HELSINKI INSTITUTE OF PHYSICS INTERNAL REPORT SERIES

HIP-2012-04

Scalar Mediated Fermion Masses for Technicolor

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ACADEMIC DISSERTATION

To be presented, with the permission of the Faculty of Science of the University of Helsinki, for public criticism in the Small Auditorium (E204) at Physicum, Gustaf Hällströmin katu 2a, Helsinki, on Tuesday, August 21st, 2012 at 12 o'clock.

Helsinki 2012

ISBN 978-952-10-5337-5 (printed version)
ISSN 1455-0563
ISBN 978-952-10-5338-2 (pdf version)
http://ethesis.helsinki.fi
Unigrafia
Helsinki 2012

M. Antola: Scalar Mediated Fermion Masses for Technicolor,
University of Helsinki, 2012, 69 pages,
Helsinki Institute of Physics Internal Report Series, HIP-2012-04
ISSN 1455-0563
ISBN 978-952-10-5337-5 (printed version)
ISBN 978-952-10-5338-2 (pdf version)

Keywords: Bosonic Technicolor, SUSY

Abstract

This thesis begins by presenting the general features of technicolor models from a model building perspective. Next, a bosonic technicolor model, based on the Next-to-Minimal Walking Technicolor theory, is reviewed. In this type of model, fermion masses arise from scalar exchange with the techniquark condensate. The model passes flavor changing neutral current limits, direct search limits, and oblique constraints, in the parameter region where the fundamental scalar is heavy compared to the composite one. However, its mass is unnaturally small compared to the Planck scale.

Supersymmetry can be used to naturalize fundamental scalars. After discussing general features of supersymmetric models, a supersymmetric bosonic technicolor model, based on the Minimal Walking Technicolor model, is introduced. This model has a special property: in the absence of coupling with the MSSM, the supersymmetric technicolor sector has an approximate $\mathcal{N} = 4$ supersymmetry. We find that this flavor extension drastically changes the condensate and low-energy spectrum compared to the naive Minimal Walking Technicolor effective theory. The model passes flavor changing neutral current limits, and oblique constraints, but in most of the otherwise viable parameter points, the Higgs particle is heavy.

Finally, the $\mathcal{N} = 4$ extended MSSM model is presented.

Acknowledgments

This thesis is based on research carried out at the Helsinki Institute of Physics and the Department of Physics at the University of Helsinki. The financial support from the Vilho, Yrjö and Kalle Väisälä Foundation, and from the Magnus Ehrnrooth Foundation for travels, is gratefully acknowledged.

First I want to thank my supervisor Kimmo Tuominen. He is a very dependable person and his support on practical matters has been absolutely invaluable. Also he has a constructive way of arguing, which has made our collaboration very enjoyable to me. Thanks are due to the other co-authors of my publications as well: Francesco Sannino, Matti Heikinheimo and Stefano Di Chiara. Especially Francesco has been very kind to me and a great source of motivation and ideas. The referees of this thesis, Jukka Maalampi and Chris Kouvaris, did their part wonderfully and I thank them for their comments.

I have greatly enjoyed my time at the University of Helsinki. From the undergraduate period I have to mention the excellent lectures of Juha Honkonen and Claus Montonen. I also want to thank Katri Huitu, Esko Keski-Vakkuri, and Keijo Kajantie for their guidance and advice. However, the most important factor that has made my daily life at the university interesting, is the social circle I have found myself in. I want to thank Ville, Olli, Janne, Samu, Timo, and Mikko, for so many pleasant discussions, and especially the heated arguments on almost any imaginable topic.

Finally, I want express my deep gratitude to my family, friends, and significant other, for always being there for me.

Helsinki, July 2012

Matti Antola

Contents

List of Publications

The content of this thesis is based on the following research articles:

I. M. Antola, M. Heikinheimo, F. Sannino and K. Tuominen,
"*Unnatural Origin of Fermion Masses for Technicolor*"
JHEP 03 (2010) 050, arXiv:0910.3681 [hep-ph]

II. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*Minimal Super Technicolor*"
Eur. Phys. J. C 71, 1784 (2011), arXiv:1001.2040 [hep-ph]

III. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*Supersymmetric Extension of Technicolor & Fermion Mass Generation*"
arXiv:1111.1009 [hep-ph], accepted for publication in NPB

IV. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*N=4 Extended MSSM*"
Nucl. Phys. B 856, 647 (2012), arXiv:1009.1624 [hep-ph]

The Author's Contribution to the Joint Publications

I. The present author did most of the calculations, wrote the first draft of the paper, and participated in editing the final draft.

II. The present author carried out all numerical and analytical calculations independently and rewrote some sections of the paper for the final draft.

III. The present author did the original calculations and wrote the first draft of the paper excluding the section on scalar phenomenology.

IV. The present author carried out all numerical and analytical calculations independently and participated in the writing process.

Chapter 1

Introduction

The Higgs boson is the last unconfirmed elementary particle predicted by the Standard Model (SM) of particle physics. If a scalar Higgs-type particle is discovered at the LHC, one must still determine the nature of it. If the discovered Higgs particle is indeed fundamental, then it is the first fundamental scalar particle found in nature. Alternatively, the Higgs boson could be composite, in analogue to the bound states of QCD.

In an effective field theory formalism, all phenomena with momentum exchange $k < \Lambda$ can be described by an action S_Λ in which all particles have masses $m < \Lambda$. From this viewpoint, the SM at the electroweak (EW) scale is simply a low-energy effective theory $S_{\Lambda_{EW}}$ that should be derived from other theories describing higher energy physics. One could imagine deriving the SM by beginning at the Planck scale where the theory of quantum gravity is mapped into a quantum field theory. If the Higgs boson is fundamental, its mass at the Planck scale is given by a dimensionless function of quantum gravity parameters multiplied by the appropriate dimensional parameter Λ^2_{Planck}. As the cutoff Λ of the action S_Λ is decreased from Λ_{Planck}, heavy particles are integrated out of the effective action. Because the Higgs mass term is not protected by any symmetry, if the heavy particles couple even indirectly to the Higgs field, they will contribute to its mass term as $(k/16\pi^2)^n m^2$, where k is a function of coupling constants, m is the heavy particle's mass, and n is the loop order in which the term appears. Since the Higgs mass should be $\mathcal{O}(\Lambda_{EW})$, the contributions of the quantum gravity theory and the heavy particles that were integrated out have to cancel in a surprising and delicate way. If instead there are no new particles between the EW and Planck scales, then the Higgs mass has to be unnaturally small at the Planck scale, $m_0^2/m^2_{Planck} \sim 10^{-34}$.

The terms "natural", "unnatural", and "technically natural" can be used to explain the degree of mismatch between the predicted and observed value of any given parameter. For example, based on the above paragraph, the Higgs mass parameter is predicted to be $\mathcal{O}(1)$ in units of the scale of new physics. Therefore, either the SM is not a complete description of nature, and new physics enters at the electroweak scale, or the SM Higgs mass parameter is unnaturally small. In contrast, the Yukawa couplings of the SM are multiplicatively renormalized, and can take any perturbative value. Since the theory does not favor any particular value, the Yukawa couplings are technically natural, see Table 1.1.

Dimensionless parameter	Definition	Example
Natural	Known mechanism explains the measured value	$\Lambda_{QCD}/\Lambda_{Planck}$
Unnatural	Known mechanism contradicts with measured value	$m_{Higgs}/\Lambda_{Planck}$
Technically Natural	Free parameter	y_t, y_e

Table 1.1: This table explains the usage of the terms natural, unnatural, and technically natural in this thesis. Natural parameters are those whose experimental value is understood and explained via a theoretical framework. Unnatural parameters are those whose experimental value is disfavored in the given theoretical framework. Technically natural parameters are completely free parameters within the theoretical framework, and while they can be fitted to experiments, there is no physical explanation for any particular value.

In technicolor (TC) [1, 2], the Higgs particle is naturalized by making it composite. The high energy Lagrangian contains new strongly interacting fermions (techniquarks) which form a low-energy condensate. The electroweak symmetry is broken by this condensate, and the smallness of the Higgs mass is a result of the coupling constant running: $\Lambda_{EW}/\Lambda_{Planck} \sim \exp(k/\alpha_{Planck} - k/\alpha_{EW})$. The would-be composite Goldstone bosons of the technicolor chiral symmetry are absorbed to form the longtitudinal degrees of freedom of the electroweak gauge bosons.

In the Standard Model, the other task of the Higgs field is to give mass to the SM fermions. In generic technicolor models, one must add another sector, called the flavor extension in this thesis, specifically for this task. This thesis focuses on the question: "How are fermion masses explained in technicolor?" and approaches the question via bosonic technicolor models. In these models, the fermion masses arise from scalar mediated interactions with the technicolor condensate.

Chapter 2 reviews the features of technicolor and generic flavor extensions. Chapter 3 summarizes the results of Paper I on the bosonic Next-to-Minimal Walking Technicolor model. Chapter 4 introduces Supersymmetry (SUSY) and the problems in the Minimal Supersymmetric Standard Model (MSSM) that supersymmetric technicolor can solve. Chapter 5 introduces the Minimal Supersymmetric Conformal Technicolor (MSCT) model and summarizes the results of Paper III. Chapter 6 introduces the $\mathcal{N} = 4$ Extended MSSM model and summarizes the results of Paper IV. The main results of Paper II are fused into Chapters 4-6. Chapter 7 contains a summary and relevant conclusions. In this introductory part, we will omit many technical details, which can be found in the papers included in the Appendix.

Chapter 2

Developing Technicolor

2.1 Effective Lagrangian for QCD

Quarks (3 families)	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
u_R	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{2}{3})$
d_R	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{1}{3})$

Table 2.1: Representations of quarks in QCD

In technicolor, electroweak symmetry breaking (EWSB) is modeled on the precedent of QCD. In isolation, QCD is an $SU(3)$ gauge theory with $N_f = 6$ massive quarks in the fundamental representation of the gauge group. The theory is coupled to other particles of the Standard Model (SM) via the Yukawa operators and gauging the six quarks as three doublets of the electroweak symmetry, $SU(2)_L \times U(1)_Y$. In this section it is shown that, because of the gauging, the spontaneous symmetry breaking of the flavor symmetry simultaneously breaks the electroweak symmetry. The methods used in constructing the effective Lagrangians are also introduced.

It seems that by chance, two of the quarks (up, down) are very light. Therefore the Lagrangian[1]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{q}_L\gamma_\mu D^\mu q_L + \bar{q}_R\gamma_\mu D^\mu q_R + \bar{q}_L m q_R + h.c. \tag{2.1}$$

approximately satisfies a global chiral symmetry $SU(2)_L \times SU(2)_R$, under which

$$q_{L/R} \to G_{L/R}q_{L/R}\ ,\ G_{L/R} \in SU(2)_{L/R}. \tag{2.2}$$

[1]If we have removed the Higgs field, the quarks ought to have no hard mass term. Because of the mass term, the model does not fully conserve even the gauged symmetry $SU(2)_L \times U(1)_Y$, and the Goldstone bosons of the symmetry are therefore missing. However, the mass term serves a purpose to introduce the spurion method and is therefore included.

Here $q_R = (u_R, d_R)$ and $m = \mathrm{Diag}(m_u, m_d)$. This symmetry is dynamically broken at low energies by the chiral condensate, which transforms as

$$\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \to \left\langle \bar{q}_L G_L^\dagger G_R q_R + \bar{q}_R G_R^\dagger G_L q_L \right\rangle . \tag{2.3}$$

Only the diagonal $SU(2)_V$ subgroup, given by $G_L = G_R$, is conserved by this condensate.

Consider the modeling of the spontaneous symmetry breaking in the QCD sector with an effective Lagrangian. To not break the Lorentz symmetry we want to study the composite scalar sector. It is obvious that the only $SU(3)_C$ singlet scalar composite field of two quarks we can write is given by $M \sim q_L \bar{q}_R$, from which we can read the transformation properties of M under $SU(2)_L \times SU(2)_R$:

$$M' = G_L M G_R^\dagger . \tag{2.4}$$

The covariant derivative for a fully gauged $SU(2)_L \times SU(2)_R$ is automatically given by the transformation properties. However, only a subgroup of the global chiral symmetry is equated with the gauged electroweak symmetry: $SU(2)_L$ is identified with the gauge symmetry but only the diagonal generator of $SU(2)_R$ is identified with hypercharge. We use the ansatz

$$D_\mu M = \partial_\mu M + i g_L W_\mu M - i g_Y M B_\mu \tau_3 , \tag{2.5}$$

where we denote $W \equiv W^i \tau_i$ and τ_i are the $SU(2)$ generators, i.e. $\tau_i = \sigma_i/2$ in terms of the Pauli matrices σ_i. Requiring $(D_\mu M)' = G_L (D_\mu M) G_R^\dagger$, one easily finds

$$\begin{aligned} W'_\mu &= G_L W_\mu G_L^\dagger - \frac{1}{i g_L} (\partial_\mu G_L) G_L^\dagger \\ B'_\mu &= B_\mu - \frac{1}{i g_Y} (\partial_\mu G_R) G_R^\dagger . \end{aligned} \tag{2.6}$$

We want to construct the simplest possible model to study chiral symmetry breaking, so we will use the minimal linear representation:

$$M = \frac{1}{\sqrt{2}} (s + 2 i \pi_M) , \tag{2.7}$$

where we denoted $\pi \equiv \pi^i \tau_i$ and s is a scalar. The ansatz (2.7) is minimal since, as shown below, it is closed in form under the symmetry group $SU(2)_L \times SU(2)_R$, and also it includes the Goldstone bosons of this symmetry. It also contains the least amount of dynamical fields given these conditions. For example, under an infinitesimal transformation of $SU(2)_L$, M transforms to

$$G_L M = \frac{1}{\sqrt{2}} \left(1 + i g_L G_L^j \sigma_j\right) \left(s I_{2\times 2} + i \pi_M^i \sigma_i\right) \tag{2.8}$$

$$= \frac{1}{\sqrt{2}} \left(s - g_L G_L^i \pi_M^i + i \sigma_k (g_L s G_L^k + \pi_M^k - g_L G_L^i \pi_M^j \epsilon_{ijk})\right) , \tag{2.9}$$

which is of the same form as (2.7).

The effective Lagrangian for this theory at low energies is given by the most general one consistent with the global symmetries and given particle content. The symmetry of the

Lagrangian (2.1) would be $SU(2)_L \times SU(2)_R$, if m transforms as $m \to G_L m G_R^\dagger$. Therefore we can formally construct the low-energy effective Lagrangian by assuming this transformation property for m. Since m is small compared to the natural QCD mass scale of around 1 GeV, we can later remove all but the lowest order terms in m. This method is called the spurion method.

For simplicity, we require the dimension of the operators to be less than four and the symmetry breaking part to be lowest order in the spurion field m:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\mathrm{Tr}\left[D_\mu M D^\mu M^\dagger\right] - \mathcal{V} \\ \mathcal{V} &= -\frac{m_M^2}{2}\mathrm{Tr}\left[M^\dagger M\right] + \frac{\lambda}{24}\mathrm{Tr}\left[M^\dagger M\right]^2 + c_1 \mathrm{Tr}\left[Mm\right] + h.c. + \dots \end{aligned} \tag{2.10}$$

The negative mass of M signifies spontaneous symmetry breaking. From experiment, we find $\langle s \rangle = f_\pi = 73$ MeV.

Unknown coefficients appearing in an effective Lagrangian are determined from experiments or by matching the effective low-energy theory with the underlying one. The order of magnitude of otherwise unknown coefficients can also be estimated by Naive Dimensional Analysis (NDA) [3, 4, 5]. According to NDA, the coefficients depend on Λ, the mass of some low lying non-Goldstone state, and $h_N \lesssim 4\pi$, which estimates the size of loop corrections in the theory. The rules will be discussed in detail in Section 3.2. The result for the couplings of (2.10) are: $(m_M^2)_{NDA} = \Lambda^2$, $(\lambda)_{NDA} = h_N^2$, $(c_1)_{NDA} = \Lambda^2/h_N$. We also have an additional condition $f_\pi = 73$ MeV, which can be used instead of one of these estimates; usually m_M^2 is solved from the extremum condition of (2.10) instead of using the NDA estimate.

It is interesting to test the NDA estimates for the parameters against the values found in actual phenomenological models of QCD; see e.g. [6]. In Table 2.2 some typical values from a global fit are shown. In these models one fits all data and therefore it is important to consistently include all fields lighter than the heaviest field included. Therefore the spectrum is chosen to contain also the CP partners of the meson field M, vector mesons, and baryons. Since the particle content is different, the parameters in Table 2.2 are not one-to-one to the ones in the effective Lagrangian (2.10), e.g. there are two independent quartic couplings while there is only one in (2.10). Still, the NDA rules give the correct order of magnitude for all parameters. For the NDA estimate in Table 2.2 we set

$$m_M^2 = \frac{\lambda f_\pi^2}{6} + \frac{\sqrt{2}c_1(m_u + m_d)}{f_\pi}. \tag{2.11}$$

	$m_M/(m_M)_{NDA}$	$\lambda/(\lambda)_{NDA}$	$c_1/(c_1)_{NDA}$	f_π
NDA	0.2	1	1	93 MeV
[6]	0.77	-0.14 and 3.2	0.14	93 MeV

Table 2.2: Comparison of values found from NDA and an actual fit to data.

To calculate the W and Z boson masses, we expand the Lagrangian around $\langle s \rangle = f_\pi$. From the kinetic term,

$$\frac{1}{4}g_L^2 f_\pi^2 \mathrm{Tr}\left[W_\mu^i \tau_i W_j^\mu \tau^j\right] = \frac{1}{8}g_L^2 f_\pi^2 W_\mu^i W_i^\mu, \tag{2.12}$$

we find

$$m_W = \frac{g_L f_\pi}{2} \sim 30 \text{ MeV}. \tag{2.13}$$

Importantly, we retain the phenomenologically successful relation

$$\frac{m_W}{m_Z} = \frac{g_L}{(g_L^2 + g_Y^2)^{1/2}} = \cos\theta_W, \tag{2.14}$$

where θ_W is the usual weak mixing angle defined in terms of coupling constants. This result is a direct consequence of the custodial $SU(2)_V$ symmetry that is satisfied by the model.

2.2 Fermion Masses and Flavor Changing Neutral Currents

The model of EWSB introduced in Section 2.1 is deficient because

1. The QCD pions are absent from the physical spectrum
2. The mass of the W boson becomes 30 MeV whereas it should be 80 GeV
3. There is no source of hard mass for quarks or leptons

Technicolor, at its simplest, assumes that the Higgs sector is replaced with a copy of QCD with a rescaled pion decay constant $f_\pi \to v = 246$ GeV, so that the mass of W is given correctly. This immediately fixes the first two problems. However, one cannot write a renormalizable operator to give mass to the SM fermions with just the TC spectrum. Therefore technicolor models need new dynamics at some higher energy scale to complete the model. This situation is common to all TC extensions of the SM: the SM fermion masses must be mediated by a yet unspecified new sector, which we will generically call the flavor extension of TC.

Traditionally technicolor model builders strived to avoid fundamental scalars and therefore only the Extended Technicolor (ETC) models were considered [7, 8]. A large gauge group G_{ETC}, defined at some high energy, is broken, possibly in many steps, to the low-energy groups $G_{TC} \times G_{SM}$ by an unspecified mechanism. The group G_{SM} contains at least the electroweak group.

$$G_{ETC} \stackrel{E\sim\Lambda_{ETC}}{\longrightarrow} G_{TC} \times G_{SM} \ . \tag{2.15}$$

This elevates the masses of the coset gauge bosons of $G_{ETC}/(G_{TC} \times G_{SM})$ to $g\Lambda_{ETC}$. These massive gauge bosons are then integrated out, and they will generically lead to four fermion interactions including the ones necessary to generate SM fermion masses.

Since there is no particularly successful model of ETC, that would explain in detail the mechanism and specify why the gauge symmetries are broken, we must somehow parametrize our ignorance on the specific ETC theory. This is done by assuming an effective Lagrangian below the ETC scale with the following terms:

$$a_{ab}\frac{\bar{Q}_L T^a Q_R \bar{Q}_R T^b Q_L}{\Lambda_{ETC}^2} + b_{ab}\frac{\bar{Q}_L T^a Q_R \bar{f}_R T^b f_L}{\Lambda_{ETC}^2} + c_{ab}\frac{\bar{f}_L T^a f_R \bar{f}_R T^b f_L}{\Lambda_{ETC}^2} \ . \tag{2.16}$$

Here and throughout this thesis Q refers to techniquarks and f to SM fermions. The matrices T^a denote the generators corresponding to the integrated gauge bosons. The couplings a, b, and c should be calculated from the full ETC theory and hence they should be proportional to g_{ETC}^2, the square of the gauge group coupling from two ETC gauge boson vertices. The first type of terms break the technicolor chiral symmetry and induce masses to any otherwise massless Goldstone boson. The second type of term gives masses to SM fermions. The c terms mediate Flavor Changing Neutral Currents (FCNC) between SM fermions [9]. At tree level the SM has no FCNCs.

A useful summary of FCNC constraints is found in [10], in which the authors use various data to limit coefficients of FCNC inducing four fermion operators. The summary table is reproduced in Table 2.3. These limits can be used to limit the largest possible fermion mass in generic ETC models. For example, looking at $(\bar{b}_L\gamma_\mu d_L)^2$, one has, assuming a typical common value $c_{ab} \sim c$,

$$\frac{c}{\Lambda_{ETC}^2} \lesssim \frac{1}{\Lambda_{exp}^2}. \tag{2.17}$$

This can be used to give an upper bound for the largest possible fermion mass, if one assumes that the couplings b and c are equal:

$$m_F \sim \frac{b\left\langle \bar{Q}Q\right\rangle}{\Lambda_{ETC}^2} \lesssim \frac{\left\langle \bar{Q}Q\right\rangle}{\Lambda_{exp}^2} \sim 1 \text{ MeV}, \tag{2.18}$$

where we have used, in the last estimate, $\left\langle \bar{Q}Q\right\rangle = 4\pi v^3$, $v = 246$ GeV, and $\Lambda_{exp} \sim 500$ TeV. This result is a factor of 10^5 lower than the top mass, and thus unrealistic. However, it is based on some crude approximations, e.g. assuming that the values of the b_{ab} and c_{ab} are equal. Usually in fact the ETC gauge group is broken down in many steps, with only the last step generating the top mass. In these models the couplings can be very different as there are multiple ETC scales. Also the situation is improved in the so called walking models, which will be discussed in Section 2.3. Still, it is clear that any ETC model will have difficulties in explaining the suppression of FCNCs. There is much ongoing research in ETC model building [11, 12, 13, 14, 15, 16, 17, 18].

This thesis focuses on an alternative to ETC, the so called Bosonic Technicolor (BTC). Bosonic Technicolor models were pioneered in a series of papers by Simmons, Kagan and Samuel, and Carone and Georgi [19, 20, 21, 22, 23, 24, 25, 26, 27]. In these models the SM fermion masses are mediated by scalars, as explained below. The Lagrangian of the BTC model is

$$\mathcal{L}_{BTC} = \mathcal{L}_{SM}\Big|_{Higgs=0} + \mathcal{L}_{TC} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}\,. \tag{2.19}$$

The Yukawa sector contains Yukawa couplings for all fermions, that is, the usual ones plus a coupling for the techniquarks. When the techniquarks condense, their Yukawa term $y_Q\bar{Q}HQ$ becomes a linear term in the Higgs potential. Therefore the Higgs potential is of the form:

$$V = -m_M^2 s^2 + \lambda_M s^4 + c_1 y_Q s h + m_H^2 h^2 + \lambda h^4, \tag{2.20}$$

where s is the techniquark composite scalar and h the fundamental one. The tree-level term tilts the Higgs potential and induces, even for a positive Higgs mass, a vacuum expectation

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

Table 2.3: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV. Observables related to CP violation are separated from the CP conserving ones with semicolons. Table is taken from [10].

value (vev) proportional to $y_Q \langle s \rangle / m_H^2$, therefore giving mass to SM fermions via their Yukawa terms. This is demonstrated in Fig. 2.1.

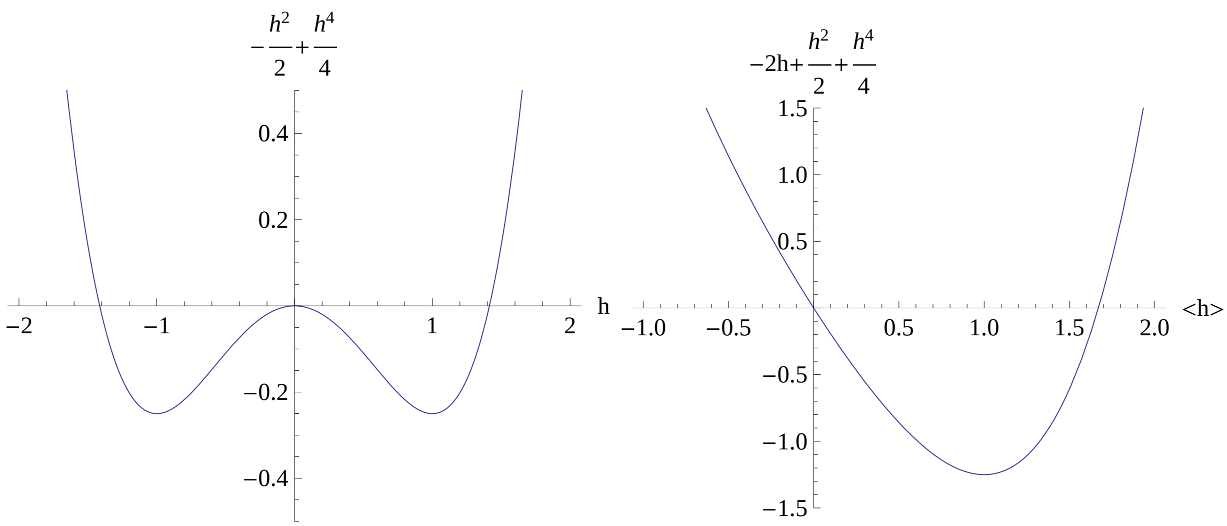


Figure 2.1: The left hand side panel shows the regular Mexican hat potential corresponding to a negative mass term and a positive quadratic term. The right hand side panel shows a potential with a linear term, a positive mass term, and a positive quadratic term.

The authors studied several possible scenarios, depending on the parameters of the Higgs potential (2.20):

1. The fundamental Higgs field is very heavy. In this case the Higgs self coupling can be ignored: $\lambda = 0$

2. The fundamental Higgs field is massless: $m_H^2 = 0$

The previous authors have essentially constructed suitable effective Lagrangians, and then applied experimental constraints from FCNCs, oblique parameters, and collider signals. A BTC model we studied in Paper I will be discussed in Chapter 3. This type of model has been investigated recently also in [28, 29, 30, 31].

One benefit of BTC models is the GIM mechanism, which works just like in the SM. However, additional FCNC contributions arise from extra $SU(2)_L$ triplet scalar states. The low-energy theory contains more than one triplet of $SU(2)_L$, because the fundamental Higgs multiplet includes one and there is at least one triplet also from the technicolor sector. One triplet will be absorbed by the W and Z gauge bosons, but there are at least three physical states left, that are exactly like QCD pions, except heavier. The relevant $\Delta s, b = 2$ box diagrams [21] are shown in Fig. 2.2. Because of the physical pion propagator, the contributions are essentially suppressed by the Higgs mass. For Higgs masses in the multi-TeV range, the contributions are small.

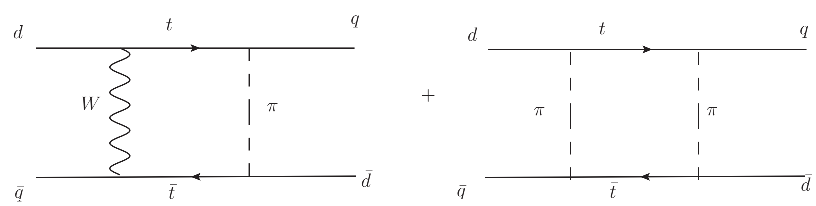


Figure 2.2: Box diagrams contributing beyond the SM to $\Delta q = 2$ FCNC interactions for $q = s\,, b$. Of the possible quark flavors running in the loops the top quark provides the dominant contribution due to its large Yukawa coupling to the scalar degrees of freedom.

BTC models also allow to analyze the backreaction of the flavor extension on the technicolor theory. For example, it was noted in [23] that if the hard quark mass grows large, so that the techniquarks become more like the c quark than the u and d quarks, the technicolor theory begins to look less like QCD, and the spurion expansion used to construct the effective Lagrangian breaks down. If the techniquarks become very heavy then they should decouple from the infrared dynamics instead of condensing.

Just like in the SM, in BTC models the Higgs field suffers from the unnaturalness problem. Therefore models without any further mechanisms to naturalize the theory are unnatural. BTC could be naturalized by compositeness, i.e. by assuming the fundamental Higgs particle is, in fact, a composite particle bound by another interaction. In this case it is natural that the Higgs particle is much heavier than the electroweak scale. The other option is to naturalize the Higgs field with supersymmetry (SUSY). SUSY theories can have natural light scalars, and therefore provide a compelling ultraviolet completion of BTC models. A specific SUSY BTC we studied in Paper III will be reviewed in Chapter 5.

Supersymmetry must be broken at low energies. In phenomenological models this is achieved by so called soft SUSY breaking terms, which yet again mediate FCNC transitions via off-diagonal terms in the scalar mass matrices. This will be discussed in Section 4.2.

2.3 Walking Theories and the S Parameter

It is natural that the large amount of data collected on QCD guides our intuition of technicolor dynamics. However, it is useful to cartograph the variety of different models available. We therefore consider all parameters of QCD that can be altered for technicolor:

1. The gauge group may be $SU(N)$ for general N. Also the symplectic or orthogonal groups can be considered.

2. The representation of the quarks may be different from the fundamental one, e.g., the adjoint. The number of quark flavors can change. There might even be quarks in many different representations.

3. The chiral symmetry might be explicitly broken to a larger degree than in QCD. Compare to QCD without the very light up and down quarks.

The first two points are conveniently discussed using the phase diagram of field theories in the (N, N_f) plane and as a function of different representations of the quarks Q. The quark representation has an effect on the screening properties of matter, and indeed leads to a very different phase diagram [32]. The prevalent feature of the phase diagram is the conformal window. Theories lying inside the conformal window display long distance conformality.

- The upper boundary of the conformal window is the limit where asymptotic freedom is lost. Below this limit the theory has a Banks-Zaks conformal fixed point [33] at values of the coupling that can be reliably calculated in perturbation theory.

- The lower boundary of the conformal window corresponds to conformal theories with large values of the conformal coupling. There are various approximate methods to estimate the number of flavors at which the conformal window ends: the Scwhinger-Dyson equation combined with the two loop beta function [34, 35], the all orders beta function [36], fixed point annihilation [37], and first principle studies on the lattice (see e.g. the recent papers [38, 39] and references therein). All results are compatible.

- Just below the lower boundary, the theory is nearly conformal, but still develops an infrared scale. In this regime, unlike QCD, the coupling walks for a large energy range before chiral condensation occurs [40, 9, 41].

In QCD, the scaling symmetry is broken spontaneously by the low-energy chiral condensate. Since there is no corresponding observed Goldstone boson, this symmetry must also be broken explicitly by the trace anomaly. Instead, walking theories nearly conserve conformality, and therefore it has been argued that the dilaton[2], i.e. s appearing in (2.7), should be light [42, 43]. There are also arguments based on large-N scaling that lead to the same conclusion. This is very interesting in light of the recent LHC data [44, 45], suggesting that even if the Higgs particle is found to be light, it can still be composite.

Note that the vastly different models this phase diagram covers correspond to only a few effective Lagrangians. This is because the global symmetry breaking patterns govern the

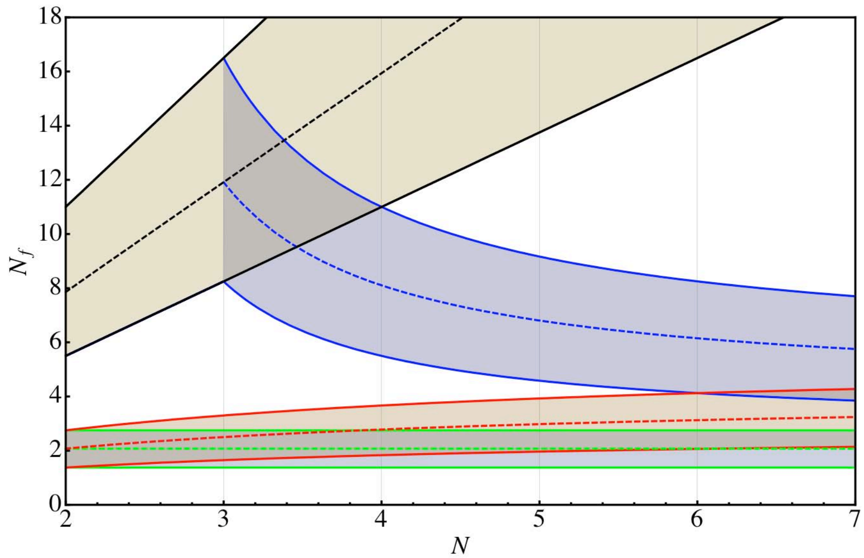


Figure 2.3: Phase diagram or $SU(N)$ gauge theories with fermions in the fundamental, two-index antisymmetric, two-index symmetric, and adjoint representation, counting from top to bottom. In between the upper and the lower solid curves, the theories are expected to develop an infrared fixed point according to the all orders beta function [36]. The area between the upper solid curve and the dashed curve corresponds to the conformal window obtained in the ladder approximation. Taken from [36].

Quark Representation	Global Symmetry Breaking Pattern
Complex	$SU(N)^2 \to SU(N)$
Real	$SU(2N) \to SO(2N)$
Pseudoreal	$SU(2N) \to SP(2N)$

Table 2.4: Possible global symmetry breaking patterns, in the absence of a flavor extension, depending on the nature of the quark representation [46].

low-energy theories of technicolor. Different possibilities are summarized in Table 2.4 and depend only on the nature of the quark representation.

Let us now consider how a walking coupling affects the maximum fermion mass allowed in the theory. Since this applies equally to BTC and ETC models, we calculate the effect with a simplified calculation assuming a dimension one field X, that can thus be scalar or gauge field, with mass m_X:

$$V \sim \frac{1}{2} m_X^2 X^2 - g_{X\bar{Q}Q} X\bar{Q}Q - g_{X\bar{F}F} X\bar{F}F. \tag{2.21}$$

Below the scale m_X one can integrate the particle out by solving X from $\partial V/\partial X = 0$. Now we are interested in the SM fermion mass term, which is given by the term

$$V \sim g_{X\bar{F}F} g_{X\bar{Q}Q} \frac{\bar{Q}Q\bar{F}F}{m_X^2}. \tag{2.22}$$

[2]The Goldstone boson of the scaling symmetry is called the dilaton.

Next we consider how this term changes under the renormalization group evolution towards low energies where the mass is measured. The couplings $g_{X\bar{F}F}$ and $g_{X\bar{Q}Q}$ are assumed to be constant. Therefore only the techniquark bilinear is renormalized.

The renormalization group equation for the techniquark bilinear gives

$$\langle\bar{Q}Q\rangle_{m_X} = \exp\left(\int_{\Lambda_{TC}}^{m_X} \frac{\mathrm{d}\mu}{\mu}\gamma_m(\alpha(\mu))\right)\langle\bar{Q}Q\rangle_{\Lambda_{TC}}. \tag{2.23}$$

In QCD, asymptotic freedom sets in quickly above Λ_{QCD} and the coupling behaves as $\alpha \propto 1/\log\mu$; if this were true in technicolor, then the anomalous dimension would behave above Λ_{TC} approximately as $\gamma_m = \gamma_m^0/\log(\mu/\Lambda_{TC})$, where $\gamma_m(\alpha(\Lambda_{TC})) \equiv \gamma_m^0$. Therefore one finds the enhancement factor

$$\exp\int_{\Lambda_{TC}}^{m_X} \frac{\mathrm{d}\mu}{\mu}\frac{\gamma_m^0}{\log\frac{\mu}{\Lambda_{TC}}} = \exp\gamma_m^0\int_0^{\log\frac{m_X}{\Lambda_{TC}}} \frac{\mathrm{d}x}{x} = \left(\log\frac{m_X}{\Lambda_{TC}}\right)^{\gamma_m^0}. \tag{2.24}$$

Instead, if $\alpha = \alpha^*$ is approximately constant between Λ_{TC} and m_X, then the corresponding ratio gives (using $\gamma_m^* \equiv \gamma_m(\alpha^*)$)

$$\exp\int_{\Lambda_{TC}}^{m_X} \frac{\mathrm{d}\mu}{\mu}\gamma_m^* = \left(\frac{m_X}{\Lambda_{TC}}\right)^{\gamma_m^*}, \tag{2.25}$$

a much larger enhancement. Therefore we arrive at the following estimate for fermion masses:

$$m_F = g_{X\bar{F}F}\,g_{X\bar{Q}Q}\frac{4\pi v^3}{m_X^2}\omega, \tag{2.26}$$

where we set the condensate at the electroweak scale to $\langle\bar{Q}Q\rangle_{TC} = 4\pi v^3$, and

$$\omega = \begin{cases} \left(\frac{m_X}{\Lambda_{TC}}\right)^{\gamma_m^*}, & \text{walking theory} \\ \left(\log\frac{m_X}{\Lambda_{TC}}\right)^{\gamma_m^0}, & \text{running theory} \end{cases}. \tag{2.27}$$

The largest possible fermion mass for a given m_X in a walking theory is

$$m_F \sim \left(\frac{g_{X\bar{F}F}}{4\pi}\right)\left(\frac{g_{X\bar{Q}Q}}{4\pi}\right)\left(\frac{3\ \mathrm{TeV}}{m_X}\right)^{2-\gamma_m^*} \times 3\ \mathrm{TeV}. \tag{2.28}$$

This equation gives the promised improvement to the result we had in (2.18). To compare with (2.18), we must estimate the couplings g. Taking $g_{X\bar{F}F} = g_{X\bar{Q}Q} = g_{ETC} = 4\pi$ we find

$$m_F \lesssim \left(\frac{3\ \mathrm{TeV}}{4\pi\Lambda_{exp}}\right)^{2-\gamma_m^*} \times 3\ \mathrm{TeV}. \tag{2.29}$$

To generate the strange quark mass from this formula, and using $\Lambda_{exp} \sim 500$ TeV, requires an anomalous dimension $\gamma \sim 0.65$. To generate the top quark mass one would need a huge anomalous dimension $\gamma \sim 1.6$. Therefore in ETC phenomenology, a walking coupling and a large anomalous dimension of the fermion bilinear are clearly desired.

Next we discuss an experimental input that limits the technicolor sector in itself, independent of the sector used to generate fermion masses. In [47] it was discovered that a large class of beyond the SM models could be analyzed just by how they affect the electroweak gauge boson self energies, and these effects could be codified in a few parameters: S, T, and U. Later a few more have been introduced as well [48]. The parameters were chosen so that they carry some intuitive significance: the S parameter counts the size of new electroweak interacting sectors; the T parameter counts isospin breaking. These parameters have become very important in BSM analysis. For technicolor, especially the S parameter is important. It is defined via two point functions of gauge bosons as

$$\alpha S = 4 s_w^2 c_w^2 \frac{\Pi_{ZZ}^{new}(m_Z^2) - \Pi_{ZZ}^{new}(0)}{m_Z^2}, \tag{2.30}$$

where the label *new* means that only contributions that are not in the SM are taken into account.

The expression (2.30) is impossible to calculate in a strongly interacting theory such as technicolor. In the simplest estimate, one calculates it using the perturbative expressions in terms of the techniquarks and assuming a large (compared to m_Z) dynamically generated mass for the techniquarks. This estimate gives

$$S_{naive} = \frac{N_F d(r)}{6\pi}, \tag{2.31}$$

where $d(r)$ is the dimension of the representation of the techniquarks. Various approximations [47, 49, 50, 51] also agree that this "naive" estimate (2.31) is relatively accurate. The full S parameter has also been postulated [52, 53] to be larger than (2.31). This means that one should choose the technicolor theory so that it minimizes (2.31).

Using the naive S parameter as a measure of minimality, there are several models on the market. The two nearly conformal benchmark models studied in this thesis are Minimal Walking Technicolor and Next-to-Minimal Walking Technicolor (MWT and NMWT, respectively) [32]. The MWT model has two technifermions in the adjoint representation of $SU(2)$, resulting in $S_{naive} = 1/6\pi$. Interestingly, the technicolor theory suffers from the $SU(2)$ Witten anomaly [54]. Therefore, to avoid the anomaly, one must add to the theory additional weak doublets that are not charged under the TC gauge group. The NMWT model has two technifermions in the sextet representation of $SU(3)$, giving $S_{naive} = 1/2\pi$. NMWT is introduced in detail in Section 3.1 and MWT in Section 5.1.

2.4 Summary

Summarizing, the full underlying Lagrangian of technicolor models is given by replacing the Higgs sector with a strongly interacting sector and possibly some additional degrees of freedom:

$$\mathcal{L}_{Higgs} \to -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\overline{Q}\gamma_\mu D^\mu Q + \ldots \tag{2.32}$$

Here various representations can be chosen for the techniquarks Q and also the gauge group and number of flavors can be chosen to satisfy various constraints such as those given by the

S parameter. The dots represent any additional degrees of freedom, and must include the flavor extension necessary to accommodate the SM masses. This extension can also backreact on the TC dynamics, by e.g. modifying the chiral symmetry breaking pattern (see Chapter 5) or affecting the anomalous dimension of the fermion bilinear [55].

Extensions with spin-1 mediators are denoted ETC models. The masses of the ETC gauge bosons have to be above the EW scale, since light ETC gauge bosons, associated with gauged flavor symmetries, are experimentally excluded. Models with spin-0 mediators are denoted BTC models. In BTC the scalar mediator can be light or heavy. Each case can be described by writing the terms that generate fermion masses in the effective action at the electroweak scale, as given in Table 2.5.

Spin	Model Category	$m_X \lesssim \Lambda_{EW}$	$m_X > \Lambda_{EW}$
Spin-0	Bosonic Technicolor	$y_Q \bar{Q}_L H Q_R + y_f \bar{f}_L H f_R$	$y_Q y_f \frac{\bar{Q}_L Q_R \bar{f}_R f_L}{m_H^2}$
Spin-1	Extended Technicolor	$\bar{Q} D_\mu \gamma_\mu Q + \bar{f} D_\mu \gamma_\mu f$	$g_{ETC}^2 \frac{\bar{Q}_L Q_R \bar{f}_R f_L}{m_{ETC}^2}$

Table 2.5: Possible mediators, name of model, and relevant terms for fermion masses in the effective action at the electroweak scale. Here m_X gives the mass of the mediator and $\Lambda_{EW} \sim$ TeV.

Importantly, the additional sectors very often induce FCNCs, leading to additional phenomenological constraints. The main sources of FCNCs depend on the precise nature of the flavor extension and they can be:

a) Four SM fermion interactions from integrating out heavy ETC gauge bosons

b) Extra pseudo-Goldstone states with quantum numbers of W

c) Soft supersymmetry breaking scalar mass terms with off diagonal elements, discussed in Chapter 4

Using these definitions, the FCNC sources relevant for each model stereotype, along with other major issues, are listed in Table 2.6.

Flavor Extension	$m_X \lesssim \Lambda_{EW}$	$m_X > \Lambda_{EW}$	General Problems
Unnatural BTC	b)	None	Unnatural
SUSY BTC	b)+c)	c)	SUSY breaking description
Composite BTC	b)	None	Composite interaction description
ETC	Excluded	a)	No singularly successful models

Table 2.6: This table lists the major issues in each model type. The first column gives the type of the model, the next two columns the types of FCNC mediation, the third column gives general problems.

Finally, in every technicolor model the vector mesons of the TC sector also induce FCNCs [56]. This is because the vector mesons can mix with the electroweak gauge bosons, and hence contribute via the diagrams already existing in the SM. These limits convert to limits on the parameter space of the vector mesons, and they are not considered in this thesis.

At the electroweak scale, the full Lagrangian (2.32) is mapped to a mesonic effective theory, similar to (2.10), which can be used to calculate observables. Explicit symmetry breaking is taken into account with the spurion method. Higher excitations can be included, such as baryonic states or vector and axial vector particles, which are important for actual discovery of generic technicolor models and for unitarizing WW scattering [57]. Some coefficients can be estimated by Naive Dimensional Analysis, which will be discussed in Section 3.2, and in the spin-1 sector also by the Weinberg sum rules [58]. In essence, the Lagrangian is

$$\mathcal{L}_{SM}|_{H=0} + \mathcal{L}_{TC} \to \mathcal{L}_{SM} + ... \tag{2.33}$$

where the EW scale effective theory contains, in addition to the operators already in the Standard Model, some new higher dimensional operators and also some completely new fields.

Chapter 3

Bosonic NMWT

Bosonic TC models are theoretically appealing because they are complete up to the scale of quantum gravity. They also allow us to directly write the operators giving mass to SM fermions and to study in detail the interplay of the technicolor sector and the extension. In this chapter, we will show that current experimental limits allow a bosonic technicolor model where the technicolor sector corresponds to NMWT. The fundamental Higgs field is kept relatively light, and an important feature of this model is that it can be viable even with a small anomalous dimension of the technicolor condensate.

3.1 Effective Lagrangian

In Paper I, we studied a bosonic technicolor model based on Next to Minimal Walking Technicolor [32, 42, 59, 60]. The NMWT extension of the SM has the following gauge group:

$$SU(3)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y \tag{3.1}$$

while its particle spectrum features all the SM particles besides the Higgs scalar, plus an EW techniquark doublet in the two-index symmetric representation of the technicolor gauge group, $SU(3)$. The two flavors are arranged into one doublet of $SU(2)_L$, and taking the technicolor degree of freedom into account implies that we are adding six doublets of $SU(2)_L$ and there is no Witten anomaly. The techniquarks are denoted as

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \qquad U_R^a\ , \quad D_R^a\ , \qquad a = 1 \ldots 6, \tag{3.2}$$

with a being the color index of $SU(3)$. The following hypercharge assignment is anomaly free:

$$Y(U_L) = Y(D_L) = 0, Y(U_R) = \frac{1}{2}, Y(D_R) = -\frac{1}{2}. \tag{3.3}$$

The meson field, transforming under the technicolor global chiral symmetry $SU(2)_L \times SU(2)_R$ as $M \to LMR^\dagger$, is written similarly to the field in Section 2.1:

$$M = \frac{1}{\sqrt{2}} \left(sI_{2\times 2} + 2i\pi_M \right) \propto Q_L \bar{Q}_R, \quad \langle s \rangle \equiv f. \tag{3.4}$$

The conserving potential of this sector alone is then

$$\mathcal{L}_{TC} = \frac{1}{2}\mathrm{Tr}\left[DM^{\dagger}DM\right] + \frac{1}{2}m_M^2\mathrm{Tr}\left[M^{\dagger}M\right] - \frac{\lambda_M}{4!}\mathrm{Tr}\left[M^{\dagger}M\right]^2. \tag{3.5}$$

The Higgs field is written in a form analogous to M:

$$H = \frac{1}{\sqrt{2}}\left(hI_{2x2} + 2i\pi_H\right), \quad \langle h\rangle \equiv v. \tag{3.6}$$

The Higgs field transforms under the gauge group $SU(2)_L \times U(1)$ as $H \to g_L H g_Y^{\dagger}$, where the U(1) is generated by τ_3. The Higgs Lagrangian is:

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{2}\mathrm{Tr}\left[DH^{\dagger}DH\right] - V_H,\ V_H = \frac{1}{2}m_H^2\mathrm{Tr}\left[H^{\dagger}H\right] + \frac{\lambda_H}{4!}\mathrm{Tr}\left[H^{\dagger}H\right]^2. \tag{3.7}$$

We include the effects of the Yukawa interactions in the effective Lagrangian via the spurion technique. The Yukawa terms are of the form

$$\mathcal{L}_{\mathrm{Yukawa}} = -\sum_{i=q,l,Q}\bar{\Psi}_{L,i}HY^i\Psi_{R,i}\ ,\ \ Y^i = y^i I_{SU(2)} + \delta y^i\sigma_3, \tag{3.8}$$

where the sum is over SM quarks, leptons, and also techniquarks. Of current interest is the Yukawa term of the techniquarks:

$$-\bar{Q}_L HY_Q Q_R, \tag{3.9}$$

To preserve the TC chiral symmetry, the spurion field HY_Q should transform as $HY_Q \to G_L HY_Q G_R^{\dagger}$ under $G_{L/R} \in SU(2)_{L/R}$.

We include all possible terms up to and including dimension four operators consistent with symmetries. We omit terms of order $\mathcal{O}(Y_Q^2)$ and higher, which are assumed to be small. Because the Higgs field is not integrated out we have implicitly assumed it to be light or that its running effects are not important. We arrive at the following effective low-energy Lagrangian for the technicolor sector and its coupling with the fundamental Higgs field:

$$\begin{aligned}
\mathcal{L}_{TC} - \bar{Q}_L HY_Q Q_R &\to \frac{1}{2}\mathrm{Tr}\left[DM^{\dagger}DM\right] + c_3\mathrm{Tr}\left[DM^{\dagger}DHY_Q\right] - V_M \\
V_M &= \frac{1}{2}m_M^2\mathrm{Tr}\left[M^{\dagger}M\right] + \frac{\lambda_M}{4!}\mathrm{Tr}\left[M^{\dagger}M\right]^2 \\
&\quad - c_1\mathrm{Tr}\left[M^{\dagger}HY_Q\right] - \frac{1}{6}c_2\mathrm{Tr}\left[M^{\dagger}M\right]\mathrm{Tr}\left[M^{\dagger}HY_Q\right] \\
&\quad - \frac{1}{6}c_4\mathrm{Tr}\left[H^{\dagger}H\right]\mathrm{Tr}\left[M^{\dagger}HY_Q\right] + \mathrm{h.c.}
\end{aligned} \tag{3.10}$$

The dimensionless coefficients c_1 ... c_4 are taken to be real to preserve the CP symmetry. The W mass is given by

$$m_W^2 = \frac{g^2}{4}\left(f^2 + v^2 + 2c_3 y_Q f v\right). \tag{3.11}$$

3.2 Naive Dimensional Analysis

We apply Naive Dimensional Analysis (NDA) [3] to estimate some of the coefficients of (3.10). NDA estimates arise from the expectation that in a strongly coupled theory, each loop order in the strong coupling should contribute equally [4], independent of what field variables one uses to describe phenomena. NDA was further developed in [5]. According to the NDA rules, the magnitudes of diagrams depend on Λ, the mass of some low lying non-Goldstone state, and $h_N \lesssim 4\pi$, which estimates the size of loop corrections in the theory:

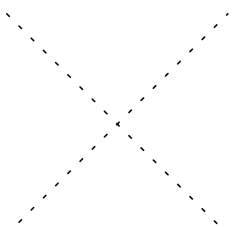
1. For each perturbative coupling g in the underlying theory one defines a rescaled coupling $\hat{g}$ by requiring the overall magnitude of the interaction vertex to be $h_N^{E-2}\hat{g}$, where E is the number of legs on the vertex.

2. The magnitude of any diagram in the effective theory is estimated to be h_N^{E-2} times any hatted perturbative coefficients needed to draw the diagram in the underlying theory, and the dimension is fixed by multiplying with appropriate powers of Λ.

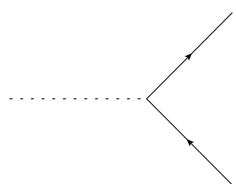
We will now apply these rules to fix the coefficients of the Lagrangian (3.10). First look at the interaction terms:

$$\frac{\lambda_H}{4!}\mathrm{Tr}\left[H^\dagger H\right]^2 - \bar{Q}_L H Y_Q Q_R \ni \frac{\lambda_H}{4!}h^4 - \bar{U}_L \frac{h}{\sqrt{2}} y_Q U_R. \tag{3.12}$$

From the Feynman rules (I have omitted the usual imaginary unit and ignore factors of $\sqrt{2}$), we define corresponding hatted couplings:



$$= \lambda_H \overset{NDA}{\sim} \hat{\lambda}_H h_N^2,$$

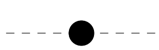


$$= \frac{y_Q}{\sqrt{2}} \overset{NDA}{\sim} \hat{y}_Q h_N,$$

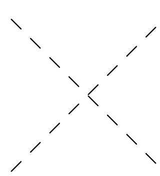
where the dotted line corresponds to h and the fermion line to U. Next, apply rule two to the symmetry conserving Lagrangian (3.5)

$$\frac{1}{2}m_M^2 \mathrm{Tr}\left[M^\dagger M\right] - \frac{\lambda_M}{4!}\mathrm{Tr}\left[M^\dagger M\right]^2 \ni \frac{1}{2}m_M^2 s^2 - \frac{\lambda_M}{4!}s^4. \tag{3.13}$$

These operators contain no perturbative coefficients, so their NDA value is directly given by the number of external legs in the corresponding graph:



$$= m_M^2 \overset{NDA}{\sim} \Lambda^2,$$

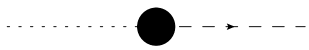


$$= \lambda_M \overset{NDA}{\sim} h_N^2,$$

Finally, we apply the rule 2 to the symmetry breaking part of the Lagrangian. First:

$$-c_1\mathrm{Tr}\left[M^\dagger H Y_Q\right] + c_3\mathrm{Tr}\left[DM^\dagger DHY_Q\right] \ni -c_1 y_Q sh + c_3 y_Q \partial s \partial h, \tag{3.14}$$

and then we compare the Feynman rules of the effective theory to the corresponding tree level graphs in the underlying theory:

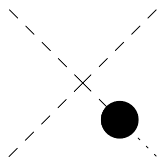


$$\begin{aligned} &= c_1 y_Q + k^2 c_3 y_Q \overset{NDA}{\sim} \Lambda^2 \hat{y}_Q + k^2 \hat{y}_Q \\ &= \Lambda^2 y_Q/h_N + k^2 y_Q/h_N. \end{aligned}$$

The other terms (3.10) are given by

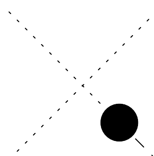
$$-\frac{1}{6}c_2\mathrm{Tr}\left[M^\dagger M\right]\mathrm{Tr}\left[M^\dagger H Y_Q\right] - \frac{1}{6}c_4\mathrm{Tr}\left[H^\dagger H\right]\mathrm{Tr}\left[M^\dagger H Y_Q\right] \ni -\frac{1}{6}c_2 y_Q s^3 h - \frac{1}{6}c_4 y_Q h^3 s \tag{3.15}$$

with corresponding graphs



$$= c_2 y_Q \overset{NDA}{\sim} \hat{y}_Q h_N^2 = h_N y_Q,$$

and



$$= c_4 y_Q \overset{NDA}{\sim} \hat{\lambda}_H \hat{y}_Q h_N^2 = \lambda_H y_Q/h_N.$$

Therefore one finds: $(c_1)_{NDA} = \Lambda^2/h_N$, $(c_2)_{NDA} = h_N$, $(c_3)_{NDA} = 1/h_N$, $(c_4)_{NDA} \sim \lambda_H/h_N$. In Paper I we used Georgi's version of NDA [4], not the one we introduced in [5]. In this estimate $(c_4)_{NDA} = 1/h_N$. The effect on final results of this difference is likely small, since $\lambda_H \sim 1$.

In walking theories the dilaton is expected to be light. This is not taken into account correctly in the NDA rules, so instead of using the NDA estimates for the symmetry conserving couplings, we simply scan over the whole parameter space of λ_M and m_M^2.

3.3 Oblique Corrections

The oblique corrections S and T, previously discussed in Section 2.3, are given by

$$\begin{aligned} S &= -16\pi\Pi'_{3Y}(0), \\ T &= \frac{4\pi}{s_w^2 c_w^2 M_Z^2}(\Pi_{11}(0) - \Pi_{33}(0)), \end{aligned} \tag{3.16}$$

In any BTC model, the theoretical determination of S and T contain large uncertainties that stem from both the non-perturbative nature of the TC sector and the coupling of the technicolor sector to the Higgs sector. Specifically, one must handle these contributions correctly:

- The naive estimate
$$S_{naive} = \frac{N_F d(r)}{6\pi} \tag{3.17}$$
calculated via the high energy spectrum only, should correspond to the full contribution of the technicolor sector for a given reference Higgs mass.

- The full S and T to be compared to experiments should contain also the contribution from the other scalar sector as well as the interplay among the two. This is possible only when evaluating S and T via the low-energy spectrum.

- If M is the technicolor composite matrix which transforms transforms under $SU(2)_L \times SU(2)_R$ as $M \to LMR^\dagger$, and $W^{\mu\nu}$ and $B^{\mu\nu}$ are the field strength tensors of $SU(2)_L$ and $U(1)_Y$, then at dimension six there would appear tree level contributions to S via the operator
$$\mathrm{Tr}\left[M^\dagger W^{\mu\nu} M B_{\mu\nu}\right]. \tag{3.18}$$

In many of the original articles it was assumed that the contribution from (3.18) should correspond to (3.17), and that in addition one should calculate the perturbative one-loop contribution of the scalar sectors. However, this calculation likely gives an overestimate of S, because in principle, S could be fully evaluated via the low-energy technihadron spectrum only. The low-energy spectrum is, in turn, connected to the UV properties of the underlying theory via dispersion relations. Of course, if we introduce explicitly only the scalar states in the low-energy effective theory, it is impossible to estimate the effects of other resonances, such as the vector mesons.

In Paper I, we considered two different limits when determining S and T. First, if all technicolor composites other than the scalar mesons are decoupled, one can compute the full contribution of the model within the effective theory, given in (3.7) and (3.10). This special limit of the parameter space corresponds to the generic two Higgs doublet model and this is the limit we use to compare to experiments. Second, if the vectors mesons do not decouple, their effect is estimated by adding $S_{\mathrm{naive}} = 1/\pi$ on the top of the S computed within the effective theory. In this case we are overestimating the contribution of the technicolor sector.

To calculate the contributions of the physical scalar triplet of $SU(2)_L$, one has to evaluate the diagrams in Fig. 3.1 for the vacuum polarization Π_{3Y} contributing to the S parameter, and the diagrams shown in Fig. 3.2 for the vacuum polarizations Π_{11} and Π_{33}, affecting the T parameter.

Finally, note that the origin of the (S, T)-plane corresponds to the SM with a given value of the mass of the Higgs particle denoted by m_{ref}. This is taken into account in the theoretical calculation of S by noting that

$$S = S_{\mathrm{SM}}(m_{\mathrm{ref}}) - S_H(m_{\mathrm{ref}}) + S_{\mathrm{new}} = S_{\mathrm{new}} - S_H(m_{\mathrm{ref}}), \tag{3.19}$$

where, by definition, $S_{\mathrm{SM}}(m_{\mathrm{ref}}) = 0$, and S_{new} contains all possible contributions calculated within the new model. Similar considerations apply also to T. Because the SM is only logarithmically sensitive to the Higgs mass, the one-loop calculations of S and T make the results practically independent of the reference Higgs mass m_{ref}. This is demonstrated in Fig. 3.3.

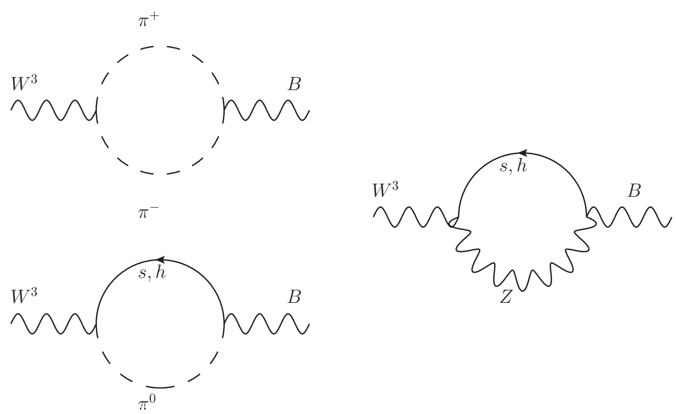


Figure 3.1: The diagrams required for the perturbative evaluation of the vacuum polarization Π_{3Y} within the effective theory.

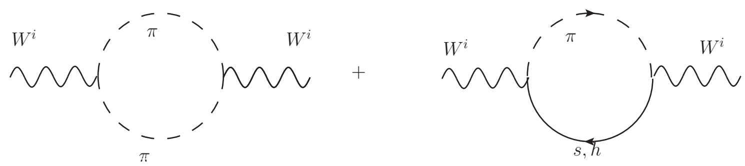


Figure 3.2: Diagrams required for the perturbative evaluation of the vacuum polarization Π_{11} within the effective theory. The vacuum polarization Π_{33}, also needed for T parameter, is obtained from similar diagrams with replacement $W^1 \to W^3$.

3.4 Random Parameter Scan

In order to be able to compare the bosonic NMWT model with experimental results we have to

- diagonalize the kinetic terms and the potential
- solve the extremum equations of the potential
- go to the unitary gauge, in which the electroweak gauge bosons are massive and only three physical pions remain in the spectrum
- generate randomly a dense set of values for the free couplings
- calculate masses and apply experimental constraints

The left panel of Fig. 3.4 is an S-T plot with the a reference Higgs mass $m_{\text{ref}} = 117$ GeV. All points pass the FCNC requirements, but the light red diamonds correspond to parameter values that are excluded on the basis of direct searches, soon to be discussed. The leftmost points are the result of the calculation within the effective theory. To these we add S_{naive}, representing the effects of vector mesons, to obtain the rightmost set of points, but we now focus on the leftmost set of points. The right panel of Fig. 3.4 shows the projection of the full data set presented in the left panel, with both red and black triangles, onto the (m_h, m_s)

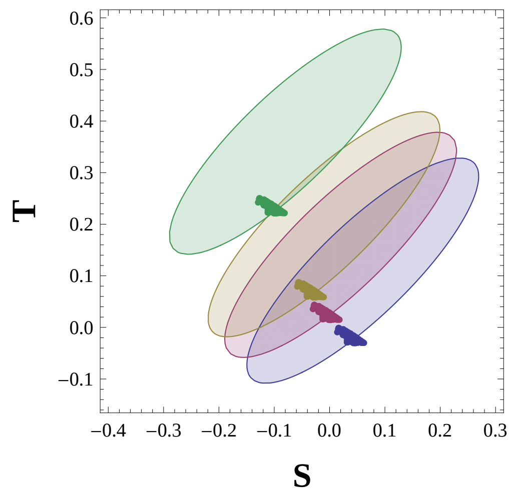


Figure 3.3: This figure shows the independence of S and T from the reference Higgs mass m_{ref}. The figure shows the 3σ ellipse and some sample points from our calculation in Paper I for different values of m_{ref}.

plane. In this figure the black triangles denote the points consistent with the 90% confidence contour of the S-T plot. The blue circles correspond to the points within the larger ellipsis of the S-T plot. Finally, the light red diamonds correspond to points still farther from the experimentally allowed region.

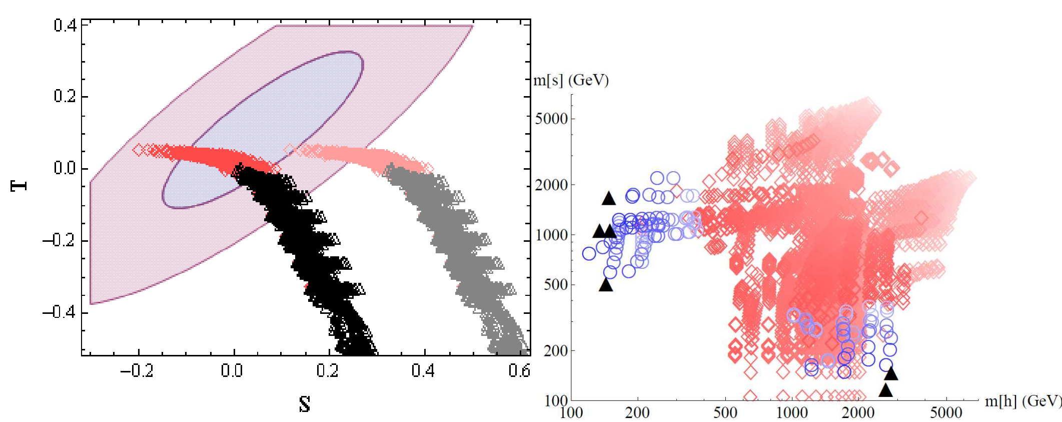


Figure 3.4: In both plots all points pass the FCNC requirements. Left: The results of the model and the 90% confidence limit contour allowed by all electroweak data for $m_{\text{ref}} = 115$ GeV. The light red diamonds are excluded by direct observations while the black triangles are not. Right: Black triangles show the points consistent with the 90% S-T confidence limit. Blue circles correspond to triangles in the left panel that are within the larger ellipse and the red diamonds correspond to to triangles even farther out. Points shaded in a lighter color are also farther from the experimentally allowed region. We have not required the LEP direct search limit $m_H > 114$ GeV in the right side panel.

Direct searches were taken into account with the newest Higgs mass limits during the time of writing: the LEP direct search limit $m_H > 114$ GeV [61]. The constraint coming from the lightest scalar decay into $b\bar{b}$ was taken into account in the simple approximation in which the coupling is the same as in the SM. This leads to an overestimate of the lightest scalar decay width into fermions. This means that the allowed regions would extend to lower values of the physical scalar masses, especially for the mostly composite scalar, s. It is also possible that the lighter scalar could evade detection below the LEP direct search limit, if its coupling to two Z-bosons is suppressed. However, we found that the points favored by the electroweak precision data do not have a suppressed scalar-ZZ coupling.

Finally, imposing the FCNC constraints forbids the area of parameter space where the value of the condensate v and the mass of the physical pion m_π are both small. This is evident when looking at the left panel of Fig. 3.5. In the right panel of Fig. 3.5 we show the allowed values of the condensates $|f|$ and $|v|$. We see that the allowed region for these parameters for each condensate changes roughly in the range of 50 to 400 GeV.

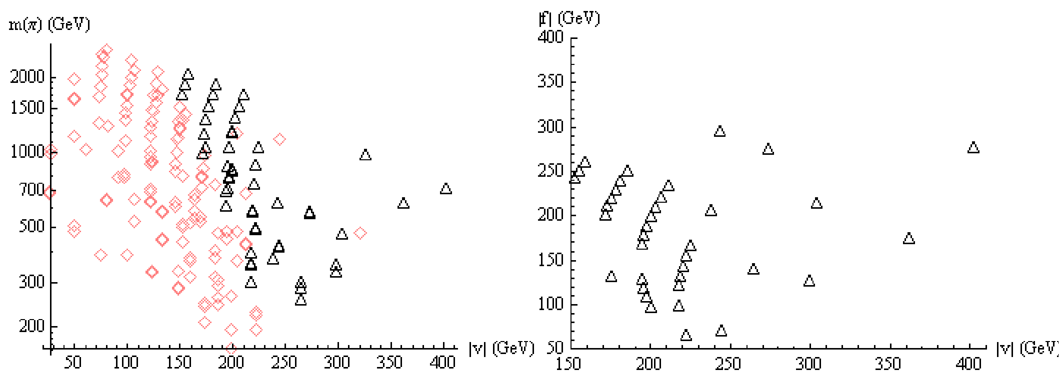


Figure 3.5: Left: The FCNC constraints on parameters m_π and $|v|$ on points satisfying direct search and S-T 90% confidence limit. Light red diamonds are disallowed, while black triangles are allowed. Right: The allowed values of the condensates $|f|$ and $|v|$ after taking all constraints into account.

More recently this model has also been studied in [31]. In that paper the authors have included vector resonances in the model and calculated their contribution to the FCNCs. The results were very similar where applicable.

The basic finding of this study, evident from Fig. 3.4, is that one of the scalars should be light and the other heavy, if no other contributions exist. This seems to guarantee a small contribution to the precision data. This parameter region is also relevant for SUSY TC theories, which will be introduced in Section 4.4.

Chapter 4

Supersymmetry

Supersymmetry is extremely appealing theoretically, as it is the only possible extension of space-time symmetries in field theories. Supersymmetry is also seen as a requirement for a string theory ultraviolet completion, and it has led to a plethora of nontrivial results in field theory. Under supersymmetry, bosons and fermions are paired. Importantly, because of non-renormalization theorems, supersymmetry stabilizes scalar masses, hence allowing naturally light scalars. Therefore, SUSY has been postulated to explain the lightness of the EW scale compared with the Planck scale.

In most phenomenological models, one assumes soft SUSY breaking terms to describe SUSY breaking. These terms emulate the true mechanism behind SUSY breaking. If soft SUSY breaking terms are relevant for EWSB, then measurements from LEP and LHC should find superpartners, the supersymmetric counterparts of the ordinary SM particles. The lack of such a finding is called the "little hierarchy problem", since it relates to why the weak scale is so much smaller than the SUSY breaking scale (instead of the Planck scale for the "large hierarchy problem"). Furthermore, unless the soft SUSY breaking terms are postulated to satisfy flavor symmetries, they also contribute to FCNCs.

In SUSY TC models, one combines the paradigms of technicolor and SUSY [62, 25, 63, 64, 65, 66, 27, 24]. In these models, technicolor explains the little hierarchy. On the other hand, SUSY naturalizes the masses of the scalars in the bosonic technicolor flavor extension. The fundamental Higgs fields do not participate in electroweak symmetry breaking but they simply act as messengers between the symmetry breaking sector and the quarks and leptons. To illustrate, QCD operates within the MSSM much like TC does in these models.

4.1 Supersymmetry and How to Break it Softly

Supersymmetry is a space-time symmetry that relates bosons and fermions. The supersymmetry generator Q should change fermionic states with bosonic states, as seen in Table 4.1. Therefore particles and their superpartners form multiplets under supersymmetry. In the minimal case, the multiplets consist of e.g. a fermion and a scalar, or a fermion and a vector boson. If there are $\mathcal{N}$ SUSY generators Q_i, where $\mathcal{N}$ is larger than one, then the supersymmetry is called extended, and in this case the schematic action is displayed in Table 4.2. In this case the multiplets under SUSY are larger. In the extreme case of $\mathcal{N} = 4$, one multiplet

State	Spin
$\lvert s\rangle$	s
$Q\lvert s\rangle$	$s-\frac{1}{2}$

Table 4.1: The action of the supersymmetry generator Q on a state of spin s decreases spin by 1/2.

State	Spin
$\lvert s\rangle$	s
$Q_1\lvert s\rangle$	$s-\frac{1}{2}$
$Q_2\lvert s\rangle$	$s-\frac{1}{2}$
$Q_1Q_2\lvert s\rangle$	$s-1$

Table 4.2: Assuming two supersymmetry generators Q_1 and Q_2, one can find three states with different spins starting from one single particle state.

contains the full gauge theory with scalars, fermions and vector bosons. Traditionally $\mathcal{N}=4$ symmetry is thought to be irrelevant for phenomenology, since the SM is a chiral theory while $\mathcal{N}=4$ symmetry allows only vector-like matter. This is circumvented in a model presented in Chapter 5, which has an explicitly broken approximate $\mathcal{N}=4$ symmetry.

The supersymmetry algebra is an extension of space-time symmetries, as is evident from the following algebra, given for one supersymmetry generator [67]:

$$\left\{Q_\alpha, Q^\dagger_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu \quad \{Q_\alpha, Q_\beta\} = 0 = \left\{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\right\} \tag{4.1}$$

$$[Q_\alpha, P^\mu] = 0 = \left[Q^\dagger_{\dot\alpha}, P^\mu\right]. \tag{4.2}$$

Here we have used the notation of two component complex spinor indices, where $\alpha = 1,2$ and $\dot\alpha = 1,2$. One can intuitively think that dotted indices refer to right handed fields and undotted to left handed ones. Since

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha}, \tag{4.3}$$

it is possible to describe all field theories with only left handed degrees of freedom, and using their complex conjugates as the right handed ones. In the rest of this thesis we will not need to write the spinor indices explicitly.

The Lagrangian of a supersymmetric theory depends on the particle content and the superpotential. The Lagrangian in terms of component fields is given by

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{g-Yuk} + \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_{P-Yuk} + \mathcal{L}_{soft}, \tag{4.4}$$

where the labels refer to the kinetic terms, the Yukawa terms given by gauge and superpotential interactions, the D and F scalar interaction terms, and the soft SUSY breaking terms. All these terms can be expressed in function of the elementary fields of the theory with the help of the following equations:

$$
\begin{aligned}
\mathcal{L}_{kin} &= -\frac{1}{4}F_j^{\mu\nu a}F^a_{j\mu\nu} - i\bar{\lambda}^a_j\bar{\sigma}^\mu D_\mu\lambda^a_j - D^\mu\phi_i^{a\dagger}D_\mu\phi^a_i - i\bar{\chi}^a_i\bar{\sigma}^\mu D_\mu\chi^a_i \ , && (4.5)\\
\mathcal{L}_{g-Yuk} &= \sum_j i\sqrt{2}g_j\left(\phi_i^\dagger T^a_j\chi_i\lambda^a_j - \bar{\lambda}^a_j\bar{\chi}_i T^a_j\phi_i\right) \ , && (4.6)\\
\mathcal{L}_D &= -\frac{1}{2}\sum_j g_j^2\left(\phi_i^\dagger T^a_j\phi_i\right)^2 \ , && (4.7)\\
\mathcal{L}_F &= -\left|\frac{\partial P}{\partial\phi^a_i}\right|^2 \ , && (4.8)\\
\mathcal{L}_{P-Yuk} &= -\frac{1}{2}\left[\frac{\partial^2 P}{\partial\phi^a_i\partial\phi^b_l}\chi^a_i\chi^b_l + h.c.\right], && (4.9)
\end{aligned}
$$

where i, l run over all the scalar field labels, while j runs over all the gauge group labels, and a, b are the corresponding gauge group indices. The superpotential is a function of the chiral superfields of the theory, but not of their conjugates:

$$P = P(\Phi). \tag{4.10}$$

We normalize the generators in the usual way, by taking the index $T(F) = \frac{1}{2}$, where

$$\mathrm{Tr}T^a_R T^b_R = T(R)\delta^{ab}, \tag{4.11}$$

with R here referring to the representation (F=fundamental).

The Lagrangian of an $\mathcal{N} = 4$ supersymmetric gauge theory can be written in terms of three $\mathcal{N} = 1$ chiral superfields $\Phi_i,\ i = 1, 2, 3$ and one $\mathcal{N} = 1$ vector superfield V, all in the adjoint representation of $SU(N)$. The superpotential for this Lagrangian reads (see [68] and references therein)

$$P = -\frac{g}{3\sqrt{2}}\epsilon_{ijk}f^{abc}\Phi^a_i\Phi^b_j\Phi^c_k,\ i, j, k = 1, 2, 3; a, b, c = 1, \cdots, N^2 - 1; \tag{4.12}$$

where g is the gauge coupling constant, and f^{abc} are the structure constants. The $\mathcal{N} = 4$ symmetry requires that the coefficient of this term is equal to the gauge coupling. This superpotential is invariant under $SU(3)$ flavor transformations. In terms of the component fields the full Lagrangian can be expressed as

$$
\begin{aligned}
\mathcal{L} \ =\ & -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - D^\mu\phi_i^{a\dagger}D_\mu\phi^a_i - i\bar{\psi}^a_i\bar{\sigma}^\mu D_\mu\psi^a_i\\
& +\sqrt{2}gf^{abc}\left(\phi_i^{a\dagger}\psi^b_i\lambda^c + \bar{\lambda}^c\bar{\psi}^b_i\phi^a_i\right) + \frac{g}{\sqrt{2}}\epsilon_{ijk}f^{abc}\left(\phi^a_i\psi^b_j\psi^c_k + \bar{\psi}^c_k\bar{\psi}^b_j\phi_i^{a\dagger}\right)\\
& -\frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\phi_i^{b\dagger}\phi^c_i\phi_j^{d\dagger}\phi^e_j,
\end{aligned} \tag{4.13}
$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu,\ D_\mu\xi^a = \partial\xi^a - gf^{abc}A^b_\mu\xi^c,\ \xi = \lambda, \psi_i, \phi_i. \tag{4.14}$$

Here λ is the gaugino field and ψ_i and ϕ_i are respectively the fermionic and scalar component of the superfield Φ_i. To make the $SU(4)$ R-symmetry of the Lagrangian explicit, the following change of variables provides useful:

$$\varphi^a_{rs} = -\varphi^a_{sr},\ \varphi^a_{i4} = \frac{1}{2}\phi^a_i,\ \varphi^a_{ij} = \frac{1}{2}\epsilon_{ijk}\phi^{a\dagger}_k,\ \eta^a_i = \psi^a_i,\ \eta^a_4 = \lambda^a;\ r,s = 1,\cdots,4. \tag{4.15}$$

Now Eq.(4.13) becomes

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} - \mathrm{Tr}D^\mu\varphi^{a\dagger}D_\mu\varphi^a - i\bar{\eta}^a_r\bar{\sigma}^\mu D_\mu\eta^a_r \\ & -\sqrt{2}gf^{abc}\left(\varphi^{a\dagger}_{rs}\eta^b_r\eta^c_s + \bar{\eta}^c_r\bar{\eta}^b_s\varphi^a_{rs}\right) \\ & -\frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\mathrm{Tr}\varphi^{b\dagger}\varphi^c\mathrm{Tr}\varphi^{d\dagger}\varphi^e.\end{aligned} \tag{4.16}$$

Under $SU(4)$ φ^a transforms as a $\mathbf{6}$, η^a as a $\mathbf{4}$, and A^a_μ as a $\mathbf{1}$, leaving the Lagrangian in Eq.(4.16) unchanged.

The world is not supersymmetric at observable energy scales. In particular, the superpartner of the electron, the selectron, should have the same mass as the electron since it is in the same supermultiplet. The absence of this particle is only explained by breaking supersymmetry. If supersymmetry would be broken, at the Lagrangian level, by dimensionless couplings, then it would be broken explicitly at all energy scales, hence defeating the purpose of SUSY in the first place. Instead, couplings with positive mass dimension correspond to superrenormalizable operators whose effects only show at low energies, thereby emulating spontaneous SUSY breaking. This mechanism is called soft SUSY breaking. The most general form of SUSY breaking terms are given as follows:

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + c.c. - (m^2)^i_j\phi^{j*}\phi_i, \tag{4.17}$$

$$\mathcal{L}_{\text{maybe soft}} = -\frac{1}{2}c^{jk}_i\phi^{*i}\phi_j\phi_k + c.c. \tag{4.18}$$

$$\mathcal{L} \;=\; -M^a_{\text{Dirac}}\lambda^a\psi_a + \text{c.c.} \tag{4.19}$$

Here λ are gauginos, ϕ spartners, and ψ chiral field fermions.

4.2 The MSSM and FCNCs

We now specify the particle content of the Minimal Supersymmetric Standard Model. There is a superpartner for each particle of the SM. In addition, there are two Higgs multiplets. The second Higgs field must be added e.g. because otherwise one cannot write mass terms for all SM fermions. The particle content is given in Tables (4.3) and (4.4).

Both Higgs fields contribute to the Z-boson mass, which is given by

$$m^2_Z = \frac{g^2_L + g^2_Y}{4}\left(v^2_u + v^2_d\right), \tag{4.20}$$

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$\bar{u}_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$\bar{d}_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$\bar{e}_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

Table 4.3: Chiral supermultiplets in the MSSM. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions. Taken from [69].

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

Table 4.4: Gauge supermultiplets in the MSSM. Taken from [69].

where $H_u^0 = (h_u^0 + i\pi_u^0)/\sqrt{2}$ and $\langle h_u^0\rangle = v_u$ and similarly for H_d. This fixes $v_u^2 + v_d^2 = v^2$ and we define a related parameter

$$\tan\beta = v_u/v_d. \tag{4.21}$$

The MSSM superpotential is given by

$$\begin{aligned} W_{\text{MSSM}} &= \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_uH_d\,. \\ &\approx y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) - y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu H_uH_d, \end{aligned}$$

where we show only the largest terms. From these equations we find the relation $m_t/m_b = 40.7 = y_t/y_b\tan\beta$. Since also the charm quark is heavier than the strange, it is natural to expect a rather large value for $\tan\beta$ such as $\tan\beta\sim 10$. In this case the Yukawa coupling of the bottom quark is almost as large as that of the top quark, in contrast to the SM where $y_t/y_b = 40.7$.

The mass terms from the soft SUSY breaking sector are:

$$\begin{aligned} L &= -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + h.c.\right) \\ &\quad -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger \\ &\quad -\widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger - m_{H_u}^2H_u^*H_u - m_{H_d}^2H_d^*H_d - (bH_uH_d + h.c.)\,. \end{aligned} \tag{4.22}$$

where $\mathbf{m_Q^2}$, $\mathbf{m_{\overline{u}}^2}$, $\mathbf{m_{\overline{d}}^2}$, $\mathbf{m_L^2}$, and $\mathbf{m_{\overline{e}}^2}$ are Hermitian 3×3 matrices in family space. Additionally there are the trilinear soft SUSY breaking a-terms from (4.17) which we will not discuss further. It is possible that the Higgs fields have positive masses at the high scale of SUSY breaking and quantum running produces $m_{H_u}^2 \ll m_{H_d}^2$ at the EW scale: in these models, electroweak symmetry breaking is actually driven by quantum corrections; this mechanism is therefore known as radiative electroweak symmetry breaking.

Off diagonal terms in the mass matrices of (4.22) induce FCNCs. For example, the gluino-squark-quark vertices are fixed by supersymmetry to be of the QCD interaction strength. This gives contributions to $K^0 \leftrightarrow \overline{K}^0$ mixing. Comparing to the limit on $\Delta m_K \equiv m_{K_L} - m_{K_S}$, one finds various limits [70], e.g.:

$$\frac{|\mathrm{Re}[(m^2_{\tilde{s}^*_R \tilde{d}_R})^2]|^{1/2}}{m^2_{\tilde{q}}} \lesssim 10^{-1} \left(\frac{m_{\tilde{q}}}{1\ \mathrm{TeV}}\right). \tag{4.23}$$

Here $m^2_{\tilde{s}^*_R \tilde{d}_R} = (\mathbf{m^2_d})_{21}$ is treated as a perturbation. This limit says that if the left hand side is $\mathcal{O}(1)$, the soft SUSY breaking scale should be $\mathcal{O}(10)$ TeV - $\mathcal{O}(100)$ TeV. This is a typical result, and shows that if the squark mass matrices are flavor blind, then the soft SUSY breaking scale must be much higher than the electroweak scale. An alternative explanation, that allows the SUSY breaking scale to be low, is to assume that the squark and slepton squared-mass matrices are flavor-blind and real, each proportional to the 3×3 identity matrix in family space. In this case, squarks and sleptons with the same electroweak quantum numbers have equal masses, and can be rotated into each other at will, thereby trivializing squark and slepton mixing angles. This is the hypothesis of soft supersymmetry breaking universality:

$$\mathbf{m^2_Q} = m^2_Q \mathbf{1}, \quad \mathbf{m^2_{\overline{u}}} = m^2_{\overline{u}} \mathbf{1}, \quad \mathbf{m^2_{\overline{d}}} = m^2_{\overline{d}} \mathbf{1}, \quad \mathbf{m^2_L} = m^2_L \mathbf{1}, \quad \mathbf{m^2_{\overline{e}}} = m^2_{\overline{e}} \mathbf{1}. \tag{4.24}$$

These relations are postulated to be the result of the SUSY breaking mechanism at some high energy scale, and the terms should first be renormalized to the electroweak scale before comparing with experiments.

4.3 The Little Hierarchy Problem in MSSM

The Higgs potential in the MSSM is given by:

$$V = (|\mu|^2 + m^2_{H_u})|H^0_u|^2 + (|\mu|^2 + m^2_{H_d})|H^0_d|^2 - (b\, H^0_u H^0_d + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H^0_u|^2 - |H^0_d|^2)^2. \tag{4.25}$$

Note that μ is a SUSY conserving parameter and hence should naturally be $\mathcal{O}(1)$ in Planck units. Such a large value would make the origin $H^0_u = H^0_d = 0$ stable. This so called μ-problem can be solved by extending the MSSM to make μ absent unless SUSY is broken [71]. Then it is automatically given by the soft SUSY breaking scale.

There are two independent extremum conditions arising from the potential (4.25), of which the second one can be written in terms of the Z-mass and expanded in powers of $1/\tan\beta$ as follows:

$$m^2_Z = -2\left(m^2_{H_u} + |\mu|^2\right) - \frac{2}{\tan^2\beta}\left(m^2_{H_u} - m^2_{H_d}\right) + \mathcal{O}\left(1/\tan^4\beta\right). \tag{4.26}$$

This relation shows that the natural scale for $m^2_{H_u}$ and $|\mu|^2$ is given by m^2_Z. Moreover, since the μ parameter is tied to the Higgsino mass, typical viable solutions for the MSSM therefore have both $-m_{H_u}^2$ and $|\mu|^2$ larger than m_Z^2, resulting in a fine tuning parametrized by $m_Z^2/m_{H_u}^2$. We can also calculate loop corrections to $m_{H_u}^2$: this parameter is protected from large corrections only up to non-SUSY physics, and thus it can receive large contributions from the mass difference between the top squark and the top quark. We find

$$\Delta m_{H_u}^2 = \frac{3}{4\pi^2} y_t^2 m_t^2 \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right), \tag{4.27}$$

where we assumed $m_{\tilde{t}}^2 = m_{\tilde{Q}_3}^2 = m_{\tilde{t}_R}^2$ and used $m_t = y_t v_u/\sqrt{2}$. The experimental bounds for $m_{\tilde{Q}_3}^2$ and $m_{\tilde{t}_R}^2$ are model dependent. For $m_{\tilde{t}}^2 = 500$ GeV one finds

$$\Delta m_{H_u}^2 \sim 0.7 m_Z^2. \tag{4.28}$$

This also illustrates the little hierarchy problem: for the phenomenologically viable parameters, it seems mysterious why the Z-boson should be so light.

To be more explicit, one should study a specific corner of the full parameter space of the MSSM, such as the Constrained MSSM (CMSSM). This model is defined at the unification scale by four parameters: $m_{1/2}$, the gaugino mass, m_0, the scalar mass, $\tan\beta$, and the sign of μ[1]. The fine tuning in this model, in light of the new LHC results, is illustrated in [72]. Within this model, taking into account running effects, the Z mass is predicted to be

$$m_Z^2 \approx 4.7 m_{1/2}^2 + 0.2 m_0^2 - 2\mu^2, \tag{4.29}$$

where the μ term is renormalized to the electroweak scale. The parameter $\tan\beta = 3$ is chosen so that the value of the top Yukawa coupling renormalized at the unification scale is $\lambda_t(M_{\text{GUT}}) \approx 0.5$. We can fix the overall SUSY mass scale via (4.29), so that the model has two free adimensional parameters: the ratios $m_{1/2}/\mu$ and m_0/μ. Such parameter space is plotted in Fig. 4.1:

- In the left light-gray regions one would have $m_Z^2 < 0$ which means that the true minimum of the scalar potential is at $v = 0$; in the bottom-right region the potential is unstable.

- The red region in the middle is experimentally excluded. The darker red shows the region excluded only by the LHC.

- The green region is allowed.

The smallness of the allowed region is a manifestation of the "little hierarchy problem": it is close to the boundary where $M_Z = 0$ and thereby has $M_Z \ll m_0, M_{1/2}, \mu$.

4.4 Supersymmetric Technicolor

To solve the little hierarchy problem, one needs a mechanism to generate the little hierarchy m_{SUSY}/m_Z. A possible mechanism could be technicolor. The relevant scales of the problem

[1]Sometimes the trilinear coupling A_0 is non-zero, in the current section it is chosen to vanish.

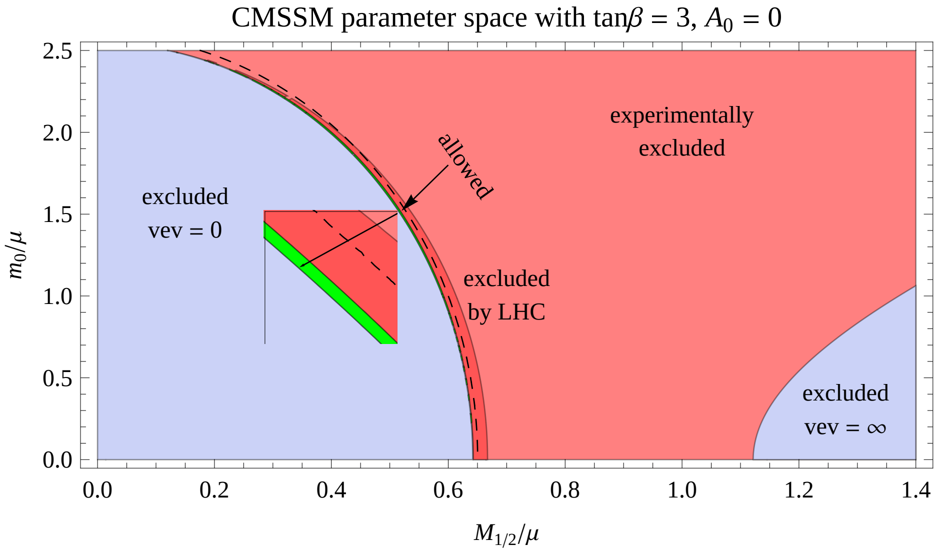


Figure 4.1: A typical example of the parameter space of the CMSSM model. The green region is allowed (see it in the enlarged box). Taken from [72].

are the supersymmetric breaking scale m_{SUSY} and the EW scale which we identify with the dynamically generated low-energy strongly coupled scale of the TC theory $m_{TC} \sim 4\pi v \sim 3$ TeV. The one loop relation for these scales reads

$$\beta = -\frac{\alpha^2}{\pi} b_1 \quad \to \quad m_{TC} = m_{SUSY} \exp\left[\frac{\pi}{b_1}\left(\frac{1}{\alpha_c} - \frac{1}{\alpha_{SUSY}}\right)\right]. \tag{4.30}$$

In this equation α_{SUSY} is the TC coupling value at the scale of the soft SUSY terms, m_{SUSY}. The other parameters are m_{TC}, which gives the electroweak scale, $b_1 > 0$, the first coefficient of the perturbative beta function, and α_c, the critical coupling for condensate formation. This relation can naturally explain a hierarchy $m_{SUSY} \gg m_{TC}$.

To estimate the maximal SUSY breaking scale, we use equation (2.3), which, for the top quark, reads:

$$m_t \sim \left(\frac{y_{H\bar{Q}Q}}{4\pi}\right)\left(\frac{y_{H\bar{t}t}}{4\pi}\right)\left(\frac{3\ \text{TeV}}{m_H}\right)^{2-\gamma_m^*} \times 3\ \text{TeV}. \tag{4.31}$$

Here m_H is the Higgs mass, which we identify with the SUSY breaking scale to avoid fine tuning. In Fig. 4.2 the value of γ_m^* is plotted as a function of m_H, assuming $m_t = 175$ GeV. This gives, for a given γ_m^* and for given values of coupling constants, the highest possible SUSY breaking scale that one can consider.

In the following we list pros and cons SUSY BTC models have, depending on the SUSY breaking scale:

1. Heavy regime, $m_{TC} \lesssim m_{SUSY} \lesssim 10$ TeV. In this regime the model brilliantly solves the little hierarchy problem. FCNCs related to additional scalar states are absent but the soft SUSY breaking terms cannot be completely arbitrary without violating FCNC limits. In Chapter 5 we review Paper III which discusses a specific SUSY TC model in this region.

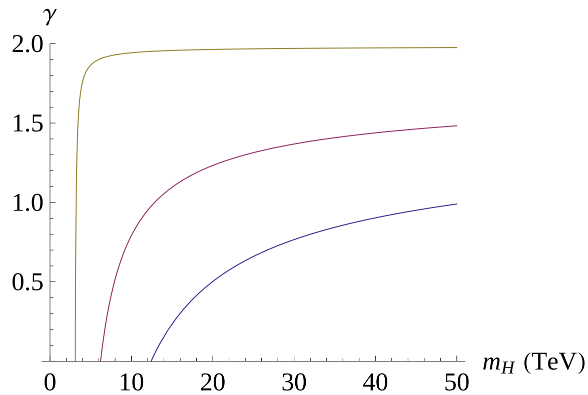


Figure 4.2: Maximum allowed mediator mass within walking TC models, given an anomalous dimension of the fermion bilinear at the approximate fixed point, the requirement to generate the top mass, and the values of the Yukawa couplings $y_{H\bar{t}t}$ and $y_{H\bar{Q}Q}$. For this illustration the coupling values are set to, counting from the bottom to top, 4π, 2π, and π.

2. Super heavy regime, $m_{SUSY} \gtrsim 10$ TeV. In this regime the theory can also, with little fine tuning, explain the absence of FCNCs induced by the soft SUSY breaking sector. However, to generate the top quark mass, the technicolor theory should be conformal and the techniquark bilinear anomalous dimension should be large.

3. Light regime $m_{SUSY} \lesssim m_{TC}$. The phenomenology in this regime is reminiscent of that in Chapter 3. There are experimental constraints from FCNCs and oblique corrections induced by extra scalar states. The soft SUSY breaking terms must conserve flavor symmetries to avoid large FCNCs. There is a problem of scales: why would the technicolor and SUSY breaking scales coincide? It is argued in [73] that this can be explained if the SUSY TC sector is at a strong conformal fixed point above the soft SUSY breaking scale. In this case chiral symmetry breaking could be triggered immediately at the soft SUSY breaking scale.

4. One can consider the tachyonic regime, i.e. the regime where the Higgs fields condense because of negative mass terms, even in the absence of interactions with the TC sector. However, this regime suffers from an elevated little hierarchy problem, since the soft SUSY breaking scale is lower than in the MSSM.

Chapter 5

Minimal Supersymmetric Conformal Technicolor

In this section we study a specific SUSY BTC model, the Minimal Supersymmetric Conformal Technicolor model (MSCT), which has the added interest that the SUSY TC sector is approximately $\mathcal{N} = 4$ supersymmetric. We then quantify how much the MSCT flavor extension modifies the vacuum, spectrum and dynamics of the original TC model, MWT. The modification of the TC dynamics and consequently of the phenomenology is substantial. We consider the SUSY breaking scale $m_{SUSY} \gtrsim 5$ TeV. We also consider the phenomenology of the scalar sector.

5.1 Minimal Walking Technicolor

We will supersymmetrize the Minimal Walking Technicolor model, which we now introduce in detail. The MWT extension of the SM has the following gauge group:

$$SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y , \tag{5.1}$$

with the particle spectrum of the SM particles besides the Higgs scalar, plus an EW techniquark doublet in the adjoint representation of $SU(2)_{TC}$ as well as its right-handed components:

$$Q_L^a = \left(\begin{array}{c} U^a \\ D^a \end{array} \right)_L , \qquad U_R^a \ , \quad D_R^a \ , \qquad a = 1,2,3 \ , \tag{5.2}$$

with a being the adjoint color index of $SU(2)_{TC}$. The left handed fields are arranged in three doublets of the $SU(2)_L$ weak interactions in the standard fashion. A new weakly charged fermionic doublet, which is a TC singlet [42], is added to cancel the Witten topological anomaly:

$$L_L = \left(\begin{array}{c} N \\ E \end{array} \right)_L , \qquad N_R \ , \ E_R \ . \tag{5.3}$$

The following hypercharge assignment is anomaly-free:

$$Y(Q_L) = \frac{y}{2} \ , \qquad Y(U_R, D_R) = \left(\frac{y+1}{2}, \frac{y-1}{2}\right) \ , \tag{5.4}$$

$$Y(L_L) = -3\frac{y}{2} \ , \qquad Y(N_R, E_R) = \left(\frac{-3y+1}{2}, \frac{-3y-1}{2}\right) \ . \tag{5.5}$$

The parameter y should be real, and electric charge is given by $Q = T_3 + Y$, where T_3 is the diagonal weak isospin generator.

The global symmetry of this theory is $SU(4)$. The following vector transforms according to the fundamental representation:

$$\eta = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 U_R^* \\ -i\sigma^2 D_R^* \end{pmatrix} , \tag{5.6}$$

where U_L and D_L are the left handed techniup and technidown fields, respectively, U_R and D_R are the corresponding right handed fields, and σ^2 is a Pauli matrix. Assuming the standard breaking to the maximal diagonal subgroup, $SU(4)$ spontaneously breaks to $SO(4)$. Such a breaking is driven by the following condensate:

$$\langle \eta_i^\alpha \eta_j^\beta \epsilon_{\alpha\beta} E^{ij} \rangle = -2\langle \overline{U}_R U_L + \overline{D}_R D_L \rangle \ , \ E = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \ . \tag{5.7}$$

where the indices $i, j = 1, \ldots, 4$ denote the components of the tetraplet of η, and the Greek indices indicate the ordinary spin. Here $\epsilon_{\alpha\beta} = -i\sigma^2_{\alpha\beta}$ and $\langle U_L^\alpha {U_R}^{*\beta} \epsilon_{\alpha\beta} \rangle = -\langle \overline{U}_R U_L \rangle$. This symmetry breaking pattern leaves us with nine broken generators with associated Goldstone bosons.

The most general $SU(2)_{TC}$ singlet fermion bilinear is $M \sim \eta\eta^T$. Therefore M transforms under the full $SU(4)$ group according to

$$M' = UMU^T \ , \qquad \text{with} \qquad U \in \mathrm{SU}(4). \tag{5.8}$$

The covariant derivative, which follows from the symmetry properties, is

$$DM = \partial M - i\left[GM + MG^T\right] \ , \tag{5.9}$$

with

$$G = g_L W^a L^a + g_Y B Y_M , \tag{5.10}$$

and

$$L^a = \begin{pmatrix} \frac{\sigma^a}{2} & 0 \\ 0 & 0 \end{pmatrix} , \qquad Y_M = \mathrm{diag}\left(\frac{y}{2}, \frac{y}{2}, -\frac{y+1}{2}, -\frac{y-1}{2}\right). \tag{5.11}$$

The connection between the composite scalars and the underlying techniquarks can be derived from the transformation properties under SU(4), by observing that the elements of the matrix M transform like techniquark bilinears:

$$M_{ij} \sim \eta_i^\alpha \eta_j^\beta \varepsilon_{\alpha\beta} \qquad \text{with } i, j = 1 \ldots 4. \tag{5.12}$$

Using this expression, the scalar fields can be related to the wave functions of the techniquark bound states.

The minimal linear representation for M is:

$$M = \left[\frac{\sigma + i\Theta}{2} + \sqrt{2}(i\Pi^a + \widetilde{\Pi}^a)\, X^a\right] E \ , \qquad \langle\sigma\rangle = f. \tag{5.13}$$

The nine generators of $SU(4)$ that do not leave the vacuum invariant are denoted X^a and can be found from [74]. Notice that it is necessary to introduce the "tilde" fields in the matrix M because the form of the matrix $M = \left(\sigma/2 + i\sqrt{2}\Pi^a\, X^a\right) E$ is not consistent under a general $SU(4)$ transformation. This is in contrast to the case of an $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ chiral group discussed in the introductory chapter (2.1), but is similar to the case of an $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ chiral group.

The effective Lagrangian for M, up to $SU(4)$ breaking terms, and prior to introducing a flavor extension, reads:

$$\mathcal{L}_{MWT} = \frac{1}{2}\mathrm{Tr}\left[DM^\dagger DM\right] - \mathcal{V}_M, \tag{5.14}$$

where

$$\mathcal{V}_M = -\frac{m^2}{2}\mathrm{Tr}\left[M^\dagger M\right] + \frac{\lambda}{4}\mathrm{Tr}\left[M^\dagger M\right]^2 + \lambda'\mathrm{Tr}\left[M^\dagger M M^\dagger M\right] - 2\lambda''\left[\det M + \det M^\dagger\right]. \tag{5.15}$$

Notice that the determinant terms explicitly break the axial $U(1)$ symmetry, and give mass to Θ, which would otherwise be a massless Goldstone boson. $SU(4)$ breaks spontaneously to $SO(4)$ for positive m^2. Stability of the potential furthermore requires

$$\lambda > 0, \lambda' > 0, \lambda + \lambda' > \lambda'' > 0. \tag{5.16}$$

The potential $\mathcal{V}(M)$ is SU(4) invariant. It produces a vev which parametrizes the techniquark condensate. The mass of the W boson is given by

$$m_W = \frac{g_L f}{2}, \tag{5.17}$$

from where it follows that $f = v$. In terms of the model parameters the vev is

$$v^2 = \langle\sigma\rangle^2 = \frac{m^2}{\lambda + \lambda' - \lambda''}, \tag{5.18}$$

Spontaneous symmetry breaking generates masses for most of the scalars. In terms of the mass eigenstates M can be written

$$M = \begin{pmatrix} i\Pi_{UU} + \widetilde{\Pi}_{UU} & \frac{i\Pi_{UD} + \widetilde{\Pi}_{UD}}{\sqrt{2}} & \frac{\sigma + i\Theta + i\Pi^0 + A^0}{2} & \frac{i\Pi^+ + A^+}{\sqrt{2}} \\ \frac{i\Pi_{UD} + \widetilde{\Pi}_{UD}}{\sqrt{2}} & i\Pi_{DD} + \widetilde{\Pi}_{DD} & \frac{i\Pi^- + A^-}{\sqrt{2}} & \frac{\sigma + i\Theta - i\Pi^0 - A^0}{\sqrt{2}} \\ \frac{\sigma + i\Theta + i\Pi^0 + A^0}{2} & \frac{i\Pi^- + A^-}{\sqrt{2}} & i\Pi_{\overline{UU}} + \widetilde{\Pi}_{\overline{UU}} & \frac{i\Pi_{\overline{UD}} + \widetilde{\Pi}_{\overline{UD}}}{\sqrt{2}} \\ \frac{i\Pi^+ + A^+}{\sqrt{2}} & \frac{\sigma + i\Theta - i\Pi^0 - A^0}{2} & \frac{i\Pi_{\overline{UD}} + \widetilde{\Pi}_{\overline{UD}}}{\sqrt{2}} & i\Pi_{\overline{DD}} + \widetilde{\Pi}_{\overline{DD}} \end{pmatrix}. \tag{5.19}$$

The off-diagonal 2×2 matrices contain technimesons ($\bar{Q}Q$-type states) and the diagonal 2×2 matrices contain baryons (QQ-type states). The linear combination $\lambda + \lambda' - \lambda''$ corresponds to the Higgs particle's self-coupling, while its mass is

$$M_\sigma^2 = 2\ m^2. \tag{5.20}$$

The three pseudoscalar mesons $\Pi^\pm$, Π^0 correspond to the three massless Goldstone bosons which are absorbed into the longitudinal degrees of freedom of the $W^\pm$ and Z boson. The remaining six uneaten Goldstone bosons are technibaryons, which are at this stage still massless:

$$M^2_{\Pi_{UU}} = M^2_{\Pi_{UD}} = M^2_{\Pi_{DD}} = 0. \tag{5.21}$$

The remaining scalar and pseudoscalar masses are

$$\begin{aligned} M_\Theta^2 &= 4v^2\lambda'' \\ M^2_{A^\pm} = M^2_{A^0} &= 2v^2\left(\lambda' + \lambda''\right) \end{aligned} \tag{5.22}$$

for the technimesons, and

$$M^2_{\widetilde{\Pi}_{UU}} = M^2_{\widetilde{\Pi}_{UD}} = M^2_{\widetilde{\Pi}_{DD}} = 2v^2\left(\lambda' + \lambda''\right) \tag{5.23}$$

for the technibaryons.

To give mass to the SM fermions, and to give mass to the yet massless Goldstone bosons, we must introduce a flavor extension of TC. In terms of the low-energy theory, this extension will explicitly break the $SU(4)$ symmetry. In the most naive extension we choose to preserve the full $SU(2)_L \times SU(2)_R \times U(1)_V$ subgroup of $SU(4)$. Also assuming parity invariance, we write:

$$\mathcal{L}_{\rm FE} = \frac{m^2_{\rm FE}}{4}\ {\rm Tr}\left[MBM^\dagger B + MM^\dagger\right] + \cdots \tag{5.24}$$

where the ellipses represent possible higher dimensional operators, $B \equiv 2\sqrt{2}S^4$ commutes with the $SU(2)_L \times SU(2)_R \times U(1)_V$ generators, and FE stands for flavor extension. Here we assume $m^2_{FE} \gtrsim 0$. With this flavor extension, the mass eigenstates are unchanged, and given by (5.19). The baryons that corresponded to Goldstone boson states now have a mass:

$$M^2_{\Pi_{UU}} = M^2_{\Pi_{UD}} = M^2_{\Pi_{DD}} = m^2_{\rm FE}. \tag{5.25}$$

Also the masses of the other baryons have changed:

$$M^2_{\tilde{\Pi}_{UU}} = M^2_{\tilde{\Pi}_{UD}} = M^2_{\tilde{\Pi}_{DD}} = m^2_{\rm FE} + 2v^2\left(\lambda' + \lambda''\right). \qquad (5.26)$$

Details concerning the vector boson sector or the incorporation of fermion masses in this naive extension are given in [74].

5.2 Microscopic Lagrangian

To build a supersymmetric technicolor theory we must supersymmetrize the additional technicolor sector, given by MWT, and also the SM. If we choose the hypercharge parameter (5.5) to be $y = 1$, we can construct an approximately $\mathcal{N} = 4$ supersymmetric technicolor sector, which is explicitly broken down to $\mathcal{N} = 1$ SUSY only by EW gauge and Yukawa interaction terms.

We start by noting that the fermionic and gluonic spectrum of MWT fits perfectly in an $\mathcal{N} = 4$ supermultiplet, provided that we also include three scalar superpartners. In fact the $SU(4)$ global symmetry of MWT is equivalent to the well known $SU(4)_R$ R symmetry of the $\mathcal{N} = 4$ Super Yang-Mills (4SYM) theory. The following $\mathcal{N} = 1$ supermultiplets have fermionic components already included in MWT:

$$\left(\tilde{U}_L,\ U_L\right) \in \Phi_1, \quad \left(\tilde{D}_L,\ D_L\right) \in \Phi_2, \quad \left(\tilde{\bar{U}}_R,\ \bar{U}_R\right) \in \Phi_3, \quad \left(G,\ \bar{D}_R\right) \in V. \qquad (5.27)$$

Here we use a tilde to label the scalar superpartner of each fermion. We indicate with Φ_i, $i = 1, 2, 3$ the three chiral superfields of 4SYM and with V the vector superfield. The superfields associated with the remaining MWT fermions N and E (the new leptons) are given by:

$$\left(\tilde{N}_L,\ N_L\right) \in \Lambda_1, \quad \left(\tilde{E}_L,\ E_L\right) \in \Lambda_2, \quad \left(\tilde{\bar{N}}_R,\ \bar{N}_R\right) \in N, \quad \left(\tilde{\bar{E}}_R,\ \bar{E}_R\right) \in E. \qquad (5.28)$$

The quantum numbers of the superfields appearing in Eqs.(5.27,5.28) and of those labeled by H_u and H_d, which contain each a Higgs scalar weak doublet, are given in Table 5.1.

Superfield	$\mathrm{SU}(2)_{TC}$	$\mathrm{SU}(2)_{\rm L}$	$\mathrm{U}(1)_{\rm Y}$
Φ_L	Adj	F	1/2
Φ_3	Adj	1	-1
V	Adj	1	0
Λ_L	1	F	-3/2
N	1	1	1
E	1	1	2
H_u	1	F	1
H_d	1	F	-1

Table 5.1: MSCT model in terms of $\mathcal{N} = 1$ superfields. Here Adj and F denote the adjoint and fundamental representations, respectively. All fields are uncharged under $\mathrm{SU}(3)_c$.

The renormalizable lepton and baryon number conserving superpotential for the MSCT is

$$P = P_{MSSM} + P_{TC}, \tag{5.29}$$

where P_{MSSM} is the MSSM superpotential, and

$$P_{TC} = -\frac{g_{TC}}{\sqrt{2}}\epsilon^{abc}\Phi_L^a \cdot \Phi_L^b \Phi_3^c + y_U \Phi^a \cdot H_u \Phi_3^a + y_N \Lambda \cdot H_u N + y_E \Lambda \cdot H_d E + y_R E \Phi_3^a \Phi_3^a. \tag{5.30}$$

Contraction between the $\mathrm{SU}(2)_L$ doublets by the antisymmetric two-index Levi-Civita tensor ϵ is indicated by a dot symbol $(\cdot)$. Gauge invariance and $\mathcal{N} = 1$ supersymmetry do not ensure the Yukawa coupling in the first term to be equal to g_{TC}, however, setting it to this value amounts to the $\mathcal{N} = 4$ limit. In general, the coupling should be written as a general Yukawa coupling y_{TC}, where $y_{TC} = g_{TC}$ can be chosen at one scale only, and the value of y_{TC} at other scales should be solved from the renormalization group equations. We have investigated the running of the coupling and shown that y_{TC} tends towards g_{TC} at low energies, as also discussed in [75]. This result justifies our choice to set it equal to the technicolor gauge coupling itself. Therefore the exact $\mathcal{N} = 4$ supersymmetry is an automatic infrared limit of the technicolor sector when its couplings with the MSSM are removed: $y_U = y_R = g_Y = g_W = 0$. As long as these couplings are small, it is reasonable to assume the $\mathcal{N} = 4$ supersymmetry is approximately valid.

To this Lagrangian we add the soft SUSY breaking terms:

$$\begin{aligned}
\mathcal{L}_{soft} &= -\Big[a_{TC}\epsilon^{abc}\tilde{U}_L^a\tilde{D}_L^b\bar{\tilde{U}}_R^c + a_U\left(\tilde{H}_1\tilde{D}_L^a - \tilde{H}_2\tilde{U}_L^a\right)\bar{\tilde{U}}_R^a + a_N\left(\tilde{H}_1\tilde{E}_L - \tilde{H}_2\tilde{N}_L\right)\bar{\tilde{N}}_R \\
&+ a_E\left(\tilde{H}_1'\tilde{E}_L - \tilde{H}_2'\tilde{N}_L\right)\bar{\tilde{E}}_R + a_R\bar{\tilde{U}}_R^a\bar{\tilde{U}}_R^a\bar{\tilde{E}}_R + \frac{1}{2}M_D\bar{D}_R^a\bar{D}_R^a + c.c.\Big] - M_Q^2\bar{\tilde{Q}}_L^a\tilde{Q}_L^a \\
&- M_U^2\bar{\tilde{U}}_R^a\tilde{U}_R^a - M_L^2\bar{\tilde{L}}_L\tilde{L}_L - M_N^2\bar{\tilde{N}}_R\tilde{N}_R - M_E^2\bar{\tilde{E}}_R\tilde{E}_R.
\end{aligned} \tag{5.31}$$

This model constitutes our fundamental description in terms of the elementary degrees of freedom and forces. The relevant scales of the problem are the supersymmetric breaking scale m_{SUSY} and the EW scale which we identify with the low-energy strongly coupled regime of the TC theory $\Lambda_{TC} \sim 4\pi v_w$, which for $v_w \simeq 246$ GeV implies $\Lambda_{TC} \sim 3$ TeV. We will consider here the following order

$$m_{\mathrm{SUSY}} > \Lambda_{TC}. \tag{5.32}$$

With this order the EW symmetry is broken dynamically. We arrange the spectrum in the following way:

1. The soft scalar masses of the fundamental scalars are taken to be of the order of m_{SUSY}.

2. The gaugino masses are also taken to be of the order of m_{SUSY} with the exception of the technigaugino mass M_D taken to be lighter than m_{SUSY}. If D_R was taken to be very heavy compared to Λ_{TC}, it could not participate in EWSB.

3. In our model the μ parameter, which gives the mass of the Higgsinos, is much larger than Λ_{TC}. Therefore the Higgsinos are ignored in low energy phenomenology.

4. The composite states acquire a dynamical mass of the order of Λ_{TC}.

5.3 Effective Lagrangian

To determine the Lagrangian incorporating the effects of this flavor extension we need the following microscopic MSCT Lagrangian terms:

$$\begin{aligned}
-\mathcal{L}_{\mathrm{Y,MSCT}} &= \tilde{H}_u \cdot F_u + \tilde{H}_d \cdot F_d + \mathrm{h.c.} \\
F_u &= q^i_{Lu} Y^i_u \bar{u}^i_R + y_U Q_L \bar{U}_R + y_N L_L \bar{N}_R \\
F_d &= q^i_{Ld} Y^i_d \bar{d}^i_R + l^i_L Y^i_l \bar{e}^i_R + y_E L_L \bar{E}_R \; .
\end{aligned} \tag{5.33}$$

The fields $\tilde{H}_u$ and $\tilde{H}_d$ are the MSSM Higgs doublets. Here we will denote the scalar components of a chiral supermultiplet with a tilde. The flavor index is denoted by $i = 1...3$ and it is summed over. The matrices Y_u, Y_d, and Y_l are diagonal, and the CKM matrix V is hidden in the definitions of the vectors

$$q^i_{Lu} = (u^i_L, V^{ij} d^j_L) \; , \qquad \text{and} \qquad q^i_{Ld} = (V^{\dagger ij} u^j_L, d^i_L) \; . \tag{5.34}$$

Furthermore the Yukawa interaction between technisquarks and techniquarks, stemming from superpotential and gauge interactions, is given as in the $\mathcal{N} = 4$ supersymmetric case, (4.13).

The MSSM Higgs field potential is

$$V_{\mathrm{MSSM}} = \left(m^2_{\mathrm{SUSY}} + |\mu|^2\right) |\tilde{H}_u|^2 + \left(m^2_{\mathrm{SUSY}} + |\mu|^2\right) |\tilde{H}_d|^2 - \left(b\tilde{H}_u \tilde{H}_d + \mathrm{h.c.}\right) + ... \tag{5.35}$$

The Higgs states are diagonalized via:

$$\begin{pmatrix} \tilde{H}_u \\ \tilde{H}^c_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \tilde{H}_1 \\ \tilde{H}_2 \end{pmatrix} \; , \tag{5.36}$$

where $\tilde{H}^c_d = \epsilon \tilde{H}^*_d$. Their tree-level physical masses $m^2_1 = \mu^2 + m^2_{\mathrm{SUSY}} - b$ and $m^2_2 = \mu^2 + m^2_{\mathrm{SUSY}} + b$ are traded for two convenient parameters θ and m_s as follows:

$$\frac{1}{2}\left(\frac{1}{m^2_1} + \frac{1}{m^2_2}\right) = \frac{c^2_\theta}{m^2_s} \quad , \quad \frac{1}{2}\left(\frac{1}{m^2_1} - \frac{1}{m^2_2}\right) = \frac{c_\theta s_\theta}{m^2_s}. \tag{5.37}$$

In terms of the original potential parameters (5.35) we have

$$m^2_s = \left(\mu^2 + m^2_{\mathrm{SUSY}}\right) \frac{(\mu^2 + m^2_{\mathrm{SUSY}})^2 - b^2}{\left(\mu^2 + m^2_{\mathrm{SUSY}}\right)^2 + b^2} \quad , \quad \tan(\theta) = \frac{b}{\mu^2 + m^2_{\mathrm{SUSY}}}. \tag{5.38}$$

Decoupling the heavy scalars leads to the following intermediate scale interaction Lagrangian for the fermions:

$$\begin{aligned}
\mathcal{L}_{4fermi} &= \frac{c^2_\theta}{m^2_s}\left(F^\dagger_u F_u + F^\dagger_d F_d\right) - \frac{c_\theta s_\theta}{m^2_s}\left(F_u \cdot F_d + \mathrm{h.c.}\right) \\
&\quad -\frac{1}{2} M_D \left(D_R D_R + \mathrm{h.c.}\right) + \frac{g^2_{\mathrm{TC}}}{m^2_{\mathrm{SUSY}}} \epsilon_{abc}\epsilon_{cde} \eta^{\alpha a}_i \eta^b_{j\alpha} \eta^{*d}_{i\beta} \eta^{*\beta e}_j \; .
\end{aligned} \tag{5.39}$$

We have retained the operators in mass dimension less or equal to six. The indices i and j indicate $SU(4)$ flavor; the first letters of the alphabet are reserved for the adjoint $SU(2)$

technicolor indices, while the Greek indices are for $SL(2,C)$ spinors. The color indices are contracted and suppressed, while the TC indices, running from 1 to 3, are written explicitly only in the last term.

Note that the terms in (5.39) do not contribute to e.g. $K-\bar{K}$ mixing at tree level, as a consequence of the unitarity of the CKM matrix. The other important outcome is that we have a tunable parameter, $\tan\theta$, to control the mass difference of the up and down type SM fermions.

The SM fermion masses as well as the fourth lepton family ones arise from the following four-fermion operator

$$\eta^T Z \eta \ , \tag{5.40}$$

with

$$Z_{ij} = \frac{y_U c_\theta \omega}{m_s^2} \left[\delta_{ik} c_\theta \left(q_{Lu}^{*k} Y_u^* \bar{u}_R^* + y_N^* L_L^{*k} \bar{N}_R^*\right) - \epsilon_{ik} s_\theta \left(q_{Ld}^k Y_d \bar{d}_R + l_L^k Y_l \bar{e}_R + y_E L_L^k \bar{E}_R\right)\right] \delta_{3,j}, \tag{5.41}$$

upon condensation of the techniquarks[1]. The spurion Z transforms as $Z \to u^* Z u^\dagger$ under $\mathrm{SU}(4)_R$.

Having derived the four-fermion theory just below the SUSY breaking scale we need now to evolve the techniquark condensates down to the EW scale. This is achieved by taking the techniquark Yukawa coupling y_U renormalized at the SUSY breaking scale, and by simultaneously having introduced the dimensionless techniquark renormalization factor

$$\omega = \frac{\langle U_L \bar{U}_R\rangle_{m_{\mathrm{SUSY}}}}{\langle U_L \bar{U}_R\rangle_{\Lambda_{TC}}} = \left(\frac{m_{\mathrm{SUSY}}}{\Lambda_{TC}}\right)^\gamma , \tag{5.42}$$

written with the assumption of a constant anomalous dimension γ for the techniquark mass operator.

The four-fermion term involving solely techniquarks is

$$\frac{y_U^2 c_\theta^2}{m_s^2}\omega^2 (Q_{Li}\bar{U}_R)(Q_{Li}^*\bar{U}_R^*) = W_{ijkl}\eta_i^\alpha \eta_{j\alpha}\eta_{k\beta}^*\eta_l^{*\beta} \ \ , \ \ W_{ijkl} = \frac{y_U^2 c_\theta^2}{m_s^2}\omega^2\left(\delta_{ik1}+\delta_{ik2}\right)\delta_{jl3}, \tag{5.43}$$

where α and β are spin indices. For the term to be invariant, the spurion W must transform as $W_{ijkl} \to u_{im} u_{jn} u_{ko}^* u_{lp}^* W_{mnop}$ under $u \in \mathrm{SU}(4)$. To estimate the effects of renormalization, we simply assumed factorization, leading to a multiplicative factor of ω^2.

The last term in Eq.(5.39) derives from decoupling the techniquarks. Decoupling these three complex scalars yields a four techniquark operator respecting the full $SU(4)$ symmetry, since it arises from the $\mathcal{N}=4$ sector per se. This four-techniquark operator appears in the Lagrangian as

$$\frac{g_{TC}^2}{m_{\mathrm{SUSY}}^2}\omega^2 \epsilon_{abc}\epsilon_{cde}\eta_i^{\alpha a}\eta_{j\alpha}^b \eta_{i\beta}^{*d}\eta_j^{*\beta e}, \tag{5.44}$$

where we have assumed the same renormalization enhancement factor ω^2 as in (5.43). This operator induces a shift in the coefficient m^2 of the $\mathrm{Tr}\left[\mathrm{MM}^\dagger\right]$ operator. The sign is such that it contributes to chiral symmetry breaking. We will keep m^2 as a free parameter.

[1]Notice that the indices i and j in Eq.(5.41) run from 1 to 4, while $k = 1, 2$ is the weak isospin.

It is also useful to introduce the spurion Δ related to the soft SUSY breaking mass operator:

$$\frac{1}{2}M_D\bar{D}_R\bar{D}_R = \eta^T\Delta\eta, \tag{5.45}$$

with

$$\Delta = \mathrm{diag}\left(0, 0, 0, \frac{M_D}{2}\right) \tag{5.46}$$

transforming under SU(4) as $\Delta \to u^*\Delta u^\dagger$.

Collecting the information above we write the following new operators emerging at the effective Lagrangian level at the lowest order in the spurions:

$$\mathcal{L}_{FE} = -c_1\Lambda_{TC}^2\mathrm{Tr}\left[M\Delta\right] + c_2\Lambda_{TC}^2\mathrm{Tr}\left[MZ\right] + c_3\Lambda_{TC}^4 W_{ijkl}M_{ij}M_{kl}^* + cc. \tag{5.47}$$

The powers of Λ_{TC} are inserted to make the coefficients dimensionless. The coefficients c_1, c_2, c_3 are estimated by NDA as $c_1 \sim 1/h_N, c_2 \sim 1/h_N, c_3 \sim 1/h_N^2$.

The anomalous dimension of MWT has been under much research recently. These investigations are based on the approximation of the technicolor sector in isolation, without taking into account the backreaction of the flavor extension. We can attempt to estimate γ_m in this model by using the Schwinger-Dyson equation. Because of the mixing between the up and down type Higgs particles, the techniquarks couple directly to both of the fields with a coupling proportional to y_U (5.30). We calculate the mass gap equation in the limit that the down type Higgs field is decoupled, and in this case the Yukawa coupling $y = y_U$.

We use the Schwinger–Dyson equation in the rainbow approximation similarly as in the gauged Nambu Jona-Lasinio model [76, 77]. In this approximation the contributions are given by three graphs, one with a gauge boson and two with scalars, either the techniscalar or the Higgs particle, Figures 5.1 and 5.2. The coupling between two techniquarks and a technisquark is found from the $\mathcal{N} = 4$ symmetric Lagrangian given in (4.16).

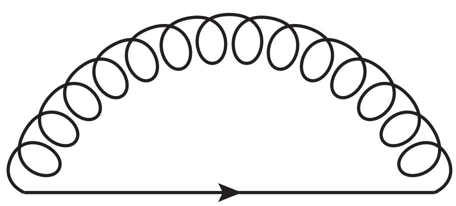


Figure 5.1: Gauge boson contribution, proportional to g_{TC}^2.

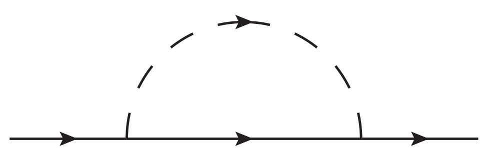


Figure 5.2: Technisquark contribution, also proportional to g_{TC}^2 from two Yukawa interactions. The same diagram also gives the Higgs particle contribution, proportional to y^2.

We find the following equation for the self energy of the U techniquark in the ladder approximation:

$$\Sigma(p^2) = \left(\frac{3g_{TC}^2}{m_\varphi^2\pi^2} + \frac{y^2}{16\pi^2 m_h^2}\right)\int \mathrm{d}q^2 q^2 \frac{\Sigma(q^2)}{q^2+\Sigma(q^2)^2} + \frac{3g_{TC}^2}{8\pi^2}\int \mathrm{d}q^2 \frac{q^2\Sigma(q^2)}{q^2+\Sigma(q^2)^2}\frac{1}{\mathrm{Max}(p^2,q^2)}. \tag{5.48}$$

We then identify $m_\varphi \sim m_h \sim m_{SUSY}$ and read off the result from standard literature [78]. We find that the values of the couplings required to achieve dynamical symmetry breaking are given by

$$\frac{3g_{TC}^2}{\pi^2} + \frac{y^2}{16\pi^2} = \frac{1}{4}(1+\Omega)^2 \quad , \quad \Omega = \sqrt{1 - \frac{3g_{TC}^2}{2\pi^2}}, \tag{5.49}$$

and the formula for the anomalous dimension is given by

$$\gamma_m = 1 + \Omega(-1+2r) \quad , \quad r \equiv \frac{y^2 + 48g_{TC}^2}{4\pi^2(1+\omega)^2}. \tag{5.50}$$

This formula is valid even away from the critical line, which is defined by $r = 1$. We therefore find a very large anomalous dimension above the critical line, $\gamma_m \geq 1.75$, as illustrated in Fig. 5.3. In the case with only the gauge boson exchange diagram, Fig. 5.1, the equivalent analysis gives $\gamma_m = 1$. Therefore, in this analysis, the Higgs Yukawa coupling has a smaller effect than the technisquark one on the estimate of γ_m.

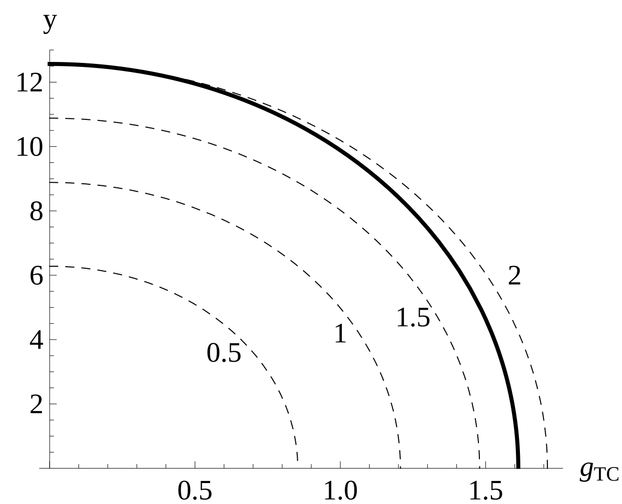


Figure 5.3: The critical line $y(g_{TC})$ for the U techniquark is plotted as a thick line. For a given value of g_{TC}, $y(g_{TC})$ is interpreted as the minimum strength of the four fermion coupling in order to achieve dynamical EWSB in the infrared. The dashed contours correspond to constant values of the anomalous dimension γ_m.

5.4 Typical Spectrum

The components of the matrix $M \sim \eta^T \eta$ can be described either in terms of wave functions of the underlying techniquarks, the transformation properties of the composites under $SU(2) \times U(1)$, or in terms of the mass eigenstates. In terms of the transformation properties under $SU(2) \times U(1)$, we have the states listed in Table (5.2).

Field	$\mathrm{SU(2)_L}$	$\mathrm{U(1)_Y}$
$\Delta \sim Q_L Q_L$	$\Box\Box$	1
$\sigma_U \sim Q_L \bar{U}_R$	$\Box$	$-\frac{1}{2}$
$\sigma_D \sim Q_L \bar{D}_R$	$\Box$	$\frac{1}{2}$
$\delta^{--} \sim \bar{U}_R \bar{U}_R$	1	2
$\delta^{-} \sim \bar{U}_R \bar{D}_R$	1	1
$\delta^{0} \sim \bar{D}_R \bar{D}_R$	1	0

Table 5.2: Transformation properties of the component fields of the matrix M under $\mathrm{SU(2)} \times \mathrm{U(1)}$.

In this notation, we have

$$M = \begin{pmatrix} \sqrt{2}\Delta^{++} & \Delta^{+} & \sigma_U^0 & \sigma_D^+ \\ \Delta^{+} & \sqrt{2}\Delta^0 & \sigma_U^- & \sigma_D^0 \\ \sigma_U^0 & \sigma_U^- & \sqrt{2}\delta^{--} & \delta^- \\ \sigma_D^+ & \sigma_D^0 & \delta^- & \sqrt{2}\delta^0 \end{pmatrix}. \tag{5.51}$$

To go to the mass basis, we must consider the modification of the low-energy effective potential induced by the flavor extension. Also the ground state of the theory must be consistently redetermined. We start by searching for a new ground state parametrized according to the following vacuum expectation value form of the matrix M:

$$\langle M \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & v_1 & 0 \\ 0 & \sqrt{2}v_2 & 0 & v_3 \\ v_1 & 0 & 0 & 0 \\ 0 & v_3 & 0 & \sqrt{2}v_4 \end{pmatrix}. \tag{5.52}$$

Minimizing the scalar potential given in (5.15) and (5.47) leads us to the following relations for the parameters:

$$\begin{aligned} c_1 \Lambda_{TC}^2 M_D &= 2\left(v_2^2 - v_4^2\right)\left(2v_4\lambda' - \frac{v_1^2\lambda''}{v_2}\right),\ m_{FE}^2 = 2\left(v_1^2 - 2v_2^2\right)\left(\lambda' - \frac{v_4\lambda''}{v_2}\right), \\ v_3 &= 0,\ m^2 = \lambda\left(v_1^2 + v_2^2 + v_4^2\right) + 4\lambda' v_2^2 + \frac{2v_1^2 v_4 \lambda''}{v_2}, \end{aligned} \tag{5.53}$$

where

$$m_{FE}^2 = c_3 y_U^2 c_\theta^2 \frac{\Lambda_{TC}^4 \omega^2}{m_s^2}. \tag{5.54}$$

This minimum is a global one.

The physical diagonalized masses at a generic point in the parameter space cannot be presented in analytic form. Therefore we choose to study the special case

$$M_D = v_2 = v_4 = 0 \tag{5.55}$$

and it turns out that the qualitative features of the spectrum remain intact if a small M_D is added. At this special point in the parameter space, minimizing the scalar potential gives:

$$v_1^2 = \frac{m^2 + m_{FE}^2}{\lambda + 2\lambda'} \; , \; v_3 = 0, \tag{5.56}$$

with m_{FE} defined in (5.54). The composite matrix M resumes a simple form in terms of mass eigenstates:

$$M = \begin{pmatrix} i\Pi_{UU} + \tilde{\Pi}_{UU} & \frac{i\Pi_{UD}+\tilde{\Pi}_{UD}}{\sqrt{2}} & \frac{\sigma+i\Pi^0}{\sqrt{2}} & A^+ \\ \frac{i\Pi_{UD}+\tilde{\Pi}_{UD}}{\sqrt{2}} & i\Pi_{DD} + \tilde{\Pi}_{DD} & i\Pi^- & \frac{A^0+i\Theta}{\sqrt{2}} \\ \frac{\sigma+i\Pi^0}{\sqrt{2}} & i\Pi^- & i\Pi^*_{UU} + \tilde{\Pi}^*_{UU} & \frac{i\Pi^*_{UD}+\tilde{\Pi}^*_{UD}}{\sqrt{2}} \\ A^+ & \frac{A^0+i\Theta}{\sqrt{2}} & \frac{i\Pi^*_{UD}+\tilde{\Pi}^*_{UD}}{\sqrt{2}} & i\Pi^*_{DD} + \tilde{\Pi}^*_{DD} \end{pmatrix} . \tag{5.57}$$

The deviation of off-diagonal 2x2 matrix from the form (5.19) is due to the different flavor extension sector. The mass of the Higgs boson σ is

$$M_\sigma^2 = 2m^2 + 2m_{FE}^2. \tag{5.58}$$

The masses of the pseudo-Goldstone baryons are given by:

$$\begin{aligned} M^2_{\Pi_{UU}} = M^2_{\Pi_{UD}} &= m^2_{FE} \\ M^2_{\Pi_{DD}} &= m^2_{FE} - 2v_1^2\left(\lambda' + \lambda''\right). \end{aligned} \tag{5.59}$$

The masses of the other scalar and pseudoscalar states are

$$\begin{aligned} M^2_\Theta &= m^2_{FE} - 2v_1^2\left(\lambda' - \lambda''\right) \\ M^2_{A^\pm} &= m^2_{FE} \\ M^2_{A^0} &= m^2_{FE} - 2v_1^2\left(\lambda' + \lambda''\right), \end{aligned} \tag{5.60}$$

for the technimesons, and

$$\begin{aligned} M^2_{\tilde{\Pi}_{UU}} &= m^2_{FE} + 4\lambda' v_1^2 \\ M^2_{\tilde{\Pi}_{UD}} &= m^2_{FE} \\ M^2_{\tilde{\Pi}_{DD}} &= m^2_{FE} - 2v_1^2\left(\lambda' - \lambda''\right), \end{aligned} \tag{5.61}$$

for the technibaryons. These expressions show that this flavor extension modifies the spectrum more subtly than the naive one introduced in Section 5.1.

The states given in (5.57) mix further when $M_D \neq 0$, and their masses will get relevant contributions as a result of this mixing. To demonstrate this effect, we present in Fig.5.4 two example mass spectra for the non-SM particles, corresponding respectively to a heavy (1 TeV) and light (150 GeV) lightest neutral Higgs scalar. These example mass spectra correspond to points inside the parameter space we have studied:

$$\begin{aligned}
&0.5 < c_{1,2}h_N < 5,\ 0.5 < c_3 h_N^2 < 5,\ \gamma = 1.5,\ 246\ \geqslant v_1\ \mathrm{GeV}^{-1} \geqslant 246/\sqrt{2},\\
&(2\pi)^2 > \lambda > 0.1,\ 2\pi > y_{t,N,E,U} > 0.1,\ 4\pi > g > \pi,\ m_t = 172\ \mathrm{GeV},\\
&120\ \mathrm{GeV} < 2\left(\frac{v_4}{v_2} - \frac{v_2}{v_4}\right)\left(2v_2v_4\lambda' - v_1^2\lambda''\right) < 10\ \mathrm{TeV},\\
&|v_4| > \sqrt{\frac{v_w^2 - v_1^2}{2}}\ ,\ m_{\mathrm{SUSY}} > 5\ \mathrm{TeV}.
\end{aligned} \tag{5.62}$$

We furthermore require the potential to be stable and physical masses to be positive.

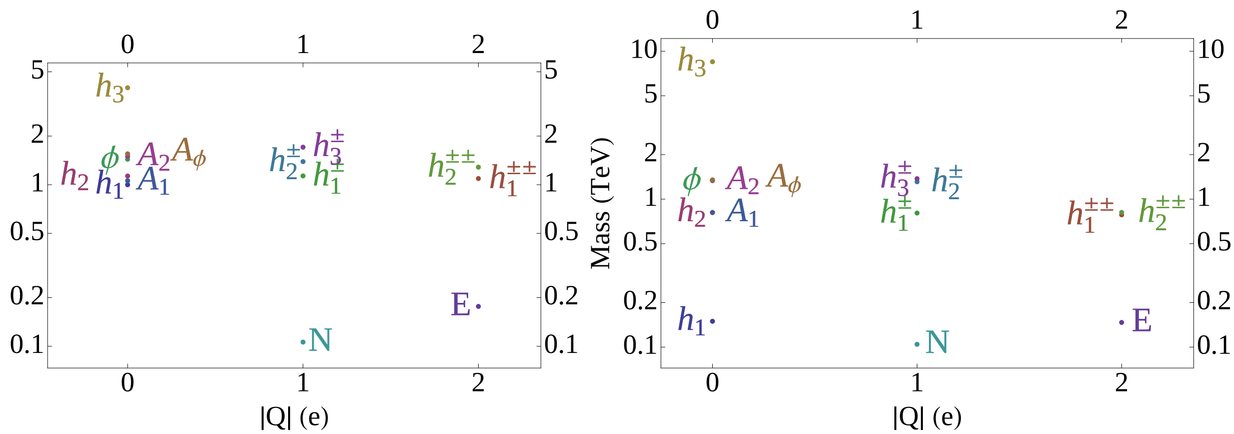


Figure 5.4: These plots contain the full scalar and heavy lepton mass spectrum as a function of the EM charge Q corresponding to a heavy (light) composite Higgs particle, with $m_{h_1} \simeq 1$ TeV (left panel) and $m_{h_1} \simeq 150$ GeV (right panel). The absolute value of the EM charge is on the x axis. Scalars are denoted by h_i where the sub index labels the mass eigenstates, and the possible electric charge is given as a super index. Pseudoscalars are similarly denoted by A_i.

Note that the masses of the upper component u and the lower component d of a generic EW fermion doublet are given by

$$m_u = c_2\Lambda_{TC}^2 \frac{c_\theta^2 y_U y_f \omega}{m_s^2}\frac{v_1}{\sqrt{2}}, \qquad\qquad m_d = \frac{y_d}{y_u} t_\theta m_u\ , \tag{5.63}$$

with $t_\theta = \tan\theta$, which is a free parameter. We will always choose the values of m_s and Yukawa couplings to yield the correct SM mass spectrum.

5.5 Oblique Corrections

We now determine the EW precision parameters which receive contributions from the vacuum structure (5.52), from the intrinsic TC dynamics, and from the new leptons N and E.

For the vev defined by Eqs.(5.52,5.53) the $W^{\pm}$ and Z bosons squared masses are readily computed:

$$m_Z^2 = \frac{1}{4}\left(g_L^2 + g_Y^2\right)\left(v_1^2 + 4v_2^2\right), \; m_{W^{\pm}}^2 = \frac{1}{4} g_L^2 \left(v_1^2 + 2v_2^2\right). \tag{5.64}$$

From these expressions one determines the value of the T parameter at tree level:

$$T_{tree} = -\frac{2v_2^2}{v_w^2}, \qquad v_w^2 = (\sqrt{2}G_F)^{-1} = v_1^2 + 2v_2^2 = (246 \text{ GeV})^2, \tag{5.65}$$

where G_F is the Fermi coupling constant. We note that extensions of the SM with a non-zero tree-level T parameter can be phenomenologically viable [79]. The tree-level T parameter is a result of custodial symmetry breaking induced by the technigaugino soft mass term. Also the coupling with the fundamental Higgs scalar breaks custodial symmetry, but its effect on the T parameter is only at loop level.

At one loop there are two distinct contributions to both S and T, one generated by the heavy leptons N and E with non-degenerate masses [80], and the other one generated by the techniquarks U^a and D^a. The latter contribution is usually accounted for by the naive values $S_{naive} = (6\pi)^{-1}$, $T_{naive} = 0$, associated to each EW doublet, that are obtained in the limit of U^a and D^a having a dynamically generated degenerate mass $m_{dyn} \to \infty$. We now estimate S and T by taking into account the full expressions for the masses of the left-handed Weyl spinors U_L^a, $\bar{U}_R^a$, D_L^a, $\bar{D}_R^a$. In these expressions we include a finite dynamical Dirac mass term, m_{dyn}. We use the formulas and notation as given in [81], with the mass parameters multiplying the operators $U_L\bar{U}_R$, $D_L\bar{D}_R$, and $D_R D_R$ in the Lagrangian given by

$$\frac{y_U^2 \omega^2 c_\theta^2}{m_s^2}\langle U_L \bar{U}_R\rangle_{\Lambda_{TC}} + m_{dyn}\;, \quad m_{dyn}\;, \quad M_D, \tag{5.66}$$

These masses are identified respectively with m_ζ, m_D and m_R of [81]. Similar estimates, albeit without any knowledge of the FE dynamics, have been used in the literature [82]. We assume, as a crude estimate for the dynamical mass,

$$m_{dyn} \simeq \frac{h_N\, v_w}{2}\;, \qquad \langle U_L \bar{U}_R\rangle_{\Lambda_{TC}} \simeq h_N v_1^3. \tag{5.67}$$

The experimental bounds on S and T depend on the reference mass of the SM Higgs boson, m_{ref}. The only non-zero contributions of the scalars h_i to S and T are associated with the diagrams of the kind given in Fig.5.5. We will find that phenomenology dictates v_2 to be small, so we will work in the simplifying limit $v_2 = 0$, so that σ_U^0 is a linear combination of just two mass eigenstates,

$$\sigma_U^0 = c'_j h_j, \quad i = 1, 2, \tag{5.68}$$

with the sum of the squares of the coefficients c'_j normalized to one. In the limit of $m_{h_i} \gg m_Z$, the resulting expressions for S and T in MSCT corresponding to the SM Higgs particle contributions are

$$S \approx \frac{1}{12\pi}\sum_i c_i'^2 \log\frac{m_{h_i}^2}{m_{\text{ref}}^2}\;, \quad T \approx \frac{-3}{16\pi c_w^2}\sum_i c_i'^2 \log\frac{m_{h_i}^2}{m_{\text{ref}}^2}, \tag{5.69}$$

with c_w the cosine of the Weinberg angle, and $m_{\text{ref}} = 117$ GeV.

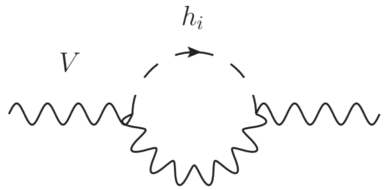


Figure 5.5: Higgs scalar h_i contribution to vector boson V's vacuum polarization amplitude.

To summarize, the full contribution to the EW parameters is given by

$$S = S_{U,D} + S_{N,E}\ , \qquad T = T_{tree} + T_{U,D} + T_{N,E}, \tag{5.70}$$

where T_{tree} is the tree level contribution of Eq.(5.65), $S_{U,D}$ and $T_{U,D}$ are the one loop contributions of the techniquarks calculated by using the formulae given in [81], and $S_{N,E}$ and $T_{N,E}$ are the one loop contributions of the heavy leptons [80].

5.6 Higgs Particle Constraints From the LHC

The sector of the MSCT effective Lagrangian relevant for Higgs particle production and decay at LHC is given by the coupling terms

$$\begin{aligned}\mathcal{L}_{int} &= -2m_{h^\pm}^2 h^+h^-\frac{\lambda_i}{v_w}h_i - m_f\psi\bar{\psi}\frac{c_j}{v_1}h_j + 2m_W^2 W_\mu^+W^{\mu-}\left(\frac{v_1}{v_w^2}c_jh_j + \frac{2v_2}{v_w^2}d_jh_j\right) \\ &+ m_Z^2 Z_\mu Z^\mu\left(\frac{v_1}{v_w^2+2v_2^2}c_jh_j + \frac{4v_2}{v_w^2+2v_2^2}d_jh_j\right),\end{aligned} \tag{5.71}$$

where we suppressed indices and sums over the MSCT fermions and charged composite scalars, represented in the equation above by ψ and $h^\pm$, and the sum over $j = 1,2,3$. The physical Higgs particles are denoted h_i and the coefficients $c_j,\ d_j$ are found by the projections on the these eigenstates:

$$\sigma_U^0 = c_jh_j, \quad \Delta^0 = d_jh_j. \tag{5.72}$$

We calculated numerically c_j, d_j, and λ_j, for each viable point, by expressing the Lagrangian in terms of the mass eigenstates, and reading off the relevant couplings.

The main production processes of a light neutral Higgs scalar (H) at LHC (see [83] by Gunion et al., and [84] by Djouadi for comprehensive reviews on Higgs particle phenomenology at LHC) are $q\bar{q} \longrightarrow W/Z + H$, $qq \longrightarrow qq + H$, $gg \longrightarrow H$, $gg \longrightarrow q\bar{q} + H$, with the corresponding Feynman diagrams shown in Fig. 5.6.

The Higgs scalar will then decay to the whole spectrum of massive particles and, through loop induced processes involving charged (colored) massive particles, to photons (gluons) as well, as shown in Fig. 5.7, where φ represents any spin-0 particle.

From Figs. 5.6 and 5.7, and (5.71), and from the fact that the fermion couplings to gauge bosons are fixed by their quantum numbers, it follows that the dominant production rates

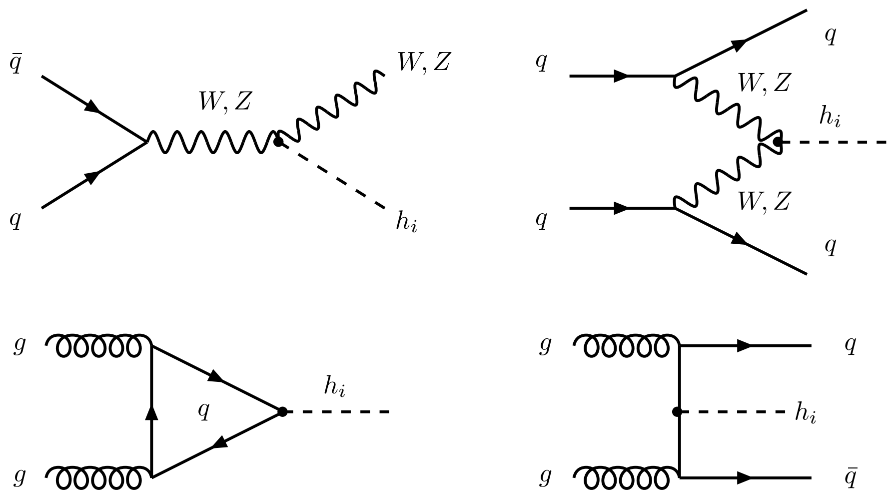


Figure 5.6: Dominant neutral Higgs scalar h_i production processes at LHC.

as well as the decay rates to fermions, W/Z bosons (both on and off shell), and gluons, can be expressed in function of the corresponding SM rates by

$$\frac{\sigma_{q\bar{q}\to Wh_i}}{\sigma_{q\bar{q}\to Wh_{SM}}} = \frac{\sigma_{qq\to q'q'h_i}}{\sigma_{qq\to q'q'h_{SM}}} = \frac{\Gamma_{h_i\to WW}}{\Gamma_{h_{SM}\to WW}} = \left[\frac{v_1}{v_w}c_i + 2\frac{v_2}{v_w}d_i\right]^2 \equiv \hat{\Gamma}_i^W , \tag{5.73}$$

$$\frac{\sigma_{q\bar{q}\to Zh_i}}{\sigma_{q\bar{q}\to Zh_{SM}}} = \frac{\sigma_{qq\to qqh_i}}{\sigma_{qq\to qqh_{SM}}} = \frac{\Gamma_{h_i\to ZZ}}{\Gamma_{h_{SM}\to ZZ}} = \left[\frac{v_1 c_i}{v_w + 2v_1/v_w} + \frac{4v_2 d_i}{v_w + 2v_1/v_w}\right]^2 \equiv \hat{\Gamma}_i^Z , \tag{5.74}$$

$$\frac{\sigma_{gg\to h_i}}{\sigma_{gg\to h_{SM}}} = \frac{\sigma_{gg\to q\bar{q}h_i}}{\sigma_{gg\to q\bar{q}h_{SM}}} = \frac{\Gamma_{h_i\to gg}}{\Gamma_{h_{SM}\to gg}} = \frac{\Gamma_{h_i\to f\bar{f}}}{\Gamma_{h_{SM}\to f\bar{f}}} = \frac{v_w^2}{v_1^2}c_i^2 \equiv \hat{\Gamma}_i^f , \tag{5.75}$$

where q' represents a quark with weak isospin different from that of the quark q.

Among the three neutral composite Higgs scalars available in MSCT, we pick the one with the strongest coupling to W, which we denote H. In Fig. 5.8 we show the numerical values of $\hat{\Gamma}_H^W$ (left panel) and $\hat{\Gamma}_H^Z$ (right panel), defined respectively in Eqs.(5.73,5.74) (where i for the state H is the one having the largest value of $\hat{\Gamma}_i^W$), for each of the 10^4 viable points defined in the previous section. For the same sample of points we show in Fig. 5.9 the numerical values of $\hat{\Gamma}_H^f$ (left panel) and the H decay rate to two photons (right panel) for both MSCT (yellow dots) and SM (black line).

From Figs. 5.8 and 5.9 it is clear that the total decay rate Γ_{tot} of H for $m_H < 1$ TeV is of the same magnitude as that of the SM Higgs particle, which is equal to roughly 1 TeV for $m_{h_{SM}} = 1$ TeV [84]. Since Γ_{tot} is of $O(m_H)$ when $m_H > 1$ TeV, the narrow width approximation cannot be used in this regime, and a full calculation of the cross section would be necessary. For most points in the region $m_H < 1$ TeV, H corresponds to the lightest neutral scalar h_1, and therefore decays to other scalars and pseudoscalars, mostly off shell, have a negligible branching ratio. The contribution of the charginos N and E to Γ_{tot} is expected to be one order of magnitude smaller than $\Gamma_{H\to WW}$. From Figs. 5.8 and 5.9 (left panel) it is evident that for $m_H \lesssim 500$ GeV the H production rate, accounted for mostly by the two processes in Fig. 5.6 with a gluon-gluon initial state, is almost identical in value to that of

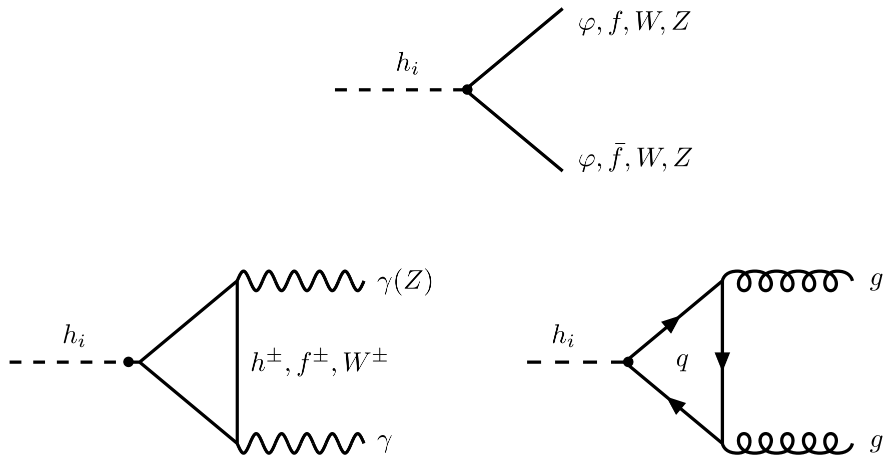


Figure 5.7: Decay channels for the neutral Higgs scalar h_i.

the SM Higgs particle. Since for the same mass range the H decay rate to WW is roughly the same as that of the SM Higgs particle, while the H decay rate to ZZ is in general greater than the corresponding SM rate, we expect most of the MSCT parameter space featuring $144\,\text{GeV} < m_H < 460\,\text{GeV}$ to be ruled out by the 1fb^{-1} to $2.3\ \text{fb}^{-1}$ ATLAS or CMS results, as it is the case for h_{SM} [45, 44].

The upcoming LHC results for $5\ \text{fb}^{-1}$ are expected to cover the range $0.5 \lesssim m_{h_{SM}}/\text{TeV} \lesssim 1$ as well, since in that mass range H features a decay rate to WW comparable to that of the SM Higgs particle, and a generally larger one to ZZ (with a relative increase as large as 50%), therefore H can be either observed or ruled out for $0.5 \lesssim m_H/\text{TeV} \lesssim 1$ with the same amount of data. In principle H could be distinguished from h_{SM} by measuring the ratio of couplings to Z and W.

From Fig. 5.9 (right panel) it is clear that the H decay to two photons cannot be used to search for a light H, while Fig. 5.9 (left panel) shows that the H decay rates to $b\bar{b}, \tau\bar{\tau}$, and gg, are almost identical in the low mass range to those of the SM Higgs particle. In this section we considered the case of relatively heavy composite Higgs particles since the phenomenological viable points featuring $m_{h_1} < 145$ GeV is about 0.5 %.

5.7 Other Regimes of MSCT

In this section we have so far discussed the MSCT model in its most natural parameter region. Now we consider other parameter regions that are, from the MWT perspective, distinct extensions. These inequivalent extensions are differentiated, for example, by the choice of the value of the coupling constant g_{TC} of the supersymmetric technicolor sector near the EW scale. Therefore there are two basic regimes: strong and weak coupling.

In the strong coupling regime, g_{TC} is large around the weak scale. In this case it is possible that the technicolor fermion bilinear condensate triggers EWSB. In this section we have investigated the region $m_{SUSY} > 5$ TeV, but have omitted the possibility that the SUSY breaking scale could be as low as $m_{SUSY} \sim m_{EW}$. It is argued in [73] that

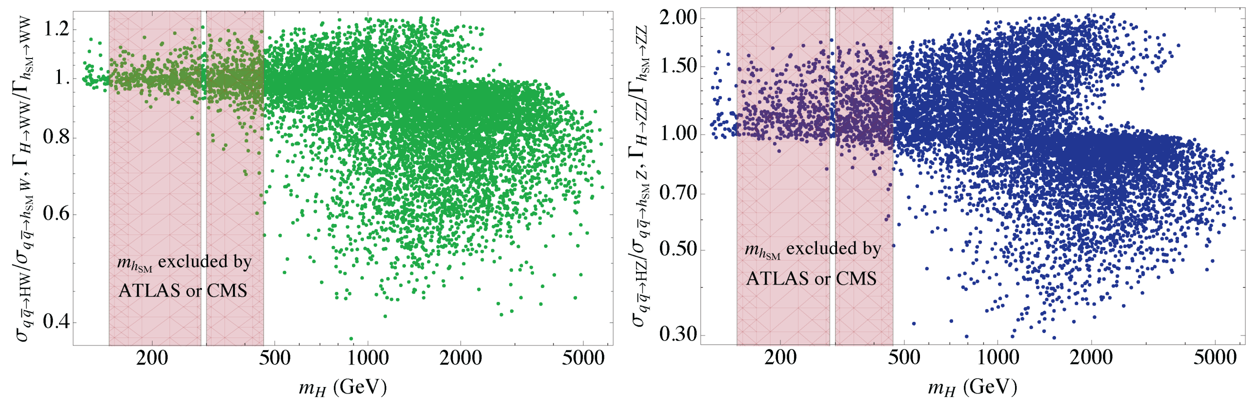


Figure 5.8: The MSCT neutral Higgs particle that has the largest coupling to WW is denoted by H. On the left (right) panel, the ratio between the decay rates of H and of the SM Higgs particle to WW (ZZ) is shown. The red bands show the range of $m_{h_{SM}}$ excluded either by ATLAS or CMS [44, 45].

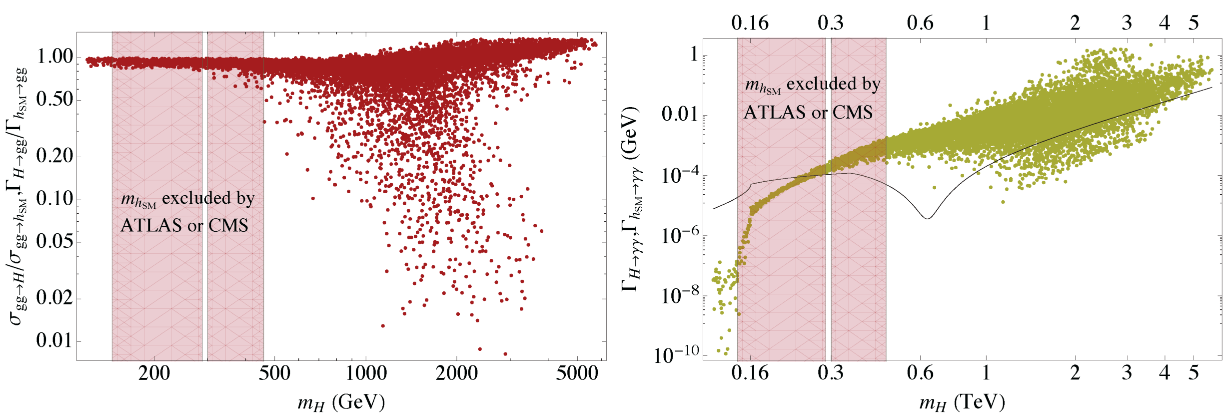


Figure 5.9: The MSCT neutral Higgs particle that has the largest coupling to WW is denoted by H. Left panel: ratio between the production rate of H and of the SM Higgs particle, via gluon fusion. Right panel: decay rate to $\gamma\gamma$ of H (yellow dots) and SM Higgs particle (black line). The red bands show the range of $m_{h_{SM}}$ excluded either by ATLAS or CMS [45, 44].

the latter regime is natural, if the technisquarks are decoupled at the electroweak scale, and the value of the gauge coupling at that scale is above the critical value. In this case, electroweak symmetry breaking is in fact triggered by soft supersymmetry breaking effects. It is a phenomenologically interesting possibility that technicolor composites and SUSY partners would have similar masses. The model still solves the little hierarchy problem because of the strong coupling nature of the TC sector, but it requires soft SUSY breaking universality. We have shortly investigated this regime in Paper II and Paper III. Finally, there is the case of tachyonic mass terms at the electroweak scale. We have not studied this case since the little hierachy problem is worse than in the MSSM.

As a toy model, it might be interesting to also study MSCT in the case that SUSY is not broken and the supersymmetric theory is strongly coupled at the EW scale. Then we must use non-perturbative methods to investigate the effects of the new sector on the MSSM dynamics and vice versa. For example, we can no longer use the single particle state interpretation in terms of the underlying degrees of freedom of the supersymmetric technicolor model but rather we must use the unparticle language [85] given that the supersymmetric technicolor model is exactly conformal, before coupling it to the MSSM. If no SUSY breaking terms are added directly to the 4SYM sector, then conformality will be broken only via weak and hypercharge interactions. An important further point is that one can use the machinery of the AdS/CFT [86] correspondence to make reliable computations in the nonperturbative sector, considering the effects of the EW interactions as small perturbations. The model resembles the one proposed in [87] in which, besides a technicolor sector, one has also coupled to the SM a natural unparticle composite sector.

In the weak coupling regime of MSCT, the gauge coupling g_{TC} is small at the electroweak scale. Such a theory has nothing to do with technicolor, as there can be no dynamical quark condensate. It is still an interesting model, since it provides a direct connection between a $\mathcal{N}=4$ theory and electroweak scale physics. It is also interesting since it is the weak coupling limit of the unparticle model. We will investigate this model in the next chapter, Chapter 6.

In Paper II we investigated also different SUSY BTC models based on MWT for different hypercharge values y. For values of y not equal to ± 1 it is not possible to have the approximate $\mathcal{N}=4$ symmetry, but phenomenological viability might be easier to achieve.

Chapter 6

N=4 Extended MSSM

In this chapter we discuss a semi-realistic extension of the MSSM, investigated in Paper IV. In this model the electroweak symmetry is partly broken by the vev of a fundamental scalar of the $\mathcal{N}=4$ Super-Yang Mills sector (4SYM). The particle content and charge assignments are the same as in the MSCT model, given in Table 5.1, and consist of the MSSM and a $SU(2)$ $\mathcal{N}=4$ sector. The couplings between the up-type Higgs field and the 4SYM fields, and the EW gauging of the 4SYM sector, cause hard breaking of the $\mathcal{N}=4$ SUSY to $\mathcal{N}=1$. In addition, SUSY is broken by soft SUSY breaking terms. Electroweak symmetry breaking is triggered by fundamental condensates arising because of tachyonic soft SUSY breaking mass terms. We assume flavor universality for the squark and slepton soft SUSY breaking sector.

The complete scalar potential is given by

$$V = V_{N4} + V_{MSSM}, \qquad V_{N4} = -\mathcal{L}_D - \mathcal{L}_F - \mathcal{L}_{soft} - \left(\frac{1}{2} M_D \bar{D}_R^a \bar{D}_R^a + c.c. \right), \tag{6.1}$$

where V_{MSSM} is the MSSM Higgs potential and can be found in [69]. The SM squarks and sleptons do not mix with the $\mathcal{N}=4$ scalars or heavy new scalar leptons at tree-level, and therefore their mass spectrum assumes the same form as in the MSSM. The Higgs scalar fields, $\tilde{H}$ and $\tilde{H}'$, on the other hand, mix with the $\mathcal{N}=4$ scalars.

The neutral scalar potential of this model, derived from (6.1), is:

$$\begin{aligned} V_{ns} &= M_Q^2 |\tilde{D}_L^3|^2 + \left(m_u^2 + |\mu|^2\right) |\tilde{H}_0|^2 + \left(m_d^2 + |\mu|^2\right) |\tilde{H}_0'|^2 - \left(b \tilde{H}_0 \tilde{H}_0' + c.c.\right) \\ &+ \frac{1}{8}\left(g_L^2 + g_Y^2\right) \left(|\tilde{D}_L^3|^2 - |\tilde{H}_0'|^2 + |\tilde{H}_0|^2\right)^2 . \end{aligned} \tag{6.2}$$

To derive the spectrum of the theory, we first determine the ground state. We allow for a nonzero vev for each of the electromagnetically neutral scalars, which are $\tilde{D}_L$, $\tilde{H}_0$ and $\tilde{H}_0'$. Without loss of generality, we choose the vacuum expectation value of the $\tilde{D}_L$ scalar to be aligned in the third direction of the $SU(2)_{N4}$ gauge space, and hence the vevs are written as

$$\left\langle \tilde{D}_L^3 \right\rangle = \frac{v_{N4}}{\sqrt{2}}, \quad \left\langle \tilde{H}_0 \right\rangle = s_\beta \frac{v_H}{\sqrt{2}}, \quad \left\langle \tilde{H}_0' \right\rangle = c_\beta \frac{v_H}{\sqrt{2}}, \tag{6.3}$$

where $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, and all vevs are chosen to be real. We indicated the scalar component of each Higgs weak doublet superfield with a tilde. We find that the gauge group breaking follows the pattern $SU(2)_{N4} \times SU(2)_L \times U(1)_Y \to U(1)_{N4} \times U(1)_{EM}$. The second $U(1)$ on the right corresponds to the ordinary electromagnetic (EM) charge. The phenomenological constraints on a new $U(1)$ massless gauge boson were studied in [88], and their analysis shows that the operators coupling such a gauge boson to the SM fields needs to be suppressed by scales at least of the order of the EW scale. This would provide relevant constraints on our model.

There are four massless scalar states at tree level in this theory. Of these, three are absorbed by the electroweak gauge bosons and one, the A_0, gains mass at the one loop level. At an e^+e^- collider, the main production channel of the light scalar A_0 would be $e^+e^- \to Z \to h_0^0 A_0$. For a hadron collider, one has also production via gluon-gluon fusion and associated production with heavy quarks. Therefore one needs the following tree level couplings:

$$g_{h_0^0 A_0 Z} \quad : \quad -\frac{\sqrt{g_Y^2+g_L^2}}{2}\frac{c_{2\beta}\sqrt{v_{N4}^2+v_H^2}}{\sqrt{v_{N4}^2+v_H^2 c_{2\beta}^2}} \tag{6.4}$$

$$g_{A_0\bar{b}\gamma_5 b} \quad : \quad -\frac{m_b}{\sqrt{v_{N4}^2+v_H^2}}\frac{v_{N4}}{v_H} \tag{6.5}$$

$$g_{A_0\bar{t}\gamma_5 t} \quad : \quad \frac{m_t}{\sqrt{v_{N4}^2+v_H^2}}\frac{v_{N4}}{v_H}, \tag{6.6}$$

where m_f is the fermion mass. The formulae are generic for the up and down type fermions.

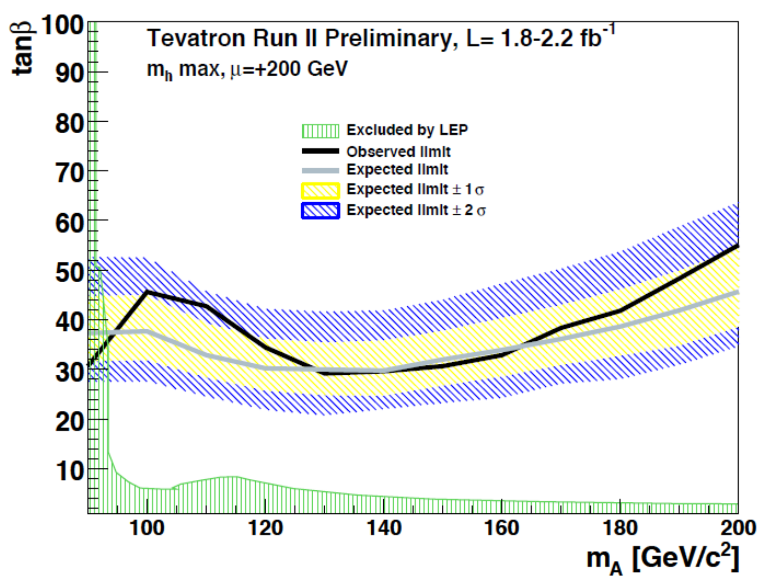


Figure 6.1: The 95% C.L. MSSM exclusion contours obtained by a combination of the CDF and DØ searches for H → τ+τ− in a maximum Higgs mass benchmark scenario, projected onto the (m_{A_0}, tanβ) plane. The regions above the solid black line are excluded, and the shaded and hatched bands centered on the lighter line show the distributions of expected exclusions in the absence of a signal. Also shown are the regions excluded by LEP searches [18], assuming a top quark mass of 174.3 GeV. Taken from [89].

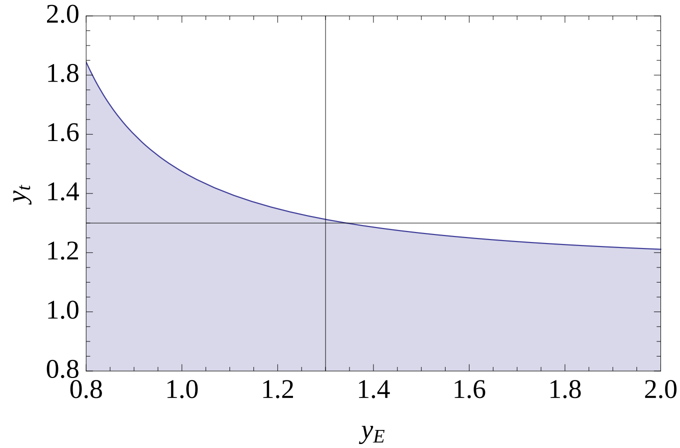


Figure 6.2: Shaded area shows the experimentally excluded values of the Yukawa couplings y_t and y_E.

We now compare the possibility of discovering the A_0 of our model to discovering the similar particle in the MSSM. For $\tan\beta \sim 1$ we find $g_{h_0^0 A_0 Z} \sim 0$ implying that there is a depletion of the A_0 production rate at e^+e^- colliders. As for the constraints from hadron colliders, if in addition $v_{N4}/v_H \simeq 1$, the couplings to quarks are of the same order as the MSSM couplings. Therefore the model parameter space has some valid region around $\tan\beta = 1$, where the A_0 would have escaped detection, see Fig. 6.1.

For the fermion sector, the lower bounds on the mass of the lightest neutralino and chargino are [61]

$$m_{\chi_0^0} > 46 \text{ GeV}\,,\; m_{\chi_0^\pm} > 94 \text{ GeV}. \tag{6.7}$$

These limits refer to the MSSM, but they are rather general, since they are extracted mostly from the Z decay to neutralino-antineutralino pair the former, and from photo-production of a chargino-antichargino pair at LEPII the latter. We use the lower bound on the chargino mass for the mass of both the singly charged particles and the doubly-charged new electron E. From these constraints we produced the interesting relation for the Yukawa couplings of the top and the doubly charged electron E:

$$y_t > \frac{173}{213}\sqrt{\frac{1}{\frac{1}{2} - \frac{94^2}{y_E^2 213^2}}}. \tag{6.8}$$

This last bound is plotted in Figure 6.2, where the shaded area shows the values of y_t and y_E excluded by the experiment: it is evident from the plot in Figure 6.2 that either y_t or y_E is constrained to be larger than about 1.3.

These Yukawa couplings are required to be rather large, so we now analyze the evolution of the gauge and Yukawa couplings using the two-loop renormalization group equations (RGE). In Figure 6.3 we plot $g_{N4}, y_{N4}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale M. The couplings are normalized at $M = m_Z$ to $y_N = 1.8$, $g_{N4} = y_{N4} = y_U = y_t = 2.3$, $y_E = 2.4$. Summarizing, g_{N4} runs towards zero at high energies, while the Yukawa couplings y_U, y_N, y_t, responsible for the mass of the heavy upper components of weak doublets, increase and flow close to an ultraviolet fixed point at around 2 TeV. For the above values of the Yukawa

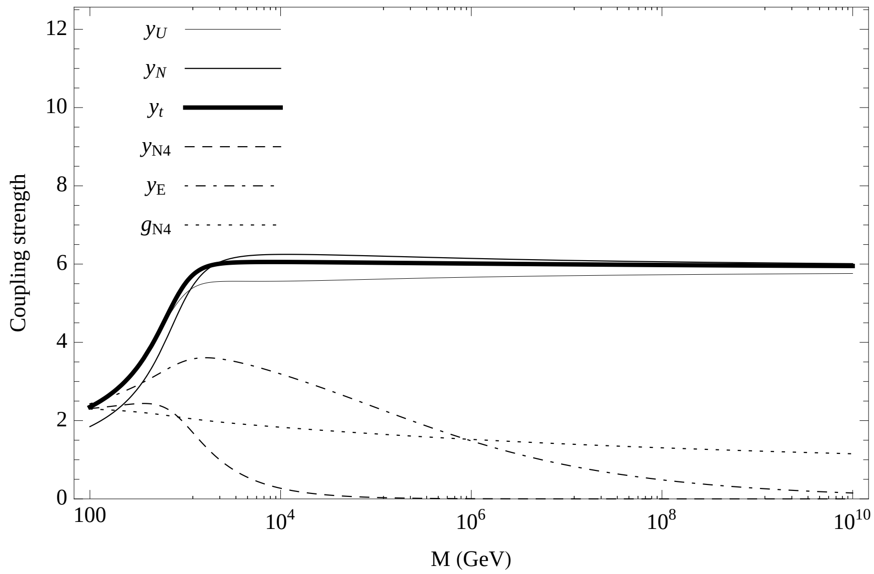


Figure 6.3: Values of $g_{N4}, y_{N4}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale M. The couplings are normalized at $M = m_Z$ to $y_N = 1.8$, $g_{N4} = y_{N4} = y_U = y_t = 2.3$, $y_E = 2.4$.

couplings we end up with the following spectrum:

$$m_{\chi_0^0} = 47 \text{ GeV}\,, \; m_{\chi_0^\pm} = 96 \text{ GeV}\,, \; m_{h_0^0} = 95 \text{ GeV}\,, \; m_{A_0} = 32 \text{ GeV}. \tag{6.9}$$

Although the mass of A_0 is low, it should be noted that this spectrum represents a sample point. Also the mass of the A_0 is low because we have not included the stop and top loops when determining the one loop effective potential.

Chapter 7

Summary and Conclusions

In Chapter 1 it was argued that the SM Higgs mass is unnaturally small. In TC models, the electroweak scale is explained naturally by logarithmic gauge coupling running.

Chapter 2 introduced technicolor. Technicolor models are not complete without an additional sector to generate the SM fermion masses. In ETC models one expects large FCNC processes, but using near conformal dynamics alleviates the FCNC problem. In bosonic technicolor models, fermion masses arise from scalar mediated interactions with the technicolor condensate. The absence of observed flavor changing neutral currents is explained by the GIM mechanism. A walking coupling is needed only if the mediator mass scale is high. One can make the coupling walk while also keeping the S parameter small by using nonfundamental representations for techniquarks.

In Chapter 3 we introduced the bosonic NMWT model. We wrote the effective Lagrangian and discussed how to estimate coefficients with NDA. We discussed the oblique corrections in the model and finally presented results. The current experimental limits do not exclude the model in the region where one of the scalars is light and SM-like. In this limit, the other scalar (the mostly fundamental one) is heavy and almost decoupled.

Chapter 4 introduced SUSY. Supersymmetry provides an elegant solution to the stabilization of the EW scale in the MSSM. This is achieved by supersymmetrizing the entire SM spectrum. By relating bosons and fermions, the symmetries protecting the fermionic sector from acquiring large quantum corrections now also apply to the scalars of the model, and consequently the EW scale stabilizes. This explanation of the stability of the Higgs mass is minimal since, in contrast to the technicolor case, one can use the same degrees of freedom to give mass to the electroweak gauge sector and to the matter fields. However, one has not seen a trace of light superpartners at the LHC experiments. Also, either the SUSY breaking sector conserves flavor to a suprisingly high accuracy, or then the SUSY breaking scale is much higher than the electroweak one. We discussed SUSY TC models in which these facts are explained by increasing the SUSY breaking scale, and correspondingly the Higgs mass, to 5-50 TeV. The electroweak scale is instead explained as in technicolor - it is analogous to the QCD scale. The fundamental Higgs fields do not participate in electroweak symmetry breaking but simply act as messengers between the symmetry breaking sector and the quarks and leptons.

Chapter 5 began by reviewing MWT. In MWT the gauge group is $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and the field content of the technicolor sector is constituted by two flavors of technifermions and one technigluon all in the adjoint representation of $SU(2)_{TC}$. The technifermions and technigluons of the Minimal Walking Technicolor form an $\mathcal{N}=4$ supermultiplet after adding only three scalar superpartners. In fact, the $SU(4)$ global symmetry of MWT is simply the well known $SU(4)_R$ R symmetry of the $\mathcal{N}=4$ Super Yang Mills theory. Supersymmetrizing MWT in this way leads to an approximate $\mathcal{N}=4$ supersymmetry of the technicolor sector that is broken to $\mathcal{N}=1$ only by EW gauge and Yukawa interactions. This specific SUSY BTC model was then studied in more detail. We wrote the effective Lagrangian, discussed the spectrum, and then proceeded to oblique corrections and Higgs scalar searches at the LHC. The backreaction of this flavor extension of the TC theory itself is significant in terms of the low-energy spectrum and anomalous dimension of the techniquark bilinear.

Finally, in Chapter 6 we studied a model in which a weakly coupled $\mathcal{N}=4$ gauge theory participates in electroweak symmetry breaking. The spectrum includes a light pseudoscalar that evades detection, because by tuning parameters it does not couple to the Z boson. This model is a simplified test laboratory for more realistic models of unparticle physics. In the large coupling limit, this model contains unparticle matter above the electroweak scale. In any case it is an interesting possibility that a $\mathcal{N}=4$ sector could be discovered at the EW scale.

This thesis has investigated bosonic technicolor models. The main conclusion is that bosonic technicolor models are viable, and that models combining supersymmetry and technicolor can be free of the problems associated with either paradigms in isolation. The second main conclusion is that the flavor extension of the technicolor sector can affect the technicolor theory itself nontrivially. Not only the anomalous dimension of the techniquark bilinear can grow, but strong attractive Yukawa interactions can have a large effect on the low-energy spectrum, and even contribute significantly to EWSB. In such technicolor theories, the possibility that the technicolor chiral symmetry is badly broken should be considered.

Bibliography

[1] Steven Weinberg. Implications of Dynamical Symmetry Breaking: An Addendum. *Phys. Rev.*, D19:1277–1280, 1979.

[2] Leonard Susskind. Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory. *Phys. Rev.*, D20:2619–2625, 1979.

[3] Aneesh Manohar and Howard Georgi. Chiral Quarks and the Nonrelativistic Quark Model. *Nucl. Phys.*, B234:189, 1984.

[4] Howard Georgi. Generalized dimensional analysis. *Phys. Lett.*, B298:187–189, 1993.

[5] Matti Antola and Kimmo Tuominen. Naive Dimensional Analysis and Irrelevant Operators. 2011.

[6] Denis Parganlija. *Quarkonium Phenomenology in Vacuum.* PhD thesis, der Johann Wolfgang Goethe-Universitï¿œt, 2011.

[7] Estia Eichten and Kenneth D. Lane. Dynamical Breaking of Weak Interaction Symmetries. *Phys. Lett.*, B90:125–130, 1980.

[8] Savas Dimopoulos and Leonard Susskind. Mass Without Scalars. *Nucl. Phys.*, B155:237–252, 1979.

[9] Bob Holdom. Raising the Sideways Scale. *Phys. Rev.*, D24:1441, 1981.

[10] Gino Isidori, Yosef Nir, and Gilad Perez. Flavor Physics Constraints for Physics Beyond the Standard Model. *Ann. Rev. Nucl. Part. Sci.*, 60:355, 2010.

[11] Thomas A. Ryttov and Robert Shrock. Ultraviolet Extension of a Model with Dynamical Electroweak Symmetry Breaking by Both Top-Quark and Technifermion Condensates. *Phys. Rev.*, D82:055012, 2010.

[12] Ning Chen, Thomas A. Ryttov, and Robert Shrock. Patterns of Dynamical Gauge Symmetry Breaking. *Phys. Rev.*, D82:116006, 2010.

[13] Thomas A. Ryttov and Robert Shrock. Infrared Evolution and Phase Structure of a Gauge Theory Containing Different Fermion Representations. *Phys. Rev.*, D81:116003, 2010.

[14] Thomas A. Ryttov and Robert Shrock. Higher-Loop Corrections to the Infrared Evolution of a Gauge Theory with Fermions. *Phys. Rev.*, D83:056011, 2011.

[15] Thomas A. Ryttov and Robert Shrock. Higher extended technicolor representations and fermion generations. *Eur. Phys. J.*, C71:1523, 2011.

[16] Thomas A. Ryttov and Robert Shrock. Generational Structure of Models with Dynamical Symmetry Breaking. *Phys. Rev.*, D81:115013, 2010.

[17] Thomas A. Ryttov and Robert Shrock. Technicolor Models with Color-Singlet Technifermions and their Ultraviolet Extensions. *Phys. Rev.*, D84:056009, 2011.

[18] Thomas Appelquist, Maurizio Piai, and Robert Shrock. Fermion masses and mixing in extended technicolor models. *Phys. Rev.*, D69:015002, 2004.

[19] Elizabeth H. Simmons. PHENOMENOLOGY OF A TECHNICOLOR MODEL WITH HEAVY SCALAR DOUBLET. *Nucl. Phys.*, B312:253, 1989.

[20] Alex Kagan and Stuart Samuel. Renormalization group aspects of bosonic technicolor. *Phys. Lett.*, B270:37–44, 1991.

[21] Christopher D. Carone and Elizabeth H. Simmons. Oblique corrections in technicolor with a scalar. *Nucl. Phys.*, B397:591–615, 1993.

[22] Christopher D. Carone, Elizabeth H. Simmons, and Yumian Su. b —> s gamma and Z —> b anti-b in technicolor with scalars. *Phys. Lett.*, B344:287–292, 1995.

[23] Christopher D. Carone and Howard Georgi. Technicolor with a massless scalar doublet. *Phys. Rev.*, D49:1427–1436, 1994.

[24] Alex Kagan and Stuart Samuel. The Family mass hierarchy problem in bosonic technicolor. *Phys. Lett.*, B252:605–610, 1990.

[25] Michael Dine, Alex Kagan, and Stuart Samuel. NATURALNESS IN SUPERSYMMETRY, OR RAISING THE SUPERSYMMETRY BREAKING SCALE. *Phys. Lett.*, B243:250–256, 1990.

[26] Alex Kagan and Stuart Samuel. Bosonic technicolor and the flavor problem. Contribution to the Conf., Quarks, Symmetries, and Strings: A Symp. in honor of Bunji Sakita's 60th birthday, New York, N.Y., Oct 1-2, 1990.

[27] Alex Kagan. Recent developments in bosonic technicolor. To appear in Proc. of 15th Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, MD, Aug 26- 28, 1991.

[28] Vagish Hemmige and Elizabeth H. Simmons. Current bounds on technicolor with scalars. *Phys. Lett.*, B518:72–78, 2001.

[29] Christopher D. Carone, Joshua Erlich, and Jong Anly Tan. Holographic Bosonic Technicolor. *Phys. Rev.*, D75:075005, 2007.

[30] Alfonso R. Zerwekh. Two Composite Higgs Doublets: Is it the Low Energy Limit of a Natural Strong Electroweak Symmetry Breaking Sector ? *Mod. Phys. Lett.*, A25:423–429, 2010.

[31] Hidenori S. Fukano, Matti Heikinheimo, and Kimmo Tuominen. Flavor constraints in a Bosonic Technicolor model. *Phys. Rev.*, D84:035017, 2011.

[32] Francesco Sannino and Kimmo Tuominen. Orientifold theory dynamics and symmetry breaking. *Phys. Rev.*, D71:051901, 2005.

[33] Tom Banks and A. Zaks. On the Phase Structure of Vector-Like Gauge Theories with Massless Fermions. *Nucl.Phys.*, B196:189, 1982.

[34] Andrew G. Cohen and Howard Georgi. WALKING BEYOND THE RAINBOW. *Nucl. Phys.*, B314:7, 1989.

[35] Thomas Appelquist, Kenneth D. Lane, and Uma Mahanta. ON THE LADDER APPROXIMATION FOR SPONTANEOUS CHIRAL SYMMETRY BREAKING. *Phys. Rev. Lett.*, 61:1553, 1988.

[36] Thomas A. Ryttov and Francesco Sannino. Supersymmetry Inspired QCD Beta Function. *Phys. Rev.*, D78:065001, 2008.

[37] Oleg Antipin and Kimmo Tuominen. Resizing the Conformal Window: A beta function Ansatz. *Phys.Rev.*, D81:076011, 2010.

[38] Tuomas Karavirta, Jarno Rantaharju, Kari Rummukainen, and Kimmo Tuominen. Exploring the conformal window: SU(2) gauge theory on the lattice. 2012.

[39] Thomas Appelquist et al. Approaching Conformality with Ten Flavors. 2012.

[40] Koichi Yamawaki, Masako Bando, and Ken-iti Matumoto. Scale Invariant Technicolor Model and a Technidilaton. *Phys. Rev. Lett.*, 56:1335, 1986.

[41] Thomas W. Appelquist, Dimitra Karabali, and L. C. R. Wijewardhana. Chiral Hierarchies and the Flavor Changing Neutral Current Problem in Technicolor. *Phys. Rev. Lett.*, 57:957, 1986.

[42] Dennis D. Dietrich, Francesco Sannino, and Kimmo Tuominen. Light composite Higgs from higher representations versus electroweak precision measurements: Predictions for LHC. *Phys. Rev.*, D72:055001, 2005.

[43] Dennis D. Dietrich, Francesco Sannino, and Kimmo Tuominen. Light composite Higgs and precision electroweak measurements on the Z resonance: An update. *Phys. Rev.*, D73:037701, 2006.

[44] Atlas Collaboration. An update to the combined search for the standard model higgs boson with the atlas detector at the lhc using up to 4.9 fb^{-1} of pp collision data at $\sqrt{s} = 7$ tev. March 7, 2012.

[45] CMS Collaboration. Combined results of searches for a higgs boson in the context of the standard model and beyond-standard models. March 7, 2012.

[46] Michael E. Peskin. The Alignment of the Vacuum in Theories of Technicolor. *Nucl. Phys.*, B175:197–233, 1980.

[47] Michael E. Peskin and Tatsu Takeuchi. A New constraint on a strongly interacting Higgs sector. *Phys. Rev. Lett.*, 65:964–967, 1990.

[48] C. P. Burgess, Stephen Godfrey, Heinz Konig, David London, and Ivan Maksymyk. Model independent global constraints on new physics. *Phys. Rev.*, D49:6115–6147, 1994.

[49] Michael E. Peskin and Tatsu Takeuchi. Estimation of oblique electroweak corrections. *Phys. Rev.*, D46:381–409, 1992.

[50] D. C. Kennedy and Paul Langacker. Precision electroweak experiments and heavy physics: A Global analysis. *Phys. Rev. Lett.*, 65:2967–2970, 1990.

[51] Guido Altarelli and Riccardo Barbieri. Vacuum polarization effects of new physics on electroweak processes. *Phys. Lett.*, B253:161–167, 1991.

[52] Francesco Sannino. Mass Deformed Exact S-parameter in Conformal Theories. *Phys. Rev.*, D82:081701, 2010.

[53] Francesco Sannino. Magnetic S-parameter. *Phys. Rev. Lett.*, 105:232002, 2010.

[54] Edward Witten. An SU(2) anomaly. *Phys. Lett.*, B117:324–328, 1982.

[55] Hidenori S. Fukano and Francesco Sannino. Conformal Window of Gauge Theories with Four-Fermion Interactions and Ideal Walking. *Phys. Rev.*, D82:035021, 2010.

[56] Hidenori S. Fukano and Francesco Sannino. Minimal Flavor Constraints for Technicolor. *Int. J. Mod. Phys.*, A25:3911–3932, 2010.

[57] Roshan Foadi, Matti Jarvinen, and Francesco Sannino. Unitarity in Technicolor. *Phys. Rev.*, D79:035010, 2009.

[58] Steven Weinberg. Precise relations between the spectra of vector and axial vector mesons. *Phys. Rev. Lett.*, 18:507–509, 1967.

[59] Dennis D. Dietrich and Francesco Sannino. Walking in the SU(N). *Phys. Rev.*, D75:085018, 2007.

[60] Thomas A. Ryttov and Francesco Sannino. Ultra Minimal Technicolor and its Dark Matter TIMP. *Phys. Rev.*, D78:115010, 2008.

[61] Claude Amsler et al. Review of particle physics. *Phys. Lett.*, B667:1–1340, 2008.

[62] Michael Dine, Willy Fischler, and Mark Srednicki. Supersymmetric Technicolor. *Nucl. Phys.*, B189:575–593, 1981.

[63] Bogdan A. Dobrescu. Fermion masses without Higgs: A Supersymmetric technicolor model. *Nucl. Phys.*, B449:462–482, 1995.

[64] Hitoshi Murayama. Technicolorful supersymmetry. 2003.

[65] Jared A. Evans, Jamison Galloway, Markus A. Luty, and Ruggero Altair Tacchi. Flavor in Minimal Conformal Technicolor. *JHEP*, 04:003, 2011.

[66] Tony Gherghetta and Alex Pomarol. A Distorted MSSM Higgs Sector from Low-Scale Strong Dynamics. 2011.

[67] Rudolf Haag, Jan T. Lopuszanski, and Martin Sohnius. All Possible Generators of Supersymmetries of the s Matrix. *Nucl. Phys.*, B88:257, 1975.

[68] Nick Dorey and S. Prem Kumar. Softly-broken N = 4 supersymmetry in the large-N limit. *JHEP*, 02:006, 2000.

[69] Stephen P. Martin. A Supersymmetry Primer. 1997.

[70] Marco Ciuchini et al. Delta M(K) and epsilon(K) in SUSY at the next-to-leading order. *JHEP*, 10:008, 1998.

[71] M. Maniatis. The Next-to-Minimal Supersymmetric extension of the Standard Model reviewed. *Int.J.Mod.Phys.*, A25:3505–3602, 2010.

[72] Alessandro Strumia. The fine-tuning price of the early LHC. *JHEP*, 04:073, 2011.

[73] Aleksandr Azatov, Jamison Galloway, and Markus A. Luty. Superconformal Technicolor. 2011.

[74] Roshan Foadi, Mads T. Frandsen, Thomas A. Ryttov, and Francesco Sannino. Minimal Walking Technicolor: Set Up for Collider Physics. *Phys. Rev.*, D76:055005, 2007.

[75] Michela Petrini. Infrared stability of N = 4 super Yang-Mills theory. *Phys. Lett.*, B404:66–70, 1997.

[76] Yoichiro Nambu and G. Jona-Lasinio. Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. 1. *Phys.Rev.*, 122:345–358, 1961.

[77] Yoichiro Nambu and G. Jona-Lasinio. DYNAMICAL MODEL OF ELEMENTARY PARTICLES BASED ON AN ANALOGY WITH SUPERCONDUCTIVITY. II. *Phys.Rev.*, 124:246–254, 1961.

[78] Koichi Yamawaki. Dynamical symmetry breaking with large anomalous dimension. 1996. 44 pages, LateX, To appear in Proc. 14th Symposium on Theoretical Physics "Dynamical Symmetry Breaking and Effective Field Theory", Cheju, Korea, July 21-26, 1995 Report-no: DPNU-96-10.

[79] Stefano Di Chiara and Ken Hsieh. Triplet Extended Supersymmetric Standard Model. *Phys. Rev.*, D78:055016, 2008.

[80] Hong-Jian He, Nir Polonsky, and Shu-fang Su. Extra families, Higgs spectrum and oblique corrections. *Phys. Rev.*, D64:053004, 2001.

[81] Mads T. Frandsen, Isabella Masina, and Francesco Sannino. Fourth Lepton Family is Natural in Technicolor. *Phys. Rev.*, D81:035010, 2010.

[82] Christopher T. Hill and Elizabeth H. Simmons. Strong dynamics and electroweak symmetry breaking. *Phys. Rept.*, 381:235–402, 2003.

[83] John F. Gunion, Howard E. Haber, Gordon L. Kane, and Sally Dawson. THE HIGGS HUNTER'S GUIDE. *Front. Phys.*, 80:1–448, 2000.

[84] Abdelhak Djouadi. The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model. *Phys. Rept.*, 457:1–216, 2008.

[85] Howard Georgi. Unparticle Physics. *Phys. Rev. Lett.*, 98:221601, 2007.

[86] Ofer Aharony, Steven S. Gubser, Juan Martin Maldacena, Hirosi Ooguri, and Yaron Oz. Large N field theories, string theory and gravity. *Phys.Rept.*, 323:183–386, 2000.

[87] Francesco Sannino and Roman Zwicky. Unparticle & Higgs as Composites. *Phys. Rev.*, D79:015016, 2009.

[88] Bogdan A. Dobrescu. Massless gauge bosons other than the photon. *Phys. Rev. Lett.*, 94:151802, 2005.

[89] K. Nakamura et al. Review of particle physics. *J. Phys.*, G37:075021, 2010.

I. M. Antola, M. Heikinheimo, F. Sannino and K. Tuominen,
"*Unnatural Origin of Fermion Masses for Technicolor*"
JHEP 03 (2010) 050, arXiv:0910.3681 [hep-ph]

II. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*Minimal Super Technicolor*"
Eur. Phys. J. C 71, 1784 (2011), arXiv:1001.2040 [hep-ph]

III. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*Supersymmetric Extension of Technicolor & Fermion Mass Generation*"
arXiv:1111.1009 [hep-ph], accepted for publication in NPB

IV. M. Antola, S. Di Chiara, F. Sannino and K. Tuominen,
"*N=4 Extended MSSM*"
Nucl. Phys. B 856, 647 (2012), arXiv:1009.1624 [hep-ph]