Thermocapillary Convection in an Inhomogeneous Porous Layer *

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(Received 15 August 2007)

A new model consisting of an inhomogeneous porous medium saturated by incompressible fluid is investigated. We focus on the effects of inhomogeneity for the streamline patterns and instabilities of the system. Influences of the 'mean porosity' and gradient of distributions of porosity are also emphasized. The results cannot be obtained by studying the media with constant porosity as carried out by other researchers, and have not been discussed before.

PACS: 47.55.Mh, 47.27.Te, 47.20.Dr

Thermal convection of fluids in porous media has been extensively investigated, mainly because of its engineering and technological importance. The onset of cellular convection in a porous layer, free at the upper surface without any deformation and heated from bottom, is attributed to the combined effects of buoyancy and surface tension.[1] The classical Rayleigh-Bénard instability problem[2,3] for a single free fluid has its equivalence for a porous media. It is the Horton–Rogers–Lapwood problem.[4,5] On the other hand, since the pioneering work of Pearson's[6] on a pure liquid with a free surface heated from the opposite boundary, when buoyancy can be left aside, surface tension gradient could also lead to convective instabilities. In 1997, Hennenberg *et al*[7] studied the Bénard–Marangoni problems in a porous layer with Brinkman's model,[8] and showed the instability curves for the onset of convective motion, i.e., the Marangoni number against wavenumber curves. In addition, many studies investigated the onset of convection in a superposed fluid film overlying a homogeneous porous layer, except the inhomogeneous ones.[9−14] They usually used Brinkman's model or Darcy's law to describe the porous medium and Navier–Stokes equations for the liquid film. They have studied the coupled effects of the convection in these two layers, but the instability problem in porous media is still basic and crucial, and this is one of the motives in the present study.

Porous media in common nature environment does not have the constant porosity usually. Therefore, it is of vital importance to study the fluid convective instability problems in inhomogeneous porous media. In this Letter, we investigate the Marangoni–Bénard problems in a vertically inhomogeneous porous layer, as schematically shown in Fig. 1.

Cartesian coordinates are used with the origin at the ground boundary with the z axis vertically upward. The variation of porosity against its depth is a function of the distance away from the top surface. In the present study, we choose the linear function and the trigonometric function, respectively. The linear variation of porosity is the function $\varphi(z) = \varphi_0[\frac{\varphi_z}{\varphi_0}(z-1)+1]$, and the trigonometric one is $\varphi(z) = \varphi_0[\frac{\varphi_z}{\pi\varphi_0}\sin(\pi(z-1))+1]$, where φ_z is the porosity variation coefficient at the top free surface and φ_0 the porosity at the upper surface which we set to be 0.7.

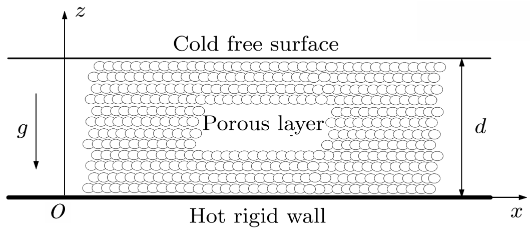

Fig. 1. Geometrical configuration.

The system is heated from bottom, and assumed to be infinite in the horizontal direction. The upper surface of the system is free without any deformation. The vertical component of velocity there equals to zero. The bottom wall is considered as rigid and perfectly heat conduction boundary. The surface tension at the upper surface is considered to be a linear function of temperature:

$$\sigma = \sigma_0 - \sigma_T(T - T_0), \tag{1}$$

where σ_0 is the surface tension of the fluid at the reference temperature T_0 and the constant rate of change of surface tension with temperature, σ_T, is supposed to be positive. We can defined a mean surface tension σ_m:[7]

$$\sigma_m = \frac{\int \sigma_l dS_l + \int \sigma_s dS_s}{\int dS_l + \int dS_s}. \tag{2}$$

For an isotropic medium, we obtain

$$\sigma_m = \sigma_l\varphi + \sigma_s(1-\varphi). \tag{3}$$

*Supported by the National Natural Science Foundation of China under Grant Nos 10772185, 10372105 and 10432060, and the Knowledge Innovation Programme of Chinese Academy of Sciences (KGCX-SW-409).
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For any physical property one has $(\ )_m = (\ )_l\varphi + (\ )_s(1-\varphi)$, where the subscript m denotes a mean property, s a property of the solid matrix in the porous media and l a property of the liquid. We use the Brinkman–Forchheimer equation[15] with Boussinesq approximation, i.e., only the density of fluid ρ_l is dependent on the temperature:

$$\rho_l = \rho_0[1-\alpha(T-T_0)]. \tag{4}$$

For a reference motionless state (the subscript r denotes a reference property)

$$T_r = -\frac{\Delta T}{\int_0^d {k_m}^{-1} dz}\int_0^z \frac{1}{k_m}dz + T_0,$$

where T_r is the reference temperature, T_0 the basic temperature of ground rigid wall.

The Brinkman–Forchheimer equation is

$$\rho_0\Big[\frac{1}{\varphi}\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{\varphi}(\boldsymbol{v}\cdot\nabla)\Big(\frac{\boldsymbol{v}}{\varphi}\Big)\Big] = -\frac{1}{\varphi}\nabla(\varphi p) + \mu_e\nabla^2\boldsymbol{v} - \frac{\mu_l}{K}\boldsymbol{v} - \rho_0 g[1-\alpha(T-T_0)]\boldsymbol{e}_z. \tag{5}$$

The energy equation is

$$(\rho c)_m\frac{\partial T}{\partial t} + \rho_0 c_l(\boldsymbol{v}\cdot\nabla)T = \nabla(k_m\nabla T). \tag{6}$$

The continuity equation is

$$\nabla\cdot\boldsymbol{v} = 0. \tag{7}$$

For convenience, the variables will be expressed in dimensionless forms. The length is scaled by the total thickness d, the temperature by ΔT, the time by d^2/κ_l, and the velocity by κ_l/d where $\kappa_l = k_l/(\rho_0 c_l)$ is the thermal diffusivity. In order to obtain the perturbation equations, we introduce perturbations of velocities, pressure and temperature: $\boldsymbol{v} = \boldsymbol{v}_r + \boldsymbol{v}'$, $p = p_r + p'$, $T = T_r + T'$, into the above three equations. After eliminating the pressure and the horizontal components of velocities, the dimensionless equations for the linearized perturbed variables take the simplified expressions and involve only two unknown quantities, namely, the vertical component of the velocity and the corresponding temperature field.

According to the normal mode technique,[16] we seek solutions for the vertical velocity component and temperature in the form

$$\begin{pmatrix} w \\ T \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \end{pmatrix}\exp[\lambda t + iax]. \tag{8}$$

The amplitudes $W(z)$ and $\Theta(z)$ describe the variation with respect to z of the vertical velocity and the temperature, and a is the dimensionless wave number in the x direction. Finally, λ is the complex growth rate of the disturbance. Then, we obtain

$$\begin{aligned}\Lambda\Big[&(D^2-a^2)^2 + \frac{1}{\varphi}\frac{d\varphi}{dz}D(D^2-a^2)\Big]W \\ &- Da^{-1}f_1(z)[(D^2-a^2) + \frac{1}{\varphi}\frac{d\varphi}{dz}D]W, \\ &+ Da^{-1}f_1(z)f_2(z)DW - a^2Ra\Theta \\ &= \lambda\varphi^{-1}\mathrm{Pr}^{-1}(D^2-a^2)W, \end{aligned} \tag{9}$$

$$\begin{aligned}&-Sf_3(z)f_4(z)W + Sf_3(z)X^{-1}f_5(z)(D^2-a^2)\Theta \\ &+ f_6(z)D\Theta = \lambda\Theta, \end{aligned} \tag{10}$$

where D stands for d/dz. The dimensionless parameters are defined as follows:

$$\begin{aligned}&\mathrm{Pr} = \nu_l/\kappa_l, \quad \Lambda = \mu_e/\mu_l, \quad Da = K(\varphi_0)/d^2, \\ &\mathrm{Ra} = \alpha g\Delta T d^3/\nu_l\kappa_l, \\ &S = \rho_0 c_l/[(\rho_0 c_l)\varphi_0 + (\rho_s c_s)(1-\varphi_0)], \\ &X = k_l/[k_l\varphi_0 + k_s(1-\varphi_0)],\end{aligned}$$

where $K(\varphi_0)$ is the permeability with porosity equal to φ_0. For pure thermocapillary convection discussed in here, we set $Ra = 0$. The functions $f_1(z)$ to $f_6(z)$ are defined as

$$\begin{aligned}f_1(z) &= K[\varphi(z)]^{-1}\cdot K(\varphi_0), \\ f_2(z) &= \frac{1}{K(\varphi(z))}\frac{dK(\varphi(z))}{dz}, \\ f_3(z) &= \frac{(\rho_0 c_l)\varphi_0 + (\rho_s c_s)(1-\varphi_0)}{(\rho_0 c_l)\varphi(z) + (\rho_s c_s)[1-\varphi(z)]}, \\ f_4(z) &= -d\Big\{[k_l\varphi(z) + k_s(1-\varphi(z))] \\ &\quad \cdot\int_0^d \frac{dz}{[k_l\varphi(z) + k_s(1-\varphi(z))]}\Big\}^{-1}, \\ f_5(z) &= \frac{k_l\varphi(z) + k_s[1-\varphi(z)]}{k_l\varphi_0 + k_s[1-\varphi_0]}, \\ f_6(z) &= \frac{k_l - k_s}{\kappa_l\cdot[(\rho_0 c_l)\varphi(z) + (\rho_s c_s)(1-\varphi(z))]}\frac{d\varphi(z)}{dz}.\end{aligned}$$

The above two equations give rise to a sixth order system with the following associated boundary conditions.

At $z = 0$, one has

$$W = 0, \quad \Theta = 0, \quad DW = 0. \tag{11}$$

At $z = 1$, one has

$$W = 0, \quad D\Theta + Bi\Theta = 0, \quad D^2W + a^2Ma\Theta = 0, \tag{12}$$

where Bi is the Biot number, and we set it to be zero in this study. $Ma = -\frac{\partial\sigma_{m0}}{\partial T}d\Delta T/\mu_e\kappa_l$ is the Marangoni number in the porous media,[7] where $\sigma_{m0} = \sigma_l\varphi_0 + \sigma_s(1-\varphi_0)$.

The calculations are carried out using the physical properties of water at temperature $T = 297.15\,\mathrm{K}$. The Prandtl number of the fluid is 6.34. The solid matrix consists of glass beads with a nominal diameter of 3 mm.[18] The permeability $K[\varphi(z)]$ of such a porous medium is obtained using the Kozeny–Carman relation:[17]

$$K[\varphi(z)] = \frac{d_1^2}{172.8}\frac{\varphi(z)^3}{[1-\varphi(z)]^2}, \qquad (13)$$

where d_1 is the diameter of the glass beads and φ the porosity. The depth of the porous layer is 4 cm. The Brinkman–Forchheimer model rests on an effective viscosity μ_e denoted Λ in dimensionless form. We take $\Lambda = 1.0$ in the following numerical computation.

Table 1. Physical properties of water and glass.

$\rho_0 = 0.997 \times 10^3\,\mathrm{kg/m^3}$	$\rho_s = 2.5 \times 10^3\,\mathrm{kg/m^3}$
$c_l = 4.16 \times 10^3\,\mathrm{J/kg{\cdot}K}$	$c_s = 0.84 \times 10^3\,\mathrm{J/kg{\cdot}K}$
$\kappa_l = 0.145 \times 10^{-6}\,\mathrm{m^2/s}$	$\kappa_s = 0.201 \times 10^{-6}\,\mathrm{m^2/s}$
$\nu_l = 0.919 \times 10^{-6}\,\mathrm{m^2/s}$	

The linear equations (9) and (10) together with their boundary conditions (11) and (12) are discretized using the spectral method (Tau–Chebyshev)[19] and then are resolved as the general eigenvalue problem. This method has been verified by analysing the Marangoni–Bénard instability in two-layer systems.[20,21] The complex time growth rate λ is computed in complex double precision. The computational solutions has also been verified in comparison with the analytical solutions.[6]

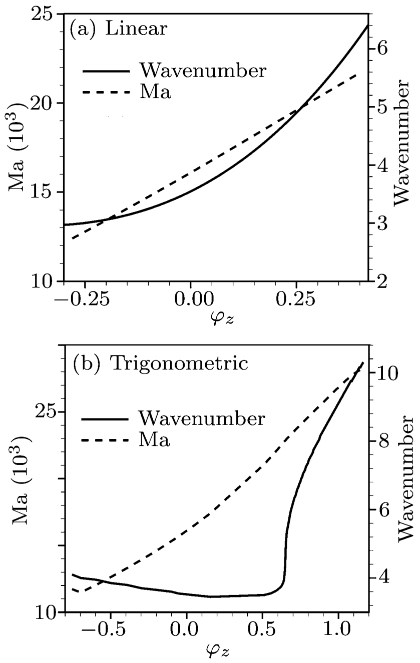

Fig. 2. Critical wavenumber and Marangoni number vs φ_z in the media where the distributions of porosity is linear (a) and trigonometric (b).

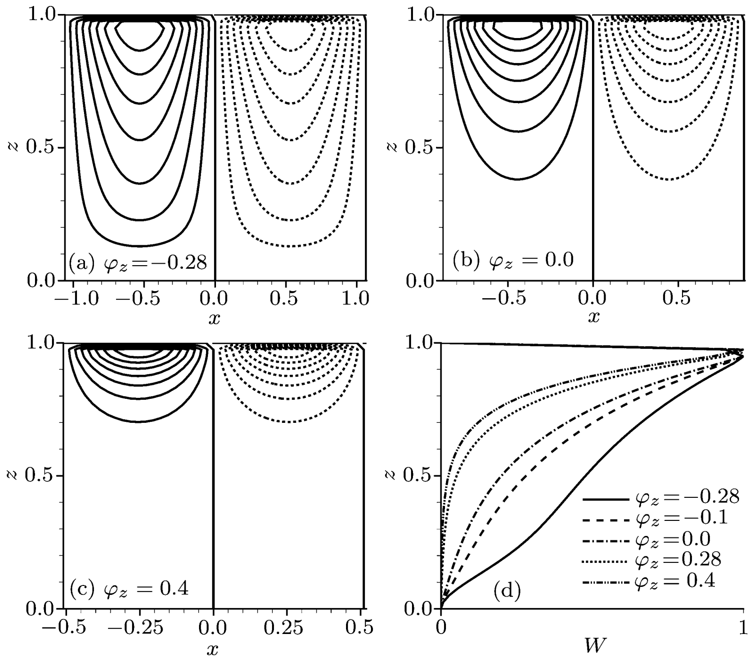

Fig. 3. Streamline patterns [(a), (b), (c)] and W distribution of pure Marangoni convection (d) in the media with linearly distributed porosity.

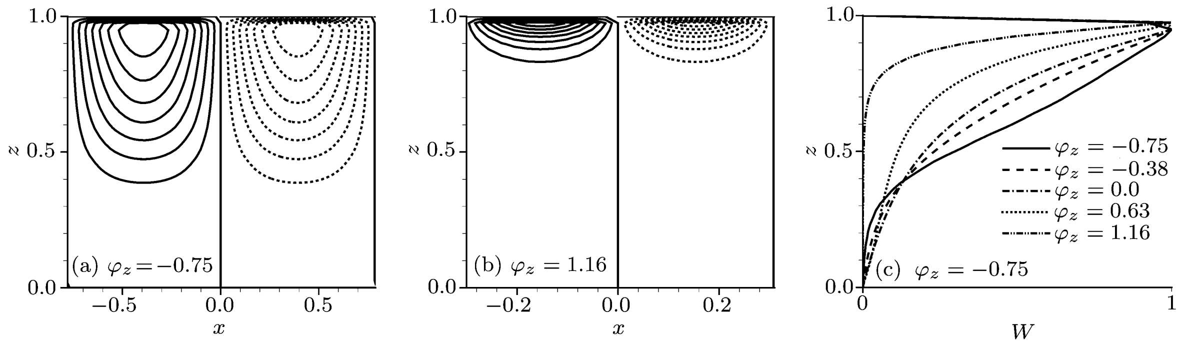

Fig. 4. Streamline patterns [(a) and (b)] and W distribution of pure Marangoni convection (c) in the media with trigonometrically distributed porosity.

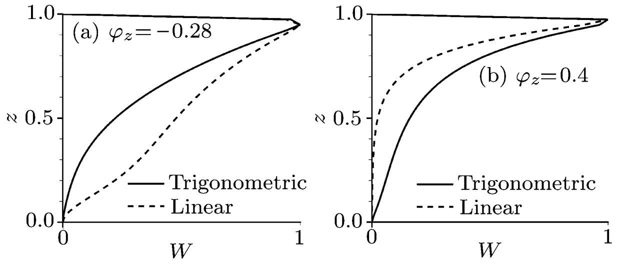

Fig. 5. Comparison of W distributions.

From Fig. 2, we find that the critical Marangoni number grows as φ_z is added up, in the media with linearly or trigonometrically distributed porosity. It can be explained as follows. If φ_z is positive, the porosity at bottom or central part is less than that on top. Then, the mean porosity of the layer is less than the one with constant porosity φ_0. From both the numerical results and physical interpretations, the less the porosity is, the more difficult the fluid is to flow, then the system is more stable, and vice versa. The curves of critical wavenumber against φ_z are also shown in Fig. 2. In the media with linearly distributed porosity, the corresponding curve is monotonic. While in the media where the porosity is trigonometrically distributed, the curve decreases slowly at first, then increases rapidly when φ_z goes through 0.5. This means that the wavenumber becomes rather sensitive to φ_z when $\varphi_z > 0.5$.

Streamline patterns and distributions of $W(z)$ of the convection in the media with linearly distributed porosity are shown in Fig. 3, and those in the media where the porosity is trigonometrically distributed are shown in Fig. 4. No matter what kind of porosity of the media is, the centre of each cell is always in the vicinity of the top surface. All the cells penetrate downwards from top for any case. This is caused by the mechanism of Marangoni instability, i.e. only when the fluctuations of surface tension can be sustained under the sufficient temperature gradient, the convection occurs.

The cells in negative φ_z case penetrate more deeply than they do when φ_z is positive. For the negative φ_z, the larger the increase of the porosity in the direction towards the bottom, the more deeply the convective cells penetrate downwards. Different from the Marangoni convection in pure liquid layer, the vortex centre in porous layer is near the top surface ($z = 1.0$). The growth rate of velocity is extremely rapid from zero at the upper surface to the maximum. It is caused by the additional viscous drag of the solid matrix which strengthens the dissipative effect of the system. When the liquid start to move from the top surface, its kinetic energy is much more easily to be dissipated by the solid matrix than that by its own viscosity only, and then it is more difficult to move downwards. The less the porosity is, the more drag the solid matrix can offer, then the more rapidly the kinetic energy of particles is to be dissipated, i.e., the more rapidly their velocity decreases, and finally the further the centre of cell departs from the centre line and the closer it is to the top. The comparisons of the distributions of $W(z)$ with the same φ_z are shown in Fig. 5. The penetrability of the cells can also be seen from this figure.

In summary, from the results described above, the porosity variation coefficient, i.e. φ_z, and the distributions of $\varphi(z)$ do have significant influence on the system. The influence is not only on the instability of system, but also on the streamline patterns and the distributions of $W(z)$ as well. The system which is denoted by the critical Marangoni number becomes more stable with the increase of φ_z, and the variation characteristic of the corresponding critical wavenumber is somewhat complicated. When φ_z is negative, the cells penetrate more deeply; when positive, the cells appear only in the vicinity of the top surface of the media. The results indicate that the stability of the system depends on the mean porosity of the medium, while the critical wavenumber and the penetration of the cells depend on the distribution characteristics of the porosity.

In the present study, we only focus on the surface-tension-driven instability of a liquid layer in inhomogeneous porous media. The influence of anisotropy of the media and coupled effect of Rayleigh and Marangoni instabilities will be studied in the near future.

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