

## Set-up costs and the existence of competitive equilibrium when extraction capacity is limited

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### **Abstract:**

Although set-up costs are prevalent and substantial in natural resource extraction, it is known that a Walrasian competitive equilibrium cannot exist in simple extraction models with set-up costs. This paper demonstrates that this result is sensitive to the assumption of unlimited extraction capacity and derives sufficient conditions for existence. An equilibrium exists if extraction is limited such that each firm earns sufficient surplus to cover its set-up costs or if firms choose extraction capacity subject to non-increasing returns. The resulting competitive equilibrium price either grows at the rate of interest when total extraction is below industry capacity or is constant when industry capacity is fully utilized. In the equilibrium, identical deposits are opened simultaneously, and set-up costs for new deposits are incurred when the industry has excess capacity rather than when capacity is fully utilized.

**Keywords:** Natural resource extraction; Exhaustible resources; Extraction constraints; Set-up costs; Competitive equilibrium; Non-convexity; Capacity installation

### **Article:**

#### **1. Introduction**

Set-up costs of natural resource extraction include costs of tunneling, drilling wells, building pipelines, searching for deposits, and removing overburden from strip mines.<sup>1</sup> These “lumpy” costs must be incurred before any production can take place and thus induce non-convexities in the production possibilities set.<sup>2</sup> With non-convexities, the well-known theorems on the existence of Walrasian competitive equilibria first developed by Arrow and Debreu [1] are no longer applicable. Although competitive equilibria may exist in some non-convex economies,<sup>3</sup> Hartwick et al. [16] and Fischer [14] have shown that a competitive equilibrium cannot exist if extraction requires set-up costs.<sup>4</sup> Without existence of a competitive equilibrium, analysis of competitive natural resource markets with significant set-up costs is greatly hindered.<sup>5</sup>

The difficulties of analyzing set-up costs have led to rule-of-thumb analysis or to neglect of set-up costs.<sup>6</sup> The well-known tools of competitive equilibrium analysis would be helpful if they could be applied. For example, the efficiency properties make competitive equilibria useful for project analysis and for testing the efficiency of markets. Competitive equilibrium price paths are useful for comparative static analysis and for analysis of taxes and subsidies. Finally, empirical tests of the competitive theory of exhaustible resources have largely neglected set-up costs.<sup>7</sup> Extending the tools of competitive equilibrium analysis to these questions could allow tractable analysis of natural resource extraction with significant set-up costs.

This paper derives sufficient conditions for the existence of a competitive equilibrium in natural resource extraction with set-up costs. The literature demonstrating non-existence has modeled set-up costs as exogenous and unrelated to extraction capacity. This paper demonstrates that if set-up costs arise from the installation of extraction capacity, then—despite non-convex production sets—the existence of a competitive equilibrium can be saved by recognizing that extraction capacity may be limited. In particular, a competitive equilibrium can exist if current extraction is sufficiently limited or if firms choose their extraction capacity.

Section 2 describes the problem of non-existence with set-up costs and argues that the results may not generalize when extraction capacity is limited. Section 3 then demonstrates sufficient conditions for existence

of a competitive equilibrium when set-up costs are costs of installing capacity. The result is shown by defining a price and allocation path and then by proving that they form a competitive equilibrium. Section 4 characterizes the price path for heterogeneous deposits and discusses other extensions of the results. Section 5 concludes by discussing several applications of competitive equilibrium analysis in exhaustible resources.

## 2. Non-existence and extraction capacity

In the literature on non-existence, Hartwick et al. and Fischer argue that extraction from identical deposits with set-up costs and constant marginal extraction costs should be strictly sequential.<sup>8</sup> However, strictly sequential extraction requires each firm to install extraction capacity large enough to satisfy demand. If capacity installation is costly, simultaneous extraction from multiple deposits with set-up costs may well be efficient. Since the arguments demonstrating non-existence by Hartwick et al. and Fischer rely on the efficiency of strictly sequential extraction, their results do not demonstrate non-existence when extraction capacity is limited.

There is, however, a more fundamental difficulty for existence which does not depend on the strict sequencing of extraction, namely: a price-taking firm has an incentive to delay incurring its set-up costs beyond the efficient date. To illustrate, consider a market with a single, price-taking firm owning one deposit which requires set-up costs and has constant marginal extraction costs.<sup>9</sup> If exploiting the deposit is efficient, the set-up costs should be incurred immediately. Once the set-up costs are sunk, consumption in each period should be exactly as if there were no set-up costs, i.e., the marginal benefit should grow at the rate of interest as discovered by Hotelling [20]. The First Welfare Theorem then implies that a competitive equilibrium price path—if one exists—must yield this efficient allocation.<sup>10</sup> The consumer's optimization implies that the Hotelling price path is the only potential competitive equilibrium price path. Are the Hotelling price path and efficient extraction consistent with a profit maximizing firm? Recall that under the Hotelling rule, the firm earns equal present value profit on extraction in any two periods. If the set-up costs were delayed and extraction increased in later periods, the present value of extraction profit would be unchanged and the set-up costs would be delayed. Since this deviation would yield higher total profit, the firm was not maximizing profit in the proposed equilibrium. Therefore the proposed price path is not an equilibrium price, and a competitive equilibrium does not exist.<sup>11</sup>

Analysis of multiple deposits with set-up costs introduces an additional difficulty for existence: discontinuities in the efficient marginal benefit path. Hartwick et al. showed that the marginal benefit path should jump down each time a new deposit is opened and then should grow at the rate of interest.<sup>12</sup> Facing a price path identical to the efficient marginal benefit path, price-taking firms now have an incentive to deviate from the efficient extraction program by extracting too early at the higher price. This deviation can be profitable if the set-up cost is incurred an instant too early and a large amount is extracted at the higher price. Note that each of these profitable deviations may not be feasible if extraction capacity is limited. This paper derives conditions under which extraction capacity is sufficiently limited that there are no profitable deviations from efficiency, and a competitive equilibrium exists.

## 3. Model of extraction capacity and set-up costs

Consider extraction of an exhaustible resource from heterogeneous deposits. Assume there are two types of deposits called shallow and deep deposits. The deep deposits require set-up costs before any of the deposit can be extracted, but stock from the shallow deposits can simply be extracted.<sup>13</sup> To emphasize analysis of the set-up costs, let there be only one shallow deposit with initial stock  $S_0$  and  $N$  deep deposits with initial stocks  $S_1 = S_2 = \dots = S_N \equiv S$ . Let  $q_i(t)$  be extraction at time  $t$  from deposit  $i$ . The shallow deposit can be extracted at constant marginal cost  $c_0$ .<sup>14</sup> Stock can only be extracted from the deep deposits after extraction capacity has been installed. Let  $F(\bar{q}_i)$  be the cost of installing  $\bar{q}_i$  units of extraction capacity in deep deposit  $i$ .<sup>15</sup> Assume extraction is at constant marginal cost  $c$  after the capacity is installed and is subject to  $q_i(t) \leq \bar{q}_i$ . Finally, let consumption be  $Q(t)$  in period  $t$ , and consumer surplus in each period be  $U(Q)$ , where  $U' > 0$  and  $U'' < 0$ .<sup>16,17</sup> Consumers and producers discount at the common rate,  $r$ .

To proceed further, the set-up costs and extraction technology must be described. Two types of technology are analyzed below: *fixed* and *endogenous* capacity. For the fixed capacity technology, the cost of installing

capacity is given by  $F(0) = 0$ ,  $F(\bar{q}) = \tilde{F}$  for  $0 < \bar{q} \leq \tilde{q}$  and  $F(\bar{q}) = \infty$  for  $\bar{q} > \tilde{q}$ . Clearly with the fixed technology, it will be efficient either to install capacity  $\tilde{q}$  at cost  $\tilde{F}$  or to install no capacity.<sup>18</sup> For the endogenous capacity technology, the set-up costs are given by the function  $F$ , where  $F$  is differentiable,  $F(0) = 0$ ,  $F' > 0$ , and  $F'' > 0$ . The endogenous technology allows firms to choose their capacity subject to decreasing returns.

### 3. 1. The fixed capacity technology

To demonstrate existence of a competitive equilibrium, arbitrary time paths of prices, consumption, and extraction can be defined. If these paths characterize a competitive equilibrium, then a competitive equilibrium exists. Define the price, production, and consumption paths as the optimizers of the following constrained optimization problem:<sup>19</sup>

$$\begin{aligned} \max_{q_0(t), q(t), T} & \int_0^T e^{-rt} [U(q_0(t)) - c_0 q_0(t)] dt - e^{-rT} NF \\ & + \int_T^\infty e^{-rt} [U(Q(t)) - c_0 q_0(t) - cq(t)] dt \end{aligned} \quad (1)$$

subject to the constraints

$$\begin{aligned} \int_0^\infty q_0(t) dt &\leq S_0, \\ \int_T^\infty q(t) dt &\leq NS, \\ q(t) &= 0 \quad \forall t < T, \\ q(t) &\leq N\bar{q} \quad \forall t \geq T, \end{aligned} \quad (2)$$

where  $Q(t) = q_0(t) + q(T)$ ,  $F = \tilde{F}$ ,  $\bar{q} = \tilde{q}$ , and the price path is defined by  $p(t) \equiv U'(Q(t))$ .

To provide intuition for this optimization, note that it looks very much like a planner's problem with two deposits. The objective function is the present value of consumer surplus less extraction and set-up costs. The first constraint ensures that extraction from the shallow deposit does not exceed its initial stock. The second constraint ensures that the total extracted from all the deep deposits does not exceed the initial stock of the deep deposits. The remaining two constraints ensure that extraction from the deep deposits does not occur before the set-up costs are incurred and thereafter does not exceed capacity. Note, however, that the optimization is not the social planner's problem since it does not allow the flexibility of installing different capacity in each deep deposit at different times.

The Kuhn–Tucker first order and complementary slackness conditions for  $q_0$  and  $q$  are

$$\begin{aligned} q_0(t) &\geq 0 \quad e^{-rt}[p(t) - c_0] - \lambda_0 \leq 0 \text{ C.S.} \quad \forall t, \\ q(t) &\geq 0 \quad e^{-rt}[p(t) - c] - \lambda - \mu(t) \leq 0 \text{ C.S.} \quad \forall t \geq T, \\ \mu(t) &\geq 0 \quad N\bar{q} - q(t) \geq 0 \text{ C.S.} \quad \forall t \geq T, \end{aligned} \quad (3)$$

where  $\lambda_0$  is the Lagrange multiplier for the first stock constraint,  $\lambda$  is the multiplier for the second stock constraint, and  $\mu(t)$  is the multiplier for the extraction constraint. Define the *augmented marginal cost* as the marginal extraction cost plus the scarcity cost of the stock, i.e.,  $AMC_0(t) \equiv c_0 + \lambda_0 e^{rt}$  and  $AMC(t) \equiv c + \lambda e^{rt}$ .<sup>20</sup> The first equation implies that extraction from the shallow deposit is optimal if  $p(t) = AMC_0(t)$  when extraction is positive. The second equation implies that, after capacity has been installed, extraction from the deep deposits is optimal if extraction is positive when  $p(t) \geq AMC(t)$  and at capacity when  $p(t) > AMC(t)$ .

To compute the optimal time to install the capacities, first note that the optimal extraction paths need not be continuous at  $T$ : Let  $q_0^- \equiv \lim_{t \uparrow T} q_0(t)$ ,  $q_0^+ \equiv \lim_{t \downarrow T} q_0(t)$ , and  $q^+ \equiv \lim_{t \downarrow T} q(t)$  be extraction immediately before and after  $T$ : Similarly, let  $Q^+ \equiv \lim_{t \downarrow T} Q(t)$  be consumption immediately after  $T$ : Differentiation of the Lagrangian yields the first-order condition for  $T$ :

$$U(q_0^-) - (c_0 + \lambda_0 e^{rT})q_0^- + rNF = U(Q^+) - (c_0 + \lambda_0 e^{rT})q_0^+ - (c + \lambda e^{rT})q^+ \quad (4)$$

Define *augmented net surplus* as gross consumer surplus less augmented marginal costs, i.e.,  $ANS(t) = U(Q) - AMC_{0q_0} - AMCq$ .<sup>21</sup> Eq. (4) states that augmented net surplus after  $T$  must exceed the augmented net surplus before  $T$  by the interest payment on the set-up cost,<sup>22</sup> i.e.,  $ANS^- + rNF = ANS^+$ . This condition implies that there must be a discontinuous increase in augmented surplus when the capacities are installed. The following lemma derives conditions under which  $p(t)$  is continuous. If  $p(t)$  is continuous, consumer surplus is continuous and the jump occurs solely in augmented producer surplus.

**Lemma 1.**  $p(t)$  is continuous if and only if  $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ . If  $p(t)$  is continuous, then  $p(T) = c + \lambda e^{rT} + \frac{rF}{\bar{q}}$ . If  $p(t)$  is not continuous, capacities are installed after the shallow deposit is exhausted.

Proof. See Appendix A.

Lemma 1 defines a trigger price,  $p(T)$ ; at which all capacities should be installed if  $p(t)$  is continuous.<sup>23</sup> This trigger price is illustrated in Fig. 1 where the set-up cost is small such that  $p(T) = c + \lambda e^{rT} + rF/\bar{q} < U'(N\bar{q})$ . The augmented net surplus before  $T$ ;  $ANS^-$ ; is the area above the augmented marginal cost curve and below the demand curve, i.e.,  $Area(A + B + C)$ . Since  $AMC_{0(T)} > AMC(T)$ ; production from the constrained deposits generates additional surplus. At  $T$ ; this additional surplus is  $Area(D) = rNF$ : Thus the additional surplus,  $ANS^+ - ANS^-$ , accrues entirely to the producers.

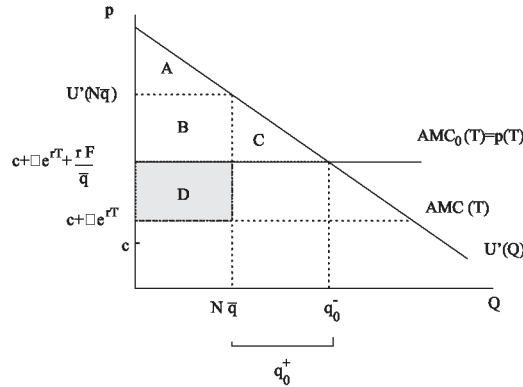


Fig. 1. Augmented marginal cost and net surplus at  $T$  if the set-up cost is small and  $p(t)$  is continuous. The increase in augmented surplus,  $Area(D) = ANS^+ - ANS^- = rNF$ , accrues entirely to producers and consumer surplus does not change at  $T$ .

**Lemma 2.** If  $c + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$  and  $S \geq S^{\min}$  then  $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ , where

$$S^{\min} = \frac{\bar{q}}{r} \left( \ln(U'(N\bar{q}) - c) - \ln \left( U'(N\bar{q}) - c - \frac{rF}{\bar{q}} \right) \right) + \frac{1}{N} \int_0^{\infty} D(c + (U'(N\bar{q}) - c)e^{rt}) dt \quad (5)$$

Proof. See Appendix A.

Lemma 2 translates the condition in Lemma 1 into two sufficient conditions on exogenous parameters. Note that the trigger price in Lemma 1 is increasing in average cost but decreasing in deposit size. The first condition in Lemma 2 ensures average cost is less than the price when all firms extract at capacity. The second condition ensures that the stock is large enough that the capital costs are recovered before the stock is exhausted.<sup>24</sup>

The price and extraction paths which will be a competitive equilibrium can now be described. Under the sufficient conditions in Lemma 2, the continuous  $p(t)$  can be written

$$p(t) = \begin{cases} c_0 + \lambda_0 e^{rt} & \text{if } t \in [0, \bar{T}], \\ U'(N\bar{q}) & \text{if } t \in [\bar{T}, \hat{T}], \\ c + \lambda e^{rt} & \text{if } t \in [\hat{T}, \infty), \end{cases} \quad (6)$$

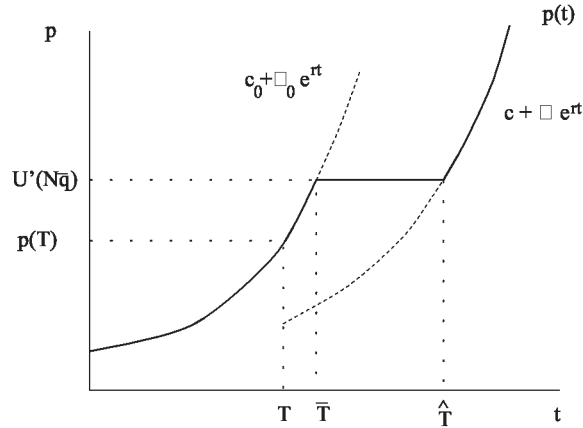


Fig. 2. Continuous  $p(t)$  for optimization problem if  $c + \lambda e^{rT} + \frac{rF}{q} < U'(N\bar{q})$ .

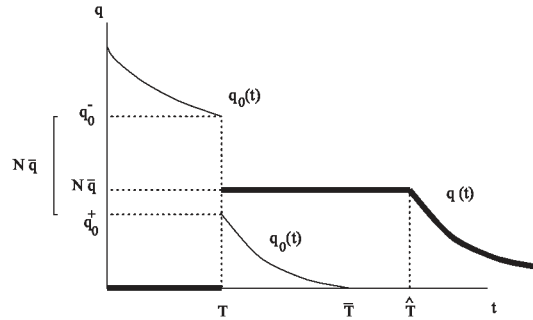


Fig. 3. Optimal extraction paths if the marginal benefit path is continuous.

where  $\bar{T}$ ; and  $\hat{T}$  are defined by the continuity of  $p(t)$ .<sup>25</sup> Let  $D(p)$  be the inverse marginal utility schedule, i.e., the demand curve. Optimal extraction from the shallow deposit is

$$q_0(t) = \begin{cases} D(p(t)) & \text{if } t \in [0, T], \\ D(p(t)) - N\bar{q} & \text{if } t \in (T, \bar{T}], \\ 0 & \text{if } t \in (\bar{T}, \infty) \end{cases} \quad (7)$$

and extraction from the deep deposits is

$$q(t) = \begin{cases} 0 & \text{if } t \in [0, T], \\ N\bar{q} & \text{if } t \in (T, \hat{T}], \\ D(p(t)) & \text{if } t \in (\hat{T}, \infty). \end{cases} \quad (8)$$

The price path is illustrated in Fig. 2 and the extraction paths are illustrated in Fig. 3. Note that the price and extraction paths resemble Hotelling's competitive equilibrium. On  $[0, \bar{T}]$ , extraction from the shallow deposits is positive while their net price grows at  $r$ . The shallow deposit is exhausted at  $\bar{T}$  after which its net price grows more slowly. Also, the deep deposits are extracted while their net price grows at  $r$ . The following proposition demonstrates that the price and extraction paths shown in Eqs. (6), (7) and (8) do indeed characterize a competitive equilibrium.

**Proposition 1.** *A competitive equilibrium exists with the fixed capacity technology if  $c + \frac{rF}{q} \leq U'(N\bar{q})$  and  $S \geq S^{\min}$  where  $S^{\min}$  is defined in Eq. (5).*

Proof. See Appendix A.

The competitive equilibrium in Proposition 1 resembles equilibria in other exhaustible resource models. The price is either constant or grows at the rate of interest net of extraction costs. Simultaneous extraction occurs from multiple deposits only if the deposits have the same costs or if some deposits extract at capacity. However, in contrast to the models of Hartwick et al. and Fischer, all set-up costs for identical deposits are incurred simultaneously, not sequentially. The following application of the First Welfare Theorem demonstrates that this simultaneous installation of capacities is efficient.

**Corollary 1.** *If  $c + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$  and  $S \geq S^{\min}$  then it is efficient to incur all set-up costs simultaneously for the fixed capacity technology.*

Proof. The conditions in Proposition 1 imply existence of an equilibrium with simultaneous capacity installation. The equilibrium is efficient by the First Welfare Theorem.

If the sufficient conditions in Proposition 1 do not hold, simultaneous capacity installation may not be efficient. In this case, the efficient marginal benefit path may follow a sawtooth pattern as in Hartwick et al. and a competitive equilibrium may not exist.

### 3.2. The endogenous capacity technology

For the endogenous capacity technology, the set-up costs are assumed to be a differentiable function of capacity with  $F(0) = 0$ ,  $F' > 0$ ; and  $F'' > 0$ ; i.e., subject to decreasing returns. To show existence of a competitive equilibrium, a price path and allocation are defined from the following constrained optimization:

$$\begin{aligned} \max_{q_0(t), q(t), \bar{q}, T} & \int_0^T e^{-rt} [U(q_0(t)) - c_0 q_0(t)] dt - e^{-rT} NF(\bar{q}) \\ & + \int_T^\infty e^{-rt} [U(Q(t)) - c_0 q_0(t) - cq(t)] dt \end{aligned} \quad (9)$$

subject to the constraints in Eq. (2). As above, the optimization in Eq. (9) is similar to a social planner's problem except the optimization does not allow different capacities to be installed in the deep deposits at different times.

Defining  $p(t)$  as above, the Kuhn–Tucker conditions are as in Eq. (3) and the condition for optimal capacity installation is  $ANS^- + rNF(\bar{q}^*) = ANS^+$ . Additionally, the Kuhn–Tucker condition for optimal capacity is

$$\bar{q}^* \geq 0 \quad -e^{-rT} NF'(\bar{q}^*) + \int_T^\infty N\mu(t) dt \leq 0 \quad C.S., \quad (10)$$

where  $\bar{q}^*$  is the optimal capacity installed. Eq. (10) implies that the marginal cost of installing capacity should equal the discounted sum of the shadow values of the extraction constraint over all periods in which it binds.<sup>26</sup>

As above, the following lemmas are used to describe  $p(t)$ :

**Lemma 3.** *If the optimal capacity choice is positive, then  $c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} \leq U'(N\bar{q}^*)$ .*

**Proof.** See Appendix A.

**Lemma 4.** *The optimal capacity choice is positive if and only if  $c + rF'(0) < U'(0)$ .*

Proof. See Appendix A.

If positive capacities are installed, Lemmas 3 and 1 imply that the marginal benefit path is continuous. Lemma 4 shows that if the choke price is higher than the extraction cost plus the interest payment on installation of a marginal unit of capacity, then it is efficient to install positive capacity. The proposition on existence can now be shown:

**Proposition 2.** A competitive equilibrium exists for the endogenous capacity technology.

Proof. See Appendix A.

Proposition 2 shows that a competitive equilibrium exists for the endogenous capacity technology. The equilibrium is described by Eqs. (6), (7), (8), and (10). In the equilibrium all capacities are installed simultaneously while the price is rising and aggregate extraction is less than industry capacity. Firms choose extraction capacities which are small enough that there is a period of simultaneous extraction with the unconstrained deposit. By choosing small capacity, a firm can decrease its capacity installation cost. However, with a small flow capacity, it takes longer to extract the stock. The equilibrium capacity choice balances these costs and benefits and thus ensures that profits are non-negative.

Corollary 2. With the endogenous capacity technology, it is efficient to incur set-up costs in identical deposits simultaneously.

Proof. The first welfare theorem.

#### 4. Extensions and generalizations

The existence results can be generalized to analyze set-up costs in resources with various sizes and costs by noting that the shallow deposit can model all deposits for which set-up costs have already been sunk. An equilibrium with many deposits has the following properties: (a) the price is alternately flat or grows at the rate of interest, (b) the price is flat when industry capacity is fully utilized, (c) the price rises when the industry has excess capacity, (d) the marginal producer, for whom net price equals scarcity cost, has excess capacity, (e) set-up costs are incurred when the price reaches the trigger price for that deposit, (f) each trigger price is increasing in average cost but decreasing in deposit size, (g) additional capacity is installed when industry capacity is not fully utilized, (h) set-up costs are incurred simultaneously for identical deposits, and (i) extraction from a deposit is only below capacity when the price—net of extraction costs for that deposit—grows at the rate of interest.

If current extraction is exogenously constrained, the sufficient conditions in Proposition 1 can be generalized to a variety of cost structures.<sup>27</sup> For example, the results may still obtain even if marginal extraction costs are decreasing in current extraction or if set-up costs are unrelated to the costs of capacity installation. An exogenous extraction constraint can ensure that firms earn sufficient surplus even under these cost structures.

The case of increasing marginal extraction costs deserves special comment. Note that the fixed capacity technology with constant marginal costs approximates a marginal cost curve which is relatively flat for small quantities but then is very steep at larger quantities. The sufficient conditions in Proposition 1, which ensure that producers earn sufficient surplus to cover their capital costs, could be extended since firms earn surplus on their inframarginal extraction when marginal cost is upward sloping.

Proposition 2 demonstrates existence when set-up costs are a function of capacity and firms choose capacity. If set-up costs are a function of recoverable stock (e.g., when set-up costs are costs of exploration) the endogeneity of the set-up costs may not ensure existence. In this case, the firm equates the marginal cost of expanding the stock with the shadow value of the deposit. This ensures scarcity rents can cover the set-up costs, but does not avoid the problems for existence discussed in Section 2.

#### 5. Conclusion

The sufficient conditions for existence of competitive equilibrium allow the tools of competitive equilibrium analysis to be applied to problems in natural resource extraction with significant set-up costs. For example, first, the competitive equilibrium prices and allocations can be compared to observed market outcomes to estimate inefficiencies from market failures.<sup>28</sup> Second, the equilibrium trigger price for efficient capacity installation can be used in project evaluation. Third, the effects of taxes and subsidies on natural resource extraction in competitive markets can be analyzed.<sup>29</sup> The equilibrium characterized above shows that subsidizing (taxing) set-up costs would induce a competitive firm to install excessive (insufficient) capacity at too low (high) a price. Fourth, competitive equilibria can be useful for comparative static analysis. Schennach [29] observed that

extra bankable pollution permits during a transition period can be modeled as an exhaustible resource and derived comparative statics results on fuel switching. Competitive equilibrium analysis would have allowed simple, general derivations of many of the comparative statics and enabled analysis of the installation of pollution control technologies.<sup>30</sup> Fifth, the competitive equilibria can be regulatory equilibria. A regulator with sufficient information could impose the equilibrium price path as a price ceiling. This ceiling would then induce firms to install the efficient amount of capacity at the proper time and to extract efficiently.

Finally, empirical tests of the competitive theory of exhaustible resources have largely neglected set-up costs.<sup>31</sup> Krautkraemer [22] describes in situ tests and Hotelling valuation tests of natural resource theory. In situ tests rely on the assumption that the in situ value (shadow value) of a resource is the difference between the price and its marginal extraction cost.<sup>32</sup> The competitive equilibrium characterized above illustrates that, with set-up costs, the difference between price and marginal extraction cost may include the shadow value of capacity as well as the in situ value of the resource.<sup>33</sup> Similarly, Hotelling valuation tests rely on the principle that the market value of the firm should equal the product of the firm's current net price and its reserves.<sup>34</sup> With set-up costs, the equilibrium value of the firm is given in Eq. (A.3). Clearly the Hotelling valuation principle would need to be modified for industries with significant set-up costs.

Although set-up costs are prevalent and substantial in natural resource extraction, the previous literature has shown that their inclusion in the analysis implies that a competitive equilibrium cannot exist. However, non-existence is sensitive to the assumption of unlimited extraction capacity. When models correctly account for limited extraction capacity, the sufficient conditions imply existence of a competitive equilibrium. The sufficient conditions can facilitate use of the tools of competitive equilibrium analysis in analyzing natural resource markets with set-up costs.

#### Notes:

1 Set-up costs are significant empirically. In Cicchetti's [8] study of the Trans-Alaska oil pipeline, the initial capital cost was \$2 billion while annual operating costs were only \$95 million. Similarly, in Holland and Moore's (forthcoming) study of imported water use in Arizona, the set-up cost for building dams, aqueducts, and pumping stations was \$2.9 billion while the total annual operating costs were only \$54 million.

2 This paper analyzes sunk set-up costs. See Farzin [12] and Lozada [23] for analysis of reversible capital investment.

3 Farrell [10] first showed that a competitive equilibrium can exist in a non-convex economy with many agents, since the non-convexities yield only small discontinuities in aggregate demand or supply. Starr [30] later provided a rigorous treatment of this result. Aumann [2] showed the existence of a competitive equilibrium in a non-convex economy with a continuum of agents.

4 In a related literature, Eswaran et al. [9] showed that a competitive equilibrium does not exist if there are fixed costs of extraction which can be avoided by extracting nothing in a given period. See also [13,21,24,26].

5 In an industry with significant set-up costs, non-cooperative analysis may sometimes be appropriate. See Hogan and Holland [18] on the difficulties of identifying subgame perfect equilibria in natural resource extraction.

6 Cicchetti simply assumes the set-up costs of the Trans-Alaska Pipeline are spread evenly across the entire deposit. Influential studies of natural resource extraction by Nordhaus [27] and Chakravorty et al. [5] do not explicitly model set-up costs.

7 See Krautkraemer [22] or Chermak and Patrick [7] for a review of the literature on empirical tests of exhaustible resource theory.

8 Strictly sequential extraction is efficient since any extraction plan with simultaneous extraction from two deposits is dominated by a plan which delays the set-up cost of one of the deposits and reallocates extraction between the two deposits such that consumption is unchanged. This reallocation dominates the initial plan since extraction costs and consumption are unchanged but the set-up cost is delayed. With non-constant marginal extraction costs, the proposed reallocation would change the present value of extraction costs and thus would not necessarily dominate the plan with simultaneous extraction. Non-constant marginal extraction costs will be discussed in Section 4.



9 Clearly, a single, unregulated firm would be expected to exert market power and would not be a price taker. Although this example is intended purely for illustrative purposes, the fact that each firm has an incentive to delay set-up costs also is true for multiple firms and has the same consequences for existence of a competitive equilibrium.

10 Recall that both welfare theorems require price-taking behavior and complete markets, and the Second Welfare Theorem additionally requires convexity. Thus in the models under consideration here, the First Theorem holds, but the Second Theorem is not applicable.

11 In a static natural monopoly, a competitive equilibrium price cannot exist since firms have negative profit under marginal-cost pricing. In natural resource extraction, firms earn scarcity rents, and thus may earn positive profits from efficient pricing if set-up costs are small. However, the preceding argument implies that a competitive equilibrium does not exist, even if firms could earn positive profits by pricing competitively.

12 The Hartwick et al. result is that a set-up cost should be incurred when it generates a sufficient increase in surplus to cover the interest payment on the set-up cost. Since producers never receive any surplus in the constant marginal cost model, it is consumer surplus which must increase discontinuously, and thus the marginal benefit path jumps down.

13 The shallow deposits model deposits—possibly owned by many owners—which do not require set-up costs. Since set-up costs are sunk costs, the extensions of the basic model in Section 4 interpret the shallow deposits as deposits for which set-up costs have already been incurred.

14 The results can be extended to cost structures where extraction costs depend on current and/or cumulative extraction. See Section 4 for a discussion of extending the model to non-constant marginal extraction costs. See [19] for a similar model where extraction costs depend on cumulative extraction.

15 Since set-up costs are only incurred once, the production set is non-convex, and the usual existence theorems are not applicable. Models of capacity installation by Switzer and Salant [31], Olsen [28] and Lozada [23] allow for capacity to be expanded at any time and have convex production sets.

16 If the choke price,  $U'(0)$ , is finite, then efficiency requires that the deposit be exhausted when the price reaches the choke price. If the choke price is infinite, then exhaustion need not occur in finite time.

17 Consumer surplus is assumed constant over time. The results may not generalize to extreme demand shifts over time since it may not be possible to find a trigger price which signals efficient investment.

18 The fixed capacity technology describes a scenario where other factors have constrained the firm's range of capacity choices. Such constraints might be, for example, engineering, environmental or political.

19 Since the optimization is of a continuous function over a compact domain, a maximum exists.

20 Note that price equal to augmented marginal cost is the well-known Hotelling rule that net price grows at the rate of interest.

21 “Augmented” net surplus is smaller than the usual surplus measure since the costs are augmented by the scarcity cost. Note that ANS measures the flow of surplus and thus the set-up costs are not included in the definition of surplus.

22 This condition on augmented surplus is identical to the Hartwick et al. condition on the Hamiltonian, namely:  $H + rNF = H^+$ .

23 The static ( $\lambda = 0$ ) intuition for the condition  $c + \lambda e^{rT} + rF/\bar{q} \leq U'(N\bar{q})$  is that the average cost be less than the marginal benefit when production is at capacity. Note that this is precisely the condition which ensures existence of a competitive equilibrium in a static model with fixed costs.

24 To understand the intuition behind these two sufficient conditions, consider the case where the first condition holds with equality, i.e.,  $c + rF/\bar{q} = U'(N\bar{q})$ . In this case, the maximum producer surplus that a competitive firm can earn while producing at capacity is  $rF$ . If the firm is to earn sufficient producer surplus to cover the entire set-up cost, the firm would need to produce forever. Thus the “minimum” deposit size would need to be infinite, i.e., a backstop.

25 The endogenous shadow values are fully specified by the equations insuring continuity at  $\bar{T}$  and  $\hat{T}$ ; the first order condition in Eq. (A.1), and the stock constraints:  $\int_0^\infty q_0(t) dt = S_0$  and  $\int_0^\infty q(t) dt = NS$ .

26 In peak-load pricing problems, a similar condition holds which requires that the capacity cost be spread across all the peak periods in which the capacity binds. Campbell [4] gives a similar result in an exhaustible resource model.

- 27 Chermak and Patrick [6] argues that periodic extraction from gas wells is constrained by past extraction. Farrow [11] also mentions discussions of “rated capacity” in the mining engineering literature.
- 28 Borenstein et al. [3] use competitive equilibrium benchmarking to evaluate market power in the deregulated California electricity markets.
- 29 The models of Hartwick et al. and Fischer make no predictions about firm behavior and thus cannot be used as a basis for such analysis.
- <sup>30</sup>Typical SO<sub>2</sub> pollution control technologies have significant set-up costs on the order of 20 times annual operating costs, US Environmental Protection Agency [32] but have limited capacities.
- 31 Empirical tests have struggled with non-convexities from declining average costs (see [6]) but have not dealt with non-convexities arising from set-up costs.
- 32 See, for example, [7,11,15].
- 33 Farrow [11] extended his analysis by correcting the in situ value when extraction was at full capacity. However, modeling the capacity constraint explicitly did not allow him to avoid rejecting the Hotelling theory for his data, i.e., his estimate of the discount rate was still negative.
- 34 See Miller and Upton [25] for a seminal Hotelling valuation test. Krautkraemer describes several extensions to the Hotelling valuation principle.
- 35 The Herfindahl result implies that for two deposits with different marginal costs,  $c_1$  and  $c_2$ , and shadow values,  $\lambda_1$  and  $\lambda_2$ , the price path is continuous and is given by  $p(t) = \min\{c_1 + \lambda_1 e^{rt}; c_2 + \lambda_2 e^{rt}\}$ .

<sup>36</sup>If the choke price,  $U'(0)$ , is finite, then all deposits must be exhausted at time  $T^c$  when the price path reaches the choke price, i.e.,  $c + \lambda e^{rT^c} = U'(0)$ . Thus the amount extracted while the constrained deposits produce below capacity can be written

$$A = \int_0^{T^c - \hat{t}} D(c + (U'(N\hat{q}) - c)e^{rt}) dt.$$

Note that since  $e^{r(T^c - \hat{t})} = \frac{U'(0) - c}{U'(N\hat{q}) - c}$ ,  $A$  is still a function solely of exogenous parameters.

<sup>37</sup>The marginal profit from an extra unit of stock is  $\lambda$  since the unit must be extracted when capacity is not binding. Thus the firm's shadow value equals the shadow value from the optimization of the constrained planner's problem.

## Appendix A

**Lemma A.1.**  $p(t)$  is continuous if and only if  $c + \lambda e^{rT} + \frac{rF}{q} \leq U'(N\bar{q})$ . If  $p(t)$  is continuous, then  $p(T) = c + \lambda e^{rT} + \frac{rF}{q}$ . If  $p(t)$  is not continuous, capacities are installed after the shallow deposit is exhausted.

**Proof.** When extraction is from the shallow deposit,  $p(t) = c_0 + \lambda_0 e^{rt}$  is continuous. When extraction is only from the deep deposits,  $p(t) = \max\{U'(N\bar{q}), c + \lambda e^{rt}\}$ . After capacities have been installed, the Herfindahl [17] result<sup>35</sup> applied to the optimization implies that  $p(t) = \min\{c_0 + \lambda_0 e^{rt}, \max\{U'(N\bar{q}), c + \lambda e^{rt}\}\}$  which is continuous. Thus if  $p(t)$  is not continuous, capacities must be installed after  $S_0$  is exhausted, and the discontinuity occurs when the capacities are installed. At  $T$ ,  $p(t)$  cannot jump up since if  $p(t)$  jumped up at exhaustion of  $S_0$ , the objective function could be increased by increasing  $\lambda_0$ . Thus, the only possible discontinuity is for  $p(t)$  to jump down at  $T$ .

If  $p(t)$  jumps down at  $T$ , then  $U'(q_0^-) > U'(Q^+)$  and  $q_0^- < Q^+$ . If  $c + \lambda e^{rT} > U'(N\bar{q})$ , then the result follows trivially. If  $c + \lambda e^{rT} \leq U'(N\bar{q})$ , then consumption is at capacity after  $T$ , i.e.,  $Q^+ = N\bar{q}$ . Then by Eq. (4),  $ANS^- = ANS^+ - rNF = U(N\bar{q}) - (c + \lambda e^{rT} + \frac{rF}{q})N\bar{q}$ . Since consumer surplus is increasing  $ANS^- = U(q_0^-) - U'(q_0^-)q_0^- < U(N\bar{q}) - U'(N\bar{q})N\bar{q}$ . Therefore  $c + \lambda e^{rT} + \frac{rF}{q} > U'(N\bar{q})$ .

If  $p(t)$  is continuous at  $T$ , then  $p(t) = U'(q_0^-) = U'(Q^+)$ . The assumptions on  $U$  then imply that  $U(q_0^-) = U(Q^+)$  and also  $q_0^- = Q^+$ . Since this implies that  $q_0^- = q_0^+ + N\bar{q}$ , Eq. (4) can be written

$$p(T) = c_0 + \lambda_0 e^{rT} = c + \lambda e^{rT} + \frac{rF}{q}. \quad (\text{A.1})$$

Since  $q_0^+ \geq 0$ , it follows that  $Q^+ \geq N\bar{q}$  and  $U'(Q^+) \leq U'(N\bar{q})$  and thus  $c + \lambda e^{rT} + \frac{rF}{q} = c_0 + \lambda_0 e^{rT} = p(T) = U'(Q^+) \leq U'(N\bar{q})$ .  $\square$

**Lemma A.2.** If  $c + \frac{rF}{q} \leq U'(N\bar{q})$  and  $S \geq S^{\min}$  then  $c + \lambda e^{rT} + \frac{rF}{q} \leq U'(N\bar{q})$ .

**Proof.** When  $S$  is smaller, its shadow value,  $\lambda$ , and scarcity cost,  $\lambda e^{rt}$ , are larger. Therefore, there is a minimum stock,  $S^{\min}$ , for which the condition holds with equality, i.e., for which  $c + \lambda e^{rT} + \frac{rF}{q} = U'(N\bar{q})$ . Using this equation and the fact that  $c + \lambda e^{r\hat{T}} = U'(N\bar{q})$ , it follows that

$r(\hat{T} - T) = \ln(U'(N\bar{q}) - c) - \ln(U'(N\bar{q}) - c - \frac{rF}{q})$ . Next note that when extraction from the constrained deposits is below capacity, i.e.,  $t \geq \hat{T}$ , the amount extracted from these deposits is independent of  $S$ . This amount,  $A$ , can be written as a function of the exogenous parameters:<sup>36</sup>

$$A = \int_{\hat{T}}^{\infty} D(c + \lambda e^{rt}) dt = \int_0^{\infty} D(c + (U'(N\bar{q}) - c)e^{rt}) dt.$$

Noting that  $S^{\min}$  equals  $\bar{q}(\hat{T} - T) + \frac{1}{N}A$  completes the derivation.  $\square$

**Proposition A.1.** A competitive equilibrium exists with the fixed capacity technology if  $c + \frac{rF}{q} \leq U'(N\bar{q})$  and  $S \geq S^{\min}$  where  $S^{\min}$  is defined in Eq. (5).

**Proof.** Consider the price path,  $p(t)$ , as presented in Eq. (6), the production allocations as in Eqs. (7) and (8), and the implied consumption path. Lemmas 1 and 2 imply that  $p(t)$  is continuous. Since  $p(t)$  equals the marginal benefit of consumption in every period, consumers are optimizing. Next consider the owner of the shallow deposit  $S_0$ . The present value of the profit per unit equals  $\lambda_0$  in each period with positive extraction and would be smaller in periods with no extraction. Therefore, the owner of the unconstrained deposit is maximizing profit given  $p(t)$ . Finally consider firm  $i$ , the owner of one of the deep deposits,  $S_i$ . Can firm  $i$  increase profit by decreasing extraction in some period and increasing extraction in another period? Firm  $i$  would want to decrease extraction after  $\hat{T}$ , since the present value of profit per unit,  $\lambda$ , is lowest then. The firm cannot increase extraction in the interval  $[T, \hat{T}]$  since extraction is already at capacity. In order to increase production before  $T$ , the capacity must be installed earlier. If firm  $n$  installs capacity at  $T_n$ , the Lagrangian for its profit maximization is<sup>37</sup>

$$L_n = \int_{T_n}^{\infty} e^{-rt} [p(t) - c]q_n(t) - \lambda q_n(t) + \mu(t)[\bar{q} - q_n(t)] dt - e^{-rT_n}F + \lambda S_n \quad (\text{A.2})$$

The derivative with respect to  $T_n$  of the Lagrangian is

$$\frac{dL_n}{dT_n} = -e^{-rT_n} [(p(T_n) - c - \lambda e^{rT_n})\bar{q} - rF].$$

Therefore, the profit maximizing time to install the capacity is when  $p(T_n) = c + \lambda e^{rT_n} + \frac{rF}{\bar{q}}$  which implies that  $T_n = T$ . Thus firm  $n$  cannot increase profits by reallocating extraction. Can the firm increase profit by never incurring the set-up cost and earning zero profit? Using a calculation from

Appendix B shows that firm  $n$ 's profit:

$$\begin{aligned}
\pi_n &= \int_T^\infty e^{-rt} [p(t) - c] \frac{1}{N} q(t) dt - e^{-rT} F = \int_T^\infty [\lambda + \mu(t)] \frac{1}{N} q(t) dt - e^{-rT} F \\
&= \lambda S + \left[ e^{-rT} \frac{F}{\bar{q}} + (\bar{T} - T)\lambda_0 - (\hat{T} - T)\lambda \right] \bar{q} - e^{-rT} F \\
&= \lambda \frac{1}{N} A + \lambda_0 \bar{q} (\bar{T} - T) \geq 0
\end{aligned} \tag{A.3}$$

is non-negative. Therefore, firm  $n$  is maximizing profit given  $p(t)$ . Thus,  $p(t)$ ,  $q_0(t)$ , and  $q(t)$  form a competitive equilibrium, and a competitive equilibrium exists.  $\square$

**Lemma A.3.** *If the optimal capacity choice is positive, then  $c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} \leq U'(N\bar{q}^*)$ .*

**Proof.** Suppose  $c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} > U'(N\bar{q}^*)$ . Lemma A.1 then implies that  $p(t)$  is discontinuous at  $T$ , and extraction is strictly sequential. The integral over the shadow value of the capacity constraint is then given by the second calculation in Appendix B:

$$\int_T^\infty \mu(t) dt = \frac{1}{r} [e^{-rT} (U'(N\bar{q}^*) - c) - \lambda] - (\hat{T} - T)\lambda.$$

But then

$$\begin{aligned}
\int_T^\infty \mu(t) dt &< \frac{1}{r} \left[ e^{-rT} \left( c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} - c \right) - \lambda \right] - (\hat{T} - T)\lambda \\
&= e^{-rT} \frac{F(\bar{q}^*)}{\bar{q}^*} - (\hat{T} - T)\lambda \leq e^{-rT} F'(\bar{q}^*).
\end{aligned}$$

This strict inequality implies, by the Kuhn–Tucker condition in Eq. (10), that  $\bar{q}^* = 0$ .  $\square$

**Lemma A.4.** *The optimal capacity choice is positive if and only if  $c + rF'(0) < U'(0)$ .*

**Proof.** Note that in Lemma A.2, the limit of the minimal stock as the capacity becomes smaller is zero, i.e.,  $\lim_{\bar{q} \rightarrow 0} S^{\min} = 0$ . Note also that  $\lim_{\bar{q} \rightarrow 0} \frac{F(\bar{q})}{\bar{q}} = F'(0)$ . Thus if  $c + rF'(0) < U'(0)$ , there exists a small capacity,  $\bar{q}'$ , such that  $c + \frac{rF(\bar{q}')}{\bar{q}'} \leq U'(N\bar{q}')$  and  $S \geq S^{\min}$ . But then Lemma A.2 implies that installing capacity  $\bar{q}'$  and extracting the constrained deposits yields a higher maximand to the constrained planner's problem than installing zero capacity. Hence the optimal capacity must be positive.

Conversely, if  $c + rF'(0) \geq U'(0)$ , then  $\bar{q}^* = 0$ . To show this, suppose the optimal capacity were greater than zero, i.e.,  $\bar{q}^* > 0$ . But then

$$c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} \geq c + \frac{rF(\bar{q}^*)}{\bar{q}^*} \geq c + rF'(0) \geq U'(0) > U'(N\bar{q}^*).$$

This contradicts Lemma A.3, hence  $\bar{q}^* = 0$ .  $\square$

**Proposition A.2.** *A competitive equilibrium exists for the endogenous capacity technology.*

**Proof.** If  $c + rF'(0) \geq U'(0)$ , the price path  $p(t) = \min\{c_0 + \lambda_0 e^{rt}, U'(0)\}$  is a competitive equilibrium price path, where no capacity is installed and the constrained deposits are not extracted.

If  $c + rF'(0) < U'(0)$ , consider the price path  $p(t)$  defined from the above constrained planner's problem. Lemma A.4 ensures that positive capacity is installed. Lemma A.3 together with Lemma A.1 imply that  $p(t)$  is continuous and hence is as defined in Eq. (6) and illustrated in Fig. 2. As above, consumers and the owner of the unconstrained deposit are optimizing given  $p(t)$ . The Lagrangian for an owner of a constrained stock is in Eq. (A.2) where the set-up cost is  $F(\bar{q})$  and the firm chooses  $q_n(t)$ ,  $\bar{q}_n$ , and  $T_n$ . As argued in the proof of Proposition A.1, profits cannot be increased by changing  $q_n(t)$  given the capacity and time of installation. The first-order conditions for profit maximizing  $T_n$  and  $\bar{q}_n$  are

$$\frac{dL_n}{dT_n} = -e^{-rT_n}[(p(T_n) - c - \lambda e^{rT_n})\bar{q}_n^* - rF(\bar{q}_n^*)] = 0$$

and

$$\frac{dL_n}{d\bar{q}_n} = \int_{T_n}^{\infty} \mu_n(t) dt - e^{-rT_n} F'(\bar{q}_n^*) = 0.$$

These are precisely the first-order conditions for the constrained planner's problem. Thus,  $T_n = T$ ,  $\bar{q}_n^* = \bar{q}^*$ , and firms are maximizing profits given  $p(t)$ . Therefore, this price and allocation is a competitive equilibrium, and hence a competitive equilibrium exists.  $\square$

## Appendix B

To calculate the integral of the shadow value of the capacity constraint if all capacities are installed simultaneously and  $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ , first note that Eq. (3) implies  $\mu(t) = e^{-rt}(p(t) - c) - \lambda$  when there is extraction from the constrained deposit. Substituting for  $p(t)$ , recalling that  $p(T) = c + \lambda e^{rT} + \frac{rF}{\bar{q}}$ , and noting that  $\mu(t)$  is continuous yields:

$$\mu(t) = \begin{cases} e^{-rT} \frac{rF}{\bar{q}} & \text{if } t = T, \\ e^{-rt}(c_0 - c) + \lambda_0 - \lambda & \text{if } t \in [T, \bar{T}], \\ e^{-rt}(\bar{p} - c) - \lambda & \text{if } t \in [\bar{T}, \hat{T}], \\ 0 & \text{if } t \in [\hat{T}, \infty], \end{cases}$$

where  $\bar{p} \equiv U'(N\bar{q})$ . Thus

$$\int_T^{\infty} \mu(t) dt = \int_T^{\bar{T}} e^{-rt}(c_0 - c) + (\lambda_0 - \lambda) dt + \int_{\bar{T}}^{\hat{T}} e^{-rt}(\bar{p} - c) - \lambda dt$$

$$\begin{aligned}
&= \frac{1}{r}(e^{-rT} - e^{-r\bar{T}})(c_0 - c) + (\bar{T} - T)(\lambda_0 - \lambda) \\
&\quad + \frac{1}{r}(e^{-r\bar{T}} - e^{-r\hat{T}})(\bar{p} - c) + (\hat{T} - \bar{T})(-\lambda) \\
&= \frac{1}{r}[\mu(T) - \lambda_0 + \lambda - [\mu(\bar{T}) - \lambda_0 + \lambda]] \\
&\quad + \frac{1}{r}[\mu(\bar{T}) + \lambda - [\mu(\hat{T}) + \lambda]] + (\bar{T} - T)\lambda_0 - (\hat{T} - T)\lambda \\
&= \frac{1}{r}[\mu(T)] + (\bar{T} - T)\lambda_0 - (\hat{T} - T)\lambda \\
&= e^{-rT} \frac{F}{\bar{q}} + (\bar{T} - T)\lambda_0 - (\hat{T} - T)\lambda.
\end{aligned}$$

If  $c + \lambda e^{rT} + \frac{rF}{\bar{q}} > U'(N\bar{q})$ , Lemma A.1 implies that optimal extraction has no phase of simultaneous extraction between the unconstrained deposit and all the constrained deposits, i.e.,  $T = \bar{T}$ . The integral of the shadow values is then

$$\begin{aligned}
\int_T^\infty \mu(t) dt &= \int_T^{\hat{T}} e^{-rt}(\bar{p} - c) - \lambda dt \\
&= \frac{1}{r}(e^{-rT} - e^{-r\hat{T}})(\bar{p} - c) + (\hat{T} - T)(-\lambda) \\
&= \frac{1}{r}[\mu(T)] - (\hat{T} - T)\lambda \\
&= \frac{1}{r}[e^{-rT}(\bar{p} - c) - \lambda] - (\hat{T} - T)\lambda.
\end{aligned}$$

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