SALTER, JONATHAN R., D.M.A. Chaos in Music: Historical Developments and Applications to Music Theory and Composition. (2009)
Directed by Dr. Kelly Burke. 208 pp.

The Doctoral Dissertation submitted by Jonathan R. Salter, in partial fulfillment of the requirements for the degree Doctor of Musical Arts at the University of North Carolina at Greensboro comprises the following:

1. Doctoral Recital I, March 24, 2007: Chausson, *Andante et Allegro*; Tomasi, Concerto for Clarinet; Bartók, *Contrasts*; Fitkin, *Gate*.

2. Doctoral Recital II, December 2, 2007: Benjamin, *Le Tombeau de Ravel*; Mandat, *Folk Songs*; Bolcom, Concerto for Clarinet; Kovács, *Sholem-alekhem, rov Fiedman*!

3. Doctoral Recital III, May 3, 2009: Kalliwoda, *Morceau du Salon*; Shostakovich, Sonata, op. 94 (transcription by Kennan); Tailleferre, *Arabesque*; Schoenfield, Trio for Clarinet, Violin, and Piano.

4. Dissertation Document: Chaos in Music: Historical Developments and Applications to Music Theory and Composition.

Chaos theory, the study of nonlinear dynamical systems, has proven useful in a wide-range of applications to scientific study. Here, I analyze the application of these systems in the analysis and creation of music, and take a historical view of the musical developments of the 20th century and how they relate to similar developments in science. I analyze several 20th century works through the lens of chaos theory, and discuss how acoustical issues and our interpretation of music relate to the theory. The application of nonlinear functions to aspects of music including organization,

acoustics and harmonics, and the role of chance procedures is also examined toward suggesting future possibilities in incorporating chaos theory in the act of composition. Original compositions are included, in both sheet music and recorded form.

Chaos in Music: Historical Developments and Applications to Music Theory and Composition

by
Jonathan R. Salter

A Dissertation Submitted to
the Faculty of The Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Musical Arts

Greensboro
2009

Approved by

Dr. Kelly Burke

Committee Chair

To Anne, for equanimity in the face of chaos.

Approval Page

This dissertation has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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Acknowledgments

The contributions to this work by Dr. Kelly Burke cannot be overstated; it was primarily under her advice that I embarked upon this research. Thanks also to Dr. Gavin Douglas, Dr. Greg Carroll, and Dr. Steven Stusek for their patience, understanding, assistance in the process and suggestions for revisions. Dr. Alejandro Rutty and Dr. Mark Engebretson were incredibly helpful in guiding my compositional process. In addition, many kind colleagues and professors at the University of North Carolina at Greensboro played a role in helping me flesh out ideas and theories; alas, far too many to name individually, though they all deserve my thanks. Dr. Edward Burger, Professor of Mathematics at Williams College, served as an outside reader for this document and contributed many helpful remarks and a few corrections.

I also owe a debt of gratitude to Eli Eban, who incidentally referred to *chaos* quite frequently in our lessons (though, unfortunately, it was in reference to my clarinet playing), as well as Alan Kay, Susan Martula, and Michèle Gingras, who each played an essential role in my development as a clarinetist and musician.

Preface

"A journey of a thousand miles begins with a single step."

— Lao-Tzu

Chaos, though applied to many different areas of study, is inherently a mathematical idea. While this dissertation does not require an extraordinary amount of mathematical background to grasp the main concepts, experience with some of the basic tools of mathematics (functions, systems, equations, etc.) is assumed. Here, I will not shy away from using explicit mathematical formulas, and those less accustomed to reading mathematics should feel free to skim the mathematical sections and spend more time on the analysis and application of these mathematical ideas.

Some background in music analysis is also assumed, though again, those readers less familiar with music theory may wish to pay more attention to the application of the analysis than the analysis itself.

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Definitions

Chaos The absence of order. In mathematics, *chaos* is used to describe functions that are deterministic but unpredictable.

Chaos Theory The collection of accumulated information on nonlinear dynamical (chaotic) systems.

Entropy In information theory, a measure of the uncertainty inherent in data.

Information Theory The study of the communication and transmission of data.

Fractal A geometric object with infinite levels of detail which exhibits self-similarity.

Function A mathematical formula which relates input values to their resultant values, such as: $y = x^2$ or $x_{n+1} = x_n + 5$.

Iterative Function A mathematical function which is repeatedly applied to its own result to produce a sequence of values.

Nonlinearity A nonlinear function is one which is not of the form $y = ax + c$; that is, it contains at least one nonlinear term, such as $y = x^2$ or $y = \sin c$. Until chaos theory, most of mathematics and physics was concerned with reducing nonlinear functions to their linear approximations to solve the system.

Nonlinear Dynamical Systems A collection of mathematical functions which exhibits nonlinearity: small changes in initial conditions can result in unpredictably large differences in the result.

Orbit A sequence of numbers which result from the iteration of a function. A periodic orbit returns to its initial value eventually, while a quasi-periodic orbit consistently returns to nearby values from its initial point.

Stochastic Function A function which utilizes chance or randomness.

Torus A doughnut-shaped geometric surface, technically the result of revolving a circle around a coplanar nonintersecting axis.

Introduction

"In the nature of things nothing accidental is granted, but all things are determined by the necessity of the divine nature for existing and working in a certain way. In short, there is nothing accidental in nature."

— Benedict de Spinoza

A butterfly flaps its wings in Brazil, and a month later tornadoes ravage Oklahoma. A split second decision as a traffic light turns yellow causes you to miss the accident you would have been in if instead you had zoomed through the light. Gavrilo Princip pulls his trigger, shooting Archduke Ferdinand in 1914. This one little motion of a single finger sparks a series of events which leads to a war that rages across Europe and changes the course of history for the rest of the 20th century. There are a wide variety of small decisions we make on a daily basis; we are all familiar with the immense consequences these seemingly insignificant decisions can have. This is one of the most basic ways in which we interact with chaos in our everyday lives.

The modern sense of the word *chaos* is informally used to indicate the absence of order. It is then somewhat of a misnomer that modern science uses this term to describe what is known as *chaos theory*, since the scientific use of *chaos* is entirely deterministic; that is, once the initial conditions of a chaotic system are known, all of the future states can be predicted; this is hardly the absence of order. It is the inability to measure and know all of the initial conditions for any system that makes a potentially deterministic universe so unpredictable.

Chaos theory, the study of nonlinear dynamical systems, is not a "theory" in the sense that it is an unproven hypothesis, but rather that it is a cohesive group of ideas which fall under a single designation. The various ideas and techniques which fall under the label of chaos theory are not intended for any particular area of science. Like algebra or calculus, chaos theory involves the study of universal mathematical systems which match a certain set of criteria and exhibit certain behaviors. The application of chaos theory is wide-ranging, providing results that range from mathematics, physics, biology, chemistry, astronomy, physiology, sociology, psychology, even art and music, and can be found in many of the most accurate mathematical models of natural phenomenon. In other words, chaos theory analyzes the complexity that underlies our universe and all of its parts. Isaac McPhee sums up the role of chaos theory as "not a specific genre of mathematics as much as it is a statement of the limitations of human knowledge."[1]

With the widespread application of chaos theory and its implications throughout the scholarly community, it is necessarily beyond the scope of this document to outline every application of chaos theory, even when limited to musical applications. This document is not intended to serve as a comprehensive summary of all of the research into chaos theory and music, but instead is focused on three primary goals:

1. To provide a common background for musicians and mathematicians in order to foster interdisciplinary communication;

2. To show examples of the application of chaos theory to musical issues;

3. To provide a toolbox for the incorporation of chaotic principles to music composition.

Part I will be concerned with the development of chaos theory as well as the simultaneous developments in the history of music, outlining the similar trends oc-

[1] Isaac M. McPhee. *Chaos Theory and Water Droplets*. Accessed 1/7/2009. URL: `http://mathchaostheory.suite101.com/article.cfm/chaos_theory_and_water_droplets`.

curring in both of these areas. Part II will address the application of chaos theory to our understanding of music, through analysis of music as well as issues of acoustics. Finally, Part III will examine the implications of chaos theory for performance issues, as well as applications of chaotic algorithms to the process of music composition.

Part I

History

Chapter I
The Development of Chaos

"Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry."

— Richard Feynman

While chaos theory revolutionized the way science understood the role of nonlinear dynamics in modeling natural phenomena, a historical look at the development of the theory itself also changes the way we understand how scientific revolutions can occur. Unlike other revolutionary discoveries, such as Newton's Laws of Motion or Einstein's Theory of General Relativity, chaos theory did not have a single individual proponent who explained and propagated his revolutionary views. Rather, the development followed a nonlinear and unpredictable course from discovery to acceptance. The theory was born out of smaller discoveries in isolated fields, and emerged from these smaller ideas to form a cohesive, universal theory.

Certainly, many individuals played an important role in the discovery and development of this theory, and here I will explore the contributions of many of the important theorists who led the way to the acceptance and incorporation of nonlinear dynamics into our scientific understanding. Like any history, the focus here must be on the people themselves as well as the ideas they advanced, and so this chapter will proceed chronologically (where possible) through the many contributors whose ideas culminated in the development of the theory, though a truly comprehensive summary of all of the contributors is beyond the scope of this document. I will explore each of the mathematical ideas as they occur, and conclude with the modern definition of chaos theory after each stone of the foundation is set in place.

1.1 Pythagoras and the Greeks

"Numbers rule the Universe."

— Pythagoras

The word "chaos" originates in Greek mythology. The Greeks imbued this word with two meanings: the primeval emptiness of the universe, and the abyss of the underworld.[1] This sense of chaos was held in contrast to the ordered universe the Greeks themselves inhabited, and this is where the sense of chaos as "disorder" and later "unpredictability" originated; the first chaotic system was the vast universe which surrounds us.

Much is different between our universe and theirs. To begin with, the Greeks would find our modern system of scholarship strange. All too often, modern scholars are pigeon-holed into their respective areas of expertise, and the student who is an avid learner in a variety of disciplines is encouraged to specialize for the purposes of employment. Pythagoras was not a mathematician, philosopher, religious leader, politician, or music theorist; he was all of these.

Or perhaps none. It is difficult to know much about Pythagoras himself, since he surrounded himself with a cult of secrecy which did not allow the transmission of ideas to the outside world. He himself wrote nothing, and the few surviving accounts

[1] Garnett P. Williams. *Chaos Theory Tamed*. Washington, D.C.: Joseph Henry Press, 1997, p. 17; Ian Stewart. *Does God Play Dice*. 2nd ed. Malden, Mass: Blackwell Publishers, 2002, p. 1

of Pythagoras are put into doubt by modern scholars. Most of what we know was written by Dicaearchus, a pupil of Aristotle.[2]

It is thought today that almost all of the important accomplishments attributed to Pythagoras were actually discovered by his followers, the Pythagoreans, and most of the important work (especially with regard to mathematics and philosophy) was probably accomplished long after his death. They attributed these ideas to Pythagoras due to his godlike status among this cult. Hermann writes:

> In truth, we cannot credit the founder of the Pythagorean movement with any of its philosophical advances, unless we consider the transmigration of the soul, immortality, musical harmony, magic, vegetarianism, purification rites, and initiations to be proper philosophical pursuits. If anything, it is only these rather esoteric interests that can be traced to the Great Sage.[3]

Of course, musical harmony's status as an "esoteric interest" is arguable, and Hermann's own account of Pythagorean beliefs show how important Pythagoras's ideas about music theory become to the Pythagoreans that followed him. "The original idea of looking to number for universal answers seems to have been hatched by the old sage himself, and it came, we are told, from his study of harmony."[4]

Pythagoras founded a religious brotherhood for the purpose of studying mathematics around 531 B.C.. The primary accomplishments of the Pythagoreans revolve around the idea of *Number*. Numbers, here meaning what we now call *natural* numbers, were the key to understanding reality, and numerology was considered a central part of mathematics. "In fact, according to Aristotle, certain Pythagoreans believed

[2]Diané Collinson and Kathryn Plant. *Fifty Major Philosophers*. 2nd ed. New York: Routledge, 2006, p. 7.

[3]Arnold Hermann. *To Think Like God*. Las Vegas: Parmenides Publishing, 2004, p. 17.

[4]Ibid., p. 93.

that things either *were* numbers, or were *made* of numbers."[5] Each number was given extra-numerical significance. Ten, for example, represented perfection.

Pythagoras also believed in *metempsychosis*, or the transmigration of the soul (reincarnation), a belief probably obtained from the Egyptians.[6] He claimed to have memories from all of his prior lives.

> The best attested part of Pythagoras' teaching is that which concerned the souls of men and their destiny. The soul is a unity which is immortal; it is rational and responsible for its actions. Its fate is determined by those actions, as it lives through successive incarnations in human bodies or those of other animals or plants. By keeping itself pure, that is, free from the pollution of the bodily passions which beset it in these incarnations, it can eventually rise to its true or proper god-like state. But if it sins, it is punished and purified by prolonged suffering in more miserable incarnations. In other words, the soul is not at home in the body and must be kept apart from it as far as possible.[7]

Music played a vital role in this philosophy, as the proportion of musical notes of a scale epitomized the idea of *unit* and a proper proportional relationship between these *units*. Pythagoreans "used music to purify the soul,"[8] and felt that music was responsible for the ascent of the soul to the divine.[9]

The proportional relationship found in music on earth was thought to also be present in the proportions of the heavens.

[5]Ibid., p. 16.

[6]Jonathan Barnes. *Early Greek Philosophy*. 2nd ed. New York: Penguin Books, 2001, p. 33.

[7]Edward Hussey. *The Presocratics*. New York: Charles Scribner's Sons, 1973, p. 64.

[8]George L. Abernethy and Thomas A. Langford. *Introduction to Western Philosophy: Pre-Socratics to Mill*. Belmont, California: Dickenson Publishing Company, Inc., 1970, p. 10.

[9]Hermann, op. cit., p. 104.

> He thought that the heavens were like a musical scale, that the stars produced harmonies, and that souls at their best must be harmonious with the heavens. That musical scales can be expressed numerically was another reason for regarding number as fundamental and primary in the cosmos.[10]

Hermann writes: "Numbers, then, proved that music was more than just a pleasing noise; it reflected a hidden but rigorous order."[11] This obsession with the discovery of order in a seemingly disordered universe would pervade scientific reasoning throughout history, as science grew primarily concerned with uncovering the laws and reasons for the phenomena of our universe.

The sound of the heavens was thought to be present throughout our lives, such that it became background noise that could no longer be recognized, much like any persistent noise becomes ignored after a while. Aristotle refuted the idea that the heavens made a sound, but to some degree the main point of the so-called "Music of the Spheres" is not the actual sound itself, but the idea that the same proportions found here on Earth would be present in the heavens, a sort of universalism of proportionality obtained through an analogy to music.

> The ratio for the octave is 1:2, because the string that is half as long will move twice as fast as the other string. Additional intervals are set at the ratio of 3:2 for the fifth, and 4:3 for the fourth. Thus, the basic intervals of a musical scale were expressible in only four numbers: 1, 2, 3, and 4. When 1+2+3+4 are added up, we arrive at 10, the number considered by the Pythagoreans as most perfect and divine.
>
> The discernment between agreeable or disagreeable sounds was no longer the exclusive domain of one's ears. One also could determine these ratios by entirely *intelligible* means.[12]

[10] Collinson and Plant, op. cit., p. 9.
[11] Hermann, op. cit., p. 95.
[12] Ibid., p. 94.

It is significant that the string was chosen as the representative instrument, since a string is fixed at both ends and thus will only vibrate at wavelengths which are whole number ratios of the original length. I will return to the idea of dissonance and consonance in human hearing in Section 4.3.

The Pythagoreans looked at the world in the context of oppositions. As might be expected, there were ten central pairs of oppositions, since ten represented perfection:

1. limited and limitless
2. odd and even
3. unit and multiple
4. right and left
5. male and female
6. resting and moving
7. straight and crooked
8. light and darkness
9. good and bad
10. square and oblong

These indivisible *units* would prove problematic, as a very simple proof showed that the square root of 2 is a value that is *irrational*, that cannot be expressed as the proportion of two whole numbers.[13] This was quite disturbing to the Pythagoreans, upsetting the belief that the entire universe was based on proportions. Thus, progress in mathematics proved essential for revising our philosophy of the world, and not for the last time.

To say that the Pythagoreans were important to the development of mathematics, especially number theory, arithmetic, and geometry, is an understatement, though to

[13]See Appendix A for an example of this proof.

credit them singly exaggerates their contribution. We credit Greek culture in general with this progress, and recognize the role of the Pythagoreans within this general movement.[14]

1.2 Isaac Newton

> *"I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."*
>
> — Isaac Newton

Isaac Newton's significant contributions to mathematics and science assure him a place in any the history of science. As author of several influential books on physics, particularly the *Principia Mathematica*, Newton codified the "Laws of Motion" which are still taught to undergraduates today. Finding insufficient existent mathematics to accomplish these tasks, he invented calculus as a tool for understanding these motions.[15] These formulas set in place linear algorithms relating distance, speed, force, acceleration, and momentum, describing the motion of all things, big and small, with a single set of universal laws. Newton was the first to show that the motion of the planets and the motion of things on Earth all operate in fundamentally the same way. He also showed that motion and energy were quantifiable and measurable ideas, and as such that the laws of the natural world could be understood through the study of mathematics.

It is worth pausing, for a moment, to contemplate how our world was different before the *Mathematical Principles of Natural Philosophy* was published. We often

[14] Hermann, op. cit., p. 107.
[15] Gottfried Wilhelm von Leibniz may also claim simultaneous credit for this innovation.

take for granted the central principle of this important work, summarized by Stewart as follows: "Nature has laws, and we can find them."[16] The view of the universe as a form of clockwork, with regular laws and principles that can be summarized and understood, is relatively new in the vast span of human history. However, this view does not capture the whole picture.

Newton's equations worked well when considering, for example, two large bodies (such as planets) under the mutual effects of gravity. However, when you added a third body to the system, with each body mutually attracting the other two, working out the motion of each body in the system became an impossible task. Indeed, it remains beyond the ability of today's mathematicians to write a complete solution to this problem. As we shall see, the three-body problem is one of the most basic examples of chaos.

This famous statement by the Marquis Pierre Simon de Laplace, a mathematician who lived after Newton, affirms the impact that Newton had on the course of science:

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate Nature and the mutual positions of the beings that comprise it, if this intellect were vast enough to submit its data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes.[17][18]

[16]Stewart, op. cit., p. 3.

[17]A. B. Çambel. *Applied Chaos Theory: A Paradigm for Complexity.* New York: Academic Press, Inc., 1993, p. 7.

[18]Stewart, op. cit., p. 6.

Determinism was the currency of science at that time. After Newton, it was believed that natural laws could be understood, written down, and applied to the real world in order to understand it. It would be many generations before it became clear that this was not actually the case.

In a way, Newton was advancing the same agenda as the Pythagoreans: trying to develop an understanding of the mechanics of the universe through an understanding of mathematics. The "music of the spheres" was the Pythagoreans' attempt to relate the motion of objects on Earth to the motions found in the heavens, and Newton's own laws of motion provided a solution to that same problem. The attempt to find universal truth through the study of mathematics is as old as the idea of universal truth itself, and remains a goal of science to this day.

1.3 George William Hill

"The most troublesome problem in this program was the determination of the mutual perturbations of Jupiter and Saturn. Newcomb recognized Hill's ability, and assigned him the task of developing the theory of the motion of these planets. The result of this assignment is comprised in the 577 pages of volume III of Hill's Collected Works, and required seven and one-half years of steady computation for its completion.... It seems probable that science lost much because Newcomb caused Hill to spend about eight years of the prime of life on this work."

— Forest Ray Moulton

The story continues with a discussion of astronomy in the mid-19th century, where the solution to the aforementioned three-body problem still eluded mathematicians and astronomers. This system could not be solved directly, but a solution could be approximated using the "perturbation" method, whereby the system would be solved for two of the bodies (this was easy), and then the difference caused by adding the third body to the system would be found incrementally. This method worked fairly well, and solved the major differences between prediction and observation of the planetary orbits of our own solar system. That is, except for one: Uranus. Using an inverse of the perturbation method, John C. Adams and Urbain J. J. LeVerrier in France predicted that there must be an unknown planet, and even predicted its location. This led to the discovery of Neptune, and the perturbation method was proven essentially useful in dealing with complex systems.[19]

But the perturbation method did not really solve the system; it only approximated the solution. Mathematicians continued their attempts to solve the generalized three-body problem. Toward the later part of the 19th century, an American astronomer and mathematician named George William Hill took on the problem. Hill was trying to solve the motion of the moon under the influence of the Earth and the Sun, and made the following approximations: first, one of the bodies (the Moon) had no mass and did not affect the motion of the other two by gravity; second, the larger bodies would move in circular orbits rather than ellipses; and third, that the motion of all three bodies was in a single plane.[20][21]

This system still defies general solution, but particular solutions are computable. It was this reduced model that would inspire Henri Poincaré and continue the development of a general theory of dynamical systems and eventually chaos theory.

[19]Edward N. Lorenz. *The Essence of Chaos*. Seattle: University of Washington Press, 1993, p. 113.
[20]Ibid., p. 114.
[21]Ernest W. Brown. *Biographical Memoir of George William Hill*. Available at `http://books.nap.edu/html/biomems/ghill.pdf`. Washington: National Academy of Sciences, 1916, p. 283.

1.4 Henri Poincaré

"The most interesting facts are those which can be used several times, those which have a chance of recurring. . . . Which, then, are the facts that have a chance of recurring? In the first place, simple facts."

— Henri Poincaré

Around this time, the precursors to chaotic dynamics started to appear. The physicist James Clerk Maxwell reportedly said, in 1873, that "when an infinitely small variation in the present state may bring about a finite difference in the state of the system in a finite time, the condition of the system is said to be unstable. . . [and] renders impossible the prediction of future events, if our knowledge of the present state is only approximate, and not accurate."[22] Jacques Hadamard said, in 1898, that an error or discrepancy in initial conditions can make a system unpredictable.[23] These ideas were around when Henri Poincaré began to examine differential equations and found a new way to analyze these difficult systems.

Before Poincaré, it was believed that the science of complexity and the science of reductionism could not be reconciled.[24] There were two separate established systems, one referred to as *deterministic* and the other *stochastic*, with no way seen to bridge the gap. Poincaré attempted to reconcile this problem, replacing the study of differential equations, which are difficult and often impossible to solve, with the study of iterated maps. For example, the orbit of a moon around a planet might be understood by watching when the moon passed through a single plane through the planet, rather than calculating its position throughout its entire orbit. This method, now referred to as the *Poincaré section*, translated differential equations into "dif-

[22]Williams, loc. cit.
[23]David Ruelle. *Chance and Chaos*. Princeton: Princeton University Press, 1991, pp. 47–49.
[24]Stewart, op. cit., p. 48.

ference" equations, which were far easier to solve and understand. This meant that continuous systems could be treated as stepwise or iterative systems.

Basically, Poincaré's idea was as follows: If a system is observed in a particular state at a particular time, and later the system returns to that exact state, then the system must be periodic.[25] Thus, if we observe a system using a Poincaré section of its phase space (transforming a differential equation into a difference equation), we should expect a periodic system at some point in time to return to its initial state. Thus, we have a simple, effective method of determining whether a system is periodic or not, and this method is solvable, even for many complex systems.

I will now return to our discussion of Newton's nemesis, the three-body problem. Poincaré used Hill's Reduced Model, which meant that he was considering the motion of a massless body under the mutual gravitation of two larger bodies. One would think that the small mass, dramatically described by Stewart as a "grain of interstellar dust" between Neptune and Pluto, for example, to follow a periodic orbit under the influence of the other two planets.[26] But it does not! Poincare writes, in his *New Methods of Celestial Mechanics*, volume three:

> When one tries to depict the figure formed by these two curves and their infinity of intersections, each of which corresponds to a doubly asymptotic solution, these intersections form a kind of net, web, or infinitely tight mesh; neither of the two curves can ever cross itself, but must fold back on itself in a very complex way in order to cross the links of the web infinitely many times. One is struck with the complexity of this figure that I am not even attempting to draw. Nothing can give us a better idea of the complexity of the three-body problem.[27]

[25]Ibid., p. 59.
[26]Ibid., p. 61.
[27]From: ibid., pp. 62-63

Using this method, Poincaré discovered that small differences in initial conditions could lead to large differences later, making prediction impossible.[28] Poincaré looked at asymptotic solutions to these equations, and demonstrated that the presence of a fixed point is enough to imply the existence of an infinite number of sequences of all periods, and also an infinite number of sequences which are not periodic.[29] This is the *essence* of chaos.

Poincaré was perhaps the first person to describe chaos explicitly (though the use of the word *chaos* would have to wait). But mathematics was not ready to deal with chaos: Mathematics still wanted simplicity and looked for order. Edward Lorenz (whom we shall meet momentarily) speculated that the development of chaos theory had to wait because finding disorder was of no use to science at the time, as you could not understand disorder.

> Poincaré was not seeking chaos. He sought to understand the orbits of the heavenly bodies, and he found chaos. To him it was the phenomenon that rendered the three-body equations too complex to be solved, rather than the principal subject of a future field of investigation.[30]

Poincaré was building on a long scientific tradition of reductionism, where a phenomenon is understood when it can be written in a simple equation of a few variables—a tradition which stretched at least as far back as Isaac Newton. James Gleick put it more simply: "The whole point of oversimplifying was to model regularity. Why go to all that trouble just to see chaos?"[31] The culture of scientific discovery at the

[28]Williams, loc. cit.
[29]Lorenz, op. cit., p. 118.
[30]Ibid., p. 121.
[31]James Gleick. *Chaos: Making a New Science*. New York: Viking, 1987, p. 65.

time was not interested in chaos and unpredictability—they wanted to find out what could be known. Chaos promised only to tell us what we could *never* know.

1.5 Relativity and Quantum Mechanics

"Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one.' I, at any rate, am convinced that He does not throw dice."

— Albert Einstein

The progress toward an understanding of dynamical systems and chaos received no further attention for some time, but meanwhile huge revolutions in science in the early 20th century would forever change our view of the world we inhabit and set the stage for the understanding of the role of chaos in our universe. While Newton's laws accurately predict the motions of objects in our everyday experience, the inaccuracies of the equations became significant under extreme conditions, such as near the speed of light or at the molecular scale. Where Newton's laws failed, a new theory was needed.

Albert Einstein had been a promising student of science, but had failed to find a job after graduating in 1900. He wound up working as an assistant examiner in the patent office of Berne, Switzerland. In the course of examining patents for various electromagnetic devices, Einstein came up with several of his famous thought experiments concerning the nature of space, time, and light, and in 1905 published four articles concerning the quantum nature of light (a revision of Max Planck's work), Brownian motion, *Special Relativity*, and mass-energy equivalence (the famous $E = mc^2$).

A detailed account of these discoveries is beyond the scope of this paper. Briefly, Special Relativity implies that the nature of the passage of time is relative to the spacial reference frame in which one is traveling. The mass-energy equivalence implied that a small amount of mass could produce an immense amount of energy, an idea which was later used to create the nuclear bomb (one of Einstein's greatest regrets). And Quantum Mechanics posits that every particle is subject to a wave function which describes its position and velocity. According to Werner Heisenberg and Niels Bohr's "Copenhagen Interpretation" of Quantum Mechanics, this wave function provides a set of probabilities for the particle's position and location. Further, the Heisenberg Uncertainty Principle puts limits on the amount of knowledge about a particle that we can possibly know. The act of measuring a particle's position is thought to cause the waveform to collapse to a single possibility; in other words, the act of measuring something changes it.

This is not the only interpretation of quantum mechanics; another interpretation was put forward by physicist David Bohm, called the "causal interpretation" (or "Bohmian"). This interpretation says that the position and velocity of particles are "hidden" variables, fixed but unmeasurable. This interpretation certainly seems more in accord with our intuition, and seems more easily reconciled with a deterministic (and hence chaotic) view of the universe, though experimental research has not yet provided a reason at this point to prefer one interpretation over another.

These ideas had a tremendous impact on our understanding of our world. The relative nature of time and space, and the idea that the act of observing could cause microscopic changes in our environment, these were disturbing, but none so much as the idea that there were limits to how much we could possible know about our universe. As we shall soon see, if there are limits to how much we can know, then there are limits to how much we can predict, no matter how intricate our models become, due to chaotic effects of nonlinear systems.

1.6 Edward Lorenz

"The first mathematically generated chaos that I encountered was produced by a very crude model of the global weather system, which contained not thousands or millions of variables, but just twelve."

— Edward Lorenz

The huge revolutions in science at the beginning of the 20th century had two major effects: first, science departed from the realm of common understanding and intuition and second, it became more specialized. Scientists themselves also became more specialized and focused on certain areas of a field, rather than the generalist "natural philosopher" model which lingered even until the late 19th century. This meant that generalist theories which would apply to all fields of science were less likely to be discovered in the first place, and more likely to be ignored after they were discovered.

Thus, it took a long time before the world was ready to hear about chaos theory. First, we had to overcome a general bias in science that existed before the 1960s, a bias against new and unproven fields of study. Edward Lorenz summarized it this way:

> One may argue that the absence of an early outburst was not caused by a prevailing lack of interest; it was the lack of interest. To some extent this is true, yet it may have been caused by the priorities of the leaders in the field. One of the quickest ways for a young scientist to gain recognition, and perhaps a prize, is to solve a problem that has become well known

> because the leading scientists of an earlier generation have tackled it and failed. One who is seeking such recognition may have little incentive to start out in a totally new direction, even though history indicates that the vast unexplored territory surrounding new problems sometimes holds the key to the solution of older ones. Certainly Poincaré and Birkhoff and most other leaders did not suggest that the problems of the future would lie in chaos theory.[32]

At the time, chaos did not seem to provide answers, just more questions, and who would want to deal with unsolvable questions when one's career is on the line? To a certain extent, this problem remains true today. Novel approaches and theories are not necessarily the safest investments for grant-giving institutions; the only reason chaos has become such a popular area of research is because it has gained recognition as an area which provides results. Who knows what innovations remain unfunded because science is devoted to the sure-thing?

Lorenz himself first came into contact with chaos in his study of meteorology. Weather prediction at the time was an important area for the application of mathematical modeling. Lorenz had designed a very simple weather model, reduced to only three variables, that he had coded into an early personal computer. His model, now referred to as the Lorenz Attractor, was as follows:

$$\begin{aligned}\frac{dx}{dt} &= 10(y-x)\\ \frac{dy}{dt} &= x(28-z)-y\\ \frac{dz}{dt} &= xy-\frac{8}{3}z\end{aligned}$$

He had the computer print out the results as a list of numerical values, with a simple graph created by adding a variable number of spaces before a printout of a letter. At

[32]Lorenz, op. cit., p. 125.

one point, Lorenz became interested in continuing an earlier computation. In order to save time, rather than starting from the beginning of his computation, he entered a value from the middle of the previous run. But not long after the computation started did he realize that the new computation was completely different than the previous run. After some subsequent checking of the computer system, he realized the problem was with his entry: he had used three digits of accuracy (the level of detail he provided on the printouts) rather than using the six digits of accuracy that the computer was using internally.

Up to this point, it was generally considered that in a simple system of equations, if one used an input with a certain amount of accuracy, the system would provide a result with an equally controllable and predictable amount of possible error. In other words, in Lorenz's model, a small error in the 4th decimal place should not have caused such a massive difference between the new run and the previous one. As it turned out, relatively simple systems can have very complex results.

Gleick notes that science at the time held some very basic misunderstandings about the role of complexity in mathematical modeling.

> Traditionally, when physicists saw complex results, they looked for complex causes. When they saw a random relationship between what goes into a system and what comes out, they assumed that they would have to build randomness into any realistic theory, by artificially adding noise or error.[33]

But complex results do not actually imply complex causes; this is the fundamental insight which makes chaos theory so important and relevant. As Lorenz discovered, a simple system of three variables can have very complex results—we shall see even

[33] Gleick, op. cit., p. 8.

simpler examples of chaotic systems later. According to Gleick, this was just one of three wrong beliefs held by science at the time, summarized as follows: that simple systems behave in simple ways, that complex behavior implies complex causation, and that different systems behave differently.[34] As it turns out, simple systems can behave in very complex ways, and many different systems can share much in common. Chaos theory studies the complexity that occurs in all kinds of systems.

While Lorenz had made a critically important discovery, he published his results in the only journal available to him: the Journal of the Atmospheric Sciences.[35] Most mathematicians, physicists, and other scientists were not frequent readers of journals on meteorology—they had enough to read in their own disciplines, not to mention the difficulties in communication across disciplines that had only increased since the beginning of the century. Among its many other revolutionary facets, chaos theory would begin the dissolution of the many walls built between scientific disciplines, making interdisciplinary communication not only possible, but necessary.

One key element of chaos theory's discovery by Edward Lorenz was his use of the computer, which may now be ubiquitous but at the time was new and somewhat rare. This machine allowed computations to be carried out with tremendous speed—speed that might have saved someone such as George William Hill eight years of his life. Due to the speed of the computation, many similar computations can be made using the same model to give an idea of the complexity found within it. Lorenz remarks: "Without the computer the needed time for computation alone would have been years instead of months, and, with other problems to occupy much of my time, I would probably not have continued."[36] The development of the computer and the

[34] Ibid., p. 303.

[35] Edward N. Lorenz. "Atmospheric Predictability as Revealed by Naturally Occurring Analogues". In: *Journal of the Atmospheric Sciences* 26 (July 1969), pp. 636–646.

[36] Id., *The Essence of Chaos*, p. 128.

possibilities afforded by the exponential increase in computational power as a result has had an immense impact on science, music, and society as a whole.

1.7 Mandelbrot

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

— Benoît Mandelbrot

Benoît Mandelbrot was born in 1924 and did most of his work at the IBM Thomas J. Watson Research Center in Yorktown Heights, NY. While there, he was interested in problems within a variety of fields, including information theory, economics, cosmology, and fluid dynamics. As a generalist, Mandelbrot was a risky investment for IBM, and his success stands as a testament against the general trend toward specialization and focus in our society.

In 1967, Mandelbrot published "How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension," which discussed how the length of the coastline changes depending on the size of the ruler that is used, a first example of what would later be called "fractal dimension." I will return to the idea of fractal dimension in Section 4.4. In 1975, he began his study of Julia sets, based on the work of Gaston Julia and Pierre Fatou from the previous century, and created what is now known as the Mandelbrot set, which incorporates all of the possible Julia sets within it.

Mandelbrot's set concerns the iterative quadratic polynomial $z_{n+1} = z_n^2 + c$, using $z_0 = 0$ as the starting value. For most real numbers, the sequence expands without bounds, as for example 0.5 produces the iterative sequence: $\{0, .5, .75, 1.062, 1.628, 3.153, 10.444, 109.567...\}$. But in the plane of *complex* (or *imaginary*) numbers,

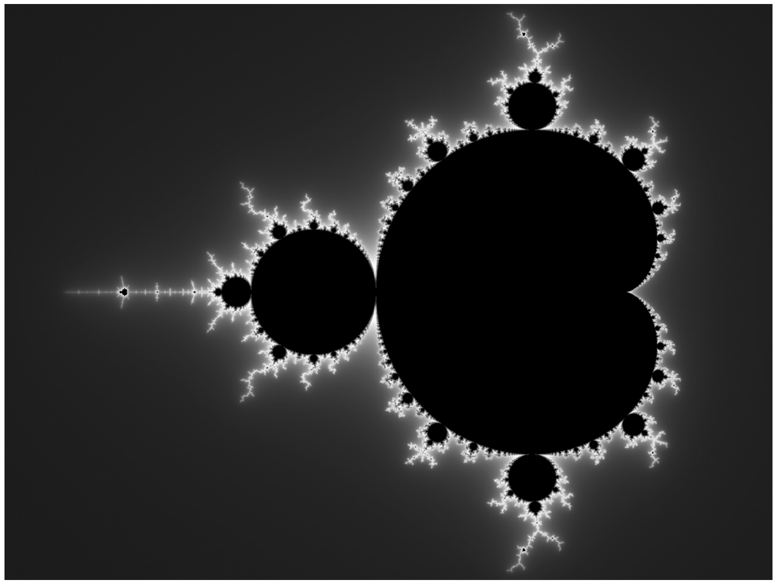

Figure 1.1: The Mandelbrot Set. Note the self-similarity of the entire set to the protrusions which surround it; this self-similarity continues at all levels of detail.

the question becomes far more interesting: for which complex values of c does the sequence remain bounded, and for which values of c is the sequence unbounded?

The Mandelbrot Set (Figure 1.1) is the set of all of the complex values of c for which the previously mentioned sequence remains bounded. The boundary of this graph is surprisingly complex: at all levels of magnification, it remains full of twists and turns. This is one example of a *fractal*, the word coined by Mandelbrot himself in 1976.

A fractal is an object that contains infinite levels of detail and which exhibits self-similarity at all detail levels.[37] In the Mandelbrot set, the shape of the entire set is repeated over and over throughout all of the levels of detail. Fractals must also contain some irregularity; that is, the number line contains infinite levels of detail and

[37]Note that self-similarity does not imply exact replication. A certain amount of flexibility is included in this definition.

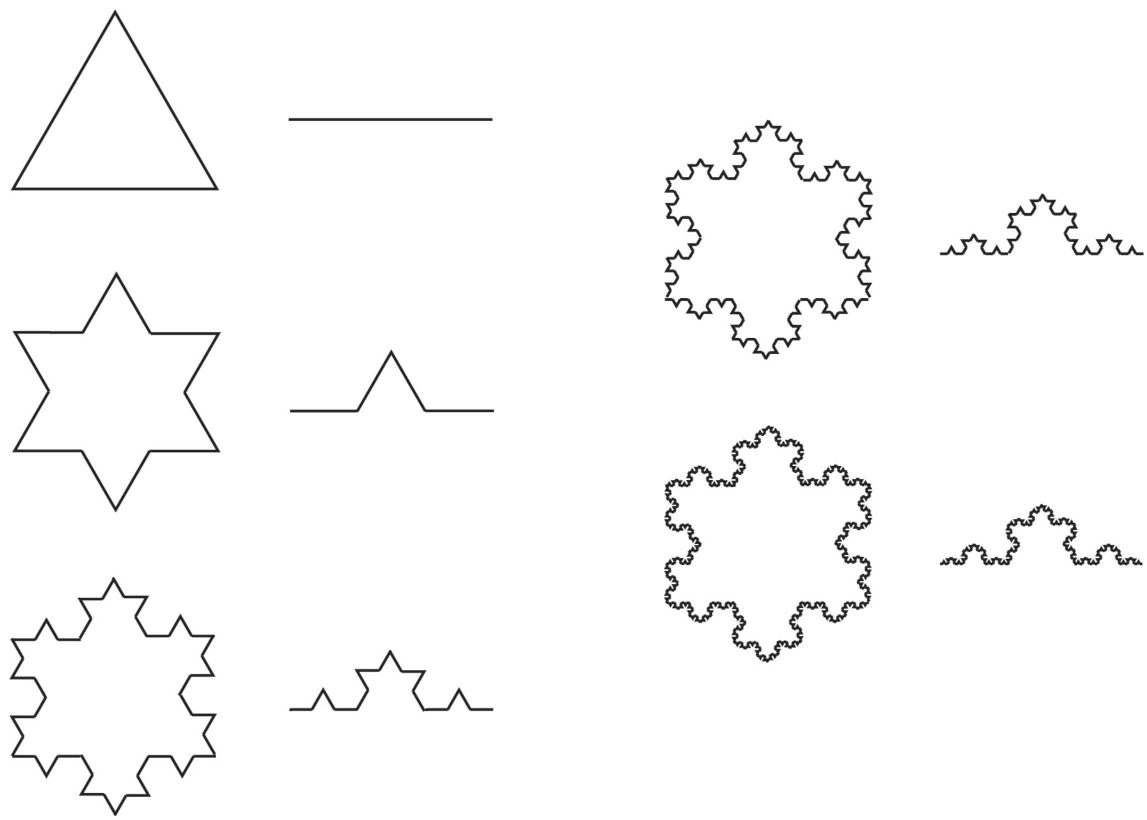

Figure 1.2: The Koch Curve and Snowflake. The construction involves an algorithmic alteration which is then repeated at all levels of detail.

is remarkably self-similar, but is not a fractal. Examples of fractals in the real world include coastlines, snowflakes, trees, ferns, broccoli, mountain ranges, and other self-similar objects, but unlike true fractals, these are only self-similar for a finite range of detail levels.

Fractals can be generated using a variety of different approaches. Section 6.5 will present the Cantor Set and Sierpinski Triangle. Here, I will examine another common fractal construction, called the Koch curve.

The Koch curve begins with a line segment, as shown in Figure 1.2. This line segment is divided into three parts, and the middle third of the line is transformed

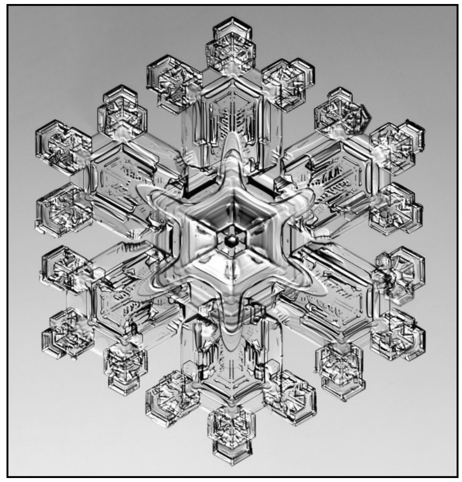

Figure 1.3: An Actual Snowflake. Note the similarity of this snowflake image to the Koch Snowflake in Figure 1.2. This image was captured by Kenneth G. Libbrecht (`www.snowcrystals.com`) using a specially designed snowflake photomicroscope. Used with permission.

into the base of an equilateral triangle, and then removed. This process is then repeated on the four segments which remain. When you match three Koch curves to form an equilateral triangle, the shape is known as the Koch snowflake, due to its similarity to actual snowflakes, such as the photo in Figure 1.3.

There are a variety of interested questions to ask at this point concerning this geometry. For example, what is the length of this curve? If the starting segment is of length 1, then we are increasing the length by 1/3 at each stage of the creation of the Koch curve. In other words, each stage multiplies the length of the entire curve by 4/3, resulting in an infinite length for the idealized mathematical object. This infinite length is bounded within a finite space. This is part of the reason why true fractals are so hard to find in reality.[38]

[38]Perhaps the main reason we don't observe true fractals has to do with the limitations of our own observational abilities at infinitely small distances; the fractals may be there, but we can't see or measure distances smaller than a certain distance known as the Planck length.

What is the area bounded by the Koch snowflake? It is apparent that we are adding 1/3 of the area of the initial triangle at the first step, and then 4/3 of the area of the smaller triangles with each subsequent step, an addition which gets infinitely small at infinite steps. This area can be expressed as:

$$A\left(1+\sum_{n=0}^{\infty}\frac{3\cdot 4^{n-1}}{9^{n}}\right)$$

where A represents the area of the original triangle. This series is geometric, and converges to $\frac{8}{5}A$. Thus, we have a finite area surrounded by an infinite boundary. This strange result speaks more to the difficult nature of determining the boundary of this geometric object. In other words, the length of the boundary depends on the size of the ruler that you use. This is a real-world difficulty in determining the actual length of coastlines, for example, which might be considered to have fractal structure.

As we shall see, fractals and chaos are ideas that may seem far apart but are, in reality, intricately entwined. Chaotic functions often incorporate fractals into their phase space geometry.

1.8 What is Chaos Theory?

"One is led to a new notion of unbroken wholeness which denies the classical analyzability of the world into separately and independently existing parts. The inseparable quantum interconnectedness of the whole universe is the fundamental reality."

— David Bohm

As chaos theory developed, it became important to define exactly what *chaos* is. Many different approaches to this definition were simultaneously developed, and there is no established consensus even today. These definitions range from the simple to the complex. For example, G. Williams defines it this way:

> Chaos is sustained and disorderly-looking long-term evolution that satisfies certain special mathematical criteria and that occurs in a deterministic nonlinear system.[39]

This definition is useful, as it does not rely entirely on mathematical jargon. Essentially, a chaotic system is one that contains disorder and unpredictability while being simultaneously deterministic. For those who want a more specific approach, I will describe a system as chaotic when it has the following three traits:

1. Exhibits sensitive dependence on initial conditions
2. Is topologically mixing
3. Contains dense periodic orbits

These are the most commonly used indicators of chaos. Of the three, the first is most relevant to our discussion, as it is perhaps the easiest to see, and will be our primary focus. This term is what is known popularly as the "Butterfly Effect," a name which comes from Edward Lorenz's presentation to the 1972 meeting of the American Association for the Advancement of Science in Washington, D.C., which was entitled "*Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?*"[40]. The idea is that the fluttering of a butterfly's wings changes the air just slightly enough to set in motion a change in the atmosphere that, months later, causes a tornado to form on the opposite side of the planet that wouldn't otherwise have occurred.

[39] Williams, op. cit., p. 9.

[40] R. C. Hilborn. *Sea gulls, butterflies, and grasshoppers: a brief history of the butterfly effect in nonlinear dynamics.* Vol. 72. 4. 2004, pp. 425–427, p. 425; Lorenz, op. cit., p. 14

In mathematical terms, sensitive dependence is expressed as follows: around a point x, a system is *sensitively dependent* if there exists an $\epsilon > 0$ so that, for all $\delta > 0$, there exists $y > 0$ and $t > 0$ such that $|x - y| < \delta$ and $|x(t) - y(t)| > \epsilon$. Essentially, for every small region around x, there exists a y in that region that will eventually diverge from the trajectory that x follows. This definition is mathematically "weak," since many mathematicians specify that the trajectories should diverge exponentially fast.

The second trait, being topologically mixing, means that any particular region of the space will eventually overlap with any other region, so that the system mixes the space. Chaotic functions are often considered to operate in two ways simultaneously: stretching and folding (think of raisins in dough, for example). This trait is simply saying that the process of stretching and folding will eventually cause any two regions to overlap. This is one way that chaotic functions are relevant to fractals: an infinite regress of stretching and folding creates a space of infinite detail and self-similarity.

The third trait means that at any point in the space you are never far from a periodic orbit; that aperiodicity and periodicity are found within any small region of the space. This implies an infinite level of detail at every area of the space, which is again a reference to fractal geometry.

Sometimes, the appearance of chaos might be misleading. For example, is $\sqrt{2}$ chaotic? As we saw in Section 1.1, this number did not fit with the Pythagoreans' worldview because of its irrational implications, that it could not be expressed as a proportion of whole numbers. The decimal expansion of $\sqrt{2}$ certainly looks chaotic:

$$1.4142135623730950488016887242096980785696718753769480731766797379\ldots$$

To confirm this to yourself, look at the expansion and attempt to predict the next number in the expansion.[41] Looking a little further, what exactly is the square root of 2? For one, $\sqrt{2}$ is the algebraic answer to the question, "What is a value of x if $x^2 = 2$?" It's also the answer to the question, "What is the value of x if $x = 2 \times \sin(45°)$?" $\sqrt{2}$ is also the result of computing this quantity:

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\ddots}}}}$$

Musically speaking, $\sqrt{2}$ is the frequency ratio of two pitches that have the interval of a tritone. It represents the length of the hypotenuse of a right triangle where the sides are both of unit length (see Figure 1.4). If I define a unit of length that has a length equal to the hypotenuse of a right triangle with sides of length 1 inch, then is a single length of that unit still irrational? It's certainly an irrational length *in inches*.

The key here is the matter of perspective. In some representations, $\sqrt{2}$ seems normal and everyday. The length seen in Figure 1.4 is certainly nothing to inspire the idea of chaos. But some uses of $\sqrt{2}$ inspire a chaotic interpretation: its irrationality (see Appendix A), its decimal expansion, or the sound of the tritone harmony as a dissonance, for example.[42]

Chaos, then, is subjective. Chaos is not a general feature of any particular number or sequence. Rather, chaos is observed through a certain lens from a certain

[41] It's 9.

[42] See Section 4.3 for more on dissonance and chaos.

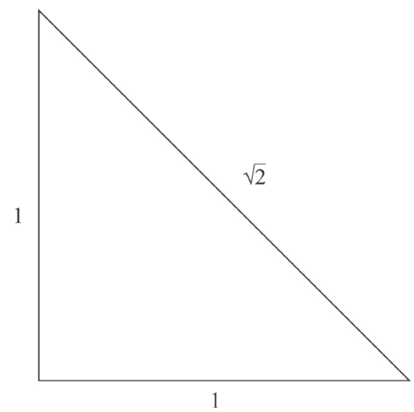

Figure 1.4: Hypotenuse of a Right Triangle is $\sqrt{2}$. This length as drawn does not seem "chaotic" in any way, but its length in proportion to the lengths of the other two sides is incommensurate, and thus this proportion can be seen under certain perspectives to be embodying chaos.

perspective, and if the same data is looked at differently, the chaos may not be seen at all.

1.9 Summary

"Chaos is a name for any order that produces confusion in our minds"

— George Santayana

The developments of science from the Pythagoreans to the modern age were primarily concerned with making sense of our universe. The Pythagoreans found proportion on earth and in the universe, and found a way to relate all things to the study of these proportions. Newton and later mathematicians formalized this relationship, and scientists in the 20th century realized the importance of complexity, understanding the limited applications of approximation and reductive science and the surprisingly subtle nature of fundamental relationships of the universe, as in quantum mechanics and relativity. Chaos theory showed that there were limitations to human knowledge; that without absolute precision even seemingly simple systems would behave unpredictably.

Chapter II
20th Century Music History and Chaos

"Chaos often breeds life, when order breeds habit"

— Henry Brooks Adams

From the history of science, I now will turn to the revolutions in music which were occurring during the same period. This chapter will explore the general trends in Western art music during the early 20th century, and then focus on certain trends which are most applicable to our discussion of chaos theory. Though our discussion will necessarily be linear and somewhat chronological, no history can possibly encompass the entirety of musical thought during this period, and as such I will emphasize the ideas that are most relevant, and omit by necessity many important contributors to the development of music in the 20th century. For those familiar with music history, this chapter will provide a very general review; for those unfamiliar, it will provide a practical background for further musical discussions.

During the late Romantic period in music, the "rules" of tonality began to be broken apart. The Romantic trend of chromaticism had weakened the classical system of tonality almost to the breaking point. Music became ambiguous and unpredictable. Brahms varied his thematic repetitions in an organic and new way. Wagner stretched the limits of tonality, creating sonorities that defied categorization by the analysis tools of the day, and flitting from key area to key area without emphasizing any of them, or extending a single key area for hundreds of bars.

The 20th century continued the trends of the Late Romantics, and started asking more fundamental questions about the nature of music; this is not unlike the fundamental questions science was exploring at this same time. Composers wondered what constitutes music, and what transforms sound into music. This led to a time of great experimentalism, sparked perhaps by the Paris Exposition of 1889.[1] This exhibition exposed European composers to a great number of non-Western performances, and European composers such as Debussy and Mahler were profoundly affected. This exposure to other musics of the world called into questions many profound assumptions, and arguably lit the fuse for the explosion of experimentalism in the 20th century.

Here are just of the few experimental ideas which came to prominence during this time. Arnold Schoenberg took a new approach to the ordering of pitches and the creation of tonal centers. Igor Stravinsky emphasized the importance of rhythm and syncopation in his “Primitivism.” Erik Satie questioned the role of music with his “furniture music,” meant to exist in the background, unnoticed. John Cage questioned the nature of composition in his works which attempted to remove the composer from the act of composition. Harry Partch grew bored with the tuning systems of conventional instruments and invented his own, while Henry Cowell experimented with unconventional ways of playing existing instruments, and, along with Lou Harrison and Alan Hovhaness, incorporated non-Western approaches to music in his own compositions. Béla Bartók researched and utilized folk music of Eastern Europe in his compositions. Charles Ives elevated the role of the amateur musician, and wrote pieces which incorporated the simultaneity of multiple pieces being played at the same time; both ideas involve an acknowledgment of the act of performance within the performance itself. Edgard Varèse experimented with electronic instruments and

[1] Elliott Schwartz and Daniel Godfrey. *Music Since 1945: Issues, Materials, and Literature.* New York: Schirmer Books, 1993, p. 9.

new sounds, as well as innovations in notation. Karlheinz Stockhausen invented a ritualized vocabulary for the performances of his own music, considering heavily the visual element as well as the audio.

These are only a few of the numerous innovative composers who made the 20th century such a productive and unique time period in music history, one frequently cited as a "paradigm shift" in terms of music history, and I have by necessity left out many vitally important contributors.[2] It's also worth noting that, though the composers had (for the most part) embraced these new ideas and perspectives, the audience was more reluctant to change, creating a point of friction which exists in Western art music to this day—Milton Babbitt may dismiss the audience, but the audience plays a vital role in the musical process.[3] This is not unlike the friction which prevented chaos theory from being developed at an earlier period of time—the scientific and musical communities were simply not ready yet.

For the rest of this chapter, I will discuss some specific experimental ideas explored in the 20th century in more depth, and how they relate to the study of chaos. Chapter 5 will explore individual pieces in more detail.

2.1 Serialism

"Great art presupposes the alert mind of the educated listener"

— Arnold Schoenberg

In the early 20th century, Schoenberg wanted to extend the chromaticism of Wagner but found he was unable to do it using the tonal tools available to him. Schoenberg had to invent an entirely new method of dealing with pitch. *Serialism* involves the

[2] So too, have I by necessity generalized the many, many innovations each of these individual composers contributed!

[3] Milton Babbitt. "Who Cares if You Listen?" In: *High Fidelity* (Feb. 1958).

ordering of pitch material in a work using what is known as a *tone row*. A tone row is simply an ordered set of pitches, which, along with its inversions and retrogradations, forms the pitch material for the entire work. This creates an ordering of sound which does not necessarily follow the rules of tonality in its resolution of tendency tones and organized harmony, though through clever construction of a tone row many tonal ideas can survive. In a way, Serialism may be the first organized system to use a strictly algorithmic approach to music composition.[4]

Thus, from a certain perspective, Serialism characterizes the utmost "order" in music.[5] It creates a world in which pitch is balanced, with no particular pitch necessarily functioning as more important than another, and every pitch presented in each work the same number of times (though in many works, this balance of pitch was altered through choices of duration in the actual composition process in order to create a sense of tonal center). At the same time that the composition was highly ordered and determined, it also became completely unpredictable and "disordered" in practice, as audiences could no longer practicably follow the train of melody and harmony in order to predict what would come next. Perhaps an audience trained solely on serialism might be trained to understand and hear tone rows in all of their forms, and in that case a new level of complexity would be needed to create unpredictability. We see again that the "chaos" present in this system is subjective, not intrinsic.

[4]Algorithmic approaches to music certainly existed as early as the Medieval period; even the idea of *form* might be considered an algorithmic approach to musical composition. To be specific, Serialism might be considered the first to break with musical tradition entirely and leave the musical choices entirely to the algorithm, though this point is entirely dependent on your definition of what constitutes an algorithm.

[5]Schwartz and Godfrey, op. cit., p. 78.

In Section 5.2, I present a method for incorporating our understanding of chaotic systems into Serialist works. In Section 6.3, I present a chaotic approach to the Serialist method.

2.2 Randomness or Chance Procedures

"**Guildenstern:** *Syllogism the second: One, probability is a factor which operates within natural forces. Two, probability is not operating as a factor. Three, we are now held within un-, sub- or super-natural forces. Discuss."*

— *Rosencrantz and Guildenstern are Dead*, Tom Stoppard

"Random," like chaos, has many meanings. In common language, the two words seem to carry the same connotations of disorder and confusion. In fact, the idea of chaos (as in nonlinear dynamics) and randomness are opposite in meaning, but in another way are closely related. Gleick writes:

> 'Chaotic' is not just a fashionable new name for 'random'. Not at all. Chaos is a different kind of beast. It may sometimes masquerade as chance, but in essence chaos and chance are poles apart.[6]

Many things we consider to be "random" are, in fact, just unpredictable. Take the quintessential example of a coin flip. Newton's Laws of Motion certainly describe how to predict, if the initial angular velocity, height, dimensions, etc. of a coin are known, how the coin will fall and land, but in practice we find our measurements are not accurate enough to provide for a complete prediction. Thus, a coin flip is deterministic, but not "random" in the common sense of the word.[7]

[6] Gleick, op. cit., p. 279.

[7] For more about the randomness of a coin toss, see J. Ford. "How Random Is A Coin Toss". In: *Physics Today* 36.4 (Apr. 1983), pp. 40–47

On further consideration, it becomes difficult to provide examples of random events. Çambelnotes that for a process to be truly "random" it must have "no causal relation between an observation at some present time, t_n, and a past observation at an incremental time, t_{n-1}, or an observation at an incremental future time, t_{n+1}."[8] In other words, issues of causality are crucial for determining whether a system is acting randomly or acting unpredictably. Williams hypothesizes: "A strong case can be made that there isn't any such thing as true randomness, in the sense of no underlying determinism or outside influence."[9] Instead, Williams replaces random events with chaotic ones, and makes less of a case for separating the two ideas:

> Descriptions of chaos as 'random-like behavior' are mostly justified. Chaotic time series not only look uncorrelated or unsystematic, they often pass every statistical test for randomness. To that extent, therefore, chaotic data are both random and deterministic.[10]

The question of "randomness" thus becomes a question of determinism and philosophy, rather than a quality inherent in data.

When we use this term, "random," we have have many common notions about randomness, part of our ability to deal with the unpredictability of the world around us. For example, consider the following two outcomes of 10 flips of a coin (H represents heads and T represents tails):

1. HHHHHTHHHH
2. HTTHHTHTHT

[8]Çambel, op. cit., p. 6.
[9]Williams, op. cit., p. 15.
[10]Ibid., p. 170.

In principle, both of these outcomes have the same likelihood of occurring, but most people would say that the second option "looks" more random.[11] We have a general conception of what we expect random data to look like. First, it tends to conform to a model of probability. That is, we expect after ten coin flips to have about 5 heads and 5 tails. Second, we expect random events to lack apparent patterns or periodicity.

When a data series has patterns, we tend to attribute causation in what is known as the "Texas Sharpshooter Fallacy," where the supposed sharpshooter simply shoots haphazardly at the side of a barn and draws the bullseye in the location where most of the shots happened to cluster. But random data may also exhibit patterns; or, perhaps it is the nature of human intelligence to seek patterns even when they are not there, as when our ancestors found constellations in the stars.

In Tom Stoppard's "Rosencrantz and Guildenstern Are Dead," the title characters witness a long series of coin flips that all result in heads. One of Guildenstern's theories to account for this is that it should come as no surprise, since each individual coin is as likely to come down heads as tails; that the probability of each flip is the same regardless of the history of previous coins.

Though 90 flips coming up all heads is equally likely as any other specific outcome, we expect to see about 45 heads and 45 tails. Random events over time do tend to conform to probabilistic analysis. Using a standard pseudo-random number generator (a fairly accurate digital representation of a coin flip), a series of 30 flips in a row was observed only after 1.4 billion attempts, making Rosencrantz and Guildenstern's observations seem rather rare and improbable. Indeed, the chance of getting any particular sequence of flips is $\frac{1}{2^{90}}$, which is approximately 8×10^{-28}.

[11] From a certain perspective, this perception is true: see Section 5.1 for more on the issue of entropy and predictability in data series.

Number of Heads	Number of Possible Outcomes of Coin Flips
0	1
1	10
2	45
3	120
4	210
5	252
6	210
7	120
8	45
9	10
10	1

Table 2.1: Probability Distribution of Coin Flips. The right column designates the number of "heads" results, and the left contains the number of different possibilities which result in this number of heads. This probability is based on a simple count of the number of heads that occurs and the number of possible outcomes; though each outcome has equal likelihood, we find it possible to generalize based on the general features (number of heads) of the outcome rather than its specific ordering, and thus probability is one way we deal with the unpredictability of chaos.

All seems lost, but if we count in terms of the total number of heads or tails, the distribution becomes Gaussian and our seemingly flawed intuition is actually correct.[12] Specifically, the number of outcomes (each of equal probability, in theory) for ten flips of a coin is found in Table 2.1. Thus, we shouldn't expect to ever see 90 heads in a row, just as we shouldn't expect to see any other specific outcome, but should not worry about generalize over the entire set of outcomes.[13]

This intuition regarding randomness is one way that humans have adapted to life in an unpredictable world. Chaotic series are completely unpredictable, but they do

[12] A Gaussian distribution is the standard bell curve.

[13] This ability to generalize over a long series of outcomes actually follows directly from our definition of what probability means. When we say that a coin flip of heads has a 50% probability, we are actually saying that, over time, we expect approximately 50% of the coin flips to be heads. Thus, it's not an intuition at all.

follow laws of distribution and probability. Thus, while mathematicians and statisticians (as well as die rollers or card players) may use idealized versions of these physical events, generalizing over large numbers of repeated outcomes in order to understand and create probabilities, the chaos theory view of these events views them as physical events in their entirety, without discrete probabilities or results but with a continuous, fractal-like set of possible results which sensitively depend on the precise conditions of the event. Thus, a die roll or coin flip is not random, but precisely determined by the exact circumstances of its physical actuality; the randomness results from our inability to measure accurately.

The element of chance played a huge role in the compositions of John Cage. Many of Cage's works depend on choice-making or on the outcome of physical events, such as tossing a coin. Thus, perhaps we can consider aleatoric works which depend on chance procedures in some sense chaotic, as these types of activities are deterministic rather than random. This is not to say that Cage's ideas weren't innovative; it was the removal of the composer from the process that was the primary motivating factor for Cage's aleatoricism, and not the emphasis on random processes:

> And what is the purpose of writing music? One is, of course, not dealing with purposes but dealing with sounds. Or the answer must take the form of paradox: a purposeful purposelessness or a purposeless play. This play, however, is an affirmation of life—not to bring order out of chaos nor to suggest improvements in creation, but simply a way of waking up to the very life we're living, which is so excellent once one gets one's mind and one's desires out of the way and lets it act of its own accord.[14]

Cage recognized the individuality of each experience of sound. He once wished to experience true silence, and subjected himself to a anechoic chamber in order to

[14] John Cage. *Silence*. Middletown, CT: Wesleyan University Press, 1961, 1973, p. 12.

remove all sound. He found that he was still able to hear sounds, perhaps the resident sounds of his nervous system or tinnitus, along with his own heartbeat. Cage wrote:

> Until I die there will be sounds. And they will continue following my death. One need not fear about the future of music.[15]

To end this section, I will examine the distinction between aleatoric music and chaotic music. Aleatoricism, broadly defined, is incorporation of chance into the process of the creation of music (or art). Aleatoricism, then, depends on randomness or stochasticism. As discussed, these processes may originate in profoundly complex nonlinear dynamical systems rather than true randomness; it is difficult to know for sure. Certainly the data resulting from actual coinflips (if this is indeed "random") is indistinguishable from nonlinear simulations of coinflips. Whether truly random or simply chaotic thus becomes a matter of perspective, and we might consider all aleatoric processes as examples of chaos in music.

2.3 Minimalism

"In serial music, the series itself is seldom audible... What I'm interested in is a compositional process and a sounding music that are one in the same thing."

— Steve Reich

The minimalism movement in America began with *In C* by Terry Riley and the experiments of Steve Reich with two pieces exploiting the "phasing" of tape loops: *It's Gonna Rain* and *Come Out*. For these pieces, Steve Reich looped a short segment of a

[15] Ibid., p. 8.

person's voice, and simultaneously coupled it with the same segment of recording only on a slightly longer tape. The result is a tape that slowly phases out of synchronization with itself, transforming the vocal audio into something entirely different.

The idea drawn from this was not necessarily to emphasize musical phase as a compositional device, but instead to use musical processes that were simple, audible, and recognizable by the audience. This is the essence of minimalism, a reaction in part to the indecipherable excesses of Serialism.

Essentially, this is a strong example of algorithmic approaches to music composition, though in this case the algorithm is intended to be comprehended as a part of the aesthetic of the piece. As we have seen, very simple algorithms can have remarkably complex results, and the simple nature of the looped audio phase of *It's Gonna Rain* should be contrasted with the incredibly complex sound-world of the result.

2.4 Pointillism and Independence

"The collision of hail or rain with hard surfaces, or the song of cicadas in a summer field. These sonic events are made out of thousands of isolated sounds; this multitude of sounds, seen as totality, is a new sonic event."

— Iannis Xenakis

Iannis Xenakis was a Greek composer and music theorist who often rebelled against what he felt were the artificial restrictions of the music-theoretical system. Pape writes:

> Every day of his life, Xenakis tried as a composer to erase everything—to begin again as if he had never composed, as if no one had ever composed, as if each new piece were the first piece of music ever written.[16]

[16] Gerard Pape. "Iannis Xenakis and the "Real" of Music Composition". In: *Computer Music Journal* 26.1 (2002), pp. 16–21, p. 16.

His music reflects his training as an architect and a mathematician. Xenakis is widely recognized for his pioneering work in the application of mathematical set theory and stochastic functions to the composition of music.

Xenakis was initially interested in the application of stochastic phenomena to the process of musical composition. He began with works composed using processes of chance, different from other composers of the time (such as John Cage) in that the chance procedures were completed once the piece was finished, so the finished work was not based on chance itself (i.e. not aleatoric), but complete and repeatable in its own right.[17] This meant that, while the construction of the piece might have involved chaos, the net result would remain consistent from performance to performance, within the limits of any other notated piece of music.

Xenakis writes: "I was attracted to the sheer complexity of stochastic phenomena."[18] This complexity was similar in sound and degree to that attained by the serialists, but Xenakis did not share a common ground with them: "In the 1950s, serialism was very powerful. I did not care for it at all. The dodecaphonic language was too restrictive... It offended my sensibility about music, which I thought had to be different."[19] If serialism was considered too controlling and restrictive, free improvisation was considered by Xenakis to be limited by the conditioning of the practitioner rather than by compositional choice.[20] This would lead Xenakis to electronic music as a means to negate the cultural conditioning of the performer.

[17]Christopher Butchers. "The Random Arts: Xenakis, Mathematics, and Music". In: *Tempo* new ser. 85 (1968), pp. 2–5, p. 4.

[18]Bridgette Robindoré and Iannis Xenakis. *Eskahaté Ereuna: Extending the Limits of Musical Thought: Comments on and by Iannis Xenakis*. Vol. 20. 4. 1996, pp. 11–16, p. 15.

[19]Ibid., p. 13.

[20]Judy Lochhead. "Hearing Chaos". In: *American Music* 19.2 (2001), pp. 210–246, p. 228. For more about free improvisation and chaos, see: David Borgo. *Sync or Swarm*. New York: The Continuum International Publishing Group, 2005

Xenakis's interest in probabilities, led by his interest in the nondeterministic processes of nature, predictably led to other applications of mathematics in the creation of musical algorithms, and he used aspects of physics (granular sound-clouds) and mathematical set theory to sculpt his musical ideas, which were intended to evoke natural events such as "the collision of hail or rain with hard surfaces, or the song of cicadas in a summer field."[9] Through his use of stochastic methods, Xenakis was attempting to recreate the chaos of natural sound worlds.

Many of Xenakis's ideas are useful to further applications of chaos theory to music. Xenakis conceived of sound as a summation of individual sonic "grains," each with their own pitch and duration:

> All sound is an integration of grains, of elementary sonic particles, of sonic quanta. Each of these elementary grains has a threefold nature: Duration, frequency, and intensity. All sound, even all continuous sonic variation, is conceived as an assemblage of a large number of elementary grains adequately disposed in time. So every sonic complex can be analyzed as a series of pure sinusoidal sounds even if the variations of these sinusoidal sounds are infinitely close, short, and complex.[21]

Out of "sound clouds" consisting of these grains, Xenakis created complex musical textures, rather like Georges Seurat used pointillism (Figure 2.1). These sound-clouds might be thought of in terms of phase states, and thus the behavior of these individual grains might be modeled using phase state diagrams and nonlinear dynamical systems. Modern computing would allow for Xenakis's method to be enacted, thus arriving closer to Xenakis's goal of imitating the natural world.

[9] xenakis2

[21] Iannis Xenakis. *Arts/Sciences: Alloys. The Thesis Defense of Iannis Xenakis.* New York: Pendragon Press, 1985, p. 43.

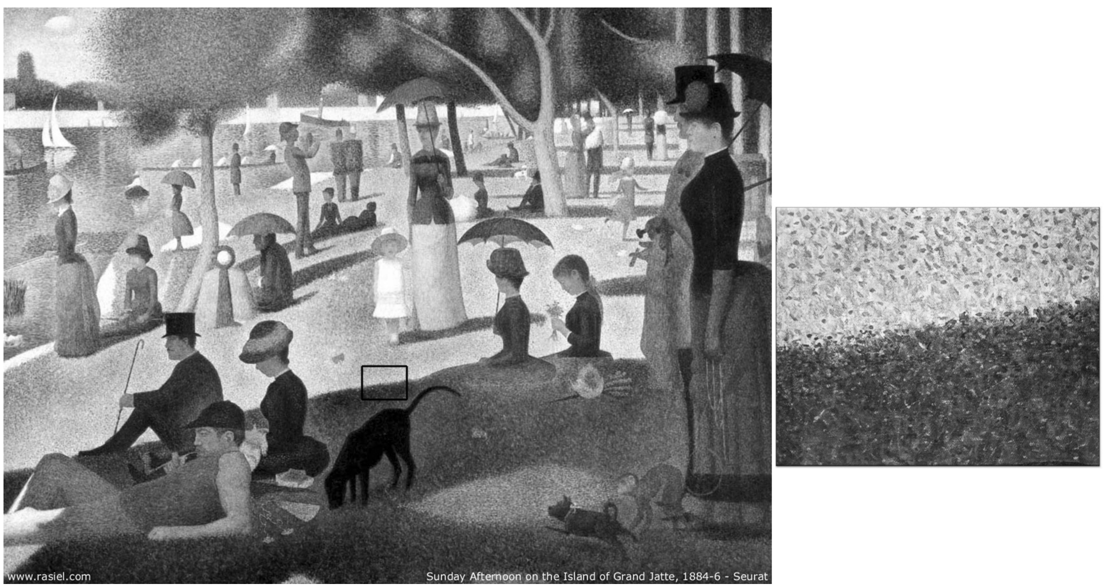

Figure 2.1: Georges Seurat, Sunday Afternoon of the Island of La Grand Jatte. Inset closeup to show pointillistic technique. The pointillistic approach can also be applied to a musical interpretation, as well as a visual one.

However, Xenakis himself might not be pleased by such an action. His music generally used stochastic algorithms which were then controlled and shaped by his own musical perspective and creativity. James Harley writes: "[Xenakis's] process takes (in effect) random data and, instead of filtering it, shapes it in particular ways which do not rely on pre-existing models but on the composer's imagination or musical 'vision.' "[22] Xenakis himself states the importance of the composer's point of view in his work:

> . . . what I did was a matter of feeling, rather than a purely logical approach. There were struggles of course, but it was the sound itself that absorbed my attention—the changes in the sound. Even if I wrote a predetermined piece, it could have parts that did not follow the exact theoretical path, but which depended more on the acoustical result. This freedom was much more interesting and important for me.[23]

Thus, while computation has its place in the process, Xenakis feels it is important not to neglect the role of the composer in the creation of the work, and does not cede control of the music to the process (as Steve Reich or John Cage might). Xenakis' own sound world was populated by the sounds around him, natural or artificial, and he sought to break with tradition and discover a new approach to music:

> But freedom of thought, for me, could not come from there. I was convinced that one could invent another way of writing music. I set myself to imagining sound phenomena, using drawings to help me: a spiral, intersecting planes. . . .

[22] James Harley. "Generative processes in algorithmic composition: Chaos and music". In: *Leonardo: Journal for the International Society for the Arts, Sciences, and Technology* 28.3 (1995), p. 221.

[23] Robindoré and Xenakis, loc. cit.

> And then, I always adored the sound of nature, the sea, crickets. During the Occupation, the demonstrations against the enemy brought together hundreds of thousands of people in Athens who shouted slogans, who planted mines. Apart from these scenes which marked me politically, the sound phenomena are engraved in me. During the street fighting of December 1944 there were scattered explosions, tracer bullets, bombings: extraordinary sounds.[24]

While both Cage and Xenakis seemed, in the end, to attempt to exempt the composer's own perspective and taste from the process of music composition, their motives are disparate. Whereas Cage was attempting to compose music which was expressionless, Xenakis attempted to write music which was powerfully expressive, and would never separate the composer completely from the process.

2.5 Electronic Music

"Suddenly, one day, it seemed clear to me that full flowering of music is frustrated by our instruments ...In their range, their tone, what they can render, our instruments are chained fast and their hundred chains must also bind the composer"

— Feruccio Busoni

Just as computers revolutionized the world of mathematical modeling, the new processing power was just as important to the development of music in the later 20th century. It began with electronic instruments such as the *Trautonium*, *Ondes Martinot* and the *Theremin* in the late 20s, and the *Rhythmicon* in the early 30s. These early instruments used electricity to control the emitted sounds. The next generation of synthesizers in the 60s and 70s included the ARP and Moog synthesizers,

[24] Iannis Xenakis, Roberta Brown, and John Rahn. "Xenakis on Xenakis". In: *Perspectives on New Music* 25 (1987), pp. 16–63, p. 21.

and by this time the electric guitar had begun its eventual climb to dominance over Western popular music.

Feruccio Busoni was a piano prodigy, composer, teacher, and philosopher. His music was characterized by a neoclassic aesthetic — small ensembles and simple, direct forms along with the rejection of the sentimentality that characterized late Romantic works.

> Exhaustion surely waits at the end of a course the longest lap of which has already been covered. Whither then shall we turn our eyes? In what direction does the next step lead?...I believe that all efforts must be directed towards the virgin birth of a new beginning.[25]

Busoni became very interested in the Dynamophone, invented by Thaddeus Cahill in the early 1900s, and deduced that the future of music would be in electronics: "...I almost think that in the new great music, machines will also be necessary and will be assigned a share in it."[26]

He was right. The modern computer has revolutionized the music world several times over. Providing a revolution in the mechanisms of distribution through the internet, the MP3 has become ubiquitous; the digitized and compressed medium of modern music. Studio processed and produced albums have replaced live musicianship as the standard, particularly in American popular music.[27]

But, more central to the issues at hand, technology has leant its remarkable computing power to the issue of composition, particularly algorithmic composition. Tech-

[25]Herbert Russcol. *The Liberation of Sound: An Introduction to Electronic Music.* Englewood Cliffs, NJ: Prentice-Hall, 1972, p. 32.

[26]Ibid., p. 38.

[27]Even as I write this, it's clear that statements regarding "current" trends in music will become obsolete as rapidly as the computer on which I write them.

nology has unlocked many possibilities with respect to the application of chaotic functions to the generation of music. Since most chaotic functions are computationally extensive, the development of computers has put many of these complex systems in the hands of composers through their desktop PC.

Most of the music generated has been done so through composer-programmers who have been adept both in music and computer science. There has yet to emerge any particular standard approach, and most researchers in this area have written their own programs to aid in this application.

Rick Bidlack examines the use of computers and chaotic functions as a means of generating musical events (specifically, note generation) in his research and compositions.[28] For Bidlack, chaotic processes provide a way to emulate natural processes of the world through complex systems of equations, thus "endowing computer-generated music with certain natural qualities not attainable by other means."[29] The generation of successive steps of chaotic equations (i.e. any of the difference equations of Poincaré from Section 1.4) are trivial for a computer, and Bidlack uses the output of these functions to control pitch in his work. Similarly, output could be used to control timbre, rhythm, duration, dynamics, and any other conditional aspect of music. Bidlack uses the Lorenz attractor (from Section 1.6) to compose his "Dodecanon I" for MIDI synthesizers.[30]

One interesting point Bidlack makes is that the conversion from a mathematical space (such as the surface of a torus) to a musical space is generally from a space which is continuous and periodic to one that is not.[31] For example, shapes which

[28]Rick Aaron Bidlack. "Chaotic systems as simple (but complex) compositional algorithms". In: *Computer Music Journal* 16.3 (1992), pp. 33–47.

[29]Ibid., p. 33.

[30]Rick Aaron Bidlack. "Music from Chaos: Nonlinear Dynamical Systems as Generators of Musical Materials". PhD thesis. University of California, San Diego, 1990.

[31]Id., "Chaotic systems as simple (but complex) compositional algorithms", p. 38.

might form a continuous oval on the surface of a torus might, due to the boundary constraints of the mapping, form a very discontinuous shape in terms of frequency. Bidlack accepts this flaw, but it might be a better approach to create a system of pitch ordering which is continuous and periodic in the same way that a torus is, for example, using a sine curve to map torus values to frequencies, creating a space which is continuous and periodic in the same way that theta values on a torus travel from 0 to 2π, though this mapping would not be isomorphic. However, this way the overall shape and continuity of a potential curve is maintained no matter where it falls with respect to arbitrary boundary lines.

Other researchers have focused on fractal geometry,[32] synthesis of acoustical sounds through nonlinear algorithms,[33] use of chaotic oscillators directly,[34] or the use of chaos in algorithmic composition,[35] as well as many other applications of computer-aided composition.

By necessity, this section leaves out many vitally important contributors in the development of electronic music, and is meant not as a summary but more as an introduction. The central issue to our purposes is simply that early electronic devices and later computers played a vital role in the development of chaos theory, and simultaneously opened up new ways of thinking about music as well. The computational power of computers created new opportunities for algorithmic approaches to problem solving, be they mathematical, musical, or otherwise.

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[32] Joseph Rothstein. "FracTunes MIDI Graphics Software for IBM PCs". In: *Computer Music Journal* 15.4 (1991), pp. 123–124.

[33] James Woodhouse. "Physical Model of Bowed Strings". In: *Computer Music Journal* 16.4 (1992), pp. 43–56; Teresa Wilson and Douglas H. Keefe. "Characterizing the Clarinet Tone: Measurements of Lyapunov Exponents, Correlation Dimension, and Unsteadiness". In: *Journal of the Acoustical Society of America* 104.1 (July 1998), pp. 550–561

[34] Dan Slater. "Chaotic Sound Synthesis". In: *Computer Music Journal* 22.2 (1998), pp. 12–19.

[35] Kenneth McAlpine, Eduardo Miranda, and Stuart Hogger. "Making Music with Algorithms: A Case-Study System". In: *Computer Music Journal* 23.2 (1999), pp. 19–30; Jeremy Leach and John Fitch. "Nature, Music, and Algorithmic Composition". In: *Computer Music Journal* 19.2 (1995), pp. 23–33

2.6 Summary

"Order is repetition of units. Chaos is multiplicity without rhythm"

— M. C. Escher

Music history in the 20th century seems to have undergone a similar trend as science, embracing complexity and the role of the individual to experiment and redefine the underlying "rules" of music. Experiments regarding the content, context, and composition of music asked critical questions about what it meant to make music in the age of Relativity and complexity. Works of surprising complexity and depth arose, for which chaos theory provides an excellent analogy and basis of understanding. The next chapter will address the similarities between the trends in science and the trends in music during this time.

Chapter III
Music, Science and Society

"Even if music can't express anything anyway, or means nothing (and this last is incorrect), music is in any case a reflection of society."

— Louis Andriessen

Through our explorations of science and music to the end of the 20th century, I have demonstrated that something new was happening during this time. For the first time, scientists and musicians both recognized that complexity was something to be appreciated in its own right. They were drawn away from reductive approaches to music analysis and science and began to embrace the whole picture. Simplifications and approximations were replaced by *Gestalt* approaches and attempts to understand music and science on multiple levels.

Science searched for the "theory of everything" in order to reconcile the complexities found both at small scales and large speeds, since traditional models didn't work. Composers created works of astounding complexity and intricacy; works which could no longer be analyzed using traditional models. The abandonment of these traditional models in both music and science was a search for a new approach and a new answer. Of course, both scientific revolutions and complex approaches to music had to wait until the society was ready to adopt these new ideas and accept complexity as something worth our attention.

3.1 Metacognition

"The richness of human life is that we have many lives; we live the events that do not happen (and some that cannot) as vividly as those that do; and if thereby we die a thousand deaths, that is the price we pay for living a thousands lives"

— Jacob Bronowski

There was an underlying change that occurred in the 20th century that was demonstrated by the changes that occurred in the arts and sciences. This change seems much more revolutionary than other periods of transition in human history, and certainly the *fin de siècle* period has proved a fruitful area of study, as a great amount of research has been spent on this time in history. What was so new about our society in the 20th century?

As we have seen for music, the arts at this stage entered a time of great experimentalism, as if the act of creating art was no longer simply an act of creation but a statement of purpose; art and music were explorations not *in* genre but *of* genre. Composers were not just expressing their own musical perspectives and creativity, but also what music should be (or could be). In the fine arts, many of the innovations of Cubism, Surrealism, or other types of abstract art were not only artworks themselves but also simultaneously expressed questions about what Art itself is. In a way, both John Cage's *4′33″* (Figure 3.1) and Marcel Duchamp's *Fountain* (Figure 3.2) express the same sentiment about music and art respectively, though John Cage was interested in the creation of art for personal enlightenment, where Duchamp was more revolutionary; a political activist attempting to undermine political hierarchies.

This is the world of the "meta." Poems about poetry, music about music and art about art; so too, science about science. Science was not just concerned with finding the the structures and laws that govern the natural world, but was concerned with

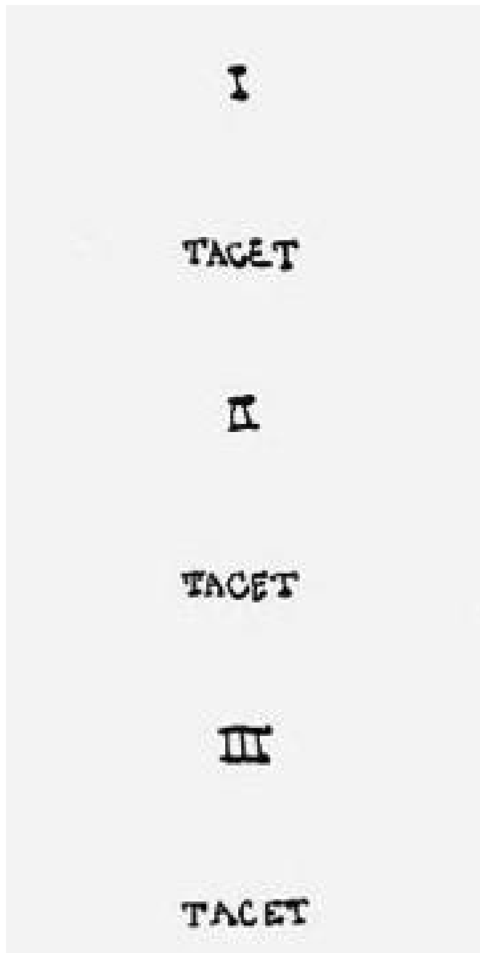

Figure 3.1: John Cage, 4'33". The work involves a set of durations to be observed, leaving the sound-scape of the composition completely dependent upon the specific circumstances of its performance.

Figure 3.2: Marcel Duchamp's *Fountain*. Submitted to Society of Independent Artists under the pseudonym "R. Mutt" for their exhibition in 1917. It was not shown at the exhibition. The Blind Man No. 2, page 4. Editors: Henri-Pierre Roche, Beatrice Wood, and Marcel Duchamp. Published in New York, May 1917 Fountain by Marcel Duchamp. 1917. Photograph by Alfred Stieglitz. Source: `http://sdrc.lib.uiowa.edu/dada/blindman/2/04.htm`

```
A poem should be palpable and mute
As a globed fruit,

Dumb
As old medallions to the thumb,

Silent as the sleeve-worn stone
Of casement ledges where the moss has grown-

A poem should be wordless
As the flight of birds.

                 *

A poem should be motionless in time
As the moon climbs,

Leaving, as the moon releases
Twig by twig the night-entangled trees,

Leaving, as the moon behind the winter leaves,
Memory by memory the mind-

A poem should be motionless in time
As the moon climbs.

                  *

A poem should be equal to:
Not true.

For all the history of grief
An empty doorway and a maple leaf.

For love
The leaning grasses and two lights above the sea-

A poem should not mean
But be.
```

Figure 3.3: Ars Poetica by Archibald MacLeish, 1926. This is a metapoem: A poem about poetry.

what science itself could tell us, and the limitations of scientific inquiry. Rather than "what do we know," the question became "what can we know?"

Of course, none of these ideas were "new" in the strictest sense. Philosophers had been asking similar questions since the origin of history. What was new about this in the 20th century was how widespread the phenomenon became. Western society was finally ready for the shift. Of course, put in such general terms the issues here are rendered somewhat simplistic; the history of the 20th century does not reflect an instant embracing of these ideas but rather a tension, a push-and-pull between those on the fringes of society who are challenging it, and those who remain traditionalists. Art and music may have experienced experimentalist revolutions but were simultaneously also being encapsulated and institutionalized throughout the 20th century—there is more subtlety in this history than I have portrayed. Still, for our purposes, the connection between changes in perspectives toward music, art, and science, points to an underlying revolution which caused these new perspectives to become accepted, and simultaneously planted the seeds toward a general acceptance of complexity and chaos theory in late 20th-century society.

Is the world tending toward complexity or toward increasing disorder? Pierre Teilhard de Chardin would have us believe that the universe is headed toward greater and greater forms of complexity (the "Omega Point"), while entropy theory has the universe (and any other closed system) tending toward disorder.[1] Perhaps these two theories are reconcilable, and the increase in disorder is also an increase in complexity; thus, perhaps we should conclude that the universe is headed toward chaos.

[1]See Section 5.1 for more about entropy

3.2 Free Will

"Life is like a game of cards. The hand you are dealt is determinism; the way you play it is free will."

— Jawaharlal Nehru

As mathematics and science understands more and more of how the world works, we must come face to face with the dire consequences that result from believing in a world that behaves so deterministically. The existence of free will has been debated by philosophers, skeptics, and religious authorities since the dawn of history, and a deterministic look at the universe might imply that our own sense of free will is an illusion; that we, too, are subject to the deterministic laws of nature. Before I leave the history of chaos theory, I must briefly examine this central issue of philosophy and how we can live in a chaotic and unpredictable, yet deterministic world.

As we've seen from the Heisenberg Uncertainty Principle and Quantum Mechanics, at a quantum level making a measurement causes changes in that which is being measured, and there is a fundamental limit to how accurate measurements can be. A. B. Çambel uses this to argue that modern science has found a solution to the problem of free will:

> In Laplace's deterministic world there would be no uncertainty, no chance, no choice, no freedom, and no free will. Everything would be predetermined. We know from personal experience that this cannot be...
>
> From the scientific viewpoint, strict determinism must be ruled out because measurements are affected by the presence of the observer. Even the so-called noninvasive measurements affect the system at least microscopically. We also know that the number of particles constituating any system is horrendously large, about 2.7×10^{19} particles per cubic centimeter, so that their coordinates and momenta cannot be specified except statistically. Even on the scale of the world's population, namely about

> 5.3×10^9, a much smaller number, we cannot tell the whereabouts, nor the activities of, individuals. There is always going to be some uncertainty.[2]

With the knowledge of chaos theory, we know that these minuscule alterations, at the smallest known scale, will cause large changes in the system in which they play a part, if that system is nonlinear. Thus, we should say that, even if we were to somehow make measurements to an extreme degree of accuracy, violating the Uncertainty Principle, the act of measuring itself would cause the system to be unpredictable. The illusion of free will may be sufficient, as absolutely predictability is impossible.

Douglas Hofstadter posits that intelligence itself may be a result of the mixing of cognition and metacognition in the brain, that the existence of "choice" (and thus free will) is a result of a obfuscation in the brain of the mechanisms of decision.[3] This is one way to reconcile the illusion of free will with the inevitability of deterministic processes in the brain. Is there a way to distinguish between the vague notion of "decision-making" in the brain from the actual firings of neurons which results in the decision being made? Does it matter if the brain has a deterministic (and yet unpredictable) method for making decisions if we still feel as if we are in control? Would randomness in decision-making be more comforting, or less?

Research by computational neuroscientists is pointing to an increasingly nonlinear view of brain computation.[4] This would both explain the unpredictable nature of human interactions, as well as explain our own sense of unpredictability in our own decision-making and inspiration. Luckily, all is not lost for believers in free will:

[2] Çambel, op. cit., pp. 7-8.

[3] Douglas R. Hofstadter. *Gödel, Escher, Bach: An Eternal Golden Braid*. New York: Basic Books, 1979, pp. 697, 710–713.

[4] Michael London and Michael Häusser. "Dendritic Computation". In: *Annual Review of Neuroscience* 28 (2005), pp. 503–32.

Edward Lorenz resolves the issue of free will directly, with an approach reminiscent of Pascal's Wager:

> We must then wholeheartedly believe in free will. If free will is a reality, we shall have made the correct choice. If it is not, we shall still not have made an incorrect choice, because we shall not have made any choice at all, not having a free will to do so.[5]

3.3 Summary

"It turns out that an eerie type of chaos can lurk just behind a facade of order—and yet, deep inside the chaos lurks an even eerier type of order"

— Douglas Hofstadter

A similar trend in scientific development and the corresponding history of music toward complexity and metacognition supplies evidence for an understanding of a shift in society toward the embracing of these ideas. This underlying shift caused many of these ideas, particularly chaos theory, to be accepted for the first time, in spite of the fact that the background research and discovery of chaos had already taken place generations earlier. Society plays a role in selecting which ideas will be accepted and incorporated into the collective understanding, and which ideas will remain fringe elements or simply ignored.

[5] Lorenz, op. cit., p. 157.

Summary of Part I

"Not everything that can be counted counts, and not everything that counts can be counted."

— Albert Einstein

As history embraced the 20th century, science and music, perhaps as a result of a similar trend in the underlying society, moved more and more toward an understanding of complexity and a non-reductionist view of their respective fields. Science developed a revolutionary new view of the universe for the first time since Isaac Newton, and music thrived in a new era of experimentalism. Both of these trends essentially depended on the culture in which they fermented; Einstein in 1600 would have been considered heretical or insane where Newton was considered brilliant, and John Cage would certainly not have been hired as a court composer at Esterháza where Haydn worked. Having the right idea *at the right time* proves essential, and meant that the world had to wait until it was culturally ready for for both chaos theory and 20th century music

It's important to emphasize the role that computers played in the later part of the 20th century. Music was heavily influenced by the new electronic medium, and chaos theory would never have been discovered and thought worthy of study without the great number of calculations required to map a chaotic space. The computer facilitated the difficult computations required to solve these problems, and as computation power increased exponentially toward the end of the century, even more complex problems could be solved.

In the following chapter, I explore the combination of science and music by investigating the application of chaos theory to music theory and acoustics.

Part II

Music Theory

Chapter IV
Understanding Harmony

"For most of us, there is only the unattended
Moment, the moment in and out of time,
The distraction fit, lost in a shaft of sunlight,
The wild thyme unseen, or the winter lightning
Or the waterfall, or music heard so deeply
That it is not heard at all, but you are the music
While the music lasts."

— T. S. Eliot

This chapter will focus on the application of chaos theory to various elements of acoustics and harmony. We begin with a general survey of human hearing, because music-making is ultimately a human act that is meant to be performed and heard by humans, and it is important to understand the context and limitations of such an activity. From this end, I explore the act of listening to music from a biological, neurological, and phenomenological standpoint, and discuss elements of modern music including noise, consonance and dissonance, the difference between recorded music and live music, and the inherent complexity of sound.

4.1 Fourier Analysis and the Anatomy of the Ear

"Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them."

— Joseph Fourier

Before addressing how music is heard, I will address what sound is composed of before it becomes the series of nervous impulses that are interpreted by the brain. For this, it is necessary to present a brief mathematical diversion on Fourier Analysis.

Joseph Fourier was a French mathematician in the early 19th century. He hypothesized that any function could be transformed into a series of sine functions. As it turns out, he was not quite correct, but it is often the case that this is possible, particularly with periodic functions. The branch of mathematics concerning this idea is called *Fourier Analysis*, and a series created in this way is known as a *Fourier Series*.

Essentially, a Fourier Series takes a complex waveform, such as a sound wave, and replaces this complexity with a sum of its sine wave components. As it turns out, sound waves turn out to be a prime topic for the application of Fourier series due to the structure of human anatomy. The anatomy of the ear functions as a tube which converts the data of sound frequencies into the perceptional tonal realm, converting from the linear domain of frequency values to the logarithmic domain of pitches. The sound waves are converted to vibrations by the eardrum and bones of the inner ear, which convey those vibrations to the fluid in the cochlea, a long tube which is curled up like a snail shell (see Figure 4.1. Small follicles line the cochlea, and the follicles respond like a Fourier analysis, vibrating sympathetically with waves that have a wavelength equal to the distance from the origin of the cochlea to the follicle. In other words, the act of hearing sounds is in fact a biological act of Fourier Analysis,

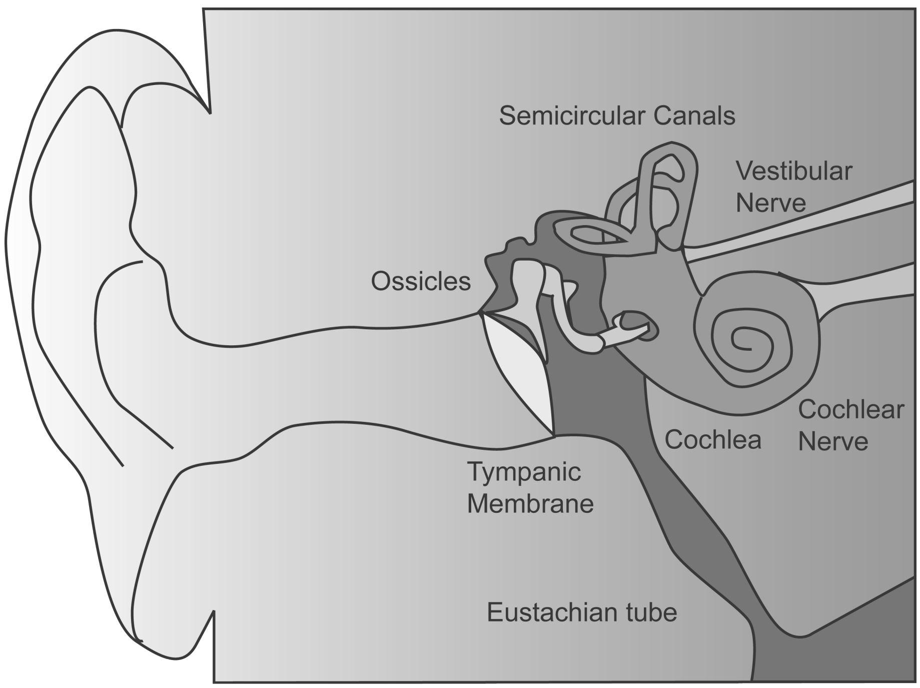

Figure 4.1: The Anatomy of the Ear. Note the small snail-shaped cochlea; this shape is responsible for the mechanisms of human hearing and its relationship to Fourier analysis.

though there are some other neurological factors in our perception of sounds which make our own sense of hearing a little less predictable.[1]

All acoustic instruments make sounds which follow a certain pattern in harmonic spectrum. This is known as the *harmonic series*, and it was recognized by the Pythagoreans as a property of music. The harmonic series is given as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \dots \tag{4.1}$$

This corresponds to all of the possible wavelengths of vibration for a string with fixed ends, expressed as a ratio of the whole length of the string. In other words, when a string is plucked (or bowed), it vibrates at all of the possible frequencies available to it, and these frequencies are limited by the fixed endpoints of the string. The harmonic series in music is expressed in terms of these frequencies, which in Figure 4.2 is based on C: 32.702Hz.[2]

This is the source of our perception of *timbre*, or tone color; that is, when all of these pitches are sounding simultaneously, the fundamental (lowest note) is the pitch that is perceived, but the overtones are experienced as timbre, which is the reason we can tell a clarinet sound from a flute or oboe.[3] Timbre is based on the relative amplitudes of the harmonic series. A clarinet, for example, has very strong odd-numbered harmonics and relatively weak even-numbered harmonics due to its cylindrical bore; Figure 4.3 shows an example of a Fourier Analysis of a clarinet sound.

[1] R. Duncan Luce. *Sound & Hearing: A Conceptual Introduction.* Hillsdale, NJ: Lawrence Erlbaum Associates, 1993, p. 182.

[2] Since *Hertz*, the unit of frequency, is measured in $\frac{1}{seconds}$, the harmonic series from Equation 4.1 becomes a series based on the multiples of the original fundamental frequency, rather than their reciprocals.

[3] Timbre is determined by overtones spectrum, but the quality of the attack of the instrument also plays a vital role in discerning instrument timbre and recognition. Incidentally, this attack is often full of noise and nonlinearity.

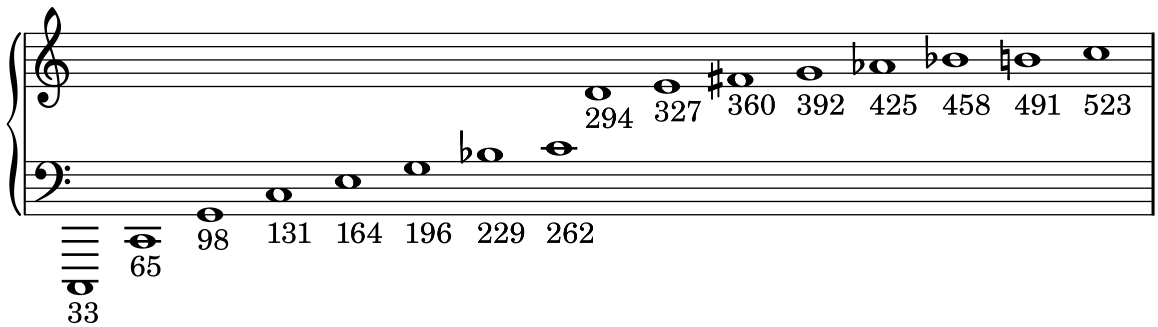

Figure 4.2: The Harmonic Series with Approximate Frequencies (Hz). The linear sequence of frequencies (shown in Hertz) is perceived as a logarithmic sequence of pitches (as notated).

This pattern of odd-numbered harmonics is what gives the clarinet its characteristic sound. Figure 4.3 also shows what a Fourier Analysis of an aperiodic sound yields.

The harmonic series is responsible for much of our history of harmony. The fundamental with its first three overtones, for example, form the major triad.[4] It is a quirk of human hearing that this linear progression of frequencies is heard as a logarithmic series of pitches. The fact that 220Hz, 440Hz, and 880Hz are heard as the same pitch is not a natural feature of the universe around us, just as the wavelengths of visible light is not an abstract property of the universe. These are human adaptations. Thus, the recognition and relative importance of the overtone series in music, and its function as the underpinning of tertian harmony, is actually a result of the anatomy of the ear, not any fundamental aspect of nature abstracted from humanity.

A natural next question is: How accurate is the human ear? Can the human ear hear to the same sensitivity as a microphone? A generally agreed measure of the

[4]Each member of the harmonic series is considered a "partial," whereas they are also known as "overtones" over the fundamental frequency. Thus, the first three *overtones* are *partials* two through four.

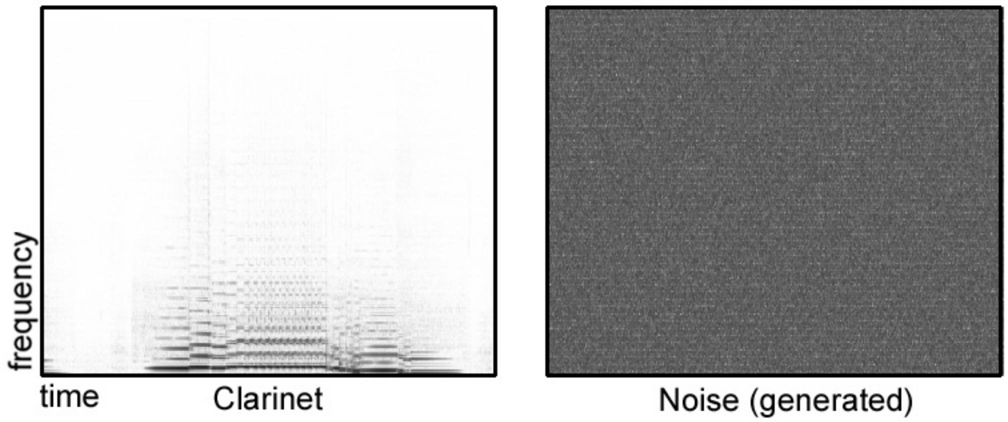

Figure 4.3: Fourier Analysis of Clarinet and Generated Noise. Created using Max 5 by Cycling '74. The clarinet sound was recorded, and the noise was computer generated. Note the linearly distributed harmonic overtones in the clarinet sound, and that the noise is measured as equal amplitudes at all frequencies.

human ear's sensitivity is approximately from 0dB to 140dB in loudness (2×10^{-5} to 200 pascals), and from 10Hz to 20,000Hz in frequency (though below 20Hz is not perceived as tone).[5] For reference, 60dB is approximately the loudness of the human voice in conversation. Luckily for us, this threshold of hearing is well above the pressure changes caused by Brownian motion of the molecules of the air; imagine if we could hear every molecule's motion as sound![6]

When audio is recorded into digital form, it is first passed through an anti-aliasing filter to remove frequencies above the *Nyquist* frequency of the recording device (for CD quality audio, the Nyquist frequency is 22.05 kHz). This is above the limits of human hearing, but may not be above the limits of human perception. The non-

[5]Stanley A. Gelfand. *Hearing: an Introduction to Psychological and Physiological Acoustics.* 4th ed. New York: Marcel Dekker, Inc., 2004, p. 279; C. Daniel Geisler. *From Sound to Synapse: Physiology of the Mammalian Ear.* New York: Oxford University Press, 1998, p. 20

[6]David Green. *An Introduction to Hearing.* Hillsdale, NJ: Lawrence Erlbaum Associates, 1976, p. 37.

linearity of the signal is also lost in digital conversion, as the digital data will be an approximation, with 44,100 samples per second, of the actual waveform. Microphones also generate a certain amount of electrical noise themselves, generally on the order of 7-15 dB, and so any sound softer than this level will be lost. On the reproduction end, a speaker will also generate its own noise from its circuitry, and will operate within certain limits; a standard household speaker produces up to 90–100dB of sound before distortion sets in.

It is possible that the primary difference between an acoustic sound, as from a musical instrument, and a digitally reproduced sound is the nonlinear aspects of this sound. Any nonlinear oscillations in the original acoustic sound would be trimmed off by the frequency response of the recording. While we do not perceive these nonlinear oscillations as pitch, since our ears would experience them as "noise," they do contribute to our experience of acoustic phenomena.

James Woodhouse writes:

> It is never safe to assume that because a particular effect is small in terms of physical measurements, it will not be significant to a skilled performing musician.[7]

Here we see the musical equivalent of sensitive dependence to small differences. The difference between a skilled performing musician and an amateur is small details, but these sometimes imperceptible details create a huge difference of opinion in a listener, and similarly, the small differences between recorded and reproduced sound and live acoustic sound can have a tremendous effect on a listener.

[7] Woodhouse, op. cit., p. 43.

Duration and frequency also play a role in human perception of hearing, in that a sustained tone is eventually ignored, and the acuteness of hearing is more or less sensitive in different ranges of frequencies. Also, the interplay between two different sounds can cause one or the other to be less audible, an effect called *masking*.[8] It's clear that our phenomenological measurements of the accuracy of human hearing are based on the interpretations of the subjects, and that our perception of hearing is not the same as hearing itself; thus, what is beyond the level of human hearing may not be completely beyond the level of human perception, conscious or unconscious.

4.2 Aperiodic Waveforms

"In all chaos there is a cosmos, in all disorder a secret order."

— Carl Gustav Jung

It's clear how periodic waveforms are interpreted by the ear. Now, I will consider aperiodic waveforms. An aperiodic waveform is perceived as "noise," because the Fourier analysis of an aperiodic waveform results in an equal amplitude measured at all (or most) frequencies. Thus, our ear also perceives this waveform in the same way; the white noise found between radio stations, for example, is simply an aperiodic waveform, interpreted by our auditory sense as "white noise." We can't hear non-linear oscillations as such. Our ears perform a Fourier transform, and the aperiodic components are transformed into an equal-amplitude white noise across all frequency spectra. Thus, this single complex waveform becomes an infinite conglomeration of sinusoidal parts.

As seen in Figure 4.3, the clarinet sound has strong harmonic components at the various resonant frequencies, but also has a certain amount of sound generated

[8] Gelfand, op. cit., p. 313.

at frequencies between these harmonics. These "noise" characteristics are nonlinear aspects to the clarinet sound. Lorenz theorized that instruments would have chaotic vibrations within their sounds:

> A string or a column of air, or to a lesser extent a membrane, usually vibrates with a strong periodic component, corresponding to a fundamental pitch. Typically there are overtones which contribute to the instrument's characteristic sound, but there is often an irregular component that further modifies the tone, and that in some instances seems to be chaotic rather than truly random. While recently visiting Douglas Keefe of the Department of Music at the University of Washington, I was rather surprised to learn that the normal tone of the saxophone is not chaotic. Chaos seems to be abundant, however, in a multiphonic tone, produced when the saxophone is played so that two distinct pitches are perceived simultaneously.[9]

Actually, studies have now found nonlinearity in the vibration of the column of air in wind instruments.[10] as well as in the vibration of a string.[11]

In clarinet playing, several studies have found nonlinearity in the flow of air over the reed and the reed's vibration. Wilson, in particular, comes to the conclusion that the nonlinearity present in the sound of the clarinet is a result not of the physical mechanisms involved but rather in the "inherent performance fluctuations," though for our purposes it doesn't matter where the chaos is found, but only that it is there.[12] Two articles by Rodet and Vergez attempt to recreate the nonlinearity in

[9]Lorenz, op. cit., pp. 149-150.

[10]Wilson and Keefe, op. cit.; C. Maganza, R. Caussé, and F. Lafoë. "Bifurcations, period doublings, and chaos in clarinetlike systems". In: *Europhysics Letters* 1 (1986), pp. 295–302; M. E. McIntyre, R. T. Schumacher, and James Woodhouse. "On the oscillations of musical instruments". In: *Journal of the Acoustical Society of America* 74 (1983), pp. 1325–1345

[11]M.E. McIntyre, R. T. Schumacher, and James Woodhouse. "Aperiodicity in Bowed-String Motion". In: *Acustica* 49 (1981), pp. 13–32.

[12]Wilson and Keefe, op. cit., p. 560.

clarinet sound for purposes of better synthesis as well as increased understanding of the nonlinearity of acoustical instruments.[13]

It is possible to approach nonlinearity in sound vibrations directly using analog synthesizers. Dan Slater has created a chaotic oscillator using a Moog analog synthesizer, coupling a pair of frequency modulated oscillators and stringing them into a feedback loop.[14] By varying the input, a wide variety of sound possibilities are produced, from single sine waves to white noise.

Slater used the following difference equations to control his oscillators:

$$\begin{aligned} L_{n+1} &= L_n e^{-i(k_1 Real(R_n)+2\pi(\frac{f_1}{s}))} \\ R_{n+1} &= R_n e^{-i(k_2 Real(L_n)+2\pi(\frac{f_2}{s}))} \end{aligned}$$

Here L_0 and R_0 are set to 1, $f_1 = -4050T$, $f_2 = 800T + 200$, $k_1 = 10,000T$, $k_2 = 20,000T$, $s = 44,100$ (sample rate) and T is an independent control variable. The real components of these calculations were used to produce the stereo output. Slater notes the similarity of his explorations to fractals:

> Chaotic FM methods, like other chaotic systems, are closely related to fractal structures, which can have exceedingly fine detail. This is rather like looking through a microscope at a large object with fine detail. there is quite a bit of acoustic detail in the chaotic FM algorithm, and one must be careful not to miss the subtleties as the coefficients are varied.

[13]Xavier Rodet and Christophe Vergez. "Dynamics in Physical Models: Simple Feedback-Loop Systems and Properties". In: *Computer Music Journal* 23.3 (1999), pp. 18–34; Xavier Rodet and Christophe Vergez. "Dynamics in Physical Models: From Basic Models to True Musical-Instrument Models". In: *Computer Music Journal* 23.3 (1999), pp. 35–49

[14]Slater, op. cit.

In a way, this is another approach to pointillism, the experience of the sound world through a combination of a multiplicity of different individual timbres, only in this case we are only able to discern the fine structure and will have trouble envisioning the resulting larger picture, much like trying to assemble *La Grand Jatte* by looking at single dot at a time.

4.3 Consonance and Dissonance

"Agreeable consonances are pairs of tones which strike the ear with a certain regularity; this regularity consists in the fact that the pulses delivered by the two tones, in the same interval of time, shall be commensurable in number, so as not to keep the ear drum in perpetual torment."

— Galileo Galilei

Consonance and dissonance serve as a vitally important aspect of music, playing a crucial role in the creation of tension and its resolution or release. Consonance and dissonance are generally understood to be a function of harmonic intervals, where generally speaking the tritone is considered the most dissonant and the octave the most consonant. While cultural conditioning and training can play a role in the interpretation of harmony as consonant or dissonant, these two intervals in particular often retain their dissonant and consonant status no matter what the training.

According to Marc Jude Tramo, interpretations of consonance and dissonance exist as a difference between rates of information transmission to the brain from the ear. When the rate of activity in the cochlear nerve is regular and rhythmic, it is heard as consonant or pleasing. When the rate is irregular, unpredictable, or aperiodic, the sound is heard as dissonant. "The input from a minor second is very chaotic."[15] Thus, since $\sqrt{2}$ represents the proportion of frequencies which characterize

[15] Jad Abumrad and Robert Krolwich. *"Music and Language." Radiolab. WYNC.* Apr. 2006; also see: Mark Jude Tramo et al. "Neurobiological Foundations for the Theory of Harmony in Western

a tritone, this is yet another way to see this quantity as a representation of chaos. This, incidentally, is not unlike what Galileo was alluding to in the above quote, nor is it unlike what Hermann von Helmholtz discovered: "These are rough and annoying to the auditory nerve, since any intermittent excitation affects our nervous system more heavily than a steady one... Consonance is a continuous, dissonance an intermittent tone sensation."[16] The difference, here, is that Tramo has actually measured it.

Thus, consonance might be thought of as "order" in the brain, whereas dissonance is chaos. In a way, then, we might discuss all tonal function as existing on a continuum of order and chaos, rather than in terms of intervals and harmony—a redefinition which allows for other, possibly nonharmonic sounds to function in a piece of music. John Cage predicted this very idea in the *Credo* of his *Future of Music*, first delivered as a speech in 1937:

> ...Whereas, in the past, the point of disagreement has been between dissonance and consonance, it will be, in the immediate future, between noise and so-called musical sounds.[17]

Incidentally, while the preference for consonant intervals has been demonstrated with humans even as infants[18], primates don't seem to exhibit such a preference in terms of consonance and dissonance according to experiments by McDermott and

Tonal Music". In: *Annals of the New York Academy of Sciences* 930 (June 2001), pp. 92–116, pp. 102-103

[16] H. v. Helmholtz. *On the Sensations of Tone as a Physiological Basis for the Theory of Music.* New York: Dover, 1954; cited in: E. Terhardt. "The Concept of Musical Consonance: A Link between Music and Psychoacoustics". In: *Music Perception* 1 (1984), pp. 276–295, p. 284

[17] Cage, op. cit., p. 4.

[18] Marcel R. Zentner and Jerome Kagan. "Perception of Music by Infants". In: *Nature* 383.6595 (Sept. 1996), p. 29; Sandra E. Trehub. "Musical Predispositions in Infancy". In: *Annals of the New York Academy of Sciences* 930 (June 2001), pp. 1–16

Hauser.[19] They posit that there might have been an evolutionary preference for music in our evolutionary heritage. It is also important to note that, though some preferences can be detected in human infants, a part of our perception of consonance and dissonance is tied to our cultural background and experience. Both biological and cultural influences shape our reaction to consonance and dissonance.

4.4 Fractal Dimension of Sound

"The opposite of a correct statement is a false statement. The opposite of a profound truth may well be another profound truth."

— Niels Bohr

Fractal Dimension is somewhat of a difficult concept to understand. Dimension is so integral to our conception of the world, that a fractional (or fractal) dimension doesn't make much sense. In essence, Fractal Dimension is a measure of how well a geometric object fills up space. To keep it relevant to our initial conception of dimension, we note that a line will be dimension 1, a plane dimension 2, and so on. The question becomes, what do we do with a line which fills more space than a simple straight line? What if the line wrapped around with such complexity that it filled the entire plane?

Fractal dimension is one way to measure the intermediate values of plane/space-filling for this line. Essentially, the way to measure this quantity is to divide the line

[19] Josh McDermott and Marc Hauser. "Are consonant intervals music to their ears? Spontaneous acoustic preferences in a nonhuman primate". In: *Cognition* 94.2 (2004), B11–B21.

into segments of length ϵ, creating N sections. The *correlation dimension* D is:[20]

$$N = \frac{1}{\epsilon^D}$$

Solving for D yields:

$$D = \frac{log(N)}{log(\frac{1}{\epsilon})}$$

Consider a straight line. If we halve the size of ϵ, then it should take twice as many segments of length ϵ to fill the line's length, thus we see a linear relationship between N and ϵ, and $D = 1$.

Now consider a plane. When we halve the size of ϵ, we need four boxes with sides of length ϵ to fill the plane. That is, we see a power-of-2 relationship between N and ϵ, so $D = 2$.

The fractional dimensions are less clear. In sound, the fractal dimension roughly correlates to the number of overtone structures in the sound, and is a measure of the sound's complexity. Modern approaches to measuring and utilizing this concept of fractal dimension have been fruitful, but often utilize different standards and units of measure, making comparisons and generalizations difficult.

4.5 Summary

"Real discoveries come from chaos."

— Chuck Palahniuk

We've seen in this chapter how acoustics are ultimately human-derived, and how chaos theory and related mathematics can help inform our understanding of acoustics

[20] P. Grasberger and I. Procaccia. "Measuring the Strangeness of Strange Attractors". In: *Physica* D9 (1983), pp. 189–208. Note that *fractal dimension* refers to correlation dimensions with fractional values.

and harmony. Nonlinear oscillations in audio waveforms form an instrumental part of all sound. Consonance and dissonance are also related to issues of periodicity and aperiodicity; thus chaos in a waveform (repeated in our brain's interpretation of that waveform) creates dissonance. A hierarchy of consonance and dissonance might be created using the concept of fractal dimension of sound, rather than the typical subjective (or experimental/phenomenological) definition, though the latter approach remains a useful abstraction for the purposes of understanding Western art music.

Chapter V
Chaotic Analysis

"In plain words, Chaos was the law of nature, Order was the dream of man."

— Henry Brooks Adams

Having seen how chaos plays a role in the mechanism of human hearing, I will now address the application of nonlinear dynamical systems in our understanding of 20th century musical compositions. I will begin with some general notes about chaos in music theory, and then analyze a variety of specific pieces and composers to help shed light on how this mathematical theory plays a role in music.

Of course, a comprehensive summary of all of the ways chaos plays a role in musical compositions would be impossible; it is not even possible to address all of the works composed during the 20th century, let alone the many ways chaos might play a role in each of these works. The intention here is to provide a few relevant examples of the many ways chaos plays a role in musical works. From these examples, it should be readily possible to find chaos in other ways; like a fractal, the closer you look, the more you will find.

One application of chaos theory to musical structure is through Schenkerian analysis and its analogy to the fractal structure of music. Essentially, Schenkerian Analysis involves a hierarchical view of a piece of music, decomposing its musical "foreground" (which involves all of the details of the piece) into its structural "background." The idea of motive pervades all levels of the work, and often the same motivic idea can function at many different musical levels, reminiscent of the Mandelbrot set of Figure 1.1 which included itself on many different levels of detail. In another perspective, the background becomes the "attractor" around which the foreground orbits in a phase-space view of music. However, this limited application of fractals to Schenker misses many of the main points of Schenker; that of the preferential hierarchy of harmonic function and the role of harmony to propel music forward. Thus Schenkerian theory encompasses far more than the application of fractals to music.[1]

For a more general view, Judy Lochhead divides applications of chaos to music into three modes: Ontological Chaos, Denotative Chaos, and Expressive Chaos.[2] The first of these, *Ontological Chaos*, deals with the distinction of order and chaos in music and the use of chaos to create unpredictability. Lochhead cites Integral Serialism of the Second Viennese School and Pierre Boulez, the process-oriented music of Steve Reich and the complex polyphony of Ligeti, who himself had studied chaos theory.[3] These works all employ highly rigorous order to create chaos, resulting in music which is deterministic but unpredictable.

Denotative Chaos uses sound to denote or imitate aurally the chaotic behaviors of the world, and is not necessarily procedural in its use of chaos. To Lochhead, Denotative Chaos is exemplified by Elliot Carter's Double Concerto for Harpsichord and Piano with Two Chamber Orchestras. For this piece, Carter cited two literary works which, for him, shared the same goals as his orchestral piece (but he does not go so far as to link the literary works as an influence, rather he "conceives them

[1] Two dissertations have been written on the application of fractals to music:Steven John Holochwost. "The Fractal Nature of Musical Structure". PhD thesis. Rutgers University, 2005, and David J. Weisberg. "Fractals and Music". PhD thesis. Rutgers University, 2000, as well as a number of books including Charles Madden. *Fractals in Music. Introductory Mathematics for Musical Analysis*. Salt Lake City: High Art Press, 1999, Robert Sherlaw Johnson. "Composing with Fractals". In: *Music and Mathematics: From Pythagoras to Fractals*. Ed. by John Fauvel, Raymond Flood, and Robin Wilson. New York: Oxford University Press, 2003.

[2] Lochhead, op. cit.

[3] Richard Steinitz. *Dynamics of Disorder*. Vol. 137. 1839. 1996, pp. 7–14, p. 7.

along with his music as creative works that reveal an equivalent conception of the world.").[4] Here, we see Carter's compositional intent to evoke, in his words, "the formation of the physical universe by the random swervings of atoms, its flourishing, and its destruction."[5] Thus, in this work Carter intends to evoke the chaos present in the natural world, even if the word "chaos" was not in his vocabulary in 1961. Iannis Xenakis is also cited as an example of the mode of Denotative Chaos, as addressed in Section 2.4.

The last mode is *Expressive Chaos*, which uses chaos to free itself from the "social and intellectual structures that are comprehended as constricting."[6] For this category, chaos is not used as a direct algorithmic procedure or an artistic goal but instead a method by which the limitations of composition are bypassed. Lochhead cited John Cage as an example of Expressive Chaos through his utilization of Asian philosophical and religious thought to shape his journey through the application of chaotic algorithms (chance procedures) in music.[7]

> While composing against a background of chaos as creative force, Cage also articulated an aesthetic of liberation that was resonant with not only other events in music but also the American culture of the 1960s in general. His proclamation of liberation—to "free" sound of "individual taste and memory (psychology) and also of the literature and 'traditions' of art music"—echoes in the rhetoric of Joseph Ford, one of the earliest scientists defining the new paradigm: "Dynamics freed at last from the shackles of order and predictability... Systems liberated to randomly explore their every dynamical possibility."[8]

[4]Lochhead, op. cit., p. 226.

[5]Elliott Carter. *The Writings of Elliott Carter: An American Composer Looks at Modern Music.* Bloomington: Indiana University Press, 1977, p. 329, as quoted in Lochhead, loc. cit. Carter is describing text which he says parallels his conception of his work, and thus his description of the text can also describe his piece.

[6]Ibid., p. 217.

[7]Ibid., p. 229.

[8]Ibid., p. 234.

Thus, through his employment of chance procedures, Cage was liberating his music from the structures he felt inherent in the processes of composition and the background of the composer.

In the following sections, I explore chaos operating on many different levels, from structure and form to rhythm and tonality, and how it pertains to specific works and/or specific composers.

5.1 Information Theory and the Mozart Concerto

"Neither a lofty degree of intelligence nor imagination nor both together go to the making of genius. Love, love, love, that is the soul of genius."

— Wolfgang Amadeus Mozart

Mozart's Clarinet Concerto predates chaos theory by almost two centuries, but we can still find elements of chaos present in this work. In the first movement, the form is clearly based on conventional Sonata form, but Mozart hides many surprises in the phrase structure of this movement.

In this chapter, I will provide a general background on information theory, and apply it to one aspect of the Mozart Concerto in in order to show how this process can yield valuable conclusions. Information theory can be applied to many different measurable quantities, and results in a measurement of the predictability or unpredictability of the data being measured. When data is deterministic, as in a fixed piece of music or a physical quantity in a dynamical system, and yet unpredictable, this indicates the presence of chaos. Note here that we are measuring predictability with reference to music as if one did not have the musical score; like physical quantities, if the underlying conditions of a system are known to infinite precision (as would be with a musical score), the system becomes completely predictable.

I have demarcated the phrase lengths from the first movement of the Mozart Concerto in Table 5.1, and the phrase lengths of the first movement of the Weber Concerto for comparison in Table 5.2. Note that the formal labels for the Mozart Concerto are from Colin Lawson's analysis[9], though the phrase lengths of the Mozart and the phrase lengths and formal analysis of the Weber are my own. It is apparent right away that the Mozart is less predictable in its structure than the Weber, with phrase lengths that change repeatedly. The Weber, on the other hand, uses a phrase length of 4 bars primarily throughout the movement.[10]

There is a way to quantify the phrase-length analysis for the purposes of comparison. We shall approach this from a general perspective, and then apply information theory for a more rigorous solution. First, suppose that, after hearing a phrase of length x, an audience expects to hear a corresponding phrase of the same length (x) follow it. Thus, we should divide the number of "surprises" by the number of phrases to get a measure of the "surprise factor" of a phrase. In this case, the Mozart has a surprise factor of 80% and the Weber has a surprise factor of 47%.

This measure is somewhat crude, however, since after hearing a chain of 4 measure phrases we would still expect to hear 4 bar phrases after the pattern is broken. In both the Weber and Mozart, 4 and 8 bar phrases might be considered "normal" or "typical" while phrases with odd lengths might be considered unexpected. Counting this way, the Mozart has a surprise factor of 53% and the Weber has a surprise factor of 24%. While both of these approximations are rough, it's clear from a surface level that the Mozart contains greater variety and unexpectedness than the Weber. This is certainly apparent when listening to this music.

[9] Colin Lawson. *Mozart: Clarinet Concerto*. Cambridge: Cambridge University Press, 1996.

[10] In fact, often where the phrase length is not 4 bars, it is done for a specific dramatic effect, i.e. a written-out ritardando. The same is occasionally true of the Mozart, but with far less frequency. This element has not been taken into account in my discussion of either work, as its effect in the results would be small.

Orchestra Ritornello		Solo Exposition		Ritornello		Development	
8	1–8	8	57–64	10	154–163	8	172–179
7	9–15	8	65–72	8	164–171	8	180–187
9	16–24	3	73–75			4	188–191
6	25–30	2	76–77			8	192–199
4	31–34	8	78–85			1	200–200
4	35–38	8	86–93			9	201–209
4	39–42	4	94–97			6	210–215
6	43–48	2	98–99			4	216–219
8	49–56	4	100–103			7	220–226
		8	104–111				
		3	112–114				
		1	115–115				
		8	116–123				
		4	124–127				
		6	128–133				
		7	134–140				
		4	141–144				
		3	145–147				
		6	148–153				

Ritornello		Recapitulation		Ritornello	
12	227–238	8	251–258	9	343–351
8	239–246	8	259–266	8	352–359
4	247–250	3	267–269		
		2	270–271		
		6	272–277		
		5	278–282		
		5	283–287		
		8	288–295		
		7	296–302		
		8	303–310		
		5	311–315		
		6	316–321		
		2	322–323		
		5	324–328		
		5	329–333		
		9	334–342		

Table 5.1: Mozart Clarinet Concerto, Allegro: Phrase Lengths. Formal diagrammatic labels from Lawson. Phrase lengths are my own, and are somewhat subjective.

First Part		Second Part		Third Part		Recap-ish		Coda	
4	1 - 4	4	84 - 87	4	145 - 148	4	223 - 226	4	258 - 261
7	5 - 11	4	88 - 91	4	149 - 152	4	227 - 230	4	262 - 265
4	12 - 15	4	92 - 95	4	153 - 156	4	231 - 234	7	266 - 272
4	16 - 19	4	96 - 99	4	157 - 160	2	235 - 236	5	273 - 277
4	20 - 23	4	100 - 103	5	161 - 165	4	237 - 240	4	278 - 281
4	24 - 27	2	104 - 105	4	166 - 169	8	241 - 248	6	282 - 287
4	28 - 31	4	106 - 109	4	170 - 173	4	249 - 252		
6	32 - 37	4	110 - 113	4	174 - 177	5	253 - 257		
4	38 - 41	4	114 - 117	4	178 - 181				
6	42 - 47	4	118 - 121	2	182 - 183				
4	48 - 51	4	122 - 125	4	184 - 187				
4	52 - 55	4	126 - 129	4	188 - 191				
4	56 - 59	4	130 - 133	4	192 - 195				
4	60 - 63	3	134 - 136	2	196 - 197				
4	64 - 67	4	137 - 140	4	198 - 201				
6	68 - 73	4	141 - 144	4	202 - 205				
4	74 - 77			4	206 - 209				
6	78 - 83			5	210 - 214				
				4	215 - 218				
				4	219 - 222				

Table 5.2: Weber, Clarinet Concerto No. 1, Allegro: Phrase Lengths. Formal analysis and phrase lengths are my own, and again, perception of phrase lengths is somewhat subjective.

To continue our analysis more formally, we require another mathematical diversion. information theory has become a crucial area of research since the dominance of computers in recent decades. Briefly, information theory attempts to quantify the amount of information contained in a stream of data. This is often considered in terms of *bits*, which are binary values (1 or 0), due to the application of information theory to computer messages. For example, if message "a" is encoded by 0, and message "b" is encoded by 1, then the following data stream has 6 bits of data:

010110

If there are more bits of data than information, then there is some redundancy in the data stream, and thus some predictability. For example, if "a" is coded by 0001, and "b" is coded by 1010, then the data stream

000110100001101010100001

only contains 6 bits of data, in spite of the 24 bits above. As soon as you receive a 0 or a 1, you can predict what the next three data bits would be. This redundancy is often useful, for example for error correction. All CDs use a code which has some redundancy, just in case there is a mistake or scratch in the CD. The redundancy can be used to correct the error, just as if the above stream of data was

000110100001101**1**10100001

we could conclude that the erroneous **1** in this case was the source of the error, since changing 1110 to 0001 involves assuming there were far more errors.

Entropy is a measure of the unpredictability of the data. If you received, for example:

1111111111111111111111111111111111111111111...

then the data is completely redundant and predictable, and entropy is 0. If you received, for example:

$$1011101000110101000010101011011011111010111\ldots$$

then the data is completely unpredictable, and entropy is set equal to 1. One way to interpret this is, when you receive a bit of data, in this case you have received a full bit's worth of information (the whole stream is worth a full 43 bits of data), whereas in the former case you didn't receive any information at all. Formally, entropy (H) is computed as follows:

$$H(P) = \sum_{n=1}^{k} p_n * \log_2\left(\frac{1}{p_n}\right),$$

where H represents entropy, P the probability distribution of the data, k the number of options, and p_n the probability for a particular data possibility. I use $\log_2$ to measure the information in "bits," though other units are possible.

In our first case, the probability of receiving a 1 was 100%, and the probability of receiving a 0 was 0%, so the equation would be:

$$\begin{aligned} H(P) &= \sum_{n=1}^{k} p_n * \log_2\left(\frac{1}{p_n}\right) \\ &= 1 * \log_2\left(\frac{1}{1}\right) + 0 * \log_2\left(\frac{1}{0}\right) \\ &= \log_2(1) \\ &= 0. \end{aligned}$$

Phrase Length	Percentage
1	0.00%
2	5.88%
3	1.47%
4	75.00%
5	5.88%
6	7.35%
7	2.94%
8	1.47%

Table 5.3: Weber Probability Distribution. This is simply a count of the number of phrases of each length, compared with the total number of phrases.

For the second example, the probability seemed approximately equal: 50% for a 0, 50% for a 1. The equation works out as follows:

$$\begin{aligned} H(P) &= \sum_{n=1}^{k} p_n * \log_2\left(\frac{1}{p_n}\right) \\ &= .5 * \log_2\left(\frac{1}{.5}\right) + .5 * \log_2\left(\frac{1}{.5}\right) \\ &= .5 * \log_2(2) + .5 * \log_2(2) \\ &= .5 * 1 + .5 * 1 \\ &= 1. \end{aligned}$$

Returning to the Mozart Concerto, Information theory can help us to measure the entropy of the data, which can be thought of as a measurement of the predictability. This will eliminate the rough approximations and try to quantify directly the amount of variety in the data. First, I will use the Weber as a baseline for the probability of different phrase lengths. In the Weber, there are 68 phrases, and the probability distribution is found in Table 5.3.

Phrase Length	Percentage
1	3.33%
2	6.67%
3	6.67%
4	16.67%
5	8.33%
6	11.67%
7	6.67%
8	30.00%
9	6.67%
10	1.67%
11	0.00%
12	1.67%

Table 5.4: Mozart Probability Distribution. As in Figure 5.3, this is a comparison of the number of each phrase length with the total number of phrases.

Phrases of length 4 or 8 are found 76% of the time, so I will approximate that 76% of the time, the phrase length will correspond to the typical phrase length of a piece of music from this time period, and 24% of the time it will not. So we should expect to see 76% of the Mozart's phrases of length 8 or 4. The actual distribution of the Mozart is in Table 5.4.

If we combine the two most frequent phrase lengths (8 and 4), we get a total of 46.67% phrases of typical length, and 53.33% phrases of unexpected length.

For our first approximation towards an information theory analysis, we'll consider the case where we do not care about the specific length of the phrase, and only whether it satisfies our idea of "typical" phrase length or "unusual." With the phrase distribution of 76% normal and 24% unusual, the entropy of the Weber Concerto's first movement is

$$H(P_W) \quad = \quad p_t * \log_2\left(\frac{1}{p_t}\right) + p_u * \log_2\left(\frac{1}{p_u}\right),$$

where H represents entropy, P_W the probability distribution of the Weber Concerto's first movement, and p_t and p_u represent the probability of typical phrase length and unusual phrase length respectively. Again I use $\log_2$ to measure the information in "bits." Filling in these values gives us

$$\begin{aligned} H(P_W) &= p_t * \log_2\left(\frac{1}{p_t}\right) + p_u * \log_2\left(\frac{1}{p_u}\right) \\ &= 0.7647 * \log_2\left(\frac{1}{0.7647}\right) + 0.2353 * \log_2\left(\frac{1}{0.2353}\right) \\ &\approx 0.296 + 0.491 \\ &= 0.787. \end{aligned}$$

So the entropy of the first movement of the Weber Concerto is 0.787. Note that the entropy of a system where the probabilities are equal for two events (such as a fair coin flip) is 1, and that the entropy of a system where only one possible outcome is possible (double-headed coin or blank die) is 0.[11]

The entropy measured for Weber's concerto is hard to interpret on its own, so let's compare it to the Mozart:

$$\begin{aligned} H(P_M) &= p_t * \log_2\left(\frac{1}{p_t}\right) + p_u * \log_2\left(\frac{1}{p_u}\right) \\ &= 0.4667 * \log_2\left(\frac{1}{0.4667}\right) + 0.5333 * \log_2\left(\frac{1}{0.5333}\right) \\ &\approx 0.513 + .484 \\ &= 0.997. \end{aligned}$$

In the end, this measurement is very similar to our coarse approximation. What these calculations are telling us is that the amount of information (about phrase lengths)

[11] 1 and 0, here, measured in bits. Thus, the result of a coin flip can be represented by a single bit, whereas the result of a double-headed coin flip doesn't need representation at all

conveyed by each phrase in the Mozart is greater than that of the Weber; i.e., that the Weber is more predictable. In terms of phrase lengths, the Mozart objectively exhibits unpredictability, nearly as much as a coin toss.

What happens when we eliminate the approximation and compute the entropy of the entire system? The entropy of the entire system is formulated:

$$H(P) \;=\; \sum_{i=0}^{n} p_i * \log_2\left(\frac{1}{p_i}\right).$$

This represents the sum of all of the measured probabilities, rather than just the "typical" versus "unusual," and thus gives no preference to any particular phrase length.

In the case of the Weber and Mozart, there are 7 and 11 possible phrase lengths represented in the first movements of each concerto (respectively). Thus, the entropy of the Weber and Mozart, measured this way, will have a different base of comparison. In terms of bits, the Weber is measured against $\log_2(7)$ bits whereas the Mozart will be measured against $\log_2(11)$ bits. These measurements represent the information found in, for example, the rolling of a 7- or 11-sided die.

The entropy of the Weber system is found to be approximately 1.39, where $\log_2(7) \approx 2.81$. The entropy of the Mozart system is found to be approximately 3.02, where $\log_2(11) \approx 3.46$. Again, we see that the Mozart system contains nearly as much information (and so is nearly as unpredictable) as a fair die roll, where the Weber system is far more predictable. To be very specific, in terms of "typical" phrase lengths versus "unusual ones," Weber tends to use typical phrase lengths whereas Mozart uses almost an equal amount of typical and unusual; what is unpredictable in Mozart is whether the phrase length will be typical, not the length itself. Again, this is something we intuitively grasp from our first approximation, but information theory allows for an objective analysis of the information conveyed by each phrase length, rather than relying on intuition.

Again, the unpredictability we are talking about here is on the level of whether the next phrase will be of typical length, not unpredictability of the phrase lengths themselves. Mozart most often uses a typical phrase length; it is his distribution of unusual phrases which is of interest.

Obviously, while phrase lengths are an appreciable and important aspect of music composition, certainly there is far more to the story than the durations of phrases. The information theory approach is generalizable to quantifiable musical information of any sort, whether pitch, rhythm, timbre, or other formal designs. The application of information theory here allows a measurable way to analyze the expectations of a listener.[12]

Could our experience of unpredictability in Mozart be tied to our admiration of his music? Certainly Mozart is hailed far above Weber in the annals of music history, though this valuation is entirely subjective. If inventiveness and the interplay of expectation and surprise in music contribute to our appreciation of the composer, certainly an information theory approach to music analysis would be a useful model for understanding this music.

5.2 Serialism

"Great art presupposes the alert mind of the educated listener."

— Arnold Schoenberg

[12]Of course we are considering a generalized listener and drawing assumptions at how this listener would listen; particularly that what the listener hears would lead him or her to expect more of the same. This is practical for our purposes, and suitable model, but not applicable for predicting the reaction of any specific listener.

As we addressed in Section 2.1, Serialist works could be considered some of the most ordered and controlled pieces composed, and yet when one is listening to Serialism it is difficult to hear this control. It would take an intensive training course to be able to recognize the algorithms involved in the composition of these works at first hearing, and as such Serialism represents the maximum of unpredictability and disorder.

Two points of view shape an information theory approach to Serialism. In essence, after the first 12 pitches of the work are played, we can predict, within a few pitches of each statement of the row, what the rest of the pitches will be (though not in practice as a listener). Thus the information content of the pitch material declines after the row is initially stated. From another point of view, the probability of any particular pitch is approximately equal at the start of the row, and then each remaining pitch is again approximately equally likely until all pitches have been used; thus, the entropy for each statement of the row is approximately 0.91 (the last pitch is completely predictable though the others might be more predictable depending on the symmetry of the row).

But information theory can only tell us so much; for example, the above approach ignored the issue of dynamic, register, articulation, tone color, or other essential features of Serialism. These ideas could be coded and quantified, but again here we miss the point; Serialism is best understood in its gestalt, as a sonic effect, not a series of data.

5.3 Olivier Messiaen

"[Listeners] will be responsive to [complexity] the day their ears are accustomed to it. It's not essential for listeners to be able to detect precisely all the rhythmic procedures of the music they hear, just as they don't need to figure out all the chords of classical music. That's reserved for harmony professors and professional composers. The moment [listeners] receive a shock, realize that it's beautiful, that the music touches them, the goal is achieved."

— Olivier Messiaen

One of the ways Messiaen differentiated himself from other musicians in his time was through his study of numerology, particularly his obsession with prime numbers, palindromes, and symmetry. He explored an additive process; instead of the usual Western approach of dividing measures into beats and subdividing beats into parts of beats, he accumulated smaller rhythmic values into larger ones, creating rhythms which didn't necessarily coincide with any particular beat structure or meter.

> Rhythm is the primordial and perhaps essential part of music; I think it most likely existed before melody and harmony, and in fact I have a secret preference for this element.[13]

In the *Quartet for the End of Time*, Messiaen employs many algorithmic approaches to composition. The first movement, written last of the 8 movements, employs a technique first used in the 14th century in isorhythmic motets and masses, though apparently Messiaen was unaware of this when he redeveloped his motivic

[13]Olivier Messiaen. *Music and Color: Conversations with Claude Samuel*. Trans. by E. Thomas Glasow. Portland, Oregon: Amadeus Press, 1994, p. 67.

ideas. Instead, it is likely that Messiaen discovered this approach to rhythm from Indian sources.[14]

The cello and piano each have a separate pattern in pitch and rhythm. The movement is then constructed out of a repetition of these patterns. The rhythmic pattern of the cello line is constructed out of a 5-note pitch pattern which is repeated over a 15-duration rhythm pattern.[15] Simultaneously, the piano part plays a cycle of 29 pitches over 17 durations, a pattern which uses two prime numbers and thus would only coincide after 493 durations or chords had gone by (29 repetitions of the duration pattern, 17 repetitions of the pitch pattern).

In a way, the difference between the cello line and the piano line is one of the degree of complexity. The pattern in the cello line is relatively simple, and the short pitch pattern aids the listener in recognition. The piano line, however, will never repeat during the movement, and the patterns are of sufficient duration to avoid deconstruction by listeners. In this way, the piano line remains unpredictable even while it is deterministic, though the measure of this judgment remains firmly relative to the abilities of the listener.

Julian Hook applies an algebraic approach to Messiaen's rhythm, and of particular interest is his discussion of *generative* rhythm.[16] Hook identifies a certain rhythmic *seed* which is then transformed through the application of *rules of generation* to produce an output. Of particular interest to our purposes is the recursive generation of rhythm, which Messiaen employed loosely but which could in theory be expanded to produce a "fractal" approach to rhythm.

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[14]Julian L. Hook. "Rhythm in the Music of Messiaen: an Algebraic Study and an Application in the Turangalîla Symphony". In: *Music Theory Spectrum* 20.1 (1998), pp. 97–120, p. 98; Robert Sherlaw-Johnson. *Messiaen.* University of California Press, 1989, p. 10; Messiaen, op. cit., pp. 75–79

[15]Robert Sherlaw-Johnson. "Rhythmic Technique and Symbolism in the Music of Olivier Messiaen". In: *Messiaen's Language of Musical Love.* Ed. by Siglind Bruhn. New York: Garland Publishing, Inc., 1998, p. 125.

[16]Hook, op. cit., p. 105.

Hook cites a particular rhythm from the *Turangalîla Symphony*:

2 1 1 1 2 3 2 1 1 1 2 4 2 1 1 1 2 3

2 1 1 1 2 4 2 1 2 4 2 1 1 1 2 3 2 1

1 1 2 4 2 1 2 1 1 1 1 2 1 2 4 2 1 1

1 2 3 9 3 2 1 1 1 2 4 2 1 2 1 1 1 1

At first, this sequence seems to lack any clear patterns. It is certainly aperiodic, seemingly unpredictable, and doesn't conform to any particular ordering. It is in fact a sequential statement of a hierarchical structure, which Hook writes as follows:[17]

								x		3		x								
							4	x		3		x	4							
				2	1	2	4	x		3		x	4	2	1	2				
1	1	1	1	2	1	2	4	x	3	9	3	x	4	2	1	2	1	1	1	1

Here, x represents the sequence 2,1,1,1,2; Hook makes this substitution because this particular sequence plays a larger role in the piece as a whole. Note that the additive process of the algorithm is closely related to x, that the two added segments: 2,1,2 and 1,1,1,1 might be thought of as a palindromic partition of x into segments 2,1 and 1,1.

Roig-Francolí calls this a "recursive progression."[18] It is the recursive nature of this process that is so essential to the idea of fractals and chaos. Each rhythm is similar to (and generated from) the one above, but with added detail and expanded length.

It is possible then that the rhythmic pattern of the cello in the 'Liturgie de cristal' may be the final result of the same sort of recursive generation. The pattern is presented as:

[17] Ibid., p. 109.

[18] Miguel A. Roig-Francolí. *Understanding Post-Tonal Music*. New York: McGraw-Hill, 2007, p. 262.

4 3 4 | 4 1 1 3 1 1 1 1 3 1 1 4

Both parts of the melody show *non-retrogradability*, that is, they are the same when reversed.[19] We can assume Messiaen started in the middle of his pattern and rewrite the rhythm as a single non-retrogradable rhythm:[20]

1 1 3 1 1 4 4 3 4 4 1 1 3 1 1

A similar construction to the pattern explored by Hook could have resulted in such a sequence. For example,

						4	3	4						
		3			4	4	3	4	4			3		
1	1	3	1	1	4	4	3	4	4	1	1	3	1	1

Certainly a flavor of the additive process is in both rhythmic patterns, and the procedure could be generalized for a fractal approach to rhythm.

The chaos here lies in the combination of an algorithmic approach to rhythm coupled with an aperiodic and unpredictable result, compounded by the fact that the rhythm is presented sequentially in time, where a fractal is presented as a graphic on a plane. If organized as above, the rhythm becomes easily understood, where if viewed from within, a fractal appears chaotic.

[19] Messiaen often expressed his terminology by impossibility, that a non-retrogradable rhythm is one which does not change when retrograded.

[20] Anthony Pople. *Messiaen:* Quatuor *pour la fin du Temps*. Cambridge: Cambridge University Press, 1998, p. 21.

5.4 John Cage

"I gave up making choices. In their place I put the asking of questions. The answers come from the mechanism, not the wisdom of the I Ching, the most ancient of all books: tossing three coins six times yielding numbers between 1 and 64."

— John Cage

As discussed in Section 2.2, chance procedures can be considered a form of chaos. In a way, this renders almost the entirety of John Cage's output as incorporating chaos in one form or another. Certainly when *Imaginary Landscape No. 4* is programmed, no one can predict how the piece will actually sound, though there is a certain identity retained by the piece; it will fall within a certain bound of sound possibilities, depending on the radio stations within range of the concert venue and the time of the concert.

Cage's output varies quite a bit on the issue of control and intent. For example, works such as *4'33"* or *0'0"* leave almost the entire work to chance, providing only a structural framework which identifies the piece as belonging to Cage. Other works, such as *34'46.776"*, were composed using chance operations but are explicit and detailed as to rhythmic relationships and methods of performance.

Imaginary Landscape No. 4 and *Music of Changes* were written using the *I Ching*, a chart of 64 hexagrams which carry their own interpretation and meanings, but the primary purpose of the incorporation of chance was to distance the composition from Cage's own perspective, to use an algorithmic approach rather than one based on intuition or compositional taste.[21] *Imaginary Landscape No. 4*, in particular, was written because, after *Music of Changes*, Henry Cowell told Cage he had not yet freed himself from his own sense of taste.[22] This piece involves 12 radios and 24

[21] Details of Cage's approach are outlined in Cage, op. cit., pp. 57-59.

[22] John Cage and Richard Kostelanetz. "His Own Music". In: *Perspectives on New Music* 25 (1987), pp. 88–106, p. 94.

performers, with each performer controlling either the frequency of the tuning of a radio, or its volume or tone. “It is thus possible to make a musical composition the continuity of which is free of individual taste and memory (psychology) and also of the literature and “traditions” of the art. The sounds enter the time-space centered within themselves, unimpeded by service to any abstraction, their 360 degrees of circumference free for an infinite play of interpenetration.”[23]

This piece incorporates chaos on a variety of levels. First, as in most aleatoric pieces, the resulting sound of the composition is impossible to predict. Second, the piece is sensitively dependent on its initial conditions, particularly its starting time, which determines from that point onwards what the piece will sound like. Last, the piece itself is completely controlled and planned; each performance should include identical actions by performers which will have remarkably different results from performance to performance. In a way, each of these performances become iterations of the original piece, and though these iterations are identical the results of the iterations are unique. The piece was also composed using the tossing of coins to produce an ordered series of tempi, durations, sounds, and dynamics, and, as we have seen in Section 2.2, the process of flipping coins might be thought of as a chaotic process rather than a random one. Then, of course, on a literal level, the radios themselves will produce “noise” when tuned in-between stations. It’s possible, were this piece to be performed at a far enough location from any radio antennas, that the sonic materials of the piece would consist entirely of white noise, the result of nonlinear electromagnetic waves in the cosmos—the radio, too, performs a sort of Fourier analysis on radio waves, and cannot produce nonlinear radio waves as anything other than white noise.

[23]Cage, op. cit., p. 59.

5.5 Iannis Xenakis

"Mathematics gives structures that are too regular and that are inferior to the demands of the ear and the intelligence. The great idea is to be able to introduce randomness in order to break up the periodicity of mathematical functions, but we're only at the beginning."

— Iannis Xenakis

Xenakis encapsulates chaos on many different levels in his compositions. He was very inventive, and his different approaches to the incorporation of chaos reflect a mind keen on innovation.

Xenakis wrote *La Légende d'Er* in 1977–78, and in the audacious tradition of Wagner also designed the space in which this piece would be performed, *Le Diatope*, a combination of hyperbolic paraboloids which was "a kind of enveloping form, closed and opened to the world at the same time by the convergence of its geometrical construction"[24], and also designed a system of lights and lasers which would illuminate the space in synchrony with the music. Xenakis used a combination of acoustic and electronic sounds on a seven-track tape which was distributed to 11 speakers. The electronic sounds were generated by the combination of sine waves in additive synthesis:

> In short, just as our universe is formed from grains (of matter) and straight lines (photon radiation) ruled by stochastic laws (probability), this spectacle offers a reflection of it which is miniature but symbolic and abstract. So music and light unite together. In some sense, this is a kind of cosmic "harmony of the spheres" which, by means of art, becomes one with that of thought.[25]

[24]Xenakis, Brown, and Rahn, op. cit., p. 35.
[25]Ibid., p. 36.

Xenakis also wrote electronic compositions, and *Gendy3* was produced using the GENDYN program. This program operates on the waveforms of sound directly, using stochastic mathematical formulas.[26] Even the composer has no idea what the work will sound like when completed, and the initial conditions of the stochastic functions determine the entirety of the piece. Xenakis, of course, set the initial parameters and chose which result was the best, and as such his compositional perspective remains important to the work. There is no live performance variable to the work however; all of the computation is done prior to the performance. This means, perhaps, that the perception of chaos might diminish with each subsequent hearing.

Le Fleuve de Désir VI involves another approach to chaos, the use of chaos on a formal level. Pape writes: "On a formal level, the piece moves from order to disorder and back to order."

> Between the first two sections of the piece, transitions are smooth and gradual, but between sections three and four, there is an abrupt transition to chaos. As turbulence sets in, changes lose their smooth, continuous quality and become perceptually unexpected or jarring.
>
> In sections five and six, as the sound becomes ever more disorderly and unpredictable, gradually disintegrating into unstable noise, one has perceptual chaos proper—that is, sound that is simultaneously continuous and discontinuous. . . . The transition back to order, between sections six and seven, is again chaotic, because there can be no smooth transition between unstable noise and an orderly sound structure such as the harmonic series.

Actually, there's no reason to think that chaos cannot support a transition; one of the most common routes to chaos involves sequential bifurcations of periodic orbits (i.e.

[26]Pape, op. cit., p. 17.

points of equilibrium) until eventually the entire range of values becomes enmeshed in chaos.[27]

These three pieces encompass three different approaches to the incorporation of chaos in music, all from a single composer. It is no coincidence that Xenakis's music covers such a wide range of styles and genres. Carrying in the tradition of Stravinsky, Xenakis experimented and innovated throughout his life in search of the ultimate freedom of expression.

Many of Xenakis's students have continued his work, especially with regard to using chaotic algorithms with electronic music.

5.6 György Ligeti

"Yes, fractals are what I want to find in my music. They are the most complex of ornaments in the arts, like small sea horses, like the Alhambra where the walls are decorated with geometric ornaments of great minuteness and intricacy, or like the Irish Book of Kells, those marvelously decorated borders and capitals. The most complicated ornaments—perhaps not art, perhaps geometry. It is a very complex music, difficult to describe. I only want to give a metaphysic for my music. After all, music is not a science."

— György Ligeti

I will analyze one of his short piano works from Études, Book 1: No. 1, *Désordre*, written in 1985.

The first Étude is titled *Désordre*, French for "disorder."[28] That the piece *sounds* chaotic is certainly obvious, though the sense of chaotic here may not be the mathematical one. The two hands seem to be in completely different places, and the

[27] See Section 6.2 for an example of this kind of bifurcation structure in the logistic map.

[28] My analysis draws from the algorithmic approach found in: Tobias Kunze. *An Algorithmic Model of György Ligeti's Étude No. 1, Désordre (1985).* Accessed 1/14/2009. 1999. URL: `http://ccrma.stanford.edu/~tkunze/pbl/1999_desordre/ligeti.html`.

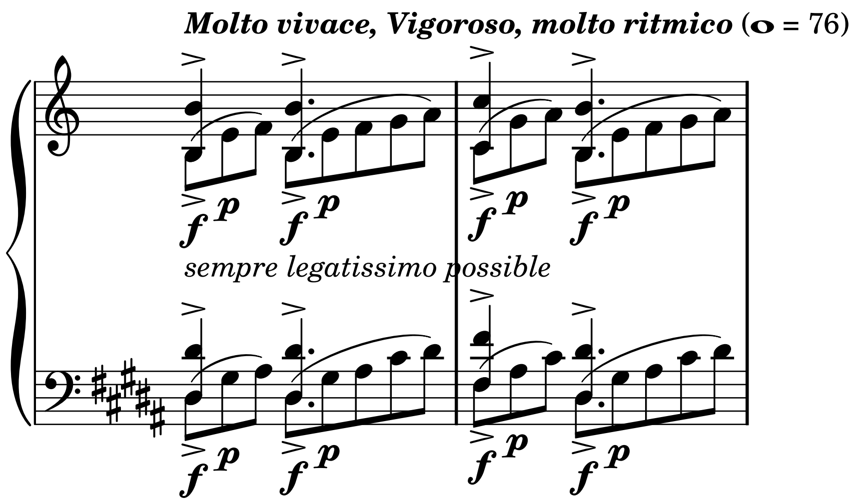


Figure 5.1: Ligeti, Études, Book 1, No. 1 *Désordre*, measures 1–2. This excerpt provides an example of the texture of the movement; from here, the pattern of accents in the right hand and left hand separate.

pattern of accents defies metric analysis. Underlying this seeming "*Désordre*" is a rigorous structure. The disorder is created through juxtaposition of the piece's highly ordered components. It could be argued that the algorithms involved "determine" the piece, making the work both deterministic and yet unpredictable; it actually is what it sounds like: *chaotic*.

In order to show chaos, Ligeti must first show order, so he starts the piece at a period of rhythmical stasis: both piano hands are playing identical rhythms in parallel contours (see Figure 5.1). Pitchwise, Ligeti is contrasting the diatonic and pentatonic scales by utilizing only the white keys in the right hand and the black keys in the left hand. Thus, while the intervals change between the hands, the contour remains similar.

There are two components to the cyclical pattern: rhythm, and pitch. The pitch patterns are held constant throughout the piece, and repeats 14 times in the upper voice and 11 times in the lower voice. The rhythmic cycles coincide with the pitch cycles, but are not periodic; Ligeti varies the rhythmic cycle, causing a general accelerando throughout the first section, a fast rhythm through the second section, and a return to the slower rhythm of the opening in the third section.

The rhythmic pattern is shown in Table 5.5 for the first six cycles. The first section in the right hand Treble line shows a gradual accelerando. The second section continues in the shorter rhythm, and the third section returns to the rhythmic cycles of the first section. The same general pattern applies to the left hand Bass part. In Table 5.5, note that the rows across from Treble to Bass generally coincide, but do not match up exactly. The only cycles shown in the table which begin exactly together are the first and the 5th (4th in Bass).

The displacement between the parts is caused by the variance in rhythmic values. In the beginning, both parts play a similar rhythmic pattern. In eight notes, this pattern is $\{3, 5, 3, 5, 5, 3, 7\}$ in the right hand. The 7 is replaced with 8 in the left hand, causing a displacement between the two hands, a phasing effect which is reminiscent of Steve Reich, though the piece is on a much smaller scale than his phase pieces. A figurative analogy to sensitive dependence on initial conditions is relevant here, as this small initial difference between the parts could be looked at as having very large results.

The lengths of the upper voice cycles in eighth notes are $\{109, 108, 109, 78, 42, 42\}$ for the first six cycles, and the lower voice cycles are $\{144, 144, 116, 55, 54\}$. Note that 109 is a prime number, and so 144 and 109 are relatively prime, meaning that, were they to continue unaltered, the initial cycles would not coincide until after 144 cycles of the top voice (109 cycles of the bottom). That's roughly 25 minutes of music before the cycles would coincide again.

Treble							
3	5	3	5	5	3	7	
3	5	3	5	5	3	7	
3	5	3	5	5	3	3	4
				5	3	3	5
3	5	3	4	5	3	8	
3	5	3	4	5	3	8	
3	5	3	4	5	3	3	5
				5	3	3	4
3	5	3	5	5	3	7	
3	5	3	5	5	3	7	
3	5	3	5	5	3	3	4
				5	3	3	5
3	5	3	4	5	2	7	
2	4	2	4	4	2	5	
2	3	2	3	3	1	1	3
				3	1	1	3
1	2	1	2	2	1	3	
1	2	1	2	2	1	3	
1	2	1	2	2	1	1	2
				2	1	1	2
1	2	1	2	2	1	3	
1	2	1	2	2	1	3	
1	2	1	2	2	1	1	2
				2	1	1	2

Bass							
3	5	3	5	5	3	8	
3	5	3	5	5	3	8	
3	5	3	5	5	3	3	5
				5	3	3	5
3	5	3	5	5	3	8	
3	5	3	5	5	3	8	
3	5	3	5	5	3	8	
3	5	3	5	5	3	3	5
				5	3	3	5
3	5	3	5	5	3	8	
3	5	3	5	5	3	8	
3	5	3	5	5	2	7	
3	4	3	4	4	2	2	4
				4	2	2	3
2	3	1	3	3	1	4	
1	3	1	2	2	1	3	
1	2	1	2	2	1	3	
1	2	1	2	2	1	1	2
				2	1	1	2
1	2	1	2	2	1	3	
1	3	1	2	2	1	3	
1	2	1	2	2	1	3	
1	2	1	2	2	1	1	2
				2	1	1	2
1	2	1	2	2	1	2	

Table 5.5: Pattern of Accents in Ligeti, *Désordre*, first 6 cycles. These numbers represent eighth-note durations of the accented line. The cycles in each hand roughly correspond as horizontally displayed in the table.

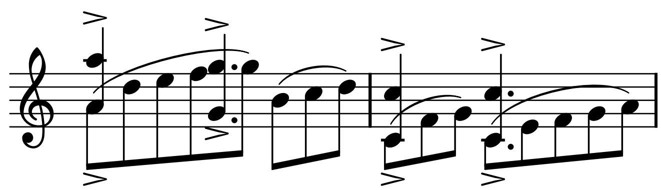


Figure 5.2: Ligeti, Études, Book 1, No. 1 *Désordre*, measures 14–15. In measure 14, the accent pattern does not line up with the background subdivision. This occurs irregularly in the treble part, though it does not occur in the bass part. Measure numbers are listed as counted in the treble

Note that the rhythm of accents in the right hand does not always correspond with the underlying eighth note rhythm. For example, in measure 14 in the treble line (see Figure 5.2, the underlying eighth note rhythm is 5+3, where the accent rhythm is 3+5.

These examples demonstrate how the rhythmic pulse is quasi-periodic. The general shape of the rhythmic pattern is present, but with minor variations. Figure 5.3 shows the first six cycles of the right hand Treble part in graph form, where the y axis represents pitch along the diatonic scale used in the right hand, and the x axis represents time, quantized to eighth notes. Figure 5.4 makes all of the parts proportional to emphasize the slight deviations between the repetitions of the rhythmic pattern, and Figure 5.5 shows the parts normalized at the beginning of the cycle to show how some of the rhythmic cycles accelerate. This quasi-periodicity makes the algorithmic composition seem less calculated, and more unpredictable.

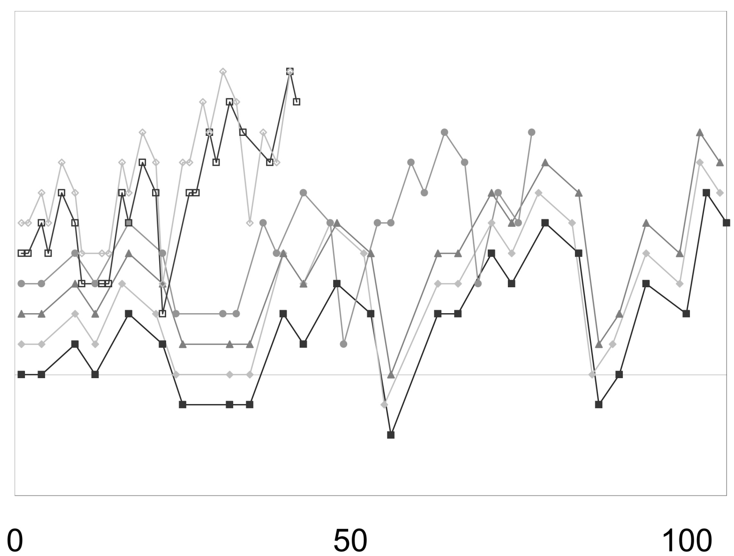


Figure 5.3: Cycles in Ligeti, *Désordre*, first 6 cycles. The horizontal axis represents time, and the vertical axis represents pitch. Note how the later (higher) cycles also get faster.

5.7 Steve Reich

"In the process of trying to line up two identical tape loops in unison in some particular relationship, I discovered that the most interesting music of all was made by simply lining the loops up in unison, and letting them slowly shift out of phase with each other. As I listened to this gradual phase shifting process, I began to realize that it was an extraordinary form of musical structure. This process struck me as a way of going through a number of relationships between two identities without ever having any transitions. It was a seamless, uninterrupted musical process."

— Steve Reich

It can be difficult to look at such determined music as unpredictable, but what is unique about phase music is its constant variation and the irrational juxtaposition of its parts. In *Come Out*, for example, rhythm is not present as a set of ordered beats. Instead, the slow phase breaks apart the rhythm in a large-scale version of the "beat"-tuning phenomena. Our experience of this piece is to understand its process

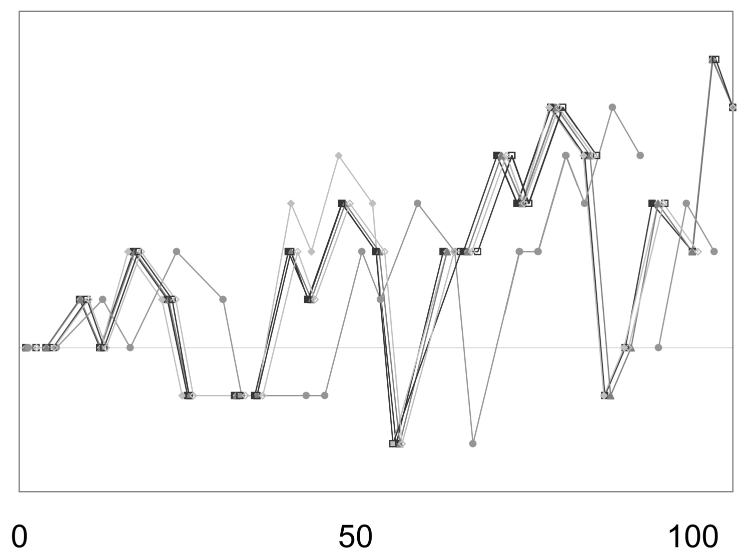


Figure 5.4: Cycles in Ligeti, *Désordre*, first 6 cycles: Proportional. Pitch is now graphed relative to each cycle's starting note. This graph shows how there are slight alterations to the proportions of the rhythms in each cycle as the rhythms accelerate.

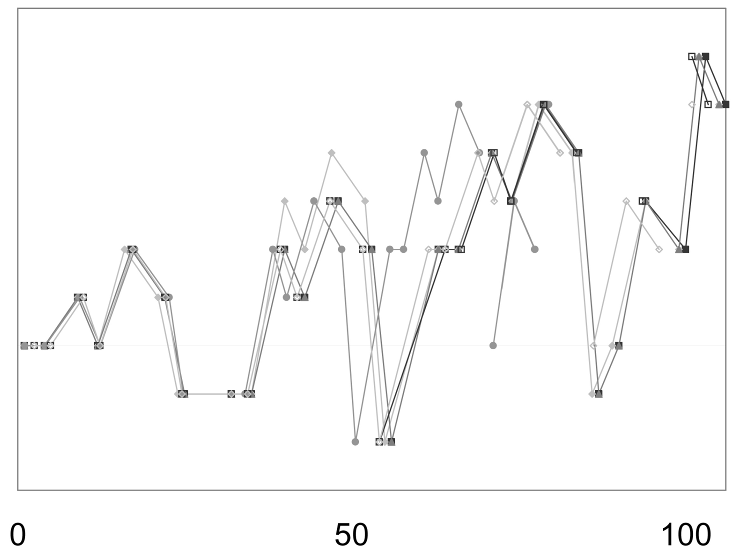


Figure 5.5: Cycles in Ligeti, *Désordre*, first 6 cycles: Normalized to Beginning. In this graph, it is easy to see the acceleration that occurs during the later cycles.

directly, but the result is completely unexpected. Reich described this process as "a kind of controlled chaos."

Reich's *New York Counterpoint* involves a single instrumentalist playing in synchronization with a set of pre-recorded excerpts from the piece; in other words, a player who plays in phase with him- or herself. In the first movement, the phasing is happening on a discrete scale; each voice in the phase is added gradually, building up from single notes as if slowly emerging. The end result is a multiphonic texture of complex rhythmic texture, where every eighth-note pulse has an attack, but the overall pitch material stays static. The effect of this gradual process is similar to a bifurcation-type relationship, where the entry of each line causes a doubling in the complexity of the resultant texture.

5.8 Summary

"Chaos is inherent in all compounded things."

— Buddha

I have outlined many applications of chaos theory toward furthering our understanding of 20th century musical works. These works invoke chaos theory on many different levels, sometimes even simultaneously within the same work. Chaos can be evoked literally, figuratively alluded to, and can serve functionally or formally within a piece of music. It can be used to make algorithmic works seem less artificial, and can be applied to rhythm, pitch, timbre, or any other aspect to a piece of music.

Summary of Part II

"[The theory of relativity] occurred to me by intuition, and music was the driving force behind that intuition. My discovery was the result of musical perception."

— Einstein

Through a discussion of the role of chaos theory in music acoustics, harmony, and an analysis of its specific roles in the production of works, we have seen how complexity and chaos is already an intrinsic part of music, from a certain perspective. I will continue to explore many of these same ideas in their applications to music composition, providing mechanisms for composers to incorporate these concepts into their own music.

Part III

Composition

Chapter VI
Creating Chaos in Music

"It is a recurring experience of scientific progress that what was yesterday an object of study, of interest in its own right, becomes today something to be taken for granted, something understood and reliable, something known and familiar — a tool for further research and discovery."

— J. R. Oppenheimer

I have shown how some composers in the 20th century incorporated chaos into their compositional approaches, either intentionally or indirectly. In this chapter, I will present my own approach to the use of chaos in musical composition, through several examples which each use chaos in a different way. It is a significant feature of my approach that these techniques result in music playable by acoustic instruments and performers; it is of the utmost importance to me that my music remains compelling and listenable, and to incorporate chaos too much or too directly into a piece of music risks losing much of what I value most in the musical experience.

For many of these works, I use Lilypond (`www.lilypond.org`) as a mechanism for generating musical scores. This program provides a text-based interface and a logical LaTeX-like structure which I produce using programs written in Perl or C (though any computer language could easily produce Lilypond code). Incidentally, Lilypond is also responsible for the typesetting of the musical examples in this document.

A fundamental question arises when one considers the implications of allowing algorithms to produce the content of a work of music. Who is responsible for the creation of this work? Can a composer assert authorship when the the work was created using an approach which separates the composer from the materials of composition? To Cage, "the composer resembles the maker of a camera who allows someone else to take the picture," but Cage's intention was to remove the composer from the so-called act of composition.[1] Instead, I consider these algorithms and approaches to be tools, that the act of composition is a control on a higher level of organization than with specific notes and rhythms. The resulting piece is always subject to my own artistic aims as a composer, and while I may not be writing individual notes and rhythms in a musical notation (itself a set of tools), I have simply created a new "notation" with which to write.

6.1 Randomness and Chance Procedures

"I can't understand why people are frightened of new ideas. I'm frightened of the old ones."

— John Cage

Section 2.2 and Section 5.4 previously dealt with Randomness and Chance procedures with respect to the development and history of these processes, and their use by composers in the 20th century. This section will specifically deal with new approaches toward using these procedures to create significant music, informed by chaos theory and the principles of randomness.

From a certain philosophical point of view, either all music composition is random or none of it is. The source of creativity in the human brain has not been traced, and could just as easily be a random/quantum-mechanical event or simply a result of a chaotic process stemming from the precise state of the brain of the composer. In this way, human creativity could be considered entirely deterministic or entirely random,

[1] Cage, op. cit., p. 11.

and from an external point of view nothing would change. Thus, the compositional act as an improvisatory medium, sparked from the center of creativity in the brain (wherever it may be), could be considered either random or the result of deterministic processes. Your point of view on this is largely dependent on your perspective from Section 3.2 on free will, but in either case, chaos certainly plays a role in the act of composition, as a single look at any composer's desk would instantly reveal.

Many of the following works employ chaotic functions in order to create a feeling of randomness. While randomness and chaos occupy opposite poles of the continuum, from within a system it is impossible to determine which of these is active. Thus, if it is a composer's goal to create randomness, pseudo-random aperiodicity is close enough.

6.2 Chaos in Form

"Music, even in situations of the greatest horror, should never be painful to the ear but should flatter and charm it, and thereby always remain music."

— Wolfgang Amadeus Mozart

I discussed chaos in form in Section 5.1. Here I will use chaos in the creation of form. Leach and Fitch have utilized chaos as a way of organizing events in algorithmically generated works, such that the values of a chaotic function determine whether the next event will be a variation of the previous or some new information.[2] I took a slightly different approach, letting the chaotic function dictate the order of events themselves, rather than a measure of the event's importance or newness.

Chaos Drumming, composed in 2008, used a function called the logistic map to generate an ordering of its rhythmic content (see Appendix B for a full score). The

[2]Leach and Fitch, op. cit.

rhythms themselves were written in imitation of *Taiko* drumming, but the order in which they appear is chaotically determined.

The logistic map is of the form:

$$x_{n+1} = rx_n(1 - x_n),$$

where r represents a given fixed value. Values for x are **Real** numbers within the range of 0 and 1. This equation is commonly used to model population under fixed resources. When x_n is large (the population is high), the subsequent x_{n+1} is strongly determined by the small value of $(1 - x_n)$ and thus will be small. When x_n is small, $1 - x_n$ will have little impact and the subsequent x_{n+1} will be strongly determined by the constant r which is considered as a net rate of reproduction, taking both births and deaths into account.

This simple equation has surprisingly complex results for certain values of r. For small r values, the population will die, as in our analogy the reproduction rate is not great enough to sustain the community. For r values between 1 and 3, the population has a single point of equilibrium at $\frac{(r-1)}{r}$. For larger r values, we see a pattern of bifurcation and eventually chaos erupt. Figure 6.1 shows the points of equilibrium for various values of r, and Figure 6.2 shows a detailed view from $r = 3.8$ to 3.9. As seen on these graphs, the points of equilibrium of the logistic map, sometimes called an orbit map, form a fractal structure.

I chose to portray disorder by first starting with order. First, I wrote 24 rhythmic patterns which were based on rhythms heard during *Taiko* drumming performances. Taiko is a Japanese drumming tradition, and while the piece is not a Taiko piece, it should be considered an *homage* to Taiko style and drumming approach. These patterns were constructed to change gradually from one to the next, resulting in a

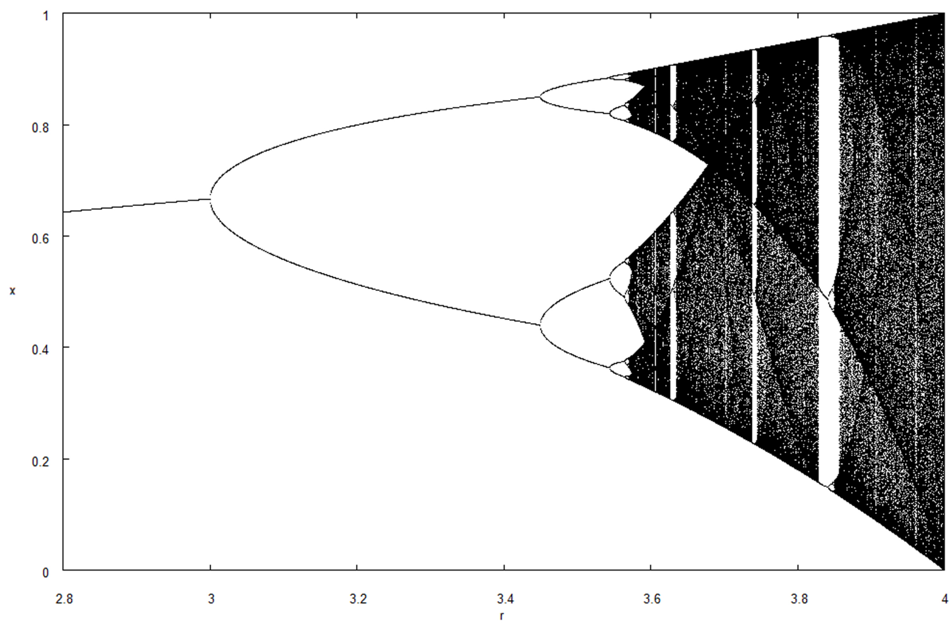


Figure 6.1: Points of Equilibrium on the Logistic Map. The points graphed represent periodic orbits on the logistic map. As r (horizontal) increases, the graph follows a pattern of bifurcation and eventually chaos emerges. The graph was created using data points generated by a C program written by the author and graphed using GnuPlot. Note the fractal self-similarity of this graph, as every point of bifurcation provides a point of reference to see its similarity to the entire graph.

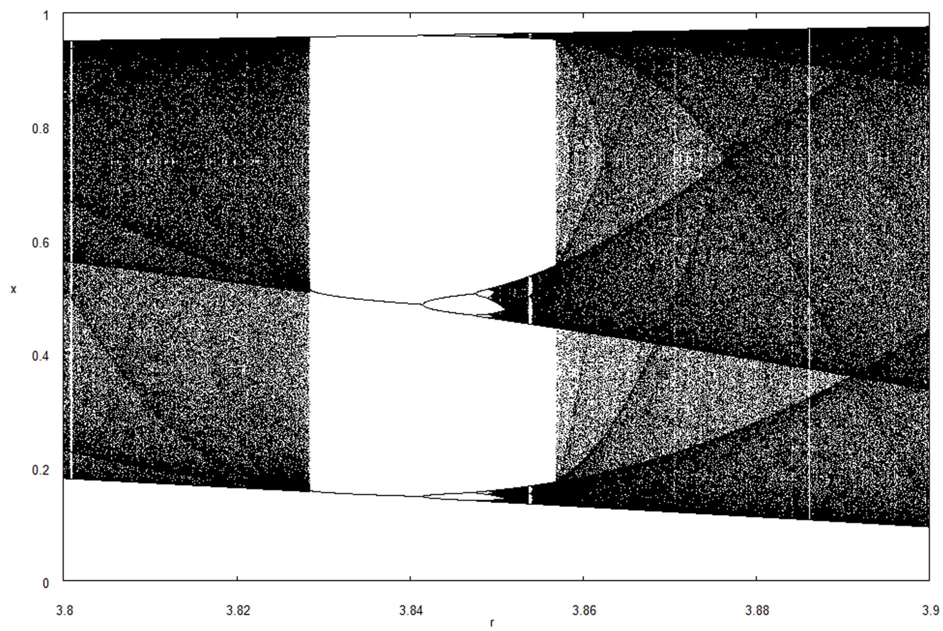


Figure 6.2: Points of Equilibrium on the Logistic Map: Closeup. This graph enhances the data between r values 3.8 and 3.9 to demonstrate the values of r which have 3 periodic orbits. Again, the pattern of bifurcation is found. The vertical white stripes in this graph and Figure 6.1 are not graphical errors, but instead are very small areas with similar structure to that shown here, with a different, relatively small number of periodic orbital points.

rhythmic space that is intended to be heard as roughly continuous: nearby rhythms share common traits, whereas rhythms that are far apart are different. The limitation of 24 rhythms, rather than a fully continuous rhythmic space, was to aid in the performance. At high tempos, the performers need to be able to recognize the rhythms quickly, and the visual aspects of the piece demanded a limited repertoire of rhythms. With synthesized or mechanical performers, it would be possible (and preferable) to use a more continuously defined rhythmic space.

The form of the piece was then determined through use of the logistic map, and control of the variable r. Each percussion part used its own initial x_0 value, but r was uniform throughout the piece. At the beginning of the piece, the x values were $\{0.25, 0.54, 0.739, 0.985\}$, and r was 3.5. I removed the first ten iterations of the formula, and then set r to 3.4 and stepped through 30 iterations, using these values to determine the initial rhythms. Through the next 16 iterations, I gradually stepped r to 3.9985, and used 60 iterations at this chaotic region. Finally, I used 15 iterations to reduce r to a value where there are three points of attraction, $r = 3.828428$, and the final 60 iterations returned to this area of relative stasis. You can see this area of stasis, with three points of equilibrium, in Figure 6.2.

The results are striking, and capture aurally many of the features of chaos that I have outlined throughout this document. In Figure 6.3, I have graphed each of the percussion parts throughout the piece, where the x axis represents the number of patterns rather than measures (each pattern is two measures long). In the beginning, the percussions converge on two points of equilibrium, and eventually arrive at x values that are quite close to one another. On the introduction of chaos, these small differences in initial conditions lead to extraordinarily different paths in the chaotic region. Finally, the stasis found within the chaos, at a r value with three points of equilibrium, is where the piece concludes.

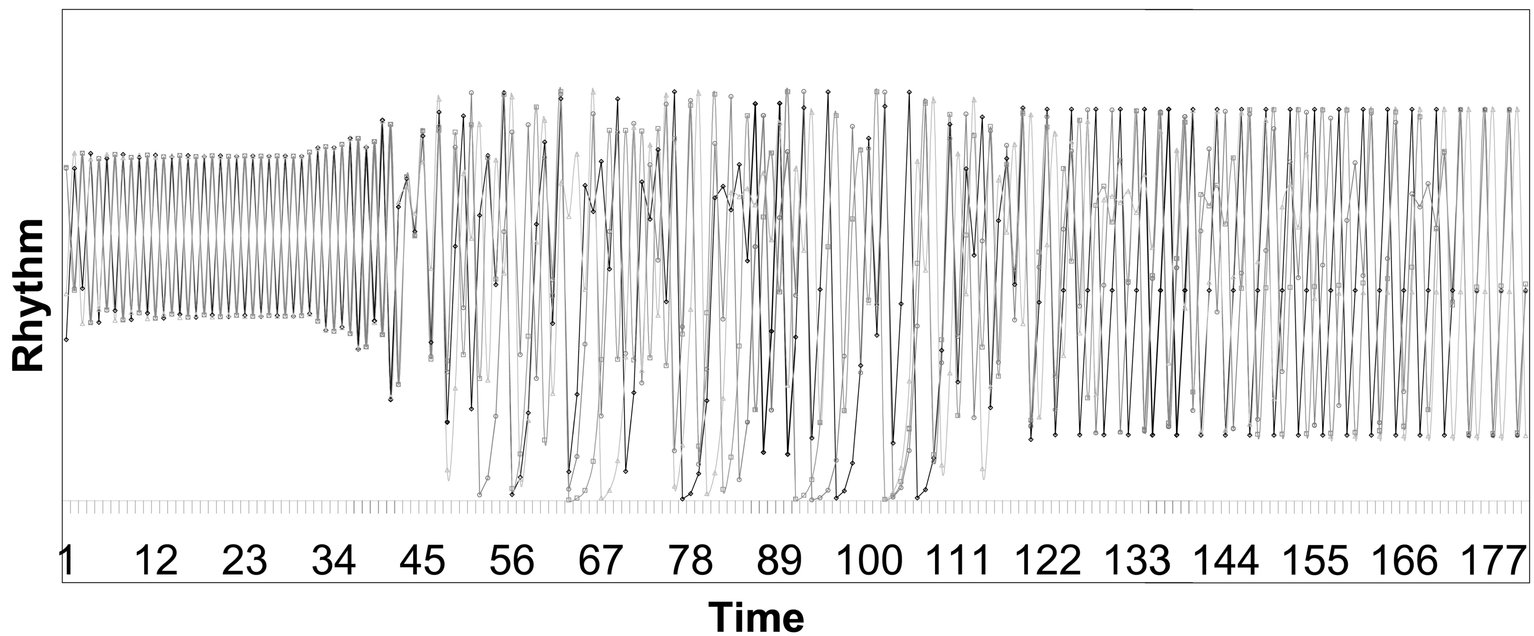


Figure 6.3: Chaos Drumming. The horizontal dimension represents the number of the pattern (each pattern is two measures long). The beginning of the piece exhibits two points of equilibrium, and the end exhibits three. Chaos is found in the middle. This graph was created using a program written in C to generate the data and Excel to graph it.

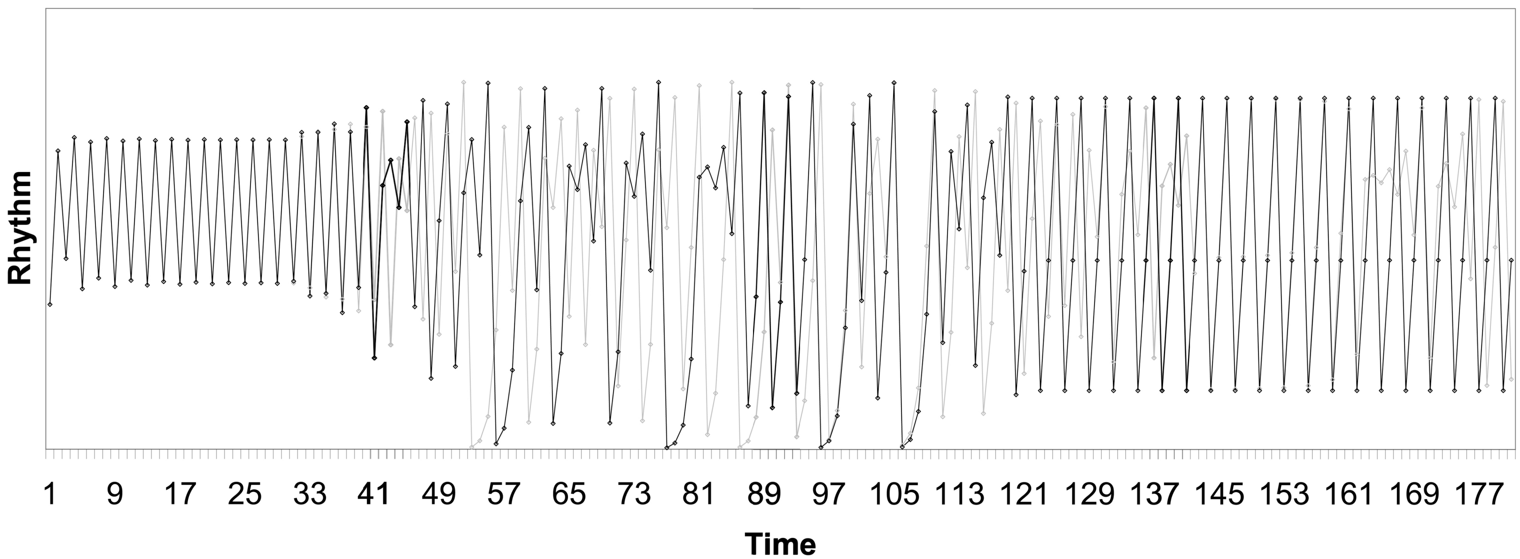


Figure 6.4: Chaos Drumming: Sensitive Dependence. The difference created between the two sample computations (near pattern 35) is 1×10^{15}.

It is worth pointing out that this piece, if calculated using a different floating point architecture than the one I used, would result in a piece alike in form but significantly different in content. My program utilized 15 digits of accuracy (53 binary digits) in resolving each of the x values, and the use of 14 or 16 digits of accuracy would have remarkable effects on the resulting piece. The strong dependence on initial conditions makes these seemingly slight calculation differences result in quite different paths for the percussionists. Figure 6.4 is a graph of the first percussion part with a difference of 1×10^{-15} introduced immediately before the chaotic part. It is immediately apparent what such a minuscule difference would have on each percussion part.

So then, one might be concerned that the limitations of only 15 digits of accuracy might cause rounding errors, and limit the flexibility of the algorithm to stay unique. If this piece continued long enough, would it be doomed to repetition? In the chaotic region, for example, if two of the percussionists happened to hit on the exact same value to the computer program's 15 digits of accuracy, they would continue along identical paths from this point onward. In testing this problem, I determined that, for my starting values, this effect does not occur for at least 2 billion iterations, which is enough music for about 70 years of nonstop playing. Thus, this level of accuracy in my computations, while not fully an authentic reproduction of the original function (if that is indeed ever possible), is at least accurate enough to observe the chaotic effects. The function I used might be written:

$$x_{n+1} = \mathrm{round}_{15}(rx_n(1 - x_n)),$$

and while this function is not as well studied as the previous, it also has the appearance of chaos that we seek. In my experiments to find the best starting values, however, this effect was observed in the initial phase of the piece, where the paths of the parts are strongly attracted to two points of equilibrium. When two parts hit on the exact same value, it meant that the two parts would remain on the exact same course throughout the entire piece, which was considered undesirable. Finding starting values that worked was not difficult, but also not guaranteed.

6.3 Chaotic Serialism

"Music expresses that which cannot be put into words and that which cannot remain silent"

— Victor Hugo

Section 2.1 dealt with the history and development of Serialism, and Section 5.2 presented a method of incorporating chaos theory into the analysis of Serialism. In this section, I present a new sort of Serialism which incorporates nonlinear algorithms into the tone row.

Standard serialism involves the use of a set or row of pitches which dictate the ordering of pitch material throughout the piece of music. This set generally contains all twelve pitches, which are used throughout the piece in their entirety. Unfortunately, this means that standard Serialist pieces have a flat pitch distribution: all pitches are presented an equal number of times. While the composer can circumvent this problem through rhythm and orchestration, another approach might lend the Serialist technique more relevance in terms of the natural world.

Rather than using an ordering of pitches which creates an equal number of each, we might use an ordering of pitches which more accurately represents distributions found in the natural world. A chaotic function would allow for all pitches to be used and present, but some pitches at different times to receive more emphasis than others.

Similar to my approach with *Chaos Drumming*, Jeff Pressing has applied the logistic map to the pitch domain and produced some interesting results:

> In musical terms, the overall effect is like a variation technique that inserts and removes material from a motive undergoing mildly erratic pitch transformations, in the style of an adventurous but development-oriented free jazz player, perhaps.[3]

I have used the logistic map to produce a brief piece of chaotic serialism which can be found in Appendix C. In this case, I used the exact same parameters as with Chaos Drumming, applying each of the percussionists' paths instead to duration, dynamic, articulation and pitch. The "chaotic serialism" per se does not begin until measure 12 (rehearsal letter B). My application serves simply as an example, and is overly simplistic and coarse-grained. A fully synthesized version, perhaps generated using Max5 directly, could explore the more subtle differences in duration, pitch, and articulation possible using finer detail from the logistic map.

This sort of Serialism may sound similar to the original Serialist approach in terms of its unpredictability and atonality, but operates at a much higher dimension of complexity than the simple transformative space used for Serialist works. This is not to say that the resulting work *is* more complex (whatever that would mean), and certainly not to extend a measure of value based on this measure of complexity, but merely to say that, when the technique for generation is analyzed, this approach uses a more intricate and complicated organizational scheme. This cross-application of Serialist ideas and chaos theory remains a very interesting area for future compositional direction.

[3] Jeff Pressing. "Nonlinear Maps as Generators of Musical Design". In: *Computer Music Journal* 12.2 (1988), pp. 35–46, p. 38.

6.4 Thematic Transformation

"I have great belief in the fact that whenever there is chaos, it creates wonderful thinking. I consider chaos a gift."

— Septima Poinsette-Clark

Leach and Fitch use chaotic algorithms in an attempt to create sequences of events which model the natural world.[4] McAlpine, et al. have used cellular automata (another chaotic system), applying them to model motivic transformations.[5] In both cases, the advantage is that chaotic functions can create both variety and repetition, self-similarity and unpredictability; these are all features that have been always been present in music.

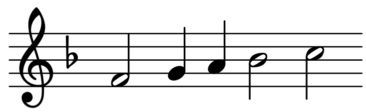
In this case, I will explore the transformation of a motive through iterative functions. A typical musical transformation space would include augmentation, transposition, inversion, and pitch multiplication (modulo 12), though there are other transformations which might be included. Here, I will define another rhythmic transformation, "folding," which incorporates augmentation but limits the transformation to a result that is the same duration as the original motive. This folding is an important attribute of chaotic functions, which can often be interpreted as transformations that invoke stretching and folding operations. You may recall from Section 1.8 that these operations played a role in the requirement of a chaotic function to be "topologically mixing."

To fold, I will note the attack points of the second half of the motive and reverse them, overlapping them with the first half of the motive. As one example, let us consider the typical chaotic functional description of stretching and folding. In this case, the stretching is represented in time by augmentation, and the folding will involve

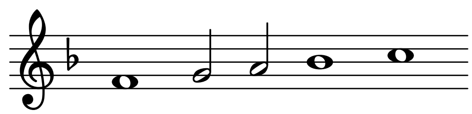
[4] Leach and Fitch, op. cit.

[5] McAlpine, Miranda, and Hogger, op. cit.

reordering the motivic elements in time, a sort of temporal inversion or reflection. A motive such as:

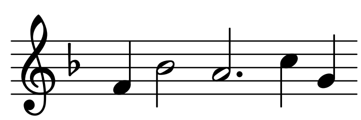


Stretching to double length results in the following:



And then folding in half would produce:

Keep in mind that I are concerned with attack points, not with duration. Here are the next three iterations:



Perhaps that result was somewhat surprising. Here's another motive:

The following sequence is the stretching and folding of this motive:

The first fold will stack areas of rhythmic symmetry, and then the subsequent foldings will form a periodic set. Note again that this folding function is with respect to attack points, not durations, since the folding with respect to total duration eventually produces long held chords of all of the notes of the motive; it is destructive to rhythm.

This approach generalizes: after the first fold, the rest of the folds will produce a periodic orbit. Most of the transformations work similarly: transposition is limited to at most 12 steps, inversion and multiplication to 2 steps.

The next step is to utilize a chaotic function to use these motivic transformations to construct a melody line out of the motivic transformations. Instead, an entire piece could be composed using the sequence of motivic transformations as inspiration.

All of our transformations each explore only a limited motivic space. Is it possible to escape from this space using a combination of these motives? If so, what kind of ordering of transformations is required to escape? A thorough exploration of the dynamics of musical-transformation spaces would provide an excellent further area of research.

6.5 Rules Music

"Big fleas have little fleas
on their backs to bite them,
and little fleas have lesser fleas,
and so ad infinitum."

— Jonathan Swift

A chaos game is one way to explore iterative chaos free from numerical computation. Each game is played by a system of rules, and the rules are iterated at each step of the game. These games have surprising results, as a simple set of rules can yield surprising complexity.

For example, let's begin with a short line segment. This segment is divided into three parts each having equal length, and the middle part is removed. Then, we successively and recursively apply this rule into the three sub-parts we just created (see Figure 6.5).

The set is the result after applying this rule an infinite number of times, resulting in a very small collection of individual points with (relatively) large gaps between them. Certain points are easily seen to be members of this set: assuming the original segment lies from 0 to 1 in the number line, the following points are examples of members of the Cantor set: 0, 1, $\frac{1}{3}$, $\frac{6}{27}$.

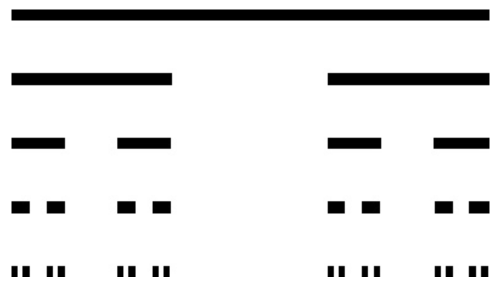


Figure 6.5: First five levels of the Cantor set. This represents the process of the set's creation; the actual set is the idealized scattering of points after following the process an infinite numbers of times.

The endpoints of the line segments can be easily expressed, and remain in the Cantor set throughout the infinite process of generating it. These endpoints are fractions which have an integral numerator and a denominator that is a power of 3. However, these represent a countably infinite subset of the uncountably infinite set; there are far more points (such as $\frac{1}{4}$) that lie in between these endpoints than the endpoints themselves. The process that generates the Cantor set can also be considered a demonstration of its fractal structure, including self-similarity and infinite levels of detail.[6]

A similar process is used in a two-dimensional figure to obtain what is known as the Sierpinski Triangle. Rather than removing the middle third of a line segment, we remove the middle portion of an equilateral triangle, again recursively applying it to the remaining three sub-triangles. The result is found in Figure 6.6.

To generate this figure, however, I used a rather unorthodox method. Starting with any point inside the triangle, I computed half the distance to one of the vertices

[6]The entire set represents points with ternary expansion $0._3d_1d_2d_3\ldots$ where $d_i \in \{0, 2\}$ for all i; i.e., the ternary expansion contains no ones. Another interesting point about the Cantor set: it has uncountably infinite points, but is of measure 0; it has no length, but instead represents an infinitely diffuse scattering of points.

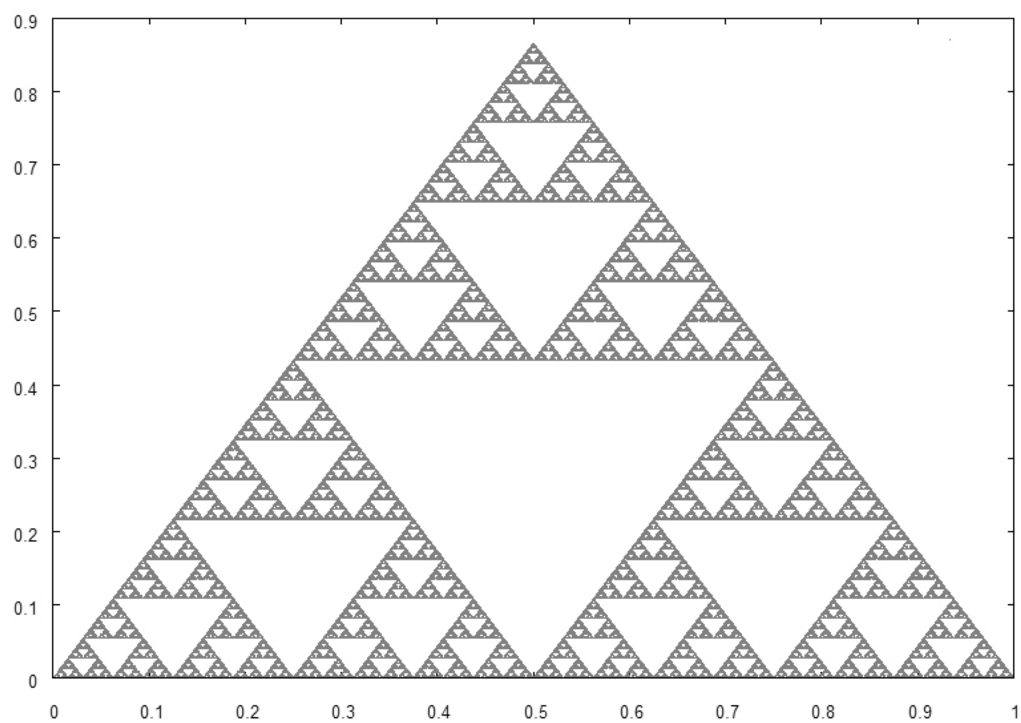


Figure 6.6: The Sierpinski Triangle. An example of a fractal created using an algorithm. Data generated by a C program, graphed in GnuPlot.

and marked that point on the graph. Using that point as the next starting point, I repeated this procedure. The fact that this process results in the Sierpinski Triangle is somewhat surprising, since the two methods of obtaining this image seem to have nothing to do with one another. This process only works if the choices of vertices for each step of the process form an aperiodic (i.e. chaotic) sequence.[7]

We will explore two applications of this concept to the domain of music composition, using these rules to control pitch patterns and signal processing. Through a simple set of rules, it is possible to generate a sequence of pitches which behave chaotically. As one example, I used the following rules:

1. There are three possible pitches, ordered into a list. We'll call them *do*, *re*, and *mi*, though it could be any pitches.
2. If *do* is played, add *mi* and *re* to the end of the list.
3. If *re* is played, add *do* and *mi* to the end of the list.
4. If *mi* is played, add *re* and *do* to the end of the list.

At this point, I have created a quasi-periodic sequence, which, while it has patterns that recur, does not ever become periodic. For example, starting with the pitch *do*, the sequence is:

do, mi, re, re, do, do, mi, do, mi, mi, re, mi, re, re, do,...

Substituting 1 for *do*, 2 for *re*, and 3 for *mi*, and extending the pattern for the first 479 pitches, we see:

1 3 2 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2
3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2
1 1 3 1 3 3 2 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 3 2 2 1 2 1

[7] Actually, it has to be more than just chaotic. For example, the logistic map does not generate the Sierpinski Triangle—it is not chaotic *enough*.

1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1...

Dropping the first starting pitch, let's now categorize by the sets of two: {3 2}, {1 3}, and {2 1}. We'll call these patterns A, B, and C: It's not surprising that these two-pitch patterns encompass the entirety of the output, since that's how the sequence was generated.

A C B B A A C A C C B C B B A C B B A B A A C B A A C A C C B B A A C A C C B A C C B C B B A A C C B C B B A C B B A B A A C A C C B C B B A C B B A B A A C C B B A B A A C B A A C A C C B C B B A B A A C B A A C A C C B B A A C A C C B A C C B C B B A C B B A B A A C B A A C A C C B B A A C A C C B A C C B C B B A B A A C A C C B A C C B C B B A A C C B C B B A C B B A B A A C B A A C A C C B A C C B C B B A A C C B C B B A C B B A B A A C A C C B C B B A C B B A B A A C ...

You'll note that, replacing A with pitch 1, B with 2, and C with 3 results in the original sequence. To further illustrate this, we can examine the following meta-sequences: {C B}, {A C}, and {B A}. We note that these meta-sequences resemble the previous sets, and we will replace {C B} with 1, {A C} with 2, and {B A} with 3, after again removing the first A:

1 3 2 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 1 3 3 2 3 2 2 1 3 2 2 1 2 1 1 3 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 3 2 2 1 2 1 1 3 2 1 1 3 1 3 3 2 2 1 1 3 1 3 3 2 ...

And the process could continue. The generalization of this results in a fractal-like structure of pitch patterns. Essentially, this structure is revealed by replacing each set of two pitches by the pitch that generated them, thus transforming each part of the sequence into an earlier part of the sequence. Thus, the transformation detailed above shows an infinite regress of self-replication, here created linearly using a set of rules. In a way, though the sequence is not periodic, it is self-similar, just stored in a compressed format.

I have notated the musical analog to this sequence in Figure 6.7, here using rhythmic notation not to denote time but instead to demonstrate the multiple levels on which the sequence is self-similar, with each pair of notes being derived from the corresponding note in the previous measure. This sequence might remind you of the Cantor set or the Sierpinski triangle. Here, we begin with a single pitch (F, in Figure 6.7), which is broken up into three pitches (A, G, and F) and one is removed (the original pitch: F). Stated this way, each generation exists as a smaller musical fragment within its parent pitch, much like the fractal forms of the Sierpinski triangle and Cantor Set. Figure 6.8 shows this relationship graphically. Again, the rhythm denotes the level of the fractal, not the temporal relationships, though using the rhythm as notated would create a analogy between size in the Cantor set and Sierpinski triangle and duration in the musical example.

Unfortunately, the sequence requires an arbitrarily long memory in order to prevent it from becoming periodic. Also, with only three pitches, the resulting sound of the sequence is not particularly compelling as a musical idea. But the idea of using

Figure 6.7: Chaos Game. The rhythm as notated is not for performance, but is intended to suggest the derivation of the sequence; each pair of notes was generated from the note above it.

quasi-periodic or a-periodic sequences to organize pitch structures in music certainly has great potential.

Another set of rules which produce chaotic behavior operates on a two-dimensional grid. This "Game of Life" was invented by John Conway in 1970, and is one of the most famous uses of a model known as a cellular automata.

The rules of this game are as follows:

1. Generations are discretely computed.
2. Any cell with 3 neighbors is born or continues to live.
3. Any cell with 2 neighbors continues to live if it is alive.
4. Any cell with fewer than 2 neighbors or greater than 4 neighbors dies.

Like the logistic map, this model can be thought of as simulating population growth, where a deficit of resources for reproduction as well as overcrowding can cause a cell to die. This game results in some surprising complexity, with the ability to create patterns which are periodic with any given period, and includes a certain amount of unpredictability, though like the other algorithmic processes once the initial state is known the entire future is determined. For some examples, the formation in Figure 6.9 emits self-replicating patterns called "gliders" (Figure 6.10) at a steady rate. The relatively compact formation in Figure 6.11 gives rise to a surprising complex result which does not achieve a state of stasis until over a thousand generations have gone by, but yet is only a single cell different than the glider, one of the most predictable patterns. Incidentally, using a toroidal world of 76 by 76 cells, the pattern lasts for over 3,000 generations, with the resulting formation found in Figure 6.12.

Predicting which cells would be part of the stable formation based on the original Pentomino formation shows how, though the generation of each step is completely determined, prediction of the end state from the beginning state is often impossible.

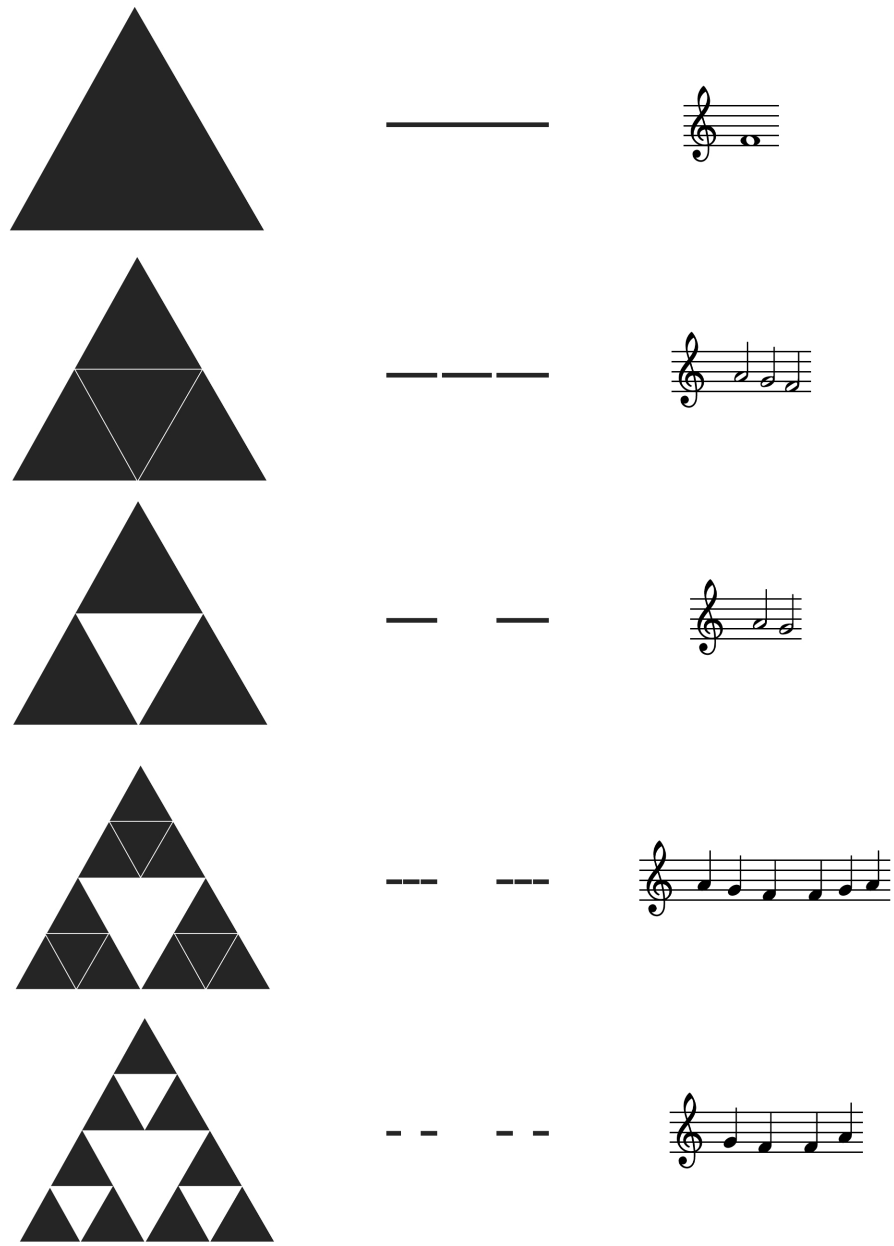


Figure 6.8: Analogy between Sierpinski Triangle, Cantor Set, and Musical Sequence. Each of these constructions follow a set of rules for dividing and removing parts of the original. The analogy to music is provided, again using rhythmic values to show derivation, rather than duration.

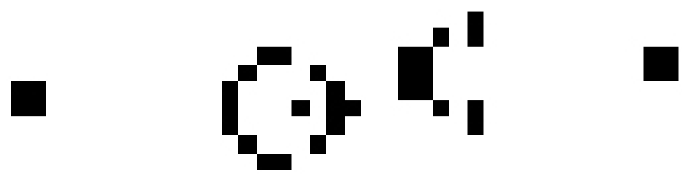


Figure 6.9: The Gosper Glider Gun. This Game of Life object will emit Gliders periodically.

Indeed, there is no algorithm other than the computation of each successive generation which will guarantee to produce the final stable end-state of any beginning state. In a deterministic universe, this is as close to "unpredictable" as anything can be.

The Game of Life has been applied to music before by McAlpine, Miranda, and Hoggar, utilizing a three-dimensional field and each pattern as a compositional theme.[8] Changes to these patterns become modifications to that theme, such as repetition, transposition, inversion, augmentation, or others.

In my application of the Game of Life to music, called *Living in Chaos*, I sought to further reinforce the idea of the "game," and rather than equating cells with individual sonic ideas, used the population in certain areas to control different effects throughout the piece. The game is played on a toroidal grid of 76x76 spaces, where the left side of the square-shaped grid is identified with the right, and the top with the bottom. This means that the space itself is very small, which helps keep the formations living through increased potential for interaction, and the toroidal shape

[8]McAlpine, Miranda, and Hogger, op. cit., p. 24.

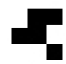


Figure 6.10: A Glider. This Game of Life object will travel in a diagonal direction across the grid.

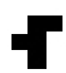


Figure 6.11: The Pentomino. This pattern, only a single cell different from the Glider, will evolve into a highly complex and long-living configuration.

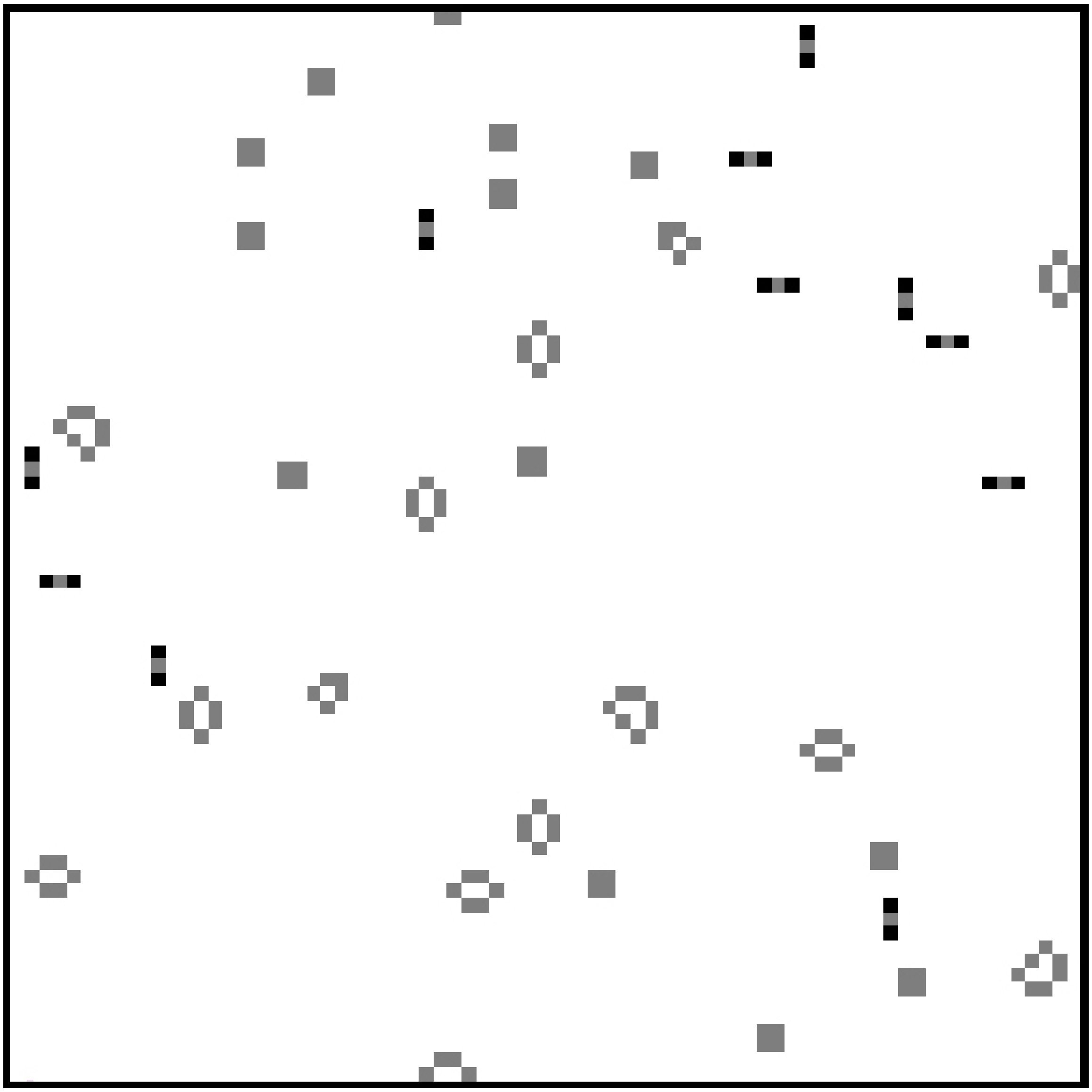


Figure 6.12: The Final Stable Formation of the Pentomino. This final state was created using a toroidal grid of 76x76 cells. This image created using a Max5 patch I created to explore musical applications of the Game of Life.

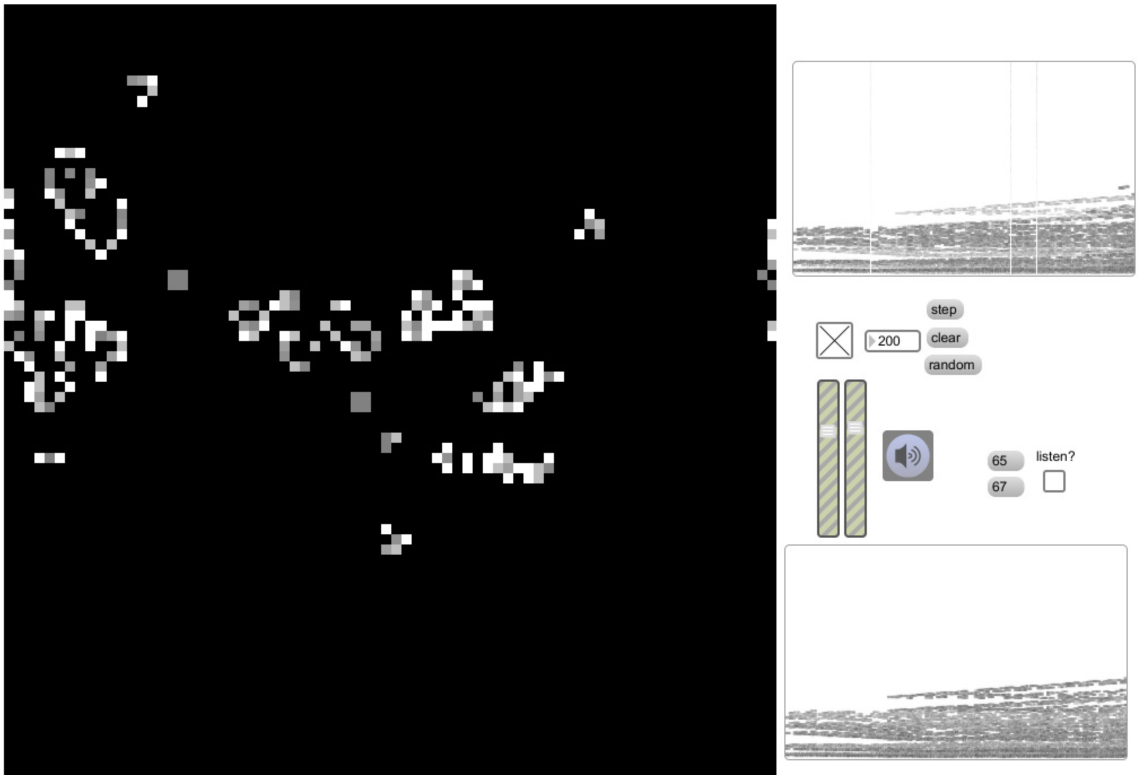


Figure 6.13: Living in Chaos. This screenshot is of the piece as implemented on Max5 using the rules from Conway's Game of Life. I recreated the rules of the game in Max5 using a Jitter matrix to hold the contents of the game and javascript to compute successive generations.

removes the possibility for odd behavior around the edges of the space, making it universally continuous.

The game begins with the Pentomino formation in the center of the grid, which is on display for the audience to see. The live performer, who has been given a repertoire of small musical segments, chooses which segment to play, providing data to the computer. These segments code for various actions with the world, such as the introduction of a glider, alteration of the rate of evolution, or other similar effects. The computer sound reflects position on the two axis, where the bottom left corresponds to a low frequency sound, and the upper right corresponds to a high frequency sound. Newborn cells are given extra harmonic overtones, which decay when the cells grow older. This ensures that the sound field does not become overly cluttered with the static elements on the grid. Furthermore, the left-right axis is affiliated with panning to help the audience aurally identify sounds with the visual elements of the piece. The synthesis is based on Ronald Keith Parks' Spectral Accumulation method using Max5, though I am using the technique not to capture sounds but to synthesize them directly.[9]

The game operates on multiple levels, existing as musical interaction between the computer and human performer as the human player (and the audience) deciphers the relationship between the various musical codes and the effects they have on the simulated world. The piece is primarily interesting because of its interactive elements and unpredictability, both built in to the Game of Life process as well as through the incorporation of human interaction to this process. Conway's Game of Life is also particularly interesting because it is destructive. It is impossible to decipher the history of a particular game simply from its end state, though it is a relatively easy task to advance the game to its next state. The game, then, operates with a certain direction of time, much like the world in which the game inhabits. This is not a coincidence at all, since it has been proven that Conway's Game of Life is a complete computational system. That is, any problem that can be solved by a computer can be solved using Life *as a computer*. There are analogous structures to computer memory and computation which can exist inside of this universe, and thus just as computer models can model the real world, the Game of Life is a model of a kind of computer.

[9]The Spec patch is available from Ronald Keith Parks' website: `faculty.winthrop.edu/parksr/promotional.htm`

And, perhaps, of life itself. The Blue Brain project in Lausanne, Switzerland is attempting to replicate the functions of the brain using an array of microchips—a very large computer. They have successfully simulated a piece of brain called a neocortical column, and hope to scale up to simulate an entire brain.[10] If possible, it would imply that computer processing, and Conway's Game of Life, are analogous to human consciousness.

[10] Jonah Lehrer. "Out of the Blue". In: *Seed* (Mar. 2008). Accessed 2/13/2009. URL: http://www.seedmagazine.com/news/2008/03/out_of_the_blue.php.

Summary of Part III

"Chaos is the score upon which reality is written."

— Henry V. Miller

Compositional approaches to chaos will generally involve the inclusion of algorithms and functions to the composition of music. Much like the ideas I explored in Section 3.1, "meta-composition" means that the composer does not operate directly on the materials of a work of music, but instead acts on the functions which create these works. This is not to say that the act of meta-composition is any less attributable to the ethos and ideas of the composer; like Xenakis versus Cage, I seek not to remove the composer from the act of composition but only to expand the tools available. Serialist composers sought a similar idea; here, I seek to expand the freedom of composition through expanding its nature.

I have applied chaos to formal design, pitch, rhythm, and used rules to create music which incorporate fractals or cellular automata. It is important, though, that the music remains compelling to an audience; unlike Babbitt, we must care if they listen. As such, we must not disregard the importance of the performer, whether electronic or human, and must use these tools to create music which still has emotional impact beyond idle curiosity.

Postlude

"What we call the beginning is often the end. And to make an end is to make a beginning. The end is where we start from."

— T. S. Eliot

I have demonstrated how the development of chaos and simultaneous innovations in musical thought can reveal and underlying shift in society toward the understanding of complexity. I presented information about acoustics and harmony toward an understanding of the role of nonlinearity in our interpretation of music, and found chaos in works from many different genres. I have also provided examples of the use of chaos in the act of composition through the application of nonlinear algorithms, emphasizing the subjective nature of "chaotic" evidence, in that any deterministic system, once one knows all, behaves completely predictably; thus, like our discussion of the $\sqrt{2}$, any application of chaos remains firmly grounded in the specific circumstances of its location, and many issues thought to be simple can reveal surprising complexity.

I must reiterate that this document serves only as an introduction to the ideas and issues chaos theory raises with regard to music. A comprehensive view of all of the applications would be well beyond the scope of any one document, as the more you search for chaos the more detail and complexity reveals itself. The phase space of scientific knowledge is no doubt an example of fractal geometry.

For one example, as performers, there is much to be learned from chaos. Chaos serves two primary roles: first, we should understand the role of chaos in our artistic lives, as well as ascertain the nonlinear aspects of the act of performing music (and hearing music, for the sake of our audiences) to better understand this act; and second, we should understand how chaos informs composition, and what we should do to become better interpreters of this music. The interpretation of algorithmic music is no trivial task, and sometimes requires an artist of the highest caliber to avoid creating music that sounds mechanical or artificial. An understanding of the role of nonlinearity in the reception of music as an audience, and the role of nonlinearity in the interpretation of music as performers, would require an understanding of the chaos in the brain, research very much in progress by neurologists today. The issues of free will and "randomness," if either exist, remain open for further interpretation.

I have endeavored to provide the background for future researches into applications of nonlinear dynamical systems to music, both in terms of analysis and composition, and have given examples to help guide readers in their understanding of the role chaos plays in music. I have attempted to provide examples of works written using chaotic algorithms to inspire future composers. From here, many different directions remain possible. Plenty of works could be written, and from there audiences educated, using nonlinear systems as an underlying structural element.

Here, we see both the end of this present document, and the potential beginning of future understanding of the role of chaos in the creation, interpretation, and enjoyment of music. From this, the future remains decidedly unpredictable.

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Appendix A
Proof of the Irrationality of $\sqrt{2}$

The Greeks probably used a geometric proof for this result. Instead, I will present one based in number theory, which would also have been available during the time of Pythagoras, and which uses the same fundamental facts about number and divisibility that a geometric proof would use.

This proof is a proof by contradiction. We will first suppose that $\sqrt{2}$ is a rational number, and should we come to a logical contradiction it will verify that $\sqrt{2}$ must be irrational. If $\sqrt{2}$ is rational, by definition it must mean that

$$\sqrt{2} = \frac{p}{q} \tag{A.1}$$

where p and q are integers and we assert that $\frac{p}{q}$ is in lowest terms (if not, reduce it to lowest terms by removing any common factors between p and q). That is, p and q share no common factors; they are *relatively prime*.

Squaring both sides of Equation A.1 and doing some algebra yields:

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2 \tag{A.2}$$

$$2 = \frac{p^2}{q^2} \tag{A.3}$$

$$q^2 * 2 = p^2 \tag{A.4}$$

Thus, since q is an integer, we know that 2 divides p; that p must be even. Since p is even, let

$$p = 2 * p' \tag{A.5}$$

where p' is the integer resulting from the division of p, which is even, by 2. Factoring this in to Equation A.4, we get

$$q^2 * 2 = (2 * p')^2 \tag{A.6}$$

$$q^2 * 2 = 4 * p'^2 \tag{A.7}$$

$$q^2 = 2 * p'^2 \tag{A.8}$$

So, by similar reasoning, we now note that, since p' is an integer, 2 must divide q and thus q is even. But p and q can't both be even, since they were originally asserted to be relatively prime. Thus, $\sqrt{2}$ is irrational.

The Pythagoreans, believing that all numbers could be expressed as the proportion of two whole numbers, would have found this very disturbing, particularly since this quantity was easy to construct using what is now known as the Pythagorean Theorem. A story, probably apocryphal, relates that Hippasus was drowned for discovering this fact.[1]

[1] According to Barnes, op. cit., p. 175, Iamblichus relates that Hippasus was drowned either for this or possibly for constructing a dodecahedron, though Iamblichus's history is not considered authentic by modern scholars.

Appendix B
Chaos Drumming

The following score and parts are provided for public performance and study of this piece. Performance for profit should be authorized through the author/composer by email: jon@jonathansalter.com.

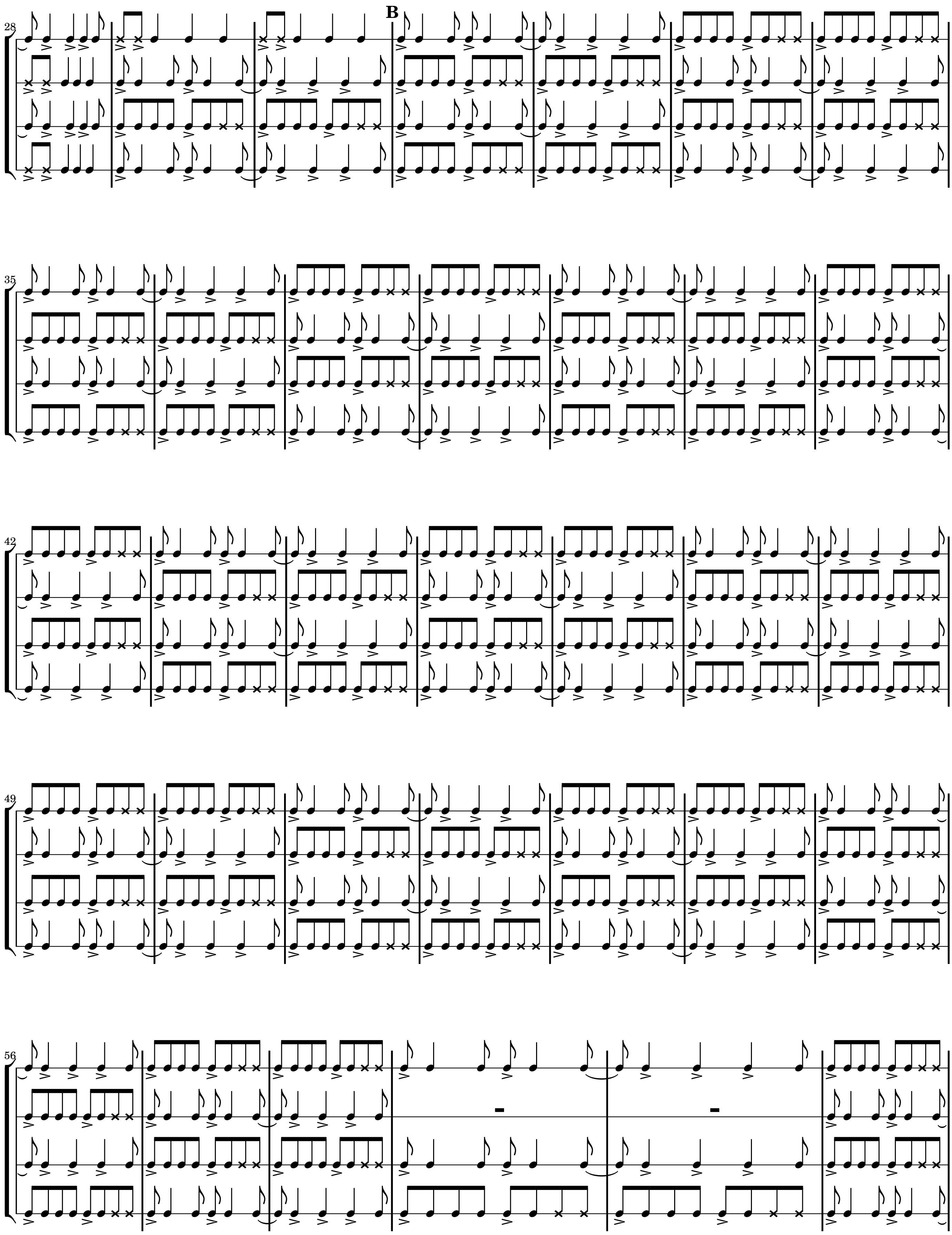
Chaos Drumming

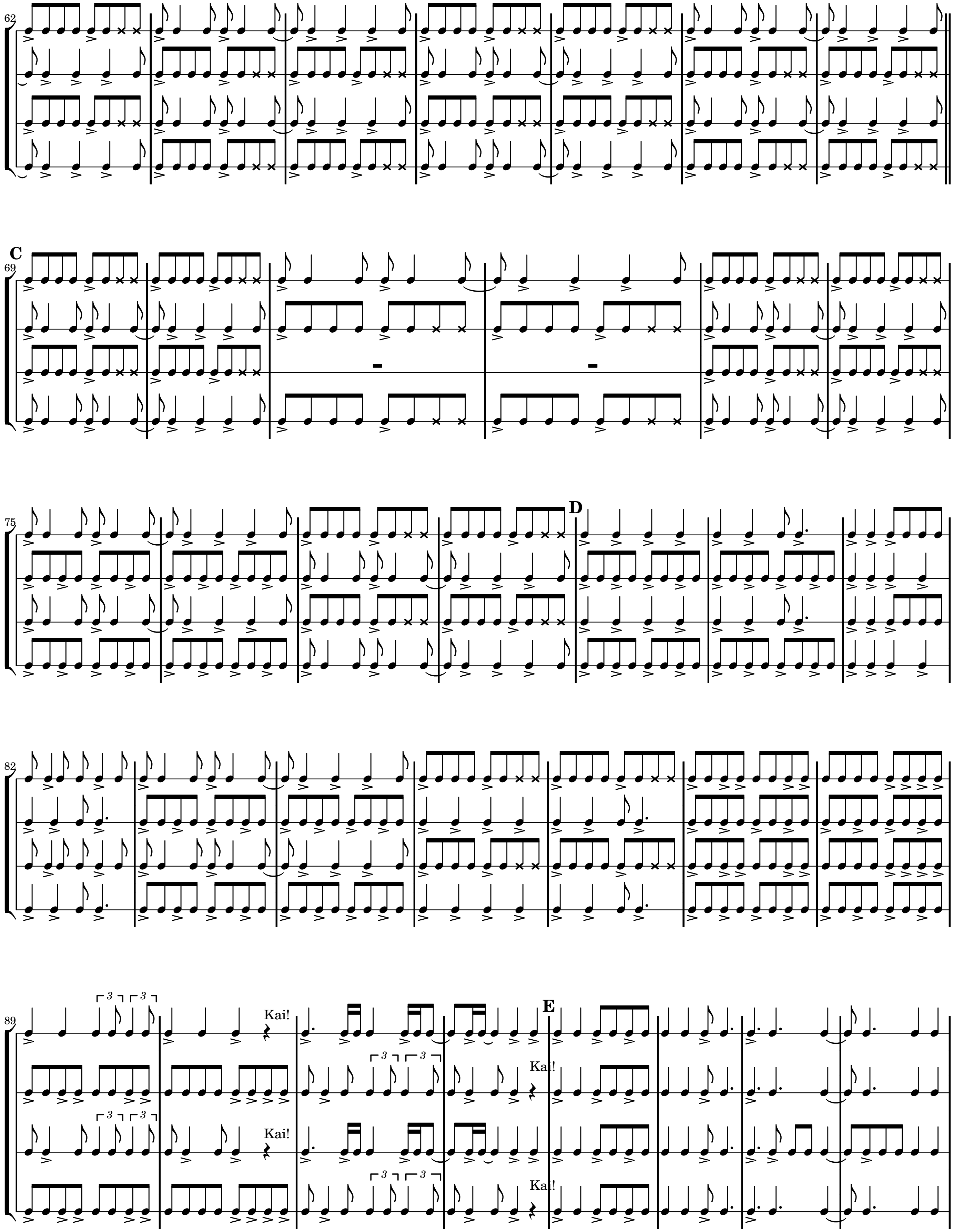
Jonathan Salter

Chaos Drumming was created using a nonlinear dynamical system called the logistic map. This iterative function provided the formal underpinning on which the piece is based, while the rhythmic material itself is inspired by traditional Taiko drumming. The piece is to be performed on Taiko drums, if available, or other deep-sounding high-impact drums. The performers should employ an aggressive stance and the ritualized motions similar to those employed in Taiko drumming. The piece is composed of the following patterns. It would be best if each of the performers observed the same motions when playing each of the pattern to help the audience understand and recognize the patterns when they occur in each of the parts.

2008


Driving (♩ = 196)
Percussion One
Percussion Two
Percussion Three
Percussion Four
A
Kai!
Kai!
Kai!
Kai!
Kai!


28
B
35
42
49
56


62
C
69
75
D
82
89
Kai!
E


97
F
103
Kai!
109
Kai!
115
Kai!
121
Kai!

126
G
Kai!
132
Kai!
138
Kai!
144
150
Kai!
Kai!


H
157
163
168
Kai!
Kai!
173
178
Kai!


184
Kai!
Kai!
Kai!
189
Kai!
194
J
Kai!
200
Kai!
Kai!
207

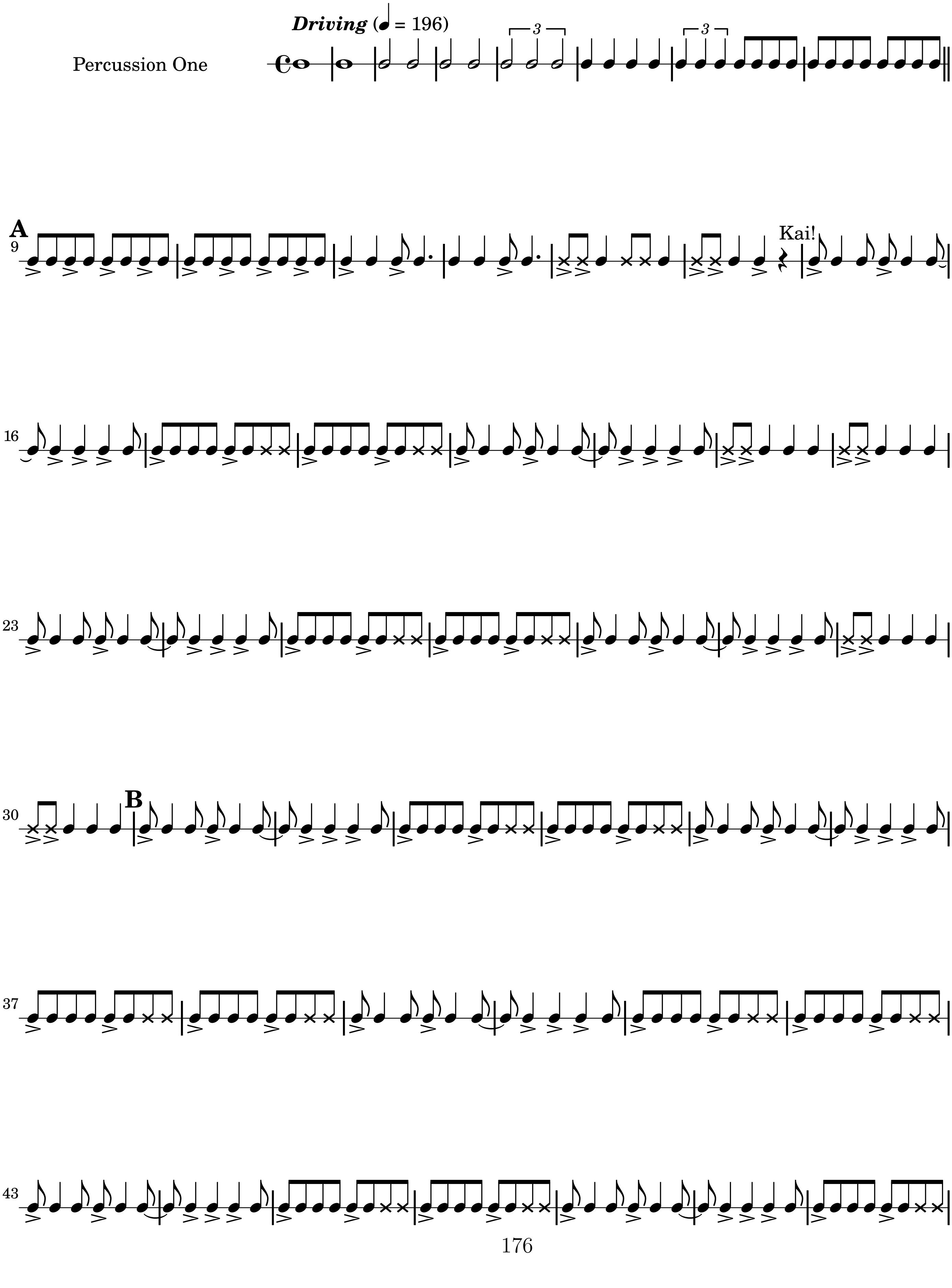

214
3
220
K
226
Kai!
233
238


245
L
251
Kai!
Kai!
258
Kai!
Kai!
264
Kai!
Kai!
Kai!
270
Kai!
Kai!
Kai!


M
276
Kai!
282
Kai!
288
Kai!
Kai!
Kai!
Kai!
294
Kai!
299
Kai!
Kai!


305
Kai!
311
Kai!
316
Kai!
322
Kai!
N
329
Kai!

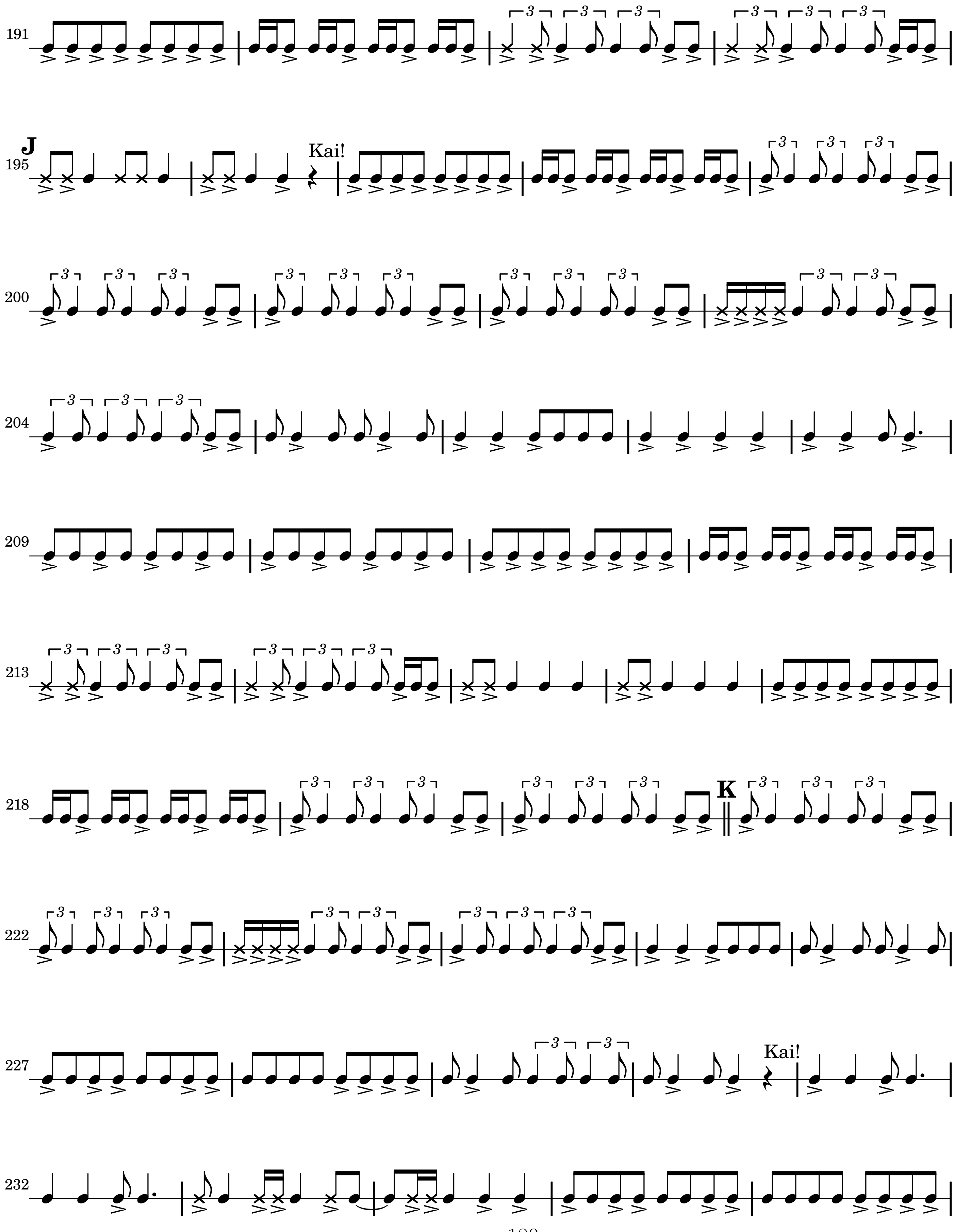

336
Kai!
Kai!
Kai!
Kai!
Kai!
343
Kai!
Kai!
Kai!
350
Kai!
O
Kai!
357
Kai!
Kai!
Kai!
Kai!
364
Kai!
Kai!
Kai!
Kai!
Kai!
Kai!
Kai!
Kai!


Driving (♩ = 196)
Percussion One
3
3
A
9
Kai!
16
23
30
B
37
43


50
57
64
C
71
78
D
85
Kai!
92
E
2


F
101
106
Kai!
111
116
Kai!
120
124
Kai!
129
G
133
Kai!
137
142


Kai!
H
Kai!
2

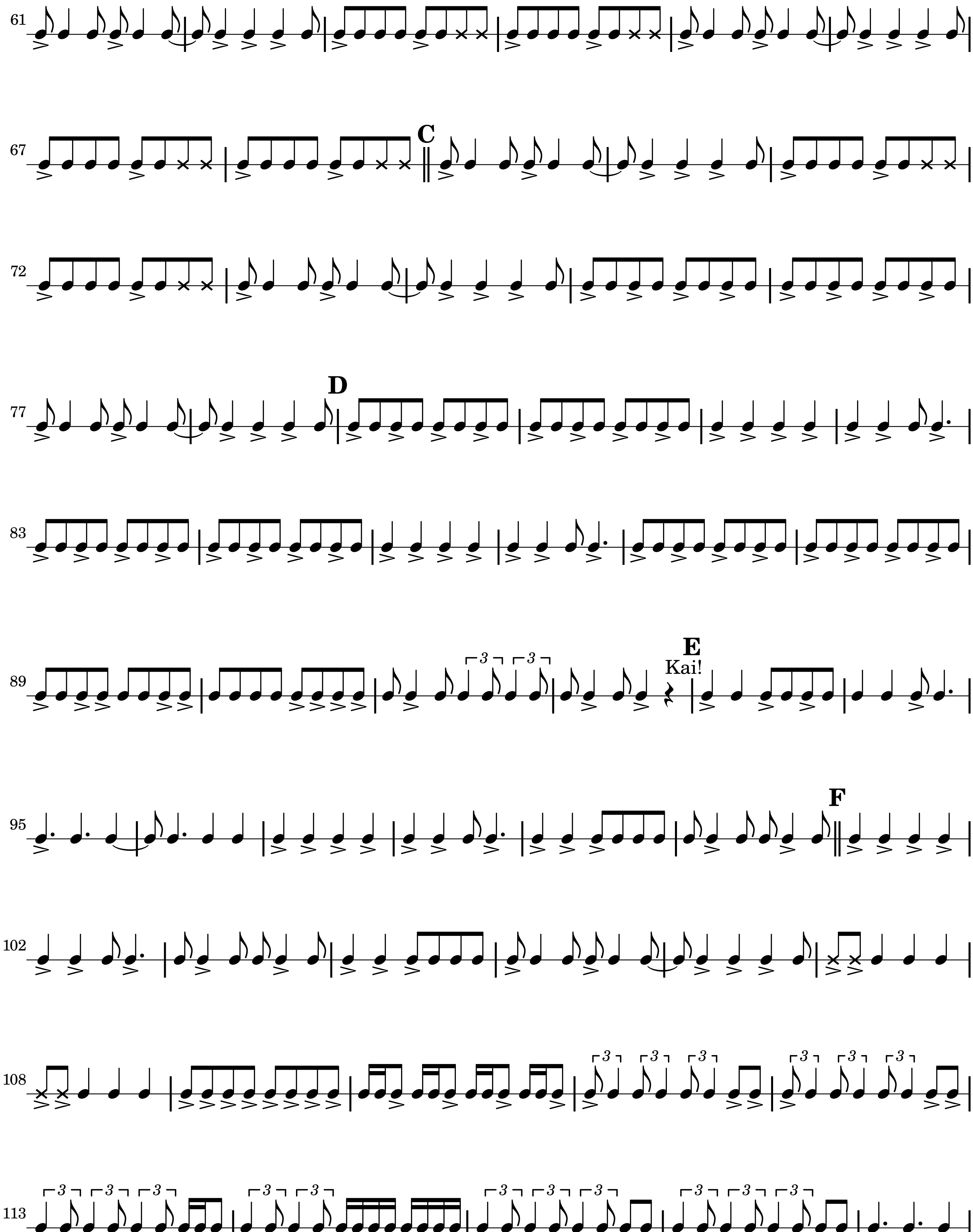

191
J
195
Kai!
200
204
209
213
218
K
222
227
Kai!
232


237
Kai!
243
Kai!
248
L
253
Kai!
258
Kai!
263
268
Kai!
273
Kai!
M
278
Kai!
283
Kai!
2


289
Kai!
294
Kai!
299
303
Kai!
307
Kai!
311
315
Kai!
320
Kai!
N
325
Kai!


330
Kai!
335
339
Kai!
344
Kai!
349
O
Kai!
354
Kai!
359
363
Kai!
367
Kai!


Driving (♩ = 196)
Percussion Two
8
A
12
Kai!
17
23
29
B
35
40
45
50
55
2


61
67
C
72
77
D
83
89
E
Kai!
95
F
102
108
113


G
Kai!
Kai!
H


173
178
184
Kai!
Kai!
191
J
196
201
208
214
219
K
224


231
238
245
L
251
Kai!
258
264
Kai!
270
Kai!
276
M
282
288
Kai!
2


297
301
305
310
315
319
323
N
Kai!
328
332

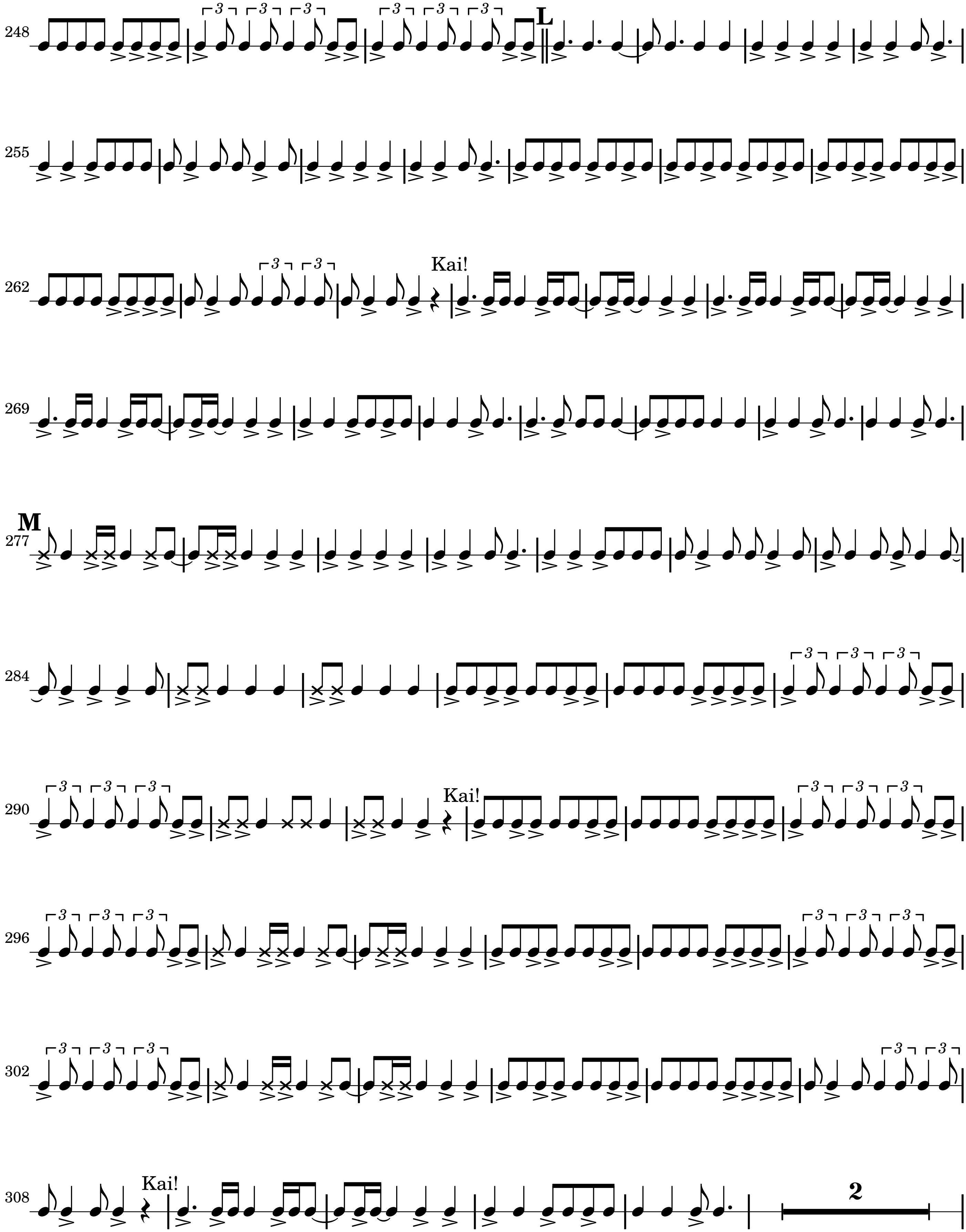

336
340
Kai!
344
348
352
O
356
Kai!
360
364
Kai!
368
Kai!


Percussion Three
Driving (♩ = 196)
8
A
Kai!
12
19
26
B
33
39
46
53
59
66
C
2


73
D
79
86
Kai!
92
E
99
F
104
Kai!
110
116
121
126
G
Kai!


Kai!
H
Kai!
2


187
Kai!
193
J
199
Kai!
206
212
217
K
224
229
Kai!
235
Kai!
241


248
L
255
262
Kai!
269
M
277
284
290
Kai!
296
302
308
Kai!
2

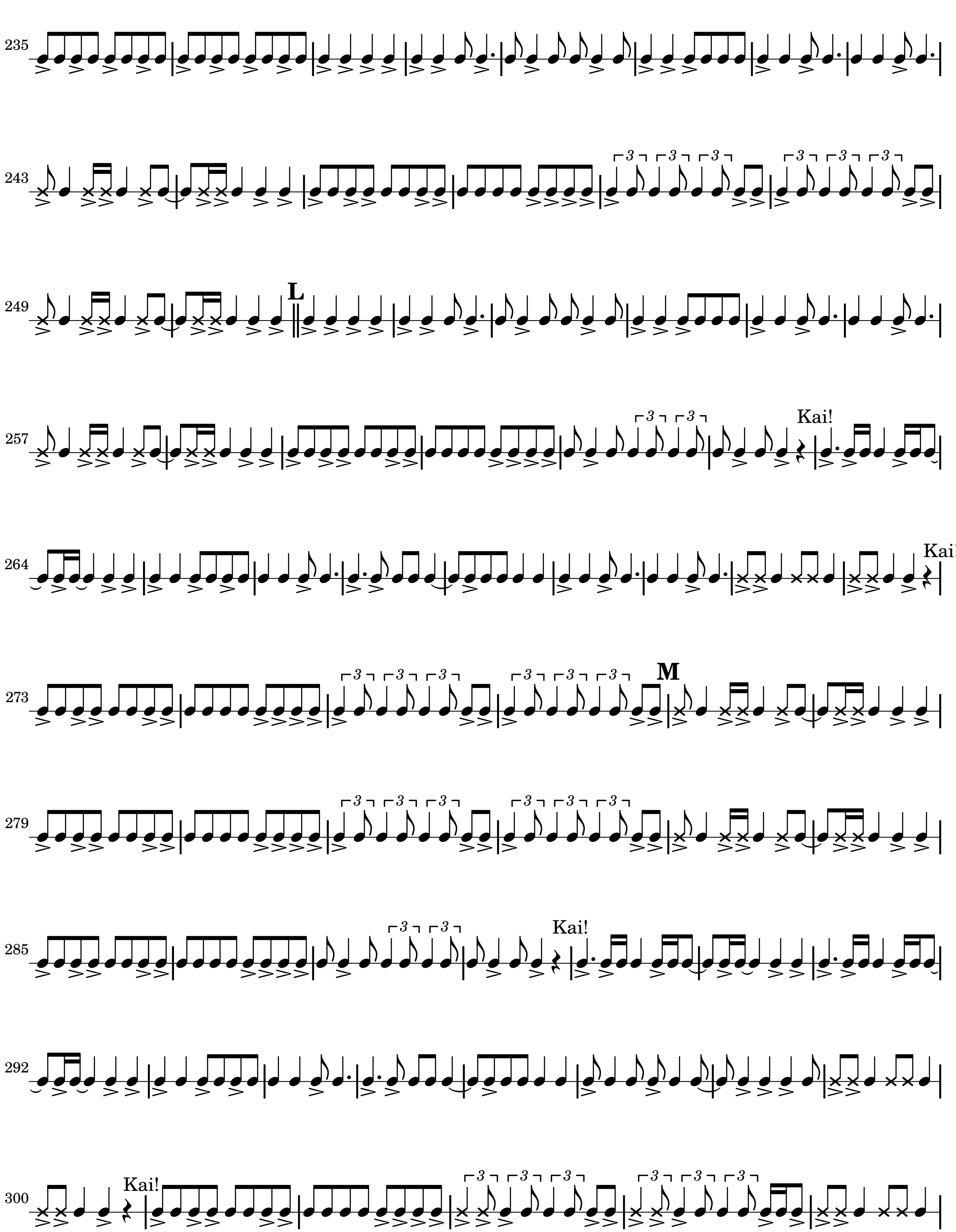

315
321
Kai!
N
326
Kai!
331
336
Kai!
342
Kai!
348
Kai!
O
354
Kai!
360
Kai!
366
Kai!
Kai!


Driving (♩ = 196)
Percussion Four
8
A
12
Kai!
18
24
30
B
36
42
48
54
60


66
C
72
78
D
84
90
E
Kai!
96
F
103
109
115
120
2


125
130
G
Kai!
135
140
147
154
H
161
166
171
176
Kai!


Kai!
J
Kai!
K
Kai!


235
243
249
L
257
Kai!
264
Kai!
273
M
279
285
Kai!
292
300
Kai!


306
Kai!
312
Kai!
318
Kai!
324
N
Kai!
Kai!
331
337
343
Kai!
351
O
358
Kai!
Kai!
365
Kai!

Appendix C
Chaotic Serialism

The following score was generated using a program written in C which computed the logistic map and outputted a score in Lilypond format. Lilypond handled the layout and respelled some of the rhythms when they crossed bar lines, though the rhythms within the bar were not corrected to normal musical standards.

The rhythmic space simply conforms to all of the subdivisions of the whole note, including the half, quarter, eighth, and sixteenth, as well as their dotted-note versions. Another application might employ all of the possible durations, measured in 32nd notes perhaps, of the whole note, though a better application may be to use a synthesizer and sequencer and use all of the possible durations within a certain time frame rather than conforming to the limitations of notation. The pitch space included 6 octaves of the piano. The dynamics and articulations each used a very limited subset, where again a directly synthesized application might explore a wider variety of possibilities.

Chaotic Serialism

Jonathan Salter


36
C
39
42
D
45
49


52
55
58
62

Appendix D
Recorded Examples

Included with this document are the following audio examples.

1. Chaotic Serialism. Created using a program written in C to generate Lilypond format musical notation, which Lilypond transformed into MIDI data. The MIDI data was then synthesized using Reason, a MIDI sequencer. See Section C.

2. Fractal Sequence. Generated using Max5. See Section 6.5 for a description of this fractal sequence.

3. Living in Chaos. Generated using Max5. This audio sample is of the evolution of the Pentomino formation, and does not include the human interaction. See Section 6.5.

4. Chaos Drumming. This is a live performance by the Philidor Percussion Group at the University of North Carolina at Greensboro on February 2, 2009. See Section 6.2 for a description of this work, and Appendix B for a score.