Competitive Location and Entry Deterrence in Hotelling's Duopoly Model

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Abstract:
This paper analyzes the problem of two firms competing in a common linear market with demand distributed continuously over the market. The firms wish to maximize their respective profits by appropriate choice of number of facilities and their locations. Equilibrium location strategies are derived for uniform and symmetric triangular demand distributions. It is shown that pioneering advantage may help a firm overcome its cost disadvantage with respect to a competitor.

Keywords: Competitive location theory, duopoly model, Stackelberg games, entry deterrence, pioneering advantage

Article:

1. INTRODUCTION

Competitive location problems were introduced by Hotelling (1929), who studied the problem of two firms selling a common product on a common linear market. In this seminal work, Hotelling concluded that at equilibrium the two firms would locate close to each other at the center of the market — a result thus dubbed 'the principle of minimum differentiation'. Hotelling's work spawned a plethora of similar works on competitive location models — for example, Lerner and Singer (1937), Rothschild (1979), Eaton and Lipsey (1975), Prescott and Visscher (1977), De Palma *et al.* (1985). D'Aspremont *et al.* (1979) showed that Hotelling's result does not hold if the price of the product is made a decision variable, as intense price competition, created by both competitors locating arbitrarily close to each other, results in lack of equilibrium. In De Palma *et al.* (1985), it was established that Hotelling's results hold when the consumers' tastes are sufficiently heterogeneous. Several models also considered oligopolistic situations — for example, Eaton and Lipsey (1975) and Lerner and Singer (1937), where it was shown that with three firms competing on a linear market, there may not be an equilibrium. However, in the same paper, Eaton and Lipsey derived conditions for equilibrium for any arbitrary number of competing firms, other than three. Other models were considered, with additional parameters; see, for example, Eiselt (1990). Comprehensive surveys of all such models in competitive locations can be found in Graitson (1982), Schmalensee and Thisse (1988) and Eiselt *et al.* (1993).

Two observations can be made about most of the models that exist in the literature: they generally assume that each competing firm locates exactly one facility, and they investigate existence, or lack thereof, of Nash equilibrium. A notable exception to the first assumption is Bonanno (1987); however, the main results in the paper are only for a few fixed number of facilities. Two well known exceptions to the second observation are Prescott and Visscher (1977) and Rothschild (1979); both study Stackelberg equilibria in their models. In their paper, Prescott and Visscher describe firms seeking Stackelberg solutions as those with 'foresight' and consider two examples where several firms independently enter the market in a given sequence and derive conditions for Stackelberg equilibrium.

In this paper, we study a competitive location problem with features that are common to Bonanno (1987) and Prescott and Visscher (1977). It can be described briefly as follows. There are two firms, designated by F1 and

F2, respectively, that are vying for a common market by opening new facilities. Both firms wish to maximize their respective profits. Assuming that one of the firms decides to locate all of its facilities first, and then the rival firm locates all of its facilities next, we investigate optimal location strategies for the two rival firms — where the term location strategy refers to the number of facilities located by the firm and their respective locations. We also seek solutions to a repeated version of this game, where each firm is given multiple turns to locate its facilities.

As in Bonanno (1987), we also assume that the competing firms are not restricted to locate only one facility each and similar to Prescott and Visscher, we seek Stackelberg solutions in our model. In contrast to the model in Bonanno (1987), we allow the setup and marginal costs of the two firms to be different and do not limit the number of facilities located by any firm. In contrast to Prescott and Visscher (1977), firms are allowed to locate multiple facilities, and we also study the repeated version of the single turn Stackelberg game associated with the model. Thus the model investigated can be viewed as a different version of the Hotelling problem, one in which both firms are assumed to have infinite foresight and are allowed to decide both the number of facilities to locate and their specific locations. This allows an exploration of a wider range of strategic responses by the planning firm and derivation of some interesting results. Our model exhibits the phenomenon of entry deterrence that enables the firms to get more profit by locating multiple facilities. It is also shown that instances of market failure, such as the first entry paradox, demonstrated in Ghosh and Buchanan (1988), do not occur in this setting.

We begin with a detailed description of the basic assumptions and parameters of the model. Some more assumptions and parameters that will be needed in special versions of the problem will be explained in the course of the paper.

Assumptions. The following assumptions are made by the model. (i) The market is assumed to be one-dimensional and bounded, and therefore, without loss of generality, the [0, 1] line segment, with the demand being inelastic and continuously distributed over the market. Two types of demand distributions will be considered in the paper: the uniform demand distribution and the triangular demand distribution that is symmetric about the center of the market (see Fig. 2). (ii) As in Eaton and Lipsey (1975), Rothschild (1979) and the basic models of De Palma *et al.* (1985), we also assume that both firms deal in a homogeneous commodity and charge the consumers a fixed mill price, which is the same for both firms. De Palma *et al.* (1985) suggest this assumption to reflect the hypothesis of homogeneity proposed by Hotelling and continued by his successors. Hence, assuming a fixed unit cost of transportation that is the same for all consumers, a consumer patronizes the firm with the cheapest delivered price, i.e. the closest facility. (iii) It is assumed that both firms cannot locate at the same point in the market at the same time. The analysis in the paper assumes that F1 is the first entrant into the market. The analysis for the other case where F2 is the first entrant is similar and is therefore not done separately. (iv) Each firm has perfect information on the fixed and marginal costs involved, both for itself and also for its competitor. Taken together with assumption (ii), this assumption therefore refers to a stationary, homogeneous product market with informed participants. Thus these characteristics presume a mature product market, where operational and transactional costs are commonly known to all firms. (v) In planning its location strategy, F1 assumes that F2 is unhampered by budgetary restrictions and hence, will locate as many facilities as are necessary to maximize its own profit. Given this, F 1 will locate its facilities only if its own optimal profit is nonnegative.

Parameters. We denote any single point in the market by x or y where $0 \leq x, y \leq 1$. Without loss of generality, we assume that the total demand of the market has been normalized to one unit. The firms are assumed to charge a mill price of p per unit of the product sold. C_i, where $i = 1, 2$, is assumed to denote the fixed cost of locating a facility by firm i and V_i, the corresponding marginal cost of production per unit of the firm. Hence the marginal profit of firm i on every unit sold is $(p - V)$, and consequently the break-even sales of firm i, which we denote by B_i, are given by $(C_i)/(p — V_i)$. Since the total demand is assumed to have been normalized, in the special case when the demand distribution is uniform, B_i also represents the length of the market that a facility of firm i needs to serve to break-even. To eliminate trivial instances of the problem, it will be assumed that marginal profit of each firm is positive and the break-even sales are less than the total demand of the market.

Taken together, assumptions (i)—(v) characterize the market backdrop of this paper. Customers are located or dispersed continuously over a line segment and the distribution of these customers maps into the demand distribution. Customers are assumed to be identical in all respects other than their location, e.g. with regard to transportation costs, willingness to pay, etc. The two firms market identical, i.e. undifferentiated, products. The only variable they control is the number and location of their distribution outlets. These assumptions can also be interpreted as setting the market context for a product line selection problem as opposed to a location problem. In this perspective, consumers are viewed as being heterogeneous in their preference for a single attribute, e.g. quality, service etc. Their location on the line segment is a reflection of their ideal point for that attribute. The distribution of demand can then be interpreted as mapping the distribution of ideal points or preferences. Firms can then be viewed as choosing the optimal assortment of products to offer to this market.

The remainder of the paper is organized into four sections as follows. In the second section, certain properties of the optimal location strategies are stated and proved. The single turn Stackelberg game is solved with F1 as the leader and F2 the follower, assuming a uniform distribution of demand. The third section deals with the repeated version of this Stackelberg game. The fourth section reconsiders the Stackelberg game of the second section, in the case where the demand distribution is symmetric triangular. The fifth section summarizes the conclusions of the paper.

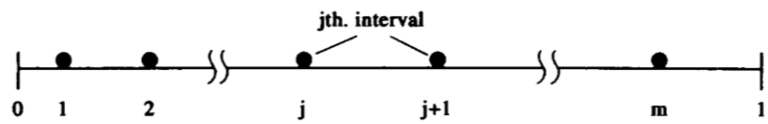

Fig. 1.

2. SINGLE TURN CASE — UNIFORM DISTRIBUTION

This section deals with the most basic version of our model, namely the following problem: assume that F1 is given the option of entering the market and then, after F1 has taken its decision, F2 is given the option to locate its facilities. Given this, it is desired to find the optimal locations strategies for F1 and F2. In the terminology of game theory, this is the single turn Stackelberg game with F1 and F2 as leader and follower, respectively.

We introduce some notation that will be used in this section. An interval is defined as a segment of the market between two adjacent facilities. When an interval contains either of the points 0 or 1, it is called a terminal interval, otherwise an interior interval. An interval is said to belong to firm i if all the customers in that interval patronize a facility of that firm. Assuming that there are m facilities and that they are numbered from the left to the right, the interval between the jth and $(j + 1)$th facility is referred to as the jth interval (note that the terminals 0 and 1 are referred to as the 0th and the $(m + 1)$th facility, respectively), as shown in Fig. 1. An entry-deterring interval, or e.d.i., of F1 at a point x, where $x \in [0, 1]$, is defined as an interval of maximum length between two adjacent facilities of Fl, the leftmost located at x, such that F2 does not have any incentive to locate in between.

As with any Stackelberg game, the first issue to be resolved is the optimal location strategy of F2, given a fixed locational configuration of Fl. That is done in the following lemma.

Lemma 1. Given any arbitrary demand distribution, F2 locates at most two facilities in any interval of F1. Moreover, when the demand distribution is uniform, F2 locates either 0 or 1 in a terminal interval and 0 or 2 facilities in an interior one.

Proof. Since the market is one-dimensional, F2 can take away any interval of F1 by locating two facilities, each of which is arbitrarily close to the two adjacent facilities of F1 that define the endpoints of this interval. Hence F2 does not need to locate any more than two facilities in any interval of Fl, and that proves the first statement.

When the demand distribution is uniform, if F2 locates one facility inside an interior interval of Fl, say the jth interval of length L, it takes away an interval of length $L/2$ from F1. In order for F2 to do so, $L/2$ should be at least B_2 which implies that L is at least $2B_2$. Hence it is optimal for F2 to locate two facilities in that interval, one to the right of the jth facility and close to it and one to the left of the $(j + 1)$th facility and close to it, thereby taking away the entire jth interval from Fl. A similar proof shows that if F2 were to locate in any of the terminal intervals of F1, it is optimal for F2 to locate exactly one facility that is arbitrarily close to the terminal facility of F1 and cut off F1 from the terminus of the market. This proves the last statement of the lemma. ■

There are two immediate consequences of Lemma 1 — the first is that when the demand distribution is uniform over the market, any terminal e.d.i. is of length B_2 and any interior e.d.i. is of length $2B2$. The second consequence is that if $B_2 < \frac{1}{2}B_2$, regardless of the distribution of demand over the market, it is optimal for F1 to not locate any facility at all. This is so because, if this condition holds, and F1 locates any facilities, then F2 can locate two facilities for every facility of F1 – one to its left and the other to its right and both arbitrarily close, and get the entire market at a positive profit. Another general result about the optimal location strategy can be stated as follows.

Lemma 2. Given any arbitrary demand distribution, if $B_1 \leq B2$, F1 will locate facilities in the optimal solution, i.e. the optimal profit of F1 is nonnegative in this case.

Proof Consider a strategy of F1 where it locates one facility at a distance of x from 0, such that the area of the demand distribution in the interval [0, x] is B2 - this is a feasible location strategy of F1 that has a nonnegative profit. Therefore, the optimal strategy must give F1 nonnegative profit too. ■

It will now be shown in the next three lemmas, that when the demand distribution is uniform, the optimal location strategy has some more properties that considerably simplify the analysis.

Lemma 3. Given a uniform demand distribution, if $B_1 \leq B_2$, it can be assumed that F2 does not locate any facilities in an optimal solution.

Proof. Suppose not. Then by Lemma 2, F1 has also located some facilities in this optimal solution. If F2 locates in the 0th interval, it can be concluded that the length of this interval is at least B_1. Then an alternative location strategy by F1, where it makes the 0th interval entry deterring to F2, by locating a minimum number of additional facilities that are required to do so, gives F1 at least as much profit and satisfies the lemma. If, however, F2 locates in an interior interval, say the jth interval, then by Lemma 1, it can be asserted that the entire jth interval belongs to F2 and is therefore at least $2B_2$ in length. In that case, an alternative location strategy that locates an additional facility at a distance of $2B_2$ from the present jth facility, and all the other facilities at the same locations, gives F1 more profit than the present strategy. ■

Lemma 4. Given a uniform demand distribution, if F1 locates any facilities, it can be assumed that all of F l's interior intervals are contiguous.

Proof. Suppose not. Because both F1 and F2 have located their facilities, it can be concluded that $B_2 < B_1 \leq 2B_2$. Hence in this optimal solution, there must exist an interior interval of F1, that F2 has captured by locating two facilities, as described in Lemma 1. Then an argument similar to the one in Lemma 3 will show that nil, is impossible. ■

Lemma 5. Given a uniform demand distribution, if both F1 and F2 locate facilities, then it can be assumed that both terminal intervals do not belong to the same firm.

Proof Since both F1 and F2 have located their facilities, it can be assumed that $B_2 < B_1 \leq 2B_2$. Suppose that both terminal intervals belong to to F2. Referring to Fig. 1, the 0th and the *m*th intervals, i.e. the left and the right terminal intervals, are each of length at least B_2. Then an alternative location strategy for F1 is to locate its first facility at a distance of B_2 from 0 and the other facilities at the same interdistance as before, yielding a higher profit and thereby contradicting the optimality of the present solution. Finally, Lemma 4 guarantees that, if both F1 and F2 locate, F2 gets a terminal interval. ■

Lemmas 3-5 above lead to the following observation about the optimal location strategy of F1: if F1 locates any facilities at all in an optimal solution, all its intervals will be contiguous, and hence, without loss of generality, to the left of F2 's intervals. Together with the idea of e.d.i., this leads us to conclude that the optimal location strategy for Fl, that we will call S^*, is as follows: F1 locates k facilities at B_2, $2B_2$, $5B_2$ etc. from 0 until the length of the *k*th interval, i.e. the right terminal interval, is less than $2B_2$. It can be readily verified that k is equal to $[(B_2 + 1)/2B_2]$. If F1 now locates another facility in the *k*th interval at a distance of B_2 from the kth facility, it can get the entire market with $(k + 1)$ facilities. Whether or not F1 does that depends on the profitability of doing so; an analysis of this issue, based on the length of the *k*th interval in relation to B_1, gives rise to the three different scenarios that we refer to as Conditions 1, 2 and 3, respectively, in Table 1.

The only remaining question now is the condition under which F1 would find it profitable to locate any facility. For this purpose we need to study the relative values of B_1 and B_2. Four cases have to be considered. Case (A): $B_1 < \frac{1}{2}B_2$; Case (B): $\frac{1}{2}B_2 \leq B_1 \leq B_2$; Case (C): $B_2 < B_1 \leq 2B_2$; Case (D): $2B_2\ B_1$. It is clear that in Case (A), F1 is very competitive relative to F2 and in Case (D) the opposite is true. For Cases (B) and (C) the two firms are close competitors. With the aid of these cases, the various solutions to the Stackelberg game can now be found — these are listed in Table 1.

Table 1. Solution to the single turn Stackelberg game

	Cases A and B		Case C			Case D
Condition	$2kB_2 > 1$ (Condition 1)	$2kB_2 \leqslant 1$ & $B_1 + (2k-1)B_2 \leqslant 1$ (Condition 2)	$2kB_2 > 1$ (Condition 1)	$2kB_2 \leqslant 1$ & $B_1 + (2k-1)B_2 \leqslant 1$ (Condition 2)	$2kB_2 \leqslant 1$ & $B_1 + (2k-1)B_2 > 1$ (Condition 3)	Not applicable
No. of facilities in S^*	k	$k+1$	k	$k+1$	k	0
Market share of F1	Entire market	Entire market	Entire market	Entire market	All but the right terminal interval	0
Total profit of F1 from S^*	$(p - V_1) - kC_1$	$(p - V_1) - (k+1)C_1$	$(p - V_1) - kC_1$	$(p - V_1) - (k+1)C_1$	$(p - V_1)(2k-1)B_2 - kC_1$	0
Is this profit of F1 guaranteed to be positive ?	Yes	Yes	No	No	No	No. It is guaranteed to be negative
What happens if total profit of F1 is negative?	Cannot happen		In all these scenarios, F1 quits and F2 gets the entire market by locating one facility anywhere in the market			

From Table 1 it can be concluded that the solution to the one turn game considered in this section depends on the relative values of the break-even sales of the two firms, which decides the value of k. Furthermore, if F1 locates at all it gets either the entire market or almost all of it, by locating k or $(k + 1)$ facilities as the case may be; F2 gets either nothing or, at best, the right terminal interval. If, however, F2 is very competitive, F1 will find it unprofitable to locate any facilities at all, and then F2 can get the entire market with just one facility. It is also worthwhile to note the similarity of S^* with the equilibrium locations in Example 2 of Prescott and Visscher (1977). There, multiple firms are assumed to enter the market in a given sequence and locate one facility each. In the optimal Stackelberg solution there, the first two firms locate at entry deterring lengths from the two terminals and then the other firms locate successively inward at entry deterring lengths from each other.

Although it is not done here, a similar analysis would show that if F2 entered the market first and located any facilities, it would locate q or $(q + 1)$ facilities, where q is equal to $[(B_1 + 1)/2B_1]$.

3. MULTIPLE TURNS

The results of the previous section will now be extended in the present one to another version of the problem — here each firm is assumed to be given multiple 'turns' to locate its facilities, with F1 being given the option to locate on every odd turn and F2 on every even one. In the terminology of game theory, this is the repeated version of the Stackelberg game with both players being given multiple turns. Then by considering the limiting case where the number of turns approaches infinity, we get the equilibrium location strategies that would be adopted by F1 and F2, given, as we have assumed, that they have infinite foresight. Although the results obtained in this section assume uniform demand distribution, the method of analysis used is independent of the demand distribution. It will also be assumed that both F1 and F2 locate all of their facilities on any one particular move; this assumption is necessary to simplify the cases that arise when multiple turns are allowed. As will be seen later, introduction of multiple moves allows the two firms to play strategic games where they can choose to postpone the locations of their facilities to a later move.

In this section, P_i is used to denote the maximum profit to firm i, if it locates all its facilities optimally at the first move, assumed to be at time zero; in other words, this is the maximum profit that the firm can earn if it were the leader in the previous version of the problem where each firm was given only one turn. The following can be inferred about P_1 and P_2 from the results of the previous section: Case (A) implies that P_1 is nonnegative and P_2 is not. Case (D) implies the reverse. Case (B) implies that P_1 is nonnegative and Case (C) implies that P_2 is nonnegative.

It has already been shown that, due to the threat of entry by the follower, the leader may have to locate multiple facilities, thereby making it unprofitable for it to enter the market. This gives rise to the following question: is it possible that it is unprofitable for both F1 and F2 to locate their facilities first, that is, as leaders, thereby making it unprofitable for any of the competing firms to enter the market first? The answer is shown to be in the negative in the following lemma.

Lemma 6. Given any arbitrary demand distribution, P_1 and P_2 cannot both be negative.

Proof. Suppose that P_1 is negative. Hence either Case (C) or Case (D) is applicable and so F1 will quit and F2 enters the market as the leader. Then F2 will be in Case (A) or (B) relative to F1, which is now the follower. But is has been shown in Lemma 2 that the initial reward for the leader is always nonnegative when Cases (A) and **(B)** are true. Hence the lemma. ■

Therefore by Lemma 6, one of the firms will always make a nonnegative profit and there will not be a case in which both firms do not locate any facilities because of competition from the rival firm. This eliminates the possibility of market failure due to competitive pressures as in the first entry paradox demonstrated in Ghosh and Buchanan (1988).

Since the version of the problem has multiple turns that are assumed to occur at different points in time, some additional parameters are needed for the analysis. These are: (i) the total number of moves (whether finite or infinite) and if finite, whether the total number of moves is odd, i.e. F1 makes the last move, or even, i.e. F2 makes the last move; and (ii) the presence or absence of a discounting factor on the profits earned at a future period in time. In addition to these two conditions, we will only consider the cases where the value of P1 and P2 is either positive or negative. This leads to several cases and the solution to this problem in all the cases is given in Tables 2-5.

Table 2. Number of moves is finite and discounting factor is zero

	F1 has last move and $P_1 > 0$	F1 has last move and $P_1 < 0$	F2 has last move and $P_1 > 0$	F2 has last move and $P_1 < 0$
$P_2 > 0$	F1 locates k or $(k + 1)$ facilities on first move. Else if it yields, F2 will locate q or $(q + 1)$ facilities on the second move	F1 yields to F2 on the 1st move. F2 then locates q or $(q + 1)$ facilities on any even-numbered move	F1 locates k or $(k + 1)$ facilities on the 1st move; else if it yields, F2 will locate q or $(q + 1)$ facilities on 2nd move	F1 yields to F2 on 1st move. F2 locates one facility on the last move and gets the entire market
$P_2 < 0$	F1 locates 1 facility on the last move and gets the entire market	Cannot happen	F1 locates k or $(k + 1)$ facilities on any odd-numbered move	Cannot happen

Note that in Tables 2-5, whether the leader locates k or $k + 1$ (or q or $q + 1$) facilities is assumed to have been determined by doing an analysis similar to the one done for F1 in Section 2. The rationale behind most of the solutions in Tables 2-5 is provided therein. The solution that is the most conspicuously different from the single turn game is the one in Table 2 where F1 has the last move and $P_1 > 0$ and $P_2 < 0$. Taking advantage of this and the fact that there is no discounting, F1 can get the entire market by locating one facility on the last move.

The solutions for Tables 4 and 5 are obtained from those of Tables 2 and 3 by taking into account that if the number of moves is infinite, there is no such thing as the last move implying that the firms would have no incentive to wait for the last move. The solutions shown in Tables 4 and 5 therefore represent equilibrium locational configurations of the game associated with this model. Thus, it can be seen that the equilibrium solution associated with this problem, as given by Tables 4 and 5, has a 'winner take all or almost all' flavor, with one of the firms adroitly locating its facilities so as to eliminate possibility of entry in any of its market intervals by its rival; which one does so depends on who locates first and the relative values of the break-even sales of the two firms.

Table 3. Number of moves is finite and discounting factor is zero

	F1 has last move and $P_1 > 0$	F1 has last move and $P_1 < 0$	F2 has last move and $P_1 > 0$	F2 has last move and $P_1 < 0$
$P_2 > 0$	F1 locates k or $(k + 1)$ facilities on 1st move. Else if it yields, F2 will locate q or $(q + 1)$ facilities on the 2nd move	F1 yields to F2 on 1st move. F2 locates q or $(q + 1)$ facilities on 2nd move — it does not wait since F1 has the last move and there is discounting	F1 locates k or $(k + 1)$ facilities on the 1st move; else if it yields, F2 will locate q or $(q + 1)$ facilities on 2nd move	F1 yields to F2 on the 1st move. F2 then compares two strategies of either locating one facility on the last move to get the entire market or locating q or $(q + 1)$ facilities on the 2nd move; it chooses the one with larger present value of profit
$P_2 < 0$	F1 compares two strategies of locating one facility on the last move to get the entire market or locating k or $(k + 1)$ facilities on the 1st move; it chooses the one with larger present value of profit	Cannot happen	F1 locates k or $(k + 1)$ facilities on 1st move. It will not wait for a future move due to discounting	Cannot happen

Table 4. Number of moves is infinite and discounting factor is zero

	$P_1 > 0$	$P_1 < 0$
$P_2 > 0$	F1 locates k or $(k + 1)$ facilities on the 1st move; else if it yields, F2 will locate q or $(q + 1)$ facilities on the 2nd move	F1 yields to F2 on the 1st move. F2 locates q or $(q + 1)$ facilities on any even numbered move
$P_2 < 0$	F1 locates k or $(k + 1)$ facilities on any odd numbered move	Cannot happen

Table 5. Number of moves is infinite and discounting factor is zero

	$P_1 > 0$	$P_1 < 0$
$P_2 > 0$	F1 locates k or $(k + 1)$ facilities on the 1st move; else if it yields, F2 will locate q or $(q + 1)$ facilities on the 2nd move	F1 yields to F2 on the 1st move. F2 locates q or $(q + 1)$ facilities on 2nd move — it does not wait for a future move due to discounting
$P_2 < 0$	F1 locates k or $(k + 1)$ facilities on 1st move — it does not wait for a future move due to discounting	Cannot happen

Two conclusions can be immediately drawn from the results of this section. The first is that if both firms can make positive profit by locating first, then the firm with the higher break-even sales, i.e. the weaker competitor, would generally be the first one to locate its facilities — i.e. the weaker competitor would be the first entrant into the market. This is so because the weaker competitor knows that if it lets the stronger firm enter the market first, then it (the weaker firm), will at most be able to get the right terminal interval of the market, and possibly no interval at all. Hence the weaker firm, by going first, can ensure that the stronger firm is either kept out of the market, or at worst, allowed a small share of the market. Thus the fact that a firm moves first can help it overcome a cost disadvantage.

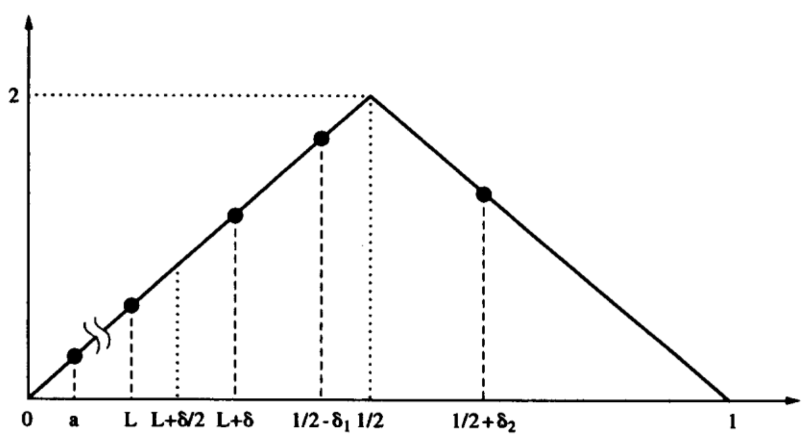

Fig. 2.

The second conclusion is that the 'winner take almost all' solution highlights the fact that it is not always optimal to try to squeeze a competitor completely out of the market. It may be better in certain situations for the leader to let a competitor coexist, albeit, in a marginal fashion.

4. SYMMETRIC TRIANGULAR DEMAND DISTRIBUTION

In this section we reconsider the Stackelberg game of Section 1 under the condition that the demand distribution is triangular and symmetric about the center of the market, as shown in Fig. 2. The corresponding repeated game where both players are given multiple turns, will not be discussed here, as an analysis similar to the one in Section 3 can be done to solve that game.

When the demand distribution is symmetric triangular, three different types of e.d.i. need to be defined, depending on whether the points 0, 1 and $\frac{1}{2}$ are included or not. An e.d.i. is defined to be left terminal if it includes the point 0, right terminal if it includes the point 1, central if it includes the point $\frac{1}{2}$ and interior otherwise. The area of an interval $[L_1, L_2]$, that we denote by $A[L_1, L_2]$, will be used to refer to the area under the demand curve in this interval. With these definitions, we now explore some of the properties of the various e.d.i.

Terminal e.d.i. In Fig. 2 the interval $[0, a]$ is the left terminal e.d.i. Since F2 can capture this interval by locating one facility, a terminal e.d.i should have an area of B_2 - hence its length is $(B_2/2)^{1/2}$.

Interior e.d.i. In Fig. 2 $[L, L + \delta]$ is an interior e.d.i. To calculate the length of an interior e.d.i. and the area under it, note that if F2 locates a facility in the interior interval shown in Fig. 2, it will do so close to $(L + \delta)$ and to its left, and will thus get the area under the interval $[L + \delta/2, L + \delta]$ Therefore, for F2 to locate two facilities in this interval, $A[L, L + \delta/2]$ should at least be B_2 − and if this is true, F2 will locate the second facility close to the right of L and take the entire interval $[L, L + \delta]$ from F1. The above implies that for this interval $[L, L + \delta]$ to be e.d.i, $A[L + \delta/2, L + \delta]$ should be B_2. Hence the equation defining the length δ of the interior e.d.i $[L, L + \delta]$ is:

$$(\tfrac{1}{2})\ (\delta/2)\ 4(L + \delta/2 + L + \delta) = B_2, \text{ implying that } \delta = [-2L + (4L^2 + 6B_2)^{1/2}]/3. \quad (1)$$

Based on (1), the following two properties of an interior e.d.i. can now be stated.

Proposition 7. The area of any interior e.d.i. is always greater than $3B_2/2$ and less than $2B_2$. Furthermore, the area of an interior e.d.i. increases as it gets closer to the center of the market.

Proof. It can be verified that the following three properties hold for any interior e.d.i: (i) $A[L + \delta/2, L + \delta] = A[L, L + \delta/2] + \delta^2$; (ii) $A[L, L + \delta/2] \geq B_2/2$ and (iii) $(B_2/2)^{1/2} \leq L \leq 1 - (B_2/2)^{1/2}$; and they prove the first statement of the proposition. Moreover, from equation (1), it is clear that as L increases, δ decreases − which implies that $A[L, L + \delta/2]$ increases as L increases. Hence the area of the interval $[L, L + \delta]$, which is equal to $A[L, L + \delta/2] + A[L + \delta/2, L + \delta]$, increases as L increases. ■

Proposition 8. Consider the location of two adjacent facilities of F1, say at x and y, where $(B_2/2)^{1/2} \leq x < y \leq \frac{1}{2}$ − i.e. $[x, y]$ is an interior interval. Let L_1 and L_2 be the lengths of the two successive interior e.d.i. that begin at x. If $y \geq L_1 + L_2$, then F2 will locate two facilities in the interval $[x, y]$.

Proof. F2 will locate two facilities in $[x, y]$ if $A[x, x + (y - x)/2]$ is at least B_2. From Proposition 7, the minimum value of $A[x, x + (y - x)/2]$ is obtained when $x = (B_2/2)^{1/2}$. By calculating L_1 and L_2 with $x = (B_2/2)^{1/2}$, it can be verified that $A[x, x + (y - x)/2]$ is greater than $3B_2/2$ if $L_1 + L_2 \leq y$. ■

Since the demand distribution is assumed to be symmetric about the center of the market, Proposition 8 also holds when $\frac{1}{2} \leq x < y \leq 1 - (B_2/2)^{1/2}$.

Central e.d.i. As mentioned previously, an e.d.i. that contains the midpoint is called a central e.d.i. — for example, the interval $[(\frac{1}{2} - \delta_1), + \delta_2)]$ in Fig. 2 is a central e.d.i. A central e.d.i. that is symmetric about the center of the market will be referred to as the symmetric central e.d.i. Similarly, a central e.d.i., one of whose endpoints is the center of the market, will be referred to as an extreme central e.d.i.

To evaluate the area under a central e.d.i., note that if F2 locates one facility in a central e.d.i, as shown in Fig. 2, it will capture some area from the interval $[(\frac{1}{2} - \delta_1),\ \frac{1}{2}]$ and some from the interval $[\frac{1}{2}, (\frac{1}{2} + \delta_2)]$. For F2 to locate a facility, the sum total of these areas should be at least B_2. Therefore, let the area that F2 captures from the intervals $[(\frac{1}{2} - \delta_1),\ \frac{1}{2}]$ and $[\frac{1}{2}, (\frac{1}{2} + \delta_2)]$ be xB_2 and $(1 - x)B_2$, respectively. From Proposition 7 above, it is known that $A[(\frac{1}{2} - \delta_1), = 2A[(\frac{1}{2} - \delta_1/2), \frac{1}{2}], - (\delta_1)^2$. This, together with the fact that $A[(\frac{1}{2} - \delta_1/2), \frac{1}{2}]$ is equal to xB_2, gives the following:

$$\delta_1 = 1 - (1 - 2xB_2)^{1/2}, \text{ implying that, } (\delta_1)^2 = 2[1 - xB_2 - (1 - 2xB_2)^{1/2}] \qquad (2)$$

Using (2), it can then be shown that

$$\begin{aligned} A[(\tfrac{1}{2} - \delta_1), &= 2A[\tfrac{1}{2} - \delta_1/2), \tfrac{1}{2} - (\delta_1)^2 \qquad (3) \\ &= 2xB_2 - 2 + 2xB_2 + 2(1 - 2xB_2)^{1/2} \\ &= 4xB_2 + 2(1 - 2xB_2)^{1/2} - 2. \end{aligned}$$

An analysis similar to (3) will show that

$$A[\tfrac{1}{2}, (\tfrac{1}{2} + \delta_2)] = 4(1 - x)B_2 + 2[1 - 2(1 - x)B_2]^{1/2} - 2. \qquad (4)$$

Summing (3) and (4) gives the total area under the central e.d.i.$[(\frac{1}{2} - \delta_1), (\frac{1}{2} + \delta_2)]$ as a function of x:

$$\begin{aligned} f(x) &= A[(\tfrac{1}{2} - \delta_1), \tfrac{1}{2}] + A[\tfrac{1}{2}, (\tfrac{1}{2} + \delta_2)] \\ &= 2[(1 - 2xB_2)^{1/2} + (1 - 2(1 - x)B_2)^{1/2}] - 4 + 4B_2. \qquad (5) \end{aligned}$$

Equation (5) can now be used to verify that $f(x)$ is concave $\forall x$. By checking the appropriate first and second order conditions it can then be seen that $f(x)$ achieves its maximum at $x = \frac{1}{2}$. This enables us to state the following.

Proposition 9. Of all central e.d.i., the symmetric central e.d.i. has the maximum area, and the extreme central e.d.i., the minimum.

Taken together, the discussion above leads to the following conclusion: of all e.d.i. the symmetric central e.d.i. has the maximum area, followed by the extreme central e.d.i., any interior e.d.i. and a terminal e.d.i., in that order.

We will now proceed to prove some properties of the optimal location strategy for FL But first, Lemma 3 will have to be reproved, since the area under the nonterminal e.d.i. is not constant in this case.

Lemma 10. Given a symmetric triangular demand distribution, if $B_1 \leq B_2$, it can be assumed that F2 does not locate any facilities in any optimal solution.

Proof Consider the optimal locational configuration: since Lemma 2 is still true it can be assumed that F1 has located facilities in this optimal solution. Suppose that F2 locates in any interval of F1. A proof analogous to the one in Lemma 3 will show that this interval cannot be a terminal interval. By proposition 7, it can also be shown that if F2 locates two facilities in the interval, the area of this interval is at least $2B_2$, which contradicts the optimality of the given solution. Thus the only remaining possibility is the case where F2 locates one facility in this interval of Fl. If that happens, by noting that the area under any nonterminal e.d.i. is at least $3B_2/2$, it can be shown that if F1 makes this entire interval entry deterring to F2, by locating the minimum number of additional facilities that are necessary to do so, its profits will increase. ■

In Lemma 4 it was shown that when the demand distribution is uniform, all interior intervals belong to F1 in the optimal solution. The corresponding result for the symmetric triangular demand distribution is Lemma 11.

Lemma 11. Given symmetric triangular demand distribution, if F1 locates at least two facilities in either $[0, \frac{1}{2}$ [or] $\frac{1}{2}, 1]$, then it can be assumed that its facilities are located such that they form contiguous interior e.d.i. in $[0, \frac{1}{2}[$ or $]\frac{1}{2}, 1]$.

Proof. Suppose not. Assume that F1 has located m facilities in $[0, \frac{1}{2}[$ that are numbered 1 through m from the left terminus of the market, i.e. 0. Because we assume the lemma not to hold, F2 has located in at least one of the interior intervals of F1 in $[0, \frac{1}{2}[$ − and let the first such interval be the nth interval. Consider an alternative strategy of F1, one in which F1 locates facilities $(n + 1)$ through m at the same locations, the nth facility at an entry deterring length away from the $(n + 1)$th facility and all its facilities 1 through n at entry deterring lengths from each other. This alternative strategy gives F1 a higher profit − as the maximum loss of F1 by adopting it is B_2, which happens if it loses the left terminal to F2; but that is also the minimum gain ensured by adopting it. ■

Although Lemma 11 above establishes the contiguity of F1's interior intervals, the issue of which firm gets the central interval is yet unresolved — that is now done in the following result.

Lemma 12. Given a symmetric triangular demand distribution, if F1 locates at least three facilities in the optimal solution, with none at the center of the market it can be assumed that F1 gets the central interval.

Proof. Because F1 has located more than two facilities and none at the midpoint $\frac{1}{2}$, it has located at least two facilities in one of the two intervals $[0, \frac{1}{2}[$and$] \frac{1}{2}, 1]$ assume that it has done so in $]\frac{1}{2}, 1]$. By lemma 11, all of F1's interior intervals in 1] form contiguous interior e.d.i. Let the two facilities of F1 in $]\frac{1}{2}, 1]$, that are closest to $\frac{1}{2}$, be the mth and (m + 1)th facilities, that are located at $(\frac{1}{2} + r_1)$ and $(\frac{1}{2} + r_2)$, respectively; and let the facility of F1 in [0, 4 that is closest to $\frac{1}{2}$, i.e., its $(m - 1)$th facility, be located at $(\frac{1}{2} - L_1)$. Then, if Lemma 12 is not true, it can be concluded that F2 has located in the central interval $[(\frac{1}{2} - L_1), (\frac{1}{2} + r_1)]$. If F2 locates two facilities in this central interval, then the market area given up by F1 is at least $2B_2$ which would make the present location strategy of F1 sub-optimal, thus eliminating this possibility.

Therefore assume that F2 has located one facility in this interval, and has thus taken away a market share of at least B_2 from F1 in this solution. Consider an alternative location strategy of F1 in which F1 locates facilities 1 through $(m - 1)$ at the same locations, the mth facility at an entry deterring length away from its $(m - 1)$th facility and all the facilities to the right of the mth facility at entry deterring lengths from each other. If in the alternative strategy, the mth facility is located to the right of $(\frac{1}{2} - r_2)$, and therefore the $(m + 1)$th facility is located to the right of $(\frac{1}{2} - r_1)$, the alternative strategy will give F1 at least as much profit as the present one. If, however, the mth facility is to the left of $(\frac{1}{2} - r_2)$ in the alternative strategy, then $A[(\frac{1}{2} - L_1), \frac{1}{2}]$ is at least equal to the area of two consecutive interior e.d.i. That would imply, from Proposition 8, that F2 has located two facilities $[(\frac{1}{2} - L_1), (\frac{1}{2} + r_1)]$. ■

Analogous to the number k in Section 2, we now define a number k^* as the maximum number of entry deterring intervals that cover the interval $[0, \frac{1}{2}]$ this is shown in Fig. 3. It is not clear how a closed form expression for the value of k^* may be obtained directly since the direct approach involves solving quadratic equation (1) iteratively. To eliminate the simple cases and also to satisfy the assumptions of Lemmas 11 and 12, it will be assumed that $k^* > 2$. Based on the definition of k^*, consider the following five location strategies of Fl.

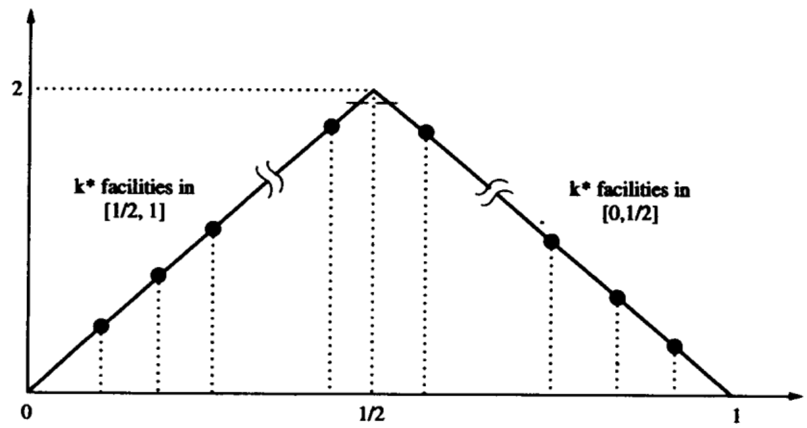

Fig. 3.

Strategy 1. Here F 1 locates $2k^*$ facilities as shown in Fig. 3 and the resulting central interval is entry deterring for F2. Therefore, F1 gets the entire market by locating $2k^*$ facilities.

Strategy 2. Same as strategy 1 except that here the resulting central interval, after having located $2k^*$ facilities, is not entry deterring. Therefore, F1 locates another facility at 1/2 and gets the entire market with $(2k^* + 1)$ facilities.

Strategy 3. Here F1 locates $(2k^* - 2)$ facilities in the following manner: two facilities are located symmetrically about the center to form a symmetric central e.d.i. Then $(k^* - 2)$ facilities are located successively on each side such that they form contiguous interior e.d.i.

Strategy 4. Here F1 locates $(2k^* - 1)$ facilities in the following manner: one facility is located at the center and two facilities are located one either side of it such that they define two extreme central e.d.i. Then $(k^* - 1)$ facilities are located in both intervals $[0, \frac{1}{2}[$ and $]\frac{1}{2}, 1]$ such that they define contiguous interior e.d.i.

Strategy 5. Here F1 locates $2k^*$ or $[(2k^* - 1)$ facilities in the following manner: one facility is located at a distance of $(B_2/2)\frac{1}{2}$ from 0 to its right such that it forms a left terminal e.d.i. The remaining $(2k^* - 1)$ or $(2k^* - 2)$ facilities are located successively such that they define contiguous e.d.i. all of which are interior e.d.i. and one is a central e.d.i.

Lemmas 11 and 12 give two important properties of the optimal location strategy of F1, namely, concerning the continuity of F1's interior intervals and when F1 captures the central interval. The remaining issue is that of profitability of locating facilities by Fl. To examine this issue consider again the relative values of B_1 and B_2. Then the different optimal location strategies can be stated as follows.

Case (*D*). As before in the case of uniform demand, F1 will quit and F2 will locate one facility anywhere in the [0, 1] interval and get the entire market.

Cases (*A*) *and* (*B*). Here $B_1 \leq B_2$ and therefore from Lemma 10, it can be concluded that F2 will not locate any facilities. Therefore, F1 will adopt either Strategy 1 or 2, depending on which one is applicable and gives larger profit. As before, Lemma 2 guarantees that F 1 's total profit is nonnegative.

Case (*C*). As with the uniform demand distribution, this case is the most involved. In Lemma 13 that follows, it is shown that in this case, F 1 will consider Strategies 1 through 5 and choose the one with the highest total profit. If this otimal profit, which is equal to P1 in this case, is negative, it will quit and F2 will take away the entire market by locating one facility.

Lemma 13. Given a symmetric triangular demand distribution, if Case (C) occurs, then it can be assumed that one of Strategies 1 through 5 is optimal for F1.

Proof. We begin with the following three observations about the optimal location strategy of F1, that are direct consequences of Proposition 7, the relative values of B_1 and B_2 in Case (C) and the general properties of the model. (i) In any optimal location strategy, Fl never lets F2 capture a market share that is greater than or equal to B_1 and hence, $2B_2$. This implies, by Proposition 7, that F1 will not locate any fewer than $k^* - 1$ facilities in either $[0, \frac{1}{2}[$ or $]\frac{1}{2}, 1]$. (ii) By the definition of k^*, F1 does not need to locate more than k^* facilities in either one of the intervals $[0, \frac{1}{2}[$ and $]\frac{1}{2}, 1]$. Hence the maximum number of facilities that F1 will need to locate in any location strategy is $2k^* + 1$, since, with as many facilities, F1 can get the entire market. (iii) By the symmetry of the demand distribution, if F 1 locates a facility at the center of the market, it can be assumed that its location strategies in $[0, \frac{1}{2}[$ and $]\frac{1}{2}, 1]$ are identical.

First, we argue that it is sub-optimal for F1 to locate fewer than $2k^* - 2$ facilities. This is proved by considering each of the different locational configurations possible, after applying condition (ii) above. It can then be seen that in each of these, F1 gives up a market share of at least $2B_2$, and hence B_1 — which violates condition (i). Since by (ii), F1 does not need to locate any more than $2k^* + 1$ facilities, this only leaves us with a few cases that we now consider individually.

If F1 locates $2k^* + 1$ facilities in any optimal solution, then it can be assumed that it will adopt Strategy 2, as it guarantees the entire market. If, however, F1 locates $2k^*$ facilities, it will adopt either Strategy 1 or Strategy 5, depending on which one is applicable and gives more profit. While Strategy 1 guarantees the entire market to F1, Strategy 5 does not. By adopting it, F1 may lose the right terminal interval to F2; however, condition (i) guarantees that this lost market share is less than B_1.

Now for the case where F1 locates $2k^* - 1$ facilities. With $2k^* - 1$ facilities, if F1 locates a facility at the center, then by condition (iii) above and Lemma 11, it can be assumed that Strategy 4 is optimal for Fl. If, however, F1 locates $2k^* - 1$ facilities and none at the center, then by condition (ii) it can be assumed without loss of generality, that it has located k^* facilities in [0, 1[and $k^* - 1$ in $]\frac{1}{2}, 1]$. Application of Lemmas 11 and 12 then readily shows that Strategy 5 is optimal for F1 with this locational configuration.

Thus the last choice to be considered is one where F1 locates $2k^* - 2$ facilities. Condition (iii) above immediately implies that in this case F1 does not locate any facility at the center. Hence, by condition (ii), it can be assumed without loss of generality, that F1 locates $k^* - 1$ facilities in each of the intervals $[0, \frac{1}{2}[$ and $]\frac{1}{2}, 1]$. By Lemmas 11 and 12, it can be seen that with this locational configuration, it is optimal for F1 to adopt Strategy 3; if not, the given solution can be perturbed, until it is similar to Strategy 3, and by the symmetry of the demand distribution, this perturbation does not reduce the total market share of Fl. ■

5. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, we have studied a variant of Hotelling's duopoly model where the number of facilities located by each of the firms is also a decision variable. The main results of the study are as follows.

(1) The firm that locates first can increase its profits by taking advantage of the phenomenon of entry deterrence – this result reflects the standard intuition for pioneering advantage or order of entry advantage; see Schmalensee (1982), Carpenter and Nakamoto (1989), Kalyanraman and Urban (1993). However, this paper makes a much more interesting observation about pioneering advantage — namely, that the weaker firm, i.e. the firm with the higher break-even sales requirement, can very effectively preempt the stronger firm by going first, and ensure that the stronger firm is either kept out of the market or, at worst, allowed a small share of the market. Thus, the fact that a firm moves first can help it overcome a cost disadvantage. To the best of our knowledge, this is a new motivation for the pioneering advantage.

(2) In this problem, the equilibrium has a 'winner take all' or 'winner take almost all' flavor with the leader firm ensuring that all its intervals are entry deterring for the follower firm. The 'winner take almost all' solution highlights the fact that it is not always optimal to try to squeeze a competitor completely out of the market. It may be better in certain situations for the leader to let a competitor coexist, albeit, in a marginal fashion.

(3) Under the fairly loose assumption that the break-even sales of the two firms are less than the total demand of the market, this model does not exhibit instances of market failure, such as the phenomenon of first entry paradox, regardless of the demand distribution.

There are a number of possible extensions and variations of this model. One is to study an oligopolistic version of this model, assuming, as in Prescott and Visscher (1977), that the firms enter the market in a given sequence. Another possibility is to investigate this model under the assumption that customers do not patronize the firm with the cheapest delivered price, but use other deterministic utility functions, such as proportional models. Yet another strand of future research may be to incorporate imperfect information into the model, by studying it under the assumption that the two firms have imperfect information about each others' fixed and marginal costs.

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