

# MANAGING TAX REVENUE VOLATILITY

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## ABSTRACT

*During the initial decade of the twenty first century a number of scholars in the American public administration arena suggested that certain social science methods, particularly those pertaining to portfolio analysis, can play an important role in managing tax revenue volatility. Several discussions involved an adaptation of Modern Portfolio Theory which indicates that investment decisions should be based on the mean-variance characteristics of "portfolios" which are collections of financial assets. This paper contributes to the technical aspect of the dialogue by outlining a procedure which may reduce some tax portfolio analysis complexities when applied to these kinds of revenue decisions.*

## INTRODUCTION

In a September | October 2007 online edition, Public Administration Review (PAR) published an article by Fred Thomson and Bruce L. Gates titled *Betting on the Future with a Cloudy Crystal Ball? Revenue Forecasting, Financial Theory, and Budgets-An expanded Treatment*. The authors' central research question is "given that we can't predict the future, how can we get a good result no matter what the economy throws at us" (Thompson and Gates, 2007, p. 48)? Their fundamental answer is that certain modern financial economic theories and social science methods may have much to offer practitioners in terms of managing the volatility often associated with tax revenues. Portfolio analysis is one of four financial tools presented when supporting their position.

The article produced several immediate responses from well-known scholars. PAR has published them in a "Commentator" section of the September | October 2007 online edition. The general consensus might best be summarized with Meyers' (2007) assertion that

at the very least, the article is the type of thought provoking-contribution that will stimulate discussion-some of which will be quite critical from a traditionalist perspective. Those who are interested in moving beyond that lens will find that the article...can apply to public administration theory and practice" (P. 74).

The resistance aspect of Meyers' declaration is likely to be true as taxation decisions involve revenue politics (Rubin, 2006). However, technocratic contests may also be expected to occur as portfolio analysis challenges existing methodologies and ways of thinking with a mean variance framework which is not necessarily intuitive. Furthermore, the technique requires a certain level of mathematical competence. As Meyers (2007) suggests "to truly understand the authors' arguments, readers will have to immerse themselves in a finance textbook" (p. 74). Thus one might presume that simplifications and clarifications will be a welcomed addition to the practitioner oriented material presented thus far.

Given the above, this paper contributes to the technical aspect of the dialogue by outlining a procedure which may reduce some tax portfolio analysis complexities. Overall, the progression of the article follows Thompson and Gates (2007) in synthesizing a body of work practitioners may find useful. The next section briefly reviews pertinent research literature before

delineating the approach. This is followed by an illustrative example. Additional comments are given in the conclusion section.

## LITERATURE REVIEW

While Thompson and Gates' (2007) article is an excellent conceptual synthesis for practitioners, the notion that portfolio analysis can be used to evaluate tax revenue volatility is not foreign to scholars interested in public budgeting and finance research oriented literature. Garret (2006), for example, employs the technique in a study examining tax revenue variability for a sample of U.S. states. Furthermore, Berg, Marlin, and Heydarpour (2000) and Mallick and Harmon (1994) use a similar approach when examining New York City and State taxes.

A common feature of the above mentioned works is that Modern Portfolio Theory (MPT) is applied. Introduced by Markowitz (1952) MPT posits that when rational investors confront risk in a financial-economic environment they should focus on moments one and two of a portfolio's return distribution as there is a positive relationship between them that can be mitigated when assets are not perfectly correlated. The first moment, or mean, is defined as expected return. The second moment, or variance, is defined as volatility (or risk). Consequently each of the investigations necessarily adopts Markowitz's assumptions, including normality, and adapts the initial constructs and methods necessary for finding objectively efficient tax portfolios.

By way of extension, objectively efficient tax portfolios are defined as mixes of components which minimize volatility for an expected percent change in revenue or that maximize the expected percent change in revenue for a given level of volatility. Mathematically sets of efficient tax portfolios can be determined by finding solutions for the following optimization problem:

$$\text{minimize } \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \sigma_{ij} \quad (1)$$

$$\text{subject to } E(R_p) = \sum_{i=1}^N \omega_i r_i; \text{ and}$$

$$\sum_{i=1}^n \omega_i = 1; \text{ and}$$

$$\omega_i \geq 0$$

where:

$\sigma_p^2$  = Variance of percentage change in revenue for a tax portfolio;

$E(R_p)$  = Expected percent change in revenue for a tax portfolio;

$n$  = Total number of tax components;

$\sigma_{ij}$  = Covariance of tax components  $i$  and  $j$ ;

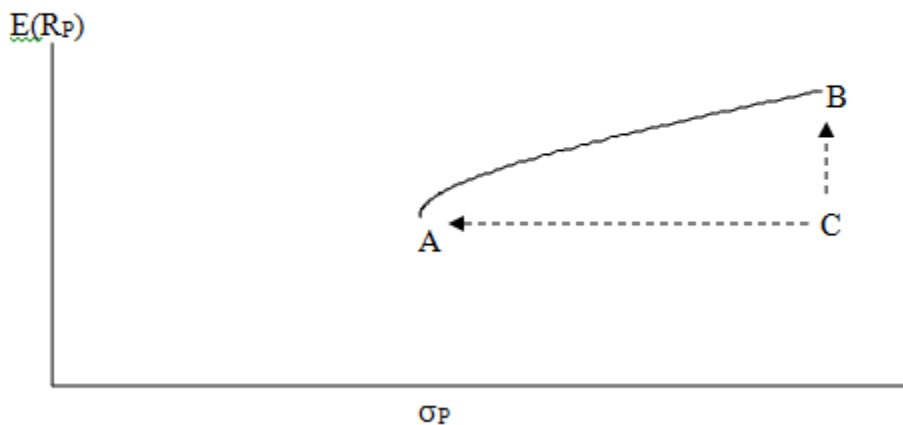
$r_i$  = Expected percent change in revenue for tax component  $i$ ; and

$\omega_i$  = Tax component  $i$ 's share of total tax revenue.

As expected, empirical studies typically rely on historical observations for a predetermined time period. Some (e.g., Garrett, 2006) concentrate on finding a minimum variance portfolio(s) by setting the first derivative of  $\sigma_p^2$  with respect to  $\omega_i$  equal to zero. Most of the researchers cited above, however, portray solutions in a two dimensional space.

A generalized result is portrayed in Figure 1 under the assumption that tax components are not perfectly correlated. The central insight is that “AB” is a frontier which reveals the expected outcomes associated with all objectively efficient tax portfolios. By comparison, tax portfolio “C” is not efficient as superior mixes exist. Portfolio A, for example, affords an identical expected percent change in revenue with much less volatility. Portfolio B, by comparison, affords a greater expected percent change in revenue with the same volatility.

**Figure 1**  
**Hypothetical Efficient Frontier**



What is not completely clear is which efficient tax mix should be chosen. Berg et al. (2000) essentially answer the query by holding  $E(R_p)$  constant for New York City. Of course this requires adjusting the existing mix of components ( $\omega_i$ ) so that a tax portfolio such as “A” in Figure 1 is achieved. Naturally the intention is to reduce  $\sigma_p^2$  for a target  $E(R_p)$  that presumably is in-line with the subjective preferences of the government. The conclusion is similar to that given in Garret (2006) if “A” is defined as the minimum variance tax portfolio. However, such a presumption may not always be correct.

Traditional financial economic analyses often incorporate subjective utility functions for risk-averse investors when answering such queries. Khan (2002) does this in a portfolio analysis that theoretically addresses Key’s (1940) expenditure side question “on what basis shall it be decided to allocate x dollars to activity A instead of activity B?” (p. 1138). Unfortunately, from the public practitioner’s point of view, computational complexities make the approach extremely problematic for even the simplest of cases (e.g., a tax portfolio with two components). Therefore methodological simplifications may be worth exploring if portfolio analysis is to become an accepted fiscal tool that is successfully implemented.

## METHODOLOGY

One possible simplification can be found in Williams' (1997) work which focuses on maximizing the likelihood of achieving an investment goal. Unlike Khan (2002) his primary contention is that probability is an appropriate measure of volatility that is consistent with risk averse-behavior and easier to work with. While not explicitly stated his probabilistic logic is rooted in Leibowitz and Henriksson's (1989) efforts which demonstrate that portfolio optimization can incorporate shortfall constraints that highlighted downside risk.

Much of Williams' article addresses econometric time series concerns but the central insight can be gleaned here by noting the following two points while referencing efficient tax portfolios A and B in Figure 1:

1. When considering tax portfolio A,  $E(R_A)$  and  $E(R_B)$  are values contained in tax portfolio A's distribution of percent changes in revenue. While the probability that  $E(R_A)$  will be realized or exceeded is fifty-percent, the probability that  $E(R_B)$  will be realized or exceeded is less than fifty-percent.
2. When considering tax portfolio B,  $E(R_B)$  and  $E(R_A)$  are values contained in tax portfolio B's distribution of percent changes in revenue. While the probability that  $E(R_B)$  will be realized or exceeded is fifty-percent, the probability that  $E(R_A)$  will be realized or exceeded is greater than fifty-percent.

Both points are based on known aspects of normal distributions and can be generalized to the entire efficient frontier. Consequently  $E(R_A)$  and  $E(R_B)$  are just two potential targets that a decision maker can choose to pursue in a variety of ways via tax mixes that are objectively efficient. Using this information the following three steps suggest a straightforward administrative procedure for incorporating subjective choice:

1. Choose a target value ( $R_T$ ) for the percent change in revenue.
2. Calculate  $Z = [R_T - E(R_P)] \div \sqrt{\sigma_P^2}$  for every tax portfolio on the efficient frontier.
3. Use each Z score in conjunction with a standardized table to calculate the probability that the percent change in revenue will be greater than or equal to  $R_T$ .

The final step is to choose the efficient tax portfolio with an acceptable probability of equaling or exceeding  $R_T$ .

## ILLUSTRATIVE RESULTS

The computations discussed above may appear complicated. However, certain technologies are available to alleviate most of these concerns. To illustrate, the information in Panel A of Table 1 is used to construct Figure 2 with various functions that are available in the Microsoft Excel spreadsheet. The data come from Berg et al.'s (2000) empirical study concerning New York City's tax mix and can be found in the third table of their publication.<sup>5</sup>

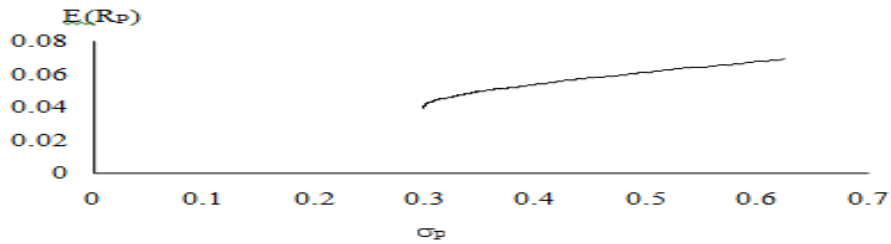
With some exceptions Panel A of Figure 2 essentially replicates the efficient frontier provided by these researchers. One modification is that the horizontal axis replaces their volatility measure,  $\sigma_p^2$ , with  $\sigma_p$  as the latter computation is more useful for a probability oriented

**Table 1**  
**Empirical Probability Frontiers Based on Berg et al. (2000)**

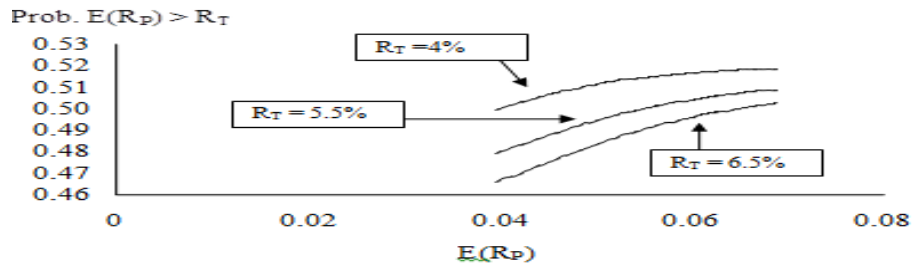
<b>Tax Portfolio</b>	<b>Least Volatile</b>	<b>Most Volatile</b>
<b>Panel A</b>		
<b>Revenue Component</b>		
Business%	1.00	0.00
Personal Income%	0.00	100
Property%	43.40	0.00
Sales%	55.60	0.00
<b>Outcomes</b>		
$E(R_p)$ [%]	.039	0.069
$\sigma_p$ [% age points]	.298	0.625
<b>Panel B</b>		
<b>Probability <math>E(R_p) \geq E(R_T)</math></b>		
$E(R_T) = 4.0\%$	49.93%	51.85%
$E(R_T) = 5.50\%$	47.92%	50.89%
$E(R_T) = 6.50\%$	46.59%	50.26%

*Note.* Both the least and most volatile portfolios are on Berg et al.'s (2000) efficient frontier. Thus any linear combination of them is also efficient. The information in Panel A is used to deduce that the correlation coefficient is approximately equal to 0.48.

**Figure 2  
Panel A: Empirical Efficient Frontier**



**Figure 2  
Panel B: Empirical Efficient Frontier**



discussion. By comparison, Panel B presents probability frontiers for three ad hoc target values: 4%, 5.5%, and 6.5%.

An important point to note is that the positions of the frontiers in the two dimensional space are consistent with intuition. For example, the frontier corresponding to  $R_T = 4\%$  lies above  $R_T = 5.5\%$  and  $R_T = 6.5\%$ . This should be expected given that the probability of equaling or exceeding a lower target value should be higher than the probability of equaling or exceeding a higher target value. Panel B of Table 2 supplements this conclusion with probability estimates for the least and most volatile tax portfolios reported in the Berg et al (2000) study.

### CONCLUSION

Overall, portfolio analysis (e.g., MPT) seems to be an important investment tool that may also be appropriate for managing the volatility often associated with tax revenues. Conceptually and empirically the connection hinges on the positive relationship between the first and second moments for an expected distribution of percentage rate changes. Another link resides in the notion of covariance and the possibility of volatility (i.e., risk) reduction with sufficient diversification. With respect to this simplification oriented paper, however, one major clarification is in order as equity is not addressed.

Obviously fairness is an important aspect of tax policy analysis and this should be employed during the course of any investigation. The mean-variance and probability models discussed here are both capable of handling such concerns by means of imposing additional constraints on any specific tax component's share of total tax revenue. In either case, such constraints are likely to a) increase the volatility for a given level of expected percent change in

revenue, or b) decrease the level of expected percent change in revenue for a given level of volatility.

Other potential concerns include issues surrounding normality and the fact that volatility can be reduced through ways other than tax mixes. These are not considered here as they are not specifically tied to the purpose at hand.

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