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LETTER TO THE EDITOR

Discussion of "Scaling laws and renormalization groups for strength and toughness of disordered materials," *Int. J. Solids Structures*, Vol. 31, pp. 291–302 (1994)

The paper represents some completely new approaches for solid mechanics, i.e. fractal geometry and renormalization group theory. As shown in this paper, these new methods can provide a rational and consistent explanation of the size scale effects on tensile strength and fracture energy of disordered media.

It is well known that dimensional analysis is an important tool for developing mathematical models of physical phenomena, and it can help us understand existing models. It is worth noting that, when fractal geometry is introduced into fracture mechanics, the dimensions of physical parameters are an important problem. Strictly speaking, the measure related to fractal geometry should be defined in Housdorff space. However, since the Housdorff measure cannot be measured directly, we still use the Euclidean measure in real application. Therefore, the dimension of a parameter should keep unchanging, otherwise, its physical meaning is not clear. For example, the length of a fractal curve is defined by (Mandelbrot, 1982)

$$L(r) = L_0^D r^{1-D} = L_0 \varepsilon^{1-D} \tag{1}$$

where r is a yardstick, L_0 is a character length of the curve (for example, the distance between the two end points of the curve), $\varepsilon = r/L_0$ is a dimensionless scaling parameter. It is obvious that the physical dimension of length $L(r)$ is still [length].

Using the same symbols with present author, the *renormalized tensile strength* σ_u^* is

$$\sigma_u^* = \frac{F_1}{l^2 \varepsilon^{d_\sigma - 2}} = \frac{F_2}{b^2 \varepsilon^{d_\sigma - 2}}. \tag{2}$$

It is easy to obtain

$$\sigma_u^{(1)} = \sigma_u^{(2)} \varepsilon^{d_\sigma}, \tag{3}$$

where, the dimensionless parameter $\varepsilon = r_{\mathrm{in}}/r_{\mathrm{out}}$, here, r_{in} and r_{out} are the inner and outer cutoff length, in which fractal or scaling exists. In formula (3), we also define $\sigma_u^{(1)} = F_1/l^2$ and $\sigma_u^{(2)} = F_2/b^2$.

Although the expression is the same as that of the original paper, the physical dimension of σ_u^* is still [force][length]$^{-2}$, not [force][length]$^{-(2-d_\sigma)}$ in the original paper.

On the other hand, it is necessary to point out that the value of ε is not equal to $1/b$ in most cases. The value of ε is dependent not only on the microscopic structure effects, but also on the macroscopic structue of specimen (Mandelbrot *et al.*, 1984). At the same time the formula (3) clearly gives its suitable range.

Similarly, keeping the physical dimension unchanging, the formulae (7), (11) and (15) etc. in the original paper may be rewritten respectively

$$\sigma_u = \sigma_u^* \varepsilon^{d_\sigma}, \tag{4}$$

$$\mathcal{G}_F^{(2)} = \mathcal{G}_F^{(1)} \varepsilon^{-d_{\mathcal{G}}}, \tag{5}$$

$$\mathcal{G}_F = \mathcal{G}_F^* \varepsilon^{-d_s}. \tag{6}$$

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