Torsional Impact Response of a Penny-Shaped Interface Crack in Bonded Materials With a Graded Material Interlayer

C. Li[1]
Department of Architectural Engineering,
Shijiazhuang Railway Institute,
Shijiazhuang 050043, P. R. China

Z. Duan
Laboratory of Nonlinear Mechanics,
Institute of Mechanics,
Chinese Academy of Sciences,
Beijing 100080, P. R. China

Z. Zou
Department of Communications Engineering,
Shijiazhuang Railway Institute,
Shijiazhuang 050043, P. R. China

In this paper, the dynamic response of a penny-shaped interface crack in bonded dissimilar homogeneous half-spaces is studied. It is assumed that the two materials are bonded together with such a inhomogeneous interlayer that makes the elastic modulus in the direction perpendicular to the crack surface is continuous throughout the space. The crack surfaces are assumed to be subjected to torsional impact loading. Laplace and Hankel integral transforms are applied combining with a dislocation density function to reduce the mixed boundary value problem into a singular integral equation with a generalized Cauchy kernel in Laplace domain. By solving the singular integral equation numerically and using a numerical Laplace inversion technique, the dynamic stress intensity factors are obtained. The influences of material properties and interlayer thickness on the dynamic stress intensity factor are investigated. [DOI: 10.1115/1.1459066]

1 Introduction

Interface crack problems of composite structures have been the important topic of fracture mechanics in recent decades. There are a large number of solutions in the technical literature for isotropic, orthotropic, and anisotropic bonded materials containing interface cracks. Some typical studies that should be mentioned are that the asymptotic analysis of the elastic fields (Williams [1]), the standard interface crack solutions (Erdogan [2], Rice and Sih [3], Willis [4] and Qu and Bassani [5]), the crack-tip contact model (Comninou [6] Achenbach et al. [7] and Rice [8]), the elastic-plastic analysis (Shih and Asaro [9]) and so on. Hutchinson and Suo [10] once gave an extensive overview on the static behavior of interface cracks. On the other hand, there are also a number of papers devoted to the dynamic fracture mechanics of interface cracks. Sih and Chen [11] studied several dynamic responses of composite materials with interface cracks, such as antiplane shear of interface rectangular cracks in layered orthotropic dissimilar materials, orthotropic layered composite debonded over a penny-shaped region subjected to sudden shear, diffraction of time-harmonic waves by interface cracks in dissimilar media. Takei and co-workers [12] and Li and Tai [13] considered the elastodynamic response of a composite with an interface crack under antiplane shear loading. Ueda and co-workers [14] reported the torsional impact response of a penny-shaped crack on a bimaterial interface. Beyond these, considerable experimental works on the dynamics of interface cracks (Lambros and Rosakis [15] and Singh, Lambros, and Rosakis [16]) and numerical simulations of dynamic interfacial crack growth (Xu and Needleman [17] and Needleman and Rosakis [18]) were also carried out. Rosakis and Ravichandran [19] recently made a rather comprehensive review on dynamic failure mechanics.

The researches mentioned above usually assumed that the dissimilar materials were bonded directly (bimaterials) or with a thin homogeneous layer which properties different from that of bonded materials. However, recent studies have indicated that in many cases an inhomogeneous interlayer exists between the bonded materials (Subramanian and Crasto [20]). This kind of interlayer may be developed as a result of certain processing techniques (Lugscheider [21] and Shiau et al. [22]) or results from intentional grading of the material composition (Kurihara et al. [23] and Jager et al. [24]). For the static problems of fracture mechanics about the inhomogeneous interlayer, there have been many theoretical studies (Delale and Erdogan [25], Ozturk and Erdogan [26], Wang et al. [27] and Fildis and Yahsi [28]). In their studies, two kind of inhomogeneous interlayer models have been proposed. One of them is the exponential function model and another is a so-called generalized interlayer model, which is a power function. These models have physical background and make the problem of stress oscillatory singularity (Williams [1]) overcome. However, as for dynamic fracture mechanic of interface cracks, there are few studies considered the effect of an inhomogeneous interlayer.

In this paper, we examine the torsional impact response of a penny-shaped interface crack in a layered composite. Although this problem is rather a theoretical problem, it also has the engineering background, such as the sudden appearance of a penny-shaped interface crack in a component under torsional loading. The main difference between our present paper and literature (Ueda, Shindo, and Astumi [14]) is that a graded material interlayer is introduced. Our main objective is to investigate whether the graded material interlayer is helpful in reducing the dynamic stress intensity factor of an interface crack in a bonded materials and how the material inhomogeneity and interlayer thickness influence the dynamic stress intensity factor. The methods used in our paper are the Laplace and Hankel integral transforms and the singular integral equation technique.

2 Formulation of the Problem

As shown in Fig. 1, consider two dissimilar half-spaces (Material-1 and Material-3) to be bonded with an inhomogeneous interlayer, which denoted as Material-2. The material properties of Material-1 and Material-3 are constant and denoted as ρ_1, μ_1 and ρ_3, μ_3 respectively, where ρ is the mass density and μ is the shear modulus.

As we have known, there are two material parameters involved in the dynamic torsional problems. They are the shear modulus μ

[1]Current address: Department of Mechanical Engineering, University of Delaware, Newark, DE 19716.

Contributed by the Applied Mechanics Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS for publication in the ASME JOURNAL OF APPLIED MECHANICS. Manuscript received by the ASME Applied Mechanics Division, June 23, 1999; final revision, June 22, 2000. Editor: A. Needleman. Discussion on the paper should be addressed to the Editor, Prof. Lewis T. Wheeler, Department of Mechanical Engineering, University of Houston, Houston, TX 77204-4792, and will be accepted until four months after final publication of the paper itself in the ASME JOURNAL OF APPLIED MECHANICS.

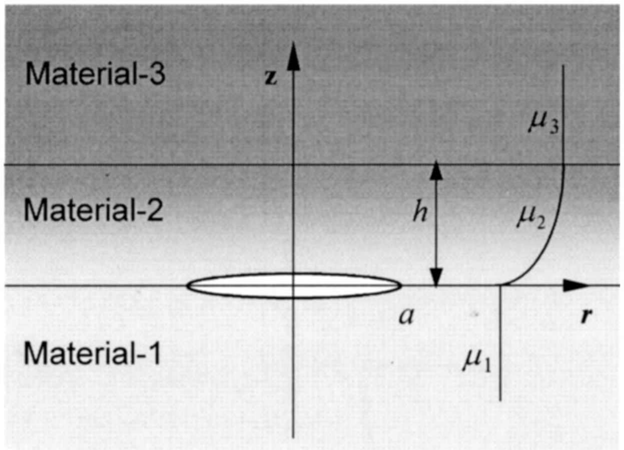

Fig. 1 A penny-shaped crack on the interface of a graded material interlayer and a homogeneous material

and the mass density ρ. For the inhomogeneous interlayer, due to the mathematical complexity introduced by the inertia term, it is necessary to assume that the shear modulus and the mass density can vary independently. Such an idealization can offer considerable simplifications to the analysis. After compared the several models for expressing the variation of the shear modulus, such as the exponential form $\mu(z)=\mu_1\exp(\alpha z)$ (Delale and Erdogan [25]), and the power form $\mu_2(z)=\mu_1(1+\alpha z)^k$ (Wang et al. [27]), we found that the variations

$$\mu_2=\mu_1(1+\alpha z)^2, \tag{1}$$

$$\rho_2=(\rho_1+\rho_3)/2, \tag{2}$$

are mathematically tractable, and still physically representative enough. In Eq. (1), the parameter α can be determined by the continuity condition of the shear modulus $\mu_2(0)=\mu_1$ and $\mu_2(h)=\mu_3$, that is $\alpha=(\sqrt{\mu_3/\mu_1}-1)/h$.

Assume a penny-shaped crack of diameter $2a$ is located at the interface of Material-1 and Material-2 and subjected to a torsional impact loading $P(r)$. For the present problem, in the cylindrical polar coordinates (r,θ,z), only the displacement $(u_\theta)_i=w_i(r,z,t)$ nonvanishes, where subscripts $i=1,2,3$ refer to materials 1, 2, and 3, and where t is the time. The nonvanishing stress components $\tau_{\theta z}$ and $\tau_{r\theta}$ are as follows:

$$(\tau_{\theta z})_i=\mu_i\frac{\partial w_i}{\partial z},\quad (\tau_{r\theta})=\mu_i\left(\frac{\partial w_i}{\partial r}-\frac{w_i}{r}\right),\quad i=1,2,3. \tag{3}$$

The governing equation of motion gives

$$\frac{\partial^2 w_i}{\partial r^2}+\frac{1}{r}\frac{\partial w_i}{\partial r}-\frac{w_i}{r^2}+\frac{\partial^2 w_i}{\partial z^2}=\frac{\rho_i}{\mu_i}\frac{\partial^2 w_i}{\partial t^2},\quad i=1,3 \tag{4}$$

$$\frac{\partial^2 w_2}{\partial r^2}+\frac{1}{r}\frac{\partial w_2}{\partial r}-\frac{w_2}{r^2}+\frac{\partial^2 w_2}{\partial z^2}+\frac{\mu_2'(z)}{\mu_2(z)}\frac{\partial w_2}{\partial z}=\frac{\rho_2}{\mu_2(z)}\frac{\partial^2 w_2}{\partial t^2} \tag{5}$$

where $\mu_2'(z)$ is the derivative of $\mu_2(z)$ with respect to z.

The boundary conditions are given as follows:

$$(\tau_{\theta z})_1(r,0^-,t)=(\tau_{\theta z})_2(r,0^+,t)=P(r)H(t),\quad 0\leqslant r<a, \tag{6}$$

$$w_1(r,0^-,t)=w_2(r,0^+,t),\quad r\geqslant a, \tag{7}$$

where $H(t)$ is the Heaviside unit step function. The continuity conditions of the displacement and the shear stress across the interfaces give

$$(\tau_{\theta z})_1(r,0^-,t)=(\tau_{\theta z})_2(r,0^+,t),\quad r\geqslant a, \tag{8}$$

$$w_2(r,h^-,t)=w_3(r,h^+,t),\quad 0\leqslant r<\infty, \tag{9}$$

$$(\tau_{\theta z})_2(r,h^-,t)=(\tau_{\theta z})_3(r,h^+,t),\quad 0\leqslant r<\infty. \tag{10}$$

Note that the standard Laplace transform on $f(t)$ is

$$f^*(p)=\int_0^\infty f(t)e^{-pt}dt \tag{11}$$

whose inversion is

$$f(t)=\frac{1}{2\pi i}\int_{\mathrm{Br}} f^*(p)e^{pt}dp \tag{12}$$

and Br denotes the Bromwich path of integration. Applying the transform (11) to Eqs. (4) and (5) results in the transformed equations

$$\frac{\partial^2 w_i^*}{\partial r^2}+\frac{1}{r}\frac{\partial w_i^*}{\partial r}-\frac{w_i^*}{r^2}+\frac{\partial^2 w_i^*}{\partial z^2}=\frac{\rho_i p^2}{\mu_i}w_i^*,\quad i=1,3 \tag{13}$$

$$\frac{\partial^2 w_2^*}{\partial r^2}+\frac{1}{r}\frac{\partial w_2^*}{\partial r}-\frac{w_2^*}{r^2}+\frac{\partial^2 w_2^*}{\partial z^2}+\frac{\mu_2'(z)}{\mu_2(z)}\frac{\partial w_2^*}{\partial z}=\frac{\rho_2 p^2}{\mu_2(z)}w_2^*. \tag{14}$$

Moreover, introducing the pair of Hankel transforms of the first order,

$$V_i(s,z,p)=\int_0^\infty w_i^*(r,z,p)J_1(sr)rdr, \tag{15}$$

$$w_i^*(r,z,p)=\int_0^\infty V_i(s,z,p)J_1(sr)sds, \tag{16}$$

where $J_1(\ )$ is the Bessel function of the first kind, then applying Eq. (15) to the Eqs. (13) and (14) yields

$$\frac{\partial^2 V_i(s,z,p)}{\partial z^2}-\left[s^2+\frac{\rho_i p^2}{\mu_i}\right]V_i(s,z,p)=0,\quad i=1,3 \tag{17}$$

$$\frac{\partial^2 V_2(s,z,p)}{\partial z^2}+\frac{2\alpha}{1+\alpha z}\frac{\partial V_2(s,z,p)}{\partial z}-\left[s^2+\frac{\rho_2 p^2}{\mu_1(1+\alpha z)^2}\right]V_2(s,z,p)=0. \tag{18}$$

Considering the displacement conditions that w_1 and w_2 vanish at $|z|\to\infty$, the solutions of Eqs. (17) and (18) can be expressed as

$$V_1(s,z,p)=A_1(s,p)\exp(\gamma_1 z) \tag{19}$$

$$V_3(s,z,p)=A_4(s,p)\exp(-\gamma_3 z) \tag{20}$$

$$V_2(s,z,p)=A_2(s,p)(1+\alpha z)^{-1/2}I_\beta\left[(1+\alpha z)\frac{s}{|\alpha|}\right]+A_3(s,p)(1+\alpha z)^{-1/2}K_\beta\left[(1+\alpha z)\frac{s}{|\alpha|}\right], \tag{21}$$

where

$$\gamma_1=\sqrt{s^2+\frac{\rho_1 p^2}{\mu_1}},\quad \gamma_3=\sqrt{s^2+\frac{\rho_3 p^2}{\mu_3}},\quad \beta=\sqrt{\frac{1}{4}+\frac{\rho_2 p^2}{\mu_1\alpha^2}} \tag{22}$$

and $I_\beta(\ )$, $K_\beta(\ )$ are the modified Bessel function of the first kind and the second kind, respectively.

From Eq. (16), we can obtain the displacements in the Laplace domain. Subsequently, the shear stresses in the Laplace transform domain $\tau_{\theta z}^*$ and $\tau_{r\theta}^*$ can be obtained from Eq. (3). Then the unknown functions A_1, A_2, A_3, A_4 can be determined from the boundary and the continuity conditions.

3 Derivation of the Singular Integral Equation

In Laplace domain, the boundary conditions become

$$(\tau_{\theta z}^*)_1(r,0^-,p)=(\tau_{\theta z}^*)_2(r,0^+,p)=\frac{P(r)}{p},\quad 0\leqslant r<a, \tag{23}$$

$$w_1^*(r,0^-,p)=w_2^*(r,0^+,p),\quad r\geqslant a, \tag{24}$$

and the continuity conditions across the interfaces become

$$(\tau_{\theta z}^*)_1(r,0^-,p)=(\tau_{\theta z}^*)_2(r,0^+,p),\quad r\geqslant a, \tag{25}$$

$$w_2^*(r,h^-,p)=w_3^*(r,h^+,p),\quad 0\leqslant r<\infty, \tag{26}$$

$$(\tau_{\theta z}^*)_2(r,h^-,p)=(\tau_{\theta z}^*)_3(r,h^+,p),\quad 0\leqslant r<\infty. \tag{27}$$

To reduce the mixed boundary conditions (23) and (24) into an integral equation, we first define the following dislocation density function on the interface of Material-1 and Material-2:

$$g(r,p)=\frac{1}{r}\frac{\partial}{\partial r}[rw_2^*(r,0^+,p)-rw_1^*(r,0^-,p)]. \tag{28}$$

From the continuity conditions and the dislocation density function, we can obtain

$$(\tau_{\theta z}^*)_2(r,0,p)=\mu_2(0)\int_0^a R(u,r,p)g(u,p)udu \tag{29}$$

where

$$R(u,r,p)=\int_0^\infty D(s,p)J_1(sr)J_0(su)sds \tag{30}$$

and

$$D(s,p)=\frac{d_{21}(sd_{32}+d_{42})-d_{22}(sd_{31}+d_{41})}{(d_{11}-d_{21})(sd_{32}+d_{42})-(sd_{31}+d_{41})(d_{12}-d_{22})}. \tag{31}$$

The coefficients d_{ij} in Eq. (31) are as follows:

$$\begin{aligned}
d_{11}&=sI_\beta\left(\frac{s}{|\alpha|}\right),\quad d_{12}=sK_\beta\left(\frac{s}{|\alpha|}\right),\\
d_{21}&=-\left(\frac{1}{2}+\beta\right)\alpha I_\beta\left(\frac{s}{|\alpha|}\right)I_{\beta-1}\left(\frac{s}{|\alpha|}\right)\frac{s\alpha}{|\alpha|},\\
d_{22}&=-\left(\frac{1}{2}+\beta\right)\alpha K_\beta\left(\frac{s}{|\alpha|}\right)-K_{\beta-1}\left(\frac{s}{|\alpha|}\right)\frac{s\alpha}{|\alpha|},\\
d_{31}&=(1+\alpha h)^{-1/2}I_\beta\left((1+\alpha h)\frac{s}{|\alpha|}\right),\\
d_{32}&=(1+\alpha h)^{-1/2}K_\beta\left((1+\alpha h)\frac{s}{|\alpha|}\right),\\
d_{41}&=-\left(\frac{1}{2}+\beta\right)\alpha(1+\alpha h)^{-3/2}I_\beta\left((1+\alpha h)\frac{s}{|\alpha|}\right)\\
&\quad+(1+\alpha h)^{-1/2}I_{\beta-1}\left((1+\alpha h)\frac{s}{|\alpha|}\right)\frac{s\alpha}{|\alpha|},\\
d_{42}&=-\left(\frac{1}{2}+\beta\right)\alpha(1+\alpha h)^{-3/2}K_\beta\left((1+\alpha h)\frac{s}{|\alpha|}\right)\\
&\quad-(1+\alpha h)^{-1/2}K_{\beta-1}\left((1+\alpha h)\frac{s}{|\alpha|}\right)\frac{s\alpha}{|\alpha|}.
\end{aligned} \tag{32}$$

Note that

$$\lambda=\lim_{s\to\infty}D(s,p)=-\tfrac{1}{2}. \tag{33}$$

$R(u,r,p)$ can be further expressed as

$$R(u,r,p)=R_n(u,r,p)+R_s(u,r,p) \tag{34}$$

where

$$R_n(u,r,p)=\int_0^\infty[D(s,p)-\lambda]J_1(sr)J_0(su)sds, \tag{35}$$

$$\begin{aligned}R_s(u,r,p)&=\lambda\int_0^\infty J_1(sr)J_0(su)sds\\
&=-\frac{\lambda}{\pi}\left[\frac{1}{u(u-r)}+\frac{-u-r+2rM(u,r)}{u(u^2-r^2)}\right],\end{aligned} \tag{36}$$

and

$$M(u,r)=\begin{cases}\dfrac{u}{r}E\left(\dfrac{u}{r}\right), & u<r,\\[2ex] \dfrac{u^2}{r^2}E\left(\dfrac{r}{u}\right)-\dfrac{u^2-r^2}{r^2}K\left(\dfrac{r}{u}\right), & u>r.\end{cases} \tag{37}$$

$E(\ )$ and $K(\ )$ are complete elliptic integrals of the second and first kind, respectively. From the boundary condition (23), we obtain a singular integral equation with a generalized Cauchy kernel,

$$\int_0^a\left[-\frac{\lambda}{\pi}\frac{1}{u-r}+R_0(u,r,p)\right]g(u,p)du=\frac{P(r)}{\mu_2(0)p},\quad 0<r<a, \tag{38}$$

where

$$R_0(u,r,p)=uR_n(u,r,p)+\frac{\lambda}{\pi}\frac{u+r-2rM(u,r)}{u^2-r^2}. \tag{39}$$

The single-valued condition can be given from the definition of $g(u,p)$,

$$\int_0^a ug(u,p)du=0. \tag{40}$$

4 Dynamic Stress Intensity Factor

Normalized the interval by the following transformation of variables:

$$u=\frac{a}{2}(1+\xi),\quad r=\frac{a}{2}(1+\eta). \tag{41}$$

The integral Eqs. (38) and (40) can be rewritten as

$$\int_{-1}^1\left[-\frac{\lambda}{\pi}\frac{1}{\xi-\eta}+\bar{R}_0(\xi,\eta,p)\right]G(\xi,p)d\xi=\frac{\bar{P}(\eta)}{\mu_2(0)p}, \tag{42}$$

$$\int_{-1}^1(1+\xi)G(\xi,p)d\xi=0, \tag{43}$$

where

$$\bar{R}_0(\xi,\eta,p)=\frac{a}{2}R_0\left[\frac{a}{2}(1+\xi),\frac{a}{2}(1+\eta),p\right], \tag{44}$$

$$G(\xi,p)=g\left[\frac{a}{2}(1+\xi),p\right], \tag{45}$$

$$\bar{P}(\eta)=P\left[\frac{a}{2}(1+\eta)\right]. \tag{46}$$

Considering the singularity at the crack tip, we assume that

$$G(\xi,p)=\frac{\bar{G}(\xi,p)}{p}\frac{1}{\sqrt{1-\xi^2}}. \tag{47}$$

Following the numerical method developed by Erdogan for singular integral equations (Erdogan [29]), expanding $\overline{G}(\xi,p)$ in forms of Chebeshev polynomials

$$\overline{G}(\xi,p)=\sum_{n=0}^{\infty} B_n T_n(\xi), \tag{48}$$

we can obtain a system of equations,

$$\sum_{i=1}^{n}\left[\frac{-\lambda}{\xi_i-\eta_j}+\pi\overline{R}_0(\xi_i,\eta_j,p)\right]\frac{\overline{G}(\xi_i,p)}{n}=\frac{\overline{P}(\eta_j)}{\mu_2(0)}, \tag{49}$$

$$\sum_{i=1}^{n}\frac{(1+\xi_i)}{n}\overline{G}(\xi_i,p)=0, \quad j=1,2,\ldots,n-1, \tag{50}$$

where ξ_i, η_j are the roots of Chebeshev polynomial of the first kind and the second kind, respectively,

$$\xi_i=\cos\left(\frac{2i-1}{2n}\pi\right), \quad i=1,2,\ldots,n,$$

$$\eta_j=\cos\left(\frac{j}{n}\pi\right), \quad j=1,2,\ldots,n-1. \tag{51}$$

Solving the system of linear algebraic Eqs. (49) and (50), the unknown function $\overline{G}(\xi,p)$ can be obtained.

If the mode III stress intensity factor in Laplace domain is defined by

$$K^*_{\mathrm{III}}(p)=\lim_{r\to a^+}\sqrt{2(r-a)}(\tau^*_{\theta z})_2(r,0,p), \tag{52}$$

then by using the properties of Chebeshev polynomials, we obtain

$$K^*_{\mathrm{III}}(p)=\lambda\mu_2(0)\sqrt{\frac{a}{2}}\frac{\overline{G}(1,p)}{p}. \tag{53}$$

The dynamic stress intensity factor in time domain can be obtained by

$$K_{\mathrm{III}}(t)=\lambda\mu_2(0)\sqrt{\frac{a}{2}}\frac{1}{2\pi i}\int_{\mathrm{Br}}\frac{\overline{G}(1,p)}{p}e^{pt}dp. \tag{54}$$

5 Results and Discussion

Suppose that the crack surface torsional loading is $P(r)=-\tau_0 r/a$. In this problem, the variables are μ_3/μ_1, h/a, and ρ_3/ρ_1. To investigate the influences of these parameters on the dynamic stress intensity factor, we analyzed some real composite materials, such as Al_2O_3/Ni, TiC/C, SiO_2/Ni, SiC/C, and so on, and found that the parameter μ_3/μ_1 may vary in a wide range but the parameter ρ_3/ρ_1 may vary in a relatively narrow range. Finally, we chose the following combinations for the analysis: $\mu_3/\mu_1=1/12,1/3,3,12$; $\rho_3/\rho_1=0.5,1.0,2.0,4.0$; $h/a=0.2,0.5,1.0,2.0$.

Solving Eqs. (49) and (50), and accomplishing the Laplace inversion (54) by the numerical procedure developed by Miller and Guy [30], the mode III dynamic stress intensity factors in different cases are obtained. The results of the normalized dynamic stress intensity factor $K_{\mathrm{III}}(t)/\tau_0\sqrt{a}$ as a function of $c_{21}t/a$ are shown in Figs. 2–4, where $c_{21}=\sqrt{\mu_1/\rho_1}$ is the shear wave velocity in material-1. A general feature of the curves is observed to be that the stress intensity factors rise rapidly and reach a peak, then oscillate about their static values with decreasing magnification. This general feature has been reported for homogeneous materials and layered composite materials.

Figure 2 shows the variations of the normalized dynamic stress intensity factor with time for various ratios of the shear modulus μ_3/μ_1 while $\rho_3/\rho_1=1.0$ and $h/a=1.0$. It can be seen that the $K_{\mathrm{III}}(t)$ factor tends to monotonically decrease with the increasing of μ_3/μ_1. The differences between the peak values of curves and the static values also decrease with increasing μ_3/μ_1. This ten-

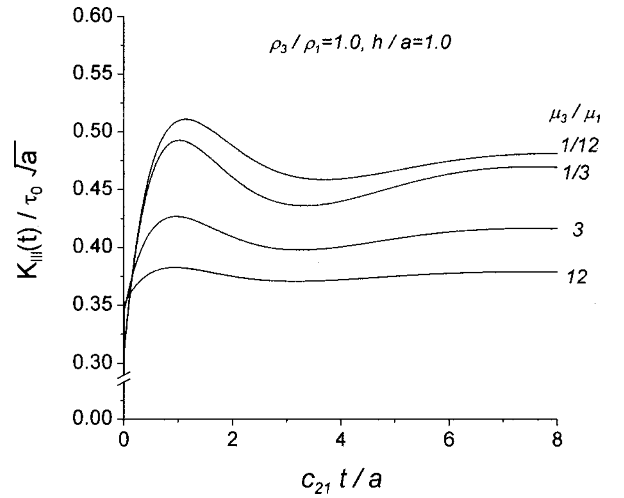

Fig. 2 The effect of the ratio of shear modulus on the normalized dynamic stress intensity factor

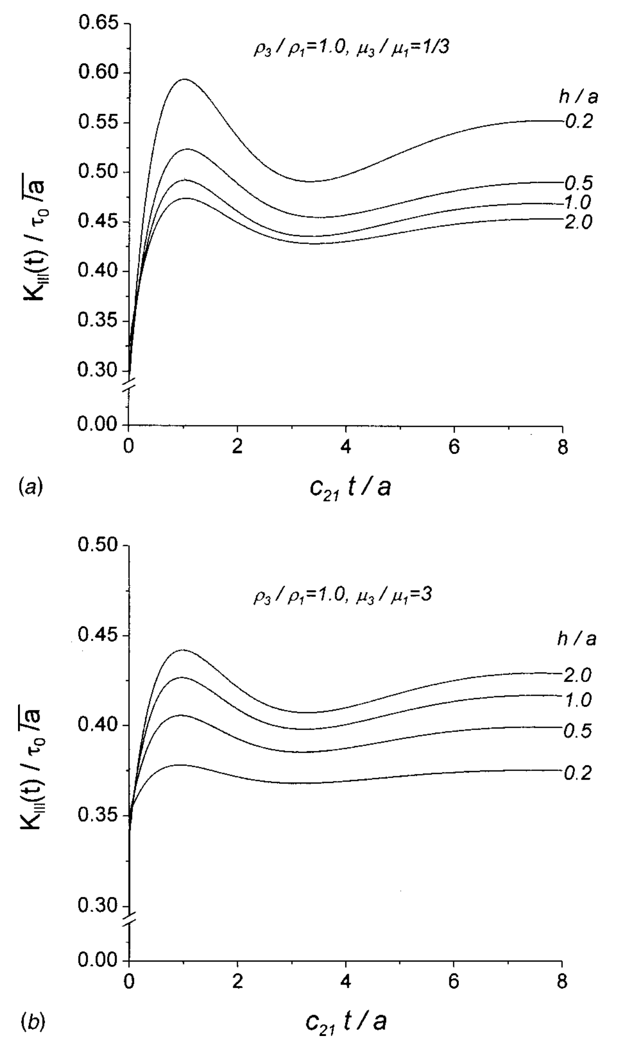

Fig. 3 The effect of the interlayer thickness on the normalized dynamic stress intensity factor

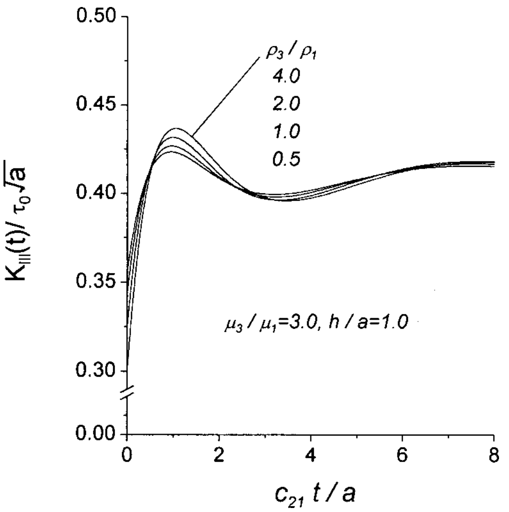

Fig. 4 The effect of the ratio of mass density on the normalized dynamic stress intensity factor

dency is somewhat different from that of two dissimilar materials bonded directly without an interlayer (Ueda, Shindo, and Astumi [14]). In the latter problem the peak values of $K_{\mathrm{III}}(t)$ factor decrease with the increasing of μ_3/μ_1, but the intersections exist during the oscillating procedure.

Figures 3(*a*) and 3(*b*) display that the $K_{\mathrm{III}}(t)$ factor is also affected by the ratio of interlayer thickness to crack radius h/a. For $\mu_3/\mu_1<1$, the dynamic stress intensity factors decrease with increasing h/a. The larger h/a is, the more the peak value goes beyond its corresponding static value. This phenomenon is depicted in Fig. 3(*a*) for $\mu_3/\mu_1=1/3$. For $\mu_3/\mu_1>1$, the opposite phenomenon can be observed from Fig. 3(*b*) for $\mu_3/\mu_1=3$ that the dynamic stress intensity factors increase with increasing h/a.

The effect of the mass density ratio ρ_3/ρ_1 on the variation of the dynamic stress intensity factor is shown in Fig. 4. This effect has not been reported before for layered composite materials. It is observed that the peak value of $K_{\mathrm{III}}(t)$ factor increases when the ratio ρ_3/ρ_1 increases. This phenomenon can be observed for an arbitrary μ_3/μ_1 and different ratios h/a, although these results are not given here as the space of the paper is limited.

As explained in Section 2, in this paper we only use the form $\mu_2(z)=\mu_1(1+\alpha z)^2$ to obtain the solution. A different choice of $\mu_2(z)$ may change the numerical values, but they should not lead to any change in the general trends of the results. We believe it can be verified in our future works by using numerical methods, such as the finite element method.

6 Conclusions

This paper presents the dynamic stress intensity factors for a penny-shaped interface crack in bonded dissimilar homogeneous half-spaces sandwiching an inhomogeneous interlayer. It is assumed that the shear modulus in the direction perpendicular to the crack surface is continuous throughout the space and the crack surfaces are subjected to torsional impact loading. A special model for describing material inhomogeneity parameter is introduced. Laplace and Hankel transforms are applied to reduce the mixed boundary value problem into a singular integral equation with a generalized Cauchy kernel. The results reveal that the dynamic stress intensity factors are affected not only by the stiffness ratio but also by the interlayer thickness and the mass density ratio. It is observed that the influences of the stiffness ratio and the interlayer thickness are stronger than the influences of the mass density ratio.

Acknowledgment

C. Li would like to thank Prof. G. J. Weng for providing him the opportunity to work at Rutgers University and giving him many suggestions in the revised manuscript. The work of C. Li was supported by the National Science Foundation of USA under CMS-9625304. Z. Duan was supported by the National Natural Science Fund of China under the Key Project No. 19891180. Z. Zou was supported by the National Natural Science Fund of China under the project No. 19772029.

References

[1] Williams, M. L., 1959, "The Stresses Around a Fault or Crack in Dissimilar Media," Bull. Seismol. Soc. Am., **49**, pp. 199–204.

[2] Erdogan, F., 1965, "Stress Distribution in Bonded Dissimilar Materials With Cracks," ASME J. Appl. Mech., **32**, pp. 403–410.

[3] Rice, J. R., and Sih, G. C., 1965, "Plane Problems of Cracks in Dissimilar Media," ASME J. Appl. Mech., **32**, pp. 418–423.

[4] Willis, J. R., 1971, "Fracture Mechanics of Interfacial Crack," J. Mech. Phys. Solids, **19**, pp. 353–368.

[5] Qu, J., and Bassani, J. L., 1993, "Interfacial Fracture Mechanics for Anisotropic Bimaterials," ASME J. Appl. Mech., **60**, pp. 422–431.

[6] Comninou, M., 1977, "The Interface Crack," ASME J. Appl. Mech., **44**, pp. 631–636.

[7] Achenbach, J., Keer, L., Khetan, R., and Chen, S., 1979, "Loss of Adhesion at the Tip of an Interface Crack," J. Elast., **9**, pp. 397–424.

[8] Rice, J., 1988, "Elastic Fracture Mechanics Concepts for Interface Cracks," ASME J. Appl. Mech., **55**, pp. 98–103.

[9] Shih, C., and Asaro, R., 1988, "Elastic-Plastic Analysis of Cracks on Bimaterials Interfaces. Part I: Small Scale Yielding," ASME J. Appl. Mech., **55**, pp. 299–316.

[10] Hutchinson, J. W., and Suo, Z., 1992, "Mixed Mode Cracking in Layered Materials," Adv. Appl. Mech., **29**, pp. 63–191.

[11] Sih, G. C., and Chen, E. P., 1981, *Mechanics of Fracture 6: Cracks in Composite Materials*, Martinus Nijhoff, The Hague.

[12] Takei, M., Shindo, Y., and Astumi, A., 1982, "Diffraction of Transient Horizontal Shear Waves by a Finite Crack at the Interface of Two Bonded Dissimilar Elastic Solids," Eng. Fract. Mech., **16**, pp. 799–807.

[13] Li, D. H., and Tai, W. H., 1991, "Elastodynamic Response of an Interface Crack in a Layered Composite Under Antiplane Shear Impact Load," Eng. Fract. Mech., **39**, pp. 687–693.

[14] Ueda, S., Shindo, Y., and Astumi, A., 1983, "Torsional Impact Response of a Penny-Shaped Crack on a Bimaterial Interface," Eng. Fract. Mech., **18**, pp. 1059–1066.

[15] Lambros, J., and Rosakis, A. J., 1995, "Shear Dominated Transonic Interfacial Crack Growth in a Bimaterial—I Experimental Observations," J. Mech. Phys. Solids, **43**, pp. 169–188.

[16] Singh, R. P., Lambros, J., and Rosakis, A. J., 1997, "Investigation of the Mechanics of Intersonic Crack Propagation Along a Bimaterial Interface Using Coherent Gradient Sensing and Photoelasticity," Proc. R. Soc. London, Ser. A, **A453**, pp. 2649–2667.

[17] Xu, X. P., and Needleman, A., 1996, "Numerical Simulations of Dynamic Crack Growth Along an Interface," Int. J. Fract., **74**, pp. 289–324.

[18] Needleman, A., and Rosakis, A. J., 1999, "The Effect of Bond Strength and Loading Rate on the Conditions Governing the Attainment of Intersonic Crack Growth Along Interfaces," J. Mech. Phys. Solids, **47**, pp. 2411–2449.

[19] Rosakis, A. J., and Ravichandran, G., 2000, "Dynamic Failure Mechanics," Int. J. Solids Struct., **37**, pp. 331–348.

[20] Subramanian, R. V., and Crasto, A. S., 1986, "Electrodeposition of a Polymer Interphase in Carbon-Fiber Composites," Polym. Compos., **7**, pp. 201–218.

[21] Lugscheider, E., 1987, "Plasma Spraying for Wear Applications," *Thermal Spray: Advances in Coating Technology*. Proceedings of the National Thermal Spray Conference. D. L. Houck, ed., ASM International, Materials Park, OH.

[22] Shiau, F. Y., Zuo, Y., Zeng, X. Y., Lin, J. C., and Chang Y. A., 1988, "Interfacial Reactions Between CO and GaAs," *Adhesion in Solids, Material Research Society Symposium*, Proc. Vol. 119, D. M. Mattox, J. E. Baglin, R. J. Gottshail, and C. D. Batich, eds., Materials Research Society, Pittsburgh, PA, pp. 171–176.

[23] Kurihara, K., Sasaki, K., and Kawarada, M., 1990, "Adhesion Improvement of diamond films," *FGM'90-Proc. of the First International Symposium on Functionally Graded Materials*, M. Yamanouchi, M. Koizumi, T. Hirai, and I. Shiota eds., Functionally Graded Material Forum, Sendai, Japan, pp. 65–69.

[24] Jager, D. A., Stover, D. and H. G. Schutz, 1991, "Plasma Spraying of Graded Composites," *Thermal Spray Coatings: Properties, Processes and Applications*. Proceedings of the National Thermal Spray Conference. T. F. Bernecki, ed., ASM International, Materials Park, OH.

[25] Delale, F., and Erdogan, F., 1988, "On the Mechanical Modeling of the Interfacial Region in Bonded Half Planes," ASME J. Appl. Mech., **55**, pp. 317–324.

[26] Ozturk, M., and Erdogan, F., 1995, "An Axisymmetric Crack in Bonded Materials With an Inhomogeneous Interfacial Zone Under Torsion," ASME J. Appl. Mech., **62**, pp. 116–125.

[27] Wang, X. Y., Zou, Z. Z., and Wang, D., 1996, "On the Griffith Crack in a

Inhomogeneous Interlayer of Adjoining Two Different Elastic Materials,'' Int. J. Fract., **79**, pp. R51–R56.
[28] Fildis, H., and Yahsi, O. S., 1996, ''The Axisymmetric Crack Problem is a Non-homogeneous Interfacial Region Between Homogeneous Half Spaces,'' Int. J. Fract., **78**, pp. 139–164.
[29] Erdogan, F., 1975, *Complex Function Technique, In Continuum Physics*, Vol. II, Academic Press, San Diego, CA, pp. 523–603.
[30] Miller, M. K., and Guy, W. T., 1966, ''Numerical inversion of the Laplace transform by use of Jacobi polynomials,'' SIAM (Soc. Ind. Appl. Math.) J. Numer. Anal., **3**, pp. 624–635.