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Tiivistelmä/Referat – Abstract

Beginning from the 1990s the relationship between pollution and income (PIR) moved to the focus of research. Various studies have found that the PIR of several pollutants takes the shape of an inverted U. This coherence became known as the Environmental Kuznets Curve (EKC). However, later studies expressed criticism on the existence of the EKC and blurred the picture. For example, more diversified evidence suggests that the EKC is valid only for short living, local pollutants, whereas long living global pollutants face a monotonically rising PIR. These facts should be considered in the theoretical research on the EKC. One can summarise the considered theoretical explanations in five groups: Behavioural changes and preferences, institutional changes, technological progress, structural change and reallocation of polluting industries. This thesis focuses on technology progress as explanation for the EKC. Particularly, I investigate how technological progress in abatement affects the EKC.

To do so, I discuss two ways how the EKC arises from learning by doing in abatement. First following the work of Brock and Taylor (2003), I present how learning by doing causes constant returns to abatement on aggregate level. Furthermore, I assume that abatement is active only of the marginal disutility of pollution exceeds the marginal utility of consumption. As long as consumption is higher rated, capital is entirely spent on consumption, otherwise abatement is active such that pollution decreases while income still grows. This model results in the EKC. The second approach based on Egli and Steger (2007) is a generalisation of the first model. Learning by doing in abatement is modelled through increasing returns to scale in abatement. Here, the EKC arises without any further assumptions regarding abatement as in the first approach. Although the concept of learning by doing in abatement suggests that environmental policy does not influence the existence of the EKC, it is shown that regulation does affect its magnitude. Therefore, the EKC is no adequate symbol against environmental policy. Both models are analysed with respect to the turning point of the EKC finding that most determinants have the same impacts. Both models provide under small adjustments potential explanations for an N-shaped PIR, a frequently found variation of the EKC. It is shown that both models are compatible with most empirical regularities on economic growth and the environment other to the EKC. Finally, criticism on the IRS model regarding potential negative pollution can be rejected if the learning by doing is assumed to lead to fading IRS in abatement.

Avainsanat – Nyckelord – Keywords
Abatement; Economic growth; Environmental Kuznets Curve; Learning by doing; pollution

The Environmental Kuznets Curve in Consideration of Learning by Doing in Abatement

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List of Symbols

A	factor productivity
a	productivity of abatement
α	elasticity of consumption
β	elasticity of environmental effort
C	consumption
$\bar{C}$	external consumption
c	consumption rate
δ	rate of depreciation
E	environmental effort
ε	elasticity of consumption
η	elasticity of
g	growth rate of the shadow price of capital
g_C	growth rate of consumption
g_K	growth rate of capital
g_Y	growth rate of income
γ	elasticity of pollution
$\mathit{\Gamma}$	technology parameter for abatement
K	capital
L	labour
λ	shadow price of capital
ω	elasticity of C in gross pollution
P	pollution
Φ	general abatement activity
ρ	discount rate
T	technology parameter for output
T_E	productive abatement technology
t	time
τ_C	tax on consumption
τ_E	subsidy for environmental effort
θ	intensity of abatement
θ^K	maximum intensity of abatement
X	gross pollution
Y	production
Y^A	production dedicated to abatement
Y^G	gross production
Y^N	net production
z	impact of pollution on utility

List of Abbreviations

CES	Constant elasticity of substitution
CO	Carbon monoxide
CO_2	Carbon dioxide
CRS	Constant returns to scale
EKC	Environmental Kuznets Curve
FDI	Foreign direct investment
FOC	First order condition
GDP	Gross domestic product
GEMS	Global Emissions Monitoring System
IRS	Increasing returns to scale
NO_X	Nitrogen oxide
OLS	Ordinary least square
PIR	Pollution-income relationship
R&D	Research and development
SO_2	Sulphur dioxide
SPM	Suspended particular matter
TVC	Transversality condition
UNEP	United Nations Environment Programme
VOC	Volatile organic compounds
WHO	World Health Organization

1 Introduction

For a long time, environmental issues were neglected in research on economic growth. Not until the Club of Rome published its famous work "The Limits to Growth" (Meadows et al., 1972), were environmental problems – first in terms of exhaustible resources, later in terms of pollution – integrated into neoclassical growth models. Although it was not considered in neoclassic theory, the environment is indeed an important factor for economic growth. It provides plenty of services to human existence not captured by income: It is a source of renewable and non-renewable resources for economic activities (*e.g.* fossil fuels as oil, nourishments as fish) and it takes the function of a sink for undesired by-products as waste and pollution. Hence, the environment does not only affect the economy, it is also affected by economic activities.

Furthermore, economic growth and its repercussions on the environment are inextricable. Since economic activities often produce ill spin-offs, they tend to intensify many environmental problems such as air and water pollution, loss of biodiversity or hazardous extraction of resources (Egli, 2005b). Thus, the extinction of single species may affect the stability of the corresponding ecosystem (Hiering, 2003; Levin and Pacala, 2003), climate change may cause lower crop yields and rising sea levels may harm people through malnutrition and permanent displacement (Stern, 2007).

Despite its negative impacts on the environment, economic growth is still important itself. Due to economic development, living standards over the last two centuries have increased tremendously (Romer, 1986; Lucas, 1988; Egli, 2005b). However, since "The Limits of Growth", the durability of economic growth has been questioned due to finite resources and a restricted carrying capacity of the earth.

Motivated by the long-term implications of the limitations on economic growth and development, the Brundtland Report of the United Nations World Commission on Environment and Development (World Commission on Environment and Development, 1988) started the discussion about sustainable development. It defines sustainable development as follows: "Sustainable development meets the needs of the present generation without compromising the ability of future generations to meet their needs." (World Commission on Environment and Development, 1988, p. 43) In economic terms, this definition can be described as the "non-declining welfare between generations" (Egli, 2005b, p. 18).

This definition in hand, a dispute about the the possibility of sustainable growth arose. Growth pessimists on the one hand state that economic growth can never be sustainable (*e.g.* Daly, 1999, 2006; Bartlett, 2006), whereas growth optimists on the other hand claim that growth actually may be sustainable (*e.g.* Beckerman, 1999). Spangenberg (2001) argues that sustainable growth is only possible if the resource productivity increases at higher rate than economic growth. However, the discussion has not yet come to an end.

The finding of the Environmental Kuznets Curve (EKC) by Grossman and Krueger (1993) changed the ongoing discussion dramatically. They found for several pollutants that environmental degradation is first increasing with income before the level of pollution decreases with higher income. This non-monotonic relationship between pollution and income suggests that with growing income the environmental pressure decreases in the long run. Thus, the EKC became a symbol for sustainable growth.

So far theoretical analysis of the EKC is scarce and therefore the underlying basis is still unknown. It is this gap I will address through investigation of the EKC with respect to learning by doing in the abatement sector. The goal of this study is to analyse whether the EKC is a real symbol for sustainable growth or only a data phenomenon. Furthermore, learning by doing in abatement is discussed as the main drive of the EKC, and the impacts on the turning point are analysed since this information may help to alleviate environmental pressure.

To do so, I first present the empirical foundations of the EKC in Chapter 2. I elaborate on, which kind of data has been used as well as the applied estimation techniques, the results and the associated criticism. I find that the EKC is found mainly for local flow pollutants. Chapter 3 deals with the theoretical explanations of the EKC and I review, critically, the five main basic approaches. In Chapter 4, I introduce learning by doing in abatement as the actual reason for the hump-shaped pattern. I also set up a dynamic one-sector model following Brock and Taylor (2003). In this model, abatement is zero until the marginal disutility of pollution exceeds the marginal utility of capital. Then abatement sets in which reduces pollution and the EKC arises. Furthermore, I discuss the turning point of this model and how single parameters affect it. In Chapter 5, a more general approach is examined. I argue that learning by doing in abatement may be modelled through increasing returns to scale as in Andreoni and Levinson (2001). Following Egli and Steger (2007) I demonstrate that increasing returns to scale in abatement is a sufficient assumption for creating the EKC in a dynamic environment. Unlike Brock and Taylor (2003), the model points out the importance of environmental policy for the relationship between economic growth and environmental degradation. In Chapter 6 I test the validity of the models by comparing their suitability to other moments concerning economic growth and the environment. Moreover, the increasing returns to scale approach faces problems with the possibility of negative pollution in the long run. These can be solved by assuming that the economies of scale fade away. Chapter 7 concludes this contribution.

2 The Environmental Kuznets Curve

2.1 The Environmental Kuznets Curve Hypothesis

The relationship between pollution and income was the topic of several studies towards the end of the 20^{th} century and has become a "stylised fact" of environmental and resource economics (*e.g.* Stokey, 1998). In these studies - here should be mentioned the seminal contributions of the World Bank (1992), Shafik and Bandyopadhyay (1992) and Grossman and Krueger (1993, 1995) - the researchers found a characteristic pattern that describes the relationship between pollution and income per capita for various pollutants depicted in Figure 1: At low levels of income, pollution of these substances rises, while at higher income levels the slope turns around and pollution declines with subsequent income growth.

Later on, this shape of an inverted U was named the Enironmental Kuznets Curve (EKC) for its similar shape to the Kuznets Curve which describes an inverted U-shape relationship between the levels of income and income inequality (Kuznets, 1955). Both curves are based on the idea that with the process of economic growth, different measures of quality of life first deteriorate before they improve again at a later time (Pearson, 1994, p. 201). The EKC hypothesis postulates that pollution initially rises with increasing income but might decline if growth continues long enough such that the relationship between income and pollution takes the form of an inverted U (Deacon and Norman, 2006). In more general terms, the behaviour of pollution at rising income can be called a pollution-income relationship (PIR), and the special case of an hump-shaped PIR is the EKC (Lieb, 2003, p. 1).

Usually, many environmentalists consider the impact of economic growth to be inevitably negative for the environment (Cole, 1999), which is synonymously to a monotonically increasing PIR. This beliefe includes that growth must sooner or later halt as the capacity of the world to assimilate pollution is limited (López, 1994) unless there is pollution-saving technological progress that allows production to increase without causing additional harm to the environment (Stokey, 1998).

The findings of the EKC by Grossman and Krueger (1993) and subsequently many other authors contradict completely the assumptions of the negative impacts of growth. They suggest that pollution would be only a transitional phenomenon during the growth process of an economy. According to the EKC, economic growth could be a panacea against environmental degradation, as pollution might be decoupled from the growth process. Therefore, there could be no conflict between economic growth and environmental degradation, because "only economic growth can provide the resources with which to tackle environmental problems" (Cole, 1999, p.91).

This again would imply that environmental policy is not effective at all and can be omitted. Hence, any government should be interested rather in growing rapidly and

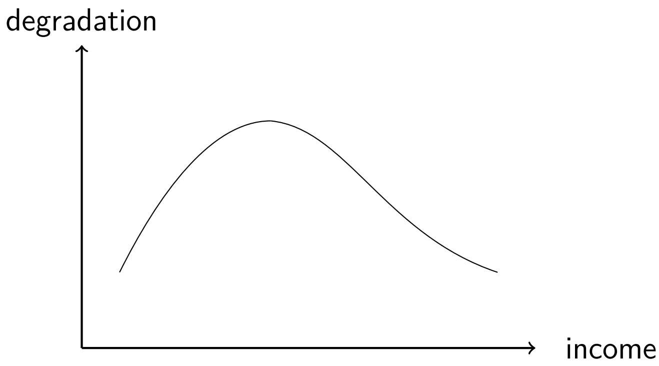

Figure 1: The Environmental Kuznets Curve

ignore environmental regulation. However, this is quite a stark statement derived from a simplistic model explaining a complex problem. Such an interpretation might be rather "false and pernicious nonsense" (Ayres, 1995, p. 97). Therefore, it is important to understand the underlying characteristics and principles of the EKC before drawing such far reaching conclusions for public policy (Andreoni and Levinson, 2001).

In several studies the estimated results provide not only a inverted U-shaped PIR but an N-shaped. This means that at the first stage of income growth, pollution increases, subsequently follow by a period when environmental quality improves with economic growth before the relationship reverses again and growth causes further deterioration. Such results imply that environmental pressure and economic growth are actually not de-linked, or at least that the decoupling effects are not persistent (de Bruyn and Heintz, 1999).

2.2 Pollution Data

Although there are data records on pollution available, for instance the Global Environmental Monitoring System (GEMS) databank collected by the World Health Organization (WHO) and the United Nations Environmental Program (UNEP), the available data contains various problems for estimating the PIR. For example, reliable and comparable data for pollution is scarce. For a vast number of countries, data is absent. In addition to its scarcity, data from developing countries is "often considered unreliable" (Auffhammer et al., 2000). This is problematic for cross-country estimations as the data might not be able to represent reality and hence the estimation results are useless. Furthermore, measuring methods vary across countries and measurement sites might be unrepresentative, making comparisons impossible (Shafik, 1994). Also, sufficiently long time series are missing since the first data on pollutants was collected in the 1960s (Lieb, 2003, p.4). Finally, many dimensions of environmental quality such as soil erosion, desertification or biodiversity loss are not recorded, therefore estimations cannot be calculated for them (Cole, 1999). Ergo, research is focused on pollutants with available data such as air or water pollutants (Lieb, 2003).

Pollution can be measured in terms of concentrations and in terms of emissions. Several surveys apply concentrations of pollutants (Shafik and Bandyopadhyay, 1992; Grossman and Krueger, 1993, 1995; Panayotou, 1997), while other take emissions into account (Selden and Song, 1994; Hilton and Levinson, 1998; Stern and Common, 2001). However, both types of measurement have advantages and disadvantages. Concentrations are more accurate than emissions, because they are ascertained directly with scientific methods, whereas emissions are not measured but constructed from estimates of fuel use and emission coefficients for various types of fuel (de Bruyn, 2000; Auffhammer and Carson, 2008). However, because of the construction from fuel combustion data, emissions are directly connected to economic activity. By contrast, concentrations may measure the local impact on the environment, but they are not related to economic activity (Lieb, 2003).

According to de Bruyn (2000), concentrations should not be used when linked them to economic variables such as gross domestic product (GDP). One of the reasons for this is that concentrations are obtained from specific locations such that the pollution variable depends on local effects. On the contrary, GDP is a nationwide variable. Combining them might cause several problems. Firstly, income per capita varies between local area where concentration is measured and the national average, but the divergence is not constant. Therefore, the site of measurement cannot be representative of the whole economy. Secondly, the area of data collection is a small geographical area such that a reallocation of industry is immediately notable in the data, even if the factory moved only a short distance: This would mean lower local pollution concentrations, the pollution output however would remain the same. Thirdly, climatic conditions may become explanatory variables. Rain, for instance, reduces the travel distance of various air pollutants in the atmosphere. Ergo, in wet seasons concentrations may be higher or lower than usual, depending if the area has usually a high or a low output of pollution.[1] Therefore, pollution should be measured in terms of emissions to make a reasonable link to economic activity.

2.3 Estimation Techniques

Various econometric techniques are used in the EKC literature. However, the dominant estimation technique is the estimation of

[1] Controlling for these phenomena is actually possible but not advisable since it makes the model very large. However, a larger size of the model may cause multicollinearity (Verbeek, 2004, p.42). Also, forecasting according to the Box-Jenkins approach requires small size models (Hamilton, 1994, p.109).

$$P_{it} = \alpha_{it} + \beta_1 Y_{it} + \beta_2 Y_{it}^2 + \beta_3 Y_{it}^3 + \beta_4 X_{it} + \epsilon_{it}, \quad (1)$$

where P is pollution in terms of emissions per capita or concentrations, α_{it} is a constant, Y is gross domestic product (GDP) per capita, i indicates the country or the monitoring station, t is a time index, X represents additional explanatory variables, β_k indicates the relative importance of the explanatory variable k and ϵ is the error term (de Bruyn and Heintz, 1999).[2] The estimation approaches are discussed in detail in Lieb (2003) and will not be further elaborated in this survey.

The presented estimation model allows for seven distinguished forms of the PIR: A monotonically rising or falling PIR, an inverted U-shaped PIR representing the EKC, a U-shaped relationship opposite the EKC, an N-shape where pollution is first rising, then falling and finally rising again, an inverted N-shaped relationship where pollution first decreases, then increases and decreases in the end, and an insignificant or flat PIR. Therefore, the EKC is only one among many possibilities (de Bruyn and Heintz, 1999) which explains the various results across studies.

Note that (1) is a reduced form that uses income as a catch-all variable (Panayotou, 1997). Therefore, it is impossible to tell if other factors that are influenced by income, for instance environmental regulation, structural change or technology, cause the EKC shaped PIR. In order to identify the impact of different effects on pollution, several scholars introduced decomposition analysis.

Grossman and Krueger (1993), for example, suggested to distinguish between the scale effect, the composition effect and the technique effect. The scale effect shows how *ceteris paribus* higher income affects pollution. Usually it is assumed that emissions increase with production, as higher economic activity implies higher resource use and hence more waste. The composition effect indicates the influence of the economy's structure on emissions. Depending on the composition of the economy, the composition effect might boost or lower pollution. It will be discussed in more detail in Chapter 3.4. The technique effect illustrates the impact of technological change. It is expected to be pollution reducing since better technology means lower emissions per unit of input in the production process.

One problem of the decomposition analysis is that in many countries output data and fuel use data - one component for estimating the technique effect - are collected on a different sectoral basis. This means that the data are not comparable and the decomposition analysis is impossible to implement (Stern, 2004).

[2] In some studies, the researchers drop the cubic term. This however is too beneficial to the EKC hypothesis. Including the cubic term on the other hand allows for both an inverted U-shaped and monotonically rising PIR (Lieb, 2003).

2.4 Estimation Results

Various types of pollutants are investigated in a vast amount of surveys. These studies focus especially on air pollutants, but also river pollutants and municipal waste are under study.[3] The group of air pollutants contains sulphur dioxide (SO_2), nitrogen oxide (NO_X), carbon monoxide (CO), carbon dioxide (CO_2), suspended particulate matter (SPM) and volatile organic compounds (VOC).

One can arrange these pollutants based upon their durability, into long-lasting stock pollutants and short-living flow pollutants. To the flow pollutants belong river pollutants (RP), SO_2, NO_X, CO, SPM, and VOC. Although all of these are actually stock pollutants, their lifetime is very short and thus they can be considered as flow pollutants: The atmospheric lifetime of SO_2 varies between one to four days, NO_X between two to five days,[4] and CO between one and three months (Intergovernmental Panel on Climate Change, 1996, p.76). The lifetime of VOC in the atmosphere is a fraction of days or months (Intergovernmental Panel on Climate Change, 2001, p.257), SPM are washed out by precipitation and thus have a short durability (Liu and Lipták, 2000, p.34). River pollutants flow with the stream such that concentrations quickly decline when emissions are reduced (Lieb, 2004, p.484). In contrast, CO_2 has a distinctly longer life time of two to four years (Liu and Lipták, 2000), and municipal waste accumulates in waste disposal sites. Therefore, these two pollutants are considered as stock pollutants (Lieb, 2004).

The results obtained provide evidence that the EKC holds for flow pollutants, whereas the PIR for stock pollutants is monotonically rising (Lieb, 2004). This is illustrated in Table 2.4. For all considered flow pollutants, the studies find very often an EKC-shaped PIR and a few times an N-shaped PIR. For stock pollutants the result are the opposite: Municipal waste is growing constantly, for CO_2 the evidence is mixed. Nevertheless, in most of the surveys on CO_2 a monotonically rising PIR is found. Hence we can conclude that the EKC is a phenomenon of short living flow pollutants.

2.5 Critiques

Although the initial findings regarding the EKC gained acceptance during the last 20 years, they are not without criticism. Objections are based on contradictory results and estimation errors. Harbaugh et al. (2002) for example suggest that the pollution-income relationship is less robust than previously thought. They re-estimated the model of Grossman and Krueger (1995) which demonstrated evidence for the EKC.

[3] There are also investigations in other dimensions of environmental quality as lack of water or deforestation. However, these do not show any clear picture and therefore they are not discussed. Their results are provided in Lieb (2003) for further interest.

[4] In all EKC studies, SO_2 and NO_X are considered as pure air pollutants. This contradicts of course the fact that they are also stock pollutants that acidify soils, wetlands or lakes.

Table 1: Empirical results for the PIR of several pollutants

	Flow pollutants						Stock pollutants	
	SO_2	SPM	NO_X	CO	RP	VOC	Waste	CO_2
Shafik and Bandyopadhyay (1992)	$\frown$	$\frown$			$\sim$		$\nearrow$	$\nearrow$
Grossman and Krueger (1993)	$\frown$	$\sim$						
Selden and Song (1994)	$\frown$	$\sim$	$\frown$	is				
Shafik (1994)	$\frown$	$\frown$			$\sim$		$\nearrow$	$\nearrow$
Grossman (1995)	$\sim$	$\frown$	$\frown$	$\frown$				
Grossman and Krueger (1995)	$\sim$	$\frown$			$\frown$			
Carson et al. (1997)	$\frown$	$\frown$	$\frown$	$\frown$		$\frown$		$\frown$
Cole et al. (1997)	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$		$\nearrow$	$\nearrow$
Moomaw and Unruh (1997)								$\sim$
Panayotou (1997)	$\sim$							
Roberts and Grimes (1997, p.192)								$\nearrow$
Kaufmann et al. (1998)	$\sim$							
Schmalensee et al. (1998)								$\frown$
Scruggs (1998)	$\frown$	$\frown$						
Torras and Boyce (1998)	$\sim$	$\frown$			$\frown$			
Wu (1998)		$\frown$						
Barrett and Graddy (2000)	$\sim$	$\frown$			$\frown$			
Cavlovic et al. (2000)	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$			$\nearrow$
Dinda et al. (2000)	$\frown$	$\sim$						
Hettige et al. (2000)				$\frown$				
List and Gerking (2000)	$\frown$		$\frown$					
Perrings and Ansuageti (2000)	$\frown$							$\nearrow$
Halkos and Tsionas (2001)								$\nearrow$
Heil and Selden (2001)								$\nearrow$
Roca et al. (2001)	$\sim$		is					$\nearrow$
Stern and Common (2001)	$\frown$ / $\nearrow$							$\nearrow$
Hill and Magnani (2002)	$\frown$		$\frown$					$\frown$ / $\nearrow$
Friedl and Getzner (2003)								$\sim$
Millimet et al. (2003)	$\frown$		$\frown$					
Egli (2005b)	is	$\frown$	$\frown$	is		is		is
Kunnas and Myllyntaus (2007)	$\frown$		$\sim$					$\nearrow$
Halicioglu (2009)								$\nearrow$
Fodha and Zaghdoud (2010)	$\frown$							$\nearrow$
Iwata et al. (2011)								$\nearrow$

Note: SPM – suspended particulate matter; RP – river pollution; VOC – volatile organic compounds; $\frown$ – EKC; $\nearrow$ – PIR monotonically rising or the turning point is out of the sample; $\sim$ – N-shaped PIR (first rising, then falling, in the end rising again); is – insignificant; $\frown$/$\nearrow$ – results from different estimations.

However, they used data revised by WHO and UNEP and adjusted by more monitoring stations and years. The results are impressive, since they reversed Grossman and Krueger's findings for sulphur dioxide and smoke, "two of the three (...) [pollutants that] exhibit the most dramatic inverse U-shaped patterns in the World Bank's report and in Grossman and Krueger" (Harbaugh et al., 2002, p. 541). Therefore they conclude, "for these pollutants, the available empirical evidence cannot be used to support either the proposition that economic growth helps the environment or the proposition that it harms the environment" (Harbaugh et al., 2002, p. 549). Taking this contradiction into account, it is important to examine, whether the findings of the EKC are a generally valid result for the PIR or if it is just a special case.

Also the traditional estimation technique of (1) is exposed to criticism. Stern (2004) summarises the critical categories in simultaneity, heteroskedasticity, omitted variable bias and contegration issues. Additionally, Lieb (2003) identifies problems regarding multicollinearity, lagged effects on the functional form of the estimation model and the lack of homogeneity across analysed countries.

Simultaneity bias arises if the explanatory variable affects and is affected by the depending variable. For example, pollution can reduce harvests or fish catches (McConnell, 1997), which are part of the production of the economy. In case of simultaneity, the results of ordinary least squares (OLS) estimations are biased and inconsistent (Stern et al., 1996). However, since there is no evidence found when testing for simultaneity (*e.g.* Cole et al., 1997; List and Gerking, 2000), both Stern (2004) and Lieb (2003) conclude that simultaneity is not a relevant critique.

Heteroskedasticity describes the case when the means of error terms are uncorrelated, but the variance varies. Given this, the OLS estimator is still unbiased but inefficient since the standard errors are incorrect which leads to misspecified significance values. This problem is frequently encountered in cross-sectional models (Verbeek, 2004, p. 82-83). Primarily discussed by Stern et al. (1996), other surveys found that adjusting for heteroskedasticity improves the estimation results (Schmalensee et al., 1998; Stern, 2002).

Another possibility that model misspecification falsifies the results arises from omitted variable bias. Stern and Common (2001) compiled several significant results that support the existence of omitted variable bias. Such bias leads to incorrect estimation results as relevant variables are excluded from the model (Verbeek, 2004, p. 55-56). Several studies find evidence for cointegration (*e.g.* Koop and Tole, 1999; Perman and Stern, 2003) which gives rise to doubts as to whether the results of the EKC model are based on econometric misspecifications (Stern, 2004).

Stern (2004) also claims that the existence of cointegration questions the estimation results. Cointegration means that all variables have stochastic trends. When cointegration is present, the conventional significance measurements are unreliable for distinguishing between long run relationships and spurious regressions (Perman and

Stern, 2003), a case where a relationship between two variables is suggested due to trends, although it actually does not exist (Verbeek, 2004, p. 313).

Another estimation problem arises from multicollinearity, *i.e.* when the explanatory variables are highly correlated. In general this is not considered a problem. If however the correlation is very high, the estimation may be unreliable with high standard errors and incorrect signs or magnitudes (Verbeek, 2004, p. 42-44). Also, small changes in data can have enormous effects on the result (Greene, 2003, p. 58). In case of the PIR, this would mean that even small measurement errors are able to change its turning point and shape dramatically. Since the standard estimation includes income in normal, squared and cubic form, multicollinearity is quite likely (Lieb, 2003).

Arguing that the driving forces behind the EKC are lagged effects and based in the past, some researchers (*e.g.* Grossman and Krueger, 1995) added lagged average GDP values in order to avoid estimating solely with current income. Unfortunately, both income and average income are highly correlated as both are include in normal, square and cubic form such that multicollinearity is found. Therefore, Lieb (2003) suggests to estimate the PIR with lagged GDP only to include the lagged effect circumventing multicollinearity.

Other functional forms than the cubic or quadratic polynomial estimation might be better to describe the PIR. Some scholars apply spline functions (Schmalensee et al., 1998; Hilton and Levinson, 1998), others a Gamma distribution as functional form (Galeotti and Lanza, 1999) and others nonparametric approaches (Azomahou and Van Phu, 2001). The results of the above mentioned studies, however, do not differ a lot from the traditional polynomial estimation[5] and often the polynomial is sufficiently accurate to produce the general shape of the PIR (Lieb, 2003).

Finally, in many PIR estimations it is assumed that all countries follow the same path and thus all countries will experience the turning point at the same level of income (Lieb, 2003). However, empirical evidence is against such assumed homogeneity (*e.g.* Koop and Tole, 1999; List and Gallet, 1999). Instead of all countries following the same PIR, each has an individual PIR. Hence, the appropriate estimation technique would actually be time series analysis, not cross-country analysis (Lieb, 2003).

[5]The differences of Azomahou and Van Phu (2001) result according to Lieb (2003) from outliers in the data, not from the functional form.

3 Theoretical Explanations

Many different theories have been established in order to explain the empirical findings of the EKC. Scholars defined various categories to structure the approaches. Pearson (1994) sorted the literature according to two factors that affect the environmental quality: supply and demand. The factors on the demand side are (i) the price for environmental quality, (ii) preferences and (iii) information and its acquisition. On the supply side, the major factors are (i) the level of population and economic activity, (ii) structures of production and consumption, (iii) efficiencies, (iv) the use of new materials and (v) external impacts. Kijima et al. (2010) classify the theoretical models as static models and dynamic models, while Copeland and Taylor (2003) subdivide into sources of growth, income effects, threshold effects and increasing returns to abatement.

According to de Bruyn and Heintz (1999), the different theoretical explanations of the EKC can be sorted as following:

- Behavioural changes and preferences
- Institutional changes
- Technological and organisational changes
- Structural changes
- International Reallocation

The different classifications of different authors coincide quite a lot. For example, preferences from de Bruyn and Heintz (1999) match quite well the demand side of Pearson (1994), while the institutional and technological changes go along with the supply side.

Lieb (2003) uses a more detailed list. In addition to the above mentioned categories, he notes separately the substitution of pollutants, increasing returns to scale in abatement, shocks and irreversibilities as reasons for the EKC. However, all additional points of this enumeration can be included in the listed items of de Bruyn and Heintz. Substitution of pollutants for example belongs to technological change.

The literature review in this paper will apply most of the structure of de Bruyn and Heintz (1999), albeit with some changes. The first subsection will refer to preferences and changes in agents' behaviour, the second to changes in institutions and environmental policy and the third to technological progress. Most of the established theoretical models are more or less based on these approaches. Moreover, the first three groups are often combined as some studies assume that technological change is policy induced (*e.g.* Smulders et al., 2011).

By contrast, the last two items are rarely discussed in theoretical models, mainly due to the simplicity of their explanation of the EKC. Moreover, structural changes and international reallocation are closely connected to each other and both are related to the effects of international trade on the environment. However, it is not yet explicitly clear why the EKC exists. Thus, the explanations should be rather treated as propositions derived from theory, empirical work or intuitive notions (de Bruyn and Heintz, 1999).

3.1 Behavioural Changes and Preferences

The first category suggests that with growing income the demand for environmental quality increases too. Hence, the downturn of the EKC is caused by changes in the demand for environmental quality. Several reasons favour this argument. Only when basic needs, *e.g.* health or education, are satisfied, additional resources from economic growth will be dedicated to combating pollution (Lieb, 2003). Eglin (1995) states that immaterial goods such as environmental quality become more important the higher the income is. Furthermore, rising income increases average education. In addition, with a higher level of education, environmental awareness (Selden and Song, 1994), fear of environmental health hazards and the concerns for a lower life expectancy increase (Gruhl, 1978). With increasing income, wages rise such that the opportunity costs of lost work-days due to health problems also increase (Shafik, 1994).

Changes caused by higher demand require that environmental quality is a normal good, if not even a luxury good. Often it is cited as a luxury good, *i.e.* that the income elasticity of demand for environmental quality is larger than one (Selden and Song, 1994; Neumayer, 1998). In this case, demand for environmental quality increases more then proportionally with growing income. However, most studies find empirical evidence that the income elasticity is positive but smaller than one (Flores and Carson, 1997; Kriström and Riera, 1996). Analysing the relationship between income and environmental R&D at different income levels, Komen et al. (1997) discover positive income elasticities, albeit elasticities that are significantly smaller than unity.

In his static model on income elasticity of demand for environmental quality, McConnell (1997) finds that the shape of the PIR does not require income elasticity equal to one. However, he claims that the higher the income elasticities, the slower the increase of pollution and the faster the decline respectively. Lieb (2002) extends the model of McConnell (1997). He disagrees with McConnell's statement about the connection between income elasticity and pollution. On the contrary, he demonstrates in a static model with pollution generated by consumption that environmental quality is necessarily a normal good if the pollution function is assumed to take a standard functional form.

The EKC is not just analysed with static models but also with dynamic ones. Some dynamic approaches take up the idea that with economic growth some constraints become non-binding, *i.e.* first, pollution increases with income until some threshold is passed, after which pollution decreases anon. John and Pecchenino (1994) consider an overlapping generations model in which economies with low income or high environmental quality do not preoccupy themselves with pollution abatement. With economic growth the environmental quality descend over time. When a certain level of pollution is obtained, the economy starts to steer towards pollution abatement and the quality of the environment will begin improving with economic growth. Their model derives an inverted V-shape, a variation of the inverse-U with a sharp turn at the breakover point where the population start to be attracted by abatement. Jaeger (1998) creates the inverse-V from changes in consumer preferences. He assumes that at low levels of pollution, consumers' need for clean air is satisfied. Thus the marginal benefits from improving environmental quality are small and consumers do not demand clean commodities. As the turning point for pollution is reached, people prefer clean goods such that the quality improves again.

3.2 Institutional Changes

de Bruyn and Heintz (1999) list also institutional changes as possible reason for the appearance of the EKC. These changes comprise policy distortions and market failures such as ill-defined property rights, subsidised energy consumption or missing internalisation of environmental externalities. Therefore, the rising arm of the EKC occurs from these distortions, while the decreasing branch of the curve results from removing the distortions and market failures. This may happen through the establishment of appropriate property rights or environmental policies to internalise the external effect (Panayotou, 1997).

Empirical results mainly support the hypothesis that changes in distortions and market failures cause the EKC. For example, Dutt (2009) demonstrates in a cross-country study that the strength of political institutions and governance lowers the emissions of CO_2. Tamazian and Rao (2010) confirm the importance of institutional quality, as they find that institutional development has a significantly positive impact on the environment. Culas (2007) shows that institutions for secure property rights shift the EKC for deforestation in Latin America downwards.

Investigating the relationship between corruption and pollution, Cole (2007) finds for SO_2 and CO_2 that corruption does have a positive impact on the emissions of both pollutants. Aiming at the same target, Leitão (2010) shows for sulphur that the degree of corruption causes the turning point of the EKC to shift to a higher income level. Thus, the EKC shifts upwards the stronger the role of corruption is.

de Bruyn and Heintz (1999) also state that the EKC may ensue from people's exertion of influence on market distortions via elections and referenda. Hence, people decide by their votes on their country's environmental policy. Moreover, they outline that "institutional changes triggered by citizens' demand for cleaner environments are more likely to occur in democratic countries" (de Bruyn and Heintz, 1999, p. 667). However, empirical evidence for this is mixed. Bernauer and Koubi (2009) figure out for SO_2 that the degree of democracy affects air quality positively. Shafik and Bandyopadhyay (1992) find that the influence of political and civil rights on several pollutants is higher, the more democratic a country is. Contrary hereto, Torras and Boyce (1998) discover opposite results when splitting the sample of Shafik and Bandyopadhyay into groups of countries with high- and low income. They detect that most pollutants are lower if the country is more democratic, but has a low income.

Aslanidis and Xepapadeas (2008) develop the idea of regime switching in a simple static model. They claim that in the real world the stringency of environmental policy depends on the stage of development of the economy. Hence, the richer the inhabitants (and thus the voters) of a country are, the stricter the environmental policy becomes due to the preferences of the voters. Aslanidis and Xepapadeas expect the slope of the PIR to be positive for lax and negative for stringent environmental policies, hence in their model the EKC takes an inverted V-shape. Analysing the data used by Harbaugh et al. (2002), Aslanidis and Xepapadeas confirm their model for SO_2 and smoke.

Kijima et al. (2011) investigate the EKC in a real options framework. They assume that many myopic, local policy makers decide between tight environmental regulation and non-regulation. Their decision depends on the trade-off between higher utility from free business in the unregulated case and lower disutility from pollution in the case of regulation and on the uncertainty about costs and benefits of regulation. Decisions are made again and again over time, thus the switching dynamics follow an alternating renewal process. The authors find numerically that the local level of pollution oscillates between the boundaries of regulation and non-regulation depending on the current regime. However, at the aggregated level of the economy the level of pollution evolves over time either in an inverted U-shape or in a N-shape for different parameters. They demonstrate that the level of pollution depends on the decision of the policy maker only, not on economic growth or utility. Hence, the EKC is not a relationship between pollution and income but between pollution and time.

Jones and Manuelli (2001) create a model that aims explicitly at environmental policy. In their model, the majority decides via elections about environmental policy in the form of emission taxes or technological minimum standards in production. Countries with low economic activity choose per capita emission taxes equal to zero. Taxation is introduced when income has increased sufficiently for crossing the threshold; here again an inverted V-shaped curve is derived. Similarly, the EKC may be associated with changes in the income distribution. If environmental policy depends on the the preferences of the median voter and if environmental quality is a normal good, then a more equal income distribution causes stricter regulations as the median voter's demand for environmental quality increases (Lieb, 2003). Kempf and Rossignol (2007) build up a simple growth model in which public expenditures enhance either growth or abatement activities. Public expenditures are chosen by democratic elections according to a simple majority rule. They demonstrate that the poorer the median voter relatively to the average individual, the higher are the preferences for growth. Hence, the larger the inequality in the distribution of income, the more the environment gets harmed. However, the empirical evidence is rather weak. Although the Gini coefficient as an indicator for the income distribution has frequently the presumed sign, it is often not significant (*e.g.* Scruggs, 1998; Torras and Boyce, 1998; Borghesi, 2000; Magnani, 2000; Gangadharan and Valenzuela, 2001). Of course, this might be due to the poor data quality on the Gini coefficient in many countries (Torras and Boyce, 1998).

3.3 Technological and Organisational Changes

The next section is about the impact of technological progress on the pollution path. Within the theoretical EKC literature there are two main emphases. The first focuses on models explaining the EKC through shifts in the use of different pollutive production technologies, whereas the second deals with the attributes of the abatement technology.

Smulders et al. (2011) analyse the EKC resulting from endogenous innovations policy induced technology shifts and intrasectoral changes. In their model, environmental degradation arises from new production technology, which is superior due to higher labour efficiency, but is at the same time more polluting. The larger pollution causes public concern such that an emission tax is introduced. This public policy causes first an immediate drop of pollution and following a constant pollution level as firms reduce their production, before at some point a clean technology is invented. The new technology reduces pollution and degradation approaches zero as more and more firms adopt it. There are two critical conditions of the model. On the one hand, the clean technology must be invented. On the other hand, the implemented tax must be high enough to pay back the adoption of the new technology.

Stokey (1998) investigates how different production technologies affect the environmental quality. In her model, at low income level clean technology is not affordable such that only the dirtiest technology is used and pollution increases linearly with economic growth. At the turning point, cleaner production technology is affordable and it is used in order to decrease the level of pollution. Applying an AK endogenous growth frame with decreasing returns to abatement[6], Stokey shows that sustainable

[6]Stokey (1998) claims to use constant returns to scale in abatement, however, Brock and Taylor (2005) demonstrate that Stokey's setting actually implies decreasing returns to scale.

growth is possible but not optimal. Rather, the growth of pollution chokes off economic growth. However, this conclusion is not surprising since Stokey combines an AK aggregate production function with strictly neoclassical attributes of abatement (Brock and Taylor, 2005).

In contrast hereto, Hartman and Kwon (2005) find a possibility of sustained growth. They build a two sector endogenous growth model with human and physical capital as inputs, where physical capital causes pollution during the production and human capital does not. In their model, the EKC arises from the marginal costs and benefits of pollution. The rising branch of the EKC is when the marginal costs of pollution exceed the marginal benefits and pollution is emitted unregulated. When this relation turns around, pollution control starts and degradation is reduced. In their model, the EKC is possible if the initial values are not too large such that the economy starts in the unregulated case. However, if the initial values make the economy controlling pollution from the very beginning, degradation simply falls over time.

The other group of models analyses the effects of abatement technology on pollution. In one of the earlier studies among those, Selden and Song (1995) describe the EKC as a result of a zero-abatement phase and a subsequently following phase with active abatement. In the first period, the marginal utility of consumption is larger than the marginal utility of a clean environment, such that abatement is not appreciated. In the following period the sign changes and the economy reduces pollution.

Dinda (2005) elaborates a one-sector model where capital is allocated in production and abatement. In his model, shifts from insufficient to sufficient abatement expenditures give the EKC, but this does not occur in the steady state: Dinda argues that the economy must be on a suboptimal growth path in order to experience the EKC.

Other studies focus on the effects of research and development (R&D) on technological progress in abatement (*e.g.* Rubio et al., 2008, 2010). In both papers, the EKC occurs from research driven technological change in abatement. First, the shadow price of clean technology exceeds the costs of developing it, hence no new abatement technology evolves. However, once the relationship turns around, firms do research on superior abatement technology. The optimal investment patterns in capital and R&D in these models support the EKC.

One point of critique should not be ignored. There is still the possibility that technological progress may cause the EKC for a single pollutant by substituting it by a different one. This happens in the development of end-of-pipe and abatement technologies as well as through insufficient environmental policy.

The consequences are overlapping EKCs (Smulders et al., 2011), a sequence of different pollutants illustrated in Figure 2, or rising levels of other environmental hazards despite the reduction of single pollutants (Dasgupta et al., 2002; Lieb, 2004).[7]

[7] Dasgupta et al. (2002) entitled this phenomenon as the "new toxics scenario".

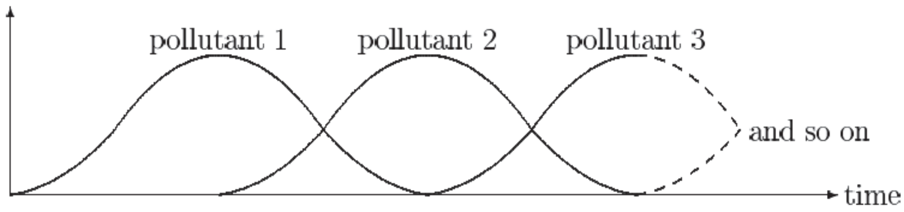

Figure 2: Pollution cycles according to Smulders et al. (2011), source: Lieb (2003)

End-of-pipe technologies for example are supposed to be the most important determinant in causing the downturn of the EKC for SO_2 (de Bruyn, 1997; Ekins, 1997). However, these technologies require energy to be produced and to be kept running. Since energy is mainly gained from fossil fuels, higher demand for energy causes higher CO_2 emissions (Faber et al., 1996). Therefore, end-of-pipe technologies actually substitute SO_2 into CO_2. This finds evidence in Stern (1998), where the author notes a shift in emissions from SO_2 and NO_X towards CO_2.

Another example is the substitution of fossil fuels by nuclear energy. Several studies found that the increasing use of energy from nuclear power plants helped to lower the emissions of SO_2, SPM and NO_X (Scruggs, 1998; Selden et al., 1999; Viguier, 1999). Iwata et al. (2011) find evidence that nuclear energy also reduces CO_2 emissions. Since nuclear power does not cause environmental pollution directly but carries along high risks for damaging the environment, the costs of environmental damages are shifted to future generations. The dimensions of such risks emerged again after the reactor accident in Japanese Fukushima Daiichi in 2011. The total scale of the degradation is not completely assessable, however, first results are discussed *e.g.* in Garnier-Laplace et al. (2011) showing higher radioactive contamination than what is considered as a safe level for terrestrial ecosystems.

The substitution of pollutants may also occur in the abating process. While reducing certain kinds of pollution, abatement technologies may cause different polluting substances themselves (Huesemann, 2001). Catalytic converters in cars for example filter CO, NO_X or hydrocarbons from the emissions. However, all these substances are mainly transformed into CO_2. Moreover, the converter increases the fuel use of the engine, which includes higher CO_2 emissions. Finally, it consists partly of platinum - a metal that accumulates in the soil (Holzbaur et al., 1996).

Finally, policy-induced technological progress might also support pollutant substitution. If public regulation concentrates on the reduction of certain pollutants, firms will substitute them in the production process by other, unregulated pollutants (Devlin and Grafton, 1994; Smulders et al., 2011). In the study of Lieb (2004) it is assumed that solely the impact of flow pollutants is immediate, while the one of stock pollutants is perceivable only in the future. The substitution of pollutants is caused by environmental policies: Myopic governments internalise only the external effects of the flow pollutants but do not regulate stock pollutants, since myopic governments ignore future damages. The study demonstrates how an EKC for flow pollutants may arise while the PIR for stock pollutants remains monotonically rising.

3.4 Structural Change

The next attempt to explain the EKC aims at changes in the structural composition of countries. Changes in the composition may arise from variations in consumer behaviour, political institutions or international competitiveness (de Bruyn and Heintz, 1999). The idea expects countries to pass through several stages of development. First, on subsistence level people do not produce any pollution. When becoming richer, the countries face first intensified agriculture and light industry, later a developing industrialisation. The pollution level increases with production. Growing further, the economy experiences a change away from industry towards the environmental benign service sector; the environmental pressure decreases again (Arrow et al., 1995; Baldwin, 1995; Lieb, 2003).

Moreover, the structure of the industry sector itself may change similarly. Syrquin (1988) states that, first, with rising income, composition of the industrial sector shifts from relative nonpolluting light manufacturing such as food production or textiles to heavy, polluting industry such as chemicals, minerals and metals or machinery. Later, at higher income levels, environmental friendly research activities and less polluting high-tech industry become the main activity of the economy (Dinda et al., 2000) .

If the EKC is derived from structural change, it is not induced by environmental policies. Instead, it would appear automatically (Gangadharan and Valenzuela, 2001). Moreover, the intuition behind the transition from an agrarian economy to a developed service economy is very clear. However, this claim does not stand without critique. De Groot (1999) challenges the automatism. He concludes that structural change may help to reduce pollution, however, it is not sufficient to cause the downturn of the EKC. In contrast, technological progress is crucial for the emerging of the EKC. The results of Cassou and Hamilton (2004) are akin. In their model, the EKC follows structural change, whereat the structural change is caused by rising taxes on pollution, *i.e.* it is policy-induced.

Furthermore, empirical evidence is ambiguous. Several studies include an additional regressor on the manufacturing share in GDP (Lieb, 2003). Some conclude that pollution moves along the structural change (Rock, 1996; Panayotou, 1997; Borghesi, 2000; Cole, 2000), others state that the effect of structural change is only small if not insignificant (Grossman et al., 1994; Grossman, 1995; Suri and Chapman, 1998). Hence, transition effects can verify at most a small part of the EKC. Moreover, according to Cole (2000) and Millimet (2001) structural change alone cannot explain the EKC. This supports the statement of de Groot (1999) claiming that policy and technological progress are crucial for the shape of the EKC.

In order to examine the impact of structural change on environmental quality more precisely, some empirical studies make use of decomposition analyses. For such analysis, the amount of emissions are split into the single components discussed in chapter 2.3, the scale effect, the composition effect and the technique effect. The scale effect – represented by GDP – describes, how income growth affects emissions. The composition effect – delineate by the output share of one sector in total output – demonstrates the impact of structural changes on environmental degradation. The technique effect finally – *i.e.* the emission intensity of one sector – analyses, in which way technological progress causes lower emissions.

Many surveys apply decomposition analyses to emissions of SO_2, CO, CO_2, SPM, NO_X, VOC and heavy metals (see *e.g.* Torvanger, 1991; de Bruyn, 1997; Selden et al., 1999; Viguier, 1999). Selden and Song (1994) and Grossman and Krueger (1995) for example argue that the EKC may indeed arise from an interaction of scale, composition and technology effects. However, most of the studies find that the composition effect is small, whereas the technique effect has always a large impact. Furthermore, while the technique effect always decreases emission, in some studies the composition effect even causes pollution.[8] Hence, decomposition analyses also demonstrate that the composition effect is not elementary for the downturn of the EKC, but the technique effect is.

Additionally, there is another often neglected point of critique about the composition effect as reason for the EKC. Even though changes in the industry-structure may lower the marginal pollution intensity of output, they cannot erase the scale effect completely, unless the polluting sector shrinks in absolute matters. Yet, this is possible only if the pollution-intensive industry produces inferior goods – a rather improbable assumption – or if this sector's products are no longer manufactured domestically but abroad and imported (Torras and Boyce, 1998). This case will be discussed in the following section.

3.5 International Reallocation

As stated above, to be able to explain the EKC with structural change it is necessary that the production of pollution-intensive goods is moved abroad and the dirty commodities become import goods. Many scholars (*e.g.* Arrow et al., 1995; Stern et al., 1996; Ekins, 1997; Rothman, 1998) point out that changes in the composition of production are linked to both consumption and trade. If consumption does not change similarly as the industrial structure does, then the appearing differences in

[8] The only exception for a clear finding of the composition effect are Ederington et al. (2004). In their study on the US economy they found that at the end of the 20^{th} century dramatic shifts towards cleaner industries have been observed.

local production and consumption need to be compensated by imports. Hence, in developed countries the polluting commodities will be imported from the developing world, which may explain the environmental improvements in the rich countries and the degradation in the poor countries in recent years (Cole and Neumayer, 2005).

Saint-Paul (1995) extends this allegation in saying that dirty industry first migrates from rich countries to middle-income countries due to their developed infrastructure in comparison to poor countries. His model leads to low pollution in high-income countries, and to high pollution in middle-income countries. This scenario would be alarming for the least developed countries, since once they are more developed, they would not have any place where to relocate their polluting industry to. Thus, these countries will need to abate emissions instead of moving it to other countries, which will be much more difficult (Lieb, 2003).

Displacement may take two forms. On the one hand, firms may actually replace their production site in developed countries with similar sites in developing countries. This happened for instance in industries with high toxic intensities (Birdsall and Wheeler, 1992) or in the leather tanning industry (Ayres, 1997). On the other hand, polluting industries may grow faster in the developing world than in developed countries (Low, 1992; Perrings and Ansuageti, 2000). Moreover, Stern (1998) finds even on the domestic level of the USA that the poorest U.S. states host relatively more polluting industry than the richest, which account for a large service sector. Hence, migration of dirty industry could explain the results of Carson et al. (1997), who observe an EKC for several pollutants across different states in the USA. However, the reallocation as explanation for the EKC is questionable. As mentioned above, the results for structural change imply that the removal of dirty industries is too slow to explain the decline of the absolute production level of these industries in developed countries. So only the fast growth of dirty production in developing countries remains as a reason. But this can solely explain a cross country EKC, not the EKC for a single growing developed country over time (Lieb, 2003).

According to the standard trade theory, countries export if they have a comparative advantage in producing one good.[9] From comparative advantages results the Pollution Haven Hypothesis. Usually, pollution-intensive goods are capital-intensive. Thus, with uniform environmental regulations, developed countries would have a comparative advantage to produce them (Cole and Elliott, 2003; Copeland and Taylor, 2004; Cole and Elliott, 2005). However, by implementing looser environmental policies, poor countries take over this advantage (Antweiler et al., 2001; Copeland and Taylor, 2004) and specialise in dirty production (Suri and Chapman, 1998; Stern, 1998; Cole and Elliott, 2003; Mani and Iha, 2006). Therefore, developed countries will have a more stringent environmental policy and ergo a comparative advantage in producing clean goods, whereas developing countries will have a comparative advantage in

[9]The idea of comparative advantage is discussed in the standard literature regarding international trade, (*e.g.* Feenstra, 2004)

dirty commodities. These countries with high concentrations of pollution are called pollution havens.[10]

Another consequence of reallocating polluting industries might be a "race to the bottom". In this scenario, dirty industries first migrate to countries with weak environmental regulations as in the Pollution Havens Hypothesis. Simultaneously, more and more capital flows out from the developed countries towards the developing countries, because the capital-intensive industries – now, due to the comparative advantage in the developing countries – attract investments. This financial pressure forces governments in the rich countries to relax their environmental standards. When relaxation happens more frequently in more and more developed countries, the EKC flattens and rises towards the highest existing level of pollution (Dasgupta et al., 2002).

Empirical evidence for the "race to the bottom" is not conclusive. As a matter of fact, the costs of regulation for polluting activities are in rich countries higher than in developing countries (Jaffe et al., 1995; Mani and Wheeler, 1998). Hence, there is an incentive for companies to relocate their dirty production sites. However, it seems that pollution-control costs are in comparison to other factors not a major determinant for moving (*e.g.* Levinson, 1997; Jänicke et al., 1997; Albrecht, 1998). Mody and Wheeler (1992) define as more important factors the distance to the market and the quality and costs of infrastructure, as suggested by Saint-Paul (1995). Furthermore, trade from high-income economies to developing countries tend to be more pollution-intensive than trade in the other direction (Mani and Wheeler, 1998; Albrecht, 1998). This suggests that the scenario of dirty industries first relocating to low-income countries and then exporting their goods back to developed countries does not hold.

According to the idea of "the race to the bottom", pollution in developing countries should increase since they are pollution havens, and pollution should increase in high-income economies because they have to relax their environmental standards in order to remain cost-competitive. Testing these prepositions for air pollution, Wheeler (2001) finds for Brazil, China and Mexico contrasting results. As foreign direct investment (FDI) into these countries increase, they should face increasing pollution, too. Moreover, air pollution in U.S. cities should worsen, since at the same time expanding industrial imports from all three countries should force the US government to slacken environmental policies. However, the opposite holds. While FDI increase, pollution in the emerging countries decline. Similarly, pollution in US cities diminish, too. Hence, the race to the bottom does not gain evidence from empirical findings.

[10]This idea gains support from the fact that environmental quality tends to be a normal good. As income increases, demand for environmental quality increases, too. Thus, environmental policy tends to be more stringent in rich countries than in poor states, which includes that rich countries have a comparative advantage in developing clean goods (Cole, 2004).

4 Learning by Doing and Induced Innovation in Abatement

As it was shown in the previous chapter, technological progress remained as the only reasonable explanation for the existence of the EKC. Behavioural change and institutional change both rely on technological progress to impose the wish for less pollution. Empirical evidence speaks against structural change and international reallocation can only explain the cross country EKC, not the EKC of a single growing country. In the following chapters, I will discuss the effects of technological progress in abatement on the EKC more closely. I assume that technological change results from learning by doing. The idea of learning by doing was primarily discussed by Arrow (1962). In his contribution, firms experience an increase in productivity through learning effects. In other words, learning by doing causes positive externalities – in the case of Arrow higher productivity per worker – that enhance output.

The basic idea of learning by doing is prominent in both growth theory and environmental economics. The output enhancing externality may come from knowledge spillover effects in aggregate R&D (Romer, 1986) or from accumulating human capital (Lucas, 1988).Thus, in endogenous growth models learning by doing is often detectable. In the simplest specification – the AK model firstly introduced by Rebelo (1991) to which I refer throughout in my text – learning by doing generates constant returns in capital accumulation on nationwide economy.

The concept of learning by doing is also important in environmental economics. Several studies point out the importance of induced innovation in solving problems concerned with pollution (*e.g.* Jaffe et al., 1995; Bramoullé and Olson, 2005). As argued in Chapter 3.3, technological progress in abatement reduces pollution output. From decomposition analyses we know that the main drive behind the EKC is technological change. Although the source of the progress is not identified, there is support that it is caused by learning by doing. Analysing the technological progress of flue gas desulphurisation[11], Bellas (1998) finds evidence that the improvements come rather from learning by doing than from efforts in R&D.

The presence of learning by doing in abatement or "learning by abating" (Bretschger and Smulders, 2007) augments the relationship between growth and environment with several new features. For instance, the costs of pollution control vary through learning by doing in abatement (Brock and Taylor, 2005). As in the model of Arrow (1962), where learning effects reduce the production costs, externalities in the abatement process can reduce the costs of abatement. If the learning effects are unbounded, then spillover effects may allow for positive growth and falling pollution levels while the abatement costs for society are decreasing. Hence, sustainable growth could dispense

[11] Flue gas desulphurisation is a technique that helps to reduce the emission of SO_2 during the combustion of coal.

with public policy as the spillover effects can solve the environmental problems by itself. If the learning effects are bounded, then the consequences of learning by abating for environmental policy are less clear. Nevertheless, Brock and Taylor argue that environmental policies are less effective because technological progress implements higher abatement regardless of the presence of public policy.

Another implication of learning by abating results in the linkage of the sectors subject to innovations. If the general knowledge stock grows and spillovers are included, then learning by doing is one option to connect innovations in abatement with technological change in production. It enables us to make the assumptions about spillover effects and technological change consistent across sectors. This is important since it is the relative rates in technological progress between output and abatement that determine the sustainability of the balanced growth path (Brock and Taylor, 2005).[12]

A final feature of learning by abating can be one form of induced innovation, although it is often modelled as a passive process implying that investments are not target-oriented. If so, it is not clear where technological progress comes from, but it is important to identify the driving factors behind innovations in abatement (Popp, 2002). These driving factors could actually be the effects of the price of pollution. If the deterioration of the environment calls for the implementation of abatement and abatement is subject to knowledge spillovers, then the technological progress arises from the higher costs of pollution, which is a clear characteristic of induced innovation (Brock and Taylor, 2005).

In the following, I will discuss the effects of learning by abating on the EKC. First, I will discuss the case when spillover effects result in constant returns to abatement such that the abatement costs for society become constant. Later in Chapter 5, I discuss the case when learning by doing leads to increasing returns to scale in abatement implying that abatement costs are diminishing on societal level. In the end I will compare the two approaches. Furthermore, spillover effects question the efficiency of environmental policy. I will discuss the need for regulations in the applied models, showing that drawing this conclusion against regulation is too simple. Instead, it will be obvious that public policy still is an important factor of the EKC hypothesis despite the existence of learning effects.

In this chapter I focus on how learning by doing in abatement leads to constant returns to abatement and the corresponding effects on the PIR. I follow the arguments of Brock and Taylor (2003) that abatement activity depends on the relative marginal price of abatement in comparison with consumption. First, I sketch the frame of the model. I demonstrate how spillover effects cause constant returns to abatement on

[12]This is easy to see when comparing *e.g.* the articles of Brock and Taylor (2010) and Stokey (1998): While Brock and Taylor (2010) allows for both technological progress in production and abatement, in Stokey (1998) technological progress is only possible in production, not in abatement. The result is that sustainable growth in Brock and Taylor (2010) is possible, whereas it is not in Stokey (1998).

a nationwide level. After that I discuss in what way this model is consistent with the EKC and sustainable growth, followed by the analysis of the turning point and its determinants.

4.1 The EKC and Constant Returns to Abatement

To discuss the effects of learning by abating on the EKC, I introduce the Kindergarten Rule model of Brock and Taylor (2003). The model is based on the assumption of knowledge spillovers in the abatement technology, which keeps pollution under control. I assume that learning by doing eliminates diminishing returns to both capital and abatement on firm level causing constant returns to capital and to abatement on aggregate level. The result is a model in which learning by abating reduces the costs of abatement but does not eliminate the drag of environmental policy completely.

Since I am interested in the consequences of technological progress on the environment and on economic growth, I use the idea of linking factor accumulation and technological progress as in Romer (1986) or Lucas (1988). For this I generate a simple one-sector model of endogenous growth and environmental degradation.

I apply a model with an infinitely living representative agent. There is one single aggregated output good Y. The two factors of production are labour L and capital K. Since this is a one-sector model, capital is meant in very broad terms including both physical and human capital. I expect the capital stock to grow with investments and decreases via a constant depreciation δ, whereas population is assumed to be constant over time. Pollution P results entirely from flow pollutants and has only local impacts. Capital K_t, consumption C_t, output Y_t, and pollution P_t vary across time, indicated by subscript t. However, I will suppress the time index if it is not needed.

Preferences

The utility of a representative agent depends on two variables, consumption C and pollution P. The utility function is

$$U \quad = U(C,P), \tag{2}$$

where consumption has a positive impact on utility such that

$$\frac{\partial U}{\partial C} > 0, \tag{3}$$

and pollution a negative one with

$$\frac{\partial U}{\partial P} < 0. \tag{4}$$

Across lifetime, the agent maximises his utility according to

$$\max \int_0^\infty U(C,P)e^{-\rho t}\,\mathrm{d}t, \tag{5}$$

where the above mentioned attributes hold. The time preference is denoted as ρ. For analyses it is useful to specify the utility function as a function of constant elasticity of substitution:

$$U(C,P) = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{zP^\gamma}{\gamma} \qquad \text{for } \varepsilon \neq 1 \tag{6}$$

$$U(C,P) = \ln C - \frac{zP^\gamma}{\gamma} \qquad \text{for } \varepsilon = 1 \tag{7}$$

with $\varepsilon > 0$ and $\gamma \geq 1$. The impact of pollution on the individual household is measured by z.

Production

Production follows standard assumptions. Each firm produces i final output good Y with the input factors labour L and capital K. The firm's production function is assumed to be strictly concave with constant returns to scale (CRS). The technology level T describes the productivity of labour and is given for every individual. Similar to Romer (1986) and Lucas (1988), I propose that the level of technology is commensurate to an economy wide measure of activity, similar to aggregate R&D in the case of Romer and to average human capital in the case of Lucas. In the specification of an AK model, the stage of technology is connected either to the aggregate capital stock or to - as assumed here - to the aggregate capital to labour ratio such that

$$T = \frac{K}{L}. \tag{8}$$

Next, I want to demonstrate the role of knowledge spillovers in the aggregated economy level. To do so, I aggregate production across firms in order to obtain an AK aggregate production function due to comprehensibility. First consider production of one single firm i where the input factor labour is affected by the general technology level T such that

$$Y_i = F^i (K_i, TL_i). \tag{9}$$

The input factors are capital K_i and labour L_i, i denotes the ith firm. The production function is assumed to be strictly concave in K_i and L_i such that

$$\frac{\partial F^i}{\partial L_i} > 0, \quad \text{and} \quad \frac{\partial^2 F^i}{\partial L_i^2} < 0$$
$$\frac{\partial F^i}{\partial K_i} > 0, \quad \text{and} \quad \frac{\partial^2 F^i}{\partial K_i^2} < 0.$$

Furthermore, it has constant returns to scale (CRS). However, labour is affected by knowledge spillovers from the general technology level T. Total production is the sum of all firms' production. Hence, the aggregate production function is

$$Y = \sum_i Y_i = \sum_i F^i (K_i, TL_i). \tag{10}$$

Next, let us assume that relative prices for labour and capital are more or less equal across firms. This homogeneity implies that all firms use the same amount of capital and labour for their production, *i.e.* $K_i = K$ and $L_i = L$ for all i. If the input factors are identical for all firms, then we can write the sum of all production function as one production function for the whole economy as $\sum_i F^i(K, TL) = F(K, TL)$. Hence, we can rewrite aggregate production as

$$Y = F(K, TL). \tag{11}$$

Since all firms are identical, efficiency requires that all firms apply the same capital intensity $\frac{K}{TL}$. Using this, we can take TL from the formula such that

$$Y = TLF\left(\frac{K}{TL}, 1\right). \tag{12}$$

Finally, we can exploit the definition of $T = \frac{K}{L}$ such that also capital vanishes from the set of production. Now, the production technology does not depend anymore on K nor TL as $F(1,1) = constant$. Thus, we can rewrite aggregate production only depending on capital and the constant production technology which I define as $F(1,1) \equiv A$:

$$Y = KF(1,1) = AK \qquad \text{where } F(1,1) \equiv A. \tag{13}$$

Here we find that the production function solves in a linear function of K. I define the constant term in the last line as $F(1,1) \equiv A$, where A represents the marginal product of capital and the general level of technology likewise. As shown above, technological progress converts diminishing returns to capital at the firm level into constant returns on the aggregate economy level. Thus, the production function is a simple AK model:

$$Y = AK. \tag{14}$$

Pollution

Environmental degradation is a function of created pollution and abated pollution, hence

$$P = P\Big(X, B(X,\Phi)\Big), \tag{15}$$

where X represents gross pollution and $B(X,\Phi)$ abated pollution. I assume that gross pollution is a by-product of economic activity, which is a standard assumption (Xepapadeas, 2005). It can arise either from production or from consumption. Likewise, abatement depends correspondingly on the creation of pollution X and on the type of abatement Φ, where Φ can be the share of total production dedicated to abatement or the environmental effort of market participants. Unless an open economy setting is assumed, both approaches lead to the same results.

Net pollution increases with gross pollution such that $P_X > 0$ and decreases with abatement $P_B < 0$. For simplicity, I assume that emitted pollution is the difference between gross pollution and abated pollution, thus:

$$P = X - B(X,\Phi). \tag{16}$$

At that point, let us think of gross pollution as a by-product of a company's gross production Y_i^G. The input factors of the abatement function are Y_i^G representing X and and expenditures for abatement Y_i^A replacing Φ such that abatement in (16) turns into $B(Y_i^G, Y_i^A)$. The abatement production function is strictly concave and has CRS such that

$$\frac{\partial B}{\partial Y_i^G} > 0, \quad \text{and} \quad \frac{\partial^2 B}{\partial Y_i^{G2}} < 0$$
$$\frac{\partial B}{\partial Y_i^A} > 0, \quad \text{and} \quad \frac{\partial^2 B}{\partial Y_i^{A2}} < 0.$$

Hence, returns to abatement are diminishing for each firm. Pollution emitted by a single firm is

$$P_i = Y_i^G - B(Y_i^G, Y_i^A). \tag{17}$$

Now, consider that – analogously to aggregate production – there is on aggregate level an abatement technology parameter Γ, which increases the marginal product of abatement. Again, it is assumed that Γ is proportional to the average abatement intensity such that

$$\Gamma = \frac{Y^A}{Y^G}. \tag{18}$$

The technology level affects the efficiency of abatement due to learning by doing. To see this, let us assume that Γ influences the capacity of cleaning Y_i^G in the abatement function of (17) such that

$$P_i = Y_i^G - B(Y_i^G \Gamma, Y_i^A). \tag{19}$$

Again, let us assume that abatement is homogenous for each firm. Then, all firms produce the same amount Y_i^G and use the same share of GDP for abatement Y_i^A. Moreover, abatement is identical for all firms due to the homogeneity. Due to that, we can factor out Y_i^G and Γ from the abatement function and rewrite the pollution function for each firm as

$$P_i = Y_i^G \left[1 - \Gamma B\left(1, \frac{Y_i^A}{Y_i^G \Gamma}\right)\right]. \tag{20}$$

Here we can see that through the elimination of $(Y_i^G \Gamma)$ from the abatement function we discover that one input factor of abatement becomes equal to one and thus constant, whereas the other becomes the ratio $\frac{Y_i^A}{Y_i^G \Gamma}$, which actually is the abatement intensity of firm i under consideration of the spillover effects through Γ. Next, we can derive aggregate pollution which is the sum of each firm's net emission:

$$P = \sum_i P_i = \sum_i Y_i^G \left[1 - \Gamma B \left(1, \frac{Y_i^A}{Y_i^G \Gamma} \right) \right]. \tag{21}$$

Since all firms are homogeneous, we can treat the whole economy as one agent. Hence, we can say that $\sum_i Y_i^G = Y^G$ and $\sum_i Y_i^G = A^A$ which allows us to rewrite the aggregated pollution function as

$$P = Y^G \left[1 - \Gamma B \left(1, \underbrace{\frac{Y^A}{Y^G \Gamma}}_{=1} \right) \right], \tag{22}$$

where the second input factor of abatement becomes constant because of the definition of $\Gamma = \frac{Y^A}{Y^G}$. Thus, abatement is not anymore a function but a constant which we can define as $B(1,1) \equiv a$. Since the productivity of abatement must be larger than one as abatement must be able to clean more than just its own negative effects, it is clear that $a > 1$. Additionally, let us define the aggregated intensity of abatement as $\theta \equiv \frac{Y^A}{Y^G}$, which is equal to Γ. Therefore, we the aggregate pollution function is

$$P = Y^G \left[1 - \theta a \right]. \tag{23}$$

In order to avoid negative pollution it is obvious that

$$\theta \leq \frac{1}{a} \tag{24}$$

must hold, because abatement can maximally reduce the flow of pollution. Otherwise pollution might be negative, which is not plausible in reality. Gross output includes all produced goods and with respect to (14) it is $Y^G \equiv Y = AK$. Therefore, we can rewrite the pollution function as

$$P = AK \left(1 - \theta a \right). \tag{25}$$

It should be pointed out that the aggregate relationship between pollution and abatement in (25) is consistent with empirical estimates which find rising marginal abatement costs at the firm level (for sulphur *e.g.* Cofala and Syri, 1998). To see this, consider again equation (17), which describes the pollution function for each individual firm depending on its output and abatement:

$$P_i \;=\; Y_i^G - B(Y_i^G \mathit{\Gamma}, Y_i^A).$$

Taking the first and the second partial derivative and rearranging them to get the partial derivative of abatement with respect to pollution gives

$$\frac{\partial Y_i^A}{\partial P_i} \;=\; -\frac{1}{\frac{\partial B}{\partial Y_i^A}} < 0, \tag{26}$$

$$\frac{\partial^2 Y_i^A}{\partial P_i^2} \;>\; 0. \tag{27}$$

This demonstrates that the marginal abatement costs are increasing for each individual firm. Or in other words, each firm faces diminishing returns to abatement.

Next, I want to show that unlike the firm level, at the society level the marginal costs of abatement are constant. For this, consider again (17). As only $\mathit{\Gamma}$ and Y_i^A affect the efficiency of abatement directly, let only $\mathit{\Gamma}$ and Y_i^A vary such that the total differential is

$$\frac{\mathsf{d}Y_i^A}{\mathsf{d}P_i} \;=\; -\frac{1}{\frac{\partial a}{\partial Y_i^A}} - \frac{\frac{\partial B}{\partial \mathit{\Gamma}}}{\frac{\partial B}{\partial Y_i^A}} \frac{\mathsf{d}\mathit{\Gamma}}{\mathsf{d}P_i} \tag{28}$$

Using the total differential of (18), divide on both sides by the differential of P_i to get

$$\frac{\mathrm{d}\mathit{\Gamma}}{\mathrm{d}P_i} = \frac{1}{Y_i^G}\frac{\mathrm{d}Y_i^A}{\mathrm{d}P_i}. \tag{29}$$

Plugging this into (28) we obtain

$$\frac{\mathrm{d}Y_i^A}{\mathrm{d}P_i} \;=\; -\frac{Y_i^A}{\frac{\partial B}{\partial Y_i^A}Y_i^A + \frac{\partial B}{\partial \mathit{\Gamma}}\mathit{\Gamma}}, \tag{30}$$

where we also exploit the fact that firms are identical and hence $Y_i^G = Y^G = \frac{\mathit{\Gamma}}{Y^A}$ must hold. Since there are CRS in the abatement sector, we can rewrite (30) as

$$\frac{\mathrm{d}Y_i^A}{\mathrm{d}P_i} \;=\; -\frac{Y_i^A}{B\left(Y_i^A, \mathit{\Gamma} Y_i^G\right)} = -\frac{1}{a} < 0 \tag{31}$$

and see that the total differential is constant as $\frac{1}{a}$ is constant. This result is the same as differentiating the aggregate relationship between pollution and abatement in (23). Hence, I can summarise that technological progress linked to aggregate abatement intensity eliminates decreasing returns to abatement – and thus rising marginal costs for abatement – on the firm level such that the marginal costs of abatement for the whole society are constant.

The planner's problem is now the aggregate production function (14), the aggregate pollution function (23) and the atemporal resource constraint connecting gross production Y^G, net output Y^N and abatement Y^A:

$$Y^N = Y^G - Y^A. \tag{32}$$

4.2 The Kindergarten Rule

Next, I concentrate on the question if and how balanced growth is feasible. For this, consider the representative agent problem (5) and apply there the utility function of (6). The dynamic maximisation problem is then

$$\max_{\{C,\theta\}} \int_0^\infty \left[\frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{zP^\gamma}{\gamma}\right] e^{-\rho t}\,\mathrm{d}t \tag{33}$$

subject to

$$\begin{aligned} \dot{K} &= AK(1-\theta) - \delta K - C && (34)\\ P &= AK(1-\theta a) && (35)\\ K(0) &= K_0, && (36) \end{aligned}$$

where $\dot{K}$ shows the evolution of the capital stock increasing with net production and decreasing with depreciation δ and consumption C. P represents the pollution function for flow pollutants. I assume that there is no pollution stock since empirical evidence supports only the EKC for flow pollutants, not for stock pollutants. Finally, I assume that the initial value of the capital stock at time $t = 0$ is K_0.

Remember from equation (24) that abatement cannot eliminate more pollution than produced and hence $\theta \leq \frac{1}{a}$. The Hamiltonian is

$$H = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{z\left[AK\left(1-\theta a\right)\right]^\gamma}{\gamma} + \lambda\left[AK\left(1-\theta\right) - \delta K - C\right]. \tag{37}$$

As corresponding first order condition (FOC) for consumption C we obtain

$$\frac{\partial H}{\partial C} = C^{-\varepsilon} - \lambda = 0, \tag{38}$$

the FOC for abatement θ is

$$\begin{aligned} \frac{\partial H}{\partial \theta} &= az\Big[AK(1-\theta a)\Big]^{\gamma-1} AK - \lambda AK = 0 \qquad (39) \\ &= az\Big[AK(1-\theta a)\Big]^{\gamma-1} - \lambda = 0. \qquad (40) \end{aligned}$$

The co-state equation for the shadow price λ is

$$\dot{\lambda} - \rho\lambda = -\frac{\partial H}{\partial K} = -\Big\{\lambda\left[A\left(1-\theta\right)-\delta\right] - z\Big[AK(1-\theta a)\Big]^{\gamma-1} A\left(1-\theta a\right)\Big\} \tag{41}$$

and the transversality condition (TVC)

$$\lim_{t\to\infty} \lambda_t K_t e^{-\rho t} = 0 \tag{42}$$

must hold. Next, I want to discuss the question of when the representative agent will abate. For this, let us define that the derivative of H with respect to θ is the function $G(\theta, \lambda)$, such that

$$\frac{\partial H}{\partial \theta} \equiv G(\theta,\lambda) = az\left[AK\left(1-\theta a\right)\right]^{\gamma-1} - \lambda. \tag{43}$$

This describes the marginal utility of abatement. The first term of the equation indicates the marginal damage of pollution; λ is the shadow price of capital, so the marginal utility of capital. Furthermore, let us consider as a starting point the case without any abatement in order to analyse which abatement behaviour is optimal for the agent. $G(\theta, \lambda)$ is then

$$G(\theta,\lambda)|_{\theta=0} = az\left(AK\right)^{\gamma-1} - \lambda. \tag{44}$$

Three different cases may occur: $G(\theta, \lambda)$ can be negative, positive or zero. If $G(\theta, \lambda) < 0$, then the marginal utility of capital exceeds the marginal damage of pollution. Therefore, the agent prefers to accumulate capital and does not care about the environment and abatement equals zero, because abating worsens the household's utility. This case I call S_{-}. In the opposite case S_{+}, the marginal damage outnumbers the shadow price of capital such that $G(\theta, \lambda) > 0$. Ergo, the household tries to abate as much as possible since pollution causes a larger utility loss than capital is able to generate. Ergo, the household chooses $\theta = \frac{1}{a} \equiv \theta^K$. Finally, in S_0 the marginal damage equals the shadow price and $G(\theta, \lambda) = 0$. This case is already optimal as the first derivation is zero. The agent is indifferent between abating and accumulating capital. Hence, abatement is set such that $\theta \in [0; \theta^K]$. The following equations define the three cases S_{-}, S_0 and S_{+} formally for any combination of $\{K, \lambda\}$:

$$\begin{aligned} S_{-} &= \Big\{\{K, \lambda\} \,|G(\theta, \lambda)|_{\theta=0} = az\,(AK)^{\gamma-1} - \lambda < 0\Big\} \\ S_0 &= \Big\{\{K, \lambda\} \,|G(\theta, \lambda)|_{\theta=0} = az\,(AK)^{\gamma-1} - \lambda = 0\Big\} \\ S_{+} &= \Big\{\{K, \lambda\} \,|G(\theta, \lambda)|_{\theta=0} = az\,(AK)^{\gamma-1} - \lambda > 0\Big\} \end{aligned}$$

Brock and Taylor (2003) entitle the maximal abatement intensity $\theta = \theta^K$ as "the Kindergarten Rule", because economies adopting the Kindergarten Rule clean up straight after creating pollution.[13] The following steps analyse the motion of the system in all three sections when $\gamma = 1$ and when $\gamma > 1$.

The Dynamics for $\gamma = 1$

First, consider the case that $\gamma = 1$, where $G(\theta, \lambda)$ is then

$$G(\theta, \lambda)|_{\gamma=1} \quad = \quad az - \lambda. \tag{45}$$

As the marginal damage from pollution az is constant, (45) depends only on λ, but not on θ. This means that the representative agent cannot influence the switching moment through abatement activity.

Now let us distinguish the motion of the system in all sections, starting with S_0. It is obvious that (45) cannot remain equal to zero, because of the motion of the shadow price given in (41). As λ is changing all the time, S_0 is unstable and switches immediately either to S_{+} or to S_{-}, depending if λ becomes larger or smaller. Hence, we know that in the case of $\gamma = 1$ abatement is either zero or it is set to the largest possible value θ^K.

[13] Brock and Taylor refer to the book *All I Really Need to Know I Learned in Kindergarten: Uncommon Thoughts on Common Things* by Robert Fulghum, which provides a list of common kindergarten rules. Among others, "clean up your own mess" – the Kindergarten Rule – is according to Fulghum one of the basic values we are taught at young age that "are the bedrock of a meaningful life" (Brock and Taylor, 2003).

Next, consider a set of $\{K, \lambda\}$ that puts us in S_+ and abatement appears at its maximum. The motion of the shadow price is then according to (41)

$$\frac{\dot{\lambda}}{\lambda} = -\underbrace{\left[A\left(1-\theta^K\right)-\delta-\rho\right]}_{\equiv g} \qquad (46)$$

where $g \equiv A(1-\theta^K)-\delta-\rho$. As long as $A(1-\theta^K) > \delta+\rho$, the shadow price becomes constantly smaller over time. This is a standard assumption in growth theory which I simply assume to hold at that point, leaving the discussion to the end of the subchapter. If $g > 0$ holds, then the growth rate of λ is negative, the shadow price for capital falls. This is plausible as the shadow price represents also the marginal utility of capital and with a growing capital stock the marginal utility decreases. We can rewrite (46) such that it gives the value λ at any period t we get:[14]

$$\lambda_t = \lambda_0 e^{-gt}. \qquad (47)$$

It is obvious that the shadow price for capital decreases all the time in S_+ as g is by definition positive. Furthermore, we can take advantage of λ_t when solving the differential equation of K_t.[15] The result is

$$K_t = e^{(g+\rho)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\,\mathrm{d}s\right]. \qquad (48)$$

We can see that K_t grows over time, because the exponent outside the brackets is positive and the exponent inside is negative. Hence, for large t the integral approaches zero and the negative term vanishes. Therefore, depending on the initial parameter values, K_t may first fall and later rise again or grow permanently.

Yet, we do not know what happens if t approaches infinity. In order to see this, I plug (47) and (48) into the TVC and find

$$\lambda_0 = [hK_0]^{-\varepsilon}, \qquad (49)$$

where $h \equiv g\left(1-\frac{1}{\varepsilon}\right)+\rho$.[16] As standard sufficient condition for the TVC it is satisfactory that $h > 0$ must hold. This means that in case $\varepsilon < 1$, the time preference of the agent needs to be large enough to offset the negativity of the first term of h.

[14] The procedure is derived in detail in Appendix A.2.1.

[15] The derivation is illustrated in Appendix A.2.3.2.

[16] The derivation of (49) is given in Appendix A.3.

Using this in (47), it is obvious that, as long as g is positive, λ_t is a strictly monotonic decreasing function of K_0 and hence sustainable growth is possible. Furthermore, the dynamics of λ_t imply that the system remains in S_+ until infinity.

Finally, I want to know if there exists a balanced growth path. For this, let us start with the growth rate of consumption. From (38) we know, that

$$C = \lambda^{-\frac{1}{\varepsilon}}. \tag{50}$$

Taking logs and differentiating with respect to time, we obtain the growth rate of consumption g_C:

$$g_C = \frac{g}{\varepsilon} > 0. \tag{51}$$

From here, it is easy to derive the steady state. In the steady state all relevant variables either remain constant or grow at a constant rate. Dividing (34) by K such that

$$\frac{\dot{K}}{K} = [A(1-\theta)-\delta] - \frac{C}{K}, \tag{52}$$

we can see that on the balanced growth path capital must grow with the same rate as consumption to keep the growth rate of capital constant. Furthermore, due to the linear production technology, it is obvious, that the growth rate of production must equal them, too. Therefore, we can conclude that the balanced growth path is the trajectory that satisfies

$$g_Y = g_K = g_C = \frac{g}{\varepsilon} > 0. \tag{53}$$

Next, suppose that the initial value for capital K_0 let us start in S_-. Then abatement equals zero and $\theta = 0$. Replacing this in (41) and (34), we can solve the differential equation for λ_t as[17]

$$\lambda_t = \lambda_0 e^{-(A-\delta-\rho)t} + \frac{Az}{A-\delta-\rho} \tag{54}$$

[17] The derivation is provided in detail in Appendix A.2.2.

and for K_t as[18]

$$K_t = e^{(A-\delta)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[(A-\delta-\rho)\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\mathrm{d}s - \left(\frac{Az}{A-\delta-\rho}\right)^{-\frac{1}{\varepsilon}}\int_0^t e^{-(a-\delta)s}\mathrm{d}s\right]. \quad (55)$$

As in case S_+, λ_t falls exponentially over time approaching zero as long as $g > 0$ holds. This attribute of λ_t is regardless its constant term. Capital increases over time as the exponents of the second and third term in squared brackets are negative, whereas the function outside the brackets has a positive power. Hence, with increasing t their negative impact on K_t becomes smaller and smaller until it vanishes as for high values of t the integrals approach zero. Then, only the positive terms are left such that K_t increases constantly.

However, the system in S_- is not stable. Since az – the border of the shadow price to switch from S_- to S_+ – is a positive number and as λ approaches zero, there exists a value for λ in finite time for which equation (43) becomes non-negative and shifts the system into the above discussed case of S_+. Therefore, S_- is not sustainable and always turns into the second stage, where abatement is active.

The Dynamics for $\gamma > 1$

Next, I discuss the more difficult case $\gamma > 1$. However, this does not change anything in dynamics in S_+ and S_- from those in case of $\gamma = 1$. Solely S_0 requires a closer look. Since some abatement occurs in S_0 such that $\theta > 0$ and as S_0 is defined such that $G(\theta, \lambda) = 0$, (43) reads as

$$G(\theta, \lambda) = az\left[AK\left(1-\theta a\right)\right]^{\gamma-1} - \lambda = 0, \quad (56)$$

where $\theta = \theta^K$ is never optimal for any positive values of λ. Therefore, abatement must take a value such that $0 < \theta < \theta^K$. Solving equation (56) for θ gives

$$\theta = \frac{1}{a}\left[1 - \frac{1}{AK}\left(\frac{\lambda}{az}\right)^{\frac{1}{\gamma-1}}\right]. \quad (57)$$

Plugging this result into (34) and (41) we obtain the state/co-state-equations:[19]

[18] Cf. Appendix A.2.3.3.

[19] In Appendix A.2.1 I show that $\dot{\lambda} = -g\lambda$ is true.

$$
\begin{aligned}
\dot{\lambda} &= -g\lambda && (58) \\
\dot{K} &= (g+\rho)\,K + D\lambda^{\frac{1}{\gamma-1}} - C, && (59)
\end{aligned}
$$

where $D \equiv \frac{1}{a}\left(\frac{1}{az}\right)^{\frac{1}{\gamma-1}}$ and recalling equation (38), $C = \lambda^{-\frac{1}{\varepsilon}}$. As before, we can solve the differential equation[20] and rewrite the shadow price depending on t only such that

$$
\lambda_t = \lambda_0 e^{-gt}. \tag{60}
$$

It is obvious that λ_t falls constantly over time. Similarly, we can solve the differential equation for K_t which gives us:[21]

$$
K_t = e^{(g+\rho)t}\left[K_0 + D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\left[g\left(\frac{\gamma}{\gamma-1}\right)+\rho\right]s}\mathrm{d}s - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(\frac{\varepsilon-1}{\varepsilon}\right)+\rho\right]s}\mathrm{d}s\right] \tag{61}
$$

We can see that in the long run, K_t increases over time since the exponent outside the squared brackets is positive but the integrals inside the brackets have a negative exponent. Thus, for large t the integrals approach zero such that all negative terms of K_t vanish.

Using the TVC yields a relationship between the initial capital stock K_0 and the initial shadow price λ_0, where we obtain[22]

$$
K_0 + \frac{D}{\alpha_1}\lambda_0^{\frac{1}{\gamma-1}} = \frac{1}{\alpha_2}\lambda_0^{-\frac{1}{\varepsilon}}, \tag{62}
$$

with $\alpha_1 \equiv \left(\frac{\gamma}{\gamma-1}\right)g + \rho$ and $\alpha_2 \equiv h = g\left(\frac{\varepsilon-1}{\varepsilon}\right) + \rho$. In order to distinguish the relationship between K_0 and λ_0, we can use the total derivative of (62):

$$
\begin{aligned}
\mathrm{d}K_0 &= -\underbrace{\left[\frac{1}{\varepsilon}\frac{1}{\alpha_2}\lambda_0^{-\left(\frac{1}{\varepsilon}+1\right)} + \frac{D}{\alpha_1}\frac{1}{\gamma-1}\lambda_0^{\frac{1}{\gamma-1}-1}\right]}_{>0}\mathrm{d}\lambda_0 && (63) \\
\Leftrightarrow \mathrm{d}\lambda_0 &= \frac{-1}{\frac{1}{\varepsilon}\frac{1}{\alpha_2}\lambda_0^{-\left(\frac{1}{\varepsilon}+1\right)} + \frac{D}{\alpha_1}\frac{1}{\gamma-1}\lambda_0^{\frac{1}{\gamma-1}-1}}\mathrm{d}K_0. && (64)
\end{aligned}
$$

[20] Cf. Appendix A.2.1
[21] Cf. Appendix A.2.3.4
[22] Cf. Appendix A.3

The total derivative indicates that λ_0 is a decreasing function of K_0, hence, the higher K_0, the lower λ_0. Thus, a large initial value of capital implies a low initial shadow price such that for K_0 we can expect to be in S_+. As λ_t falls constantly over time it is clear that the system remains in S_+ once it enters.

From (60) and (61) we know that in S_0 λ_t shrinks exponentially over time and K_t increase in the long run. Using this information on (57), we see that θ must grow steadily in the long run too, until the maximal value θ^K is reached. Then, however, the system switches to S_+ and it remains there as in the case shown above.

Finally, suppose the initial conditions put the system into S_-. Since λ_t remains the same as (54) in S_- and falls steadily over time, the system is not sustainable in S_- because once λ_t falls below the level of the stage switch and S_0 is entered.

Just as in the case of $\gamma = 1$, we know from (58) that the growth rate of consumption is $g_C = \frac{g}{\varepsilon} > 0$. As θ increases with ongoing time, the growth rates of capital and output approximate those values of the case of $\gamma = 1$. Ergo, if we are in S_0 and $\gamma > 1$ holds, then the intensity of abatement approaches the Kindergarten Rule asymptotically.

The Assumption of $g > 0$

As mentioned above, one crucial assumption for sustainable growth is $g \equiv A(1 - \theta^K) - \delta - \rho > 0$. This assumption however is quite similar to the assumption of the standard AK model: usually, the marginal product of capital must offset depreciation and impatience (*e.g.* Novales et al., 2010, p. 258). In this model, the marginal product of capital adjusted for the abatement costs must be sufficiently high. Furthermore, it must be ensured that $A(1 - \theta^K)$ is positive. Yet, this is given since abatement *per definitionem* is a productive activity. This is obvious from $\theta^K \equiv \frac{1}{a}$: As mentioned above, $a > 1$ is true because abatement must be able to clean the pollution caused by itself plus the dirt it is supposed to lower, otherwise it is not abatement. To put it differently, if abatement produces one unit of pollution, it must reduce $a > 1$ units of pollution to be effective. Therefore, the required condition, that $A(1 - \theta^K)$ is positive, holds.

Nevertheless, it is still necessary that the adjusted marginal product of capital exceeds the depreciation δ and the time preference ρ. If the reduction of pollution is not very effective, a will be close to unity and so will θ^K. This in turn means that growth will not happen. Additionally, if capital is not very productive or if the level of depreciation and/or impatience is high enough to satisfy $A(1 - \theta^K) < \delta + \rho$, economic growth is not possible. However, as said before, these are general requirements for any kind of growth wherefore the assumption of $g > 0$ seems to be appropriate.

4.3 The Turning Point

In the following section I want to discuss the turning point and the factors that influence it. The turning point is the locus, in which abatement starts to be non-zero. Hence, the turning point divides the model into two stage; in stage I, there is no abatement, whereas in stage II society reduces the emitted pollution. In order to find the turning point, let us think first about the case when $\gamma = 1$. Recall that abatement occurs when (45) is non-negative and S_+ starts.[23] Therefore, for the turning point we have $\lambda^* = az$. Moreover, from (49) we know that the TVC of stage II requires that

$$\lambda^* = [hK^*]^{-\varepsilon}. \tag{65}$$

Solving this for K^*, we find K^*

$$K^* = \frac{1}{h}\left[\frac{1}{\lambda^*}\right]^{\frac{1}{\varepsilon}} = \frac{1}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}} \tag{66}$$

and thus the turning point for $\gamma = 1$ is at

$$Y^* = AK^* = \frac{A}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}}. \tag{67}$$

The graphical solution of the PIR for $\gamma = 1$ is provided in Figure 3. If the parameters remain constant, we can see that the PIR rises with the same rate independent from the initial capital stock. Moreover, for a certain set of parameters the turning point is the same for any path. Hence, K_0 only defines the starting point of the system and thus the pollution level at the turning point: The smaller K_0, the higher is the pollution level at Y^*. For small values of Y, pollution increases due to the fact that abatement is zero. For $Y > Y^*$, abatement is at its maximum. Since I consider pollution as a flow variable, full reduction leads to a zero-pollution scenario. This gives an EKC-shaped PIR, where the turning point covers the whole down turn of the EKC. It should be mentioned that the initial values of an economy affects its pollution path. The smaller the initial value of capital, the higher is the level of pollution in the turning point. This occurs because a poor economy faces the turning point relatively late in comparison to a rich country. Since capital grows longer without abatement employed, pollution grows simultaneously to a higher peak. However, poor and rich

[23] Remember that in the case of $\gamma = 1$ S_0 is not stable and shifts immediately forward into S_+.

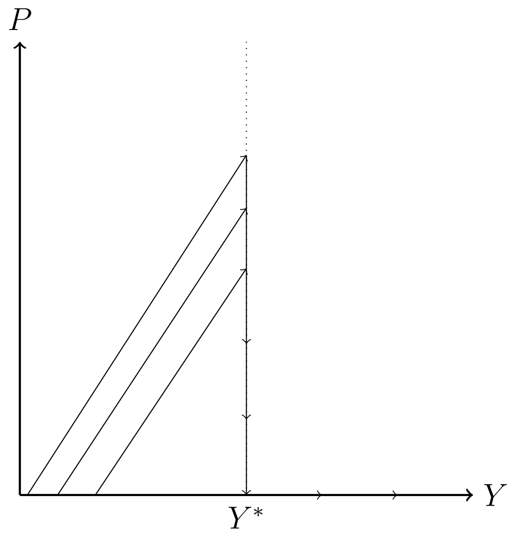

Figure 3: EKC and turning point for $\gamma = 1$

countries reduce pollution at the same level of Y, whereafter the economy experiences zero pollution regardless its initial attributes.

For $\gamma > 1$, deriving the turning point is more complex. However, the approach is the same. The change from a non-abating to an abating regime happens when

$$\lambda^* = az\,(AK^*)^{\gamma-1} \tag{68}$$

is satisfied. From the TVC we know that

$$K^* = \frac{1}{\alpha_2}\lambda^{*-\frac{1}{\varepsilon}} - \frac{D}{\alpha_1}\lambda^{*\frac{1}{\gamma-1}}. \tag{69}$$

Plugging (68) into (69) and solving for K^* gives us the turing point at

$$K^* = \left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\frac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma+\varepsilon-1}{\varepsilon}}. \tag{70}$$

Hence, the turning point is at

$$Y^* = AK_t = A\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\frac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma+\varepsilon-1}{\varepsilon}}. \tag{71}$$

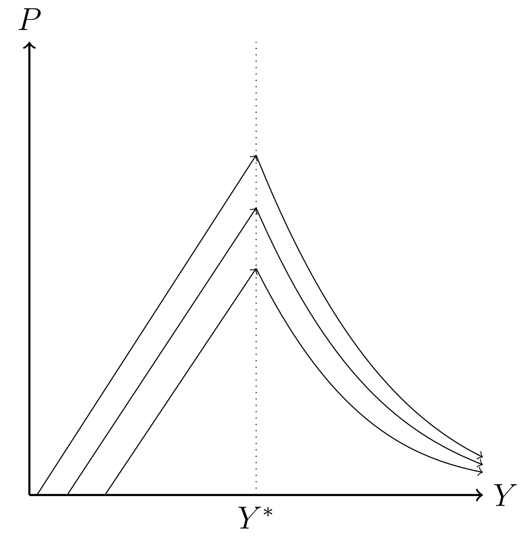

Figure 4: EKC and turning point for $\gamma > 1$

Again, the turning point is a vertical line, meaning the initial conditions of a country do not affect the application of abatement: it will always switch from stage I to state II in Y^*. However, the initial conditions do affect the amount of pollution the country faces at the turning point. The smaller K_0, the higher the peak level of P at the turning point. As in the case of $\gamma = 1$, a poorer country needs relatively more time to accumulate capital to reach the turning point. Meanwhile, pollution accrues such that at the turning point an initially poor country has a larger burden of pollution than a initially richer country. However, as t approaches infinity, the difference of pollution level fades as gross pollution falls to zero and flow pollution implies that there is no stock of pollution that needs to be reduced. The case of $\gamma > 1$ is pictured in Figure 4.

It is important to analyse the determinants of the turning point. To do so, we are interested in the comparative statics of the turning point and its determining factors, that means in $\frac{\partial Y^*}{\partial x}$ for $x \in \{A, a, z, \rho, \delta\}$ for $\gamma = 1$ and $\gamma > 1$. The calculations of the derivatives are provided in Appendix A.4. The results listed in Table 2 are however partly ambiguous. Starting the discussion with $\gamma = 1$, we still need to differentiate whether ε is larger, smaller or equal to unity.

For $\varepsilon = 1$, the results for all determinants can be derived. The derivative with respect to A is larger than zero, hence an increase in A leads to a higher turning point. The opposite holds for a, z and ρ: The larger they are, the lower the turning point. δ does not affect the turning point at all, the derivative is zero. For $\varepsilon > 1$, for all determinants but A the derivatives are clear. Any increase of a, z and ρ shifts the turning point inwards and Y^* becomes smaller. An increase in δ works the opposite direction such that Y^* shifts outwards. For A, the derivate depends on the values of the parameters. For $\varepsilon < 1$, the results for a turn around: Now Y^* becomes larger as

Table 2: Comparative static results for Y^*

$\frac{\partial Y^*}{\partial x}$ for $x \in \{A, a, z, \rho, \delta\}$						
	$\gamma = 1$, $\varepsilon = 1$	$\gamma = 1$, $\varepsilon > 1$	$\gamma = 1$, $\varepsilon < 1$	$\gamma > 1$, $\varepsilon = 1$	$\gamma > 1$, $\varepsilon > 1$	$\gamma > 1$, $\varepsilon < 1$
A	> 0	?	?	?	?	?
a	< 0	< 0	> 0	?	?	?
z	< 0	< 0	< 0	> 0	> 0	> 0
ρ	< 0	< 0	< 0	?	?	?
δ	0	> 0	< 0	> 0	?	?

a rises, whereas it shrinks with z, ρ and δ. Again, for A the results are unclear since they depend on the values of the parameters. Thus, a general statement for A is not possible.

For $\gamma > 1$ it is significantly more difficult to provide any clear statements from the comparative statics. Explicit results are possible only for a few variables: the derivative with respect to z is positive for any ε, and the derivative with respect to δ is positive for $\varepsilon = 1$. In all other cases, the turning point equation is too complex and depends on the parameters' values. Therefore, for $\gamma > 1$ the model must be solved numerically to receive proper answers. This however would go beyond the scope of this master's thesis, therefore I omit a numerical analysis here and leave it for later studies. Due to the ambiguity of the turning point of $\gamma > 1$, I will focus in the further debate exclusively on the case $\gamma = 1$.

The effect of ρ on the turning point is independent from ε. The higher the time preference ρ, the lower the turning point and the system switches away from zero-abatement. The effect is plausible: The higher the time preference, the smaller is the shadow price of capital.[24] But the smaller λ_t, the earlier the stage change due to (45) occurs.

For a and δ the analysis varies. Deriving Y^* with respect to a is negative for $\varepsilon \geq 1$ and positive otherwise. So if the elasticity of substitution of consumption is relatively high, an increase in the productivity of abatement causes the turning point to shift to the left. If, however, the substitution of consumption is difficult ($\varepsilon < 1$), then stronger abatement productivity means that the turning point shifts to the right. There are two effects on the turning point caused by a change of a. The first one comes from (45): if a increases, the shadow price at the turning point λ^* also increases, such that it is reached quicker. The second affects the initial shadow price λ_0 from (49). The larger a, the larger g. Depending on ε, h becomes larger with g ($\varepsilon > 1$), smaller ($\varepsilon < 1$) or remains the same ($\varepsilon = 1$). If h is larger, λ_0 is smaller and λ^* is met quicker. For $\varepsilon \geq 1$, both effects cause the turning point to be reached quicker. If

[24] Compare for this (47) and (49).

$(\varepsilon < 1)$, the effects work in the opposite direction and the regime switch occurs later.

In matters of δ the elasticities affect the impact on the turning point even more. Depending on ε, a higher depreciation rate shifts the turning point inwards, outwards, or not at all. The explanation requires a look at (49). Let us assume that δ becomes smaller, then g becomes larger. Depending on ε, h becomes then larger (for $\varepsilon > 1$), smaller (for $\varepsilon < 1$) or does not change (for $\varepsilon = 1$). If h is larger, then the initial shadow price λ_0 is lower. Hence, the turning point is reached quicker, abatement sets in earlier and the turning point shifts left.

For z the effects on the turning point are not clear from comparative statics at first sight. However, these uncertainties can be cleared up easily. If z increases, then the shadow price at the turning point λ^* increases too such that the system reaches the turning point earlier. If $\varepsilon < 0$, $\frac{\partial Y^*}{\partial z}$ is negative only if h is larger than zero, which is assumed. Hence, for any ε the stronger the impact of disutility, the lower is the turning point.

For A, the effects on the turning point are totally unclear. Only for $\varepsilon = 1$ it is sure that the turning point increases, any other case depends on the parameters. There are two effects considered with A. If A increases, then g increases causing a change in λ_0 depending on ε (49). Moreover, the larger A, the faster production grows (14). If $\varepsilon = 1$, the only impact of A is on the production level. Hence, when reaching the shadow price at S_0, Y^* is larger if A is larger. However, if $\varepsilon \neq 1$, then the strength of the single effects depend on the parameters such that a simple statement is not possible.

Throughout the analysis of the turning point, we find that the elasticities are important factors for the turning point. The income elasticity of marginal damage ε plays an important role in determining the income level of the turning point and thus indirectly also the pollution level at which abatement sets in and degradation declines. Also, the direction of the parameters' impacts rely on the value of the elasticities. However, the elasticities do not affect the rate of improvements nor do they determine whether improvements themselves occur.

The turning point indicates the moment when abatement begins. But is it also possible that abatement, once started, ceases again or that an economy never reaches the turning point? These questions I want to discuss now. Let us first think about of ending abatement. For this, we must be located in S_+ to be able to stop pollution reduction. Recall from (47) that λ_t is a strictly monotonic falling function during stage II. With this information it is obvious that (45) does never become negative again as the difference is already positive and the subtrahend becomes smaller while the minuend is constant. Therefore, once in the second stage, the economy remains there such that abatement never ceases.

To demonstrate that each economy must switch the stages in finite time consider the steady state loci of the stages with and without abatement with $\frac{\partial K}{\partial t} = 0$.

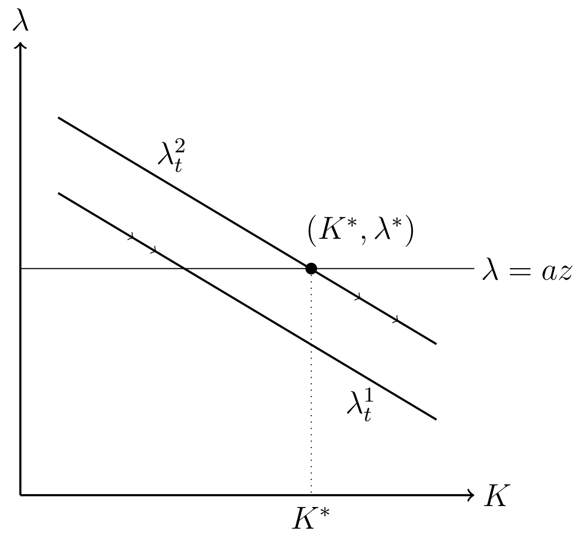

Figure 5: Phase diagram: The stage switch

In stage I, the locus must have $\lambda_t^1 = [(A - \delta) K]^{-\varepsilon}$, in the second it is $\lambda_t^2 = \left\{\left[A\left(1 - \theta^K\right) - \delta\right] K\right\}^{-\varepsilon}$.[25] In a phase diagram with $\{K_0, \lambda\}$ as in Figure 5, both loci are necessarily downward sloping. However, due to $\theta^K = \frac{1}{a} > 0$, one can see that λ_t^2 lies always above λ_t^1. The horizontal line of $\lambda = az$ indicates the shadow price at the turing point, the intersection of this horizontal line with λ_t^2 the turning point in (K^*, λ^*).

For any λ above the horizontal line the system is located in S_-, in S_+ it is for any λ below it. Since I assumed that $g > 0$, the dynamics of λ move the shadow price below the switching boarder. Thus, for any K_0 between 0 and K^*, a λ_0 can be found that carries the starting point (K_0, λ_0) in finite time t^* to the turning point. Thus, any economy starting in stage I must approach the turning point and starts abating.

The optimality of this constructed candidate solution in (K, λ) space, which joins the first and second stage, can be proven by the sufficiency theorem of Arrow and Kurz (1977, p. 43-49). First, the candidate solution meets the state/co-state equations. In stage II, the solution of the state/co-state equations satisfies the TVC. Running the dynamics of the state/co-state equations in S_- backwards in time starting from the turning point, one can see easily that the projection of λ_t^1 covers all K that meet $0 < K < K^*$. Thus, for any positive initial value of K_0, there is a well-defined candidate solution. Focusing on the locus (K^*, λ^*), one can see that in stage I, $\theta = 0$ is optimal, whereas this holds for $\theta = \theta^K$ in stage II. Ergo, the control θ optimises the Hameltonian H and so does the control C. This concludes the proof with the sufficiency theorem.

[25] The superscripts of $\lambda_t^i, i \in \{1, 2\}$ indicate the corresponding stage.

5 The EKC and Increasing Returns to Scale in Abatement

In this section, the approach of Chapter 4 will be examined in a more general way. As mentioned in the previous chapter, learning by doing may also lead to increasing returns to scale (IRS) in abatement. This context was initially discussed in Andreoni and Levinson (2001), but other scholars picked up the idea (Copeland and Taylor, 2003; Egli and Steger, 2007). Andreoni and Levinson discuss the impact of IRS in abatement on the EKC in a simple static one-sector model, showing that no other assumptions are necessary to create a hump-shaped PIR. Copeland and Taylor (2003) investigate IRS in abatement in a two-sector general equilibrium model, supporting the weak assumptions for the EKC. While the model of Andreoni and Levinson focuses on pollution and abatement driven by consumption and environmental effort respectively, Copeland and Taylor replicate the findings for a production economy. Egli and Steger (2007) extend the model of Andreoni and Levinson into a dynamic frame.

The source of IRS is not restricted to learning by doing. Andreoni and Levinson state that IRS in abatement might result from spillover effects or from factory sizes. Copeland and Taylor argue that the economies of scale are caused by industry-wide external effects. In the same direction aim Egli and Steger who assume that IRS result from positive external effects of other abating market participants. However, these externalities are actually nothing else but spillover effects, hence they are similar to learning by doing. As Bretschger and Smulders (2007) show for the model of Egli and Steger (2007), the external effects are the same as an exogenous shift in the abatement technology. Therefore, they can be treated the same way as if they were from learning by abating.

In this chapter I discuss the idea of increasing returns to scale in abatement in a dynamic model following the work of Egli and Steger (2007). For this purpose I first introduce the static model of Andreoni and Levinson (2001) as the basic framework for IRS in abatement in order to provide a reference point for the dynamic case. After that, I set up the dynamic model of Egli and Steger. At first, I present the problem for the benevolent planner in a general and a specified form. Thereon, I investigate the determinants of the time path of pollution and of the pollution-income relationship as well as the turning point. Following this, I discuss the general solution for a decentralised economy and the corresponding optimal taxation that leads to the social planner solution. Finally, I analyse in the specified model the determinants of the PIR and the turning point and the effects of public policy on both features.

5.1 The Framework of the Andreoni and Levinson EKC Model

Firstly, let us set up the model of Andreoni and Levinson (2001) as a benchmark in the following discussion on IRS in abatement. Their simple static model requires only very weak conditions for the EKC. They demonstrate that IRS in abatement is the only necessary feature to create an EKC. No other assumptions on *e.g.* technological change, policy institutions or released constraints are needed.

Furthermore, they state that IRS in abatement actually encompass these explanations. The implementation of abatement after passing a threshold could be justified by the existence of fixed costs which lead to IRS in abatement. Thinking of the work of Jones and Manuelli (2001), the fixed costs could arise through the installation of a capable environmental agency after the regime-switch towards pollution-control. These costs cause economies of scale from a societal perspective. Also, if abatement is too costly for the economy before passing the threshold as in Stokey (1998) or Brock and Taylor (2003, 2005) it might be due to fixed costs in establishing pollution abatement. Therefore, the IRS model of Andreoni and Levinson can be viewed as a reduced form of the model of Brock and Taylor presented in Chapter 4.

Preferences

The preferences are similar to those used in the previous chapter. Just as in (2) the utility function depends on consumption C and pollution P such that

$$U = U(C, P), \tag{72}$$

where consumption has a positive impact on utility $U_C > 0$ and pollution a negative one $U_P < 0$. Likewise, U is quasiconcave in C and in $-P$.

Pollution

As before, net pollution is the difference between gross pollution and abatement. However, it is modelled differently than in the previous chapter. Now, degradation depends on consumption C and environmental effort E. Again, gross pollution is assumed to be a by-product of economic activities, but now it results from consumption, not from production. Nevertheless the idea is lucid. Consumption has both a direct and an indirect impact on the environment. On the one hand, higher levels of consumption require larger input factors and generate larger amounts of polluting factors. On the other hand, consumption behaviour and consumers' choice constitute an important part of the production-consumption chain, because it is the consumer who makes the final decision about the purchase of commodities and services (Orecchia and Tessitore, 2011). Therefore, it is reasonable to think of pollution as a by-product of consumption.

Abatement results from the resource expenditures of households. The two possible resources are environmental effort E and consumption C, which affects effective abatement indirectly via its impact on pollution. The abatement function $B = \tilde{B}\Big(P(C), E\Big)$ describes the effectiveness of environmental effort in improving environmental quality. Since consumption is the only argument of pollution, we can rewrite the abatement function as

$$B(X, \Phi) = \tilde{B}\Big(P(C), E\Big) = B(C, E). \tag{73}$$

The abatement function increases in both arguments C and E such that $B_C, B_E > 0$. As before, it is strictly concave. The coherency between abatement and environmental effort is intuitively clear: The higher the environmental effort undertaken by market participants, the higher the level of abatement. Although not that obvious *prima facie*, the relationship between abatement and consumption is explained easily too: a higher consumption level means higher pollution. This in turn implies that in absolute values the abatement is also higher, or in other words that abatement is more effective. Hence, abatement increases in both arguments.

Both arguments are essential inputs for abatement such that $B(0, E) = B(C, 0) = 0$. Thus, abatement does not exist if there is no environmental effort nor consumption. Again, this is intuitive for E. For C the indirect way via P is necessary. If there is no environmental effort, there cannot be any abatement since nobody tries to reduce pollution. If there is no consumption, then there is no pollution. Since pollution cannot be negative, zero pollution displays concurrently zero abatement. Applying this information in (16), the pollution function reads as

$$P(C, E) \quad = \quad C - B(C, E). \tag{74}$$

Resource Constraint

Finally, suppose that the endowment for available resources Y is limited to expenditures for consumption C and environmental effort E only such that

$$Y \quad = \quad C + E. \tag{75}$$

Conditions for the EKC

Under two conditions the existence of the EKC is provided in this setting. According to the first one, *"the marginal willingness to pay to clean up the last speck of pollution does not go to zero as income approaches infinity"* (Andreoni and Levinson, 2001, p. 277). This rather weak condition is easily satisfied if pollution abatement is a normal good. This condition, however, was mentioned already in chapter 2.1: The normality of environmental quality, which is a synonym for pollution abatement, is *sine qua non* for the existence of an EKC (Lieb, 2002).

The second condition requires IRS in abatement in order to produce the inverse-U shape of the PIR. To demonstrate this, Andreoni and Levinson (2001) define the utility function as

$$U(C, P) \quad = \quad C - zP \tag{76}$$

and the abatement technology as

$$B(C, E) \quad = \quad C^{\alpha} E^{\beta} \tag{77}$$

with $\alpha, \beta \in (0, 1)$. Plugging (77) into (74) gives

$$P(C, E) \quad = \quad C - C^{\alpha} E^{\beta}. \tag{78}$$

Assume for simplicity that $z = 1$.[26] Substituting (78) into (76) gives then

$$U \quad = \quad C - C + C^{\alpha} E^{\beta} = C^{\alpha} E^{\beta}. \tag{79}$$

Using the resource constraint (75), we can replace E such that utility depends only on C for given Y:

$$U \quad = \quad C^{\alpha} (Y - C)^{\beta}. \tag{80}$$

The optimal level of consumption can be derived through the first derivative:

[26] Andreoni and Levinson (2001) show that for $z \neq 1$ the results remain the same despite higher complexity.

$$\frac{\partial U}{\partial C} = \alpha C^{\alpha-1}(Y-C)^{\beta} - \beta C^{\alpha}(Y-C)^{\beta-1} = 0 \tag{81}$$
$$\Leftrightarrow C^* = \frac{\alpha}{\alpha+\beta}Y. \tag{82}$$

Likewise, the optimal level of environmental effort can be derived. For this, replace C with $(Y-E)$ in (79) and take the first derivative with respect to E:

$$\frac{\partial U}{\partial E} = \beta E^{\beta-1}(Y-E)^{\alpha} - \alpha(Y-E)^{\alpha-1}E^{\beta} = 0 \tag{83}$$
$$\Leftrightarrow E^* = \frac{\beta}{\alpha+\beta}Y. \tag{84}$$

Plugging C^* and E^* into (78) we get the optimal PIR:

$$P^*(Y) = C^* - C^{*\alpha}E^{*\beta} \tag{85}$$
$$= \frac{\alpha}{\alpha+\beta}Y - \left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}Y^{\alpha+\beta}. \tag{86}$$

Taking the first derivate, which stands for the slope of the EKC gives

$$\frac{\partial P^*}{\partial Y} = \frac{\alpha}{\alpha+\beta} - (\alpha+\beta)\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}Y^{\alpha+\beta-1}. \tag{87}$$

It is obvious that the sign of the first derivative depends on the parameters α and β. When $\alpha+\beta=1$, abating pollution faces constant returns to scale and the first derivative is constant as $Y^{\alpha+\beta-1}=1$. For any combination that satisfies $\alpha>\beta$, the minuend is in each ratio larger than the subtrahend, the PIR increases constantly and monotonically with rising Y. For $\alpha<\beta$, the slope of the PIR is negative such that the PIR decreases constantly with rising Y. For $\alpha=\beta$, the first derivate is equal to zero such that the PIR remains on a constant level for any value of Y. Hence, the PIR moves constantly and monotonically with rising Y, ergo there is no turning point. If $\alpha+\beta\neq 1$, we need the second derivative of $P^*(Y)$ to analyse the slope of the PIR:

$$\frac{\partial^2 P^*}{\partial Y^2} = -(\alpha+\beta-1)(\alpha+\beta)\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}Y^{\alpha+\beta-2}. \tag{88}$$

From this, one can see that in the case that abatement features diminishing returns to scale (*i.e.* if $\alpha+\beta<1$), $P^*(Y)$ is convex, whereas in case of increasing returns to scale in abatement ($\alpha+\beta>1$), $P^*(Y)$ is concave. The corresponding graphs of the three cases are illustrated in Figure 6, where (A) shows the case of constant returns, (B) the one for diminishing returns and (C) increasing returns to scale. It is clear to see that Figure 6 (C) resembles the shape of the EKC. Thus, IRS in abatement do produce an EKC.

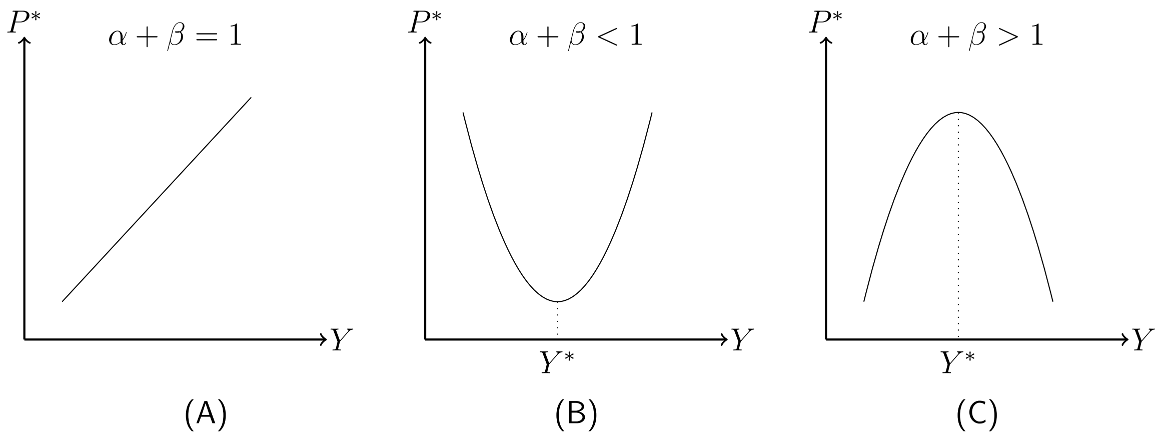

Figure 6: Optimal pollution-income relationships

As shown above IRS in abatement are crucial for the occurrence of EKC. This requires that the gross pollution function is linear. In their review on the Andreoni and Levinson paper, Plassmann and Khanna (2006) criticise the assumed linearity in the gross pollution function. Revising Andreoni and Levinson (2001), Plassmann and Khanna allow in a more general setting that the gross production function may exhibit increasing, decreasing or constant returns to scale. Furthermore, they display that, in fact, the relative magnitudes of returns to scale in abatement and gross pollution determine the existence of an EKC. It is a sufficient condition for the decline of pollution if the returns to scale in abatement are larger than the returns to scale in gross pollution. However, the assumption of a linear gross pollution function is suitable for pollution in terms of emissions, in contrast to pollution in terms of ambient concentrations or damage. Since this paper focuses on pollution as a flow variable, it is most appropriate to model pollution in terms of emissions. Thus, the assumption of IRS in abatement is in fact the sufficient cause for the EKC (Egli, 2005a).

5.2 The Dynamic Model for the Social Planner

Next, I transfer the ideas of Andreoni and Levinson (2001) into a dynamic frame following Egli and Steger (2007). Starting with the benevolent planner, I present first a general dynamic model followed by a specified model that allows one to find the PIR and the turning point of the model. The specified model is necessary to investigate analytically the shape of the PIR and the determinants of the turning point.

5.2.1 The General Dynamic Model

In this subsection, I begin with the general dynamic EKC model of Egli and Steger (2007). In this model, pollution is a by-product of consumption and it is modelled as a flow pollutant. Abatement results from expenditures on it in terms of C and E. The EKC shape results from IRS in abatement. The economy produces one homogeneous final-output good under constant returns to scale using only capital (physical and human) as an input factor. Additionally, I consider two economy-wide external effects that affect the single agent: One negative externality resulting from consumption $\bar{C}$ of other market participants and one positive from productive abatement technology T_E. The externality from technological progress results from learning by doing in abatement.

The economy contains a large number of identical households distributed on the interval [0,1]. The utility of the representative household increases with consumption C and decreases with net pollution P. Hence, the corresponding utility function reads as $U(C,P)$ with $U_C > 0, U_{CC} < 0, U_P < 0, U_{PP} < 0$. Each household can spend money on consumption C and environmental effort E.

Pollution increases with the representative agent's consumption and with consumption of all market participants such that the gross pollution function is $G(C,\bar{C})$. As before, effective abatement is again depending on gross pollution and abatement $B(C,E,T_E)$. Note, that abatement contains both the direct environmental effort of the household and the technological progress in abatement.

In order to avoid negative pollution we must assure that abatement never exceeds gross pollution. Therefore, effective abatement is

$$B\left(C,E,T_E\right) = \begin{cases} B\left(C,E,T_E\right) & \text{for } G\left(C,\bar{C}\right) \geq B\left(C,E,T_E\right), \\ 0 & \text{for } G\left(C,\bar{C}\right) < B\left(C,E,T_E\right). \end{cases} \tag{89}$$

In this setting, pollution can never become negative. This actually is an adequate attribute of pure flow pollutants (Lieb, 2004; Egli, 2005a).

The representative household maximises its lifetime utility according to

$$\max_{\{C,E\}} \int_0^\infty U(C,P)e^{-\rho t}dt, \tag{90}$$

where C and E are the control variables, ρ is the time preference and t the time index. The corresponding constraints are

$$\begin{aligned}
P(C,\bar{C},E,T_E) &= G\left(C,\bar{C}\right) - B\left(C,E,T_E\right) && (91)\\
\dot{K} &= F(K) - \delta K - C - E && (92)\\
K(0) &= K_0, && (93)
\end{aligned}$$

where the first constraint is the net pollution function, increasing with gross emissions and decreasing with abatement activity. The second constraint describes the evolution of the capital stock K: The capital stock increases with production $F(K)$ and decreases through depreciation δ and expenditures for C and E. The depreciation rate is constant and non-negative. The current value Hamiltonian for this problem is

$$H = U\left[C, P(C,\bar{C},E,T_E\right] + \lambda\left[F(K) - \delta K - C - E\right], \quad (94)$$

the corresponding state equations are

$$\begin{aligned}
\frac{\partial H}{\partial C} &= 0 \qquad \Rightarrow U_C + U_P(P_C + P_{\bar{C}}) = \lambda && (95)\\
\frac{\partial H}{\partial E} &= 0 \qquad \Rightarrow U_P(P_E + P_{T_E}) = \lambda, && (96)
\end{aligned}$$

where U_x and P_i denote the partial derivatives with respect to $x \in \{C, P\}$ and $i \in \{C, \bar{C}, E, T_E\}$ respectively. The TVC $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$ must hold. Moreover, since the interest lies on the EKC, only interior solutions are considered such that gross pollution is always larger than abatement. It is also assumed that the necessary conditions are sufficient.

The marginal utility of consumption and the marginal utility of environmental effort must equal the shadow price of capital λ along the optimal growth path. Marginal utility of consumption consists of direct utility from consumption U_C and disutility from pollution $U_P(P_C+P_{\bar{C}})$. As the social planner takes all externalities into account, (95) contains $U_P P_{\bar{C}} = U_P G_{\bar{C}}$ too. Marginal utility of abatement comprises the direct effects of abatement only. Again, due to the central planner $U_P P_{T_E} = -U_P B_{T_E}$ is included in (96).

The co-state equation is

$$\dot{\lambda} - \rho\lambda = -\frac{\partial H}{\partial K} \qquad \Rightarrow \dot{\lambda} = -\lambda(F_K - \delta - \rho), \quad (97)$$

where F_K is the marginal productivity of capital.

5.2.2 Specification of the Dynamic Model

In order to give appropriate solutions for the shape of the PIR we need to specify the general model. These are provided as follows. Subject of interest lies on defining the functions for utility, gross pollution, abatement and production. The parameterised functions are defined by the following system:

$$
\begin{aligned}
U(C,P) &= \ln(C - zP) && \text{with} && z > 0, C \geq zP && (98)\\
G(C,\bar{C}) &= C^{1-\omega}\bar{C}^{\omega} && \text{with} && 0 < \omega < 1 && (99)\\
B(C,E,T_E) &= C^{\alpha}E^{\beta}T_E && \text{with} && 0 < \alpha, \beta < 1, \alpha + \beta = 1 && (100)\\
Y &= AK && \text{with} && A > 0. && (101)
\end{aligned}
$$

Equation (98) demonstrates that utility rises with consumption C and falls with pollution P. The household's desire for a clean environment is given by z: The lower z, the lower is the loss of utility through pollution. This implies that the household benefits relatively more from consumption than from less degradation and hence it spends less on E and more on C. Since pollution is expected to harm people, z cannot be negative. Additionally, to avoid negative utility, disutility from pollution must not exceed utility from consumption.

The gross pollution function (99) includes the internal effect of consumption on gross pollution C and its external effect $\bar{C}$, ω defines the output elasticity of $\bar{C}$, $1-\omega$ the one of C. Remember that it is most suitable to measure flow pollutants in terms of emissions. Therefore, a linear gross pollution function is appropriate and ω takes a value between zero and unity.

Akin to (99), the abatement function (100) refers to both the abatement expenditures E and productivity effect T_E. To capture learning by doing in abatement I assume a positive link from environmental effort to abatement technology such that $T_E = E^{\eta}$ with $\eta \in \{0,1\}$. Abatement depends on C also, because effective abatement cannot occur without consumption. The coefficients α, β and η are again the output elasticities of the abatement input factors. Therefore, all take a value between zero and unity. Since we assume IRS in abatement $\alpha + \beta + \eta > 1$ must be satisfied.

Finally, in (101) I assume that the economy produces one homogeneous final-output good according to an AK technology.

Since the benevolent planner takes the pollution externality into account, we can set C and $\bar{C}$ equal. Given this, the net pollution function as the difference between gross pollution $G(C,\bar{C}) = C$ and abatement $B(C,E,T_E) = C^{\alpha}E^{\beta+\eta}$ is

$$\hat{P}(C,E) \;=\; C - C^{\alpha}E^{\beta+\eta}. \tag{102}$$

Furthermore, let us start analogue to Andreoni and Levinson such that $z = 1$. In this case, the utility function becomes

$$U\Big[C,\hat{P}(C,E)\Big] \;=\; \ln\Big(C - C + C^{\alpha}E^{\beta+\eta}\Big) = \ln\Big(C^{\alpha}E^{\beta+\eta}\Big). \tag{103}$$

By plugging (101) into (92) and its first order condition into (97) we receive the optimal control problem where the current value Hamiltonian is

$$H = \ln\Big(C^{\alpha}E^{\beta+\eta}\Big) + \lambda\left[(A-\delta)K - C - E\right], \tag{104}$$

while the corresponding state/co-state equations are

$$\frac{\partial H}{\partial C} = 0 \qquad \Leftrightarrow \qquad C = \frac{\alpha}{\lambda} \tag{105}$$

$$\frac{\partial H}{\partial E} = 0 \qquad \Leftrightarrow \qquad E = \frac{\beta+\eta}{\lambda} \tag{106}$$

$$\dot{\lambda} - \rho\lambda = -\frac{\partial H}{\partial K} \qquad \Leftrightarrow \qquad \dot{\lambda} = -\lambda(A-\delta-\rho) \tag{107}$$

The transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda K = 0 \tag{108}$$

must hold to make the maximisation problem solvable. Since α, β and η are constant, we know from (105) and (106) that the growth rates of C, E and λ are connected widdershins such that

$$\frac{\dot{C}}{C} = \frac{\dot{E}}{E} = -\frac{\dot{\lambda}}{\lambda}. \tag{109}$$

The time path of pollution and the PIR

In the steady state all variables must grow with the same rate such that $\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{E}}{E}$, so it is obvious that the growth rate of capital must be

$$\frac{\dot{K}}{K} = -\frac{\dot{\lambda}}{\lambda} = A - \delta - \rho. \tag{110}$$

Rewriting this we find the current value of capital depending on its initial constant value and its change over time:

$$K_t = K_0 e^{(A-\delta-\rho)t}. \tag{111}$$

Moreover, we can derive the current value of λ in terms of parameters only. Recall from (110) that

$$\frac{\dot{K}}{K} = A - \delta - \frac{C+E}{K} = A - \delta - \rho = -\frac{\dot{\lambda}}{\lambda}. \tag{112}$$

From (105) and (106) we know additionally that

$$C + E = \frac{\alpha+\beta+\eta}{\lambda}. \tag{113}$$

Plugging (113) into (112), the current value of the shadow price resolves as

$$\lambda_t = \frac{\alpha+\beta+\eta}{K_0\rho} e^{-(A-\delta-\rho)t}. \tag{114}$$

With this information in hand, we can derive the time path of pollution and the PIR. For this, plug (105) and (106) into (102) and replace λ according to (114).[27] The pollution time path is then

$$\tilde{P}(t) = \frac{\alpha\rho K_0 e^{(A-\delta-\rho)t}}{\alpha+\beta+\eta} - \left(\frac{\alpha\rho K_0 e^{(A-\delta-\rho)t}}{\alpha+\beta+\eta}\right)^{\alpha} \left(\frac{(\beta+\eta)\,\rho K_0 e^{(A-\delta-\rho)t}}{\alpha+\beta+\eta}\right)^{\beta+\eta}. \tag{115}$$

For the PIR we need to express pollution in terms of income. To do so, let us define the consumption rate as $c \equiv \frac{C}{Y}$ and the "environmental effort rate", *i.e.* the income share spent for environmental effort, as $b \equiv \frac{E}{Y}$. Replacing C and E in (102) with cY and bY respectively provides us the PIR:[28]

[27] The derivation of $\tilde{P}(t)$ is shown in Appendix A.5.

[28] This is shown in Appendix A.5.

$$P^*(Y) \;=\; cY - (cY)^{\alpha}\,(bY)^{\beta+\eta}\,. \tag{116}$$

Yet, the PIR is not solvable, because b and c are still unknown to us. However, we can still specify both in terms of parameters only, such that the PIR depends solely on constant terms. For this, consider again (105) and (106). Applying here (114) and the production function (101) gives us:

$$c \equiv \frac{C}{Y} = \frac{\frac{\alpha}{\lambda_t}}{AK_t} = \frac{\alpha\rho}{A\,(\alpha+\beta+\eta)} \tag{117}$$

and

$$b \equiv \frac{E}{Y} = \frac{\frac{\beta+\eta}{\lambda_t}}{AK_t} = \frac{(\beta+\eta)\,\rho}{A\,(\alpha+\beta+\eta)}. \tag{118}$$

Egli and Steger (2007) provide a graphical simulation of the PIR and the pollution time path.[29] Figure 7 depicts the PIR (left) and the pollution time path (right). The left figure shows that pollution first rises with growing income, later falls and possibly becomes zero. It satisfies the humped shape of an EKC due to IRS in abatement. The PIR represents a balanced growth phenomenon.[30] It can be seen that the PIR is asymmetric with an upper tail that declines relatively gradually. This EKC pattern is similar to empirical evidence by Grossman and Krueger (1995), who found asymmetric EKC shapes for different pollutants. The right picture indicates that a lot of time must pass by until pollution approaches zero again: In the simulation of Egli and Steger (2007) it takes about 250 years until pollution becomes zero.

The Turning Point

In this section, I want to analyse the determinants of the turning point. I start assuming $z = 1$, since closed-form solutions and thus analytical analyses are only possible for this case. Although it is just a special case of the model, the qualitative results remain mainly the same as can be seen in numerical analyses. Such numerical solutions for $z < 1$ provided by Egli and Steger (2007) will be discussed afterwards.

[29] The reader can find the baseline set of parameters that is used for the simulation in section 4.3. of Egli and Steger (2007).

[30] This is surprising at first glance, because a balanced growth phenomenon requires by definition that pollution grows at a constant rate what it clearly does not. However, pollution is an expression of its inputs C and E, which do grow at a constant rate. Therefore, the PIR represents a balanced growth phenomenon.

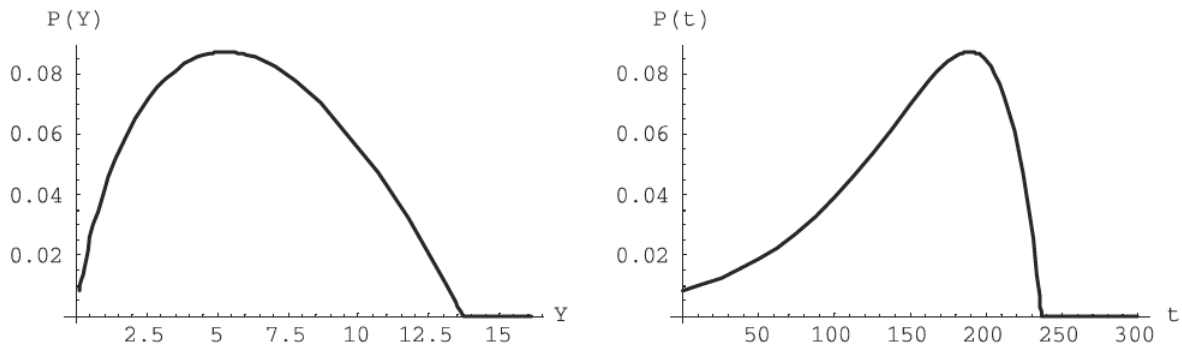

Figure 7: $P^*(Y)$ and $\tilde{P}(t)$ with IRS in abatement, source: Egli and Steger (2007)

For analysing the turning point we need to take the first differential of (115) and of (116) in order to obtain the turning point in terms of time and in terms of income respectively. The calculation of both differentials is given in Appendix A.5. Differentiating $P(t)$ with respect to t gives the time at which pollution reaches it maximum t^*:

$$t^* = -\frac{\ln\left[K_0^{\alpha+\beta+\eta-1}\alpha^{\alpha-1}(\beta+\eta)^{\beta+\eta}(\alpha+\beta+\eta)^{2-\alpha-\beta-\eta}\rho^{\alpha+\beta+\eta-1}\right]}{(\alpha+\beta+\eta-1)(A-\delta-\rho)} \tag{119}$$

At the moment t^*, pollution reaches its peak level after which it decreases again. It is determined by all parameters but z and ω. However, this is not surprising due to the assumption of $z = 1$ and $C = \bar{C}$ which eliminate the two parameters in the model. Differentiating $P(Y)$ with respect to Y gives us the turning point of the PIR independent from time:

$$Y^* = \frac{A\alpha^{\frac{1-\alpha}{\alpha+\beta+\eta-1}}(\beta+\eta)^{-\frac{\beta+\eta}{\alpha+\beta+\eta-1}}(\alpha+\beta+\eta)^{1-\frac{1}{\alpha+\beta+\eta-1}}}{\rho} \tag{120}$$

It is worth noting that Y^* is the equivalent to the solution of Andreoni and Levinson (2001). It is determined by the marginal product of capital A, the time preference ρ, the elasticities of consumption in abatement α, and environmental effort in abatement β and η. However, the depreciation rate δ and the initial value K_0 do not affect the turning point.

The comparative static results for the turning point of Y^* are listed in Table 3[31], their calculation is demonstrated in Appendix A.6. An increase in A rises Y^*, hence the turning point appears at a higher level of income. A higher value of A allows for larger capital accumulation such that more capital can be spent for C and E. For

[31] In order to simplify the notation in the table, I define $\sigma \equiv \alpha + \beta + \eta$.

Table 3: Comparative static results for Y^*

$\frac{\partial K^*}{\partial x}$ for $x \in \{A, \rho, \alpha, \beta, \eta, \delta\}$		
A	$Y^* \frac{1}{A}$	> 0
ρ	$Y^* \frac{-1}{\rho}$	< 0
α	$Y^* \frac{(\sigma-1)(-\alpha+\beta+\eta)+\alpha\sigma(\ln[\sigma]+(\beta+\eta)(\ln[\beta+\eta]-\ln[\alpha]))}{\alpha\sigma(\sigma-1)^2}$	?
β	$Y^* \frac{2+\sigma(\ln[\sigma]-2)+\sigma(\alpha-1)(\ln[\alpha]-\ln[\beta+\eta])}{\sigma(\sigma-1)^2}$	?
η	$Y^* \frac{2+\sigma(\ln[\sigma]-2)+\sigma(\alpha-1)(\ln[\alpha]-\ln[\beta+\eta])}{\sigma(\sigma-1)^2}$	?
δ	0	0

Note: $\sigma \equiv \alpha + \beta + \eta$

easier interpretation of this result, imagine that $\alpha = \beta + \eta$ such that $C = E$. In this case, pollution depends only on consumption and the effects can be analysed simply with the exponents of (116). By comparing to (117), it is obvious that a larger A causes c to fall. However, the smaller c, the larger level of income Y is required to make pollution reaching its maximum.

The second results shows that the higher ρ, the smaller Y^* is. The logic is straight forward: The higher ρ, the more important is the future for the agent's current utility. Thus, the more valuable the environmental quality of the future, the larger are the abatement activities to avoid pollution such that the turning point shifts inwards. To demonstrate this, consider again that $\alpha = \beta + \eta$. As ρ rises, c rises too, which in turn means a lower Y to reach the turning point in the updated equation (116).

The impact of the elasticities of consumption α and environmental effort β and η on the turning point are not clear. The first derivatives of (120) with respect to α and β depend on the chosen values of α, β and η. However, the static model would suggest that the stronger the economies of scale, the earlier is the turning point (cf. (87)). Ergo, the derivatives with respect to α, β and η should be all negative. Furthermore, as we will see in the next subsection, Egli and Steger (2007) solved the problem numerically and find that α, β and η have a negative effect on the turning point. An increase in the degree if IRS in abatement ergo causes a higher abatement output for each level of income, which in turn means a lower turning point.

5.3 The Dynamic Model in the Decentralised Economy

Up to now, I have considered only the case of a social planner, and have ignored the idea of market economy. However, analysing only the social planner solution is far from reality. Market imperfections are neglected and hence public policy is not necessary as demonstrated in Chapter 4. The need for public policy however has been shown already by Andreoni and Levinson (2001). In their static model, they show that in a scenario with many consumers the market solution leads to higher consumption and to smaller abatement than in the social optimum. Hence the unregulated economy is inefficient and public policy for correction needed. This clearly contradicts the statement of Brock and Taylor (2003, 2005).

Therefore, I will elaborate the thought of Andreoni and Levinson in the frame of Egli and Steger (2007). Starting with their general model, I will investigate the differences between the market solution and the one of the central planner. From there, I will derive the optimal tax that corrects these differences. Finally, I will analyse in the specified model the effects on the PIR, the determinants of the turning point and the cost effectiveness of public policies. For this, I will consult the numerical calculations of Egli and Steger.

5.3.1 The General Model

The foundations in the decentralised economy model are mainly the same is in the social planner model. The differences are concerning the capital accumulation. The households increase their capital stock through lending capital where r denotes the rental price for one unit of K. Gross expenditures for consumption and environmental effort are $(1-\tau_C)C$ and $(1-\tau_E)E$, where τ_i with $i \in \{C, E\}$ is a tax (or a subsidy for negative values of τ_i) on C or E respectively. The tax revenues T are redistributed to the household in a lump-sum manner. Moreover, T comprises all taxes such that $T = \tau_C C + \tau_E E$.

The households maximise their lifetime utility to present values

$$\max_{\{C,E\}} \int_0^\infty U(C,P)e^{-\rho t}dt \tag{121}$$

subject to

$$\begin{aligned}
P(C,\bar{C},E,T_E) &= G\left(C,\bar{C}\right) - B\left(C,E,T_E\right) &(122)\\
\dot{K} &= rK - (1+\tau_C)C - (1+\tau_E)E + T &(123)\\
K(0) &= K_0, &(124)
\end{aligned}$$

where net pollution is the difference between gross pollution and abatement, it has the same attributes as in the social planner's problem. $\dot{K}$ is the rate of change of K in period t increasing with revenues from lending capital and the redistribution of the taxes and decreasing with gross expenditures on C and E. Again, K_0 denotes the initial capital stock.

The current value Hamiltonian for this problem is then

$$H = U[C, P(C, \bar{C}, E, T_E)] + \lambda[rK - (1 + \tau_C)C - (1 + \tau_E)E + T], \quad (125)$$

the state equations according to Pontryagin's principle are

$$\frac{\partial H}{\partial C} = 0 \qquad \Rightarrow \frac{U_C + U_P P_C}{1 + \tau_C} = \lambda \quad (126)$$

$$\frac{\partial H}{\partial E} = 0 \qquad \Rightarrow \frac{U_P P_E}{1 + \tau_E} = \lambda, \quad (127)$$

and the co-state equation is

$$\dot{\lambda} = -\frac{\partial H}{\partial K} \qquad \Rightarrow \dot{\lambda} = -\lambda(r - \rho). \quad (128)$$

Again, the TVC must hold and I assume that the necessary conditions are sufficient. (126) indicates that the private marginal utility of consumption corrected by τ_C must equal λ. Similarly, from (127) we see that the private marginal utility of environmental effort corrected by τ_E must equal λ, too. Equation (128) demonstrates that the shadow price disappears at rate $r - \rho > 0$. The representative firm produces one homogenous final-output good with capital as single input. The production technology is $Y = AK$, thus it has CRS. Capital loses value with a constant depreciation rate δ. When maximising the output, the interest rate must equal the difference between the accretion of output and its depreciation, hence $r = A - \delta$.

For the ease of interpretation it is helpful to imply some simplifications. Suppose that there are zero taxes such that $\tau_C = \tau_E = 0$. Comparing (126) of the market economy with (95) of the benevolent planner it is obvious that the marginal utility of the external effect of consumption is missing. Likewise, (127) excludes the externality from productive abatement technology. Thus, without regulation the private agent ignores the external consequences of $\bar{C}$ and T_E.

The exclusion of the externalities changes the agents behaviour in the market solution. To see this, let us compare (95) and (126) and suppose that λ is constant. Then it is clear that the marginal utility of consumption in the centralised economy is larger than in the market economy as $U_P P_{\bar{C}} < 0$. Since diminishing marginal benefit of consumption is assumed, we can conclude that consumption in the market economy exceeds consumption in the centralised economy.

Similarly, comparing (96) and (127) and holding λ constant, it is obvious that the marginal utility of environmental effort in the centralised economy is smaller than in the market economy too as $U_P P_{T_E} > 0$. Hence, environmental effort is larger in the social planner case due to the assumption of diminishing marginal utility of environmental effort. All in all, in the unregulated market economy C is too high and E is too low compared to the centralised economy. However, as will be discussed next the implementation of public policy can remedy the oblivion of the externalities.

5.3.2 Optimal Taxation

In order to neutralise the imperfections of the market solution demonstrated by the differences between the decentralised and centralised economy, the government can regulate the market by implementing taxes and subsidies. The optimal tax scheme is obtained if the government sets the taxes such that the tax absorbs the external effect completely. For this purpose, one can equate (126) and (127) to (95) and (96) and solve for τ_C and τ_E respectively. The results are:

$$\tau_C{}^* = -\frac{U_P P_{\bar{C}}}{U_C + U_P(P_C + P_{\bar{C}})} > 0 \tag{129}$$

$$\tau_E{}^* = -\frac{P_{T_E}}{P_E + P_{T_E}} < 0 \tag{130}$$

Equation (129) demonstrates that the optimal tax on consumption equals the ratio of the marginal external effect of consumption on utility $U_P P_{\bar{C}}$ and the sum of private and marginal consumption effect on utility $U_C + U_P(P_C + P_{\bar{C}})$. Analogously, the optimal tax on environmental effort is the ratio of the marginal external effect of environmental effort on pollution P_{T_E} and the total marginal effect of environmental effort on pollution $P_E + P_{T_E}$.

From the utility function it is known that $U_C > 0$ and $U_P < 0$, *i.e.* utility increases with higher consumption and decreases with higher pollution. Similarly, from the abatement function we know that $P_C > 0$, $P_{\bar{C}} > 0$, $P_E < 0$ and $P_{T_E} < 0$. Applying this information to the taxation equations, one finds that $\tau_C{}^* > 0$ and that $\tau_E{}^* < 0$. Hence, in the optimal case, the government levies taxes on consumption, but grants subsidies to environmental effort.

The consequences of taxation on the representative household's decisions can be explained in a simple example. Consider again as a starting point that $\tau_C = \tau_E = 0$. Let the government introduce a consumption tax $\tau_C > 0$, the left hand side of equation (126) decreases. Holding λ constant requires that the marginal utility of consumption increases. As consumption faces falling marginal utility, an increasing marginal utility implies that the level of consumption drops. Hence, implementing a consumption tax reduces the household's level of polluting consumption.

Similarly, the household reacts on the implementation of a subsidy on environmental effort, ergo changing the subsidy to $\tau_E < 0$. This leads to a rise of the left hand side of (127). Again, λ needs to be constant such that the marginal utility of environmental effort must increase in order to compensate the change. This in turn means according to diminishing marginal utility that the actual level of environmental effort increases. Thus, a subsidy for environmental effort increases the amount the representative household spends for abatement.

5.3.3 Numerical Analysis of the Specified Model

Egli and Steger (2007) solve the specified model of the market economy numerically.[32] They investigate the dependence of Y^* on the model parameters in order to describe the effects on the PIR and the turning point. Additionally, the cost-effectiveness of different policy regimes is under study. Unlike the analytical approach to the social planner's problem, now various values of z are considered. Furthermore, the examination includes the analysis of an unregulated economy and of a semi-regulated, because those two cases are believed to describe the real world best.

Egli and Steger find several interesting outcomes. First, the shape of the PIR remains the same as in the centralised economy. However, this is not surprising since also in the market economy there are IRS in abatement. But as the effects of $\bar{C}$ are not taken into account in the unregulated case, C chosen by the agent is higher than in the social economy. For the same reason, expenditures for E are too low. Therefore, the EKC of the unregulated economy is larger in magnitude than in the economy of the benevolent planner.

In terms of the imperfectly regulated economy, the magnitude of the PIR is somewhere between the extreme cases. This is because the intervention causes lower consumption and higher abatement effort than in the unregulated case, but the imperfect policies cannot reduce C or raise E such that the level of the perfectly regulated economy is reached. Both results are in line with the findings of Andreoni and Levinson (2001).

In order to find the determinants of the turning point, Egli and Steger calculated numerically the elasticities of Y^* with respect to different model parameters such that $\frac{\Delta Y^*/Y^*}{\Delta x/x}$ with $x \in \{A, \rho, \alpha, \beta, \eta, \omega, z\}$. This has been done for $z = 1$ and $z < 1$[33] and for an unregulated and an imperfectly regulated economy. Doing so, Egli and Steger find that the results for $z = 1$ are qualitatively the same as for $z < 1$. In both cases, all elasticities have the same sign, *i.e.* apart from varying in strength all parameters keep the same effects. The same holds for the unregulated and semi-regulated economy.

The effect of each parameter on the turning point is the same regardless whether there is environmental regulation. This can be seen in Table 4. Furthermore, from Table 4

[32] For details in results and approach please compare Egli and Steger (2007).

[33] The case $z > 1$ is omitted since a higher value of z increases the disutility from P and thus increases the willingness to abate. Therefore, using $z = 1$ as a benchmark, the case of $z > 1$ is not interesting for the discussion of the effectiveness of public policy.

Table 4: Numerical elasticities of Y^* with respect to model parameters

	A	ρ	α	β	η	ω	z
unregulated economy	>0	<0	<0	<0	<0	>0	<0
semi-regulated economy	>0	<0	<0	<0	<0	>0	<0

we can see that the analytical results of the centralised economy are confirmed. A has a positive impact on the turning, whereas the opposite is true for ρ. Also, the ambiguous results for α, β and η are determined as negative. Moreover, the impact of z itself on the turning point is throughout negative. This result is not surprising because z measures the impact of pollution on utility. The larger z, the higher the disutility of pollution and the higher is the willingness to abate. Hence, the household chooses lower consumption and higher abatement which leads to a smaller EKC with a lower turning point. Finally, in the numerical approach the effects of η and ω are ascertainable. Egli and Steger find that ω has a positive impact on the turning point, whereas the effect of η is negative. Again, the turning point is independent from δ as in the centralised economy.

The interpretation of the effect of ω is straight forward. From (99) we know that ω indicates the strength of $\bar{C}$ on gross pollution. Since in neither the unregulated nor the semi-regulated case the external effect is entirely internalised as with the central planner, an increase in ω implies higher gross pollution at any level of income. This is equivalent to an outward shift of the EKC, the turning point moves to a higher level of income.

The interpretation of η is slightly more complex, but nevertheless straight forward. Two effects occur with a change in η, a substitution effect and a scale effect. First, the larger the impact of the external effect on abatement, the lower is the agent's environmental effort. To see this, suppose that $\beta + \eta$ is constant: In the centralised economy no change is caused by a variation of η, because the total degree of IRS remains the same. However, increasing η means a reduction of β, wherefore in the decentralised economy E will be lower, causing a shift of the turning point to a higher income level. Thus, the stronger the external effect in abatement, the more it crowds out each household's abatement activity. This is the substitution effect.

The scale effect is the overall higher abatement due to a larger η. To see this, consider now that β is constant. Then a rise of η increases the degree of IRS in abatement, implying that pollution has a smaller value for each level of income – the EKC shifts inwards. Since the calculated elasticities of η are always negative, the scale effect clearly dominates the substitution effect causing a lower turning point. Combining the effects of α, β and η, it is clear that the degree of IRS in abatement has a large negative impact on the turning point.

Finally, Egli and Steger discuss the cost effectiveness of public policies. In a scenario with a maximum level of pollution which must not be exceeded, the scholars compare the effects of a subsidy on environmental effort τ_E and a consumption tax τ_C. They find that the consumption tax leads at any time to lower pollution levels than the abatement subsidy. However, the abatement subsidy does not reduce consumption as much as the consumption tax. Egli and Steger argue that the welfare gain from higher consumption from the subsidy outnumbers the negative effect of higher pollution. Therefore, they conclude that the subsidy is preferable at any time.

6 Comparison of the Models

So far, two approaches to model the EKC from learning by doing in abatement were presented. However, it has not yet been discussed whether the models vary in conclusions and predictions. This section focuses on comparing the properties of the models. It will be examined in which matter the models provide the same results and where they differ from each other. Furthermore, their fit to empirical findings from reality should be under study. The aim is to analysis whether general conclusions can be drawn from the models and if so, what they are.

First, the predictions of the models of Chapter 4 and Chapter 5 are compared. Special attention is paid to the turning point and the effectivity of environmental policy with regard to the EKC. Next, I analyse how the models are compatible with empirical regularities on growth and the environment other than those presented in Chapter 2. Furthermore, the conformity of the models with other shapes of the PIR for flow pollutants will be evaluated. Thereupon, I examine the empirical evidence of economies of scale in abatement. Finally, I analyse the models with respect to negative pollution. Models with pollution modelled as the difference between gross pollution and abatement might suffer from negative pollution in the long run. I discuss whether this holds for the models used and if so how to avoid this problem.

6.1 Model Predictions

The models provide significant information about the turning point of the EKC and the effectiveness of environmental policy. The predictions are not completely identical. While the turning point analysis notes mainly the same results in both models, the statements about public policy differ substantially.

In order to compare the results of both models, we must restrict the analysis of Table 2 on $\varepsilon = \gamma = 1$, hence on the first column. This has several reasons. First, the model of Chapter 5 considers only elasticities equal to unity (cf. (98)). Ergo, for any other elasticities the models and thus their predictions are not comparable. Second, a discussion of $\gamma > 1$ is not expedient since empirical evidence finds that demand for environmental quality is not a luxury good (cf. Chapter 3.1) such that $\gamma \not> 1$. Therefore, the restriction to $\varepsilon = \gamma = 1$ in Table 2 is reasonable.

The models are quite consistent in matters of the turning point analysis. For $\varepsilon = \gamma = 1$, both models predict the same effects of the determinants of the turning point. An increase in the factor productivity A shifts the turning point to a higher level of income. This is because income grows larger before abatement reduces pollution. Hence, the turning point shifts to the right. The discount factor ρ and the pollution intensity factor z have in both models a degrading effect on the turning point. The higher the time preference, the more important is present utility to the household and the more eager it is to spend resources on abatement. The stronger pollution harms utility, the sooner the household tries to reduce pollution.

The same holds for the strength of abatement. In Brock and Taylor (2003), the strength of abatement is measured through a, whereas in Egli and Steger (2007) the degree of IRS in abatement defines the magnitude of abatement (*i.e.* α, β and η). In both cases, the stronger the abatement technology to reduce pollution, the lower is the turning point of the EKC. In both approaches, a change of the depreciation rate δ does not influence the turning point. However, the models also produce unambiguous results. For instance, as Brock and Taylor (2003) do not consider a negative externality in gross pollution, ω – the elasticity of external gross pollution – cannot be compared.

It should not be forgotten that the income elasticities of consumption and pollution are strongly linked to the turning point analysis. It can be seen from Table 2 that the elasticities have a non-negligible impact on the determinants of the turning point. Therefore, the consistency of the models depends completely on the elasticities. If they change, the results may not be uniform anymore. Hence, a general statement about the impact directions is not possible.

Despite the consistency in the turning point analysis, the predictions of the models are not uniform in all dimensions. In matters of environmental policy, the models draw very different conclusions. Brock and Taylor (2003, 2005) claim that public policy does not have any direct impact on the EKC, the hump-shaped PIR occurs without the existence of regulation. It is only needed to enforce full abatement after switching to the full-abatement regime. Also the IRS approach suggests *prima facie* that the EKC arises anyway without intervention of the government.

However, Brock and Taylor only consider the social planner's problem and hence they neglect the possibility of market failures. Thus, public policy is not required to rule out these failures. The IRS approach in contrast does not ignore the possibility of market failures. Instead, Andreoni and Levinson (2001) and Egli and Steger (2007) demonstrate that the market economy is inefficient in comparison with the social planner: Both studies show that the magnitude of the EKC is larger in the market economy. Egli and Steger even specify that subsidies for abatement are preferable to taxing polluting activities. Hence, environmental policy is necessary to correct this error. This proves Brock and Taylor wrong.

6.2 Other Empirical Regularities

Both models describe a pollution path first increasing and later decreasing as it is found in the empirical EKC literature discussed in Chapter 2.4. However, producing this outcome only is insufficient to describe reality adequately. Therefore, other empirical regularities concerning economic growth and the environment should be satisfied too in order to confirm the propriety of the EKC model.

Based on US-data for 1950-2001, Brock and Taylor (2010) show that the emission intensities, *i.e.* emissions per output (or $\frac{P}{Y}$) are declining for several pollutants over time. This is visualised in Figure 8. Beginning from 1948, Brock and Taylor report this for several short living pollutants (SO_2, NO_X, VOC, CO and particular matters PM10[34]) and for CO_2 as a long living pollutant. However, the information concerning the short living substances is more important for this contribution than the information about CO_2.

The model of Brock and Taylor (2003, 2005) produces this pattern along the balanced growth path. To see this, consider the model after crossing the threshold. Abatement sets in and reduces pollution rapidly, while output is still growing. Obviously the model exaggerates this relationship if abatement is changed immediately to its maximum: Then P becomes quickly zero. However, since the idea of the model is to simplify the reality, the magnitude of the effect is a minor concern as long as the direction is the same. Furthermore, when θ approaches θ^K over time, then P decreases slowly but steadily and the emission intensities become steadily smaller. This is consistent with the empirical findings.

In the model of Egli and Steger (2007), emission intensities follow this pattern, too. The higher Y, the more the households can spend on C and E, hence both increase. Expenditures in E increase total abatement, spendings in C give raise to both gross pollution and abatement. Because of IRS in abatement but CRS in gross pollution, the economies of scale in abatement exceed those in gross pollution such that abatement increases more than gross pollution. Hence, P grows slower than Y and the emission intensity falls.

The second stylised fact reported by Brock and Taylor refers to the findings of Vogan (1996) concerning the expenditures for pollution abatement per dollar of GDP over the period 1972-1994. The results are presented in Figure 9. It is shown that the share of spendings on abatement rises rapidly until 1975 and remains fairly constant afterwards between 1.6% and 1.8 %.

To test if the model of Brock and Taylor (2003) satisfies constant costs of abatement, consider again the balanced growth path, *i.e.* the system after the threshold. The expenditures for abatement in the maximum are $\theta^K = \frac{YA}{YG}$ which is a constant fraction of total output. Hence, in the steady state the model implies that the expenditures for abatement relative to GDP are constant over time.

The same attribute can be found in the model of Egli and Steger (2007). To see this, recall equation (118). There we can see that the ratio of environmental effort to GDP depends on parameters only. Hence, there are no dynamic changes over time and

[34] PM10 is equal to SPM.

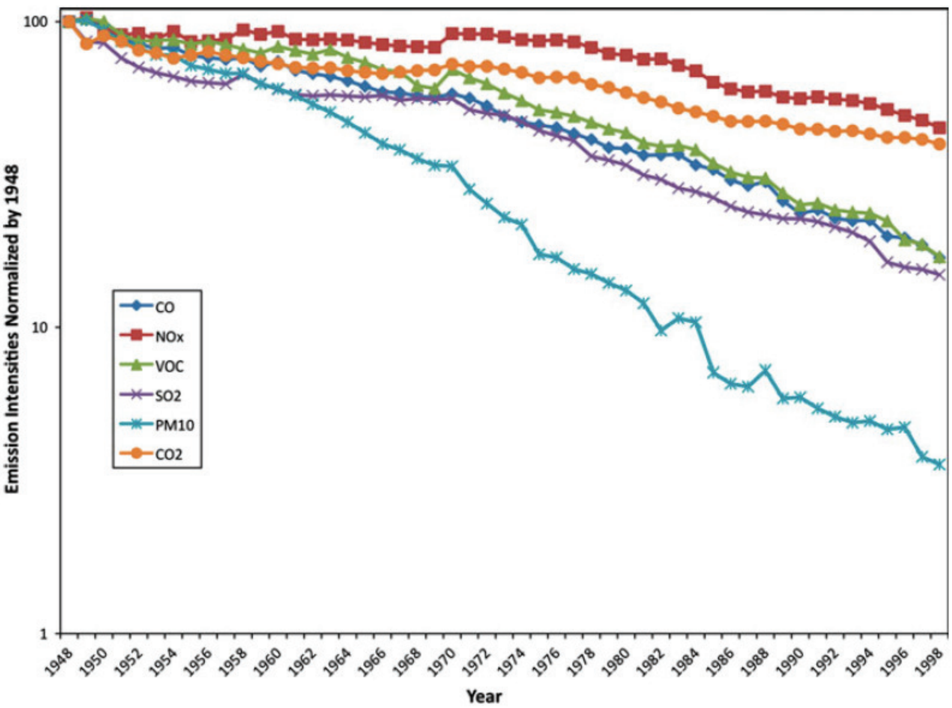

Figure 8: Emission intensities, 1948-1998, source: Brock and Taylor (2010)

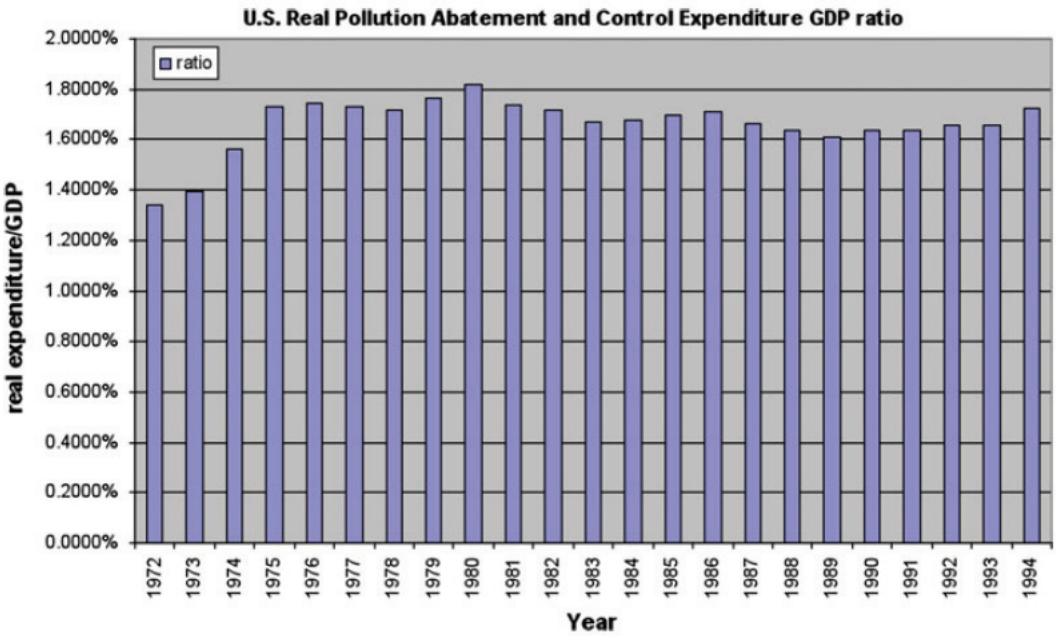

Figure 9: Pollution abatement costs, 1972-1994, source: Brock and Taylor (2010)

the fraction remains constant. Thus, this model is also consistent with the empirical evidence.

Finally, let us see if the models conform with Kaldor's facts (Kaldor, 1961). Since both models apply AK growth technology with pollution, they are both compatible with most of these stylised facts. For both, the per capita growth rate of output is positive and constant. Also, capital per capita grows over time. Additionally, the capital-output ratio is constant and the real rate of return to capital is constant, too.[35] Hence, both discussed dynamic models satisfy all the regularities concerning economic growth and the environment equally.

6.3 Other Shapes of the PIR

In Chapter 2.4 we have seen that the EKC is not the only possible shape of the PIR. Also monotonically increasing functions or N-shapes were found. Both patterns are in contradiction to sustainable growth since in the long run pollution is increasing again with economic growth. Besides the hump-shaped PIR, the N-shaped PIR is often found for flow pollutants. Thus, I will analyse now if the models of concern are able to also produce an N-shaped PIR.

Starting again with Brock and Taylor (2003), we can see that this model is actually able to cope with N-shapes. To see this, we must allow for a small change in the setting of the model. Suppose that maximal abatement does not reduce pollution completely but slightly less such that $\theta^K < \frac{1}{a}$. After crossing the threshold, abatement begins and pollution decreases. However, it can never disperse emissions completely such that at Y^* pollution falls to a lower level. Thereafter, pollution increases again with economic growth at a lower rate. (23) demonstrates this effect: pollution can only become zero if $\theta^K = \frac{1}{a}$. If it is smaller, then pollution increases with economic growth endlessly.

Figure 10 illustrates this outcome for both cases discussed in Chapter 4. In (A), abatement switches immediately to its maximum, whereas in (B) abatement approaches θ^K over time. In (A), abatement sets in at Y^* such that the emissions drop to a lower level. But since θ^K is too small to eliminate pollution completely, emissions continue to grow at lower rate on a lower level. In (B), we can observe the same if θ approaches θ^K. At the threshold Y^*, abatement sets in and becomes gradually stronger as θ becomes larger. However, once θ^K is reached, pollution starts to increase again with growing income because $\theta^K < \frac{1}{a}$. Hence, both cases of Brock and Taylor (2003) produce an N-shaped PIR for any $\theta^K < \frac{1}{a}$.

Also the model of Egli and Steger (2007) is compatible with the N-shaped PIR. To

[35]Both models do not say anything whether the capital and labour income shares are constant nor do they give information about real growth of wages, the remaining Kaldor facts.

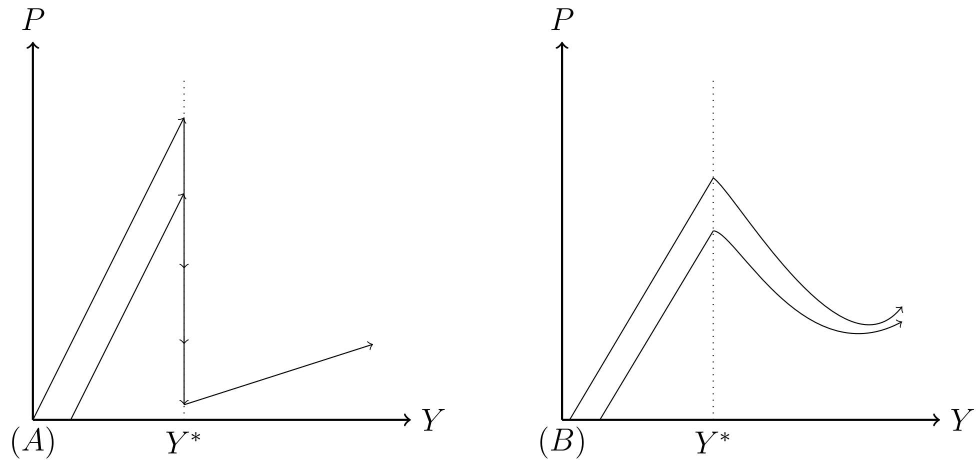

Figure 10: N-shaped PIR of the threshold model

see this, suppose first that the unregulated market economy is located in the upward sloping branch of the EKC. Let at some point the economy change to a regulated economy. As discussed before, we know that the PIR of any regulated economy has a smaller magnitude than in the unregulated economy. Therefore, the implementation of policy instruments shifts the EKC downwards and pollution diminishes. Provided that the economy is still below the turning point of the regulated economy, pollution starts to increase again. Thus, the installation of regulation results in an N-shaped PIR.

However, the N-shaped outcome is just valid at first glance. In fact, the economy experiences another turning point due to IRS in abatement. This second turning point is equal to the original turning point of the regulated economy. Thus, the economy actually faces an M-shaped PIR where the first turning point is policy induced, the subsequent rise in pollution is caused by polluting economic growth and the second turning point results from IRS in abatement. The M-shaped PIR is illustrated in Figure 11.

The implications of an M-shaped PIR are sweeping. In contrast to the N-shape, the M-shape implies that sustainable growth is actually possible. From the moment when the growth in abatement exceeds the larger emissions in the regulated economy, pollution will never rise again. Nevertheless, the N-shaped PIR is still possible if the returns to scale in abatement does not exceed unity. Then the second turning point does not exist. This would be the same as in Brock and Taylor (2003) discussed above.

Empirical evidence for an M-shaped PIR is rare. The only research so far providing evidence for an M-shaped EKC pattern is by Giles and Mosk (2003) using long-run data on methane emissions for New Zealand. However, the scarcity of empirical evidence does not exclude the M-shaped PIR from being possible since the finding

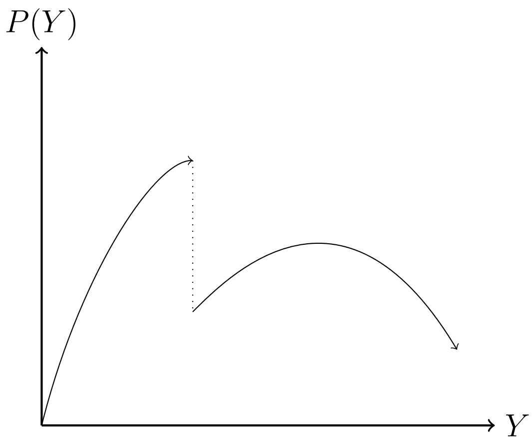

Figure 11: M-shaped PIR for IRS in abatement

of N-shaped PIR does not necessarily mean that there will not be a second turning point: It simply might have not been experienced yet in the investigated economies such that they are still on the second rising branch of the M-shaped PIR.

6.4 Empirical Evidence for Increasing Returns to Scale in Abatement

IRS in abatement is the major driver of the EKC in the model of Egli and Steger (2007). However, it is not apparent whether the assumption of IRS is reasonable in reality. Hence, empirical evidence should confirm the validity of the hypothesis. IRS in abatement have rarely been subject to empirical investigations. Andreoni and Levinson (2001) report empirical evidence for IRS in abatement on plant and US-state level. Analysing the cost efficiency of coal-fired boilers, they find that the average abatement costs decline with the size of the boiler. Furthermore, Andreoni and Levinson examine data on abatement costs across US-states and industries. They show that the size of the industry affects the abatement costs negatively. This result they find across states, across industries and over time. Those are clearly indications for IRS in abatement since on a large scale abatement is less costly than on a small scale.

Similar results are obtained by Maradan and Vassiliev (2005). They discover for CO_2 that the marginal opportunity costs of abatement decline with increasing per capita GDP on nationwide level. If the marginal opportunity costs diminish with income, then abatement is more efficient at higher income. This is synonymous to IRS in abatement. Other scholars inspect the existence of IRS in abatement directly. Managi (2006) investigates the environmental risk in the US agricultural sector between 1970 and 1997. Testing for economies of scales in abatement, he discovers that there are indeed IRS in abatement. Managi and Kaneko (2009) find evidence for IRS in abatement effort analysing data on the Chinese secondary industry on province level for the period of 1992-2003.

The results of these studies favour the assumption of IRS in abatement. This provides clear support for the models of Andreoni and Levinson (2001) and Egli and Steger (2007). However, it is an indirect support for Brock and Taylor (2003) too and thus also for learning by doing in abatement. Recall that the IRS in abatement model is a reduced form of Brock and Taylor's threshold model. The implementation of abatement at the threshold carries high fixed costs which can be seen synonymously as IRS in abatement. Thus, although Brock and Taylor assume CRS in abatement the implementation costs the costs of introduction work as a mark up to the returns to scale, hence there are IRS. Therefore, the discovery of Maradan and Vassiliev (2005) does not contradict the assumptions of Brock and Taylor. Also, the results of Maradan and Vassiliev are not in contradiction with the assumption of increasing marginal abatement costs on firm level as reported by Cofala and Syri (1998), because Maradan and Vassiliev's findings are on aggregate level. Hence, the empirical evidence of IRS in abatement suits both model approaches.

6.5 Negative Pollution

Growth models concerning the EKC often face problems with negative pollution. This happens when, in the long run, more pollution is abated than was actually created. Usually, the problem of negative pollution appears very late in the model. Nevertheless, if a model contradicts logic or natural facts in the long run, it is doubtful whether this model produces reliable results in the short and medium run (Egli, 2005a). Considering solely interior solutions for instance (*e.g.* Selden and Song, 1995; Andreoni and Levinson, 2001) is not fully satisfying due to eventual conflict.

In order to circumvent negative pollution one could apply a non-negativity constraint. However, Egli (2005a) demonstrates that this technical solution is insufficient. To see this, let us recall the static IRS model and its pollution function (74). The non-negativity constraint for pollution requires that[36]

$$P(C, E) \geq 0. \tag{131}$$

Suppose both C and E grow over time. Under the assumption of IRS in abatement, net pollution becomes sooner or later negative and the constraint becomes binding. But at that point, C and E cannot be chosen independently anymore: When $P(C, E) = 0$ is reached, the choice of E must be such that abatement does not exceed gross pollution. Hence, E becomes a function of C. This however contradicts

[36]In the dynamic model, the constraint is given in (89).

the independent choice of C and E, a fundamental assumption of the model. Thus, the non-negativity constraint of pollution is an insufficient solution to avoid negative pollution.

The Kindergarten Rule model of Brock and Taylor (2003) does not suffer from negative pollution. There, the use of pollution intensities in the gross pollution function helps to circumvent the problem. However, according to Egli (2005a) this is not because of the general correctness of using intensities but rather due to adequate modelling by Brock and Taylor. Intensities are not a universal remedy against negative pollution.

Negative pollution occurs especially in models with IRS in abatement, but empirical evidence favours the existence of IRS in abatement. Egli (2005a) tries to solve this contradiction with fading IRS in abatement. The idea is that at a low stage of the PIR, the abatement technology exhibits increasing returns to scale. However, the larger the abatement activity is, the less effective is the pollution control. The scale factor becomes less and less and approaches constant returns to scale (CRS) in the long run. This mechanism is demonstrated in Figure 12. Gross pollution increases linearly with the polluting activity. Abatement technology exhibits IRS at low levels of abatement, but the economies of scale diminish the larger abatement is. In the long run, abatement approaches a non-negative constant.

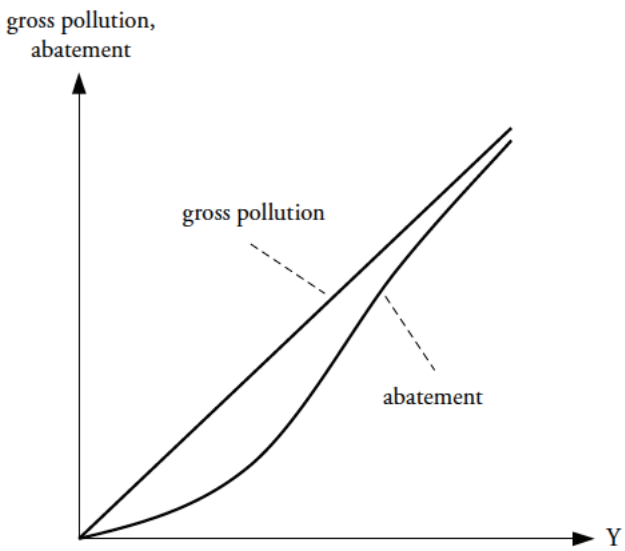

Figure 12: Fading IRS in abatement, source: Egli (2005a)

Besides the obviation of negative pollution, there are a number of other reasons supporting the concept of fading IRS. First, it is in line with empirical findings of IRS in abatement as presented in Chapter 6.4. Moreover, although Managi and Kaneko (2009) find evidence for IRS in abatement, they doubt that these are durable in the long run. They rather expect them to be be short-run results. Hence, indirectly they propose that IRS in abatement fade away.

Second, if learning by doing is the main drive behind the EKC, it is very likely that the growth of abatement effectiveness will not be constant over time. The potential gains will rather decline with cumulative activity as it is typically shown in learning curves. Bramoullé and Olson (2005) report that learning lessens potential cost reductions for infant technology more than for mature technology. Comparing the learning rates of different technologies in the energy sector, McDonald and Schrattenholzer (2001) find that for data on later technology, the learning rates are systematically lower.

Third, if the degree of IRS diminishes, pollution declines smoothly towards zero, not rapidly with a kink when reaching zero-pollution as in the case of constant IRS. This is clear from Figure 12. There, abatement moves steadily closer to gross pollution such that the difference diminishes. If IRS were constant, the abatement function would cross gross pollution from below causing a sharp break when zero-pollution is reached. Egli (2005a) illustrates this with data on SO_2 for Switzerland between 1950 and 1989. There, the rate at which emission decline decreases such that reduction slows down. The deceleration of emission reduction describe precisely the smoothing effect of fading IRS.

Fourth, fading IRS in abatement is also consistent with the EKC. To demonstrate this, Egli (2005a) revised the specific model of Egli and Steger (2007) using fading IRS in abatement. The adjusted pollution function

$$P = C - C^{\alpha} E^{1-\alpha+\frac{1}{1+E^2}} \tag{132}$$

replaces (102). The second part of the exponent of E causes the shrinkage of IRS. In the long run $\frac{1}{1+E^2}$ approaches zero such that in the limit there is CRS in abatement. Simulating the model numerically, Egli demonstrates that the PIR has the proper EKC-shape while pollution remains non-negative. However, he admits that the restraint of the degree of IRS must be specified adequately to produce a hump-shaped PIR.

It is not too far-fetched to conclude that the impact of the determinants on the turning point is the same in the fading-IRS model as previously discussed. However, this would go too far because Egli's model does not provide any information about the turning point. Furthermore, it is not possible to solve it analytically. Therefore, it is not clear whether the turning point is affected by the same parameters as in Egli and Steger (2007). But solving the model numerically in order to receive information about the turning point goes beyond the scope of this master's thesis. It is left to future research to gain information in this matter. Concluding, Egli (2005a) demonstrates that fading IRS is indeed a reasonable approach to the EKC as it eliminates the problem of IRS in abatement and the possibility of negative pollution. However, since his model still relies on accurate modelling to obtain the EKC, it cannot be seen as universally valid. Further research into this direction should clarify the suitability of the concept.

7 Conclusion

The aim of this research was to help understanding the Environmental Kuznets Curve and its relevance. The study starts with a critical review of the empirical approach and the theoretical explanations for the EKC. I find that the EKC is valid mainly for local, short living air pollutants as SO_2 or NO_X. For global, long living air pollutants such as CO_2 the results are ambiguous, rather proclaiming that the PIR for CO_2 is a monotonic increasing function. Additionally, it is revealed that the applied estimation techniques often exhibit substantial defects. Therefore, the produced results may be based on miscalculations such that the EKC remains only as a data phenomenon.

The theoretical explanations are manifold. Structuring the attempts as de Bruyn and Heintz (1999), there are five different possibilities for the EKC: behavioural changes, institutional changes, technological progress, structural change and international reallocation. However, only technological progress remains without doubt as an explanation. Behavioural and institutional change rely on technological progress to impose the adjusted desire for a cleaner environment, structural change itself cannot explain the EKC and needs the shift of dirty production abroad. But migration is usually not rapid enough to explain the EKC through the reduction of the absolute production level in the corresponding industries of a developed country. This contribution pays attention to the effects of technological progress on the EKC, namely the impact of learning by doing in the abatement sector. This is done because according to empirical research learning by doing is more important in the improvement process of abatement technique than R&D.

Yet, theoretical work on the EKC is scarce. This gap I endeavour to fill with my contribution. I discuss two approaches in modelling learning by doing by abatement in a dynamic growth model in order to describe the origin of the EKC and to make general conclusions about it and its determinants possible. In the first model, learning by doing in abatement leads to constant returns to abatement on the aggregate level of the economy, it is a so called threshold model. In the first stage, the marginal utility of capital exceeds the marginal disutility of pollution such that the representative household is not interested in reducing the environmental pressure. The focus lies completely on the accumulation of capital implying economic growth and rising pollution. At the turning point, the relationship between marginal utilities of capital and pollution switch: now each unit of pollution affects utility more than an additional unit of consumption (financed by capital) and abatement starts. Due to constant returns to abatement it is possible to reduce pollution completely. Depending on the elasticities, pollution drops immediately to zero or shrinks smoothly. The effects on the turning point are illustrated in Table 2. The determinants of the turning point rely on the elasticities of the model. Depending on the elasticities, the impacts of the determinants varies in direction or cannot be derived analytically. Hence, it is impossible to provide general statements about the determinants.

The second model is a simplification of the threshold idea. In the reduced form learning by doing is expressed as increasing returns to scale in abatement. At first I present the idea of IRS in abatement as the cause for the EKC in a static frame. Thereupon, the idea is developed towards a dynamic model. Two externalities are included: a negative externality in gross pollution caused by other people and a positive in abatement efforts caused by learning by abating. The technological progress leads to IRS in abatement. The results remain the same as in the static case, IRS in abatement are the only requirement for creating the EKC. Environmental policy is not necessary for the existence of the EKC, however, it does influence the magnitude of the EKC. The internalisation of the external effects lowers the pollution level at the turning point. Hence, regulation cannot be rejected as irrelevant as proposed by several scholars.

The determinants of the turning point are shown in Table 4. It occurs that the determinants have the same sign as those of Chapter 4 if the elasticities of both models are the same. Then an increase of the factor productivity shifts the turning point towards a higher level of income and also to a higher pollution level. The same happens if the output elasticity of the external effect on gross pollution rises. An increase in the strength of abatement, a stronger impact of pollution on utility and a higher time preference shifts the turning point towards a lower level. The depreciation rate does not have an impact at all. However, as suggested by the threshold approach, the results may vary widely with other elasticities of consumption and pollution. Thus, the results here should not be taken as universally valid.

After that follows a comparison of the models. There it is shown that both models are in line with empirical regularities other than the EKC hypothesis. Both satisfy the findings of declining emission intensities and constant expenditures for abatement relative to GDP. Additionally, the models as augmented AK models are compatible with most of the Kaldor facts. Moreover, there is empirical evidence for IRS in abatement. This supports the IRS-model directly and the threshold model indirectly. Also the idea of learning by abating receives confirmation.

With small adjustments, both approaches are able to produce an N-shaped PIR, an often found variation of the EKC for local air pollutants. In the Kindergarten model, the modification is such that maximal abatement cannot eliminate pollution completely but reduces it close to zero. Then pollution increases after the turning point on a lower level than before. In the IRS model, the implementation of environmental regulation implies a shift downwards of the EKC. If the introduction happens before the turning point of the regulated economy is reached, pollution increases. In this case the IRS model actually proposes an M-shaped PIR as at the regulated stage pollution will decline again due to IRS in abatement. However, the idea of IRS in abatement faces theoretical problems of negative pollution in the long run. This is problematic because illogical predictions in the long run undermine the general validity of the whole model. Therefore, fading IRS in abatement are presented as a possible solution. The EKC can still arise from this change. Nevertheless, further research needs to be done into that direction in order to develop the idea of fading IRS in abatement.

It has been shown in a theoretical approach that learning by doing in abatement is a reasonable explanation of the EKC. However, it would go too far to conclude that the EKC is a perfect symbol for sustainable growth. For this, too many doubts are not yet dispelled. First of all, the EKC has only been tested for pollutants for which data is available. Hence, many other dimensions of pollution without proper data are excluded and thus for them the EKC has not been proven. Moreover, the EKC has been confirmed for a few short living pollutants only. For CO_2, the key measure for the climate change (Nordhaus, 1991), the PIR is rather monotonically increasing. But if the climate change does not exhibit an inverted-U shaped PIR, economic growth continues to pollute.

Furthermore, the models do not reject the possibility of pollutant substitution since they only explain the EKC for single substances. The abatement process often replaces one substance by another. Therefore, abating one pollutant could actually imply the boost of a different one. Especially the contrasting results for short living and long living air pollutants supports this hypothesis. But if abatement exchanges only the substances instead of eliminating them, economic growth creates pollution continuously despite the abatement efforts. This clearly contradicts the definition of sustainable growth. Therefore, the existence of the EKC does not imply the possibility of sustainable growth.

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A Appendix

In the appendix I would like to add several calculations of the main part which I did not discuss in detail in the main text.

A.1 Optimal Control Theory

At first, I want to introduce optimal control theory. In my notations I follow Novales et al. (2010). Let us consider the dynamic optimisation problem

$$\max_{v_t} \int_0^T f\left(x_t, v_t, t\right) \text{ d}t$$

with the constraint condition

$$\dot{x_t} \quad = \quad h\left(x_t, v_t, t\right)$$

and initial value x_0 given. Then we can call v_t the *control* variable and x_t the *state* variable. The constraint describes the evolution of the state variable with respect to time. Hence, for each value of the control variable at any time, the constraint evolves differently.

We can solve this problem through its *Hamiltonian*

$$H(x_t, v_t, \mu_t, t) \quad = \quad f(x_t, v_t, t) + \mu_t h(x_t, v_t, t),$$

where μ_t is the *co-state* variable representing the shadow price of the control variable or the marginal value of one additional unit of the co-state variable at time t in utility units at time 0.

In order to solve the maximisation of the Hamiltonian with respect to the state variable we have to apply Pontryagin's principle. This includes three optimality conditions:

1. The *state equation*
$$\frac{\partial H}{\partial v_t} = 0 \Leftrightarrow \frac{\partial f}{\partial v_t} + \mu_t \frac{\partial h}{\partial v_t} = 0,$$

2. the *co-state equation*
$$\dot{\mu_t} = -\frac{\partial H}{\partial x_t} \Leftrightarrow \dot{\mu_t} = -\frac{\partial f}{\partial x_t} - \mu_t \frac{\partial h}{\partial x_t},$$

and 3. the transversality condition. The transversality condition is necessary as often in economic applications the state variable (*e.g.* the capital stock) $x(T) = x_T$ is restricted in sign. Then, the transversality condition requires that

$$x_T \geq 0, \; x_T \mu_T = 0$$

hold. This implies that either $x_T = 0$ or $\mu_T = 0$. In case x_T is not restricted, the transversality condition is $\mu_T = 0$. For the infinite horizon, the transversality condition of the planning problem is

$$\lim_{T\to\infty} x_T \geq 0, \; \lim_{T\to\infty} x_T \mu_T = 0$$

for x_T restricted in sign and

$$\lim_{T\to\infty} \mu_T = 0$$

for x_T unrestricted.

A.2 First-Order Linear Differential Equations

This subsection deals with different first-order differential equations which appear in the models. The explanations are based on Chapter 15 of Chiang and Wainwright (2005).

A.2.1 Homogeneous Case with Constant Coefficient

Recall from (46) that the growth rate of the shadow price is known as

$$\frac{\dot{\lambda}}{\lambda} = -g.$$

Integrating on both sides with respect to t we obtain

$$\int \frac{\dot{\lambda}}{\lambda} \mathrm{d}t = -\int g \mathrm{d}t.$$

By the substitution rule and the log rule, the left side gives us

$$\int \frac{\frac{\mathrm{d}\lambda}{\mathrm{d}t}\mathrm{d}t}{\lambda} = \int \frac{\mathrm{d}\lambda}{\lambda} = \ln \lambda_t + c_1,$$

where c_1 is a constant term. The right hand side yields

$$-\int g\mathrm{d}t = -gt + c_2,$$

where c_2 is another constant term. Combining those, we find that

$$\ln \lambda_t = -gt + \overbrace{c_2 - c_1}^{\equiv \zeta}$$

where I merge the constant terms as ζ. Next, I take the antilog of $\ln \lambda_t$, *i.e.* I use the e-function on both sides such that

$$\begin{aligned} e^{\ln \lambda_t} &= \lambda_t = e^{-gt+\zeta} \\ \Rightarrow \lambda_t &= e^{-gt} \underbrace{e^{\zeta}}_{\equiv \lambda_0} \\ \Rightarrow \lambda_t &= \lambda_0 e^{-gt}. \end{aligned}$$

Note that ζ is a constant and therefore e^{ζ} is a constant too. Thus, we can rewrite e^{ζ} simply as the constant λ_0. This gives us the solution equal to (47).

For S_0 with $\gamma > 1$, we know from (57) that

$$\theta = \frac{1}{a}\left[1 - \frac{1}{AK}\left(\frac{\lambda}{az}\right)^{\frac{1}{\gamma-1}}\right].$$

Plugging this into (41) gives us:

$$
\begin{aligned}
\dot{\lambda} - \rho\lambda \quad &= -\frac{\partial H}{\partial K} = -\left\{ \lambda \left[A\left(1-\theta\right) - \delta \right] - z \Big[AK(1-\theta a) \Big]^{\gamma-1} A\left(1-\theta a\right) \right\} \\
&= -\left\{ \lambda \left[A - \frac{A}{a} + \frac{A}{aAK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} - \delta \right] \right. \\
&-Az \left[AK - AK\frac{a}{a} + \frac{aAK}{aAK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} \right]^{\gamma-1} \\
&\left. +Az \left[AK - AK\frac{a}{a} + \frac{aAK}{aAK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} \right]^{\gamma-1} \frac{a}{a} \left[1 - \frac{1}{AK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} \right] \right\} \\
&= -\left\{ \lambda \Big[A \Big(1 - \underbrace{\frac{1}{a}}_{=\theta^K} \Big) - \delta \Big] + \frac{\lambda}{aK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} - \frac{Az}{Az} \frac{\lambda}{aK} \left(\frac{\lambda}{az} \right)^{\frac{1}{\gamma-1}} \right. \\
&\left. -Az \left(\frac{\lambda}{az} \right) + Az \left(\frac{\lambda}{az} \right) \right\} \\
&= -\left\{ \lambda \left[A \left(1 - \theta^K \right) - \delta \right] \right\} \\
\Rightarrow \dot{\lambda} \quad &= -\left\{ \lambda \left[A \left(1 - \theta^K \right) - \delta - \rho \right] \right\} \\
\Leftrightarrow \dot{\lambda} \quad &= -g\lambda
\end{aligned}
$$

We can see that also for S_0 and $\gamma > 1$, we have the homogeneous case for λ_t. Hence, we can solve this as above such that:

$$
\dot{\lambda} = -g\lambda \qquad \Leftrightarrow \qquad \lambda_t = \lambda_0 e^{-gt}.
$$

This is the result of (60).

A.2.2 Non-Homogeneous Case with Constant Coefficient and Constant Term

For S_- with $\gamma = 1$, the differential equation of the shadow price is slightly different:

$$
\dot{\lambda} \;\; = \;\; -\lambda(A - \delta - \rho) + Az,
$$

where Az is constant. Hence, we have a non-homogeneous linear differential equation for S_-.

The reduced equation refers to the homogeneous part.

To receive the reduced equation, we can set the non-homogenous part to zero such that $Az = 0$. Then

$$\dot{\lambda} \;=\; -\lambda(A-\delta-\rho).$$

This we can solve as described in Appendix A.2.1 such that

$$\dot{\lambda} = -\lambda(A-\delta-\rho) \qquad \Leftrightarrow \qquad \lambda^c = \lambda_0 e^{-(A-\delta-\rho)t}.$$

In order to get the particular integral, we can assume that λ is a constant, hence we can set $\dot{\lambda} = 0$. Solving for λ gives then

$$\lambda^p \;=\; \frac{Az}{A-\delta-\rho}.$$

The general solution for λ_t is the sum of the complementary function and the particular integral:

$$\lambda_t = \lambda^c + \lambda^p = \lambda_0 e^{-(A-\delta-\rho)t} + \frac{Az}{A-\delta-\rho}.$$

This is the result of the differential equation given in (54).

A.2.3 Non-Homogeneous Case with Variable Term

Here I explain the solutions for the non-homogeneous linear differential equations with variable terms. First, a short introduction to exact differential equations is needed in order to solve the differential equations discussed afterwards.

A.2.3.1 Exact Differential Equations In order to derive the non-homogeneous differential equations with variable term, I want to introduce first the concept of exact differential equations. If we have a function of two variables $F(y,t)$, the corresponding total differential is

$$\mathrm{d}F(y,t) = \frac{\partial F}{\partial y}\mathrm{d}y + \frac{\partial F}{\partial t}\mathrm{d}t.$$

If this is set equal to zero, the resulting differential equation can be called exact since the right hand side is exactly the differential of $F(y,t)$. Let is define $M \equiv \frac{\partial F}{\partial y}$ and $N \equiv \frac{\partial F}{\partial t}$ such that the differential is

$$M\mathrm{d}y + N\mathrm{d}t = 0$$

Because of $\frac{\partial^2 F}{\partial y \partial t} = \frac{\partial^2 F}{\partial t \partial y}$ according to Young's theorem, the differential equation is exact if

$$\frac{\partial M}{\partial t} = \frac{\partial N}{\partial K}.$$

Finally, as the exact differential equation is

$$\mathrm{d}F(y,t) = 0,$$

its general solution is constant such that

$$F(y,t) = c$$

where c is a constant term.

A.2.3.2 Solution for S_+ and $\gamma = 1$ Let us start with the case of S_+ and $\gamma = 1$. The differential equation for K_t derived from (34) is

$$\dot{K} = \frac{\mathrm{d}K}{\mathrm{d}t} = [A(1-\theta^K) - \delta]K_t - C_t = (g+\rho)K_t - C_t.$$

This we can rewrite as

$$\mathrm{d}K + [-(g+\rho)K_t + C_t]\mathrm{d}t = 0,$$

which is an exact differential equation. To assure that this differential is indeed exact, let us insert an integrating factor I_t on both sides:

$$\underbrace{I_t}_{M}\,\mathsf{d}K + \underbrace{I_t[-(g+\rho)K_t + C_t]}_{N}\,\mathsf{d}t = 0.$$

Yet, the integrating factor is unknown. Hence, first we must define I_t such that the differential equation to be solved is exact. For this, I define $M \equiv I_t$ and $N \equiv I_t[-(g+\rho)K + C]$. Exactness requires that

$$\frac{\partial M}{\partial t} = \frac{\partial N}{\partial K}$$

is true. According to the definitions of M and N, the parts of the exactness test are:

$$\begin{aligned}\frac{\partial M}{\partial t} &= \frac{\mathsf{d}I}{\mathsf{d}t}\\ \frac{\partial N}{\partial K} &= -I_t(g+\rho).\end{aligned}$$

Setting $\frac{\partial M}{\partial t}$ and $\frac{\partial N}{\partial K}$ equal, we obtain

$$\begin{aligned}&\frac{\mathsf{d}I}{\mathsf{d}t} = -I_t(g+\rho)\\ \Leftrightarrow\quad &\frac{\frac{\mathsf{d}I}{\mathsf{d}t}}{I_t} = -(g+\rho)\\ \Leftrightarrow\quad &I_t = I_0 e^{-(g+\rho)t},\end{aligned}$$

where we can set $I_0 = 1$ as it is constant. Therefore, the integrating factor is

$$I_t = e^{-(g+\rho)t}.$$

Replacing I_t in the differential equation, we get

$$e^{-(g+\rho)t}\mathsf{d}K + e^{-(g+\rho)t}[-(g+\rho)K_t + C_t]\mathsf{d}t = 0$$

which can be solved in four steps. In step I, we write the preliminary result as

$$
\begin{aligned}
F(K,t) &= \int M \mathrm{d}K + \phi(t) \\
&= \int I_t \mathrm{d}K + \phi(t) \\
&= K_t I_t + \phi(t) \\
&= K_t e^{-(g+\rho)t} + \phi(t)
\end{aligned}
$$

In step II, we remember that due to exactness, $\frac{\partial F}{\partial t} = N$ must hold. Therefore, we can set them equal and find:

$$
\begin{aligned}
\frac{\partial F}{\partial t} &= K_t[-(g+\rho)]e^{-(g+\rho)t} + \phi'(t) = e^{-(g+\rho)t}[-(g+\rho)K_t + C_t] = N \\
\Leftrightarrow &\quad K_t[-(g+\rho)]e^{-(g+\rho)t} + \phi'(t) = -e^{-(g+\rho)t}(g+\rho)K_t + e^{-(g+\rho)t}C_t \\
\Leftrightarrow &\quad \phi'(t) = +e^{-(g+\rho)t}C_t.
\end{aligned}
$$

Knowing $\phi'(t)$, we can proceed to step III, which is integrating $\phi'(t)$ with respect to time in order to get $\phi(t)$:

$$
\phi(t) = \int_0^t \phi'(s)\mathrm{d}s = \int_0^t C_s e^{-(g+\rho)s}\mathrm{d}s.
$$

Finally, in step IV we plug $\phi(t)$ into $F(K,t)$ to get its complete form:

$$
F(K,t) = K_t e^{-(g+\rho)t} + \int_0^t C_s e^{-(g+\rho)s}\mathrm{d}s.
$$

As shown above, the general solution of the exact differential equation is equal to a constant c:

$$
K_t e^{-(g+\rho)t} + \int_0^t C_s e^{-(g+\rho)s}\mathrm{d}s = c.
$$

Solving this for K_t we have the solution for the differential:

$$
K_t = e^{(g+\rho)t}\left[c + \int_0^t C_s e^{-(g+\rho)s}\mathrm{d}s\right]
$$

Since we know from (38) that $C = \lambda_t^{-\frac{1}{\varepsilon}}$ and from (47) that $\lambda_t = \lambda_0 e^{-gt}$, we can insert those into K_t. Furthermore, we can define that $c \equiv K_0$ since c is constant.

Hence, we obtain the result of given in (48):

$$
\begin{aligned}
K_t &= e^{(g+\rho)t}\left[K_0 - \int_0^t \lambda_0^{-\frac{1}{\varepsilon}} e^{\frac{g}{\varepsilon}s} e^{-(g+\rho)s} \mathrm{d}s\right] \\
\Leftrightarrow K_t &= e^{(g+\rho)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{-\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\, \mathrm{d}s\right].
\end{aligned}
$$

A.2.3.3 Solution for S_- and $\gamma = 1$ In this case we know from (34) that for $\theta = 0$ the differential equation is

$$
\begin{aligned}
& \dot{K} = (A-\delta)K_t - C_t \\
\Leftrightarrow \quad & \mathrm{d}K + [-(A-\delta)K_t + C_t]\mathrm{d}t = 0.
\end{aligned}
$$

Adding the integrating factor I_t on both sides gives

$$
\underbrace{I_t}_{M} \mathrm{d}K + \underbrace{I_t[-(A-\delta)K_t + C_t]}_{N} \mathrm{d}t = 0.
$$

Exactness requires that $\frac{\partial M}{\partial t} = \frac{\partial N}{\partial K}$, hence we have

$$
\begin{aligned}
& \frac{\partial M}{\partial t} = \frac{\mathrm{d}I}{\mathrm{d}t} = -I_t(A-\delta) = \frac{\partial N}{\partial K} \\
\Leftrightarrow \quad & \frac{\frac{\mathrm{d}I}{\mathrm{d}t}}{I_t} = -(A-\delta) \\
\Leftrightarrow \quad & I_t = \underbrace{I_0}_{\equiv 1} e^{-(A-\delta)t} = e^{-(A-\delta)t}.
\end{aligned}
$$

Hence, the differential equation is

$$
e^{-(A-\delta)t}\mathrm{d}K + e^{-(A-\delta)t}[-(A-\delta)K_t + C_t]\mathrm{d}t = 0.
$$

Using again the four steps for solving, we obtain from step I

$$
F(K,t) = \int I_t \mathrm{d}K + \phi(t) = K_t e^{-(A-\delta)t} + \phi(t).
$$

From step II, we receive

$$\begin{aligned}
\frac{\partial F}{\partial t} &= K_t[-(A-\delta)]e^{-(A-\delta)t} + \phi'(t) = e^{-(A-\delta)t}[-(A-\delta)K_t + C_t] = N \\
\Leftrightarrow &\quad K_t[-(A-\delta)]e^{-(A-\delta)t} + \phi'(t) = -e^{-(A-\delta)t}(A-\delta)K_t + e^{-(A-\delta)t}C_t \\
\Leftrightarrow &\quad \phi'(t) = +e^{-(A-\delta)t}C_t.
\end{aligned}$$

as $\frac{\partial F}{\partial t} = N$ must hold for exactness. Integrating $\phi'(t)$ gives us $\phi(t)$ (step III)

$$\phi(t) = \int_0^t \phi'(s)\mathsf{d}s = \int_0^t C_s e^{-(A-\delta)s}\mathsf{d}s,$$

plugging $\phi(t)$ into $F(K,t)$ we obtain the complete form of the differential equation

$$F(K,t) = K_t e^{-(A-\delta)t} + \int_0^t C_s e^{-(A-\delta)s}\mathsf{d}s.$$

Again, the general solution of the exact differential equation is equal to a constant which I name K_0:

$$K_t e^{-(A-\delta)t} + \int_0^t C_s e^{-(A-\delta)s}\mathsf{d}s = K_0.$$

Solving this for K_t we have the solution for the differential:

$$K_t = e^{(A-\delta)t}\left[K_0 + \int_0^t C_s e^{-(A-\delta)s}\mathsf{d}s\right]$$

Since we know from (38) that $C_t = \lambda_t^{-\frac{1}{\varepsilon}}$ and from (54) that $\lambda_t = \lambda_0 e^{-(A-\delta-\rho)t} + \frac{Az}{A-\delta-\rho}$, the solution for K_t in S_- is

$$\begin{aligned}
K_t &= e^{(A-\delta)t}\left[K_0 + \int_0^t \lambda_s^{-\frac{1}{\varepsilon}} e^{-(A-\delta)s}\mathsf{d}s\right] \\
&= e^{(A-\delta)t}\left[K_0 + \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{\frac{(A-\delta-\rho)}{\varepsilon}s} e^{-(A-\delta)s}\mathsf{d}s - \left(\frac{Az}{A-\delta-\rho}\right)^{-\frac{1}{\varepsilon}} \int_0^t e^{-(A-\delta)s}\mathsf{d}s\right] \\
K_t &= e^{(A-\delta)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{-\left[(A-\delta-\rho)\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\mathsf{d}s - \left(\frac{Az}{A-\delta-\rho}\right)^{-\frac{1}{\varepsilon}} \int_0^t e^{-(A-\delta)s}\mathsf{d}s\right].
\end{aligned}$$

This result is the same as (55).

A.2.3.4 Solution for S_0 and $\gamma > 1$ Finally I want to solve the differential equation of capital in the case of S_0 and $\gamma > 1$, which is

$$\begin{aligned} & \dot{K} = \frac{\mathrm{d}K}{\mathrm{d}t} = (g+\rho)\,K_t + D\lambda_t^{\frac{1}{\gamma-1}} - C_t \\ \Leftrightarrow \quad & \mathrm{d}K + \left[-\,(g+\rho)\,K_t - D\lambda_t^{\frac{1}{\gamma-1}} + C_t\right]\mathrm{d}t = 0. \end{aligned}$$

Inserting the integrating factor I_t on both sides gives

$$\underbrace{I_t}_{M}\,\mathrm{d}K + \underbrace{I_t\left[-\,(g+\rho)\,K_t - D\lambda_t^{\frac{1}{\gamma-1}} + C_t\right]}_{N}\mathrm{d}t = 0.$$

Exactness requires that $\frac{\partial M}{\partial t} = \frac{\partial N}{\partial K}$, hence we have

$$\begin{aligned} & \frac{\partial M}{\partial t} = \frac{\mathrm{d}I}{\mathrm{d}t} = -I_t(g+\rho) = \frac{\partial N}{\partial K} \\ \Leftrightarrow \quad & \frac{\frac{\mathrm{d}I}{\mathrm{d}t}}{I_t} = -(g+\rho) \\ \Leftrightarrow \quad & I_t = \underbrace{I_0}_{\equiv 1}\,e^{-(g+\rho)t} = e^{-(g+\rho)t}. \end{aligned}$$

Hence, the differential equation is

$$e^{-(g+\rho)t}\mathrm{d}K + e^{-(g+\rho)t}[-(g+\rho)K_t + C_t]\mathrm{d}t = 0.$$

Using again the four steps for solving, we obtain from step I

$$F(K,t) = \int I_t \mathrm{d}K + \phi(t) = K_t e^{-(g+\rho)t} + \phi(t).$$

From step II, we receive

$$\begin{aligned} \frac{\partial F}{\partial t} &= K_t[-(g+\rho)]e^{-(g+\rho)t} + \phi'(t) = e^{-(g+\rho)t}[-(g+\rho)K_t - D\lambda_t^{\frac{1}{\gamma-1}} + C_t] = N \\ \Leftrightarrow \quad & K_t[-(g+\rho)]e^{-(g+\rho)t} + \phi'(t) = -e^{-(g+\rho)t}(g+\rho)K_t - e^{-(g+\rho)t}D\lambda_t^{\frac{1}{\gamma-1}} + e^{-(g+\rho)t}C_t \\ \Leftrightarrow \quad & \phi'(t) = -D\lambda_t^{\frac{1}{\gamma-1}}e^{-(g+\rho)t} + e^{-(g+\rho)t}C_t. \end{aligned}$$

as $\frac{\partial F}{\partial t} = N$ must hold for exactness. Integrating $\phi'(t)$ gives us $\phi(t)$ (step III)

$$\phi(t) = \int_0^t \phi'(s)\mathsf{d}s = -D\int_0^t \lambda_s^{\frac{1}{\gamma-1}} e^{-(g+\rho)s}\mathsf{d}s + \int_0^t C_s e^{-(g+\rho)s}\mathsf{d}s,$$

plugging $\phi(t)$ into $F(K,t)$ we obtain the complete form of the differential equation

$$F(K,t) \quad = \quad K_t e^{-(g+\rho)t} - D\int_0^t \lambda_s^{\frac{1}{\gamma-1}} e^{-(g+\rho)s}\mathsf{d}s + \int_0^t C_s e^{-(g+\rho)s}\mathsf{d}s.$$

Again, the general solution of the exact differential equation is equal to a constant which I name K_0:

$$K_t e^{-(g+\rho)t} - D\int_0^t \lambda_s^{\frac{1}{\gamma-1}} e^{-(g+\rho)s}\mathsf{d}s + \int_0^t C_s e^{-(g+\rho)s}\mathsf{d}s = K_0.$$

Solving this for K_t we have the solution for the differential:

$$K_t = e^{(g+\rho)t}\left[K_0 + D\int_0^t \lambda_s^{\frac{1}{\gamma-1}} e^{-(g+\rho)s}\mathsf{d}s - \int_0^t C_s e^{-(g+\rho)s}\mathsf{d}s\right]$$

Since we know from (38) that $C_t = \lambda_t^{-\frac{1}{\varepsilon}}$ and from (60) that $\lambda_t = \lambda_0 e^{-gt}$, the solution for K_t in S_0 solves as

$$\begin{aligned} K_t &= e^{(g+\rho)t}\left[K_0 + D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\frac{gs}{\gamma-1}} e^{-(g+\rho)s}\mathsf{d}s - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{\frac{gs}{\varepsilon}} e^{-(g+\rho)s}\mathsf{d}s\right] \\ \Leftrightarrow K_t &= e^{(g+\rho)t}\left[K_0 + D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\left[g\left(\frac{\gamma}{\gamma-1}\right)+\rho\right]s}\mathsf{d}s - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(\frac{\varepsilon-1}{\varepsilon}\right)+\rho\right]s}\mathsf{d}s\right] \end{aligned}$$

This result is the same as in (61).

A.3 The Use of the Transversality Condition in the Kindergarten Model

To describe the behaviour of the model for t approaching infinity, one can take advantage of the transversality condition (TVC). In the case of $\gamma = 1$ of the Kindergarten Model, this is done in order to prove that the system remains in S_+ once it has reached it. To do so, the TVC is used to derive (49). Recall the TVC:

$$\lim_{t\to\infty} \lambda_t K_t e^{-gt} = 0.$$

Furthermore, from (47) we know that

$$\lambda_t = \lambda_0 e^{-gt}$$

and from (48) that

$$K_t = e^{(g+\rho)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{-\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\,\mathrm{d}s\right].$$

Plugging (47) and (48) into the TVC and gives us (49):

$$\begin{aligned}
& \lim_{t\to\infty} e^{-gt}\lambda_0 e^{-gt} e^{(g+\rho)t}\left[K_0 - \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{-\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\,\mathrm{d}s\right] = 0 \\
\Leftrightarrow \quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}} \int_0^t e^{-\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]s}\,\mathrm{d}s \\
\Leftrightarrow \quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}} \left[\underbrace{\frac{-1}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}}_{=-\frac{1}{h}} e^{-\overbrace{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}^{=h}s}\right]_0^t \\
\Leftrightarrow \quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}} \left[-\frac{1}{h}\underbrace{e^{-ht}}_{\to 0} + \frac{1}{h}\underbrace{e^{0}}_{=1}\right] \\
\Leftrightarrow \quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}} \frac{1}{h} \\
\Leftrightarrow \quad & \lambda_0 = [K_0 h]^{-\varepsilon}
\end{aligned}$$

For $\gamma > 1$, the transversality condition remains the same as

$$\lim_{t\to\infty} \lambda_t K_t e^{-gt} = 0.$$

The shadow price develops according to (60) as

$$\lambda_t = \lambda_0 e^{-gt},$$

capital according to (61) as

$$K_t = e^{(g+\rho)t}\left[K_0 + D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\left[g\left(\frac{\gamma}{\gamma-1}\right)+\rho\right]s}\mathrm{d}s - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(\frac{\varepsilon-1}{\varepsilon}\right)+\rho\right]s}\mathrm{d}s\right].$$

Plugging λ_t and K_t into the TVC, we obtain (62):

$$\begin{aligned}
&\lim_{t\to\infty} e^{-gt}\lambda_0 e^{-gt}e^{(g+\rho)t}\left[K_0 + D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\left[g\left(\frac{\gamma}{\gamma-1}\right)+\rho\right]s}\mathrm{d}s - \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(\frac{\varepsilon-1}{\varepsilon}\right)+\rho\right]s}\mathrm{d}s\right] = 0\\
\Leftrightarrow\quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}}\int_0^t e^{-\left[g\left(\frac{\varepsilon-1}{\varepsilon}\right)+\rho\right]s}\mathrm{d}s - D\lambda_0^{\frac{1}{\gamma-1}}\int_0^t e^{-\left[g\left(\frac{\gamma}{\gamma-1}\right)+\rho\right]s}\mathrm{d}s\\
\Leftrightarrow\quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}}\left[\underbrace{\frac{-1}{g\left(1-\frac{1}{\varepsilon}\right)+\rho}}_{=-\frac{1}{\alpha_2}}e^{-\alpha_2 s}\right]_0^t - D\lambda_0^{\frac{1}{\gamma-1}}\left[\underbrace{\frac{-1}{g\left(\frac{\gamma}{\gamma-1}\right)+\rho}}_{-\frac{1}{\alpha_1}}e^{-\alpha_1 s}\right]_0^t\\
\Leftrightarrow\quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}}\left[\frac{-1}{\alpha_2}\underbrace{e^{-\alpha_2 t}}_{\to 0}+\frac{1}{\alpha_2}\underbrace{e^0}_{=1}\right] - D\lambda_0^{\frac{1}{\gamma-1}}\left[\frac{-1}{\alpha_1}\underbrace{e^{-\alpha_1 t}}_{\to 0}+\frac{1}{\alpha_1}\underbrace{e^0}_{=1}\right]\\
\Leftrightarrow\quad & K_0 = \lambda_0^{-\frac{1}{\varepsilon}}\frac{1}{\alpha_2} - \frac{D}{\alpha_1}\lambda_0^{\frac{1}{\gamma-1}}
\end{aligned}$$

A.4 The Derivatives of the Turning Point in the Kindergarten Model

In Table 2, I illustrate the marginal derivatives $\frac{\partial Y^*}{\partial x}$ with $x \in \{A, a, z, \rho, \delta\}$ in order to analyse how the parameters in x affect the turning point. The derivations are given in the following section.

A.4.1 The Results for $\gamma = 1$

Let us start with the case of $\gamma = 1$ such that the turning point is as in (67) defined by

$$Y^* \quad = \quad \frac{A}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}}.$$

where $g \equiv A(1-\theta^K) - \delta - \rho > 0$. For $\varepsilon = 1$, the derivative with respect to A is then

$$\frac{\partial Y^*}{\partial A}|_{\varepsilon=1} = \left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}} \frac{1}{\rho} > 0.$$

For $\varepsilon \neq 1$, the derivative of Y^* with respect to A is

$$\frac{\partial Y^*}{\partial A}|_{\varepsilon\neq 1} = \left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}} \frac{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right] - A\frac{\partial g}{\partial A}\left(1-\frac{1}{\varepsilon}\right)}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]^2}$$

with $\frac{\partial g}{\partial A} = (1-\theta^K) = \left(1-\frac{1}{a}\right)$ since $\theta^K = \frac{1}{a}$ and $a > 1$. The first term in brackets is positive, and so is the squared denominator. Hence, the sign depends only on numerator. Therefore we can write

$$\begin{aligned}
& g\left(1-\tfrac{1}{\varepsilon}\right) + \rho - A\left(1-\tfrac{1}{a}\right)\left(1-\tfrac{1}{\varepsilon}\right) \\
\Leftrightarrow \quad & \left[A\left(1-\tfrac{1}{a}\right)\left(1-\tfrac{1}{\varepsilon}\right)\right] - \left[A\left(1-\tfrac{1}{a}\right)\left(1-\tfrac{1}{\varepsilon}\right)\right] - (\delta+\rho)\left(\tfrac{\varepsilon-1}{\varepsilon}\right) + \rho \\
\Leftrightarrow \quad & \rho - (\delta+\rho)\left(\tfrac{\varepsilon-1}{\varepsilon}\right)
\end{aligned}$$

From here we can see that $\frac{\partial Y^*}{\partial A}$ is positive if $\rho > (\delta+\rho)\left(\frac{\varepsilon-1}{\varepsilon}\right)$. This we can rewrite such that the derivative is positive as long as

$$\begin{aligned}
\varepsilon\rho &> \delta\varepsilon + \rho\varepsilon - \delta - \rho \\
1 &> \delta\varepsilon - \delta - \rho \\
\Leftrightarrow \varepsilon &< \frac{\delta+\rho}{\delta}.
\end{aligned}$$

Otherwise the derivative is negative. Hence, for $\varepsilon \neq 1$ the impact of A on the turning point depends on the parameters.

The derivation of Y^* with respect to a is

$$\frac{\partial Y^*}{\partial a} = A\frac{\left(-\frac{1}{\varepsilon}\right)(az)^{-\left(\frac{1}{\varepsilon}+1\right)} z\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right] - \frac{\partial g}{\partial a}\left(1-\frac{1}{\varepsilon}\right)(az)^{-\frac{1}{\varepsilon}}}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]^2}.$$

Again, the denominator is positive since it is squared. Since $\frac{\partial g}{\partial a} = \frac{A}{a^2}$, the sign of the derivation depends again only on $(1-\frac{1}{\varepsilon})$. So, for $\varepsilon \geq 1$ both the first and the second term of the numerator are smaller than zero such that the derivative is smaller than zero, too. For $\varepsilon < 1$, the opposite is true: Because of $1 < \frac{1}{\varepsilon}$, both terms of the numerator become positive and so does the derivative.

The derivative of Y^* with respect to z is given by

$$\frac{\partial Y^*}{\partial z} = \frac{A}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]}(az)^{-\left(\frac{1}{\varepsilon}+1\right)}\left(-\frac{1}{\varepsilon}\right)a$$

For $\varepsilon \geq 1$, the sign of the derivative is negative. However, for $\varepsilon < 1$ the result is unclear. Depending if $g\left(1-\frac{1}{\varepsilon}\right)$ is larger or smaller than ρ, the sign is negative or positive respectively. However, in (49) I assume that $h \equiv g\left(1-\frac{1}{\varepsilon}\right)+\rho$ must be positive in order to make sustainable growth possible. Hence, as $\varepsilon > \frac{g}{g+\rho}$ holds, $\frac{\partial Y^*}{\partial z}$ remains negative.

The derivation of Y^* with respect to ρ is

$$\frac{\partial Y^*}{\partial \rho} = A\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}}\frac{-\frac{\partial g}{\partial \rho}\left(1-\frac{1}{\varepsilon}\right)+1}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]^2}.$$

The denominator is positive because it is squared. Due to $\frac{\partial g}{\partial \rho} = -1$, the derivative can be written as

$$\frac{\partial Y^*}{\partial \rho} = A\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}}\frac{-\frac{1}{\varepsilon}}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]^2} < 0.$$

Thus, regardless the value of ε the derivative is always smaller than zero.

The derivation of Y^* with respect to δ is

$$\frac{\partial Y^*}{\partial \delta} = A\left[\frac{1}{az}\right]^{\frac{1}{\varepsilon}}\frac{-\frac{\partial g}{\partial \delta}\left(1-\frac{1}{\varepsilon}\right)}{\left[g\left(1-\frac{1}{\varepsilon}\right)+\rho\right]^2},$$

where the denominator is always positive due to the square. $\frac{\partial g}{\partial \delta} = -1$ eliminates the minus in the numerator. Hence, the sign depends only on the term $(1-\frac{1}{\varepsilon})$. If $\varepsilon > 1$, the derivative of K^* with respect to δ is larger than zero, if $\varepsilon < 1$ is given, the opposite holds. For $\varepsilon = 1$, the derivative is zero.

A.4.2 The Results for $\gamma > 1$

Next, let us consider the case of $\gamma > 1$. Here we know from (71) that

$$Y^* = A\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\frac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma+\varepsilon-1}{\varepsilon}}$$

with $D \equiv \frac{1}{a}\left(\frac{1}{az}\right)^{\frac{1}{\gamma-1}} > 0$, $\alpha_1 \equiv \left(\frac{\gamma}{\gamma-1}\right)g+\rho > 0$ and $\alpha_2 \equiv g\left(1-\frac{1}{\varepsilon}\right)+\rho$.
The derivative of Y^* with respect to A is

$$\begin{aligned}\frac{\partial Y^*}{\partial A} = &\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\frac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma+\varepsilon-1}{\varepsilon}}\\
&+A^{\frac{\gamma+\varepsilon-1}{\varepsilon}}\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\frac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma-1}{\varepsilon}}\\
&\left\{\left[\frac{\partial\alpha_2}{\partial A}\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}+\frac{\alpha_2}{\varepsilon}\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}-1}(\gamma-1)azA^{\gamma-2}\right]\left[1+\frac{D}{\alpha_1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\right]\right.\\
&\left.+\alpha_2(azA^{\gamma-1})^{\frac{1}{\varepsilon}}D\frac{\alpha_1\left(\frac{1}{\gamma-1}\right)\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}-1}(\gamma-1)azA^{\gamma-2}-\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\frac{\partial\alpha_1}{\partial A}}{\alpha_1^2}\right\}.\end{aligned}$$

It can be shown that the sign of this derivation depends on the values of the parameters. For this, remember that since α_2 is assumed to be positive, all terms in the first three rows are positive. Then, the all terms but the very last division is positive. The sign of the last term however depends on the values of the parameters. To see this, consider the division of the last row. The denominator is positive due to the squared term. So let us focus on the numerator:

$$\begin{aligned}&\alpha_1\tfrac{1}{\gamma-1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}-1}(\gamma-1)azA^{\gamma-2}-(azA^{\gamma-1})^{\frac{1}{\gamma-1}}(1-\tfrac{1}{a})\tfrac{\gamma}{\gamma-1}\\
&\Leftrightarrow (azA^{\gamma-1})^{\frac{1}{\gamma-1}}\left(\tfrac{\alpha_1}{A}(azA^{\gamma-1})^{-1}(azA^{\gamma-1})-(1-\tfrac{1}{a})\tfrac{\gamma}{\gamma-1}\right)\\
&\Leftrightarrow (azA^{\gamma-1})^{\frac{1}{\gamma-1}}\left[A^{-1}\tfrac{\gamma}{\gamma-1}\left(A(1-\tfrac{1}{a})-\delta-\rho\right)+\rho A^{-1}-(1-\tfrac{1}{a})\tfrac{\gamma}{\gamma-1}\right]\\
&\Leftrightarrow (azA^{\gamma-1})^{\frac{1}{\gamma-1}}\left[\tfrac{\gamma}{\gamma-1}(1-\tfrac{1}{a})(1-1)+A^{-1}(\tfrac{\gamma}{\gamma-1}(-1)(\delta+\rho)+\rho)\right]\\
&\Leftrightarrow (azA^{\gamma-1})^{\frac{1}{\gamma-1}}A^{-1}\left[\tfrac{\gamma}{\gamma-1}(-1)(\delta+\rho)+\rho\right]\end{aligned}$$

From the term in the square brackets it is obvious that the sign depends on the values of the parameters. Thus, the we cannot solve the sign of $\frac{\partial Y^*}{\partial A}$ analytically for any value of ε.

The derivative of Y^* with respect to a is

$$\begin{aligned}\frac{\partial Y^*}{\partial a} = \quad & A^{\frac{\gamma+\varepsilon-1}{\varepsilon}} \left\{\alpha_2 \left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}} \left[1+\tfrac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma-1}{\varepsilon}} \\ & \left\{\left[\tfrac{\partial\alpha_2}{\partial a}(azA^{\gamma-1})^{\frac{1}{\varepsilon}} + \alpha_2\tfrac{1}{\varepsilon}(azA^{\gamma-1})^{\frac{1}{\varepsilon}-1}zA^{\gamma-1}\right]\left[1+\tfrac{D}{\alpha_1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\right]\right. \\ & \left.+\alpha_2(azA^{\gamma-1})^{\frac{1}{\varepsilon}}\left[\frac{\frac{\partial D}{\partial a}\alpha_1 - D\frac{\partial\alpha_1}{\partial a}}{\alpha_1^2}(azA^{\gamma-1})^{\frac{1}{\gamma-1}-1} + \tfrac{D}{\alpha_1}\tfrac{1}{\gamma-1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}-1}zA^{\gamma-1}\right]\right\}.\end{aligned}$$

The partial derivatives of α_1 and α_2 with respect to a are

$$\frac{\partial\alpha_1}{\partial a} = \frac{\partial\alpha_2}{\partial a} = \frac{\partial g}{\partial a} = \frac{A}{a^2}.$$

Because of this and due to assumption that α_2 must be positive, the first two rows are positive. The partial derivative of D with respect to a is

$$\begin{aligned}\frac{\partial D}{\partial a} &= (-1)a^{-2}(az)^{-\frac{1}{\gamma-1}} + a^{-1} - \frac{1}{\gamma-1}(az)^{-\frac{1}{\gamma-1}-1}z \\ &= -aD + a^{-1} - \frac{1}{\gamma-1}D \\ &= (-1)D\left(a+\frac{1}{\gamma-1}\right) + a^{-1}.\end{aligned}$$

Since the sign of $\frac{\partial D}{\partial a}$ depends on the parameters, a conclusion for $\frac{\partial Y^*}{\partial a}$ cannot be given. Thus, the effect of a on Y^* remains unclear for any ε.

The derivative of Y^* with respect to z is

$$\begin{aligned}\frac{\partial Y^*}{\partial z} = \quad & A^{\frac{\gamma+\varepsilon-1}{\varepsilon}} \left\{\alpha_2 \left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}} \left[1+\tfrac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma-1}{\varepsilon}} \\ & \left\{\tfrac{\alpha_2}{\varepsilon}\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}-1} aA^{\gamma-1}\left[1+\tfrac{D}{\alpha_1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\right]\right. \\ & \left.+\alpha_2(azA^{\gamma-1})^{\frac{1}{\varepsilon}}\tfrac{D}{\alpha_1}\tfrac{1}{\gamma-1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}-1}aA^{\gamma-1}\right\}\end{aligned}$$

For $\varepsilon \geq 1$ each row is positive such that the derivative has a positive sign. For $\varepsilon < 1$ however, the first row becomes unclear due to the ambiguity of α_2. But since $\alpha_2 = h$ is assumed to be larger than zero to allow for sustainable growth, the first row remains positive also for $\varepsilon < 1$. Hence, $\frac{\partial Y^*}{\partial z}$ is positive for any ε.

The derivative of Y^* with respect to ρ is

$$\begin{aligned}\frac{\partial Y^*}{\partial \rho} = & A^{\frac{\gamma+\varepsilon-1}{\varepsilon}}\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\tfrac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma-1}{\varepsilon}} \\ & \left\{\tfrac{\partial \alpha_2}{\partial \rho}(azA^{\gamma-1})^{\frac{1}{\varepsilon}}\left[1+\tfrac{D}{\alpha_1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\right]\right. \\ & \left.-\alpha_2(azA^{\gamma-1})^{\frac{1}{\varepsilon}}D(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\alpha_1^{-2}\tfrac{\partial \alpha_1}{\partial \rho}\right\}.\end{aligned}$$

The partial derivatives of α_1 and α_2 with respect to ρ:

$$\begin{aligned}\frac{\partial \alpha_1}{\partial \rho} &= \frac{\gamma}{\gamma+1}\frac{\partial g}{\partial \rho}+1=\frac{1}{\gamma+1} \\ \frac{\partial \alpha_2}{\partial \rho} &= \left(1-\frac{1}{\varepsilon}\right)\frac{\partial g}{\partial \rho}+1=\frac{1}{\varepsilon},\end{aligned}$$

where $\frac{\partial g}{\partial \rho}=-1$. The first row of $\frac{\partial Y^*}{\partial \rho}$ is always positive. However, the total result is unclear as it is unclear whether the second term in curly brackets is positive or negative due to the subtraction. Therefore, no conclusion can be drawn regarding the sign of the derivative for any ε.

Finally, the derivative of Y^* with respect to δ is

$$\begin{aligned}\frac{\partial Y^*}{\partial \delta} = & A^{\frac{\gamma+\varepsilon-1}{\varepsilon}}\left\{\alpha_2\left(azA^{\gamma-1}\right)^{\frac{1}{\varepsilon}}\left[1+\tfrac{D}{\alpha_1}\left(azA^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\right]\right\}^{\frac{\gamma-1}{\varepsilon}} \\ & \left\{\tfrac{\partial \alpha_2}{\partial \delta}(azA^{\gamma-1})^{\frac{1}{\varepsilon}}\left[1+\tfrac{D}{\alpha_1}(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\right]\right. \\ & \left.-\alpha_2(azA^{\gamma-1})^{\frac{1}{\varepsilon}}D(azA^{\gamma-1})^{\frac{1}{\gamma-1}}\alpha_1^{-2}\tfrac{\partial \alpha_1}{\partial \delta}\right\}.\end{aligned}$$

The partial derivatives of α_1 and α_2 with respect to δ are

$$\frac{\partial \alpha_1}{\partial \delta}=\frac{\partial \alpha_2}{\partial \delta}=\frac{\partial g}{\partial \delta}=-1.$$

As in the case of ρ, the first line is always positive but due to the subtraction in the second curly brackets, the derivation is unclear regarding its sign. No result can be given for any $\varepsilon \neq 1$.

For $\varepsilon=1$, we know that $\alpha_2=\rho$ and hence, we know also that $\frac{\partial \alpha_2}{\partial \delta}=0$. Then, the derivative of Y^* with respect to δ is

$$\frac{\partial Y^*}{\partial \delta}|_{\varepsilon=1} = A\gamma \left\{ \rho \left(azA^{\gamma-1} \right)^{\frac{1}{\varepsilon}} \left[1 + \frac{D}{\alpha_1} \left(azA^{\gamma-1} \right)^{\frac{1}{\gamma-1}} \right] \right\}^{\gamma-1} \rho \left(azA^{\gamma-1} \right)^{\frac{\gamma}{\gamma-1}} D\alpha_a^{-2}(-1)\frac{\partial \alpha_1}{\partial \delta} > 0.$$

Since $\frac{\partial \alpha_1}{\partial \delta} = -1$ still holds, the derivative with respect to δ is positive for $\varepsilon = 1$.

A.5 The Turning point of the IRS Model

In Chapter 5, I derived the turning point with respect to time (119) and income (120) from the pollution time path and the PIR respectively. The calculations are given here. Let us start with the pollution time path.

First, consider the pollution function of (102)

$$\hat{P}(C, E) = C - C^{\alpha} E^{\beta+\eta},$$

and the FOC with respect to C (105) and E (106):

$$C = \frac{\alpha}{\lambda}$$

$$E = \frac{\beta+\eta}{\lambda}$$

Combining these gives

$$\hat{P}(C, E) = \frac{\alpha}{\lambda} - \left(\frac{\alpha}{\lambda} \right)^{\alpha} \left(\frac{\beta+\eta}{\lambda} \right)^{\beta+\eta},$$

Using the definition of λ_t from (114)

$$\lambda_t = \frac{\alpha+\beta+\eta}{K_0\rho} e^{-(A-\delta-\rho)t},$$

we can rewrite the pollution function such that it is only depending on t:

$$\tilde{P}(t) = \frac{\alpha\rho K_0}{\alpha+\beta+\eta} e^{(A-\delta-\rho)t} - \left(\frac{\alpha\rho K_0}{\alpha+\beta+\eta} \right)^{\alpha} \left(\frac{(\beta+\eta)\,\rho K_0}{\alpha+\beta+\eta} \right)^{\beta+\eta} e^{(A-\delta-\rho)t(\alpha+\beta+\eta)}.$$

This is pollution time path discussed in (115). Deriving this with respect to t gives the turning point with respect to time:

$$\begin{aligned}\frac{\partial\tilde{P}(t)}{t} &= \frac{\alpha\rho K_0}{\alpha+\beta+\eta}(A-\delta-\rho)e^{(A-\delta-\rho)t} \\ &\quad -\left(\frac{\alpha\rho K_0}{\alpha+\beta+\eta}\right)^{\alpha}\left(\frac{(\beta+\eta)\,\rho K_0}{\alpha+\beta+\eta}\right)^{\beta+\eta}(A-\delta-\rho)(\alpha+\beta+\eta)e^{(A-\delta-\rho)t(\alpha+\beta+\eta)} = 0.\end{aligned}$$

This can be rewritten as:

$$\begin{aligned}&\frac{K_0\alpha\rho}{\alpha+\beta+\eta}e^{(A-\delta-\rho)} = \left(\frac{K_0\rho}{\alpha+\beta+\eta}\right)^{\alpha+\beta+\eta}\alpha^{\alpha}(\beta+\eta)^{(\beta+\eta)}(\alpha+\beta+\eta)e^{(A-\delta-\rho)(\alpha+\beta+\eta)t} \\ \Leftrightarrow\;& (K_0)^{1-\alpha-\beta-\eta}\alpha^{1-\alpha}(\beta+\eta)^{-(\beta+\eta)}(\alpha+\beta+\eta)^{\alpha+\beta+\eta-2}\rho^{1-\alpha-\beta-\eta} = e^{(A-\delta-\rho)(\alpha+\beta+\eta-1)t} \\ \Leftrightarrow\;& \ln\left[(K_0\rho)^{1-\alpha-\beta-\eta}\alpha^{1-\alpha}(\beta+\eta)^{-(\beta+\eta)}(\alpha+\beta+\eta)^{\alpha+\beta+\eta-2}\right] = (A-\delta-\rho)(\alpha+\beta+\eta-1)t \\ \Leftrightarrow\;& t^* = \frac{\ln\left[K_0^{1-\alpha-\beta-\eta}\alpha^{1-\alpha}(\beta+\eta)^{-(\beta+\eta)(\alpha+\beta+\eta)^{\alpha+\beta+\eta-2}\rho^{1-\alpha-\beta-\eta}}\right]}{(A-\delta-\rho)(\alpha+\beta+\eta)}.\end{aligned}$$

In order to get the turning point with respect to income, recall the pollution function (102)

$$\hat{P}(C,E) = C - C^{\alpha}E^{\beta+\eta},$$

and the definition of the consumption rate $c \equiv \frac{C}{Y}$ and of the environmental effort rate $b \equiv \frac{E}{Y}$. Combining those gives us the PIR as in (116):

$$P^*(Y) = cY - (cY)^{\alpha}(bY)^{\beta+\eta}.$$

Taking the derivative of $P^*(Y)$ this with respect to Y gives us the turning point with respect to income:

$$
\begin{aligned}
\frac{\partial P^*(Y)}{\partial Y} &= c - \alpha(cY)^{\alpha-1}(bY)^{\beta+\eta} - (\beta+\eta)(cY)^{\alpha}(bY)^{\beta+\eta} = 0 \\
&\Leftrightarrow \ c = c^{\alpha} b^{\beta+\eta}(\alpha+\beta+\eta)Y^{\alpha+\beta+\eta-1} \\
&\Leftrightarrow \ Y^{\alpha+\beta+\eta-1} = c^{1-\alpha} b^{-(\beta+\eta)}(\alpha+\beta+\eta)^{-1} \\
&\Leftrightarrow \ Y^{\alpha+\beta+\eta-1} = \left[\frac{\alpha\rho}{A(\alpha+\beta+\eta)}\right]^{1-\alpha}\left[\frac{(\beta+\eta)\rho}{A(\alpha+\beta+\eta)}\right]^{-(\beta+\eta)}(\alpha+\beta+\eta)^{-1} \\
&\Leftrightarrow \ Y^{\alpha+\beta+\eta-1} = \left(\frac{A}{\rho}\right)^{\alpha+\beta+\eta-1}\alpha^{1-\alpha}(\beta+\eta)^{-(\beta+\eta)}(\alpha+\beta+\eta)^{\alpha+\beta+\eta-2} \\
&\Leftrightarrow \ Y^* = \frac{A\alpha^{\frac{1-\alpha}{\alpha+\beta+\eta-1}}(\beta+\eta)^{-\frac{\beta+\eta}{\alpha+\beta+\eta-1}}(\alpha+\beta+\eta)^{1-\frac{1}{\alpha+\beta+\eta-1}}}{\rho}
\end{aligned}
$$

A.6 Comparative Statics of the IRS Model

In Table 3, the comparative statics of the turning point in the IRS model are presented. Here, I provide the derivations for A, ρ, α, β and η. For simplicity, let us define $\sigma \equiv \alpha+\beta+\eta$ with the partial derivatives $\frac{\partial\sigma}{\partial\alpha} = \frac{\partial\sigma}{\partial\beta} = \frac{\partial\sigma}{\partial\eta} = 1$. Recall the turning point with respect to income from (120). This we can rewrite as

$$
\begin{aligned}
Y^* &= \frac{A}{\rho}\alpha^{\frac{1-\alpha}{\sigma-1}}(\beta+\eta)^{\frac{-(\beta+\eta)}{\sigma-1}}\sigma^{\frac{\sigma-2}{\sigma-1}} \\
&= \frac{A}{\rho}e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}}.
\end{aligned}
$$

The derivative of Y^* with respect to A is:

$$
\begin{aligned}
\frac{\partial Y^*}{\partial A} &= \frac{1}{\rho}e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&= \frac{Y^*}{A}.
\end{aligned}
$$

The derivative of Y^* with respect to ρ is:

$$
\begin{aligned}
\frac{\partial Y^*}{\partial \rho} &= -\frac{A}{\rho^2}e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&= -\frac{Y^*}{\rho}.
\end{aligned}
$$

Taking the partial derivative with respect to α is:

$$
\begin{aligned}
\frac{\partial Y^*}{\partial \alpha} &= \frac{A}{\rho}\Bigg[e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}\left(\frac{1}{\alpha}\frac{1-\alpha}{\sigma-1}+\ln[\alpha]\frac{-(\sigma-1)-(1-\alpha)\frac{\partial\sigma}{\partial\alpha}}{(\sigma-1)^2}\right)e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&\quad +e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}\ln[\beta+\eta]\frac{(\beta+\eta)\frac{\partial\sigma}{\partial\alpha}}{(\sigma-1)^2}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&\quad +e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}}\left(\frac{1}{\sigma}\frac{\partial\sigma}{\partial\alpha}\frac{\sigma-2}{\sigma-1}+\ln[\sigma]\frac{(\sigma-1)\frac{\partial\sigma}{\partial\alpha}-(\sigma-2)\frac{\partial\sigma}{\partial\alpha}}{(\sigma-1)^2}\right)\Bigg] \\
&= \underbrace{\frac{A}{\rho}e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}}}_{Y^*}\Bigg[\frac{1}{\alpha}\frac{1-\alpha}{\sigma-1}+\frac{1}{\sigma}\frac{\sigma-2}{\sigma-1}+\frac{-\sigma+\alpha}{(\sigma-1)^2}\ln[\alpha] \\
&\quad +\ln[\beta+\eta]\frac{\beta+\eta}{(\sigma-1)^2}+\ln[\sigma]\frac{1}{(\sigma-1)^2}\Bigg] \\
&= \frac{Y^*}{\alpha\sigma(\sigma-1)^2}\Bigg[(1-\alpha)(\sigma-1)\sigma+(\sigma-2)(\sigma-1)\alpha \\
&\quad +\ln[\alpha](-1)(\overbrace{\sigma-\alpha}^{\beta+\eta})\alpha\sigma+\ln[\beta+\eta](\beta+\eta)\alpha\sigma+\ln[\sigma]\alpha\sigma\Bigg] \\
&= \frac{Y^*}{\alpha\sigma(\sigma-1)^2}\Bigg[(\sigma-1)(\overbrace{\sigma-2\alpha}^{-\alpha+\beta+\eta})+\alpha\sigma\ln[\sigma]+\alpha\sigma\left(\ln[\sigma]+(\beta+\eta)\ln[\beta+\eta]-\ln[\alpha]\right)\Bigg] \\
&= \frac{Y^*}{\alpha\sigma(\sigma-1)^2}\Bigg[(\sigma-1)(-\alpha+\beta+\eta)+\alpha\sigma\left(\ln[\sigma]+(\beta+\eta)\ln[\beta+\eta]-\ln[\alpha]\right)\Bigg].
\end{aligned}
$$

The partial derivative with respect to β and η are the same. They read as following:

$$
\begin{aligned}
\frac{\partial Y^*}{\partial \beta} &= \frac{\partial Y^*}{\partial \eta} = \frac{A}{\rho}\Bigg[e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}\left(\ln[\alpha]\frac{-(1-\alpha)\frac{\partial\sigma}{\partial\beta}}{(\sigma-1)^2}\right)e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&\quad + e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}\left(\frac{1}{\beta+\eta}\frac{-(\beta+\eta)}{\sigma-1}+\ln[\beta+\eta]\frac{-(\sigma-1)+(\beta+\eta)\frac{\partial\sigma}{\partial\beta}}{(\sigma-1)^2}\right)e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}} \\
&\quad + e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}}\Big(\frac{1}{\sigma}\frac{\partial\sigma}{\partial\beta}\frac{\sigma-2}{\sigma-1}+\ln[\sigma]\frac{(\sigma-1)\frac{\partial\sigma}{\partial\beta}-(\sigma-2)\frac{\partial\sigma}{\partial\beta}}{(\sigma-1)^2}\Big)\Bigg] \\
&= \underbrace{\frac{A}{\rho}e^{\ln[\alpha]\frac{1-\alpha}{\sigma-1}}e^{\ln[\beta+\eta]\frac{-(\beta+\eta)}{\sigma-1}}e^{\ln[\sigma]\frac{\sigma-2}{\sigma-1}}}_{Y^*}\Bigg[\ln[\alpha]\frac{-(1-\alpha)}{(\sigma-1)^2}+\frac{-1}{\sigma-1} \\
&\quad + \ln[\beta+\eta]\frac{-(\sigma-1)+(\beta+\eta)}{(\sigma-1)^2}+\frac{\sigma-2}{\sigma(\sigma-1)}+\ln[\sigma]\frac{1}{(\sigma-1)^2}\Bigg] \\
&= \frac{Y^*}{\sigma(\sigma-1)^2}\Bigg[-\ln[\alpha](1-\alpha)\sigma-\sigma(\sigma-1) \\
&\quad + \sigma\ln[\beta+\eta](\overbrace{1-\sigma+\beta+\eta}^{1-\alpha})+(\sigma-2)(\sigma-1)+\sigma\ln[\sigma]\Bigg] \\
&= \frac{Y^*}{\sigma(\sigma-1)^2}\Bigg[\sigma\ln[\sigma]-2\sigma+2+\sigma(\alpha-1)\Big(\ln[\alpha]-\ln[\beta+\eta]\Big)\Bigg] \\
&= \frac{Y^*}{\sigma(\sigma-1)^2}\Bigg[2+\sigma\Big(\ln[\sigma]-2\Big)+\sigma(\alpha-1)\Big(\ln[\alpha]-\ln[\beta+\eta]\Big)\Bigg].
\end{aligned}
$$