## ACTIVITY RECOGNITION USING GREY-MARKOV MODEL

## By

Kirke Shouse<br>A Thesis<br>Submitted to the<br>Faculty of the Graduate School<br>of<br>Western Carolina University<br>in Partial Fulfillment of the Requirements for the Degree of<br>Master of Science in Technology

Committee:
Director
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A thesis presented to the faculty of the Graduate School of Western Carolina University in partial fulfillment of the requirements for the degree of Master of Science in Technology.

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Kirke Shouse

Director: Dr. James Zhang<br>Associate Professor<br>Department of Engineering and Technology

Committee Members: Dr. Peter Tay, ECET
Prof. Paul Yanik, ECET

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# ABSTRACT <br> ACTIVITY RECOGNITION USING GREY-MARKOV MODEL 

Kirke Shouse, M.S.T.
Western Carolina University (December 2011)

Director: Dr. James Zhang

Activity Recognition (AR) is a process of identifying actions and goals of one or more agents of interest. AR techniques have been applied to both large and small scale activity identification. Examples of AR techniques include Genetic Algorithm, Markov Chain, and so on.

This research proposes a novel method, Grey Markov Model (GMM), for detection and prediction of pre-defined activities. There were three objectives of this research. The first objective was to establish a database of pre-defined human activities. The second objective was to establish the Grey Markov Model. The final objective was to verify the model performance using the established database.

This thesis describes the methodology of test setup and data collection, as well as the procedures of model generation. Furthermore, experimental results of model performance verification test are also reported.

## CHAPTER 1: INTRODUCTION

Activity Recognition (AR) is an ongoing field of study. Activity Recognition (AR) is a process of identifying actions and goals of one or more agents of interest. AR techniques have been applied to both large and small scale activity identification. An example of large scale recognition is traffic pattern for a major street and its intersections to help generate a better flow of traffic. Small scale recognition is where AR is most frequently seen. An example of small scale recognition is Activities of Daily Living (ADL). ADL is used to help assisted living facilities determine whether or not a patient is in need of care. This thesis focuses on small scale recognition, and in particular identifying and predicting the current position of someone's posture in a chair.

The importance of AR is becoming more and more prevalent as the life expectancy and number of elderly people increases. By being able to identify and predict someone's current state and future state, the care for the elderly can be vastly improved. For example, if the current state of a patient is one that is known to lead to one of the negative future states, such as falling down, then the healthcare provider knows that intervention is necessary to prevent injury. As sensory technology improves the non-invasive approaches can even help with predicting internal problems, such as heart and other vital organ failures.

Most methods of AR lack the speed and accuracy needed to provide the responses needed for healthcare concerns. The most predominant method of AR is considered to be Markov Model. This method is primarily used in speech recognition, and not designed to deal with AR. While it does boast decent accuracy and speed, it still lacks the precision
needed for healthcare predicaments. This thesis proposes a new method for AR that has yet to be explored in great depth using a Grey-Markov Model (GMM) as the algorithm for detection. GMM is a combination of the previously stated Markov Model and Grey Model. Grey Model was originally designed for control systems. Its inclusion with Markov Model helps to reduce error previously seen in Markov Model alone.

Through this research the generation of the GMM algorithm was constructed in MatLab. This algorithm used the Markov Model as it's main source of interpretation, but as previously stated the introduction of Grey Model was appended to eliminate error and improve detection rates. The main focus of the Markov Model is to link the states of interest with a transition matrix. A Markov chain takes a stochastic process and makes the next state dependent on the current state and not the past states. The Grey Model takes the transition matrix and transition probabilities and uses a differential equation to predict the current and future state. The predictions error is further reduced by implementing the residual error calculated from one state to any other state, and determining the most likely next state.

To test the algorithm, Dow Jones stock exchange data was used to determine if, given a specific amount of closing averages, could the following day's closing average be predicted. This was done over one month, three month, and one year time intervals. The predictions were used as a comparison between Grey Model and GMM to test the improvements. The prediction results showed that the algorithm was consistently more accurate than Grey Model by itself.

The next step was to test the algorithm on previously collected data used for AR. This data was provided by a faculty member which was obtained during his dissertation work. The data consisted of participants positioned on a couch in five unique states. The data was tested and predicted accurately what the current state was. To further test this
data set the original data was scrambled so that the original order of states was altered to confirm the accuracy of the algorithm. The results showed the algorithm was just as accurate as before.

Once the initial test of the algorithm was done with actual AR data, new data was generated using a similar concept. The new data was collected by placing four load cells underneath the legs of a chair. Four states were established and consisted of front, back, left, and right. The new data acquired was tested by the algorithm, and again the accuracy shown was sufficient for AR.

This thesis is organized as follows, chapter two provides a literature review discussing the basic concepts of AR, Markov Model, Grey Model, and GMM. In chapter three the design methodology is discussed in detail pertaining to the algorithm, data collection, and data testing. Chapter four shows the results and analysis attained through this research. Lastly, chapter five concludes the research and recommends future work.

## CHAPTER 2: LITERATURE REVIEW

### 2.1 Activity Recognition

Activity Recognition(AR) is a process of identifying actions and goals of one or more agents of interest. AR techniques have been applied to both large and small scale activity identification. Examples of AR techniques include Genetic Algorithm, Markov Chain, and so on. AR can be viewed in numerous ways dependent upon its application. For example, if the actions of interest are behavioral then AR can be classified as behavioral recognition. Other common references to AR are goal recognition, intent recognition, plan recognition, and location-based recognition [1-4].

Since the inception of the digital age and devices such as smartphones, AR has become more prevalent. Its significance has lead to vast amounts of research in the varying fields of AR. One important field that research has arisen is the medical field. The importance of AR for medical purposes is vast, but benefits to the elderly have emerged as the prevailing source of interest in the field $[2,4]$.

One such benefit is the use of AR to help with assisted living patients. These patients require consistent attention, and with more patients than healthcare workers the advent of AR to help identify and diagnose problems has been a reprieve to the healthcare providers. Preventative measures can be used to help identify potential injuries and problems before they occur. The ultimate goal would be to identify and solve potential problems autonomously if at all possible, but further progress in the field must occur for that revelation $[2,4]$.

### 2.2 Predominant Methods in Activity Recognition

There have been many different approaches to AR, but with so many facets to AR it is difficult to generalize the methods. Some prevalent methods consist of data mining, robotics, and most commonly used sensors. An example of robotic assistance in AR can be found in the work of Pollack et al. about memory loss in the elderly [4]. This research explores the assisted living of the elderly with a robot to assure activities of daily living are remembered and accomplished as planned. This may range from simple task such as using the restroom to taking important medication. The use of adaptivity in the robots algorithm allows for daily routines to be completed without the monotonus approach of an alarm clock reminder.

Want et al. uses the more commonly used approach, sensors [2]. This approach uses a badge worn by agents that tracks movement in varying locations. Two examples of where the badge was used were in an office building setting and in a hospital setting. In the office building setting the use of the badge allowed a receptionist to easily keep track of where individuals were when they were needed. In the hospital setting the badge was used to keep up to date information on patients and staff in case medical emergencies arose. This may seem that AR is not heavily relied upon, but once the badges are used for an extended period of time training occurs allowing an algorithm to not only see where someone is, but also to predict where they may be going using previous data. The problem with the badge method is that an individual's privacy may be violated with this invasive approach.

Another example of AR using a sensor approach can be seen using a non-invasive approach [3]. This approaches uses sensor data on household objects instead of the per-
son. By using these sensors on items such as a coffee-maker, the authors investigate if they could determine recognizable patterns with objects in daily life. Through decision trees which classify patterns they were able to achieve a high success rate of detection without being invasive to the members of the household.

The method used in this research was a non-invasive approach using sensors to determine a person orientation in a chair. The algorithm used to determine and predict the current and future states was a combination of Grey Prediction and Markov Model. The combination of this algorithm is known as Grey-Markov Model, and allowed for the short-comings of both to be surpassed.

### 2.3 Grey Prediction and Markov Model

Grey prediction was originally introduced by Deng [5] in 1982. Grey prediction was originally designed for use with control systems. It has been widely used in control systems since its inception. The theory states that if a system consisted of all unknowns then it was considered black. Conversely if a system consisted of all knowns it was considered white. Therefore a grey system consisted of known and unknowns. The goal of grey prediction was to whiten the system and reveal the unknowns. Since control system data tend to follow an exponential trend the method was ideal for the task. When using grey prediction in a field such as AR the exponential trend tends to propagate error through the process.

A series of papers were published in the 1960s first referencing Hidden Markov Models. One of the first applied uses of Hidden Markov Models was found in speech recognition. As people became more familiar with Hidden Markov Models, more fields of study began to use the prediction method. Some examples of fields are Biology, Stock Market, Power Consumption, and DNA research. In 1989 Lawrence R. Rabiner published a paper explaining Hidden Markov Models and their application in speech recognition. [6].

First Rabiner explains the process in constructing a Hidden Markov Model. The author further explained Hidden Markov Models for prediction using two examples. The first example used is the simple task of flipping a coin that has heads on one side and tails on the other. Rabiner sets up the experiment by explaining that you are in a room with a curtain in between you and the person flipping the coin. The person flipping the coin will not tell you anything they are doing just the outcome of every coin flip. To construct the Hidden Markov Model one must first determine what a given state will correspond to, i.e. state one would be heads, while state two would be tails. Next one must determine how many states; in this case there are two states, either head or tails. Lastly, the determination of the probability of the coin being head or tails must be decided. It is fairly obvious in the case of the coin what the probabilities would be for the coin to be either heads or tails, so Rabiner goes on to show a two coin model and a three coin model. These would allow you to generate a Hidden Markov Model that can predict a sequence of coin flips [6].

### 2.3.1 Grey Prediction Method

The first step of implementing the Grey Prediction Method is data treatment. Let the original data sequence be denoted as such:

$$
\begin{equation*}
\left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\} \tag{2.1}
\end{equation*}
$$

The original data can be reconstructed with the following equation:

$$
\begin{equation*}
\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)-D_{k+1} \tag{2.2}
\end{equation*}
$$

Where:

$$
\begin{equation*}
D_{i}=D_{i-1}+2\left[x^{(0)}(i-1)-x^{(0)}(i)\right] \tag{2.3}
\end{equation*}
$$

The second step is to build the $\operatorname{GM}(1,1)$ Model. The Accumulated Generation Operation(AGO) sequence $X^{(1)}$ can be obtained by:

$$
\begin{equation*}
X^{(1)}=\left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\} \tag{2.4}
\end{equation*}
$$

Where:

$$
\begin{equation*}
x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i), k=1,2, \cdots, n \tag{2.5}
\end{equation*}
$$

The grey $\mathrm{GM}(1,1)$ model can be made by establishing the first-order differential equation for $x^{(1)}(t)$ as:

$$
\begin{equation*}
\frac{d x^{(1)}(t)}{d t}+a x^{(1)}(t)=u \tag{2.6}
\end{equation*}
$$

Where $a$ and $u$ are the parameters to be estimated, and can be obtained by:

$$
\begin{equation*}
\binom{\hat{a}}{\hat{u}}=\left[B^{T} B\right]^{-1} B^{T} Y \tag{2.7}
\end{equation*}
$$

$$
\begin{gather*}
\text { Where } B=\left[\begin{array}{cc}
-\left[x^{(1)}(1)+x^{(1)}(2)\right] / 2 & 1 \\
-\left[x^{(1)}(2)+x^{(1)}(3)\right] / 2 & 1 \\
\vdots & \vdots \\
-\left[x^{(1)}(n-1)+x^{(1)}(n)\right] / 2 & 1
\end{array}\right]  \tag{2.8}\\
Y=\left[x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)\right]^{T}
\end{gather*}
$$

We can obtain the time response function by solving the differential equation:

$$
\begin{equation*}
\hat{x}^{(1)}(k+1)=\left[x^{(1)}-\frac{\hat{u}}{\hat{a}}\right] e^{-\hat{a} k}+\frac{\hat{u}}{\hat{a}} \tag{2.9}
\end{equation*}
$$

This can be rewritten as:

$$
\begin{equation*}
\hat{x}^{(1)}(k+1)=b e^{-\hat{a} k}+c \tag{2.10}
\end{equation*}
$$

Lastly, the residual errors must be calculated by using the following:

$$
\begin{equation*}
E=\left\{e_{1}, e_{2}, \cdots, e_{n}\right\} \tag{2.11}
\end{equation*}
$$

Where:

$$
\begin{equation*}
e_{i}=x^{(0)}(i)-\hat{x}^{(0)}(i) \tag{2.12}
\end{equation*}
$$

### 2.3.2 Grey Markov Method

The Grey-Markov Model(GMM) is an extension of Grey Model(GM) to further reduce detection and prediction errors. The first step in building the GMM is to divide the residual errors into q states where each state satisfies the equi-probability principle, and is defined as $R_{1}, R_{2}, \cdots, R_{q}$.

Next the construction of the transition matrix is done by determining the probability from state $R_{i}$ to state $R_{j}$, which results in the transition matrix P:

$$
P^{(1)}=\left[\begin{array}{lccc}
P_{11}^{(1)} & P_{12}^{(1)} & \cdots & P_{1 q}^{(1)}  \tag{2.13}\\
P_{21}^{(1)} & P_{22}^{(1)} & \cdots & P_{2 q}^{(1)} \\
\vdots & \vdots & \vdots & \vdots \\
P_{q 1}^{(1)} & P_{q 2}^{(1)} & \cdots & P_{q q}^{(1)}
\end{array}\right]
$$

The transition probability can be calculated directly as follows:

$$
\begin{equation*}
P_{i j}^{(m)}=\frac{M_{i j}^{(m)}}{M_{i}}(i, j=1,2, \cdots, L) \tag{2.14}
\end{equation*}
$$

Where $M_{i j}^{(m)}$ stands for the transition from $R_{i}$ to $R_{j}$ in $m$ steps, where $M_{i}$ is the number of state $R_{i}$.

The resultant transition probability matrix is as follows:

$$
P^{(m)}=\left[\begin{array}{cccc}
P_{11}^{(m)} & P_{12}^{(m)} & \cdots & P_{1 q}^{(m)}  \tag{2.15}\\
P_{21}^{(m)} & P_{22}^{(m)} & \cdots & P_{2 q}^{(m)} \\
\vdots & \vdots & \vdots & \vdots \\
P_{q 1}^{(m)} & P_{q 2}^{(m)} & \cdots & P_{q q}^{(m)}
\end{array}\right]
$$

Next, the residual error must be confirmed. Let the interval median in $\left[R_{i-}, R_{i+}\right]$ be residual error forecasting value as follows:

$$
\begin{equation*}
\hat{e}_{i}=0.5\left(R_{i-}+R_{i+}\right) \tag{2.16}
\end{equation*}
$$

Once the residual error $e_{i}$ and $\hat{x}^{(0)}$ are determined the prediction of the original data can be constructed:

$$
\begin{equation*}
\hat{y}(i)=\hat{x}^{(0)}(i)+\hat{e}_{i} \tag{2.17}
\end{equation*}
$$

Comparing $\hat{y}(i)$ against the original data will determine the success rate of the process.

## CHAPTER 3: METHODOLODY

In this chapter, we first introduce the algorithm for this research. The results of algorithm test with known data sets are shown. The method for obtaining data to be tested is also explained. The method for pre-processing the collected data is also shown in this chapter.

### 3.1 Algorithm

The algorithm for this research is based on the theories described in chapter two. The flow-chart of the algorithm is shown in Figure 3.1.


Figure 3.1: Algorithm Flow Chart

As can be seen in Figure 3.1 the algorithm begins with an initial steady state. Once the initial steady state has been recognized the algorithm begins to check for a transition. Once a large enough change is noticed on the data a transition is considered to have occurred. Once the transition is detected the algorithm attempts to predict the next state based on the previous steady state and the transition which was detected. After the prediction is made the algorithm confirms the predicted steady state and establishes it as the new current steady state for the prediction of the next state.

An initial performance verification test was performed using the algorithm and Dow Jones Industrial Closing Average. The test used the GMM algorithm previously constructed and the original Grey Model to verify the improvement of the proposed algorithm. Three tests were conducted using the data, the first test used the first twenty-nine days of a month to predict the thirtieth day. The next test used three months of data to predict the first day of the fourth month. The last test used one year worth of data to predicted the first day of the following year. The results of this test are shown below:



Figure 3.2: Dow Jones Industrial Closing Average Results
Table 3.1: Dow Jones Industrial Closing Average Table

|  | GM | Error(\%) | GMM | Error(\%) | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: |
| One Month | 11265 | 1.2576 | 11174 | 0.4396 | 11125 |
| Three Month | 11591 | 4.1902 | 11301 | 1.5834 | 11125 |
| One Year | 13532 | 21.637 | 12036 | 8.1898 | 11125 |

It can be seen in Figure 3.2 the original data is a solid blue line, the GMM prediction is a solid red line, and the GM prediction is a dotted black line. The results shown in Table 3.1 display the actual closing average and the percentage error associated with each prediction method. There is a noticeable increase in error which correlates to the number of data points, but an error of $8.1898 \%$ shown for GMM over a year of prediction towards a value of 11,125 is not that significant. As opposed to an error percentage of $21.637 \%$ for GM over a year this value is clearly significant. This clearly shows that the GMM method is more accurate then the original GM method. The same closing day was used for all three tests and the previous data was constructed with that in mind.

### 3.2 Test with Existing AR Data

This research was an extension of the dissertation work of a faculty member and therefore data was readily available for testing. The data set was based on the sitting position of three participants assuming five distinct positions on a couch. Four load cells were placed underneath each leg of the couch to produce a reading that would vary based on a position. The original data was separate with regards to each state being an independent data set and no transitions were available. To attempt to test this data the individual data sets of the five positions for one participant was combined into one larger data set. This new data set was then scrambled, keeping the four load cell readings for one state together, but moving the states themselves into random order. The final data sets consisted of five states randomly scrambled 1500 times and tested for 1000 iterations. The prediction rate over 1000 iterations was $89 \%$ accurate. This detection rate was satisfactory but the data was not ideal for the algorithm proposed since it was missing the transitions between states.

### 3.3 Design of AR Test

To overcome the lack of transitions in the previous existing data sets new data was collected following the same approach. Instead of a couch for the testing apparatus a chair was chosen and four distinct states were selected. The four states were sitting with your back in the chair, sitting towards the front of the chair, leaning on the right arm of the chair, and leaning on the left arm of the chair. Next a transition state diagram was designed to insure that each state and its corresponding possible transitions were accounted for. The state diagram is shown below:


Figure 3.3: Transition State Diagram

The participants began the test standing up and were told to sit down in the front position. They held each steady state for 10 seconds before transitioning to the new state.

Once all transitions were complete the participants would then get up from the chair and the data collection would end. This was done twice for each participant with a cushion in the chair and twice for each participant with no cushion in the chair. These multiple runs allowed for an averaging of each particular steady state to try and compensate for the lack of training needed for the transition probability matrix.

### 3.4 Test-bed Design and Data Collection

The design and implementation of a test-bed as well as data collection strategy are described in the following subsections.

### 3.4.1 Test Apparatus Design and Construction

The test apparatus is depicted in this subsection. The test-bed was designed similar to that used in the faculty member's dissertation work. A chair was chosen, but the same load cells, and data acquisition method were implemented to maintain consistency. The four steady states were chosen for their distinct difference from one another. The four states are shown in the figure below:



Figure 3.4: Four Steady State Positions

As can be observed in Figure 3.4, the load cells were attached to the legs of the chair and held in place to maintain consistent readings. A cushion was used in two of the four each participants data runs to insure there was no effect in distribution in the weight of the chair. The results confirmed this hypothesis and the four runs were considered consistent.

### 3.4.2 Data Collection

The data was collected using four FC23 Compression Load Cells as the sensors attached to the legs of the chair. A NI-USB-6008 DAQ was used to collect the data through a Labview program. The Labview program recorded the four load cell values and amplify their output by 10,000 . This amplification was needed due to the load cell's high end weight capacity. This amplification did not obscure the data because six significant digits were shown to maintain accuracy. Each state was maintained for a minimum of ten seconds, and the sampling rate was ten samples per second.

### 3.5 Experiment Design and Data Treatment

This section discusses the design of the experiment and data treatment.

### 3.5.1 Participant Selection

There were 14 participants chosen for this research. There were 11 males and 3 females chosen. The participants were chosen to add variation, not only to gender but weight. Shown below is a table of the 14 participants, their corresponding weight, and how many iterations of data were collected from them:

Table 3.2: Participant Information

| Participant | Gender | Weight | Runs |
| :---: | :---: | :---: | :---: |
| 1 | Female | 120 | 4 |
| 2 | Female | 125 | 4 |
| 3 | Female | 130 | 4 |
| 4 | Male | 150 | 4 |
| 5 | Male | 160 | 4 |
| 6 | Male | 170 | 4 |
| 7 | Male | 175 | 4 |
| 8 | Male | 180 | 4 |
| 9 | Male | 185 | 4 |
| 10 | Male | 205 | 4 |
| 11 | Male | 205 | 4 |
| 12 | Male | 250 | 4 |
| 13 | Male | 250 | 4 |
| 14 | Male | 315 | 4 |

As seen in Table 3.2 each participant's weight varied from 120 pounds to 315 pounds to see if a general classification of steady state and transitions could be generated. Each participant completed four iterations to combine for an average steady state value.

### 3.5.2 State transition design

A state transition matrix was designed in the algorithm to help determine the correct state. Shown below is the methodology behind that design:

$$
\begin{equation*}
\text { Reference State }(\text { ref })=\left[\right] \tag{3.1}
\end{equation*}
$$

Where $1, \cdots, n$ refers to the $n$ possible transitions, and $f$ is a notation of the reference state and holds no mathematical significance.

Then the measured/predicted state is shown as:

$$
\text { Measured/Predicted State }(\text { mea })=\left[\begin{array}{cccc}
a_{1} & b_{1} & c_{1} & d_{1}  \tag{3.2}\\
& & \vdots & \\
& & \vdots & \\
& & b_{1} & c_{1}
\end{array}\right]
$$

Where the corresponding state can be determined from below using the same size matrices for ref and mea:

$$
[\text { minval ind }]=\min (\operatorname{sum}(a b s(r e f-\text { mea })))
$$

Where ind is the state closest to the reference, and sum is notation associated with MatLab referencing Summation.

Not only was there a design needed for the steady state matrix, but one was also required for the transition matrix. That design is described below:

$$
\begin{gather*}
\text { Measured Transition }(M T)=\left[\begin{array}{cccc}
a_{1} & b_{1} & c_{1} & d_{1} \\
& & \vdots & \\
& & \vdots & \\
a_{1} & b_{1} & c_{1} & d_{1}
\end{array}\right]  \tag{3.3}\\
\text { Reference Transition }(r e f)=\left[\begin{array}{cccc}
a_{1} f & b_{1} f & c_{1} f & d_{1} f \\
a_{2} f & b_{2} f & c_{2} f & d_{2} f \\
& & \vdots & \\
a_{n} f & b_{n} f & c_{n} f & d_{n} f
\end{array}\right] \tag{3.4}
\end{gather*}
$$

$$
\operatorname{Ref} *(M T)^{\prime}=\left[\begin{array}{cccc}
a_{1} f * a_{1} & b_{1} f * b_{1} & c_{1} f * c_{1} & d_{1} f * d_{1}  \tag{3.5}\\
& \vdots & & \\
a_{n} f * a_{n} & b_{n} f * b_{n} & c_{n} f * c_{n} & d_{n} f * d_{n}
\end{array}\right]
$$

Then the corresponding state can be determined from below:

$$
[\text { maxval ind }]=\max \left(\operatorname{Ref} *(M T)^{\prime}\right)
$$

Where the ind value returned is the predicted state.
These two methods allowed for the algorithm to detect the steady state and to predict the next steady state based off the transition. The results showed that the algorithm's use of the two methods had a very high detection rate.

The transitions themselves were able to be characterized into four groups. Depending on what the current state was a combination of the four loads magnitude and direction could characterize the transition. A table describing this process is shown below:

Table 3.3: Transition Classification

| State Transition | Load Cell Trend | Coded Transition |
| :---: | :---: | :---: |
| Front $\rightarrow$ Any | $\uparrow \uparrow \downarrow \downarrow$ | $11-1-1$ |
| Any $\rightarrow$ Front | $\downarrow \downarrow \uparrow \uparrow$ | $-1-111$ |
| Back $\rightarrow$ Right <br> Left $\rightarrow$ Right <br> Left $\rightarrow$ Back | $\uparrow \downarrow \uparrow \downarrow$ | $1-11-1$ |
| Right $\rightarrow$ Back <br> Back $\rightarrow$ Left <br> Right $\rightarrow$ Left | $\downarrow \uparrow \downarrow \uparrow$ | $-11-11$ |

The arrows in Table 3.3 dictate the direction the load cell's magnitude is going. The four arrow combination is for right rear, left rear, right front, left front load cells respectively. The trouble in prediction occurs with the last two groups, but the magnitude becomes the deciding factor in determining the next steady state.

### 3.5.3 Data Treatment

The original data collected had poor results initially. This was due to a lot of high frequency contents in the transitions and steady states. To eliminate the errors associated with this problem the data was passed through a lowpass filter. An example of the original data and resultant filtered data is shown below:


Figure 3.5: Original Data versus Filtered Data

This transformation was accomplished using the Zero-Phase Forward and Reverse Digital Filtering function, filtfilt, in MatLab. The filtfilt function requires three parameters to accomplish this. The function is done using filtfilt $(\mathrm{b}, \mathrm{a}, \mathrm{x})$, where a is $[0,1], \mathrm{x}$ is the data, and $b$ is finite impulse response. The resultant differential equation is shown below:

$$
\begin{align*}
y(n)=b(1) * x(n) & +b(2) * x(n-1)+\ldots+b(n b+1) * x(n-n b)  \tag{3.6}\\
& -a(2) * y(n-1)-\ldots-a(n a+1) * y(n-n a)
\end{align*}
$$

This process allowed for the high frequency content to be eliminated and the data to be more easily interpreted.

The method's used in this section allowed for the construction of the algorithm, the data collection, the data pre-processing, and data interpretation. Using these processes, results and analysis were gathering for a conclusion to be drawn on the effectiveness of the proposed method in the field of AR.

## CHAPTER 4: RESULTS AND ANALYSIS

### 4.1 Results on Collected Data

This chapter provides results and analysis of real-world data using GMM for AR. This data consisted of the 14 participants of varying weight and gender positioned in four unique postures. Original data can be found in Appendix(D,E,F). The first step in the process was to make sure steady state detection was identifiable on its own merit without the use of transitions. An example of an easily, nominally, and poorly detectable steady state is shown in the plots below:

(a) Rear Right

(b) Rear Left

(c) Front Right

(d) Front Left

Figure 4.1: Easily Detectable Steady State

(a) Rear Right


(d) Front Left

Figure 4.2: Nominally Detectable Steady State

(a) Rear Right



Figure 4.3: Poorly Detectable Steady State

The combination of the four load cell values correlates directly to a given steady state. For example the right rear is shown in (a), the left rear is shown in (b), the front right is shown in (c), and the front left is shown in (d). There are four steady states and transitions shown in the figures. The GMM algorithm can correctly identify and mimic these changes resulting in a high detection rate for steady state. With the steady state identification alone there were 53 data sets with 13 states to be identified. The algorithm correctly identified 670 out of 689 states which equates to a $97.2 \%$ detection rate. Below is a table identifying the state and correctly identified and wrongly identified amounts out of the 53 data sets:

Table 4.1: Steady State Detection Results

| Steady State | Correct | Wrong | $\%$ |
| :---: | :---: | :---: | :---: |
| 1 | 53 | 0 | 100 |
| 4 | 53 | 0 | 100 |
| 1 | 53 | 0 | 100 |
| 2 | 51 | 2 | 96.2 |
| 4 | 53 | 0 | 100 |
| 2 | 52 | 1 | 98.1 |
| 3 | 48 | 5 | 90.6 |
| 4 | 51 | 2 | 96.2 |
| 3 | 49 | 4 | 92.4 |
| 2 | 50 | 3 | 94.3 |
| 1 | 53 | 0 | 100 |
| 3 | 51 | 2 | 96.2 |
| 1 | 53 | 0 | 100 |
| Total | 670 | 19 | 97.2 |

In Table 4.1, steady state 1 refers to the front steady state, 2 refers to the back steady state, 3 refers to the left steady state, and 4 refers to the right steady state. This is promising, but prediction, not identification, is the true goal of this research. The next step in the process was to use the steady state detection in conjunction with transition detection to produce a prediction for the next steady state.

As previously shown in Table 3.3, the transitions could be characterized into four groups. There was a two step process in equating the current state using transitions. The first step was to make a decision about which group the transition belonged to. The second step was to determine which transition in the corresponding group occurred. Once the transition was identified the steady state associated with it could be classified. The first test done for transition detection is based on this assumption. Identifying the correct groups would eliminate possible errors found when associating with transitions outside of the groups. Shown below are figures of the transition detection method used to identify the direction and magnitude of a transition from steady state to steady state:

(a) Rear Right

(b) Rear Left


Figure 4.4: Transition Detection

In Figure 4.4 (a) refers to the rear right, (b) refers to the rear left, (c) refers to the front right, and (d) refers to the front left load cells. This transition detection used a
windowing method to identify the correct direction of the transitions at each steady state. Without this windowing the detection rate would either identify too many transitions or not enough transitions. With this approach, if there were too many transitions detected in a specific window then the one with greatest magnitude detected first would be declared the correct transition. Also if there was no transition detected in the window then one would be designated with a magnitude of one as to not have a great effect. Once the transition detection method was complete all 54 data sets were applied to the algorithm to determine the detection rate for the four group identification. There were 642 out of 756 correctly identified transition groups. This equates to an $84.92 \%$ detection rate of the transition groups. Once the group detection rate was complete the next step was to identify each individual transition using the knowledge gained from the transition group detection. Once again there were 54 data sets ran through the algorithm and there were 544 out of 756 correctly identified transitions. This equates to a $71.96 \%$ detection rate for each individual transition. Shown below is a table depicting the number of transitions, correctly identified group transitions, and correctly identified individual transitions:

Table 4.2: Transition Detection Rate

| Participant | Transitions | Group | Individual | Group \% | Individual \% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 56 | 50 | 46 | 89.3 | 82.1 |
| 2 | 56 | 38 | 30 | 67.8 | 53.5 |
| 3 | 56 | 52 | 50 | 92.8 | 89.2 |
| 4 | 56 | 42 | 34 | 75 | 60.7 |
| 5 | 56 | 55 | 50 | 98.2 | 89.2 |
| 6 | 56 | 54 | 50 | 96.4 | 89.2 |
| 7 | 56 | 45 | 39 | 80.3 | 69.6 |
| 8 | 56 | 56 | 55 | 100 | 98.2 |
| 9 | 56 | 49 | 40 | 87.5 | 71.4 |
| 10 | 56 | 45 | 30 | 80.3 | 53.5 |
| 11 | 56 | 50 | 45 | 89.2 | 80.3 |
| 12 | 56 | 56 | 54 | 100 | 96.4 |
| 13 | 42 | 22 | 10 | 52.3 | 23.8 |
| 14 | 42 | 28 | 11 | 66.7 | 26.2 |
| Total | 756 | 642 | 544 | 84.92 | 71.96 |

The last step in this process was to generate new data by asking participants to repeat the data collection, but this time they were told to choose their own sequence of postures. The participant was to hold the position for the usual ten seconds, and to transition twenty times. This was done for four participants and allowed for 80 new steady states to be created at random. There were 72 out of 80 steady states predicted correctly for a detection rate of $90 \%$. This used both the transition detection and steady state detection to predict the next steady state.

## CHAPTER 5: CONCLUSIONS AND FUTURE WORK

The proposed thesis investigated the addition of Markov Model to improve the performance of the Grey Model for activity recognition. Through research a better understanding of the proposed techniques and methods for AR was achieved. The need for AR, in particular for healthcare, is paramount. The use of AR to help alleviate the problems associated with assisted living for not only the patient, the healthcare provider, but the family of the patients is unmeasurable.

The initial research identified the proposed method Grey Markov Model as an approach to be explored in great depth for AR. The success of the Grey Markov Model in other fields showed promise that its application in AR could improve performance. This thesis work achieved the three goals set for the research: data collection, algorithm development, and algorithm verification. In this research, a test apparatus was designed and implemented to collect real-world data for verification purposes. An algorithm was developed and refined to achieve the detection rates reported in this thesis. This algorithm was constructed and verified throughout multiple procedures. The use of the Dow Jones data, data collected previous data, and newly acquired data provided a relatively thorough investigation of the algorithm.

There were also algorithms developed to both detect steady state and transitions. The corresponding results were shown and the original data provided for reproduction of the experiments. Both the steady state detection and the transition detection show satisfactory results for the field of activity recognition. The last set of data gathered used the
previously collected data as training, and allowed for a true correlation between prediction and detection to be seen. The detection rate of $90 \%$ on the final data sets acquired showed that the two detection methods could predict the next state very consistently.

Future work for this research should consist of collecting more data sets for future testing. The acquisition of more data sets will allow for a training of the system to increase detection rates. Also an investigation of statistical significance of factors such as gender and weight is necessary as they pertain to the thresholds for detection. If direct correlations could be established with these factors then a more general and better threshold could be established and detection speed would be greatly increased.

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## Appendix

## APPENDIX A: ONE MONTH DJI DATA

| Day | Closing Average |
| :--- | :--- |
| 1 | 10829.68 |
| 2 | 10751.27 |
| 3 | 10944.72 |
| 4 | 10967.65 |
| 5 | 10948.58 |
| 6 | 11006.48 |
| 7 | 11010.34 |
| 8 | 11020.4 |
| 9 | 11096.08 |
| 10 | 11096.92 |
| 11 | 11062.78 |
| 12 | 11143.69 |
| 13 | 10978.62 |
| 14 | 11107.97 |
| 15 | 11146.57 |
| 16 | 11132.56 |
| 17 | 11164.05 |
| 18 | 11169.46 |
| 19 | 11126.28 |
| 20 | 11113.95 |
| 21 | 11118.4 |
| 22 | 11124.62 |

## APPENDIX B: THREE MONTH DJI DATA

| Day | Closing Avg | Day | Closing Avg | Day | Closing Avg |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 10674.38 | 26 | 10340.69 | 51 | 11020.4 |
| 2 | 10636.38 | 27 | 10387.01 | 52 | 11096.08 |
| 3 | 10680.43 | 28 | 10415.24 | 53 | 11096.92 |
| 4 | 10674.98 | 29 | 10462.77 | 54 | 11062.78 |
| 5 | 10653.56 | 30 | 10544.13 | 55 | 11143.69 |
| 6 | 10698.75 | 31 | 10526.49 | 56 | 10978.62 |
| 7 | 10644.25 | 32 | 10572.73 | 57 | 11107.97 |
| 8 | 10378.83 | 33 | 10594.83 | 58 | 11146.57 |
| 9 | 10319.95 | 34 | 10607.85 | 59 | 11132.56 |
| 10 | 10303.15 | 35 | 10753.62 | 60 | 11164.05 |
| 11 | 10302.01 | 36 | 10761.03 | 61 | 11169.46 |
| 12 | 10405.85 | 37 | 10739.31 | 62 | 11126.28 |
| 13 | 10415.54 | 38 | 10662.42 | 63 | 11113.95 |
| 14 | 10271.21 | 39 | 10860.26 | 64 | 11118.4 |
| 15 | 10213.62 | 40 | 10812.04 | 65 | 11124.62 |
| 16 | 10174.41 | 41 | 10858.14 |  |  |
| 17 | 10040.45 | 42 | 10835.28 |  |  |
| 18 | 10060.06 | 43 | 10788.05 |  |  |
| 19 | 9985.81 | 44 | 10829.68 |  |  |
| 20 | 10150.65 | 45 | 10751.27 |  |  |
| 21 | 10009.73 | 46 | 10944.72 |  |  |
| 22 | 10014.72 | 47 | 10967.65 |  |  |
| 23 | 10269.47 | 48 | 10948.58 |  |  |
| 24 | 10320.1 | 49 | 11006.48 |  |  |
| 25 | 10447.93 | 50 | 11010.34 |  |  |

## APPENDIX C: ONE YEAR DJI DATA

| Day | Closing Avg | Day | Closing Avg | Day | Closing Avg |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 9789.44 | 26 | 10285.97 | 51 | 10710.55 |
| 2 | 9771.91 | 27 | 10337.05 | 52 | 10609.65 |
| 3 | 9802.14 | 28 | 10405.83 | 53 | 10725.43 |
| 4 | 10005.96 | 29 | 10471.5 | 54 | 10603.15 |
| 5 | 10023.42 | 30 | 10501.05 | 55 | 10389.88 |
| 6 | 10226.94 | 31 | 10452 | 56 | 10172.98 |
| 7 | 10246.97 | 32 | 10441.12 | 57 | 10196.86 |
| 8 | 10291.26 | 33 | 10308.26 | 58 | 10194.29 |
| 9 | 10197.47 | 34 | 10328.89 | 59 | 10236.16 |
| 10 | 10270.47 | 35 | 10414.14 | 60 | 10120.46 |
| 11 | 10406.96 | 36 | 10464.93 | 61 | 10067.33 |
| 12 | 10437.42 | 37 | 10466.44 | 62 | 10185.53 |
| 13 | 10426.31 | 38 | 10520.1 | 63 | 10296.85 |
| 14 | 10332.44 | 39 | 10547.08 | 64 | 10270.55 |
| 15 | 10318.16 | 40 | 10545.41 | 65 | 10002.18 |
| 16 | 10450.95 | 41 | 10548.51 | 66 | 10012.23 |
| 17 | 10433.71 | 42 | 10428.05 | 67 | 9908.39 |
| 18 | 10464.4 | 43 | 10583.96 | 68 | 10058.64 |
| 19 | 10309.92 | 44 | 10572.02 | 69 | 10038.38 |
| 20 | 10344.84 | 45 | 10573.68 | 70 | 10144.19 |
| 21 | 10471.58 | 46 | 10606.86 | 71 | 10099.14 |
| 22 | 10452.68 | 47 | 10618.19 | 72 | 10268.81 |
| 23 | 10366.15 | 48 | 10663.99 | 73 | 10309.24 |
| 24 | 10388.9 | 49 | 10627.26 | 74 | 10392.9 |
| 25 | 10390.11 | 50 | 10680.77 | 75 | 10402.35 |


| Day | Closing Avg | Day | Closing Avg | Day | Closing Avg |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 76 | 10383.38 | 101 | 10895.86 | 126 | 10926.77 |
| 77 | 10282.41 | 102 | 10907.42 | 127 | 10868.12 |
| 78 | 10374.16 | 103 | 10856.63 | 128 | 10520.32 |
| 79 | 10321.03 | 104 | 10927.07 | 129 | 10380.43 |
| 80 | 10325.26 | 105 | 10973.55 | 130 | 10785.14 |
| 81 | 10403.79 | 106 | 10969.99 | 131 | 10748.26 |
| 82 | 10405.98 | 107 | 10897.52 | 132 | 10896.91 |
| 83 | 10396.76 | 108 | 10927.07 | 133 | 10782.95 |
| 84 | 10444.14 | 109 | 10997.35 | 134 | 10620.16 |
| 85 | 10566.2 | 110 | 11005.97 | 135 | 10625.83 |
| 86 | 10552.52 | 111 | 11019.42 | 136 | 10510.95 |
| 87 | 10564.38 | 112 | 11123.11 | 137 | 10444.37 |
| 88 | 10567.33 | 113 | 11144.57 | 138 | 10068.01 |
| 89 | 10611.84 | 114 | 11018.66 | 139 | 10193.39 |
| 90 | 10624.69 | 115 | 11092.05 | 140 | 10066.57 |
| 91 | 10642.15 | 116 | 11117.06 | 141 | 10043.75 |
| 92 | 10685.98 | 117 | 11124.92 | 142 | 9974.45 |
| 93 | 10733.67 | 118 | 11134.29 | 143 | 10258.99 |
| 94 | 10779.17 | 119 | 11204.28 | 144 | 10136.63 |
| 95 | 10741.98 | 120 | 11205.03 | 145 | 10024.02 |
| 96 | 10785.89 | 121 | 10991.99 | 146 | 10249.54 |
| 97 | 10888.83 | 122 | 11045.27 | 147 | 10255.28 |
| 98 | 10836.15 | 123 | 11167.32 | 148 | 9931.97 |
| 99 | 10841.21 | 124 | 11008.61 | 149 | 9816.49 |
| 100 | 10850.36 | 125 | 11151.83 | 150 | 9939.98 |


| Day | Closing Avg | Day | Closing Avg | Day | Closing Avg |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 151 | 9899.25 | 176 | 10359.31 | 201 | 10271.21 |
| 152 | 10172.53 | 177 | 10097.9 | 202 | 10213.62 |
| 153 | 10211.07 | 178 | 10154.43 | 203 | 10174.41 |
| 154 | 10190.89 | 179 | 10229.96 | 204 | 10040.45 |
| 155 | 10404.77 | 180 | 10120.53 | 205 | 10060.06 |
| 156 | 10409.46 | 181 | 10322.3 | 206 | 9985.81 |
| 157 | 10434.17 | 182 | 10424.62 | 207 | 10150.65 |
| 158 | 10450.64 | 183 | 10525.43 | 208 | 10009.73 |
| 159 | 10442.41 | 184 | 10537.69 | 209 | 10014.72 |
| 160 | 10293.52 | 185 | 10497.88 | 210 | 10269.47 |
| 161 | 10298.44 | 186 | 10467.16 | 211 | 10320.1 |
| 162 | 10152.8 | 187 | 10465.94 | 212 | 10447.93 |
| 163 | 10143.81 | 188 | 10674.38 | 213 | 10340.69 |
| 164 | 10138.52 | 189 | 10636.38 | 214 | 10387.01 |
| 165 | 9870.3 | 190 | 10680.43 | 215 | 10415.24 |
| 166 | 9774.02 | 191 | 10674.98 | 216 | 10462.77 |
| 167 | 9732.53 | 192 | 10653.56 | 217 | 10544.13 |
| 168 | 9686.48 | 193 | 10698.75 | 218 | 10526.49 |
| 169 | 9743.62 | 194 | 10644.25 | 219 | 10572.73 |
| 170 | 10018.28 | 195 | 10378.83 | 220 | 10594.83 |
| 171 | 10138.99 | 196 | 10319.95 | 221 | 10607.85 |
| 172 | 10198.03 | 197 | 10303.15 | 222 | 10753.62 |
| 173 | 10216.27 | 198 | 10302.01 | 223 | 10761.03 |
| 174 | 10363.02 | 199 | 10405.85 | 224 | 10739.31 |
| 175 | 10366.72 | 200 | 10415.54 | 225 | 10662.42 |


| Day | Closing Avg | Day | Closing Avg |
| :--- | :--- | :--- | :--- |
| 226 | 10860.26 | 251 | 11118.4 |
| 227 | 10812.04 | 252 | 11124.62 |
| 228 | 10858.14 |  |  |
| 229 | 10835.28 |  |  |
| 230 | 10788.05 |  |  |
| 231 | 10829.68 |  |  |
| 232 | 10751.27 |  |  |
| 233 | 10944.72 |  |  |
| 234 | 10967.65 |  |  |
| 235 | 10948.58 |  |  |
| 236 | 11006.48 |  |  |
| 237 | 11010.34 |  |  |
| 238 | 11020.4 |  |  |
| 239 | 11096.08 |  |  |
| 240 | 11096.92 |  |  |
| 241 | 11062.78 |  |  |
| 242 | 11143.69 |  |  |
| 243 | 10978.62 |  |  |
| 244 | 11107.97 |  |  |
| 245 | 11146.57 |  |  |
| 246 | 11132.56 |  |  |
| 247 | 11164.05 |  |  |
| 248 | 11169.46 |  |  |
| 249 | 11126.28 |  |  |
| 250 | 11113.95 |  |  |

## APPENDIX D: EASILY DETECTABLE DATA SET

|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $1.1203940 \mathrm{e}+002$ | $1.3733390 \mathrm{e}+002$ | $8.6122600 \mathrm{e}+001$ | $4.0194100 \mathrm{e}+001$ |
| 2 | $1.1153010 \mathrm{e}+002$ | $1.3784290 \mathrm{e}+002$ | $8.5612900 \mathrm{e}+001$ | $4.1721300 \mathrm{e}+001$ |
| 3 | $1.0898380 \mathrm{e}+002$ | $1.3631610 \mathrm{e}+002$ | $8.5612900 \mathrm{e}+001$ | $4.1721500 \mathrm{e}+001$ |
| 4 | $1.1203920 \mathrm{e}+002$ | $1.3835210 \mathrm{e}+002$ | $8.6122700 \mathrm{e}+001$ | $4.1212300 \mathrm{e}+001$ |
| 5 | $1.1203930 \mathrm{e}+002$ | $1.3580710 \mathrm{e}+002$ | $8.6122600 \mathrm{e}+001$ | $4.3248900 \mathrm{e}+001$ |
| 6 | $1.0847460 \mathrm{e}+002$ | $1.3733400 \mathrm{e}+002$ | $8.4084000 \mathrm{e}+001$ | $4.1721400 \mathrm{e}+001$ |
| 7 | $1.1051150 \mathrm{e}+002$ | $1.3936990 \mathrm{e}+002$ | $9.1728800 \mathrm{e}+001$ | $4.1212200 \mathrm{e}+001$ |
| 8 | $1.0949300 \mathrm{e}+002$ | $1.3936990 \mathrm{e}+002$ | $8.6122600 \mathrm{e}+001$ | $4.1212100 \mathrm{e}+001$ |
| 9 | $1.1102070 \mathrm{e}+002$ | $1.3835200 \mathrm{e}+002$ | $8.4593600 \mathrm{e}+001$ | $4.3758100 \mathrm{e}+001$ |
| 10 | $1.1051140 \mathrm{e}+002$ | $1.4089690 \mathrm{e}+002$ | $8.3064800 \mathrm{e}+001$ | $4.1721600 \mathrm{e}+001$ |
| 11 | $3.9726300 \mathrm{e}+001$ | $8.1857500 \mathrm{e}+001$ | $1.2944420 \mathrm{e}+002$ | $8.6015600 \mathrm{e}+001$ |
| 12 | $3.8707600 \mathrm{e}+001$ | $8.1348300 \mathrm{e}+001$ | $1.3607000 \mathrm{e}+002$ | $8.8052000 \mathrm{e}+001$ |
| 13 | $3.9726200 \mathrm{e}+001$ | $7.9312600 \mathrm{e}+001$ | $1.3454100 \mathrm{e}+002$ | $9.3143700 \mathrm{e}+001$ |
| 14 | $3.8198500 \mathrm{e}+001$ | $7.9821700 \mathrm{e}+001$ | $1.3097330 \mathrm{e}+002$ | $8.8052100 \mathrm{e}+001$ |
| 15 | $4.0744700 \mathrm{e}+001$ | $8.0839400 \mathrm{e}+001$ | $1.3352160 \mathrm{e}+002$ | $8.8052300 \mathrm{e}+001$ |
| 16 | $3.9217100 \mathrm{e}+001$ | $7.9821500 \mathrm{e}+001$ | $1.3708940 \mathrm{e}+002$ | $9.1106900 \mathrm{e}+001$ |
| 17 | $4.1254100 \mathrm{e}+001$ | $7.9821500 \mathrm{e}+001$ | $1.3759910 \mathrm{e}+002$ | $9.0597900 \mathrm{e}+001$ |
| 18 | $3.9217000 \mathrm{e}+001$ | $8.2366400 \mathrm{e}+001$ | $1.3657970 \mathrm{e}+002$ | $8.8561400 \mathrm{e}+001$ |
| 19 | $3.9726300 \mathrm{e}+001$ | $8.0330500 \mathrm{e}+001$ | $1.3657970 \mathrm{e}+002$ | $9.2125300 \mathrm{e}+001$ |
| 20 | $3.9726300 \mathrm{e}+001$ | $7.8294500 \mathrm{e}+001$ | $1.3250230 \mathrm{e}+002$ | $8.9579600 \mathrm{e}+001$ |


|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 21 | $1.2477010 \mathrm{e}+002$ | $1.0679630 \mathrm{e}+002$ | $1.0345130 \mathrm{e}+002$ | $5.3431400 \mathrm{e}+001$ |
| 22 | $1.2578850 \mathrm{e}+002$ | $1.0526950 \mathrm{e}+002$ | $1.0294160 \mathrm{e}+002$ | $4.8849200 \mathrm{e}+001$ |
| 23 | $1.2171480 \mathrm{e}+002$ | $1.0476050 \mathrm{e}+002$ | $9.9883600 \mathrm{e}+001$ | $4.8340100 \mathrm{e}+001$ |
| 24 | $1.2171470 \mathrm{e}+002$ | $1.0374280 \mathrm{e}+002$ | $1.0141250 \mathrm{e}+002$ | $4.9358500 \mathrm{e}+001$ |
| 25 | $1.2273330 \mathrm{e}+002$ | $1.0628730 \mathrm{e}+002$ | $1.0345130 \mathrm{e}+002$ | $5.0376500 \mathrm{e}+001$ |
| 26 | $1.2273320 \mathrm{e}+002$ | $1.0476040 \mathrm{e}+002$ | $1.0090310 \mathrm{e}+002$ | $4.8849200 \mathrm{e}+001$ |
| 27 | $1.2578850 \mathrm{e}+002$ | $1.0679630 \mathrm{e}+002$ | $1.0345110 \mathrm{e}+002$ | $4.8340100 \mathrm{e}+001$ |
| 28 | $1.2171470 \mathrm{e}+002$ | $1.0679620 \mathrm{e}+002$ | $1.0141250 \mathrm{e}+002$ | $5.1904100 \mathrm{e}+001$ |
| 29 | $1.2120550 \mathrm{e}+002$ | $1.0730530 \mathrm{e}+002$ | $1.0294150 \mathrm{e}+002$ | $4.9867500 \mathrm{e}+001$ |
| 30 | $1.2324240 \mathrm{e}+002$ | $1.0679630 \mathrm{e}+002$ | $1.0141260 \mathrm{e}+002$ | $4.7321900 \mathrm{e}+001$ |
| 31 | $6.9262400 \mathrm{e}+001$ | $1.5769290 \mathrm{e}+002$ | $9.2748400 \mathrm{e}+001$ | $6.1577700 \mathrm{e}+001$ |
| 32 | $6.9262300 \mathrm{e}+001$ | $1.5616600 \mathrm{e}+002$ | $9.1219400 \mathrm{e}+001$ | $6.0559500 \mathrm{e}+001$ |
| 33 | $6.8243800 \mathrm{e}+001$ | $1.5616600 \mathrm{e}+002$ | $9.0709700 \mathrm{e}+001$ | $6.5141500 \mathrm{e}+001$ |
| 34 | $6.9262400 \mathrm{e}+001$ | $1.5616600 \mathrm{e}+002$ | $9.2238800 \mathrm{e}+001$ | $6.5141500 \mathrm{e}+001$ |
| 35 | $7.0280700 \mathrm{e}+001$ | $1.5514810 \mathrm{e}+002$ | $9.3767700 \mathrm{e}+001$ | $6.0559400 \mathrm{e}+001$ |
| 36 | $7.0280700 \mathrm{e}+001$ | $1.5769280 \mathrm{e}+002$ | $9.3767700 \mathrm{e}+001$ | $6.2086800 \mathrm{e}+001$ |
| 37 | $7.1808400 \mathrm{e}+001$ | $1.5616600 \mathrm{e}+002$ | $9.2748400 \mathrm{e}+001$ | $6.2086800 \mathrm{e}+001$ |
| 38 | $7.0280800 \mathrm{e}+001$ | $1.5667490 \mathrm{e}+002$ | $9.2748400 \mathrm{e}+001$ | $6.2595900 \mathrm{e}+001$ |
| 39 | $6.8243700 \mathrm{e}+001$ | $1.5616590 \mathrm{e}+002$ | $9.5296800 \mathrm{e}+001$ | $6.2595900 \mathrm{e}+001$ |
| 40 | $6.7225300 \mathrm{e}+001$ | $1.5820190 \mathrm{e}+002$ | $9.2748400 \mathrm{e}+001$ | $6.5650700 \mathrm{e}+001$ |

## APPENDIX E: NOMINALLY DETECTABLE DATA SET

|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $1.3393640 \mathrm{e}+002$ | $1.4853120 \mathrm{e}+002$ | $1.0803840 \mathrm{e}+002$ | $6.7687200 \mathrm{e}+001$ |
| 2 | $1.3037160 \mathrm{e}+002$ | $1.4802220 \mathrm{e}+002$ | $1.0599970 \mathrm{e}+002$ | $6.3614100 \mathrm{e}+001$ |
| 3 | $1.2884400 \mathrm{e}+002$ | $1.4954900 \mathrm{e}+002$ | $1.1058650 \mathrm{e}+002$ | $6.5141500 \mathrm{e}+001$ |
| 4 | $1.3088110 \mathrm{e}+002$ | $1.5107610 \mathrm{e}+002$ | $1.0599950 \mathrm{e}+002$ | $6.7687000 \mathrm{e}+001$ |
| 5 | $1.3037160 \mathrm{e}+002$ | $1.4751330 \mathrm{e}+002$ | $1.06509330 \mathrm{e}+002$ | $6.5141500 \mathrm{e}+001$ |
| 6 | $1.3139030 \mathrm{e}+002$ | $1.4802230 \mathrm{e}+002$ | $1.0854810 \mathrm{e}+002$ | $6.6669000 \mathrm{e}+001$ |
| 7 | $1.2935310 \mathrm{e}+002$ | $1.4700450 \mathrm{e}+002$ | $1.0650940 \mathrm{e}+002$ | $6.2086800 \mathrm{e}+001$ |
| 8 | $1.2986250 \mathrm{e}+002$ | $1.4649550 \mathrm{e}+002$ | $1.0803830 \mathrm{e}+002$ | $6.3614100 \mathrm{e}+001$ |
| 9 | $1.2884380 \mathrm{e}+002$ | $1.5056720 \mathrm{e}+002$ | $1.1160590 \mathrm{e}+002$ | $6.4123200 \mathrm{e}+001$ |
| 10 | $1.3037170 \mathrm{e}+002$ | $1.4598650 \mathrm{e}+002$ | $1.0905760 \mathrm{e}+002$ | $6.5650600 \mathrm{e}+001$ |
| 11 | $4.4818600 \mathrm{e}+001$ | $7.2187200 \mathrm{e}+001$ | $1.7174620 \mathrm{e}+002$ | $1.0536290 \mathrm{e}+002$ |
| 12 | $4.3290900 \mathrm{e}+001$ | $7.2187200 \mathrm{e}+001$ | $1.7225590 \mathrm{e}+002$ | $1.0332600 \mathrm{e}+002$ |
| 13 | $4.5837400 \mathrm{e}+001$ | $7.2696100 \mathrm{e}+001$ | $1.7276560 \mathrm{e}+002$ | $1.0281700 \mathrm{e}+002$ |
| 14 | $4.4818600 \mathrm{e}+001$ | $7.1169300 \mathrm{e}+001$ | $1.7072690 \mathrm{e}+002$ | $9.9762300 \mathrm{e}+001$ |
| 15 | $4.7874200 \mathrm{e}+001$ | $7.2696100 \mathrm{e}+001$ | $1.7276560 \mathrm{e}+002$ | $1.0027140 \mathrm{e}+002$ |
| 16 | $4.5837100 \mathrm{e}+001$ | $7.1169200 \mathrm{e}+001$ | $1.7123650 \mathrm{e}+002$ | $1.0281700 \mathrm{e}+002$ |
| 17 | $4.4309500 \mathrm{e}+001$ | $7.2187200 \mathrm{e}+001$ | $1.7225590 \mathrm{e}+002$ | $1.0383520 \mathrm{e}+002$ |
| 18 | $4.3291000 \mathrm{e}+001$ | $6.9642500 \mathrm{e}+001$ | $1.7327520 \mathrm{e}+002$ | $1.0383540 \mathrm{e}+002$ |
| 19 | $4.3290900 \mathrm{e}+001$ | $7.2696200 \mathrm{e}+001$ | $1.7225580 \mathrm{e}+002$ | $1.0230780 \mathrm{e}+002$ |
| 20 | $4.2272500 \mathrm{e}+001$ | $7.1678200 \mathrm{e}+001$ | $1.7072670 \mathrm{e}+002$ | $1.0332620 \mathrm{e}+002$ |


|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 21 | $1.4055670 \mathrm{e}+002$ | $1.1341300 \mathrm{e}+002$ | $1.2638630 \mathrm{e}+002$ | $7.0742000 \mathrm{e}+001$ |
| 22 | $1.3546410 \mathrm{e}+002$ | $1.1544900 \mathrm{e}+002$ | $1.2587650 \mathrm{e}+002$ | $7.0232800 \mathrm{e}+001$ |
| 23 | $1.3699200 \mathrm{e}+002$ | $1.1595800 \mathrm{e}+002$ | $1.2587650 \mathrm{e}+002$ | $7.0742000 \mathrm{e}+001$ |
| 24 | $1.3597350 \mathrm{e}+002$ | $1.1493990 \mathrm{e}+002$ | $1.2944420 \mathrm{e}+002$ | $7.2269500 \mathrm{e}+001$ |
| 25 | $1.3801040 \mathrm{e}+002$ | $1.1697580 \mathrm{e}+002$ | $1.2587680 \mathrm{e}+002$ | $6.9723600 \mathrm{e}+001$ |
| 26 | $1.3648260 \mathrm{e}+002$ | $1.1290400 \mathrm{e}+002$ | $1.2842500 \mathrm{e}+002$ | $6.9723600 \mathrm{e}+001$ |
| 27 | $1.3953820 \mathrm{e}+002$ | $1.1443100 \mathrm{e}+002$ | $1.2842500 \mathrm{e}+002$ | $7.2778600 \mathrm{e}+001$ |
| 28 | $1.3953810 \mathrm{e}+002$ | $1.1290400 \mathrm{e}+002$ | $1.2536690 \mathrm{e}+002$ | $7.0742000 \mathrm{e}+001$ |
| 29 | $1.3902910 \mathrm{e}+002$ | $1.1341300 \mathrm{e}+002$ | $1.2791540 \mathrm{e}+002$ | $6.9214500 \mathrm{e}+001$ |
| 30 | $1.3851970 \mathrm{e}+002$ | $1.1188600 \mathrm{e}+002$ | $1.2893450 \mathrm{e}+002$ | $7.0232900 \mathrm{e}+001$ |
| 31 | $1.1051150 \mathrm{e}+002$ | $1.6227340 \mathrm{e}+002$ | $1.0090280 \mathrm{e}+002$ | $6.8705400 \mathrm{e}+001$ |
| 32 | $1.1203940 \mathrm{e}+002$ | $1.6227330 \mathrm{e}+002$ | $1.0294160 \mathrm{e}+002$ | $6.8196200 \mathrm{e}+001$ |
| 33 | $1.1102070 \mathrm{e}+002$ | $1.6227340 \mathrm{e}+002$ | $1.0090270 \mathrm{e}+002$ | $6.7178000 \mathrm{e}+001$ |
| 34 | $1.1458540 \mathrm{e}+002$ | $1.6176450 \mathrm{e}+002$ | $1.0039330 \mathrm{e}+002$ | $7.2269500 \mathrm{e}+001$ |
| 35 | $1.1356690 \mathrm{e}+002$ | $1.6023750 \mathrm{e}+002$ | $1.0396090 \mathrm{e}+002$ | $7.4306000 \mathrm{e}+001$ |
| 36 | $1.1254840 \mathrm{e}+002$ | $1.6227340 \mathrm{e}+002$ | $1.0090280 \mathrm{e}+002$ | $7.2778600 \mathrm{e}+001$ |
| 37 | $1.1254850 \mathrm{e}+002$ | $1.5921970 \mathrm{e}+002$ | $1.0039320 \mathrm{e}+002$ | $7.2269500 \mathrm{e}+001$ |
| 38 | $1.1458530 \mathrm{e}+002$ | $1.6023760 \mathrm{e}+002$ | $1.0447060 \mathrm{e}+002$ | $7.0232900 \mathrm{e}+001$ |
| 39 | $1.1254860 \mathrm{e}+002$ | $1.5769280 \mathrm{e}+002$ | $1.0447060 \mathrm{e}+002$ | $7.3287700 \mathrm{e}+001$ |
| 40 | $1.1356720 \mathrm{e}+002$ | $1.5820180 \mathrm{e}+002$ | $1.0599960 \mathrm{e}+002$ | $7.2778700 \mathrm{e}+001$ |

## APPENDIX F: POORLY DETECTABLE DATA SET

|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $8.2502700 \mathrm{e}+001$ | $1.1901170 \mathrm{e}+002$ | $8.3064600 \mathrm{e}+001$ | $3.8157400 \mathrm{e}+001$ |
| 2 | $8.6067600 \mathrm{e}+001$ | $1.1901170 \mathrm{e}+002$ | $8.3064800 \mathrm{e}+001$ | $3.6630000 \mathrm{e}+001$ |
| 3 | $8.5558200 \mathrm{e}+001$ | $1.2002970 \mathrm{e}+002$ | $8.1026200 \mathrm{e}+001$ | $3.7648400 \mathrm{e}+001$ |
| 4 | $8.7595300 \mathrm{e}+001$ | $1.1595790 \mathrm{e}+002$ | $8.1535800 \mathrm{e}+001$ | $3.9175700 \mathrm{e}+001$ |
| 5 | $8.2502600 \mathrm{e}+001$ | $1.1748500 \mathrm{e}+002$ | $8.2554900 \mathrm{e}+001$ | $3.8157300 \mathrm{e}+001$ |
| 6 | $8.2502700 \mathrm{e}+001$ | $1.1799390 \mathrm{e}+002$ | $8.6122800 \mathrm{e}+001$ | $4.0194000 \mathrm{e}+001$ |
| 7 | $8.5558300 \mathrm{e}+001$ | $1.1697590 \mathrm{e}+002$ | $8.3574300 \mathrm{e}+001$ | $3.7139000 \mathrm{e}+001$ |
| 8 | $8.7595300 \mathrm{e}+001$ | $1.1850280 \mathrm{e}+002$ | $8.0516400 \mathrm{e}+001$ | $3.8157300 \mathrm{e}+001$ |
| 9 | $8.4539800 \mathrm{e}+001$ | $1.1799380 \mathrm{e}+002$ | $8.2555100 \mathrm{e}+001$ | $3.7139000 \mathrm{e}+001$ |
| 10 | $8.4539700 \mathrm{e}+001$ | $1.1748480 \mathrm{e}+002$ | $8.2555000 \mathrm{e}+001$ | $3.7648400 \mathrm{e}+001$ |
| 11 | $3.6670800 \mathrm{e}+001$ | $6.6588800 \mathrm{e}+001$ | $1.1058680 \mathrm{e}+002$ | $8.0415400 \mathrm{e}+001$ |
| 12 | $3.7180200 \mathrm{e}+001$ | $6.2516900 \mathrm{e}+001$ | $1.0956730 \mathrm{e}+002$ | $7.8888100 \mathrm{e}+001$ |
| 13 | $3.8198700 \mathrm{e}+001$ | $6.4043800 \mathrm{e}+001$ | $1.0854800 \mathrm{e}+002$ | $7.9906400 \mathrm{e}+001$ |
| 14 | $3.8707800 \mathrm{e}+001$ | $6.6079700 \mathrm{e}+001$ | $1.0752870 \mathrm{e}+002$ | $7.7360700 \mathrm{e}+001$ |
| 15 | $4.0235400 \mathrm{e}+001$ | $6.3025900 \mathrm{e}+001$ | $1.0701900 \mathrm{e}+002$ | $7.7360800 \mathrm{e}+001$ |
| 16 | $3.7689300 \mathrm{e}+001$ | $6.3535000 \mathrm{e}+001$ | $1.0803850 \mathrm{e}+002$ | $8.0415400 \mathrm{e}+001$ |
| 17 | $3.8707600 \mathrm{e}+001$ | $6.3534800 \mathrm{e}+001$ | $1.1058650 \mathrm{e}+002$ | $7.8888000 \mathrm{e}+001$ |
| 18 | $3.8198500 \mathrm{e}+001$ | $6.5570800 \mathrm{e}+001$ | $1.0752870 \mathrm{e}+002$ | $7.7360800 \mathrm{e}+001$ |
| 19 | $3.9216900 \mathrm{e}+001$ | $6.4552800 \mathrm{e}+001$ | $1.0752860 \mathrm{e}+002$ | $7.7869900 \mathrm{e}+001$ |
| 20 | $3.7689200 \mathrm{e}+001$ | $6.0990000 \mathrm{e}+001$ | $1.0956730 \mathrm{e}+002$ | $7.7869900 \mathrm{e}+001$ |


|  | Load Cell A | Load Cell B | Load Cell C | Load Cell D |
| :---: | :---: | :---: | :---: | :---: |
| 21 | $7.2317700 \mathrm{e}+001$ | $1.1341300 \mathrm{e}+002$ | $8.1026200 \mathrm{e}+001$ | $4.6812800 \mathrm{e}+001$ |
| 22 | $7.4354700 \mathrm{e}+001$ | $1.1544890 \mathrm{e}+002$ | $8.0006900 \mathrm{e}+001$ | $4.7321800 \mathrm{e}+001$ |
| 23 | $7.2826900 \mathrm{e}+001$ | $1.1748490 \mathrm{e}+002$ | $8.1535800 \mathrm{e}+001$ | $5.1904000 \mathrm{e}+001$ |
| 24 | $7.6391800 \mathrm{e}+001$ | $1.1697590 \mathrm{e}+002$ | $8.0006700 \mathrm{e}+001$ | $4.9358300 \mathrm{e}+001$ |
| 25 | $7.4354700 \mathrm{e}+001$ | $1.1697590 \mathrm{e}+002$ | $8.0006900 \mathrm{e}+001$ | $4.6812800 \mathrm{e}+001$ |
| 26 | $7.3336200 \mathrm{e}+001$ | $1.1392190 \mathrm{e}+002$ | $8.2045500 \mathrm{e}+001$ | $4.8340100 \mathrm{e}+001$ |
| 27 | $7.3336300 \mathrm{e}+001$ | $1.1748490 \mathrm{e}+002$ | $8.4083800 \mathrm{e}+001$ | $4.9867400 \mathrm{e}+001$ |
| 28 | $7.4354700 \mathrm{e}+001$ | $1.1697590 \mathrm{e}+002$ | $8.4084000 \mathrm{e}+001$ | $4.7321900 \mathrm{e}+001$ |
| 29 | $7.1808500 \mathrm{e}+001$ | $1.1901180 \mathrm{e}+002$ | $8.3574400 \mathrm{e}+001$ | $4.8340100 \mathrm{e}+001$ |
| 30 | $7.2317700 \mathrm{e}+001$ | $1.1748470 \mathrm{e}+002$ | $8.3574300 \mathrm{e}+001$ | $4.8849400 \mathrm{e}+001$ |
| 31 | $7.4864000 \mathrm{e}+001$ | $1.1901160 \mathrm{e}+002$ | $8.4593700 \mathrm{e}+001$ | $3.9175600 \mathrm{e}+001$ |
| 32 | $7.5882400 \mathrm{e}+001$ | $1.2053840 \mathrm{e}+002$ | $8.4593700 \mathrm{e}+001$ | $3.8157300 \mathrm{e}+001$ |
| 33 | $7.6391700 \mathrm{e}+001$ | $1.1697580 \mathrm{e}+002$ | $8.5103200 \mathrm{e}+001$ | $4.0703200 \mathrm{e}+001$ |
| 34 | $7.3336100 \mathrm{e}+001$ | $1.1748480 \mathrm{e}+002$ | $8.8161300 \mathrm{e}+001$ | $3.7648200 \mathrm{e}+001$ |
| 35 | $7.7919400 \mathrm{e}+001$ | $1.1901180 \mathrm{e}+002$ | $8.3574400 \mathrm{e}+001$ | $4.2230600 \mathrm{e}+001$ |
| 36 | $7.4863900 \mathrm{e}+001$ | $1.1748470 \mathrm{e}+002$ | $8.6122600 \mathrm{e}+001$ | $3.9684800 \mathrm{e}+001$ |
| 37 | $7.5882600 \mathrm{e}+001$ | $1.1697590 \mathrm{e}+002$ | $8.3064800 \mathrm{e}+001$ | $4.1212400 \mathrm{e}+001$ |
| 38 | $7.2827100 \mathrm{e}+001$ | $1.1850290 \mathrm{e}+002$ | $8.5612900 \mathrm{e}+001$ | $3.9684900 \mathrm{e}+001$ |
| 39 | $7.4864100 \mathrm{e}+001$ | $1.1799380 \mathrm{e}+002$ | $8.4593700 \mathrm{e}+001$ | $4.0703000 \mathrm{e}+001$ |
| 40 | $7.3336100 \mathrm{e}+001$ | $1.1850280 \mathrm{e}+002$ | $8.4593800 \mathrm{e}+001$ | $4.0703100 \mathrm{e}+001$ |

