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SATURATED DURATION OF RECTANGULAR PRESSURE PULSE APPLIED TO RECTANGULAR PLATES WITH FINITE-DEFLECTIONS

Ya-Pu Zhao

Laboratory for Nonlinear Mechanics of Continuous Media (LNM), Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China

E-Mail: yzhao@lnm.imech.ac.cn; Fax: (86-10) 6256 1284

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Introduction

The saturated impulses were determined by Zhao, Yu and Fang (1994) for the dynamic plastic response of both simply supported and fully clamped rigid-perfectly plastic beams [1], which were subjected to rectangular pressure pulses in the medium range, the approximate square yield conditions were used, and the secondary effect of finite-deflections was taken into account. Later, Zhao, Yu and Fang (1995) extended the concept of saturated impulses to analyze the dynamic plastic response of others structures such as simply supported circular plate, simply supported and fully clamped square plates, and cylindrical shells subjected to the same kind of loading [2,3]. As a matter of fact, saturated impulse for rectangular pressure pulse is equivalent to saturated duration of the pulse when the magnitude is given, so the corresponding dimensionless saturated duration was also given for these kinds of structures [2]. Zhu and Yu have recently extended the concept of saturated impulses of rigid-perfectly plastic structures to pulse-loaded elastic-plastic

plastic square plates [4]. The saturated impulse refers to the critical value after which the final deflection of the structure will not increase with further continuously applied load [2].

By using the approximate yield conditions, the present paper extends the concept of saturated duration to analyze the dynamic plastic response of both simply supported and fully clamped rigid-perfectly plastic rectangular plates subjected to rectangular pressure pulses in the medium range. The secondary effect of finite-deflections is also taken into account.

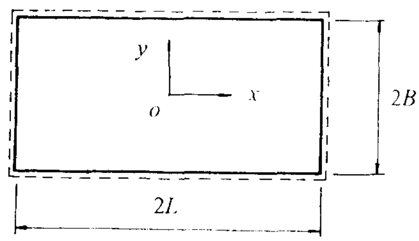

FIG. 1. Simply supported rectangular plate

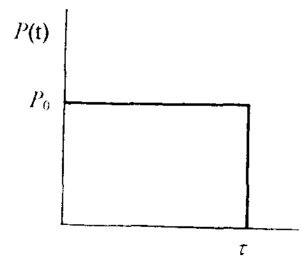

FIG. 2. Rectangular pressure pulse

Simply supported rectangular plates

Consider a simply supported, rigid-perfectly plastic rectangular plate of length $2L$, width $2B$ and thickness H (as shown in FIG. 1), the plate is subjected to rectangular pressure pulse which is sketched in FIG. 2 and which may be expressed in the form

$$p(t) = p_0\left[H(t) - H(t-\tau)\right] \tag{1}$$

where $H(t)$ is the Heaviside function, p_0 the magnitude of the rectangular pressure pulse, and τ the duration of the pulse. If the simplified Tresca yield conditions are used and the effect of finite-deflections is considered, then the dimensionless permanent central deflection of the rectangular plate for medium load is [5]

$$\frac{W_f}{H} = (3-\xi_0)\frac{\sqrt{1+2\eta(\eta-1)(1-\cos\gamma\tau)}-1}{4[1+(1-\xi_0)(2-\xi_0)]} \tag{2}$$

where $\eta = p_0 / p_c$ is the ratio of the magnitude of dynamic pressure pulse to the corresponding static collapse pressure of the simply supported rectangular plate $p_c = \frac{6M_0}{B^2(3-2\xi_0)}$, $M_0 = \frac{\sigma_0 H^2}{4}$ is the fully plastic bending moment of the plate per unit length, σ_0 the yield stress of the plate material, $\beta = B / L$ denotes the aspect ratio of the rectangular plate, other symbols in (2) are defined by

$$\xi_0 = \beta\tan\phi\,, \tan\phi = -\beta + \sqrt{3+\beta^2}\,,$$

$$\gamma^2 = \frac{24M_0}{\mu HB^2(3-2\xi_0)}\left(1-\xi_0+\frac{1}{2-\xi_0}\right) \tag{3}$$

where μ is the mass per unit central plane area of the plate. Medium load refers to $\eta \le 2$. For simply supported square plate, we have

$$\beta = 1\,, \quad \tan\phi = 1, \quad \xi_0 = 1\,, \quad \gamma^2 = \frac{24M_0}{\mu HB^2} \tag{4}$$

From equation (2) we know that the saturated duration of the rectangular pressure pulse is reached when the function $\cos\gamma\tau$ reaches its minimum value, i.e.,

$$\gamma\tau_{\text{sat}} = \pi \tag{5}$$

Substituting (3) into (5), we obtain the saturated duration of the applied rectangular pressure pulse in the medium range applied to rectangular plate as follows

$$\tau_{\text{sat}} = \pi B\sqrt{\frac{\rho}{6\sigma_0}}\sqrt{\frac{(3-2\xi_0)(2-\xi_0)}{1+(1-\xi_0)(2-\xi_0)}} \tag{6}$$

where ρ is the density of the material, the relationship between μ and ρ is

$\mu = \rho H$. For simply supported square plate $\beta = 1$, the saturated duration of the rectangular pressure pulse in the medium range is then reduced to

$$\tau_{sat} = \pi B \sqrt{\frac{\rho}{6\sigma_0}} \tag{7}$$

It should be noted that eqn (7) is the same as that given by [2]. It is well-known that the major response characteristics of a simply supported, rigid-perfectly plastic square plate are the same as those of a simply supported, rigid-perfectly plastic circular plate with a radius which just inscribes the square form [6], then eqn (7) is also the saturated duration of rectangular pressure pulse for the dynamic plastic response of a simply supported circular plate with radius B. Eqn (7) shows that the theoretical value of saturated duration of the rectangular pressure pulse depends on the density, yield stress of the material, and the radius of the circular plate (half width for a rectangular plate). Eqn (7) also shows that the theoretical value of saturated duration of rectangular pressure pulse does not depend on the magnitude of the applied load in the medium range and the thickness of the plate. It is also interesting that simply supported beam is a special case when $\beta = 0$, then from (3) and (6) we can get the saturated duration of rectangular pressure pulse for the dynamic plastic response of a simply supported, rigid-perfectly plastic beam as

$$\tau_{sat} = \pi B \sqrt{\frac{\rho}{3\sigma_0}} \tag{8}$$

where B is the semi-length of the beam.

Both eqn (7) and eqn (8) show that the saturated duration of rectangular pressure pulse in medium range applied to simply supported (or fully clamped) beam, simply supported circular plate, simply supported (or fully clamped) rectangular plate or square plate depends on three factors, the first one is the characteristic dimension of the structure (semi-length of beam, radius of circular plate and semi-width of rectangular plate), the second one is the mass density of the material and the last one is the uniaxial yield stress of the material.

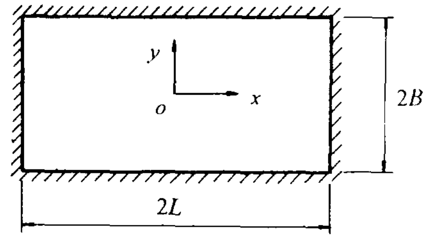

FIG. 3. Clamped rectangular plate

Fully clamped rectangular plates

For a fully clamped, rigid-perfectly plastic rectangular plate of length $2L$, width $2B$ and thickness H which is illustrated in FIG. 3, the plate is subjected to the rectangular pressure pulse in the medium range. If the simplified Tresca yield conditions are used and the finite-deflections effect is taken into account, then the dimensionless permanent central displacement of the rectangular plate is in the form [5]

$$\frac{W_f}{H} = (3-\xi_0)\frac{\sqrt{1+2\eta(\eta-1)(1-\cos\gamma\tau)}-1}{2[1+(1-\xi_0)(2-\xi_0)]} \tag{9}$$

where $\eta = p_0 / p_c$ is also the ratio of the magnitude of dynamic pressure pulse to the corresponding static collapse pressure of the fully clamped rectangular plate $p_c = \dfrac{12M_0}{B^2(3-2\xi_0)}$, other symbols are also denoted by (3). From eqns (3) and (9) we know that the saturated duration of the rectangular pressure pulse in the medium range applied to fully clamped, rigid-perfectly plastic rectangular plate is also denoted by (6). The saturated duration of the rectangular pressure pulse in medium range applied to fully clamped square plate is also given by (7). Fully clamped beam is a special case of a fully clamped rectangular plate when $\beta = 0$, so the saturated duration of the rectangular pressure pulse in medium range applied to fully clamped beam is also denoted by (8), in other words, the saturated duration of the rectangular pressure pulse for fully clamped beam is exactly the same as the simply supported beam.

Conclusions

This paper determines the saturated duration of rectangular pressure pulse in the medium range for the dynamic plastic response of both simply supported and fully clamped rigid-perfectly plastic rectangular plates by using the approximate yield conditions. It should be noted that the rigid-perfectly plastic results for simply supported beam, simply supported square plate and simply supported circular plate are all special cases of those of simply supported rectangular plate subjected to the same rectangular pressure pulses, and the rigid-perfectly plastic results for both fully clamped beam and fully clamped square plate are special cases of those of fully clamped rectangular plate subjected to the same rectangular pressure pulses. Thus the determination of the saturated duration of rectangular pressure pulse for rectangular plate has a more general meaning. The relationship among the saturated duration of rectangular plate, simply supported or fully clamped beam, square plate, simply supported circular plate as well as annular plate is illustrated in FIG. 4. The following conclusions are illustrated by Fig. 4: (1) beam is a special case of rectangular plate [7]; (2) square plate is a special case of rectangular plate; (3) dynamic plastic characteristics of square plate are the same as those of circular plate with a radius just inscribes the square form [6]; (4) the results of annular plate can be easily obtained from those of circular plate. So, the rectangular plate is the most general case among these kinds of structures.

The values of saturated duration of rectangular pressure pulse in medium range for both simply supported and fully clamped rigid-perfectly plastic beams are the same, and those of saturated duration of rectangular pulse in medium range for both simply supported and fully clamped rigid-perfectly plastic rectangular plates are the same, this is also true for the case of square plate.

It is shown that the saturated duration of rectangular pressure pulse in medium range applied to simply supported (or fully clamped) beam, simply supported circular plate, simply supported (or fully clamped) rectangular plate or square plate depends on three factors, the first one is the characteristic dimension of the structure (semi-length of beam, radius of circular plate and semi-width of rectangular plate), the second one is the mass density of the material and the last one is the uniaxial yield stress of the material. The theoretical value of the saturated duration of the rectangular pressure pulse in the medium range does not depend on the thickness of the structure and the magnitude of the pulse.

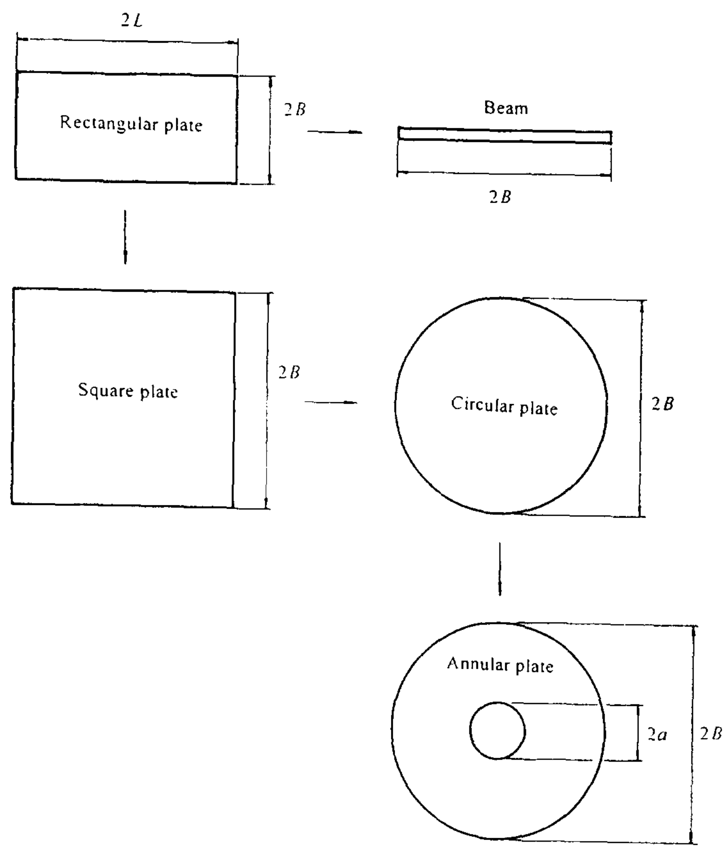

FIG. 4. Relationship of the saturated duration among several kinds of structures

From eqn (7) and eqn (8), we know that the value $B\sqrt{\rho/\sigma_0}$, which has the dimension of time, is expected to become a new important characteristic parameter of the dynamic plastic response of structures under rectangular pressure pulse, and the ratio $\tau/B\sqrt{\rho/\sigma_0}$ is expected to be an important similarity parameter for dynamic plastic modeling.

Acknowledgments

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