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MATHEMATICAL MODELING OF TWO FINNISH LAKES

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A simulation model (FINNECO) was developed and applied to two Finnish lakes. The objective was to formulate a model that would be suitable for Finnish conditions. The model is well documented. It is one-dimensional, has numerous state variables, includes the effect of wind in mixing water and simulates the freeze-up and break-up of ice. The model was applied to Lake Pyhäjärvi and to the northern sub-basin of Lake Päijänne with the data sets of two and four years, respectively. A time scale of eight hours was used. In Lake Pyhäjärvi, which has an occasionally anoxic hypolimnion, the model could only be calibrated but not verified. However in Lake Päijänne the model simulated rather well temperature, dissolved oxygen, nutrients and the level of phytoplankton biomass and it could be considered as both calibrated and verified. Despite some limitations of the model it appeared to describe reasonably well the real lake ecosystem and should be suitable for lakes in which the calculation of vertical water quality is important and which do not have extremely poor water quality e.g. anoxic hypolimnion.

Index words: Mathematical models, water quality models, ecological models, simulation models, water quality prediction, FINNECO model.

1. INTRODUCTION

Mathematical models have been developed and applied to Finnish water bodies in order to determine their suitability for solving water pollution problems. Water quality models have traditionally been developed for making a synthesis of available information on aquatic ecosystems in order to obtain better understanding of their basic mechanisms and for evaluating the effects of various measures, e.g. loading alternatives, on water quality. In particular, simulation models have been used for these purposes as they offer a rational method for taking into account the numerous reactions taking place in natural water bodies. Although simulation models typically include many state variables and a great number of mathematical equations they are nevertheless major simplifications of real ecosystems. Models must therefore be vigorously tested in various types of lakes to determine their suitability for the purposes for which they are constructed.

A new model entitled the FINNECO-model was developed. The objective of the development work was to formulate a suitable model for Finnish climatic conditions, including winter time. The model produced is relatively complex. It includes eighteen state variables, takes into account the effect of wind in mixing water and simulates the freeze-up and break-up of ice on the basis of energy balance. The model was applied to two Finnish lakes. For one of the lakes it could be only calibrated but for the other it could be both calibrated and verified. General conclusions concerning the suitability of the model are difficult to make solely on the basis of only two applications. However, despite some limitations of the model such as large data requirements and sensitivity to the values of various parameters it appears to be suitable for lakes which do not have extremely poor water quality, e.g. anoxic conditions.

2. THE FINNECO MODEL

The FINNECO model was developed on the basis of the EPAECO model (Gaume and Duke, 1975). The EPAECO model was earlier applied to a Finnish lake with promising results (Niemi, 1978). However, the model could not simulate the ice-covered season and was therefore developed further. The development work was carried out in a co-operative research project conducted by the National Board of Waters and Oy International Business Machines Ab, Finland. In developing the model about 300 computer runs were made, of which about 30 were with the final version. The detailed structure of the model has been presented by Kinnunen et al. (1982) and the users' manual by Kauranne (1983). Only a brief description of the model is therefore given here. However, as the mechanisms of the wind mixing effect and ice cover are typical of this model they are discussed to some extent. The computer program of the model is available from the author.

The FINNECO is a deterministic one-dimensional model developed for lakes and reservoirs. The main differences between the FINNECO model and the EPAECO model from which it was derived are the possibilities of simulating ice cover and the effect of wind, modified nitrogen and phosphorus cycles, a new temperature correction function and the possibility of simulating ten phytoplankton groups instead of two in the original model. Some state variables of the original model, such as fish and benthic animals, were omitted while a new one, sodium lignosulphonate, was added. This state variable is an indicator of pulp and paper mill effluents. The state variables of the model are:

- Temperature
- Dissolved oxygen
- Carbonaceous biological oxygen demand
- Alkalinity
- pH
- Total inorganic carbon
- Carbon dioxide
- Ammonia nitrogen
- Nitrate nitrogen
- Nitrite nitrogen
- Phosphate phosphorus
- Phosphorus in inorganic sediment
- Hygienic indicator
- Sodium lignosulphonate
- Phytoplankton (10 groups)
- Zooplankton
- Total dissolved solids
- Organic sediment
- Detritus

The driving variables of the model include meteorological data and data of the quantity and quality of inflowing waters such as tributaries, diffuse loading, precipitation and wastewater treatment plants. The length of the time step in simulations was eight hours.

In the absence of wind the mixing effect of water is based solely on the density gradient, as in the original model. When there is wind, however, the present model simulates turbulence in the upper layers due to wind by means of a separate routine based on simple energy considerations. Starting from a measured wind speed at a certain height above the ground the module calculates, the Froude number and wind stress coefficient and then shear stress on the basis of wind stress coefficient, air density and wind speed.

The area over which the shear stress is active is only a fraction of the lake surface area and equals the cross-sectional area of the water body at mixing depth. This implies that in the shallow fringe area of the lake, where the thickness of the wind-mixed layer exceeds water depth, the surplus turbulent energy is dissipated by the bottom stress. Horizontal energy transfer is thus ignored. From a computational point of view this means that mixing depth can be determined by considering a vertical column of water with unit cross-sectional area, the unit area being the area over which the shear stress is active. Multiplication by the friction velocity gives the amount of work acting on the water column. The program proceeds from the top downwards, taking one element into consideration at a time and comparing the turbulent energy of the wind with the change in potential energy required for mixing the water from the surface down to the element under consideration. The lowest element included in the mixing process leaves a surplus energy, which is not sufficient to mix the next deeper element completely. A fraction of the lower element, calculated from a second degree equation, is however mixed together with the upper elements. The processes are performed in the program in the following order: (i) mixing caused by negative density gradients (ii) wind mixing (iii) updating of the heat balance and solving of the differential equations for temperature.

The ice cover on a lake inhibits evaporation, prevents the exchange of gases between atmosphere and water and reduces the intensity of light in the water. These effects must be included in any model which attempts to simulate the ice-covered season. Typically the freezing mechanism begins when the heat flux across the air-water interface becomes negative. In this model ice formation takes place when the temperature of the lake reaches 0°C and the net heat loss reaches a value equivalent to the heat of fusion of a 2 cm thickness of ice. The model includes a facility for reading a different set of values for diffusion parameters for the winter as input data. These parameters become active when the water temperature at all depths falls below 4°C. Once ice formation has begun, ice thickness increases at a rate which balances the heat of fusion with the total heat loss to the atmosphere. Typically in existing models simulating ice cover, the longwave back-radiation is calculated using the same linearized form of the Stefan-Boltzmann equation as used under summer conditions. The values of the coefficients of this equation correspond to radiation from a perfect black body, which may be reasonably correct for a lake in the summer time. In winter, however, this assumption may lead to a too rapid increase in ice thickness, particularly in Finnish conditions, since the ice is typically covered by snow. Test runs indicate that the longwave back-radiation should be calculated with coefficients reduced to about 85% of the corresponding values for summer conditions. In the present model the thickness of snow cover was given as input data, based on historical snow thickness data. Separate values of heat conductivity for ice and snow were used.

The heat content of the water body affects the date when ice formation begins. Measurements indicate that the heat content of a water body increases during the early spring when there is still an ice cover, which means that there is a flow of heat from the bottom and shore areas of the lake into the water. This heat flux, due to e.g. geothermal heat and groundwater seepage, was not treated in the FINNECO model.

3. CASE STUDY LAKES

Lake Pyhäjärvi is situated near the city of Tampere in southern Finland. The detention time of the lake is relatively short and it resembles to some extent a slowly flowing river. Both municipal and industrial waste waters are discharged to the lake. The waters discharged to the lake are influenced by wastes of the pulp and paper industry. Because of heavy nutrient inputs the water quality of Lake Pyhäjärvi is poor, although during recent years slight improvement has been observed. In particular the concentrations of coliform bacteria and nutrients in epilimnion have decreased. A similar decreasing trend was not observed during the same period in the hypolimnion, because this layer is at times anoxic and nutrients are released from sediments to the overlaying water. The surface area, volume and average depth of the case study lake are 22.1 km^2, 192.1 x 10^6 m^3 and 8.7 m, respectively. The maximum depth is 46 m and the detention time 27 days. Taking into consideration the morphological characteristics and water quality of this lake it was evident that it was a difficult case study area for any model. However, serious water pollution problems exist in this area and a suitable water quality model would perhaps be valuable in solving these problems. It was therefore decided to test whether the model would be applicable to this kind of lake.

Lake Päijänne, situated near the city of Jyväskylä in southern Finland, is one of the most thoroughly investigated of Finnish lakes. A relatively good set of historical data of the water quality of this lake therefore exists. Water authorities commenced investigation of Lake Päijänne in the beginning of the nineteen-sixties. Material balances of the lake have been calculated and the lake was included in the eutrophication research conducted by the OECD (OECD, 1982). The protection of Lake Päijänne is particularly important because the city of Helsinki and its neighbouring municipalities utilize water from this lake. The lake has been regulated since 1968.

The model was applied only to the northern sub-basin of the lake. The area of this basin is 142 km^2, volume 2.53 km^3, average depth 18 m, theoretical detention time 140 days, length 28 km and breadth 16 km. The sub-basin is polluted by waste waters of the wood processing industry and municipalities. The northern part of Lake Päijänne can be considered as eutrophic, although e.g. dissolved oxygen saturation values are only slightly decreased.

4. DATA

Data of the water quality of lakes were mainly obtained from the water quality data bank of the National Board of Waters. Data of the quality and quantity of tributaries, diffuse loadings and of wastewater treatment plants were obtained from the data banks of water quality and hydrology, from special investigations carried out in the case study areas and from the regular reports of the wastewater treatment plants. Data on the quantity of precipitation and other meteorological data was obtained from the publications of the Finnish Meteorological Institute. Special investigations on the quality of precipitation were used for estimating the quality of rainwater. Because the time step used in the simulations was short, interpolation and extrapolation of observations was necessary for producing the data set required by the model. The quantity and quality of data for Lake Päijänne was better than that for Lake Pyhäjärvi. For Lake Pyhäjärvi only simulations of the open water period could be carried out due to the lack of data.

5. RESULTS

Calibration was carried out with a subjective trial and error method in which the values of one or two parameters were changed at a time, a computer run was made and the effects of the new parameter values on the simulated results were visually examined. Temperature is an important state variable in the model because the inflowing waters are positioned in the lake according to their density and because many of the reactions occurring in the model are treated as functions of temperature. The temperature program was therefore separated from the main program of the model and calibrated independently from it. This made calibration faster and saved computer time. After completion of the temperature calibration the temperature program was connected with the main program and the remainder of the state variables were calibrated.

The model was calibrated for Lake Pyhäjärvi with the data of the open water season of 1980 and verification was attempted with the corresponding data of 1981. Calibration gave reasonably good results. The model could not, however, be considered as verified. When the values of five parameters used in calibration were changed the model gave as good results with the data of 1981 as with those of 1980. The model can therefore be considered as calibrated for Lake Pyhäjärvi with two different data sets (Niemi and Eloranta 1984).

The model was calibrated for Lake Päijänne with the data set covering one year starting from May 1977 and continuing to April 1978. Verification was carried out with three corresponding data sets, each of which covered one year: namely 1974–1975, 1975–1976 and 1976–1977. During the verification runs the parameter combination obtained during calibration was kept unchanged. Results show that the model can be considered as calibrated and verified. Detailed calibration and verification results are presented by Kinnunen et al. (1982).

6. DISCUSSION

Finnish watercourses typically comprise chains of lakes which are in contact with each other through straits or rivers. In applying models to this type of lakes problems are often encountered. For example in this application both the case study areas were parts of larger lakes, the boundaries of which are hard to define. The northern sub-basin of Lake Päijänne is actually only a small part of the whole lake, while the case study area of Lake Pyhäjärvi comprised the major part of the lake. Discharges from both case study areas occur via a strait with no discharge measurement station. The output discharges had therefore to be calculated.

The question of obtaining adequate data is always a problem in large simulation models. Because the data of driving variables must be given to the model once during every time step, the observed data must be interpolated and extrapolated to produce the necessary data set required by the model. The data on the driving variables appears therefore to be more precise than it actually is, because it is measured in practice on a daily or weekly basis, sometimes even less frequently. The data of Lake Päijänne can be considered to be more precise than that of Lake Pyhäjärvi. The result obtained naturally depend on the accuracy of the input data. In these applications, however, the difference in data between the case study areas did not have a decisive effect on the results.

The calibration of the model, which was accomplished by visually comparing the calculated and observed results, is subjective because of the lack of objective criteria for agreement. In practice the method is also time-consuming, as the user of the model tends to think that the next parameter combination will give better results and continues to calibrate. It would however be important to get good overall agreement between the calculated and observed values in a short period of time.

The sensitivity of the model to the values of many parameters, e.g. half saturation constants, decay rates of nutrients, growth rates and new temperature dependence, was obvious in both applications. Laboratory or field measurements for the values or parameters are hard to obtain. The number of parameters that can be calibrated in the FINNECO model is about one hundred, for only some of which are there recommended values in the literature. Consequently the number of parameters that has to be estimated by calibration remains large and the correct combination of parameters for producing the best fit with observations, if such a combination even exists, is hard to find. Seeking this combination requires several computer runs as well as some intuition on the part of the user.

The parameters of paramount effect in this model are advection and diffusion parameters. If these are not at the correct level the model does not produce good results with any combination of the remainder of the parameters. In the FINNECO model it is therefore important first to calibrate the temperature stratification and water stage in the lake. After this, other state variables can be calibrated. A strict order of calibrating the other state variables cannot be given as many of them are interrelated. Methods of parameter estimation in ecological models have been presented e.g. by Lewis and Nir (1978), Benson (1979) and Jørgensen et al. (1981).

The model could be calibrated for Lake Pyhäjärvi but not verified. However, after changing the values of only five parameters used in the calibration run the model gave as good results as with calibration. This shows that the model was not capable of accurate simulation of such a dynamic system as Lake Pyhäjärvi. On the other hand it also shows that the structure of the model realistically describes the real ecosystem, because only five parameters out of about one hundred had to be changed. This can also be interpreted as indicating that the model is sensitive to these parameters.

The inability of the model to simulate accurately the dynamic ecosystem of Lake Pyhäjärvi may be due to several causes. When the hypolimnion becomes anaerobic, nutrients begin to be released from the sediment to the overlying water. The model takes into account the leaching of nutrients as a function of temperature and oxygen concentration. Consequently the model ought to be capable of simulating the main reactions of the anaerobic nutrient cycle if it could simulate accurately temperature and oxygen concentration, in particular the time when oxygen concentration becomes close to zero. Because the model could not simulate temperature and oxygen concentration correctly enough it therefore could not either simulate accurately the leaching of nutrients. In addition, the assumption of a horizontally homogenous water mass caused an error due to short detention times and mixing currents.

The model gave reasonably good results for Lake Päijänne with the data sets of all four years, although the results were better during the open water season than during the ice-covered season. In the model the lake froze about two weeks earlier than was observed. Therefore in the model too much energy escaped from the lake and the simulated temperature results were lower than than the observed ones. This shows that the mechanism for calculating the longwave radiation in the autumn and winter should be checked. This difference in temperature can also be observed in the simulated oxygen results in the winter. The model can be considered as calibrated and verified for Lake Päijänne although some results could ideally be numerically more precise. The main reasons for the difference of applicability of the model to the two lakes appear to be the structure of the model and the special characteristics of Lake Pyhäjärvi, and perhaps to some extent the accurracy of the input data.

The problems encountered in this application are typical of large simulation models. These problems have been discussed e.g. by Simons and Lam (1980), Fedra et al. (1981), Gentil and Blake (1981) and Beck and van Straten (1983).

General suitability of the model cannot be evaluated solely on the basis of two applications. However, the model did not seem capable of simulating a eutrophic, dynamic ecosystem with anoxic hypolimnion, while for a more stable ecosystem it could be both calibrated and verified.

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TIIVISTELMÄ

FINNECO-vedenlaatumallia sovellettiin Tampereen Pyhäjärveen ja Päijänteen ylimpään osaaltaaseen. Molempiin järviin johdetaan sekä teollisuuden että asutuksen jätevesiä ja ne ovat eutrofisia. Pyhäjärvi on suhteellisen matala, nopeasti virtaava järvi, jonka viipymä on 27 vuorokautta ja veden laatu heikko, mm. alusveden happikato on yleinen ilmiö. Päijänteen ylin osa-allas on syvempi ja sen teoreettinen viipymä on noin 140 vuorokautta. Päijänteen vedenlaatu on parempi kuin Pyhäjärven. FINNECO-mallia sovellettiin näihin järviin, jotta saataisiin selville mallin soveltuvuus kahdelle eri tyyppiselle kohdealueelle. Tulokset osoittivat, että malli soveltui Päijänteelle paremmin kuin Pyhäjärvelle. Pyhäjärvelle malli saatiin kalibroitua kahden eri vuoden aineistolla. Sen sijaan Päijänteelle malli saatiin sekä kalibroitua että verifioitua. Tutkimus osoitti, että malli jäljittelee tyydyttävästi vesiekosysteemin toimintaa. Sen numeerinen tarkkuus ei kuitenkaan näytä olevan riittävä Pyhäjärven kaltaiselle voimakkaasti kuormitetulle järvelle.

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