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LABORATORY STUDY ON CRACKS IN SATURATED SANDS*

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ABSTRACT: It has been reported[1] that when a loosely packed column of saturated sand in a vertical cylindrical container is shock loaded axially by dropping to the floor, large horizontal cracks initiate, grow and eventually fade away in the sand as it settles under gravity. This paper shows that a similar phenomenon can also be observed when shock loading is replaced by forcing water to percolate upward through the sand column. It is believed that our result sheds further light on the physics of formation of these cracks.

KEY WORDS: saturated sand, impact, steady drive, crack, liquefaction index

1 INTRODUCTION

The dynamic behavior of saturated sand was studied by W.A. Charlie, J.B. Bolton and G.E. Veyera[1~3] using a shock tube type set-up, in which the dynamic load was exerted by a piston driven by a bullet, designed to simulate shock loading on saturated sand near an exploding charge. Their aim was to establish the relationship between load and liquefaction for given initial effective stress and density. They defined the liquefaction index Y as

$$Y = p_e/\sigma_0' \tag{1}$$

where p_e is excess pore water pressure, and σ_0' the initial effective stress. $Y = 1$ indicates liquefaction and $Y < 1$ means that liquefaction has not taken place.

Based on the Poiseuille's capillary model on the relationship between the mean flow rate, the mean pressure gradient and mean drainage radius, J. Studer and L. Kok[4] analyzed the shear stress on the capillary wall at liquefaction, and pointed out that the liquefaction condition is $Y = 1$.

Recently, in an experimental study of the drainage and densification of saturated sand under impact, Zhang Junfeng and Meng Xiangyue et al.[5,6] observed that, in saturated sand cracks may form, grow and close; the pore water gathers inhomogeneously and short drainage pathways appear; and these phenomena are related closely to the size-grading and inhomogeneity in the vertical direction of the sand samples.

A possible explanation for the formation of horizontal cracks of large size could be that, when the compression wave arrives at the free surface, a reflected rarefaction wave forms so

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that the sand in the neighborhood of the free surface breaks due to its low tensile strength. Nevertheless, the characteristic time for the waves is on the order of milliseconds, and the horizontal cracks form several tens of seconds later. Therefore, crack formation can not be the result of the reflected tensile wave. In fact, percolation and consolidation subsequent to the shock are of much greater relevance.

To show that the appearance of horizontal cracks is a late stage phenomenon, a new experiment has been carried out for which the shock loading is replaced by a steady current of water forced to percolate upward through the vertical column of sand. This experiment demonstrates that among other things, this view is indeed true and, as far as the formation of such cracks is concerned, shock loading is nonessential.

2 DESCRIPTION OF THE EXPERIMENT

A schematic diagram of the experimental device is shown in Fig.1. The main part is a vertical plexiglass tube with an inner diameter of 59 mm. The tube is filled with water and contains a column of saturated sand 400 mm in height initially standing on a perforated plate. At the lower end, the tube is connected via a water duct to a reservoir which maintains a constant pressure drop across the specimen during the experiment. A flowmeter measures the flow rate U. Six equally spaced pressure holes along the column of sand are connected to pressure gauges marked by 1~6 from the top to the bottom. The vertical coordinate pointing upwards is defined as z, and the origin is placed at the bottom of the sand column. The height of the sand column H_0 is 400 mm. The coordinates of the pressure measuring holes are $h_n = (400 - n \cdot 60)$ mm, where $n = 1 \sim 6$.

In this experiment four kinds of natural sand are used. Two of them (samples 1 and 2) labeled here as well graded are composed of sand grains with a broad size distribution as shown in Fig.2. The other two (samples 3 and 4) labeled as poorly graded are composed of sand grains of much narrower size distribution.

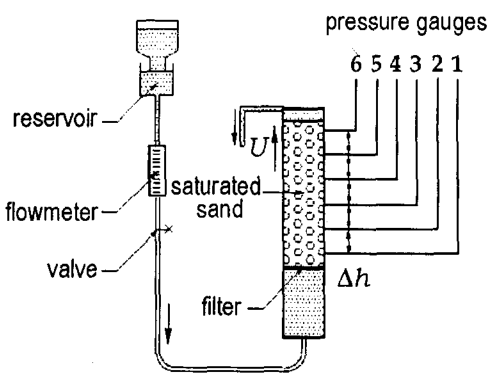

Fig.1 Setup of steady drive experiments

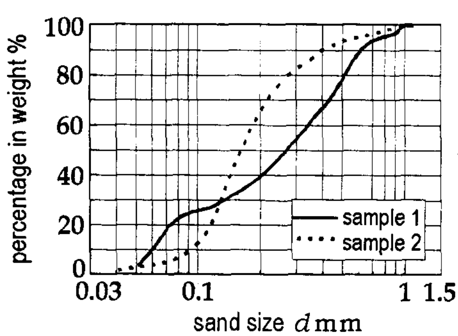

Fig.2 Size-grading of samples

Figure 2 shows the grain size distribution of the well graded sand samples 1 and 2. For the sample 1, the density of the dry sand is $\rho_s = 2.70\,\mathrm{g/cm^3}$, the nonuniformity coefficient is $c_u = d_{60}/d_{10} = 5.8$, the grading coefficient is $c_c = d_{30}^2/(d_{10}d_{60}) = 0.93$, the minimum and maximum porosity are $\varepsilon_{\min} = 0.28$ and $\varepsilon_{\max} = 0.38$, the minimum and maximum permeability are $k_{\min} = 4.3 \times 10^{-5}$ m/s and $k_{\max} = 7.3 \times 10^{-5}$ m/s respectively. For the sample 2, $\rho_s = 2.65\,\mathrm{g/cm^3}$, $c_u = 2.0$, $c_c = 1.0$, $\varepsilon_{\min} = 0.29$ and $\varepsilon_{\max} = 0.40$, $k_{\min} = 5.6 \times 10^{-5}$ m/s and $k_{\max} = 9.1 \times 10^{-5}$ m/s respectively. For the sample 3, the particle sizes distribute in the range of 0.32 mm~0.40 mm, $\varepsilon_{\min} = 0.37$ and $\varepsilon_{\max} = 0.44$ respectively. For the sample 4, the particle sizes distribute in the range of 0.15 mm~0.32mm, $\varepsilon_{\min} = 0.38$ and $\varepsilon_{\max} = 0.47$ respectively.

3 EXPERIMENTAL PHENOMENA

Experiments have been carried out for three kinds of samples, i.e.

(1) Poorly graded sand samples;

(2) Well graded sand samples;

(3) Well graded sand samples, but with a thin layer of fine sand sandwiched in between. All experiments are run under prescribed pressure drops.

The excess pore-water pressure is

$$p_e = p - \rho_w g h \tag{2}$$

where p is pore-water pressure, ρ_w—water density, and g—gravitational acceleration.

The pressure gradient J is defined as

$$J = (\mathrm{d}p_e/\mathrm{d}h)/(\rho_w g) \sim (\Delta p_e/\Delta h)/(\rho_w g) \tag{3}$$

The initial effective stress at $z = h$ is

$$\sigma_0'(h) = \int_0^h \rho_s' g \mathrm{d}h = \rho_s' g h \tag{4}$$

where $\rho_s' = (1 - \varepsilon)(\rho_s - \rho_w)$, ρ_s is the density of sand grains, and ε is porosity.

Since inertia forces are negligible, the effective stress u is equal to $\sigma_0' - p_e$ in the absence of wall friction. Within this context, one can say that $Y = 1$ is a valid necessary condition for liquefaction.

3.1 Experiments for the Poorly Graded Sand Samples (Samples 3 and 4)

In all experiments, excess porewater pressure and the flow rate of water are simultaneously measured together with continuous observation of deformation and water flow for each pressure difference applied to the ends of the sand column. The main features are:

(1) No horizontal cracks are observed;

(2) As the pressure difference is raised, vertical drainage pathways begin to appear. Furthermore, the smaller the average size of sand and consequently the smaller the permeability, the smaller the minimum flow rate necessary for the occurrence of vertical drainage pathways;

(3) For each applied pressure drop, the pressure gradient varies only a little along the sand column. For the small flow rate, Darcy's law works well and the experiment yields reproducible results.

3.2 Experiments for the Well Graded Sand Samples

In this set of experiments for the well graded sand samples (samples 1 and 2), similar measurements are made. The main features are:

(1) Horizontal cracks in addition to vertical drainage pathways occur for certain applied pressure drops;

(2) Within the range of pressure drop of our experiments, the pressure gradient along the sand column is no longer constant, and the permeation deviates from Darcy's law. This may be attributed to the following two factors, i. e. (a) The occurrence of horizontal cracks and vertical pathways causes the permeation to become nonuniform and irregular concurrent with structure damage in the sand; (b) Because of the well grading of sand, the distribution of size-grading along the sand column is not uniform;

(3) Horizontal cracks appear at the position where the pressure gradient J is larger as shown in Fig.3 by the J-z curve;

(4) Horizontal cracks appear at the position where the liquefaction index Y nearly equals to 1.0 as shown in Fig.4 by the Y-z curve.

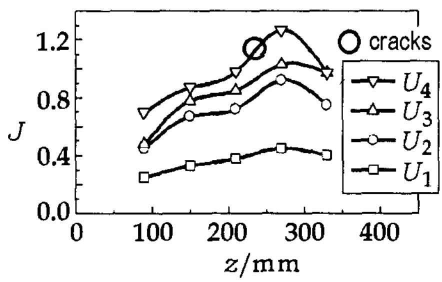

Fig.3 Pressure gradient versus z for sample 2

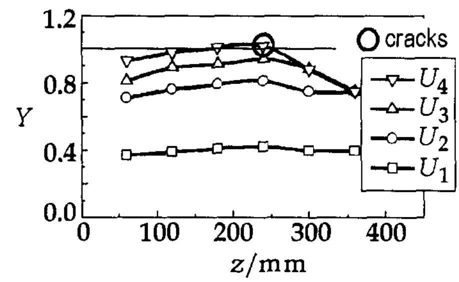

Fig.4 Liquefaction index versus z for sample 2

($U_1 = 0.06\,\mathrm{mm/s}$, $U_2 = 0.11\,\mathrm{mm/s}$, $U_3 = 0.13\,\mathrm{mm/s}$, $U_4 = 0.17\,\mathrm{mm/s}$)

3.3 Experiments for the Well Graded Sand Sample with a Thin Layer of Fine Sand Sandwiched in between

In the preparation process of the sample, a special sand loading procedure was used, i. e., the loading of sand was interrupted for 10 $\sim$ 20 seconds before reloading. By doing so, a very thin layer of fine sand settles on the top of the lower section of sand column. In this experiment, we find:

(1) The pressure gradient at the thin layer of fine sand is larger than elsewhere. If there is only a single such layer, and if it is sufficiently thick, then a crack develops there (Fig.5). When J is sufficiently large, but if the thickness of the layer is too thin, no such crack occurs, because it is easily punctured and rendered in effective by numerous capillary flows;

(2) The horizontal crack basically occurs at the position where the liquefaction index equals nearly 1.0 as shown in Fig.6. But no crack forms where $Y > 1$ indicates that at these positions, the effective wall friction may not be negligible;

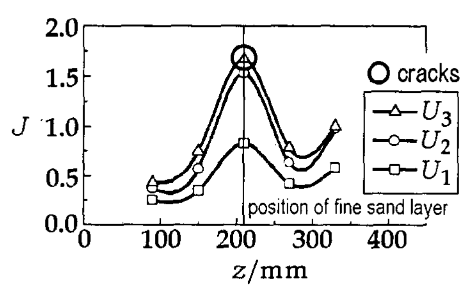

Fig.5 Pressure gradient versus z for sample 1

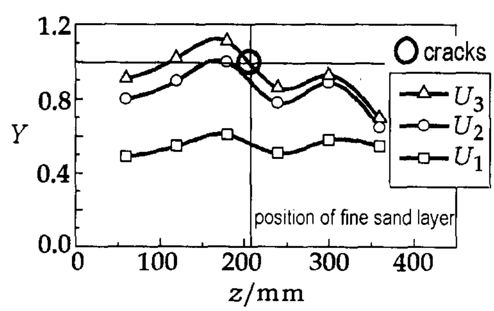

Fig.6 Liquefaction index versus z for sample 1

($U_1 = 0.08\,\mathrm{mm/s}$, $U_2 = 0.13\,\mathrm{mm/s}$, $U_3 = 0.16\,\mathrm{mm/s}$)

(3) If several thin layers of fine sands are placed in the sand column, it is not certain at which layer crack first occurs. Generally speaking, cracks occur in the upper and middle portion of the sand column, and rarely in the lower part.

4 ANALYSIS AND DISCUSSION

4.1 Relationship between Occurrence of Horizontal Cracks and the Liquefaction Index Y

In the analysis of experimental data, the formation of horizontal cracks is correlated with the pressure gradient and the liquefaction index. We find that the value of liquefaction index bears direct relationship to the occurrence of horizontal cracks. Table 1 shows the correlation with the liquefaction index.

Table 1 Liquefaction index at the position of cracks

	Sample 1				Sample 2			
charac-teristics	1/4 cracks	1/2 cracks	through cracks	fine sand cracks	1/4 cracks	1/2 cracks	through cracks	fine sand cracks
N	14	13	14	13	8	8	9	10
Y_{min}	0.74	0.89	0.98	0.91	0.91	0.93	0.94	0.90
Y_{max}	1.12	1.04	1.07	1.11	1.01	1.11	1.19	1.07
$\bar{Y}$	0.96	0.98	1.03	1.02	0.96	1.01	1.02	0.98
S_Y	0.08	0.04	0.04	0.05	0.04	0.07	0.07	0.05
$E_Y/\%$	8	4	4	5	4	7	7	5

In the table, by 1/4 cracks and 1/2 cracks we mean that the ratio of the circumferential length of horizontal cracks and the circumference of sand column are 1/4 and 1/2 respectively, by fine sand cracks we mean cracks occurring at the thin layer of fine sand, N is the number of experimental points, $\bar{Y}$ is the arithmetical mean of Y, S_Y is the standard quadratic deviation, and $E_Y(= S_Y/Y)$ is the error coefficient for the liquefaction index.

The fact that the quadratic standard variance is small leads to the conclusion that $Y = 1$ at the crack is indeed a valid necessary condition for liquefaction and crack formation justifying the assumption that the wall friction may be neglected.

4.2 Position of Horizontal Cracks

For the well graded sand samples, most of the horizontal cracks occur in the upper part of the sand column. Let $Z = z/H_0(0 \le Z \le 1)$ and $P(Z)$ denotes the probability of occurrence at the position higher than or equal to z, then the distribution function is $f(Z) = \mathrm{d}P(Z)/\mathrm{d}Z$. Figure 7 shows these functions for samples 1 and 2. These two curves are similar in shape, and possess the following common features: for $Z = 0.4 \sim 0.8$, $f > 1.0$; f takes its maximum within $Z = 0.5 \sim 0.6$; and for $Z = 0 \sim 0.3$ or $Z = 0.9 \sim 1.0$, $f < 1.0$.

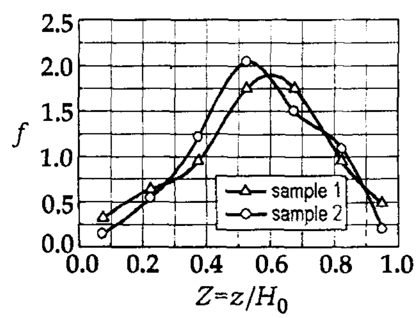

Fig.7 Distribution function for the position of cracks occurrence

4.3 A Simplified Model for Elucidating the Crack Formation at the Thin Layer of Fine Sand

When a thin layer of fine sand is located in the upper part of the sand column and sufficiently large pressure drop is applied, a horizontal crack forms at the thin layer. A simplified one-dimensional model is presented here to further elucidate the condition of horizontal crack formation in the saturated sand column.

Suppose a thin layer of fine sand exists in the sand column, and its thickness is $\Delta h = h_2 - h_1$, we assume that

(1) The variation of permeability k with height h is expressed by two step functions as shown in Fig.8, where the permeability ratio $\gamma = k_1/k_2 > 1$;

(2) Aside from k, we assume all other relevant parameters are constant throughout the column. Deformation of sand is neglected.

(3) Permeation obeys Darcy's law, i.e., the seepage velocity is proportional to pressure gradient;

(4) Both sand and pore-water are treated as incompressible, so that the seepage velocity does not depend on the height h;

(5) The saturated sand can not bear tensile stress.

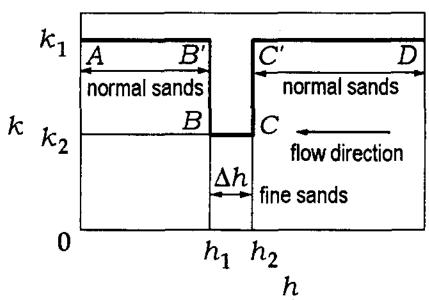

Fig.8 Permeability distribution

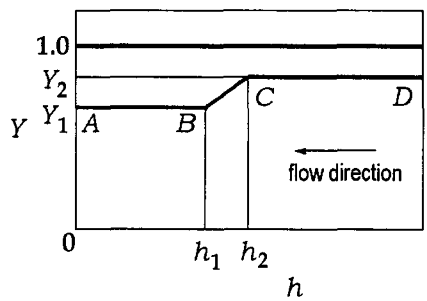

Fig.9 Liquefaction index distribution

According to Darcy's law

$$U = k \cdot J = k \cdot (\mathrm{d}p_e/\mathrm{d}h)/(\rho_w g) \tag{5}$$

integrating (5) and using $p_e = 0$ at $h = 0$, we have

$$p_e = \int_0^h (\rho_w g U/k)\mathrm{d}h = \rho_w g U \int_0^h (1/k)\mathrm{d}h \tag{6}$$

From (1), (4) and (6), the variation of liquefaction index Y with height h for given seepage velocity U is shown in Fig.9. It can be seen that the liquefaction index reaches a maximum at point $C(h = h_2)$ corresponding to the lower face of the thin layer of fine sand. Therefore, with the increase of seepage velocity, the liquefaction condition is satisfied first at the lower end of that layer. When there is only one thin layer of fine sand in the sample, the ratio of liquefaction indices at points C and point B is written as λ, and we have

$$\lambda = Y_2/Y_1 = 1 + (\gamma - 1)\Delta h/h_2 \tag{7}$$

The ratio λ is a useful parameter reflecting the characteristics of the thin layer of fine sand, for given Y_1, Y_2 reaches 1 the sooner the larger λ.

5 CONCLUDING REMARKS

By driving a water current upward through a vertical column of sand with a steady pressure drop, intensified permeation and structural change in sand are examined. Based on the measurement of excess porewater pressure, the condition and mechanism for formation of horizontal cracks are explored. It is shown that like in impact test, horizontal cracks can also appear in saturated sand under a steady fluid percolating against gravity. In both cases, liquefaction is a necessary condition for the formation of horizontal cracks. Furthermore,

(1) Horizontal cracks do not form in poorly-graded sand, but do form in well-graded sand, in agree with previous results[5].

(2) For given pressure head, when the distribution of pressure gradient is not uniform along the vertical direction, horizontal cracks tend to form at the position where pressure gradient is relatively large;

(3) The formation of horizontal cracks is closely related to the magnitude of liquefaction index. A necessary condition for the formation of horizontal cracks is that liquefaction index be equal to 1.

Lastly, we note that the formation of horizontal cracks by impact loading and by forcing a percolating water through the sand column differs in an essential way. In the former case, the whole column of sand is liquefied immediately after impact whereas in the other case, the sand is initially not liquefied.

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