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Effects of the Spanwise Characteristic Length on the Transition Feature in the Wake of a Cylinder

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Abstract: The transition features of the wake behind a uniform circular cylinder at $Re=200$, which is just beyond the critical Reynolds number of 3-D transition, are investigated in detail by direct numerical simulations of 3-D incompressible Navier-Stokes equations. The spanwise characteristic length determines the transition features and global properties of the wake.

Key words: cylinder; spanwise characteristic length; wake; transition

展向特征长度对圆柱尾迹转捩特征的影响. 熊俊, 凌国灿, 朱克勤. 中国航空学报(英文版), 2003, 16(2): 65—68.

摘　要:通过对三维不可压缩 N-S 方程的直接数值模拟详细研究了 $Re=200$,即刚超过三维转捩的临界雷诺数时的圆柱尾迹。圆柱的展向特征长度决定了尾迹的转捩特征和整体性质。

关键词:圆柱; 展向特征长度; 尾迹; 转捩

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Bluff body wakes are one of the focuses of the investigations in aeronautics. Among the different types of bluff bodies, the circular cylinder receives a great deal of attention. The 3-D transition of the wake in the regime $190 \leqslant Re \leqslant 260$ has been extensively investigated by numerical studies[1,2]. However, because of adopting different spanwise characteristic lengths L, their numerical simulations have revealed the different patterns associated with the spanwise modes in the wake and led to completely different conclusions on the route that the wake follows to the turbulence. Thus, studies on how the transition features and the global characteristics of the wake change with L are significant. The purpose of the present work is to provide detailed results about the effect of L on the supercritical 3-D transition of the wake at Reynolds number of 200, which is just beyond the onset of the transition.

1 Numerical Method

The right-handed Cartesian coordinate system is established, in which the positive direction of the x axis is the direction of incoming flow and the z axis is the axis of the cylinder. The 3-D N-S and the continuity equations governing the incompressible viscous flow are taken as follows, where all variables are normalized by the diameter of the cylinder D and the uniform stream velocity U:

$$\partial \boldsymbol{V}/\partial t = -\nabla p + \nabla^2 \boldsymbol{V}/Re + \boldsymbol{N}(\boldsymbol{V}) \tag{1}$$

$$\nabla \cdot \boldsymbol{V} = 0 \tag{2}$$

where the Reynolds number is defined as $Re = UD/\upsilon$, in which υ is the kinematic viscosity, and the nonlinear convection operator $\boldsymbol{N}(\boldsymbol{V}) = -\boldsymbol{V} \cdot \nabla \boldsymbol{V}$.

The time discretization of Eq. (1) employs a high-order splitting algorithm based on the mixed stiffly stable scheme[3]. A mixed Fourier-spectral spectral-element method is employed in the spatial

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discretization, *i.e.* the Fourier spectral method in the spanwise direction and the spectral element method in the x-y plane[1,4,5].

2 Numerical Results

The preliminary simulations in this paper employ a relatively small x-y computational domain illustrated in Fig. 1 because of the restriction of the available computer hardware. The conformity of the present results to the previous researches[2, 6] justifies their validity. The selected values of L/D and the number of Fourier modes M are listed in Table 1. For $Re=200$, the wavelength range of instability mode A band is (3.25D, 5.13D) according to the Floquet stability analysis[7]. Therefore, L/D is chosen as $4k$, where k is a positive integer, to ensure that mode A is included. The time step $\Delta t=0.01$. At $t=0$, the initial disturbance along the z direction, which is in a triangular-shaped distribution with a maximum amplitude of 0.1, is added to the streamwise velocity u in the domain $x<0$. The introduction of the initial disturbance with a finite amplitude can shorten the time needed to reach the inherent asymptotic state of the flow a great deal.

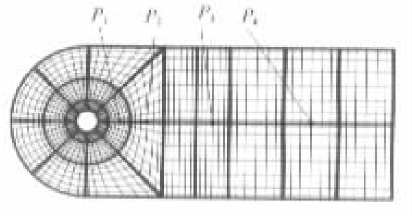


Fig. 1 The mesh in the x-y plane (Points $P_1 \sim P_4$ indicate the positions where the time series are extracted)

Table 1 Selected cases of L/D and corresponding M, the permitted discrete modes in mode A band and dominant modes.

Spanwise characteristic length	Number of Fourier modes	Permitted wave length in mode A band	Dominant mode	
			Wave length	Whether in mode A band
4	16	4	4	Yes
8	32	4	4	Yes
12	32	4	6	No
24	64	4.8,4,3.43	24	No

First, examine the temporal development of the flow by means of the time series of their kinetic energy

$$E_k(t)=(|u_k|^2+|v_k|^2+|w_k|^2)/2,\quad k=0,1,\cdots,M/2-1 \tag{3}$$

at specific (x, y) points. The $E_k(t)$ curves of typical modes at the point P_1 for $L/D=12$ are shown in Fig. 2. From the phenomenological point of view, the temporal evolution behavior of different modes can be roughly divided into two categories for the long wave and the short wave. The amplitudes of the former continuously vary in the whole time interval of simulation, such as mode $k=2$ for $L/D=12$. On the contrary, the amplitudes of the latter present marked intermittent feature, such as mode $k=13$ for $L/D=12$.

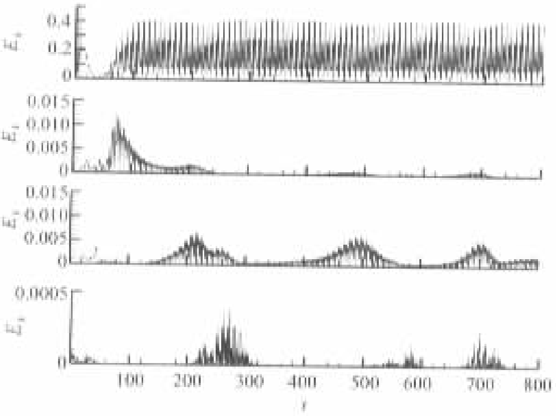
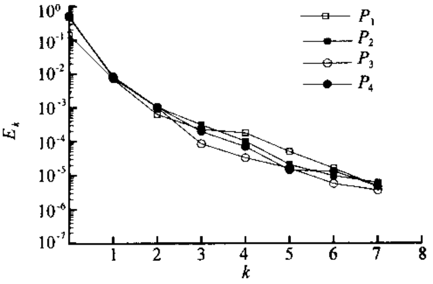


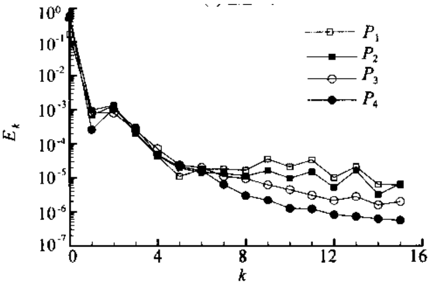
Fig. 2 $E_k(t)$ of the modes $k=0,1,2$ and 13 (from the top to down), at point P_1 for $L/D=12$

According to the linear stability theory, the linear unstable modes are induced directly by small disturbance at the early stage of the evolving process, while other linear stable modes emerge through their nonlinear interactions with the developed unstable modes. However, in the present work, it is found that peaks of marked amplitude appear in the early stage of $E_k(t)$ curves for more modes, such as mode $k=1$ for $L/D=12$ in Fig. 2, other than those linear unstable modes in mode A band. This indicates that some modes, which are predicted by linear analysis to be stable, are directly excited in a short time period from starting.

Although all the spanwise modes involved in the present computation emerge, the amplitudes of their kinetic energy differ a lot in magnitude. The dominant spanwise mode can be determined by the mean energy averaging in its equilibrium interval, which is shown in Fig. 3. The dominant modes for various L/D are listed in Table 1. It is worth noting that for $L/D=12$, mode $k=3$ is the sole linear unstable mode, but its mean kinetic energy is less than that of mode $k=2$. So in this situation the dominant mode is not mode A but the mode $\lambda=6D$. Similar case that the linear stable mode becomes the dominant mode in the wake, overwhelming mode A, can be found when $L/D=24$. The contours of the streamwise vorticity shown in Fig. 4 confirm that the dominant modes in the flow field for $L/D=4$ and 12 are modes $\lambda=4D$ and $\lambda=6D$, respectively.



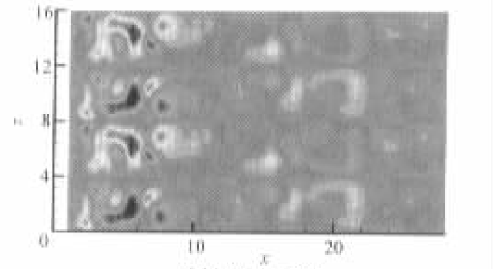
(a) L/D=4



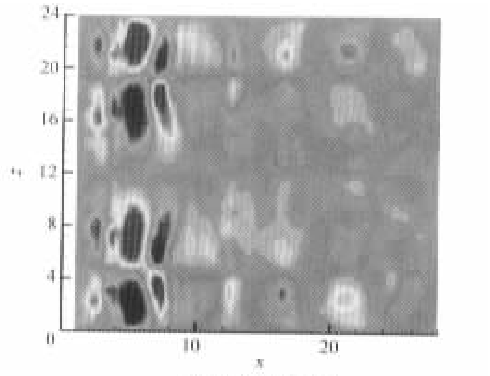
(b) L/D= 12

Fig. 3 Mean kinetic energy in the equilibrium interval at four points shown in Fig. 1

The distributions of time mean values of streamwise velocity u, denoted as $U_{\rm m}$, at several sections for $L/D = 4$ and 12 are calculated. The results are illustrated in Fig. 5. The comparison between the profiles for $L/D=4$ and 12 indicates that the irregularity of the velocity along the transverse direction increases along with the increase of L/D and decays along with the development of the flow downstream.

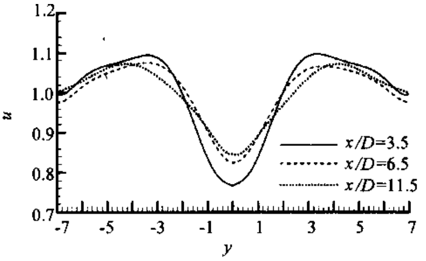


(a) L/D=4, t=200

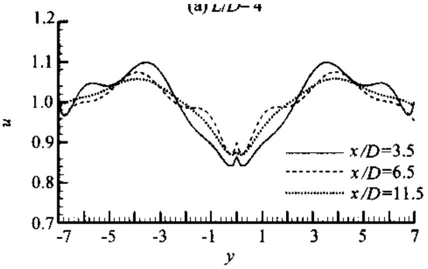


(b) L/D=12, t=200

Fig. 4 Contours of streamwise vorticity at $y=0$



(a) L/D= 4



(b) L/D=12

Fig. 5 Profiles of mean velocities at sections $x=$ 7.0, 13.0 and 23.0

3 Conclusions

Nonlinear features of the supercritical transition in the wake of a circular cylinder, especially the effect of the spanwise characteristic length, are investigated through direct numerical simulations. The flow is essentially in a 3-D quasi-periodic transitional laminar state. It has shown the irregularity in the span and transverse directions, which is suppressed by the viscosity and decays downstream.

The spanwise characteristic length has great effect on the global properties of the flow field. The dominant mode of the flow varies with the spanwise characteristic length. For some L/D, the specific mode predicted to be stable by the linear stability theory could become dominant mode in addition to the linearly instable mode. The increase of L/D can also result in the increase of the damping rate of large wave number modes downstream.

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