The Stubborn Non-probabilist—'Negation Incoherence' and a New Way to Block the Dutch Book Argument

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Abstract. We rigorously specify the class of nonprobabilistic agents which are, we argue, immune to the classical Dutch Book argument. We also discuss the notion of expected value used in the argument as well as sketch future research connecting our results to those concerning incoherence measures.

1 Introduction

Suppose you decide that your first task on a sunny Tuesday morning is to convince your friend who does not subscribe to probabilism (that is, he claims his degrees of belief need not be classical probabilities) of the error of his ways[1]. You decide to try the classical Dutch Book argument first. To your surprise you discover that your friend is not worried about the somewhat pragmatic nature of the argument, allows you to set all the stakes to 1 for convenience, and, while claiming that the set of propositions about which he holds some degree of belief is finite, he is eager to contemplate betting on virtually anything. He also considers a bet to be fair if its expected profit both for the buyer and seller is null, and even accepts the 'package principle', that is, believes a set of bets to be fair if each of the bets in that set is fair. Knowing all that, when telling your friend about how fair betting quotients are connected with the Kolmogorov axioms, and then about the identification of fair betting quotients with degrees of belief, you expect him to be immediately convinced.

To your surprise he shakes his head in opposition, saying 'I agree that fair betting quotients are exactly those which satisfy the axioms of classical probability. Still, even when we set all stakes to 1, I don't believe that these quotients are my degrees of belief.'

'But Alan', you say, 'this is standard. We went through this. We agreed that if your degree of belief in A is $b(A)$, and your degree of belief in $\neg A$ is $b(\neg A)$, then your betting quotient for the bet for A is that particular q for which the expression $b(A) \cdot (1 - q) + b(\neg A) \cdot (-q)$, that is, what you expect to be the value

[1] Why Tuesday? See [7].

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of the bet, equals 0. And it is a matter of mundane calculation that q is exactly $b(A)$. In general this means that betting quotients are your degrees of belief.'

'Still, look' – your friend responds – 'you're missing one thing. It's just that in my case $b(A) + b(\neg A)$ is in general not equal to 1. My degrees of belief are such that for each proposition A there is a non-zero number r_A for which it holds that $b(A) + b(\neg A) = r_A$; some of those numbers may be equal to 1, but none need be. And so my betting quotient for the bet for a proposition A is in general $^{b(A)}/_{r_A}$. Can you run your argument using such quotients?'

Well, *can you*? It turns out that sometimes you can – but sometimes not. It all depends on the particulars of your friend's belief state. In what follows we will specify the formal details. Notice that the way the story is set up, our friend has granted you the assumptions needed to overcome the well known flaws of the Dutch Book argument (discussed e.g. in [1,17]). Still, it seems that even then he needs not be persuaded by the reasoning. This suggests that we have here a problem for Dutch Book arguments.

The feature of a belief function described above, which we take as suggesting a way in which a nonprobabilist can resist the Dutch Book argument, was first described in print by [8] and called "negation incoherence". In this paper we go further:

- by saying something new regarding why and when a nonprobabilistic agent might not violate the norm of rationality appealed to in Dutch Book arguments;
- more explicitly, by proving a theorem describing the class of those nonprobabilistic agents which are, we believe, immune to Dutch Book arguments,
- and lastly, by discussing some different ways in which an incoherent agent might approach the task of calculating the expected value of some bet.

We take negation incoherence, then, as another reason for which "betting odds and credences come apart" ([2,13]), but which, let us note here, has nothing to do with the issues related to self-location (for the root of a big part of modern literature on that subject see [4]). The key to our idea is that while we see nothing wrong with the classical Dutch Book *theorems*, they concern betting quotients (or odds), while the Dutch Book *argument* tries to establish something about credences. If there are situations in which these two "come apart", that is, should not simply be identified with each other, we should either say that the argument is not applicable (which may be a sensible road to take in the face of self-location problems) or try to first establish a rigorous link between them and then to reevaluate the fate of the Dutch Book argument. The latter is the way we have chosen for this article.[2]

Simply *assuming* that degrees of belief are to be identified with betting quotients would amount to adopting some kind of operational approach to credences, with the details depending on how the understanding of the quotients would be fleshed out. We see little gain from this, aside from a short-lived satisfaction at a spurious connection to empirical matters. We are motivated rather by the spirit

[2] We would like to thank one of the Reviewers for pressing us on this.

of [5]; that is, we try to keep an open mind regarding what degrees of belief *are*, and investigate the relationship between them and betting quotients on a single basic assumption: that whatever they are, they can be expressed by a real number.

2 Details

Notice first that assuming that in general $b(A) + b(\neg A) = 1$ does not amount to assuming the probabilist thesis, that is, the problem is not that of pure *petitio principii*. Still, by doing so we are assuming something with which a nonprobabilist may by no means agree. We just know that by denying it, he has to hold that the additivity axiom or the normalization axiom (stating that the probabilities of tautologies equal 1) is not satisfied by his degrees of belief.

We can arrive at the problem from another direction. The traditional way of looking at the Dutch Book argument for probabilism would have it imply that possessing degrees of belief which violate classical probability axioms is a mark of irrationality. This should be puzzling if we think about the particular form of the 'normalization' axiom used in the classical axiomatization of probability. If we believe tautologies to a degree different from 1, we can apparently be Dutch-booked. Surely there's a mistake here: the choice of the number 1 as the probability of tautologies is purely conventional. The number 2 (say) would do just as well. But if we are careful about setting the betting quotients the way with which our nonprobabilistic friend would agree, then if his degree of belief in countertautologies is 0 and his degree of belief in tautologies is 2, his betting quotient for tautologies is 1, exactly the same as in the classical case.

Let us continue towards the theorem specifying the class of cases in which a nonprobabilist is not Dutch-bookable. In this paper we confine our attention to finite structures.

One of the main points of this paper is to identify/determine the conditions under which it is possible to link betting quotients with credences while arguing for probabilism. We will see that the exact identification of betting quotients and credences is possible when the agent is not "negation-incoherent", and so it should not be surprising that mathematically the betting quotient functions and the credence (degrees of belief) functions are objects of the same type. That is, they are functions from an algebra of events (propositions) defined over a given set (interpreted as a set of possible worlds, sample space or whatnot). The difference between these functions, argumentation-wise, lies in their interpretations and is justified by the way the degrees of belief of a given agent induce betting quotients in betting scenarios via the condition of fairness of bets. All this should be clear by the time the Reader reaches Definition 4 below. Let us start with the basic notions of belief and betting spaces:

Definition 1 (Belief space, betting space)**.** *A belief (betting) space is a tuple* $\langle W, Prop, b\rangle$, *(*$\langle W, Prop, q\rangle$*) where* W *is a nonempty finite set,* $Prop$ *is a Boolean algebra of subsets of* W *('propositions'), and* b *(*q*) is a function from* $Prop$ *to* $\mathbb{R}$*, called the* belief function (betting quotient function)*.*

In what follows we always assume that we are given a (finite) set W and a Boolean algebra $Prop \subseteq \mathcal{P}(W)$ of subsets of W ('propositions').

Let us now provide an intuitive description of the concept of betting quotients. We say that a bet on a proposition $A \in Prop$ consists of a stake $s(A)$ and a price $p(A)$ (both real numbers) considered by the agent to be fair for a bet regarding A with that particular stake ('fair' as in 'not favouring either side'), as well as the agent's payoffs: $s(A) - p(A)$ in the case A is true, and $-p(A)$ in the case A is false. Intuitively, the agent's betting quotient for A equals $\frac{p(A)}{s(A)}$, and is simply the price of a bet with the unit stake $s(A) = 1$ considered by the agent as fair. On our account the betting quotient is attached to a proposition, and so it is not the price $p(A)$ that the betting quotient depends on, but rather the other way round: the price that the agent considers fair is determined by her betting quotients and the announced stake. Therefore, it is already here that the Reader might observe that a thing crucial for an accurate interpretation of Dutch Book scenarios is understanding under what conditions the agent considers a given price of a given bet to be fair.

With the interpretation of the betting quotient function in hand, we are in the position to state the formal definition of Dutch Books and recall the Dutch Book Theorems that seem to constitute the main engine of Dutch Book Arguments.

Definition 2 (Dutch Book)**.** *Let W be a non-empty (finite) set and let $\mathcal{F} \subseteq \mathcal{P}(W)$ be a Boolean algebra of its subsets. Let $q : \mathcal{F} \to \mathbb{R}$ be a real-valued function. We say that q is* ***susceptible to a Dutch Book (is Dutch-Bookable; permits a Dutch Book)*** *if there exists a function $s : \mathcal{F} \to \mathbb{R}$ and $\mathcal{F}_0$, a finite non-empty subset of $\mathcal{F}$, such that for any $w \in \bigcup \mathcal{F}$ the following inequality holds:*

$$U(w) = \sum_{E \in \mathcal{F}_0 : w \in E} (1 - q(E))s(E) - \sum_{E \in \mathcal{F}_0 : w \notin E} q(E)s(E) = \sum_{E \in \mathcal{F}_0} (\chi_E(w) - q(E))s(E) < 0,$$

where χ_E is the characteristic function of the set E.

We can also say in such a case that the betting space $\langle W, \mathcal{F}, q \rangle$ permits a Dutch Book, or that it is Dutch-bookable, or that an agent with a betting quotient function q is susceptible to a Dutch Book (is Dutch-bookable).[3]

Definition 3 (Classical probability function (finite))**.** *A function p from $Prop$ to $\mathbb{R}$ is* a classical probability function *if it satisfies the following three axioms:*

1. *$p(W) = 1$ (the normalization axiom),*
2. *for any A in $Prop$ $p(A) \geq 0$ (the non-negativity axiom),*
3. *for any A and B whose intersection is empty $p(A \cup B) = p(A) + p(B)$ (the additivity axiom).*

Theorem 1 (Dutch Book Theorem - [9,10])**.** *A betting quotient function is not Dutch-bookable iff it is a classical probability function.*

[3] For a detailed discussion of defining Dutch Books in the more general context of (possibly) nonclassical spaces, including a detailed discussion of the formula for $U(w)$, see [18].

Let us notice that it is crucial to distinguish between:

- the Dutch Book Theorem (DBT), which is an established mathematical result, and
- the Dutch Book Argument (DBA) for probabilism, which only uses the DBT as one of its premises.

Let us consider that direction of the DBA which aims to establish that violating probabilism leads to violating some norm of rationality. The structure of the argument is usually as follows:

1. Assume that a given agent's belief function violates the classical probability axioms.
2. Identify the agent's credences with her betting quotients, that is, the quotients of bets fair according to her[4]—this means that the agent's betting quotient function violates probability axioms.
3. By the Dutch Book Theorem such the agent is guaranteed a sure loss.
4. Ergo, the agent's degrees of belief are irrational.

As the Reader sees in point 2, the argument identifies the degrees of belief with the betting quotients. This might seem close to obvious, as for instance [2] claim[5]:

> "All we need is for there to be a normative link between the belief and the bet. Something like 'Other things being equal (risk-neutral, utility linear with money, ...), an agent who accepts E with 50% certainty is rationally permitted to accept a bet on E that pays twice the stake or better'. This link is broadly accepted, and will be all we need."

What we intend to show in this paper is that the *broad acceptance* reported in the quote above actually deserves serious and careful scrutiny—we hope to demonstrate that it should actually be rejected and although there is a link between the credence and the quotient, it by no means has to be identity in all cases.

The only constraint that we have with respect to the nature of the above-mentioned link is to make sure that the agent expects the value of the bet for A to be 0 (assume all stakes are set to 1; nothing important in the argument depends on that) which is a natural explication of fairness of a given bet: it does not favour any of the sides. Thus, what we need to guarantee, while linking the belief function b to the betting quotient function q, is that for any event A:

$$b(A)(1 - q(A)) - b(\neg A)q(A) = 0.$$

[4] Some variation is possible at this point. Some may prefer to speak instead about bets the agent would accept. This will not be important for the topic of our paper.

[5] Note, though, that the authors talk about and agent being permitted to accept a bet if she does not expect her own loss, and so they do not use the concept of fairness as not favouring either side. This is tangential to our argument.

Notice that as mentioned above, normalization and additivity imply that for any A in $Prop$ $b(A) + b(\neg A) = 1$, that is, they imply the assumption we need for the 'classical' connection between degrees of belief and betting quotients, i.e. their identification. In our case we wish to play by our friend's rules, that is, for any A, we want to set the betting quotient for A to $b(A)/(b(A)+b(\neg A))$: this way we will make sure that indeed our friend expects the value of the bet for A to be 0. Therefore, we may then define:

Definition 4 (Induced betting quotient)**.** *A belief space $\langle W, Prop, b\rangle$ induces a betting quotient $q : Prop \to \mathbb{R}$ if for any $A \in Prop$:*

1. $b(A) + b(\neg A) \neq 0$,
2. $q(A) = \frac{b(A)}{b(A)+b(\neg A)}$.

Defined this way, $q(A)$ is the betting quotient which makes a bet for or against A such that an agent with a belief function b expects it to have value 0.[6] It follows that if a belief space induces a betting quotient function, that is, if the first condition of the above definition holds, then that function is unique.

The question which now arises is the following: are there any non-probabilistic epistemic agents (i.e. such that their degrees credences violate the classical probability axioms) that are not susceptible to Dutch Books, i.e. such that their betting quotients (induced by their credences) are not Dutch-Bookable? The answer is given by the following simple theorem:

Theorem 2. *The betting quotient function q induced by a belief space $\langle W, Prop, b\rangle$ is a classical probability function iff the following conditions hold:*

1. $b(\emptyset) = 0$,
2. *for any A in $Prop$* $b(A) \cdot b(\neg A) \geq 0$,
3. *for any A and B in $Prop$ with an empty intersection:*

$$\frac{b(A \cup B)}{b(A \cup B) + b(\neg(A \cup B))} = \frac{b(A)}{b(A) + b(\neg A)} + \frac{b(B)}{b(B) + b(\neg B)}.$$

Proof. Let q be the belief quotient function induced by a belief space $\langle W, Prop, b\rangle$.

($\Rightarrow$) Assume q is a classical probability function. By the normalization axiom $q(W) = 1$, and by the additivity axiom $q(\emptyset \cup W) = q(W) = q(W) + q(\emptyset)$, so $q(\emptyset) = 0$. Thus $b(\emptyset) = q(\emptyset) \cdot (b(\emptyset) + b(W)) = 0$.

Let $A \in Prop$. By the definition of the induced betting quotient we have $b(A) \cdot b(\neg A) = [q(A) \cdot (b(A) + b(\neg A))] \cdot [q(\neg A) \cdot (b(A) + b(\neg A))] = q(A) \cdot q(\neg A) \cdot (b(A) + b(\neg A))^2$. As the function q is non-negative and we multiply the square of $(b(A) + b(\neg A))$, the value of the entire expression is ≥ 0.

The last condition holds since it basically says that $q(A) + q(B) = q(A \cup B)$ for disjoint sets in $Prop$, which is guaranteed by the additivity axiom.

($\Leftarrow$) Assume the conditions form the statement of the theorem hold. Trivially, since $b(\emptyset) = 0$ and q is induced by b, it holds that $q(\emptyset) = 0$ and $q(W) = 0 + \frac{b(W)}{b(W)+b(\emptyset)} = 0 + \frac{b(W)}{b(W)+0} = 0 + 1 = 1$, so normalization holds.

[6] For more regarding the notion of expected value see Sect. 4 below.

Let $A \in \mathit{Prop}$. We have $q(A) = \frac{b(A)}{b(A)+b(\neg A)}$, and $b(A)$ and $b(\neg A)$ are of the same sign (or one of them is equal to 0). Thus, both the counter and the denominator of the formula defining $q(A)$ are of the same sign as well (or the counter is equal to 0). Thus, $q(A) \geq 0$.

The additivity of q follows trivially from the formulation of the third condition. □

It is worthwhile to reflect on which steps of the above proof depend on what properties of the classical probability measure on the one hand, and the induced betting quotient on the other. If b satisfies the conditions in the statement of the theorem, then the normalization axiom is implied just by the fact that $b(\emptyset) = 0$. Additivity is immediate in both directions of the reasoning. On the other hand, the same sign of the belief function on complementary events follows from the non-negativity of q, if the latter is the quotient function induced by b. However, the value $b(\emptyset) = 0$ follows from (the conjunction of) normalization and additivity of q. That is, it is not the case that the conditions in the statement of the theorem correspond directly to the respective probability axioms.

3 Discussion

To see an example of a Dutch-bookable, nonprobabilist belief space, consider the space with three atomic propositions depicted in Fig. 1.

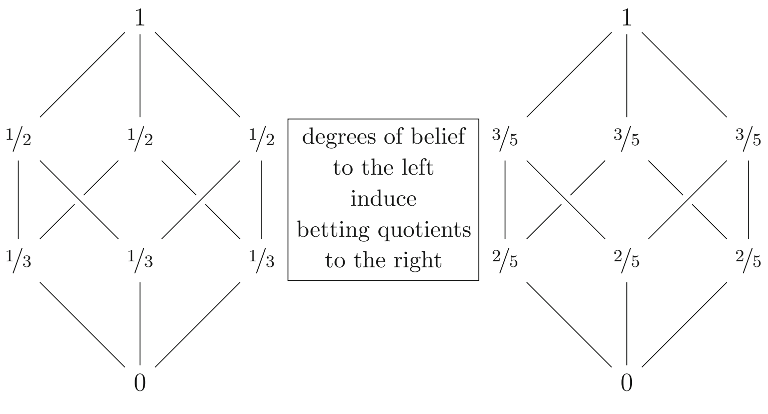

Fig. 1. A nonprobabilist, un-Dutch-bookable belief space and its induced betting quotient function.

That is, consider $\langle W, \mathit{Prop}, b\rangle$ where $W = \{w_1, w_2, w_3\}$, $\mathit{Prop} = \mathcal{P}(W)$, and $b : \mathit{Prop} \to \mathbb{R}$ takes the values $b(\emptyset) = 0$, $b(W) = 1$, $b(\{w_i\}) = \frac{1}{3}$ for each $i \in \{1, 2, 3\}$, and $b(\{w_i, w_j\}) = \frac{1}{2}$ for distinct $i, j \in \{1, 2, 3\}$. Then the induced betting quotient function q is as follows: $q(\emptyset) = 0$, $q(W) = 1$, $q(\{w_i\}) = \frac{2}{5}$ for each $i \in \{1, 2, 3\}$, and $q(\{w_i, w_j\}) = \frac{3}{5}$ for distinct $i, j \in \{1, 2, 3\}$.

(Where a similar illustration appears in the remainder of the paper we shall use the same representational convention, that is, the left algebra represents the function b defined on $\mathcal{P}(\{w_1, w_2, w_3\})$, and the right algebra represents the induced betting quotient function q.)

We can see that the betting quotient function q does not satisfy the classical probability axioms (it is not additive), therefore by Theorem 1 the belief function b is susceptible to a Dutch Book. We can see observe that the belief function b does not satisfy the third condition of Theorem 2, e.g.

$$\frac{b(\{w_1, w_2\})}{b(\{w_1, w_2\}) + b(\{w_3\})} = \frac{3}{5} \neq \frac{4}{5} = \frac{b(\{w_1\})}{b(\{w_1\}) + b(\{w_2, w_3\})} + \frac{b(\{w_2\})}{b(\{w_2\}) + b(\{w_1, w_3\})}.$$

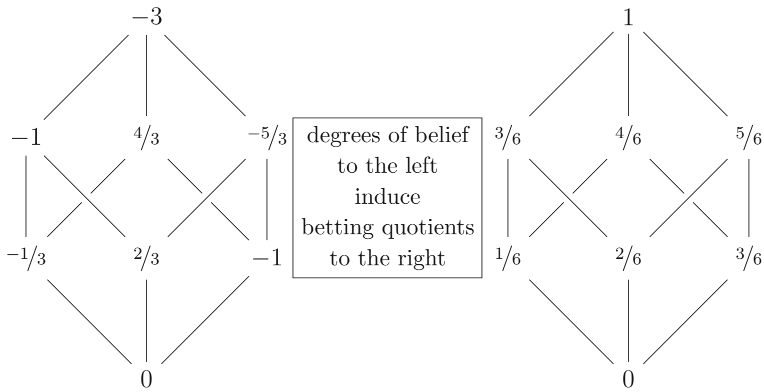

Fig. 2. A nonprobabilist, un-Dutch-bookable belief space with a "wild" belief function.

For a contrasting example, Fig. 2 depicts another nonprobabilistic belief space. As we can see, the induced betting quotient function may be a classical probability function even though the original belief function does not seem to be anything reasonable.

Note that you do need to subscribe to any particular interpretation of belief functions to accept the above argument (for a survey see the already mentioned [5]). The only two things that are needed are that you agree that degrees of belief can be expressed by a real number (so that, for example, you are not a strong operationalist) and agree to our description of the relation between them and the betting quotients.

Notice also that having negative credences—whatever this would mean—does not by itself make you prone to Dutch-Books. You might be exploitable in some ways, and so holding such credences might be irrational. But the main goal of this paper was to distill the essence of the power of the Dutch Book arguments, and from the above Theorem we see that it does not exclude negative credences as irrational.

4 How to Expect Things When You Are Incoherent

We have intentionally used expressions like "the agent expects the bet to have value 0" instead of "according to the agent the expected value of the bet is 0". Consider the following argument (notice that for clarity we have omitted the phrase "according to the agent", but it should be immediate where it is intended to figure):

1. A bet is a random variable;
2. a fair bet is defined as one that has expected value 0;
3. a fair set of bets is defined as one that the expected value of the sum of all the bets in the set is 0;
4. the expected value of a sum of finitely many random variables is equal to the sum of expected values of those random variables;
5. therefore, a finite set of bets all of which are fair is also fair;
6. therefore there are no finite Dutch Books.

Since the conclusion of this argument is false (examples of finite Dutch Books abound), we need to see where it fails. Since a bet outputs a real number (profit) given an element of the sample space (possible world), it can be thought of as a random variable, and so step 1 is true. Steps 2 and 3 are definitions. Step 4 is a basic fact about random variables. Step 5 follows from the previous four, and step 6 is just a reformulation of 5.

So, what is wrong? The culprit is step 2. Yes, it is a definition, but its application to an incoherent agent yields probably unintended results. Compare defining, for an agent with a belief function b, the expected value of a bet for A which pays 1 and costs q as

$$b(A) \cdot (1-q) + b(\neg A) \cdot (-q) \tag{1}$$

(as is done e.g. by [17]; we have used the same formula in the previous sections) with putting it as

$$\sum_{w \in A} b(\{w\}) \cdot (1-q) + \sum_{w \notin A} b(\{w\}) \cdot (-q). \tag{2}$$

These two expressions may be different for agents with a nonadditive belief function. (Notice that the former, and not the latter, is, mathematically speaking, the expected value of the bet, considered as a random variable: see e.g. Definition 4.1.1 in [14].) It seems that an incoherent agent may respond to our result in Sect. 2 by saying "that's all very nice, thank you very much for defending me, but really, I expect a bet for A to have value 0 precisedly when (2) is 0, that is, I am using this *atomic* notion of expected value just like it is employed in esteemed publications like [6, p. 615] and [12, Chap. 14]. And so my alleged Dutchbookability is a matter of a different calculation!"

But is that option really available to the agent? It seems to us that the answer is "no". When figuring the relevant betting quotient the agent considers the payoff in the case of winning the bet, the payoff in the case of losing it, and takes into account how probable he or she thinks the two outcomes are, that is, his or hers credences in the proposition (because the bet is won if the proposition is true) and its negation (because it is lost if the proposition is false). The agent's credences in the constituents of the proposition are irrelevant to this; and so, the correct formula to be used is (1). This is of course debatable: but we are willing just to say that at this point the non-probabilist, even though—as pointed out—she could appear to the existing literature, would truly become Stubborn.

4.1 Conclusions

Thus, even if we forget about all the problems of Dutch Book arguments which are usually mentioned in the formal epistemology literature (see e.g. [1,11,17]), it turns out that another one lurks in the basic step of connecting degrees of belief with betting quotients. There is a gap between betting quotients, which theorems in Dutch-Book-inspired formal epistemology are about, and degrees of belief, which those theorems are supposed to be about. This gap prevents the classical Dutch Book argument from being convincing to the target group, that is, nonprobabilists. We propose to bridge that gap using the notion of induced betting quotient; and show that susceptibility to a Dutch Book remains a nontrivial notion: **some nonprobabilists are immune, but others are not.**

5 Relation to Incoherence and Inaccuracy Measures: Some Preliminary Remarks

Since the context of this paper is an argument for probabilism, we have confined our attention to a binary notion: either the agent can be Dutch-booked, and so is irrational, or not, in which case if (s)he indeed is irrational, we need a different argument to show it, since everything seems to be fine about her or his credences at the given moment. This is fine if we are interested in norms of rationality of ideal agents and what exactly it takes to satisfy them. However, if we think of real agents, who—reason would dictate—can only aspire to the probabilistic ideal, or if we would like to compare different violations of probabilism displayed by ideal agents, some graded notion is needed: one which would aim to capture the "distance" between an agent's belief function and some maximally "close" coherent function. (The word "distance" is in quotes since the two-argument functions used in the literature may not be metrics; see e.g. [12].)

One approach would be to use a notion of Dutch Book which would enable us to ask questions regarding "how Dutch-bookable an agent is", for example, intuitively, how much money can be extorted from the agent (assuming some normalization is used). This road is taken by [15]. We have tried to make our paper acceptable to those who think one fault of the classical Dutch Book argument for probabilism from the point view of epistemology was its pragmatic nature; we have thus decided to use a relatively strict notion of a "a bet the agent considers as not favouring any side", and not something similar to "a bet the agent would accept since (s)he considers it to have a nonnegative expected value for her or him", which is the notion Schervish et al. use. We do not know yet how our approach fares if we switch from one notion to the other—this is a task for the future.

The approach by Schervish et al. has been criticised by e.g. [16] on both technical and philosophical grounds, but at least one of their incoherence measures, the "neutral/max" one, stands its ground, and we will consider it in the future. The basic question to be asked is the following. Consider the class N consisting of all nonprobabilist agents which cannot be Dutch-booked (according to a version of our argument which takes into account the class of bets interesting from the point of view of Schervish et al. described above) and the class M consisting of all nonprobabilist agents which can. Are all members of N less incoherent according to the "neutral/max" rule than all members of M?

Another route to consider would be to investigate how members of the classes N and M fare from the standpoint of alethic accuracy (on that notion consult [12]). However, it is not evident what kind of question should be asked in this context. The relationship between graded incoherence and alethic inaccuracy has not been completely worked out and research in that area is ongoing: see e.g. [3]. There seem to be no "simple" theorems to look for in this area; for example, as shown in [3], promoting one virtue does not in general result in promoting the other. Just like in the case of the issues discussed in the previous paragraph, the number of implicit quantifiers involved in researching such issues makes the number of potential formal hypotheses quite high. However, at this moment we are sceptical regarding the outlook of similar endeavours. Consider the nonprobabilist belief space from Fig. 3: on any reasonable inaccuracy measure its "distance" from the closest coherent function will be minimal, and yet it belongs to the class M: it is Dutch-bookable. We will pursue these issues further in [19].

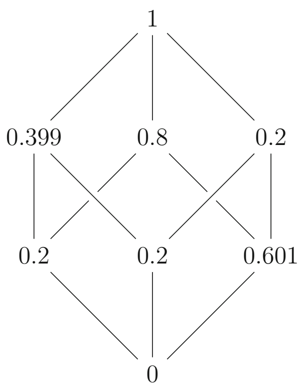

Fig. 3. A belief space which is negation coherent (and so featuring degrees of belief which *are* betting quotients) and Dutch-bookable (since the betting quotients are not additive), but intuitively *very close* to a classical space.

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