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This paper examines and compares three theorists' treatments of the overtone series and their influence on concepts of tonality such as scale construction and chord roots. Special attention is paid to Ernst Terhardt's assertions that listeners are aware of the overtone series as an auditory pattern and that their brains can recognize and complete partial instances of that pattern.

This paper also examines the Terhardt and Parncutt models for calculating chord roots and explores the concepts underlying them. Both models are comparatively evaluated based on the results they return for three different chords. Finally, this paper offers some ideas on altering the curriculum of an undergraduate music theory classroom to more closely align with the psychoacoustical research discussed throughout, including examples of how to properly contextualize and diminish the prominence of misleading analytical tools and how to introduce overtone pattern recognition principles and the Parncutt model to undergraduate students.

MUSIC AND PATTERNS

by

Benjamin Brent Rawls

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This thesis has been approved by the following committee of the Faculty of the Graduate School at the University of North Carolina at Greensboro.

Committee Chair ____________________________

Committee Members ____________________________

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CHAPTER I

INTRODUCTION

The overtone series has been implicated in the inner workings of chords and keys since Rameau’s *corps sonore*, but perceptual and cognitive data have lagged behind theory and speculation. Without a direct demonstrated connection between the overtone series and functional tonality, ideas of a tonal mandate from a higher power gained traction. The effects of tonality were often explained in scientific or quasi-religious terms, such as Hindemith’s repeated references to “Nature.” After all, why not attribute tonality to a higher power, in the absence of any other way to explain why it all fits together so neatly?

Tonality's allure is not a function of the natural world and its mandates, but a function of the human desire to perceive the world in terms of familiar patterns. This inclination is so powerful that patterns are widely perceived even in an absence of evidence to properly support them, as when stargazers inferred the existence of Martian sculptors from the sight of a "face" suggested by a configuration of craters on that planet's surface.[1] Even to an observer who knows the site is merely a crater formation, it still looks like a human face; even many

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[1] Carl Sagan, *The Demon-Haunted World: Science as a Candle in the Dark* (New York: Random House, 1996), 52-59.

NASA astronomers, who were equipped with the logical and analytical tools that should have allowed them to know better, were unable to resist the lure of a familiar pattern, or the temptation to assign fantastical, deliberate intent to that pattern's appearance.[2]

Musicians should find these paths of thought familiar, having walked down them many times themselves. J. S. Bach famously spoke about music in terms of a reaffirmation of the proper spiritual order of the world banishing "devilish hubbub" with holy consonance.[3] Boethius projected musical ratios into the realm of astronomy, postulating a *musica universalis* which, generated as it was by the ratios of orbiting planets, took precedence over the comparatively quotidian (if actually audible) tunes played by mere mortal musicians. Kepler famously took this concept of interval ratios, already elevated from music theory to high heavenly supposition, back down to Earth with a grim pronouncement subjugating the nature of life upon this planet to the cosmic ratios defined by its orbit: " The Earth sings Mi, Fa, Mi so that you may infer even from the syllables that in this our domicile MIsery and FAmine obtain."[4]

It is easy to look at patterns and infer too much, drawing unsupported conclusions of supernatural agency or guiding intent. The human tendency to

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[2] Sagan, *The Demon-Haunted World*, 53.

[3] Paul Hindemith, *Craft of Musical Composition, vol. 1, Theoretical Part* , translation by Otto Ortmann (New York: Associated Music Publishers, 1941) pp. 12-13.

[4] Johannes Kepler, *The Harmony of the World*, translated by Charles Glenn Wallis (Chicago: Encyclopedia Britannica, Inc., 1952).

search for patterns often sets the brain seeking misguidedly after underlying great secrets, divine causes, and assorted mysteries of nature. With greatest irony, this same pattern-seeking tendency distracts us from understanding the psychoacoustical underpinnings of tonality. Tonality's easy familiarity and lasting power to command listener interest is powered by the pattern-seeking tendency itself, and by each listener's subconscious drive to fit pitched stimuli into the familiar world of that pitched *ur*-pattern, the overtone series.

In this document, I examine modern theories which aim to understand tonality by examining the overtone series, with a particular emphasis on the writings of Paul Hindemith, Ernst Terhardt, and Richard Parncutt. By examining these theorists' writings on the overtone series and on human perception of that series, I hope to provide insight into why tonality is what it is: an enduring musical force with great power to move a listener to sensations of great comfort and stability, and to evoke restlessness, imbalance, and unease when it is temporarily suppressed or absent.

CHAPTER II

OVERVIEW

Viewed against the context of later theorists' writings on the nature of tonality, Paul Hindemith's remarkable assertions on the overtone series and its mandate of tonality constitute an odd digression from scholarly thought about tonality. As Rameau did before him and many theorists would do afterward, Hindemith linked tonality with the overtone series, using the latter's acoustical reality to explain the artistic prevalence of the former.[5] Hindemith extrapolated from the overtone series in order to account for the neotonal dialect he preferred in his own compositions; unsatisfied with the simple octaves, fifths, and simple triads that are readily apparent on the surface of the overtone series, he devised methods to derive ordered, fully-chromatic series from the overtones generated by (and centered around) a single pitch.

If we, armed with hindsight, find fault with Hindemith for the ways in which his reach exceeded his grasp, we should judge not that his grasp was too feeble, but that his reach was too broad. Hindemith's bold description of an overtone series both flexible enough to accommodate neotonality's digressions into

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[5] Hindemith, *Craft of Musical Composition*, *vol. I.*

chromatic *terrae incognitae* and tyrannical enough to demand a specific and proscribed tonal aesthetic by mere virtue of its existence sparked contention at both of these extremes. His critics found fault with both the chromatically inclusive nature of his models and with the idea of a concrete mandate of Nature.[6] These fierce and often technical disputes at both ends of his theory are perhaps an indication that Hindemith would have done well to narrow its scope, asserting neither a mandate from on high, nor an all-inclusive hierarchy which finds a place for each pitch class within the array generated by a single overtone series.

Later explorations of the same terrain benefit from the addition of ideas of learned associations and pattern recognition, relieving much of the strain by shifting conceptual burdens from nature to nurture. According to Ernst Terhardt, the primacy of tonality as an aesthetic and organizational system is inextricably linked to the overtone series, just as Hindemith asserted. However, this linkage is perceptual in nature, a direct result of listeners' learned but subconscious familiarity with the overtone series' widespread presence within most pitched sounds we hear, and owing much of its power to the brain's ability to recognize incomplete instances of a pattern as partial reproductions of an ideal model. This pattern recognition tendency continues to the point of imagining the missing elements required to complete the pattern, automatically creating complete

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[6] Norman Cazden, “Hindemith and Nature.” *Music Review* 15 (1954): 288-306.

overtone series wherever possible through audiation.[7]

This tendency to apply the overtone series to collections of pitches as a template means that our perception of harmony is consistently guided by the series' perceptual power. By recognizing a portion of the series comprised of high partials and extending the series downward to its (not necessarily sounded) generative tone, a listener arrives at a pitch class with great influence over the sounded pitches, because they all fit together into the same template that all listeners are conditioned to apply subconsciously.[8] This phenomenon results in certain pitch classes among commonly encountered pitch class sets (such as the diatonic collection, or individual chords) being perceived as exerting hegemony over the pitch class set at large (as the tonic of a key, or as the root of a chord).

The acquisition of conscious understanding of this perceptual process does much to clarify key tonal concepts by explaining their psychoacoustical derivation.[9] Common analytical tools, such as reordering pitch class collections into stacks of thirds to understand them collectively as a chord, present their own problems (i.e., why thirds, when octaves and fifths are much more consonant intervals, with preferential position within the overtone series and less room for Major/minor compromises?) Recasting our understanding of tonal concepts enables us to do

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[7] Ernst Terhardt, "The Concept of Musical Consonance: A Link between Music and Psychoacoustics," (*Music Perception vol. 1 no. 3*, 1984), 290.

[8] William Thomson, "The Harmonic Root: A Fragile Marriage of Concept and Percept," (*Music Perception vol. 10 no. 4*, 1993), 365-367.

[9] Ibid., 405.

without these convoluted conceptual scaffoldings, allowing us instead to rely directly upon analytical processes that elucidate our perceptions because they mirror our subconscious perceptual processes.[10] These new understandings have potential practical applications in the music theory classroom, allowing students to replace arbitrary or contrived mnemonics with more intuitive comprehension, which are based directly on the psychoacoustic basis of keys and chords.

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[10] Richard Parncutt, “Revision of Terhardt’s Psychoacoustical Model of the Root(s) of a Musical Chord,” (*Music Perception vol. 6 no. 1*, 1988), 67-68.

CHAPTER III

HINDEMITH, NATURE, AND THE SERIES I

Due in part to its natural occurrence within the overtone series and in part due to its compositional utility, Hindemith[11] insisted on the primacy of the major triad, calling it

> one of the most impressive phenomena of nature, simple and elemental as rain, snow, and wind. Music, as long as it exists, will always take its departure from the major triad and return to it. The musician cannot escape it any more than the painter his primary colors, or the architect his three dimensions.... In the world of tones, the triad corresponds to the force of gravity. It serves as our constant guiding point, our unit of measure, and our goal, even in those sections of compositions which avoid it.[12]

In perfect accord with his own assertion of this utmost primacy, Hindemith derived his hierarchies of pitches and intervals from the first six tones of the harmonic series, discarding the "out-of-tune" seventh harmonic and all that follow, removing them from consideration.[13] It is no coincidence that the first six overtones, the only ones Hindemith was willing to consider relevant, are the tones

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[11] Paul Hindemith (1895-1963), a German-born American composer, conductor, theorist, violist, and pedagogue, used his composition and theory to explore a unique neotonal idiom. After being ejected from Germany for his ideological refusal to compromise with Goebbels, Hindemith gained American citizenship and taught at Berkshire Music Center, Yale, Harvard, and the University of Zurich.

[12] Hindemith, *Craft of Musical Composition, vol. I,* 22.

[13] Hindemith, *Craft of Musical Composition, vol. I,* 37-38.

that comprise a major triad whose root is the fundamental of the overtone series in question. If all composition, no matter how chromatic and dissonant, is ultimately rooted in the overtone series, and if the overtone series is so inextricably linked with the major triad, there can be no wondering why Hindemith regarded the major triad as the ultimate source and destination of all music. Because he consistently limited his consideration to the first six tones of the overtone series, whenever he cited the overtone series as the source of his compositional ideas, Hindemith was really sourcing his concepts in the major triad. (This leaves one to wonder whether he professed the primacy of the major triad because all his theoretical ideas proceed from that triad, or whether he based his theoretical ideas on the triad because of a preexisting belief in the triad's superiority.)

Hindemith made occasional but brief references to psychoacoustics, extolling the importance of the major triad as "based on the nature of the ear itself,"[14] but he did not go as far in this direction as his successors would later attempt. Where Hindemith regarded the mandates of tonality as external factors of nature, Terhardt and Parncutt call them internal functions within each listener: not entering the ears, but generated between them.[15] Hindemith praised the ear as the only human sensory organ which cannot be deceived[16], one that unerringly

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[14] Ibid., 23.

[15] Hindemith, *Craft of Musical Composition, vol. I,* 22; Terhardt, "The Concept of Musical Consonance," 286; and Richard Parncutt, *Harmony: A Psychoacoustical Approach* (Berlin: Springer-Verlag, 1989), 48-49.

[16] Hindemith, *Craft of Musical Composition, vol. I,* 23.

recognizes ratios between stimuli in situations where analogous stimuli would confound sight with illusion.[17]

When Hindemith "suggest[ed] a new method for erecting a scale,"[18] he did so by producing two series, one ranking the twelve pitches classes, the other ranking the six interval classes, each ordered by a decreasingly consonant relationship to the pitch class that generates the overtones from which they are all derived. The methods of Hindemith's derivations are described below.

Hindemith's Series I, generated from a single pitch, is produced by two separate means: one for the first six pitch classes, and another for the remaining, more distant six. As shown in figure 1, the first six tones are derived by methodically examining each overtone generated by the original pitch, considering each as if it were a lower overtone in a harmonic series based on a different fundamental, and retaining that fundamental for the series if it occupies the range beginning at our original fundamental and ending one octave above (but not including the octave replica of the tonic). All operations are carried out in a particular order, considering lower overtones before higher overtones. Using the overtone series generated by the C at 64 Hertz, Hindemith produced the order C, G, F, A, E, Eb by these calculations.

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[17] Ibid., 23-27.

[18] Ibid,, 32.

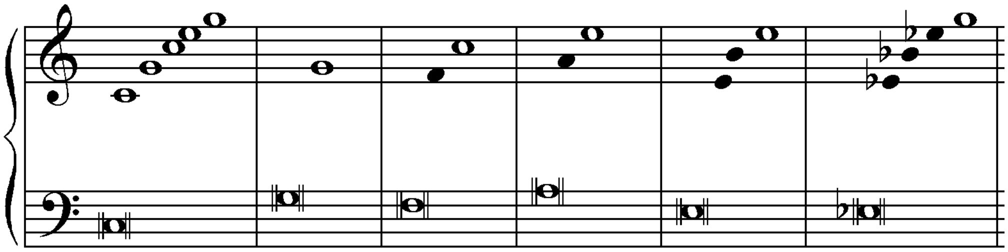

Figure 1: Deriving the first part of Hindemith’s Series I from C. New pitches are added to the series when they share common overtones with C, and pitches whose common overtones are low in the overtone series are added first.

The series begins, arbitrarily, with C 64, the C that generates all the other tones-- it is clearly more closely related to itself than any other pitch could be. It is the first partial in its own series, and it cannot be regarded as a lower partial, since no such partials exist. When one considers the second partial, C 128, as the fundamental of a series beginning on C 128, nothing useful is gained-- it merely duplicates at the octave a pitch class already in the series, and moreover it relies upon a fundamental outside of Hindemith's proscribed range: the octave beginning at C 64, but not including the octave replica of the tonic. Hindemith adds G by reinterpreting the third partial of the C 64 series as the second partial of a series beginning on G 94, a perfect fifth above the generative tonic; Hindemith likewise adds F to the series when C 256 is considered as the third partial above F 85.33. The fifth partial above the fundamental C 64 is E 320, which grants the series A 106.66 by being the third partial above that fundamental, and also E 80 by being the fifth partial two octaves above that pitch. The sixth partial, G 384, provides no non-duplicate pitch classes for our series until Eb 76.8, above which G 384 is the fifth partial. Because this is as far as he can go without resorting to the seventh overtone, which Hindemith places off-limits because of its distastefully questionable intonation, Hindemith must shift tactics to derive the remaining six pitch classes of Series I.

Figure 2 demonstrates how Hindemith fills in the remaining six pitch classes by repeating this process, which he began with C, upon the overtone series generated by the G 94, F 85.33, A 106.66, E 80, and Eb 76.8 in turn, letting C's "children" add "grandchildren" and "great-grandchildren" to this tonal genealogy. As before, only new, non-duplicative pitches within the octave above C 64 are considered. The overtone series based upon G 94 contributes D 72 (D 288 is the third partial of the G 64 series and the fourth partial of D 72). The series based on F 85.33 adds both Bb 113.78 and Db 68.27 (F 341.33, the fourth partial above this fundamental F, is the third partial above this Bb and the fifth above this Db). The overtones of E 80 produce B 120 by means of the partial B 240 which both these pitches share, the overtones of A produce nothing new within range, and the overtones of Eb and Ab produce Gb 92.16, which is rejected for suboptimal intonation, Cb, which is rejected because it is a differently-tuned enharmonic equivalent of the B already present, and Fb 81.92, which is similarly rejected for being a mistuned enharmonic of our previously-derived E. (These enharmonic duplicate near-misses are a consequence of Hindemith's attempts to sidestep the Pythagorean comma, justifying the existence of an equal-tempered chromatic scale despite the incompatibility of the pure, Pythagorean intervals within the overtone series.)

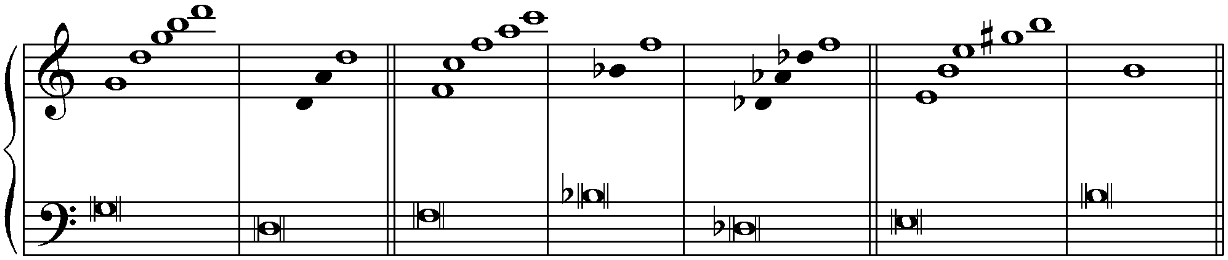

Figure 2: Further pitches of the Series I are added based on common overtones shared with secondary pitches of the series: G, F, and E.

The only gap left is the tritone above the original C, which is arrived at in two separate ways via C's "grandchildren," both shown in figure 3. Gb 91.03 comes from Db 68.27 (they share Db 273.08 in common as a third and fourth overtone, respectively), and Gb 91.02 from Bb 113.78 (they share Bb 227.56, their fifth and second overtone, respectively). As is perhaps appropriate for the pitch class related to the fundamental by this most problematic of intervals, this final and most distant pitch class is the only one that Hindemith must generate through three degrees of harmonic separation, and the precise frequencies of both derivations do not quite agree with each other. As this result is in line with the long-established perception of the tritone as the most dissonant and unstable of intervals, the listener may feel justified in considering this strange result a confirmation rather than a repudiation of Hindemith's methods of derivation.

Hindemith's Series 2 ranks interval classes from most consonant to most dissonant, judged by the properties of the combination tones produced by the simultaneous sounding of both pitches.

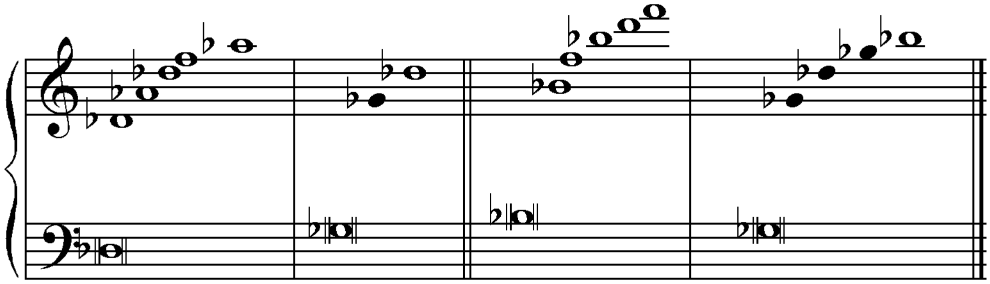

Figure 3: In Series I, the two ways of arriving at the tritone above the generative C: from the Db and from the Bb.

Hindemith's reliance upon combination tones is both unique and controversial; later theorists have called into question the perception-shaping power of tones which are inaudible under most circumstances. However, Hindemith's work with combination tones has curious resonance with the later psychoacoustic studies of Terhardt and Parncutt. Although these latter two theorists eschewed combination tones and looked instead to overtone series patterns filled in by the subconscious pattern recognition of each listener's ear, all three theorists found psychoacoustical support for tonality in the form of phantasmal bass pitches, generated by deliberately sounded notes but not deliberately sounded themselves, which offer harmonic support by reinforcing structurally important pitch classes from a low register.

CHAPTER IV

TERHARDT, SCHEMA, ROUGHNESS, AND ROOT SUPPORT

Ernst Terhardt's[19] psychoacoustic tonal conceptions rely even more heavily upon the overtone series than Hindemith's. Significantly, Terhardt asserts that the most musically relevant overtone series is not any series generated from any audible pitch, but rather the model overtone series in the mind of the listener. According to Terhardt, listeners grow accustomed to the overtone series through extensive exposure to that series as generated by each and every complex tone the ear perceives, especially including the vowel sounds of spoken language. This subconsciously memorized overtone model, or schema, is the template against which perceived sounds are constantly checked, and the brain fits pitch patterns to this template whenever possible, even to the extent of audiating pitches and overtones not actually present, in order to complete the pattern and repair an incomplete series. Even elements that suppress or eliminate a portion of the overtone series (such as telephone connections, low-quality speakers, or low-pass filters) do not cause listeners to become unaware of the true pitch of these altered

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[19] Ernst Terhardt is a retired professor of audio-communication with the Institute of Electroacoustics of the Technical University, Munich. A prolific researcher and writer, Terhardt explored speech, music, and the common communicative processes in each.

tones, or to perceptually transpose them to incorrect registers; the mind readily corrects for missing overtones by filling in missing fundamentals, keeping pitch perception intact.

Instead of extrapolating from overtones a tonal hierarchy of interval classes, as Hindemith did, Terhardt speaks of intervals not in terms of a Series II ranking, but in terms of varying degrees of roughness. Simultaneities of tones are considered relatively consonant or dissonant based on their roughness, a function of musical annoyance. Within a certain range, beats produced by pitches in close proximity cause actual sensory irritation by virtue of rapid amplitude changes in the composite waveform. The total roughness within an interval is examined by considering the degrees of roughness produced by the fundamentals and all their overtones, which is why odd configurations of sine waves and other pure tones can be more easily made to sound pleasant than similar configurations of complex tones-- the pure tones are not trailing nets of overtones to get tangled up in each other.[20]

If this is the case, humans may be alone in their use of the overtone series to distinguish consonance from dissonance in this way. Neuroscientist Josh McDermott conducted an experiment to see whether monkeys display preference for traditionally consonant or traditionally dissonant intervals. McDermott placed monkeys in a maze full of speakers whose output was controlled by the position of

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[20] Terhardt, “The Concept of Musical Consonance,” 284.

the monkeys within the maze. By choosing where to linger, the test subjects could choose whether to hear octaves, perfect fifths, minor seconds, minor ninths, or tritones: they showed no preference.[21]

Notably, this system places the entire concept of consonance and dissonance within the realm of the mind, discounting it as an external feature of raw sound, or as any product of nature (in the sense that Hindemith used the word). Instead, in this roughness-based model, dissonance is the result of the listener's subconscious awareness that the sensory irritation caused by beat interactions between overtones has crossed a line into the realm of unpleasant roughness, even though the irritation-causing beats in question, like the overtones that produce them, may never be audible to the conscious listener.

An experiment conducted in 1986 by Vanderbilt University psychology professor Randolph Blake ranked and studied the annoying qualities of sounds, with particular attention paid to the sound of fingernails on a chalkboard. Alterations made to the sound with audio filters disproved one of Blake's first hypotheses: that auditory roughness in the high frequency range was the source of the sound's irritating characteristics. In truth, the sound became much more bearable when a band-reject filter eliminated frequencies between 500 and 2,000

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[21] J.H. McDermott and A.J. Oxenham, "Music Perception, Pitch, and the Auditory System," (*Current Opinion in Neurobiology*, *vol. 18*, 2008), 452-463.

Hz, comfortably in the middle of the range of human hearing.[22]

Further investigation into natural auditory irritants within the human vocal frequency range indicates that a human scream has more energy at and around 3,000 Hz—a particularly sensitive frequency, due to the architecture of the human ear—than any other sound humans make.[23] Evolutionary hypotheses account for increased irritation from roughness within this frequency range by invoking similarity to primate screams indicating danger, and to instinctual hearing protection by means of an aversion to frequencies both highly damaging and highly unpleasant.[24] If the human capacity to be irritated by auditory roughnesses is concentrated in this frequency range, those higher frequencies—as Blake discovered contrary to his original expectations—contribute surprisingly little to the human brain's aversion to sounds. Terhardt's theories involving dissonance of harmonic intervals being sourced in roughness between overtones then serves to neatly explain why many intervals traditionally understood as dissonant seem much more consonant when played in a high register: their clashing overtones, which are higher still, are either above the frequency range where listeners are most prone to perceive them as sensory irritants, or they are above the listeners' frequency thresholds altogether—too high to be heard, even subconsciously.

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[22] Joe Palca and Flora Lichtman, *Annoying: The Science of What Bugs Us* (Hoboken: John Wiley and Sons, 2011), 48-49.

[23] Ibid., 50-51.

[24] Ibid., 51-54.

McDermott's further experiments with primates reset his above-described sound preference testing maze, replacing the choice between consonant and dissonant intervals with a choice between the sound of fingernails on a chalkboard and white noise. The monkeys displayed no preference in this matter, either, contradicting notions of auditory roughness being related to primate warning cries.[25] McDermott's alternate hypothesis to explain the unpleasantness of scraping fingernails returns us to the principle of auditory roughness in the form of rapid amplitude fluctuations.[26] These types of fluctuations can be produced by the interference of sound waves whose frequencies vary by between 20 and about 75 Hz[27]-- exactly the interactions between overtones that Terhardt's theory of dissonance via roughness relies upon.

Interestingly, this idea of interval roughness provides a certain roundabout support for certain pillars of tonality, not as a mandate of the overtone series, but as a likely (perhaps inevitable) consequence of the collision between the overtone series and the easily-annoyed brain. Maximum roughness is generated, predictably, by seconds and sevenths, and by their compound interval variants. Only one set class of cardinality three contains no such rough intervals but permits the occurrence of the especially stable perfect fifth. This set class is, naturally, [0 3

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[25] Palca and Lichtman, *Annoying*, 56-57.

[26] Ibid., 57-58. Automobile manufacturers also look for waveforms with rapid amplitude fluctuations when working to eliminate unpleasant noises their vehicle prototypes produce while in operation. In this case, a rough envelope is a primary signifier of these problematically annoying sounds.

[27] Ibid., 57-58.

7]: both the major and the minor triad.

Terhardt asserts that notes and chords drop their harmonic anchors when listeners, conditioned by a lifetime of exposure to the harmonic series, subconsciously apply pattern recognition to the sounded pitches they hear, considering each pitch as if it were an overtone and extrapolating down into the bass to find the "fundamental."[28] This process is demonstrated in figure 5, as contrasted from the overtone series in figure 4.

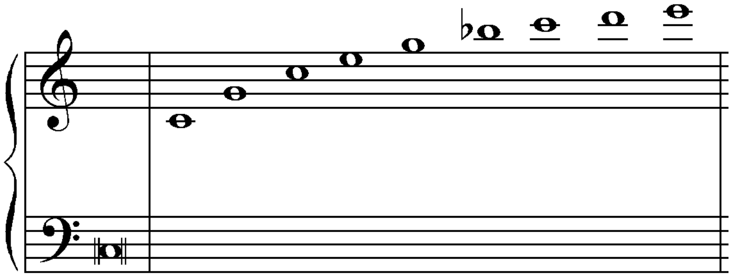

Figure 4: The overtones generated by a given fundamental.

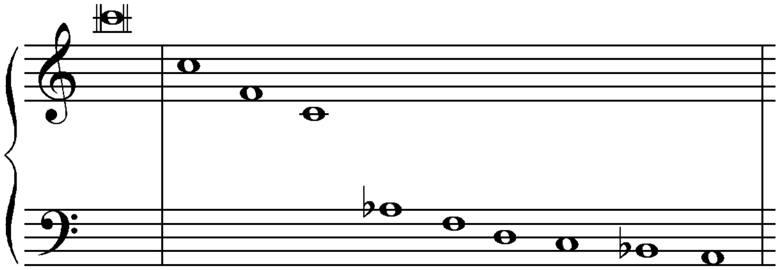

Figure 5: The potential fundamentals which could generate a given pitch as an overtone.

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[28] Terhardt, “The Concept of Musical Consonance,” 287-291.

Rather than being a product of centuries of cultural accretion, harmony is a "psychoacoustically based phenomenon that accompanies the perception of complex signals, in particular, speech;" a byproduct of the human ability to recognize the correct virtual pitch of a tone even when some of the overtones are missing or obscured.[29] According to Terhardt's system, the reason the root of a major triad is the pitch class of greatest primacy within the chord is not because of its position within a stack of thirds, but because it is the higher-octave duplicate of a lower pitch that, if sounded as a fundamental, would produce all the chord members as overtones. Figure 6 demonstrates.

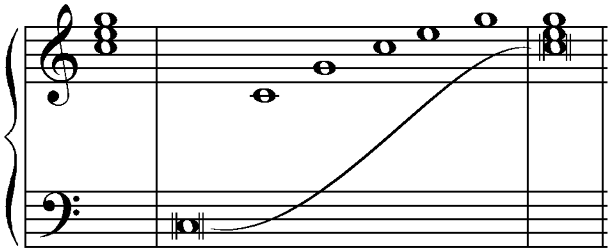

Figure 6: The ear recognizes the major triad as a subset of the overtone series, establishing a chord root by filling in the missing fundamental.

Because this system of determining harmonic roots by pattern-matching pitches against the learned overtone series accounts only for chords that are neatly contained within the overtone series, Terhardt proposes a modification to accommodate other chords. Using the first ten overtones, Terhardt extrapolates

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[29] Terhardt, "The Concept of Musical Consonance," 288.

several harmonic series down from each pitch class present in the chord, treating each chord member in turn as if it were an octave, a fifth, a major third, a minor seventh, and a major ninth above a number of potential fundamentals, as shown in figure 7.

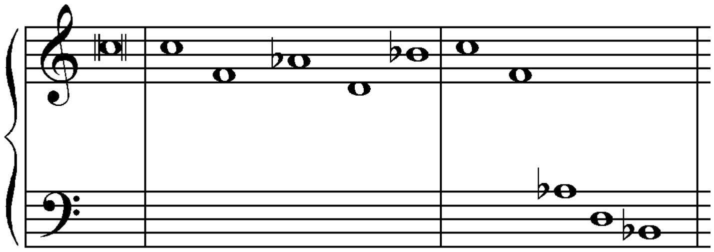

Figure 7: The pitch classes that support a given chord tone.

These potential fundamentals are candidates for the root of the chord, because they are the notes that could generate their associated chord members within the first ten overtones of harmonic series they generate. Simply put, the root of any given chord is the pitch class that shows up as a potential fundamental generating the largest quantity of that chord's tones. Figure 8 demonstrates this process for the major triad.

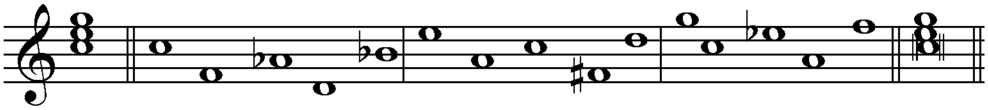

Figure 8: Using Terhardt's model to calculate the root of a major triad. Five supporting pitch classes are generated from each chord tone. For this chord, C is the root because each chord tone generates it as a support.

Unfortunately, this algorithm sometimes produces aberrant results, and in all cases returns a single pitch class as an unequivocal chord root, failing to take into account ambiguous roots or context-sensitive changes in perceived rootedness (such as the chord F A C D, the root of which is either clearly F or clearly D, depending on the tonal and harmonic context in which it appears, as well as its voicing).

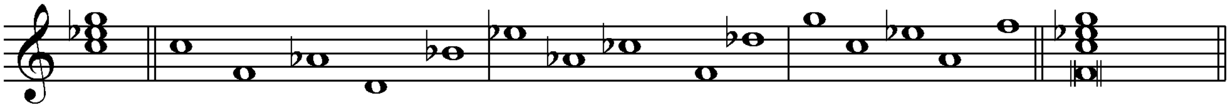

Figure 9: Terhardt's model erroneously returns F as the root of a C minor triad. All three chord tones receive support from F, but only C and G are supported by C.

Another significant defect in the Terhardt model involves its inability to determine the correct root of the minor triad. Although this method correctly returns the expected root of a major triad by means of a process that mirrors Terhardt's psychoacoustical ideas about the overtone series, the root it asserts for a minor triad does not match listener expectations at all, as shown in figure 9. The Terhardt model predicts the root of a minor chord to be a perfect fifth below conventionally-understood root (i.e., asserting that the root of a C minor triad is F, etc.), as if each minor triad were in actuality a dominant ninth chord with the root and third omitted.

As a result, anyone using the Terhardt model is obliged to set it aside when discussing these anomalies, resorting to familiar workarounds employed to make sense of the minor triad since Rameau.[30] The major triad fits neatly into the model while the minor does not, so the minor triad must be thought of as a variant of the major triad: an altered form, more distant from the overtone series. This idea carries with it the implication that the minor triad is inferior to the major. Though the contrasting emotional connotations of major and minor harmonies and tonalities are well established, no consistently documented perception or psychoacoustical data supports the notion of the minor triad as less important than the major.[31]

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[30] Parncutt, *Harmony,* 69.

[31] A 1987 study cited by Parncutt interestingly notes a correlation between a preference for minor tonality and oral dependency, associated with a desire to be nurtured and supported, but this would seem a dubious, roundabout method by which to justify suborning the minor triad to the major.

CHAPTER V

PARNCUTT, PRENATAL CONDITIONING, AND WEIGHTED ROOTS

Richard Parncutt's theories of musical conditioning assert that the overtone series is merely the first important musical pattern among many, and that any listener's understanding of music involves a process of pattern-recognition with the most emphasis placed upon the most familiar musical structures.[32] As the listening brain sorts through its catalogue of recognized patterns, "music may be regarded as a multilayered structure of more or less familiar patterns, and its perception seen as a multilayered process of pattern recognition, based on familiarity."[33] This definition acknowledges both naturally-occurring patterns such as the overtone series as well as arbitrary, composed patterns prevalent in the musical style of a culture.

The patterns with the greatest influence over a listener's perceptions—the ones most fervently sought by the subconscious ear—are the patterns most familiar to the listener. Parncutt further asserts that individuals begin acquiring

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[32] Richard Parncutt (1957 -) is a Professor of Musicology at the University of Graz in Austria. He studied music and physics at the University of Melbourne, earned his PhD at the University of New England, Austrailia, and researched with Terhardt in Munich. Dr. Parncutt also founded the Journal of Interdisciplinary Music Studies and the Conference on Interdisciplinary Musicology series.

[33] Parncutt, *Harmony*, 49.

familiarity with audible patterns *in utero*, resulting in listeners who, throughout their lives, give increased emphasis to patterns frequently encountered in the womb, notably the overtone series as experienced through speech (particularly the voice of the mother).[34]

If these theories of prenatal conditioning are correct, they account for the apparent universality of certain aspects of harmonic perception, particularly the prevalence of the octave and perfect fifth in music across cultures, since these are derived directly from the overtone series, and since listeners gain familiarity with this series prenatally, before gaining familiarity with other musical stimuli. There is much more variance in the perception of other aspects of harmony as a result of various secondary, culturally driven patterns all listeners hear throughout life, including stock chord progressions, cadences, and other compositional traditions. These learned associations can influence but not displace the underlying pattern of the overtone series, learned prenatally from complex tones, because it predates all other harmonic patterns in the listener's subconscious memory. As a result, there exist a wide variety of musical styles and traditions which share certain limited and simple core elements.[35]

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[34] Parncutt also discusses the effects of other prenatal stimuli upon musical understanding, including the footsteps of the mother as a predecessor of rhythm. Uneven fetal positioning is credited with laying a groundwork for duple meter preference because either the left or the right foot is perceived as louder, and the natural slowing of walking pace as a destination is approached is an analogue for end-of-phrase rubato.

[35] Parncutt, *Harmony*, 49-50.

It has been demonstrated that infants are sensitive to the octave, fifth, and third, to melodic contour, and to other basic musical elements.[36] Further, it has been repeatedly shown that newborns of many species are soothed and comforted by stimuli, especially audible stimuli, that are similar to sensations they have experienced prenatally.[37] Because an innate understanding of basic building blocks of melody and harmony is not associated with survival value or evolutionary advantage and is therefore unlikely to be instinctual, it must be learned. Because it is learned so early in development, it lays the foundation of auditory perception and even musical preference throughout life.[38]

For the purposes of being able to understand harmony, Terhardt cites the importance of being able to perceive a complex tone as a single entity, rather than as a simultaneity of partials, each one a distinct musical event. If Parncutt is correct in his supposition that this ability is learned at some prenatal stage from the voice of the mother, there must exist a previous prenatal period during which the fetus perceives each complex tone as a stack of separate partials, a chord of simple tones arranged according to the lower and more easily audible partials of the

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[36] Ibid., 51. Parncutt cites a 1987 study to this effect by S. E. Trehub, published in *Attention, Perception, & Psychophysics*, vol. 41.

[37] Ibid., 51-56. Lately, toymakers have exploited this property by producing stuffed animals which contain speakers programmed to play soothing sounds. By no coincidence do these soothing sounds resemble the rush of blood through uterine capillaries.

[38] Ibid., 51-52.

overtone series.[39] This realization lends strong support to the assertions of many previous theorists concerning the fundamental importance of the overtone series or the major triad, though this support may come from an unanticipated direction. Schenker's notion of the overtone series as "the chord of nature" fits perfectly with this perceptual reality. If one takes the minor leap of understanding "nature" to be the prenatal onset of sensory ability, as does Hindemith's extolling of the major triad, as presented in the lowest six partials of any complex tone, as "one of the most impressive phenomena of nature, simple and elemental as rain, snow, and wind."[40]

Like Terhardt, Parncutt uses the overtone series as a tool to determine the root of a chord, on the basis that subconscious familiarity with the overtone series is actually the cause of a chord's perceived rootedness. However, while treating each chord tone as a partial and extrapolating several fundamentals which could have generated an overtone series containing it, Parncutt rates each possible fundamental according to the closeness of its relationship within the series to the chord tone from which it is derived. This means, for example, that the presence of pitch class A within a chord might suggest both D and B among several possible roots, but D is more probably the root since A is a lower partial on its overtone series than it is on B's overtone series. In this way, the lower overtones within the series are prioritized, having a stronger effect on root calculation than higher

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[39] Parncutt, *Harmony*, 51.

[40] Hindemith, *Craft of Musical Composition, vol. I,* 22.

overtones. This result is appropriate both perceptually, because lower overtones are usually louder and more apparent, making them more likely to affect the process of understanding harmony, and conceptually, because lower overtones are related to their generative fundamentals by more harmonically stable intervals than higher overtones, except for higher overtones that are octave duplicates of pitches that occur lower in the series.

By means of a deft system of scaling, Parncutt rebalances the contributing forces attributed to each supporting interval around each pitch class present in the chord, so that the five prospective roots (below each chord-constituent pitch class, in order from strongest to weakest: the octave, the perfect fifth, the major third, the minor seventh, the whole step) are assigned the strengths $1/n$, where n represents the integers one through five, in turn. A simple table[41] is used to total each pitch class's suggested root strength, as contributed by all chord members; the highest value indicates the most probable root. This revision is all that is necessary to correct some of the most glaring anomalies from the Terhardt model (most notably the case of the minor triad, whose root the model predicts a fifth below the traditionally understood root of a minor triad). On the next page, table 1 compares the Terhardt and Parncutt models when used to examine the minor triad.

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[41] In these modern times, it could very easily be automated with a spreadsheet or very simple computer program.

Table 1: Comparing the Terhardt model to the Parncutt model, by examining the C minor triad. As shown, Terhardt's model predicts an incorrect result for a minor triad's root, but by weighting the support intervals, Parncutt arrives at the result listeners expect.

Minor Triad C-Eb-G (Terhardt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	C			Eb				G				
P5	G					C			Eb			
M3				G					C			Eb
m7			C			Eb				G		
M2		Eb				G					C	
Number of Supports	2	1	1	2	0	**3**	0	1	2	1	1	1

Minor Triad C-Eb-G (Parncutt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	1.00			1.00				1.00				
P5	0.50					0.50			0.50			
M3				0.33					0.33			0.33
m7			0.25			0.25				0.25		
M2		0.20				0.20					0.20	
m3	0.10				0.10					0.10		
Weight of Support	**1.60**	0.20	0.25	1.33	0.10	0.95	0	1.00	0.83	0.35	0.20	0.33

Another feature of the Parncutt revision of Terhardt's model is the allowance for the possibility of multiple, ambiguous root options, and the quantitative comparison of the relative strengths of multiple root candidates. When examining chords with ambiguous roots, the table results show two or more pitch classes with near values at the highest range. (Conversely, the table results also indicate, in the case of chords with relatively unequivocal roots, just how strongly those roots are asserted, by laying bare the relative weakness of a root's competition from other pitch classes.) The formula produces predictably ambiguous results for chords whose roots vary greatly depending on voicing and context, such as all qualities of diminished sevenths, and the aforementioned major triad with added sixth.

Table 2 uses two chords to compare the Terhardt and Parncutt models. Probable roots predicted by the models have their total values displayed in bold text. Note the ways that the Parncutt model both reintroduces likely root candidates that the Terhardt model fails to consider and discounts unlikely roots that the Terhardt model unduly prioritizes.

Table 2: Parncutt’s model predicts multiple possible roots for ambiguous chords. Note how for the chord F-A-C-D, Parncutt’s model actually predicts fewer roots than Terhardt’s model offers, eliminating the less probable G and Bb.

Half-Diminished Seventh Chord D-F-Ab-C (Terhardt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	1		1			1			1			
P5		1				1		1			1	
M3		1			1				1		1	
m7			1		1			1			1	
M2	1			1			1				1	
Number of Supports	2	1	2	1	2	2	1	2	2	0	**4**	0

Half-Diminished Seventh Chord D-F-Ab-C (Parncutt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	1.00		1.00			1.00			1.00			
P5		0.50				0.50		0.50			0.50	
M3		0.33			0.33				0.33		0.33	
m7			0.25		0.25			0.25			0.25	
M2	0.20			0.20			0.20				0.20	
m3			0.10			0.10				0.10		0.10
Weight of Support	1.20	0.83	**1.35**	0.20	0.58	**1.60**	0.20	0.75	**1.33**	0.10	**1.28**	0.10

Ambiguous Chord F-A-C-D (Terhardt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	1		1			1				1		
P5			1			1		1			1	
M3		1				1			1		1	
m7			1		1			1				1
M2	1			1				1			1	
Number of Supports	2	1	**3**	1	1	**3**	0	**3**	1	1	**3**	1

Ambiguous Chord F-A-C-D (Parncutt Model)												
Root Candidate	**C**	**Db**	**D**	**Eb**	**E**	**F**	**Gb**	**G**	**Ab**	**A**	**Bb**	**B**
P1	1.00		1.00			1.00				1.00		
P5			0.50			0.50		0.50			0.50	
M3		0.33				0.33			0.33		0.33	
m7			0.25		0.25			0.25				0.25
M2	0.20			0.20				0.20			0.20	
m3			0.10				0.10			0.10		0.10
Weight of Support	1.20	0.33	**1.85**	0.20	0.25	**1.83**	0.10	0.95	0.33	1.10	1.03	0.35

CHAPTER VI

CONCLUSIONS AND APPLICATIONS

Developing understanding of how the brain processes vertical harmonies provides powerful new tools for designing music teaching methods. Students should benefit from pedagogical techniques that teach them to consciously and cognitively understand harmony in the same ways that they already understand it subconsciously. Pedagogical models founded on cognitive and perceptual functions will suggest intuitive and satisfying answers to the “why” questions often asked by advanced and particularly inquisitive students, and should lay a useful foundation for students who go on to study these musical concepts in greater detail in more advanced classes.

Contrast this approach with the commonly used shortcuts employed in music classrooms for beginners and undergraduates. Students are often taught to rely on simplistic tasks, such as organizing a chord into a stack of thirds and considering the lowest pitch class in the stack to be the root. These devices are unrelated to the reasons and causes that underlie the very concepts those methods are employed to examine. Thirds do not cause a particular pitch to be perceived as a chord’s root; a given pitch class is not perceived as the root of a chord merely because of the thirds stacked above it. The interval of the third itself is of peripheral importance to the concept of a root; as both the Terhardt and Parncutt models discuss in depth, octaves and fifths do much more to impart the sense of stability associated with the root, because of their lower position within the overtone series. The chief redeeming value of the stack of thirds method is that it allows students to calculate chord roots quickly, producing accurate results under most circumstances. The danger of this method is that over the course of the countless chord identifications and calculations students perform, especially during the early phases of their studies when their grasp of fundamental concepts is unfixed, students will create such an association between thirds and roots that they will incorrectly believe that chord roots are directly caused by the interaction of thirds. Not realizing that they are simply exploiting a useful coincidence because it facilitates a convenient computational shortcut, students may incorrectly intuit a cause and effect relationship where none exists, and this false association will inhibit their understanding of the actual workings of vertical harmony and chord roots.

Instead, building the curriculum to include teaching the Parncutt model would solve several problems in the undergraduate theory classroom. Because its method of calculating chord roots mirrors the subconscious mental process that causes the ear to hear certain chord tones as roots, answers to inquisitive students' "why" questions would be built directly into the curriculum, and addressing those students' concerns would not seem like a semi-relevant tangent or digression from the task at hand. This method of teaching chord roots would fit directly into other important theory concepts, like the unique properties and stable role of perfect intervals, and the nature of complex tones and the concept of the overtone series, from which spring all of the previous topics. Teaching via the Parncutt model that even such basic concepts as the idea of chord rootedness are direct results of the overtone series would contribute to an overall sense of order and unity in the subject matter by allowing students to see the connections, through overtones, of disparate subtopics within the theory classroom. Finally, of course, students already conversant with the Parncutt model and the psychoacoustic concepts of the schema and pattern recognition that support it would be well prepared to continue their musical studies, pursuing advanced topics in theory with a confident grounding in concepts of harmony and psychoacoustics that are already familiar to them.

The ideal pedagogical sequence would present the overtone series as a foundational concept before discussing chords. Students should be familiar with the overtone series as a pervasive property of sound and aware of its presence even when studying topics that, on the surface, do not seem to be directly related to the overtone series. Placing an introduction to the overtone series towards the beginning of a music curriculum would prime students to consider and accept overtone-related explanations for the musical phenomena they would subsequently study in, at least initially, much greater depth. The principles supporting both the Terhardt and Parncutt models could then be used to introduce the concept of chords, demonstrating to students that a chord may be thought of as supported by its root by means of the overtone series, and that a chord's root is the pitch class most capable of generating all the other chord tones as overtones. Examples may be used to walk students through the process of determining chord roots in this way, so that they are familiar with the process.[42]

After this foundation has been established, students could be shown the stack-of-thirds method as a shortcut, allowing them to bypass the many steps of the psychoacoustically-based models. Secondary introduction of this concept would underscore its indirect relation to the concepts being studied; it can be viewed as merely a convenient tool with no direct connection to the phenomena it dissects, because those phenomena and their workings have already been explored by students at the outset. Later, as more unusual chord types are introduced, the Parncutt model could be revisited as a more powerful diagnostic tool. Because of their early initiation into the concept of the overtone series and its role in the perception of chords, students should grasp the importance of the model and its relationship to music cognition more readily than they would have if they were encountering these ideas for the first time.

When students, as music listeners, already address a musical phenomenon

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[42] Were it not for the commonly-occurring and problematic minor triad, I would suggest using the Terhardt model to introduce these concepts to new students. The Terhardt and Parncutt models both include the same core concepts, and since my concern here is to introduce students to those concepts, I would just as soon postpone the introduction of Parncutt's increased calculations. Further engineering may yet produce an simplified, introductory model which streamlines the required calculations but still returns correct results for most common chords.

by means of a certain subconscious process and are then asked to address the same phenomenon with a conscious and scholarly examination, this second, deliberate process of comprehension should mirror the preexisting, subconscious process. The more we learn about the ways in which the brain understands music, the more tools we will have available to develop pedagogical methods which build upon that subconscious groundwork. Such well-founded methods should produce students whose understanding of musical concepts is deeper, more comfortable, and more lasting.

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