount of equity that the bank has at any time. On one hand, this assumption represents, as a proxy of the size of the bank, its ability to attract depositors, and on the other hand it represents, in the form of a measure of solvency, its ability to obtain funding from other financial institutions.

In addition to these constraints, there are *nonanticipativity constraints* (see e.g. Birge (1995) or Dupacova (1995)), which define the structure of the scenario tree, while at the same time defining the information that the decision-maker has in different scenarios at different points in time. These constraints are defined for all decision variables in the model. The nonanticipativity constraints defining the scenario tree structure shown in Figure 1 are defined by Equation 11 where $X^{i,j}$ is a vector containing the values of all the decision variables in Scenario i at

time j . The nonanticipativity constraints state that before the crisis begins, it is impossible to distinguish a given scenario from the scenario where the financial crisis does not materialize at all. This also implies that scenarios, in which the crisis has not yet begun in a given period, cannot be distinguished from each other. It should be noted that before the crisis starts in a given scenario, also the parameters defining the economic environment are equal to those in the scenario where the crisis does not materialize at all. The nonanticipativity constraints are consistent with Figure 2, which shows the combinations of scenarios and periods where there is a financial crisis going on.

The objective function:

$$Max \sum_{i=1}^I P^i E^{i,J} \quad (1)$$

The constraints defining the positions:

$$L^{i,j} = L^{i,j-1} - M_L^{i,j} + \Delta_L^{i,j} \quad \forall i \in 1 \dots I, j \in 1 \dots J \quad (2)$$

$$D^{i,j} = D^{i,j-1} - M_D^{i,j} + \Delta_D^{i,j} \quad \forall i \in 1 \dots I, j \in 1 \dots J \quad (3)$$

$$E^{i,j} = E^{i,j-1} + r_L L^{i,j-1} - r_D D^{i,j-1} - d_L^{i,j} M_L^{i,j} \quad \forall i \in 1 \dots I, j \in 1 \dots J \quad (4)$$

The constraints defining the balance sheet identity:

$$C^{i,j} + L^{i,j} = D^{i,j} + E^{i,j} \quad \forall i \in 1 \dots I, j \in 1 \dots J \quad (5)$$

The constraints defining the amounts of maturing Loans and Deposits:

$$M_L^{i,j} = L^{i,0} / \tau_L \quad \forall i \in 1 \dots I, j \in 1 \dots \tau_L \quad (6)$$

$$M_L^{i,j} = \Delta_L^{i,j-\tau_L} \quad \forall i \in 1 \dots I, j \in (\tau_L + 1) \dots J \quad (7)$$

$$M_D^{i,j} = D^{i,0} / \tau_D \quad \forall i \in 1 \dots I, j \in 1 \dots \tau_D \quad (8)$$

$$M_D^{i,j} = \Delta_D^{i,j-\tau_D} \quad \forall i \in 1 \dots I, j \in (\tau_D + 1) \dots J \quad (9)$$

The constraints defining the maximum amount of new Deposits:

$$\Delta_D^{i,j} \leq m_D^{i,j} E^{i,j} \quad \forall i \in 1 \dots I, j \in 1 \dots J \quad (10)$$

The nonanticipativity constraints:

$$X^{i,j} = X^{I,j} \quad \forall i \in 1 \dots I-1, j \in 1 \dots J_1 + i - 1 \quad (11)$$

3 Results

3.1 Dynamic Behavior of the Bank

3.1.1 Case: Defaulting Loans

The first type of financial crisis considered is one during which some of the bank's loans default at maturity, whereas it is assumed that there is no liquidity crisis going on, i.e. the bank's ability to obtain new funding is not reduced. It is assumed that during the crisis, 25% of the loans maturing in each period default, i.e. $d_L^{i,j} = 0.25$, while the other values of the parameters presented in Table 2 remain unchanged.

Figure 3(a) depicts how the amounts of each balance sheet item develop over the planning horizon in case of the middle scenario, i.e. Scenario 12, in which the financial crisis starts in Period 48. Figure 3(b), on the other hand depicts the developments in the case that the financial crisis does not materialize at all. It can be seen, that the two scenarios are identical before Period 48, i.e. before the crisis starts. Figures 4(a) and 4(b) depict how Cash and Loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of Deposits and Equity are depicted in Figures 5(a) and 5(b), respectively.

The bank starts to accumulate Cash before the crisis begins, while at the same time lending less. By doing so, it avoids losses from Loans that would be maturing and defaulting during the crisis. A few periods into the risky phase, if the crisis has not started yet, the bank transfers the Cash, that it has been accumulating, back into Loans, since the Loans, having long maturities, will be maturing only when the potential crisis is already over. When the crisis starts, the bank's lending is reduced, since the losses it takes from defaulting Loans reduce its Equity and thus reduce its ability to obtain new funding.

3.1.2 Case: Liquidity Crisis

The second kind of crisis considered is a liquidity crisis, in which the bank's ability to obtain funding (Deposits) is drastically reduced, whereas the crisis considered here does not involve defaulting Loans. It is assumed that during the crisis $m_D^{i,j} = 0.1$, while the other values of the parameters presented in Table 2 remain unchanged.

Figures 3(c) and 3(d) depict the development of the bank's balance sheet in the middle scenario and in the scenario in which the financial crisis does not materialize at all. Figures 4(c) and 4(d) depict how Cash and Loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of Deposits and Equity are depicted in Figures 5(c) and 5(d), respectively.

The bank starts to accumulate Cash instead of Loans before the crisis. It needs the extra Cash in order to be able to repay the Deposits that mature during the crisis, since Loans, having longer maturities than Deposits, mature too slowly in order to provide a sufficient inflow of cash while the lack of liquidity reduces its ability to obtain new funding for repaying its maturing Deposits. This way, the bank can remain sufficiently prepared for the potential liquidity crisis. During the liquidity crisis, the bank's lending is reduced even further as new funding is scarcely available.

3.1.3 Case: Liquidity Crisis with Defaulting Loans

The third kind of crisis considered is a combination of the two types of crises considered above, i.e. a combination of a liquidity crisis and a crisis involving defaulting loans. It is thus assumed, that during the crisis, 25% of the loans maturing default while at the same time the bank's ability to obtain new funding is reduced. Thus, $d_L^{i,j} = 0.25$ and $m_D^{i,j} = 0.1$ during the crisis.

Figures 3(e) and 3(f) depict the development of the bank's balance sheet in the middle scenario and in the scenario in which the financial crisis does not materialize at all. Figures 4(e) and 4(f) depict how Cash and Loans, respectively, develop over the planning horizon in all the 25 scenarios of the model. The corresponding developments of Deposits and Equity are depicted in Figures 5(e) and 5(f), respectively.

The bank starts to reduce the size of its balance sheet before the crisis begins. The defaulting of Loans together with the bank's difficulty of obtaining new Deposits for repaying maturing older Deposits during the crisis, force the bank either to hold less Deposits, or to hold Cash reserves in order to avoid defaulting. As there is an implicit cost on holding Cash reserves (i.e. the cost of interest on Deposits) and the attractiveness of lending is reduced by the risk of defaults, the bank chooses to hold less Deposits while at the same time lending less. When the crisis starts, the bank's lending is further reduced as it becomes difficult to obtain new funding.

A few periods into the risky phase, if the crisis has not started yet, the bank starts to increase its lending again, since after a certain point, new Loans will be maturing only when the potential crisis is already over. This requires additional funding in the form of Deposits. Part of the new funding is held in the form of Cash, in order for the bank to be able to survive without defaulting in the event that the crisis would still materialize.

3.2 Varying the Parameters Defining the Crisis

The bank's behavior before and during the financial crisis is affected by the type of crisis considered. In this section, the values of the parameters defining the type, severity and duration of the anticipated crisis are varied, and the implications on the bank's dynamic behavior are considered.

The surface plots in Figure 6 depict the development of the amount of Loans in the bank's balance sheet in the scenario where the crisis does not materialize

at all. The development is depicted at different values of the parameters determining the type, severity and duration of the crisis. While these plots consider only the case where no crisis materializes, they nonetheless provide insight into how the bank prepares for the potential crisis at different parameter values.

Based on Figure 6, it can be concluded that crises involving more defaults, or more significant reductions in the availability of funding, induce the bank to lend less, as a precautionary measure. However, such precautionary measures appear to be taken only if the potential crisis is expected to be severe enough. If the duration of the anticipated crisis is longer, more drastic precautionary measures appear to be taken.

4 Conclusions

This paper presents a model, the purpose of which is to analyze the optimal dynamic behavior of a bank in an environment that involves stochastic elements. The model sheds light on how a bank engaging in maturity transformation adjusts its balance sheet before and during a financial crisis. The scenario tree structure, as well as a number of parameters that are specified for each scenario and period, define the uncertainties involved. In this paper, there is assumed to be uncertainty as to when the crisis is going to start and also whether it is going to materialize at all. While the sources of uncertainty considered are relatively simple, the model could also be applied to more complex scenario tree structures.

Simplifications had to be made in order to maintain sufficient transparency of the model and to be able to draw conclusions regarding the impacts of different parameters. Despite the simplifications, the model includes features that are very central to the banking firm. These features include e.g. maturity transformation, the adjustment of the size of the bank's balance sheet, and the adjustment of the share of liquid assets in the balance sheet.

The outcomes of the model show that the bank takes actions in order to prepare for the anticipated financial crisis even though it is uncertain whether the crisis is going to materialize. These actions illustrate the bank's optimal dynamic behavior subject to a number of constraints, that e.g. force the bank to keep its balance sheet in such a condition that it will not default on its Deposits. The analysis considers financial crises that involve defaulting loans, reduced availability of funding (i.e. a liquidity crisis), and combinations of these two basic types of crises.

Observations that can be made based on the outcomes of the model include e.g. the bank's tendency to reduce its lending in preparation for an anticipated financial crisis as well as its tendency to accumulate cash reserves in order to maintain sufficient liquidity. The fact that the bank holds cash reserves even though they provide zero returns contradicts the assumption sometimes made in the theory of banking that banks would only hold the minimum amount of cash required by regulations (Freixas and Rochet, 2008, p. 71). Running the model at different parameter configurations also indicates that the more

severe the anticipated crisis and the longer its duration, the more drastic are the precautionary measures taken in preparation for the crisis.

The actions taken by the bank in some of the outcomes of the model may seem unrealistically radical e.g. involving significant changes in the composition and size of the balance sheet. In reality, a bank's actions may be constrained e.g. by market illiquidity, transaction costs, policies, regulations etc. However, constraining the model as little as possible gives good insight into the incentives of a banking firm in the economic situations considered. On a general level, the phenomena observed by using a model such as the one considered here are likely to appear also in more constrained and realistic environments. Despite being perhaps somewhat radical, the bank's actions in the outcomes of the model appear to be logical and they are in line with what one might intuitively expect to observe. The model is sufficiently detailed in order to be able to capture relevant dynamics of a bank facing a financial crisis. The bank also appears to react logically to different values of the parameters determining the type, severity, and duration of the crisis.

The model presented here could be used to analyze the dynamic behavior and incentives of a generic bank in different kinds of dynamic economic and financial environments involving stochastic elements. Whereas the main purpose of this paper is to present the model and verify that it behaves logically, further research could be carried out in more realistic settings, using the same model or a model closely resembling the one presented here. This could involve redefining the scenario tree structure and using different values for the parameters defining the economic environment in the scenarios and periods of the model.

In the model presented here, the future is completely known to the bank at the moment when the financial crisis starts. A more realistic assumption would be that both the length of the crisis and the development of its severity would be unknown to the bank in advance. One approach to modeling macroeconomic developments could be to construct a Markov chain for representing the transitions from one state of the economy to another. These states could differ from each other at least for the part of the values of the parameters defining the percentage of loans defaulting and the availability of new funding. Also the interest rate parameters could be made dependent on the economic environment. Incorporating a stochastic process of this kind into the scenario tree would require a far larger number of scenarios. This might turn out to be computationally challenging and the lesser transparency of the outcomes would make it harder to interpret the results. It would however constitute a more realistic way of modeling the information that the bank has at a given moment. The development of a financial crisis could then also be more realistically specified.

One possible modification to the equations defining the optimization model would be to allow the bank to default on its own Deposits, while such defaults perhaps could be assumed to have negative impacts on the bank's utility. A benefit of this relaxation would be that the optimization model would then not turn out to be infeasible, i.e. to have no solutions satisfying all constraints, in situations where no precautionary measures could have saved the bank from defaulting. This modification would increase the flexibility of using the model,

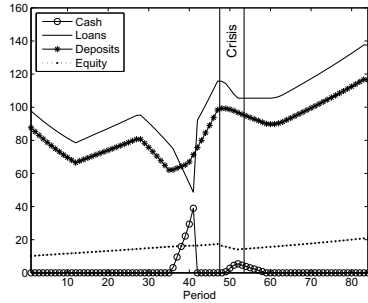
as it would allow a larger range of potential future economic developments to be considered, including crises of extreme severity. In addition, this would increase the realism of the model, as defaults do happen in the real world, and also since it is unlikely that banks have the ability or the incentives to take sufficient precautions against *every* scenario, some of which may have very low probabilities.

Furthermore, the analysis could be extended from the context of a single bank to that of an interbank market. One could for example assume that some fraction of the Loans in the balance sheet would represent lending to other banks or, alternatively, one could model interbank lending as a separate balance sheet item. In any case, the willingness of a bank to lend to other banks could partly determine the availability of funding to the other banks. Thus, the model could be extended to include the analysis of systemic risks relating to the banking sector.

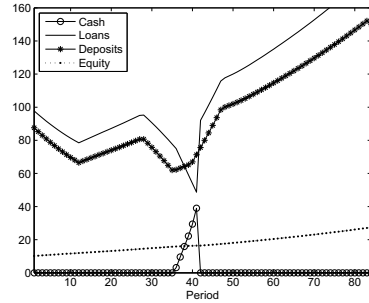
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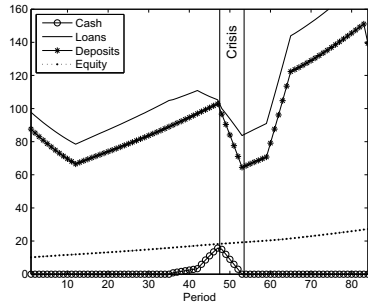
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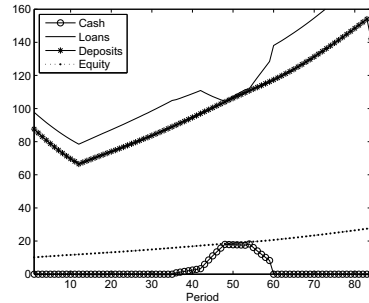
(a) Defaulting Loans: Middle scenario



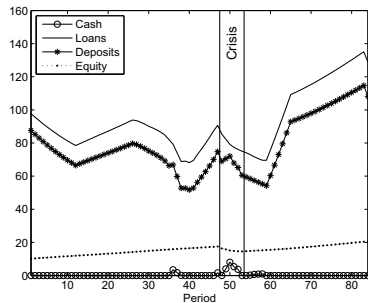
(b) Defaulting Loans: No crisis materializes



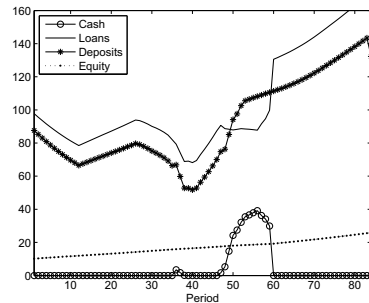
(c) Liquidity crisis: Middle scenario



(d) Liquidity crisis: No crisis materializes

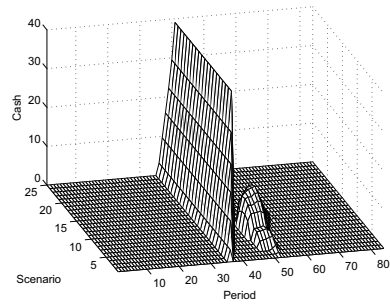


(e) Liquidity crisis with defaulting Loans: Middle scenario

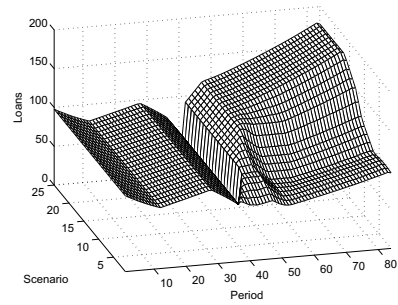


(f) Liquidity crisis with defaulting Loans: No crisis materializes

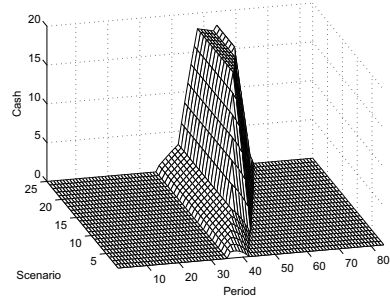
Figure 3: Development of the balance sheet in the three types of crises considered. Panels (a), (c), and (e) depict the middle scenario (Scenario 12), where the crisis begins in the middle of the risky phase (in Period 48). Panels (b), (d), and (f) depict the scenario where the crisis does not materialize at all (Scenario 25).



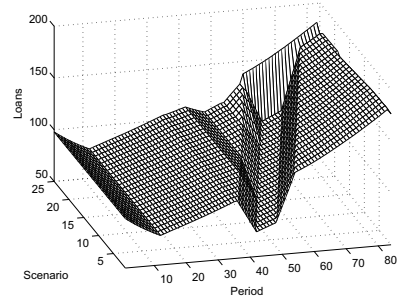
(a) Defaulting Loans: Amount of Cash



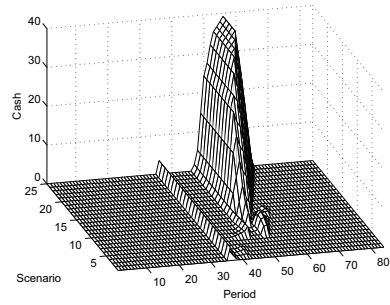
(b) Defaulting Loans: Amount of Loans



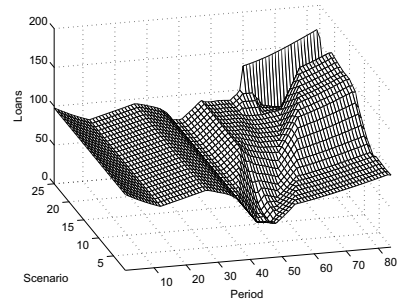
(c) Liquidity crisis: Amount of Cash



(d) Liquidity crisis: Amount of Loans

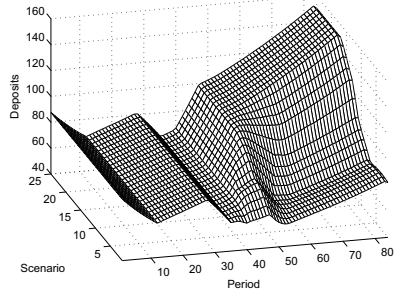


(e) Liquidity crisis with defaulting Loans:
Amount of Cash

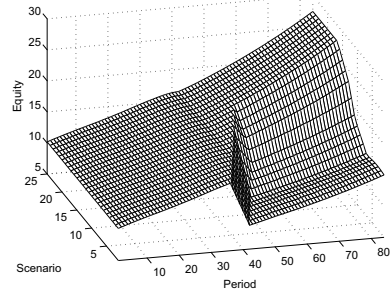


(f) Liquidity crisis with defaulting Loans:
Amount of Loans

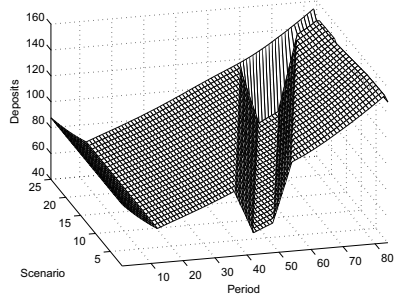
Figure 4: Development of the amounts of Cash and Loans in the three types of crises considered.



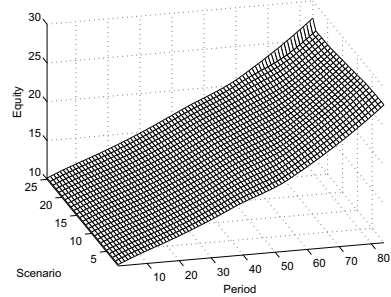
(a) Defaulting Loans: Amount of Deposits



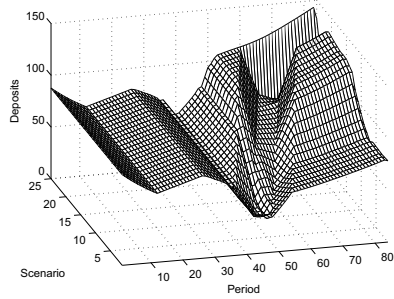
(b) Defaulting Loans: Amount of Equity



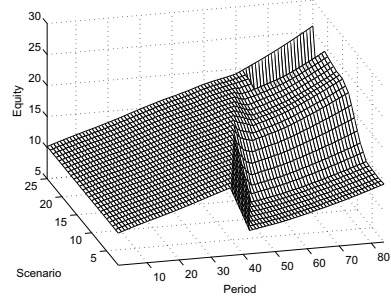
(c) Liquidity crisis: Amount of Deposits



(d) Liquidity crisis: Amount of Equity

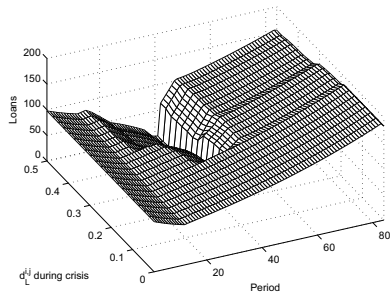


(e) Liquidity crisis with defaulting Loans:
Amount of Deposits

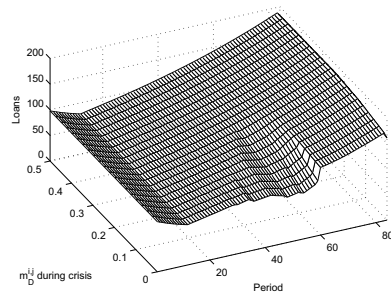


(f) Liquidity crisis with defaulting Loans:
Amount of Equity

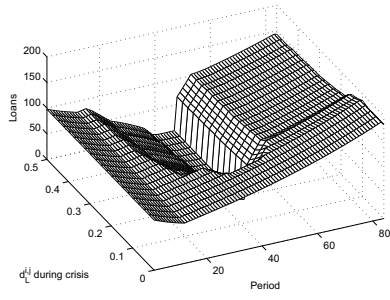
Figure 5: Development of the amounts of Deposits and Equity in the three types of crises considered.



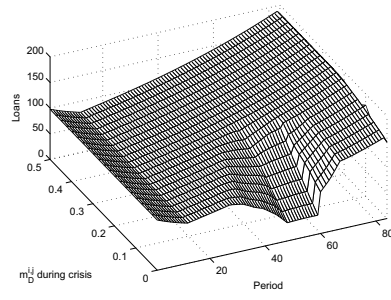
(a) Amount of defaults varied.
Duration of crisis: 6 periods.



(b) Availability of funding varied.
Duration of crisis: 6 periods.



(c) Amount of defaults varied.
Duration of crisis: 12 periods.



(d) Availability of funding varied.
Duration of crisis: 12 periods.

Figure 6: Development of Loans in the scenario where the crisis does not materialize, considered at different values of the parameters determining the type, severity and duration of the crisis.