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STATISTICAL MODELLING OF DAMAGE EVOLUTION IN SPALLATION

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Résumé - En vue de comprendre le mécanisme d'initiation de l'écaillage dans les métaux laminés, un modèle statistique unidimensionnel de l'évolution des microfissures lors de l'écaillage est proposé. Pour cette description statistique la longueur de fissure semble être la variable fondamentale. Deux processus dynamiques, la nucléation des fissures et leur croissance, sont associés dans le modèle d'évolution du dommage.
Un cas simplifié est examiné et des corrélations préliminaires avec les observations expérimentales de l'écaillage sont données.

Abstract - In order to understand the mechanism of the incipient spallation in rolled metals, a one dimensional statistical model on evolution of microcracks in spallation was proposed. The crack length appears to be the fundamental variable in the statistical description. Two dynamic processes, crack nucleation and growth, were involved in the model of damage evolution. A simplified case was examined and preliminary correlation to experimental observations of spallation was made.

1 - INTRODUCTION

Spallation resulted from tensile stress pulse in impacted materials, provides a convenient approach to the study of the damage evolution. In fact, damage is almost always dynamic and hence rate - dependent, as shown in the empirical formula in spallation /1/

$$\left(\frac{\sigma}{\sigma_o} - 1\right)^m \Delta T = K \qquad (1)$$

where σ is stress, ΔT is the duration of the tensile loading,m and K are parameters. In addition, it was shown that the parameters are dependent on the degree of damage of spalled specimens /2/. Therefore, how to reveal and describe the implication of this rate-dependent process arouses great interest in mechanical study and engineering practice.

There are some models deliberating about the damage accumulation in spallation /3/. Particularly, the statistical models of microdamage are showing greater promise /4, 5/, for instance, the approach of microstatistical fracture mechanics /6,7/. In these works, there are detailed descriptions of empirical distribution functions and the actual counts and measurements of microcrack numbers, sizes and orientations. But there is just a little mentioned about the evolution law of the system of microcracks. Therefore it seems that there is a need to establish some general formulation of the evolution of microdamage, according to the fundamentals of statistics.

In this paper, it is intended to set up a simplified one dimensional statistical model on damage due to microcracks in spallation, based on the equation of evolution. This concerns a system, consisting of a variable number of microcracks with various

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length. In addition, the statistical system of microcracks is neither Hamiltonian nor quantum ones, since the motions of microcracks are different from those of classical Newtonian particles or quantum.

In section 2, a brief review is given on the observations of microcracks occuring in the spalled rolled aluminium alloy specimens in planar impact tests. The observed phenomenon provides the initiative of the present statistical study. Then the derivation of a one dimensional statistical model of microcracks is given in section 3. In particular, some implications of the equation of evolution will be emphasized. Finally, in section 4 a very simple system of microcracks will be studied as an illustrative example.

2 - OBSERVATION OF INCIPIENT SPALLATION IN A ROLLED ALUMINIUM ALLOY

All the specimens are circular thin sheets, less than 10mm thick, and taken from a rolled aluminium alloy plate. The specimens were tested under planar impact loading with a 101mm bore light gas gun. This configuration of testing guarantees the uniaxial strain loading on the central part of the target specimens. After impact, the tested specimens were softly recovered in a specially designed catcher to prevent from secondary damage owing to undesirable hitting. The recovered specimens were sectioned and carefully polished for microscopic observation.

The damage in the spalled specimens, under tensile stress wave loading resulted from the stress wave reflection, is in the form of distributive and roughly parallel microcracks on the sectioned surface, see Fig.1. The longer and wider the microcracks are, the more severe the damage is, thereupon the tested specimens present lower residual strength /2/.

However, as far as the incipient spallation is concerned, its main feature is a number of fairly distributive microcracks, and it seems that the nucleation and growth of individual microcracks are predominant, whilst the linkage or the interaction of the microcracks appears to be negligible at this stage. Our following statistical model is just limited to this kind of incipient spallation.

Some special observations were made recently in our laboratory. The technique of very short stress pulse, hundreds ns long (see Fig.2), was used in the tests /8/, in order to reveal the mechanism of nucleation of microcracks in spallation /9/. The tests show that both period and strength of the stress pulse significantly affect the level of the incipient spallation. It also shows that at this stage of incipient spallation nearly all the microcracks nucleate within or arround the second phase particles in the aluminium alloy, see Fig.3 /9/. Therefore it is quite believable from the observation that the size distribution of the inhomogeneities in the material predominately governs the distribution of the nucleation of microcracks as well as the growing cracks in the incipient spallation.

It has been pointed out in literature, for instance in /7/, that imperfections in a variety of materials have a size distribution of the following form

$$n\ (a) = n_o \exp(-\frac{a}{a_o}) \qquad (2)$$

where n(a) da is the number of imperfections per unit volume between the imperfection size a and a + da. n_o and a_o are two characteristic parameters of particular distribution. Also, there are some theoretical explanations about this quite unique distribution function of imperfections in materials. /4/.

More interestingly, some size distribution function of microcracks has been put forward in incipient spalled specimens /6/, which has the same exponential form as formula (2). Perhaps, this, in someway, touchs the intrinsical entity of the evolution of microcracks in spallation.

One may interpret the similarity of the two size distribution functions, by taking into account the following facts and assumptions. Firstly, the nucleation of microcracks by inclusion debonding may occur so rapidly under stress pulse loading that one can consider the process instantaneous compared to the tensile stress wave loading time. Secondly, the above mentioned observations (Fig.3) of nucleating microcracks being limited to the sizes of second phase particles offer a broad hint to correlate the size distribution functions of imperfections and microcracks. Nevertheless, since not all imperfections are broken or debonded simultaneously under wave loading and the nucleated microcracks would grow in some way, the above seemingly similarity would become groundless, unless one can find the full answer to the evolution of microcracks.

In summary, the observations of incipient spallation in the rolled aluminium alloy present a need and challenge to the description of the damage in terms of a micro-statistical model of one dimensional microcracks.

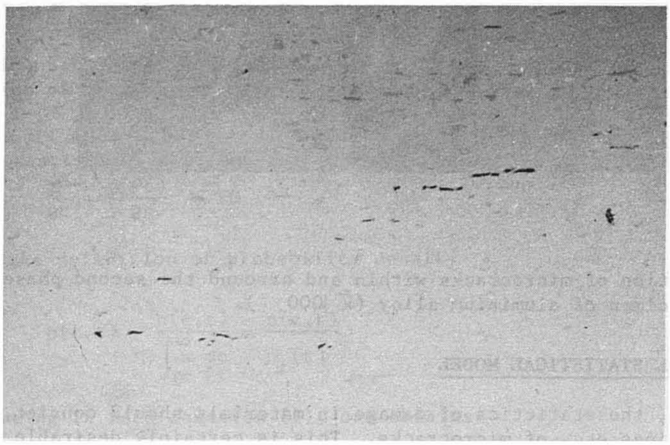

Fig.1 - Parallel microcracks on the sectioned surface of spalled specimen of aluminium alloy (x 200).

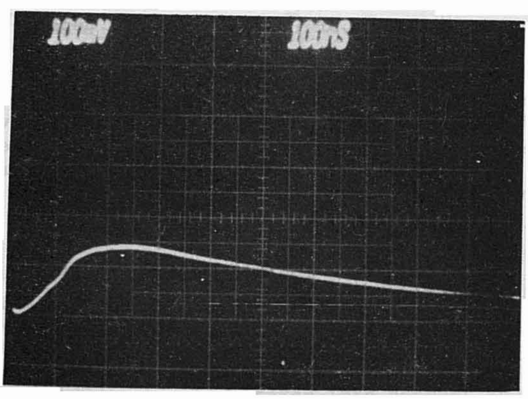

Fig.2 - The profile of input stress pulse recorded with carbon stress gauge embedded in specimen.

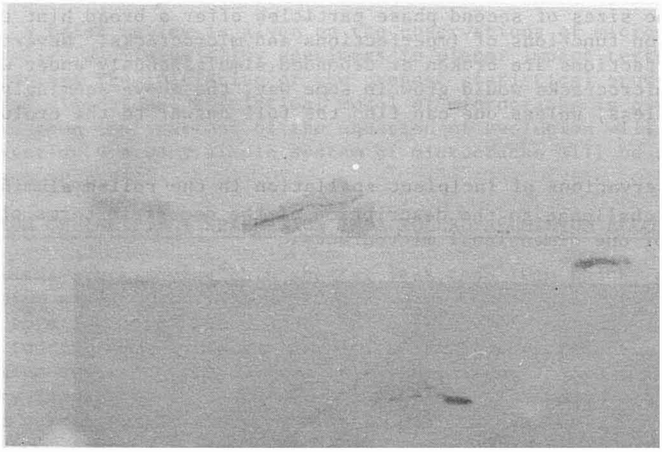

Fig.3 - The nucleation of microcracks within and arround the second phase particles in the spalled specimen of aluminium alloy (x 1000).

3 - ONE DIMENSIONAL STATISTICAL MODEL

Generally speaking, the statictics of damage in materials should consist of sizes, orientations, spacings etc. of microcracks. This is certainly desirable and worth investigating profoundly.

As a simplification, a plain one dimensional statistical model was proposed, in the light of the above observations, in order to explore some essential aspects of the evolution of micro-cracks. It is assumed in the model that all microcracks are parallel to each other and perpendicular to the tensile loading. Therefore, one stochastic variable c, the length of microcrack, is involved in the model. Obviously, variable c is continuous in the domain of $(0, \infty)$ and time dependent.

Clearly, the statistics of microcracks is somewhat different from that of Newtonian particles. In the latter, the states, namely the positions and momenta q_i and p_i ($i=1,2,\ldots\ldots N$), in 6N dimensional phase space are the stochastic variables and the number N of the particles is fixed. But in the concerned case, the number of the microcracks is variable as well, owing to the nucleation of microcracks, though the probability density ρ should satisfy the normalisation condition

$$\int dq\ dp = 1 \tag{3}$$

with the view of damage mechanics, one is interested in the evolution of the probability density $\rho(t,c)$ of stochastic variable c as well as the concentration of microcracks, i.e. the number of microcracks $n\ (t,c)\ dc$ between the length of c and $c + dc$ in unit area.

The following assumptions are adopted in the model. (1) Only nucleation and growth of microcracks are vital in the incipient spallation. (2) Microcracks can merely grow. (3) The concentration of microcracks is just determined by the adjacent states and the crack nucleation. (4) The microcrack velocity $\dot{c}$ is a deterministic function of crack length c and time t. Then in a small region of c and $c + dc$ in the phase space, the increase of the number of microcracks is controlled by two terms: the nucleation rate of microcracks and the flux of the microcracks into and out of the crack length interval (c, c + dc).

$$\left.\frac{\partial n}{\partial t}\right|_{(c+\theta dc,t)} dc = n_N\,(c+\theta dc)\,dc \qquad (4)$$

$$+\, n(c,t)\,\dot{c}\,(c,t) - n(c+dc,t)\,\dot{c}\,(c+dc,t)$$

where $0 < \theta < 1$, n_N denotes the nucleation rate at crack length c and $\dot{c}$ denotes the crack velocity. Let $dc \rightarrow 0$, one can obtain the basic equation

$$\frac{\partial n}{\partial t} + \frac{\partial (n\dot{c})}{\partial c} = n_N(c,t) \qquad (5)$$

This type of equation can be found in some literature, for instance in /5/. But the very evolution of microcracks has been seldom treated.

It has been well known that the probability density follows similar equilibrium equation in statistical physics, i.e.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho\dot{c})}{\partial c} = 0 \qquad (6)$$

Considering the definition of probability density

$$\rho(c,t) = \frac{n(c,t)}{\int_0^{\infty} n\, dc} = \frac{n(c,t)}{n_t(t)}$$

and substituting it into equation (5), one obtains

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho\dot{c})}{\partial c} = \frac{n_N}{n_t} - \frac{\dot{n}_t}{n_t}\,\rho \qquad (7)$$

When $n_N = 0$, $\dot{n}_t$ becomes zero, hence equation (7) is reduced to equation (6). In fact, the integration of the right hand side of equation (7) is

$$\int_0^{\infty} \left(\frac{n_N}{n_t} - \frac{\dot{n}_t}{n_t}\rho\right) dc = \frac{1}{n_t}\left(\int_0^{\infty} n_N\, dc - \dot{n}_t \int_0^{\infty} \rho\, dc\right)$$

$$= \frac{1}{n_t}\left(\int_0^{\infty} n_N\, dc - \dot{n}_t\right) \qquad (8)$$

$$= 0$$

in accord with the normalisation condition.

Therefore, both equations (5) and (7) are the basic equations of the evolution of damage for the system of microcracks with deterministic law of crack motion, $\dot{c}=\dot{c}(c,t)$. Clearly, this provides the simplest representation of microcracks system. Differently, a transition probability has been taken into account in the basic equation of evolution in an otherwise paper.

4 - ILLUSTRATIVE EXAMPLE

In statistical physics, the equation of probability density ρ, i.e. equation (6) will show the simple behaviour similar to incompressible fluid

$$\frac{d\rho}{dt} = 0 \tag{9}$$

namely the probability density ρ remains a constant, provided the observation is mode on a fixed 'particle' in the Hamiltonian system. This presents a clear picture for people to understand the implication of the statistics.

Here, a preliminary and illustrative example will be presented to show what may happen in the system of microcracks.

Suppose that the tensile load be a Heaviside function, hence the crack nucleation rate n_N will become a function of single variable, crack length c, with a parameter of constant stress σ, $n_N = n_N(c;\sigma)$. In the present formulation of microstatistics, the motion of microcracks are assumed to be deterministic $\dot{c} = \dot{c}(c,t;\ldots)$. As an illustrative example, we choose $\dot{c}$ = constant. This supposition does not seem to be a mere conceptual exercise, since there seems to be some hints to show that crack velocity approaches constant quickly under constant loading.

In accord with the theory of the first order partial differential equations, the characteristic equations of the partial differential equation (5) are

$$dt = \frac{dc}{\dot{c}} \tag{10}$$

$$\frac{dc}{\dot{c}} = \frac{dn}{n_N(c)} \tag{11}$$

The general solution to the original partial differential equation (5), therefore, has the form

$$\phi\left(c - \dot{c}t,\ n - \frac{1}{\dot{c}}\int n_N(c)\, dc\right) = 0 \tag{12}$$

where ϕ is an arbitarary function. The solution can be written as explicit form of n (c,t)

$$n(c,t) = \frac{1}{\dot{c}}\int n_N(c)\, dc + \varphi(c - \dot{c}t) \tag{13}$$

To determine the arbitrary function φ , one should consider the initial and boundary conditions. The initial condition is assumed that there are no microcracks at all at initial time, i.e.

$$t = 0, \qquad n(c) = 0 \tag{14}$$

Substitution of (14) into formula (13) leads to

$$\varphi(c) = -\frac{1}{\dot{c}}\int n_N(c)\, dc \tag{15}$$

So, the solution (13) has the simple formulation

$$n(c,t) = \frac{1}{\dot{c}}(-N(c) + N(c-\dot{c}t)) \tag{16}$$

$$N(c) = -\int_{\infty}^{c} n_N(c)\, dc = \int_{c}^{\infty} n_N(c)\, dc \tag{17}$$

where N(c) denotes the total number of nucleating microcracks greater than c. When crack length c tends to infinity,

$$n(\infty) = \frac{1}{\dot{c}}(-N(\infty) + N(\infty - \dot{c}t)) = 0 \tag{18}$$

This is physical sensible. Now the implication of the evolution law in this statistics of microcracks becomes quite clear. According to some empirical relations, $N(c)$ should be a decreasing function of crack length c. Hence, the concentration $n(c,t)$ of the microcracks is governed by the total number of microcracks nucleating between the crack length of $c-\dot{c}t$ and c. In this example it is obvious that the nucleation function of microcracks plays a significant role in the evolution of microcracks. Particularly, if

$$n_N = A \exp(-\frac{c}{c_o}) \tag{19}$$

then equation (17) gives

$$N(c) = c_o A \exp(-\frac{c}{c_o}) \tag{20}$$

and

$$n(c,t) = \frac{c_o A}{\dot{c}} \exp(-\frac{c}{c_o})(\exp(\frac{\dot{c}}{c_o} t) - 1) \tag{21}$$

The concentration of microcracks remains the exponential distribution, but increases with time t exponentially.

5 - CLOSURE

A simple one dimensional statistical model was proposed in the paper, in order to explore the actual evolution of microcracks occuring in incipient spallation analytically. The principal assumption involved in the model is a certain deterministic law of crack growth. Equation (5) is the fundamental one in the model. But both laws of nucleation and growth of microcracks are needed in solving the evolution equation. The assumption of constant velocity of microcracks makes the solution extremely easy. The solution, i.e. formulas (16) and (17), shows quite clear implication of the statistics: the concentration of microcracks $n(c,t)$ is equal to the difference of the total numbers of nucleating microcracks between the length $c-\dot{c}t$ and c. Provided that the rate of nucleation n_N is an exponential function of crack length, the concentration of microcracks will remain the same type. This is fairly in agreement with observations. More realistic model, as well as laws of crack nucleation and motion, are studied under way.

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