Heuristics for two-machine flowshop scheduling with setup times and an availability constraint

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ABSTRACT

This paper studies the two-machine flowshop scheduling problem with anticipatory setup times and an availability constraint imposed on only one of the machines where interrupted jobs can resume their operations. We present a heuristic algorithm from Wang and Cheng to minimize makespan and use simulation to determine the actual error bound.

ACKNOWLEDGMENTS

I would like to express my sincere appreciation to Dr. John Karlof for guidance on my thesis work. I also want to thank Dr. Matthew TenHuisen and Dr. Yaw Chang for serving on my thesis advisory committee and offering constructive ideas. Also, I want to thank all the faculty members in the Mathematics and Statistics Department who gave me valuable advice and help. I also want to thank my wife Lifang Du for her patience and help. This thesis is dedicated to my parents.

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1 INTRODUCTION

The subject of machine scheduling problems with availability constraints has attracted much research attention over the years. The two-machine flowshop scheduling problem with availability constraints was first studied by Lee [7]. Under the resumable assumption, he proved that the problem is NP-hard when an availability constraint is imposed on only one machine. He also developed two heuristics. The first heuristic is for solving the problem where the availability constraint is imposed on machine 1, which has a worst-case error bound $\frac{1}{2}$. The second heuristic is for solving the problem where the availability constraint is imposed on machine 2, which has a worst-case error bound $\frac{1}{3}$. Lee [8] further studied and developed a pseudo-polynomial dynamic programming algorithm and heuristics. For the resumable case, Cheng and Wang [3] developed an improved heuristic when the availability constraint is imposed on the first machine, and the heuristic has a worst-case error bound $\frac{1}{3}$. Breit [2] presented an improved heuristic for the problem with an availability constraint only on the second machine and showed that the heuristic has a worst-case error bound $\frac{1}{4}$. Cheng and Wang [4] considered a special case of the problem where the availability constraint is imposed on each machine, and the two availability constraints are consecutive. They developed a heuristic and showed that it has a worst-case error bound $\frac{2}{3}$. In addition, the two-machine flowshop scheduling problem with availability constraints has also been studied under the no-wait processing environment by Cheng and Liu [5,6]. For the general flowshop scheduling problem with availability constraints, Aggoune [1] proposed a heuristic based on a genetic algorithm and a tabu search.

Definition 1 *The objective is to minimize total completion time, called the makespan.*

Definition 2 *Error bound* $= (C_{Hi} - C^\star)/C^\star$

In all the above-mentioned flowshop scheduling models, setup times are not considered; in otherwords, setup times are assumed to be included in processing times. However, in many industrial settings, it is necessary to treat setup times as separated from processing times (for example [9,10]). The two-machine flowshop scheduling problem with anticipatory setup times, where an availability constraint is imposed on only one machine has been studied by Wang and Cheng [11]. They study the cases where the availability constraint is imposed on machines 1 and 2 and present two heuristics and show that their worst-case error bounds are no larger than $\frac{2}{3}$.

In this paper, we present the heuristic algorithm developed by Wang and Cheng for the two-machine flowshop scheduling problem with setup times where an availability constraint is imposed on machines 1 and 2. In section 2, we introduce the notation and present the parallel machine scheduling problem with the unavailable time on machine 1. In section 3, we present a heuristic for minimizing the makespan for the case where the availability constraint is imposed on machine 1. We first introduce the Yoshida and Hitomi algorithm [12] for the classical two-machine permutation flowshop scheduling problem with setup times and no unavailable time, then present a lemma, the heuristic algorithm of Wang and Cheng, and show that its worst-error bound is no larger than $\frac{2}{3}$. In section 4, we study the case where the availability constraint is imposed on machine 2, present a lemma and algorithm we also show that the worst-error bound is no larger than $\frac{2}{3}$. In section 5, we program the heuristic in JAVA and estimate the actual error bound by simulation for both cases.

For the problem under consideration, we introduce the following notation to be used throughout this paper.

- S = $J_1, ..., J_n$: a set of n jobs;.
- M_1, M_2 : machine 1 and machine 2;
- Δ_l = t_l - s_l : the length of the unavailable interval on M_l , where M_l is unavailable from time s_l to t_l, $0 \leq s_l \leq t_l$, $l = 1, 2$;
- s_i^1, s_i^2 : setup times of J_i on M_1 and M_2 , respectively, where $s_i^1 > 0, s_i^2 > 0$;
- a_i, b_i : processing times of J_i on M_1 and M_2 , respectively, where $a_i > 0, b_i > 0$;
- π: = [$J_{\pi(1)}$,...,$J_{\pi(n)}$]:a permutation schedule, where $J_{\pi(i)}$ is the ith job in π ;
- $\pi^\bigstar$: an optimal schedule;
- C_{H_x}: the makespan yielded by heuristic H_x;
- $C^\bigstar$: the optimal makespan.
- $F2/setup, r - a(M_i)/C_{\max}$: the makespan minimization problem in a two-machine flowshop with setup times and a resumable availability constraint on M_i.

Fig. 1 A schedule π for the example.

As an example, consider a problem instance of $F2/setup, r - a(M_1)/C_{\max}$ with $n = 3$. Let s_1^1 =3, $a_1 = 4$,s_2^1=5, $a_2 = 4$,s_3^1=4,$a_3 = 5$, s_1^2=2, $b_1 = 6$, s_2^2=4, $b_2 = 8$, s_3^2=2, $b_3 = 3$, $s_1 = 10$, and $t_1 = 15$. A schedule π= $[J_1, J_2, J_3]$ for the instance is shown in Fig. 1.

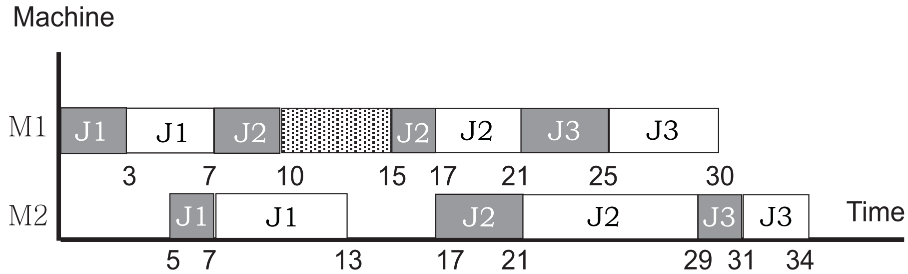

Figure 1: Example of $F2/setup, r - a(M_1)/C_{\max}$, where the 10-15 area on M_1 is the unavailable time

3 UNAVAILABLE INTERVAL ON M_1

In this section we present a heuristic for the problem $F2/setup, r - a(M_1)/C_{\max}$ and evaluate its worst-case error bound by Wang and Cheng [11]. The basic ideas of this heuristic are to combine a few simple heuristic rules and then improve the schedules by re-arranging the order of some special jobs with large setup times or large processing times on M_2 in different situations. They developed the schedules π_1, π_2, π_3, π_4, π_5 and then choose the one with the shortest makespan.

3.1 YHA algorithm(π_1)

The Yoshida and Hitomi algorithm (YHA) works in the following manner:

Divide S into two disjoint subsets A and B, where $A = \{J_i | s_i^1 + a_i - s_i^2 \leq b_i\}$ and $B = \{J_i | s_i^1 + a_i - s_i^2 > b_i\}$. Sequence the jobs in A in nondecreasing order of $s_i^1 + a_i - s_1^2$ and the jobs in B in nonincreasing order of b_i. Arrange the ordered subset A first, followed by the ordered subset B.

Let s_1^1 =9, $a_1 = 3$, s_2^1=2, $a_2 = 4$, s_2^1=3,$a_3 = 2$, s_1^2=7, $b_1 = 4$, s_2^2=1, $b_2 = 7$, s_3^2=2, $b_3 = 3$, $s_1 = 20$, and $t_1 = 25$.

Then, $J_2, J_3 \in$A, and $J_1 \in$B. Because $s_2^1 + a_2 - s_2^2 > s_3^1 + a_3 - s_3^2$,then the order in set A will be $\{J_3, J_2\}$ (nondecreasing order). The final order will be π_1 $\{J_3, J_2, J_1\}$. See Figure 2(a).

Job number	Set A	Set B
1	None	$s_1^1 + a_1 - s_1^2$=9+3-7=5>4
2	$s_1^2 + a_2 - s_2^2$=2+4-1=5<=7	None
3	$s_3^1 + a_3 - s_3^2$=3+2-2=3<=3	None

Table 1: Values considered in π_1

3.2 Decreasing ratio(π_2)

Next we sequence the jobs in nonincreasing order of $(s_i^2 + b_i)/(s_i^2 + a_i)$.

Job number	$(s_i^2 + b_i)/(s_i^1 + a_i)$
1	$(s_1^2 + b_1)/(s_1^1 + a_1) = 11/12$
2	$(s_2^2 + b_2)/(s_2^2 + a_2) = 8/6$
3	$(s_3^2 + b_3)/(s_3^1 + a_3) = 5/5$

Table 2: Values considered in π_2

Then we get $(s_2^2 + b_2)/(s_2^2 + a_2) > (s_3^2 + b_3)/(s_3^1 + a_3) > (s_1^2 + b_1)/(s_1^1 + a_1)$. So the order will be π_2 $\{J_2, J_3, J_1\}$. See Figure 2(b).

3.3 Largest job p, q on machine 2 (π_3)

Next we need find jobs J_p and J_q such that

$$s_p^2 + b_p \geq s_q^2 + b_q \geq \max\{s_i^2 + b_i | J_i \in \mathrm{S} \setminus \{J_p, J_q\}\}.$$

Job number	$s_i^2 + b_i$
1	$s_1^2 + b_2$=7+4=11
2	$s_2^2 + b_2$=1+7=8
3	$s_3^2 + b_3$=2+3=5

Table 3: Values considered in π_3

Let $p = 1$ and $q = 2$, For π_3 put job J_p first and keep other $n - 1$ jobs in the same order as π_2. Then the order will be π_3 $\{J_1, J_2, J_3\}$. See Figure 2(c).

3.4 Random sequence p(π_4, π_5)

Test if $(s_p^1 + a_p) + (s_q^1 + a_q) \leq s_1$ if not then no π_4, π_5, otherwise make two sequences

π_4: Place J_p and J_q as the first two jobs. the remaining $n-2$ jobs are sequenced randomly. π_4 $\{J_1, J_2, J_3\}$. See Figure 2(c).

π_5: Place J_q and J_p as the first two jobs. the remaining $n-2$ jobs are sequenced randomly. π_5 $\{J_2, J_1, J_3\}$. See Figure 2(d).

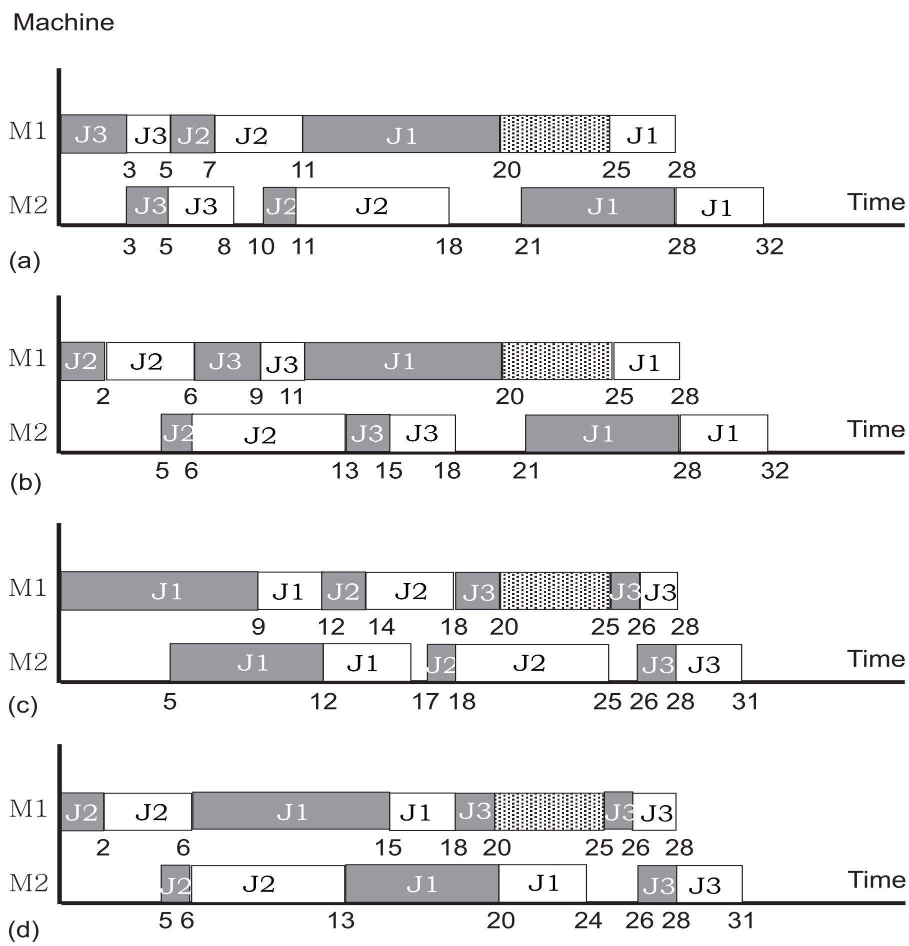

Figure 2: (a)Solution of order π_1; (b)Solution of order π_2; (c)Solution of order π_3 and π_4; (d)Solution of order π_5

3.5 Heuristic H1:

(1) Find jobs J_p and J_q such that

$$s_p^2 + b_p \geq s_q^2 + b_q \geq \max\{s_i^2 + b_i | J_i \in \mathrm{S} \setminus \{J_p, J_q\}\}.$$

(2) Sequence the jobs by YHA. Let the corresponding schedule be π_1 and the corresponding makespan be $C_{\max}(\pi_1)$.

(3) Sequence the jobs in nonincreasing order of $(s_i^2 + b_i)/(s_i^2 + a_i)$. Let the corresponding schedule be π_2 and the corresponding makespan be $C_{\max}(\pi_2)$.

(4) Place job J_p in the first position and keep the other $n-1$ jobs in the same positions as those in Step (3). Let the corresponding schedule be π_3 and the corresponding makespan be $C_{\max}(\pi_3)$.

(5) If $(s_p^1 + a_p) + (s_q^1 + a_q) \leq s_1$, then sequence jobs J_p, J_q as the first two jobs. The remaining $n-2$ jobs are sequenced randomly. Let the corresponding schedule be π_4 and the corresponding makespan be $C_{\max}(\pi_4)$.

(6) If $(s_p^1 + a_p) + (s_q^1 + a_q) \leq s_1$, then sequence jobs J_q, J_p as the first two jobs. The remaining $n-2$ jobs are sequenced randomly. Let the corresponding schedule be π_5 and the corresponding makespan be $C_{\max}(\pi_5)$.

(7) Select the schedule with the minimum makespan from the above five schedules. Let $C_{H1} = \min\{C_{\max}(\pi_1), C_{\max}(\pi_2), C_{\max}(\pi_3), C_{\max}(\pi_4), C_{\max}(\pi_5)\}$.

In the following, we analyze the performance bound of heuristic H1.

Definition 3 *Let π be a schedule for the problem F2/setup, $r - a(M_1)/C_{\max}$. We define the critical job $J_{\pi(k)}$ as the last job such that its starting time on M_2 is equal to its finishing time on M_1.*

Lemma 1 *For schedule π_2 defined in Step (3) of heuristic H1, we assume that the completion time of the critical job $J_{\pi_2(k)}$ on M_1 is t, and let $J_{\pi(v)}$ be the last job that finishes no later than time t on M_1 in a schedule π. The following inequality holds:*

$$C_{\max}(\pi_2) \le C_{\max}(\pi) + b_{\pi_2(k)} + s^2_{\pi(v+1)}.$$

Proof. For schedule π_2, its makespan is

$$C_{\max}(\pi_2) = t + b_{\pi_2(k)} + \sum_{j=k+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}). \tag{1}$$

Since on machine 2 there will be no idle time after J_k , because of the definition of the critical job.

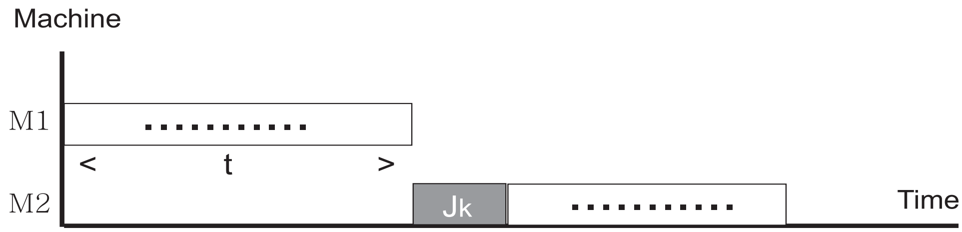

Figure 3: Illustrations of (1),$J_k = b_{\pi_2(k)}$

Under the assumption of lemma 1, $J_{\pi(v)}$ is the last job that finishes no later than time t on M_1 in a schedule π. We have

$$\sum_{j=1}^{v}(s^1_{\pi(j)} + a_{\pi(j)}) \le \sum_{j=1}^{k}(s^1_{\pi_2(j)} + a_{\pi_2(j)}),$$

and because $\sum_{j=1}^{n}(s^1_{\pi(j)} + a_{\pi(j)}) = \sum_{j=1}^{n}(s^1_{\pi_2(j)} + a_{\pi_2(j)})$, obviously

$$\sum_{j=v+1}^{n} (s^1_{\pi(j)} + a_{\pi(j)}) \ge \sum_{j=k+1}^{n} (s^1_{\pi_2(j)} + a_{\pi_2(j)}). \tag{2}$$

Since all the jobs are sequenced in nonincreasing order of $(s^2_{\pi_2(j)} + b_{\pi_2(j)})/(s^1_{\pi_2(j)} + b_{\pi_2(j)})$ in π_2, and because after critical job k on M_1, there is no idle time, we have

$$\sum_{j=k+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}) > \sum_{j=k+1}^{n} (s^1_{\pi_2(j)} + a_{\pi_2(j)}). \tag{3}$$

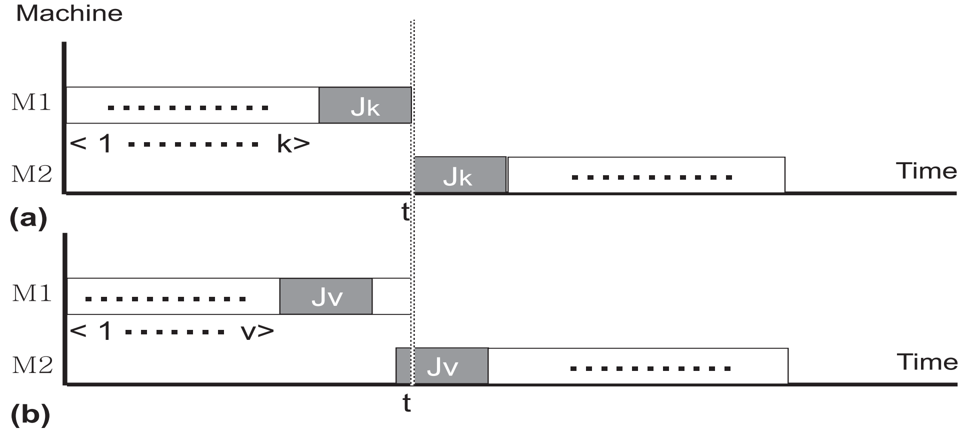

Figure 4: Illustrations of (2), (a)Order π_2; (b)Order π;

From (2) and (3)

$$\sum_{j=v+1}^{n} (s^2_{\pi(j)} + b_{\pi(j)}) \geq \sum_{j=k+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}). \tag{4}$$

For schedule π, we have

$$C_{\max}(\pi) \geq t + \sum_{j=v+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}) - s^2_{\pi(v+1)}. \tag{5}$$

Therefore, from (1), (4) and (5), we have

$$\begin{aligned} C_{\max}(\pi_2) &= t + b_{\pi_2(k)} + \sum_{j=k+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}) \\ &\leq t + b_{\pi_2(k)} + \sum_{j=v+1}^{n} (s^1_{\pi(j)} + a_{\pi(j)}) \\ &\leq C_{\max}(\pi) + b_{\pi_2(k)} + s^2_{\pi(v+1)}. \end{aligned}$$

Theorem 1 *For the problem $F2/setup, r - a(M_l)/C_{\max}$, $(C_{H1} - C^\star)/C^\star \leq 2/3$.*

Proof. If $\sum_{i=1}^{n}(s^1_i + a_i) \leq s_1$, it is obvious that $C_{\max}(\pi_1) = C^\star$ from Yoshida and Hitomi algorithm(YHA)[11]. So we assume $\sum_{i=1}^{n}(s^1_i + a_i) > s_1$.

Notice that since all the jobs are resumable for the problem $F2/setup, r - a(M_1)/C_{\max}$, then π_1 is the best schedule without an unavailable time then we have $C_{\max}(\pi_1) \leq C^\star + \Delta_1$. See figure 5.

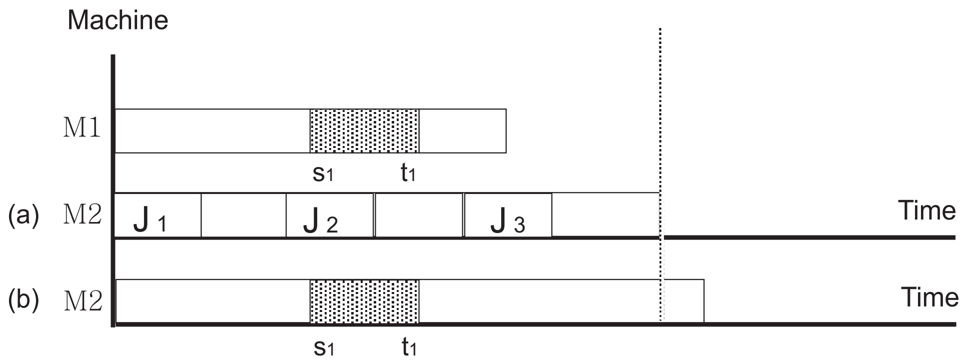

Figure 5: a is for $\pi^\star$ which is no idle time at all, b is for π_1, $\Delta_1 = t_1 - s_1$.

If $\Delta_1 \leq 2C^\star/3$, then we are done. So, in the following, we focus on the situation where $\Delta_1 > 2C^\star/3$.

Because $\Delta_1 > 2C^\star/3$ and $\sum_{i=1}^{n}(s_i^1 + a_i) + \Delta_1 < C^\star$, we have $\sum_{i=1}^{n}(s_i^1 + a_i) < C^\star/3$. Let $S' = \{J_i | s_i^2 + b_i > C^\star/3, i = 1, 2, ..., n\}$. It is obvious $\mid S' \mid \leq 2$.

Case 1:$\mid S' \mid = 0$

For an optimal schedule $\pi^\star$, according to lemma 1,we have $C_{\max}(\pi_2) \leq C^\star + b_{\pi_2(k)} + s^2_{\pi^\star(v+1)} < 5C^\star/3$.

Case 2:$\mid S' \mid = 1$

In this case, $S' = \{J_p\}$. If $s_p^2 \leq C^\star/3$ and $b_p \leq C^\star/3$,then $b_{\pi_2(k)} \leq b_p \leq C^\star/3$ and $s^2_{\pi^\star(v+1)} \leq s_p^2 \leq C^\star/3$, then from lemma 1 $C_{\max}(\pi_2) \leq C^\star + C^\star/3 + C^\star/3 \leq 5C^\star/3$, we are done. Otherwise at least one of $s_p^2 \geq C^\star/3$ or $b_p \geq C^\star/3$ will be exist, then we consider schedule π_3 of Heuristic H1.

For subcase $s_p^1 + a_p \leq s_1$, suppose that the critical job does not exist in π_3, then there is no idle time on machine 2 that implies $C_{\max}(\pi_3) = \sum_{i=1}^{n}(s^2_{\pi_3(i)} + b_{\pi_3(i)}) = C^\star$.

Otherwise, we denote the critical job as $J_{\pi_3(u)}$. If $\sum_{i=1}^{u}(s^1_{\pi_3(i)} + a_{\pi_3(i)}) \leq s_1$, see figure 6. then

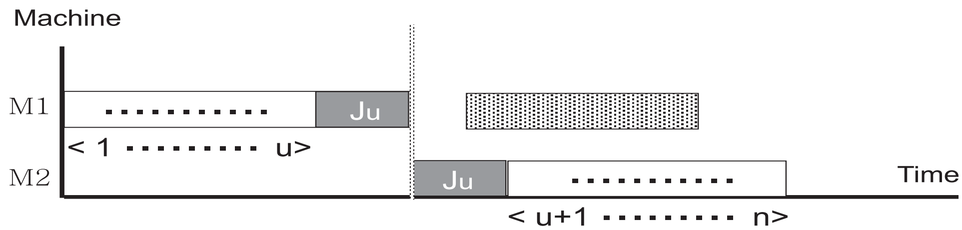

Figure 6: Illustrations of equation [6] of π_3; J_u on M_2 equal to $b_{\pi_3(u)}$.

$$
\begin{aligned}
C_{\max}(\pi_3) &= \sum_{i=1}^{u}(s^1_{\pi_3(i)} + a_{\pi_3(i)}) + \sum_{i=u+1}^{n}(s^2_{\pi_3(i)} + b_{\pi_3(i)}) + b_{\pi_3(u)} \\
&\leq C^\bigstar/3 + C^\bigstar = 4C^\bigstar/3 \qquad (6)
\end{aligned}
$$

Otherwise, let $\sum_{i=1}^{u}(s^1_{\pi_3(i)} + a_{\pi_3(i)}) > s_1$, J_p is the first job in π_3 and $s^1_p + a_p \leq s_1$, then $u > 1$. see figure 7. Thus,we have

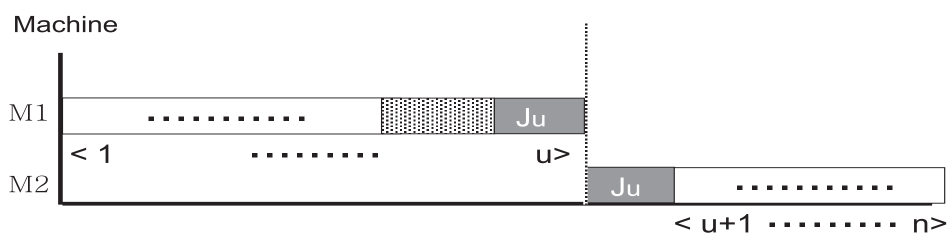

Figure 7: Illustrations of equation [7] of π_3; J_u on M_2 equal to $b_{\pi_3(u)}$.

$$
\begin{aligned}
C_{\max}(\pi_3) &= \sum_{i=1}^{u}(s^1_{\pi_3(i)} + a_{\pi_3(i)}) + \Delta_1 + \sum_{i=u+1}^{n}(s^2_{\pi_3(i)} + b_{\pi_3(i)}) + b_{\pi_3(u)} \\
&\leq C^\bigstar + 2C^\bigstar/3 = 5C^\bigstar/3 \qquad (7)
\end{aligned}
$$

For subcase $s^1_p + a_p > s_1$, we have $s^1_p + a_p + \Delta_1 + b_p \leq C^\bigstar$. If the critical job does not exist or job J_p is the critical job, see figure 8. then we have

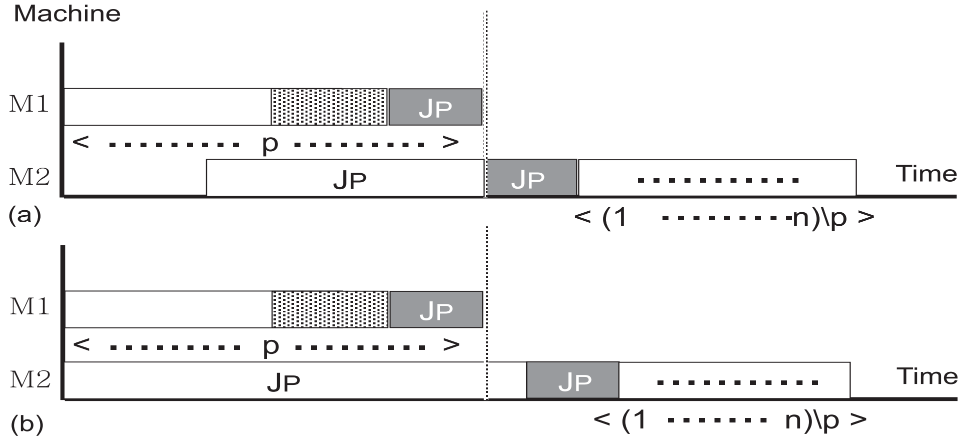

Figure 8: Illustrations of equation [8] of π_3; compare $\max\{s_p^1 + a_p + \Delta_1, s_p^2\}$. $s_p^1 + a_p + \Delta_1$ in (a), s_p^2 in (b).

$$
\begin{aligned}
C_{\max}(\pi_3) &= \max\{s_p^1 + a_p + \Delta_1, s_p^2\} + b_p + \sum_{J_i \in S \setminus J_p} (s^2_{\pi_3(i)} + b_{\pi_3(i)}) \\
&\leq C^\star + 2C^\star/3 = 5C^\star/3 \qquad (8)
\end{aligned}
$$

Otherwise, for the critical job $J_{\pi_3(u)}$, $u > 1$, see figure 9, we have

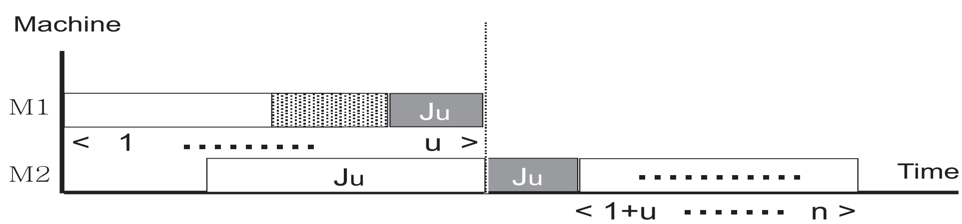

Figure 9: Illustrations of equation [9] of π_3; J_u on machine 2 equal to $b_{\pi_3(u)}$.

$$
\begin{aligned}
C_{\max}(\pi_3) &= (\sum_{i=1}^{u} (s^1_{\pi_3(i)} + a_{\pi_3(i)}) + \Delta_1) + b_{\pi_3(u)} + \sum_{i=u+1}^{n} (s^2_{\pi_3(i)} + b_{\pi_3(i)}) \\
&\leq C^\star + 2C^\star/3 = 5C^\star/3 \qquad (9)
\end{aligned}
$$

Case 3:$| S' |= 2$

Similar to case 2 to check that schedule π_2 or π_3 may yield a solution with an error bound of no more than $2C^\star/3$. In the following, we further prove that the error bound of schedule π_4 obtained in Step (5) is no more than $C^\star/3$ for this case.

For schedule π_4, if no critical job exists, see figure 9, then this is obviously

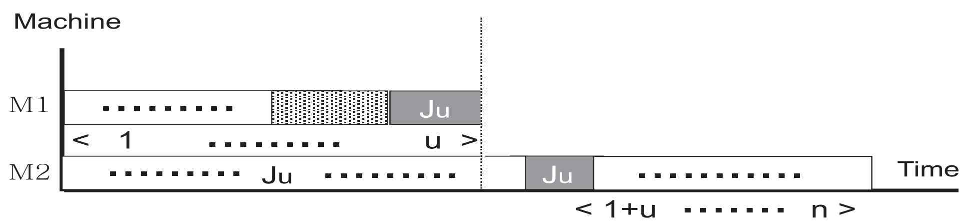

Figure 10: Illustrations of equation [10] of π_4; this means no idle time on M_2.

$$\begin{aligned} C_{\max}(\pi_4) &= \sum_{i=1}^{n}(s^2_{\pi_4(i)} + b_{\pi_4(i)}) \\ &= C^\star. \end{aligned} \tag{10}$$

Otherwise, for the critical job $J_{\pi_4(u)}$, if $u > 2$, See figure 11, we have from figure 10,

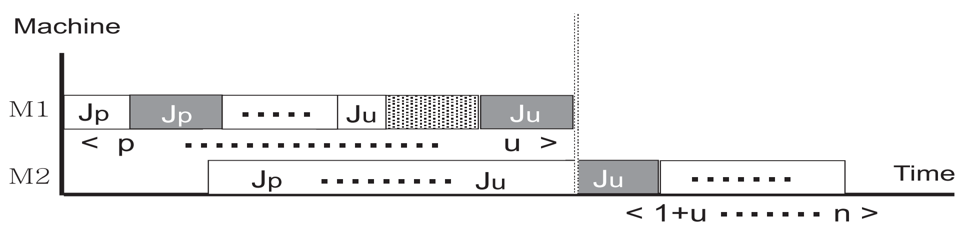

Figure 11: Illustrations of equation [11] of π_4.

because $| S' |= 2$ and $u > 2$ which means $\sum_{i=1}^{u}(s^1_{\pi_4(i)} + a_{\pi_4(i)}) + \Delta_1 < C^\star$.

$$\begin{aligned} C_{\max}(\pi_4) &= \sum_{i=1}^{u}(s^1_{\pi_4(i)} + a_{\pi_4(i)}) + \Delta_1 + \sum_{i=u+1}^{n}(s^2_{\pi_4(i)} + b_{\pi_4(i)}) + b_{\pi_4(u)} \\ &\leq C^\star + C^\star/3 = 4C^\star/3 \end{aligned} \tag{11}$$

If $u = 2$, then we obtain a contradiction.

$\sum_{i=1}^{n}(s_i^1 + a_i) < C^\star - \Delta_1 < C^\star - 2C^\star/3 = C^\star/3$

$C^\star/3 > \sum_{i=1}^{n}(s_i^1 + a_i) > (s_p^1 + a_p) + (s_q^1 + a_q) \geq \min\{s_p^2 + b_p, s_q^2 + b_q\} > C^\star/3.$

So obviously u must be equal to 1. Thus, see figure 12, we have

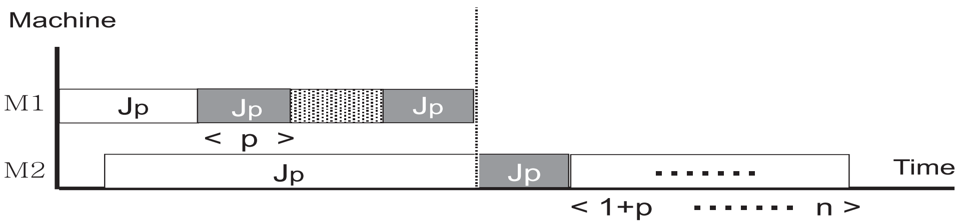

Figure 12: Illustrations of equation of π_4 [12] and π_5 [13].

$$\begin{aligned} C_{\max}(\pi_4) &= (s_p^1 + a_p) + b_{\pi_4(p)} + \sum_{i=2}^{n}(s^2_{\pi_4(i)} + b_{\pi_4(i)}) \\ &\leq C^\star/3 + C^\star = 4C^\star/3 \end{aligned} \tag{12}$$

Similarly for π_5, see figure 11, we need to change p to q.

$$\begin{aligned} C_{\max}(\pi_5) &= (s_q^1 + a_q) + b_{\pi_5(1)} + \sum_{i=2}^{n}(s^2_{\pi_5(i)} + b_{\pi_5(i)}) \\ &\leq C^\star/3 + C^\star = 4C^\star/3 \end{aligned} \tag{13}$$

From the proof of theorem 1, we see that Steps (1)...(5) of Heuristic H1 can produce a solution with an error bound of no more than $2C^\star/3$, and schedule π_4 in step (5), π_5 in step (6) can produce a solution with an error bound of no more than $C^\star/3$ in some special situations.

Although we do not know whether the bound is tight or not, the following instance shows that the worst-case error bound of H1 is no less than 1/2 . Consider an instance with $s_1^1 = h$, $a_1 = h$, $s_1^2 = 3h$, $b_1 = 7$, $s_2^1 = 3$, $a_2 = 4$, $s_2^2 = 6$, $b_2 = 3h$, $s_3^1 = m$, $a3 = m$, $s_3^2 = 1$, $b_3 = 1$, $s_1 = 8$, and $t_1 = 4h + 8$, where $h \gg 1$ and

$0 < m < 7/(3h+6)$. Applying heuristic H1, we obtain $\pi_1 = \pi_3 = [J_1, J_3, J_2]$ with $C_{\max}(\pi_1) = C_{\max}(\pi_3) = 9h+15$ (see figure 13(a)), and $\pi_2 = [J_3, J_2, J_1]$ with $C_{\max}(\pi_2) = 10h+2m+14$ (see figure 13(b)). Since $(s_p^1 + a_p) + (s_q^1 + a_q) = 2h+7 > s_1$, we need not consider Step (5) of H1. Thus, $C_{H1} = 9h+15$. It is easy to check that $\pi^\star = [J_2, J_1, J_3]$ with $C^\star = 6h+16$ (see figure 13(c)). Hence, we see that $(C_{H1} - C^\star)/C^\star$ approaches 1/2 as h approaches infinity.

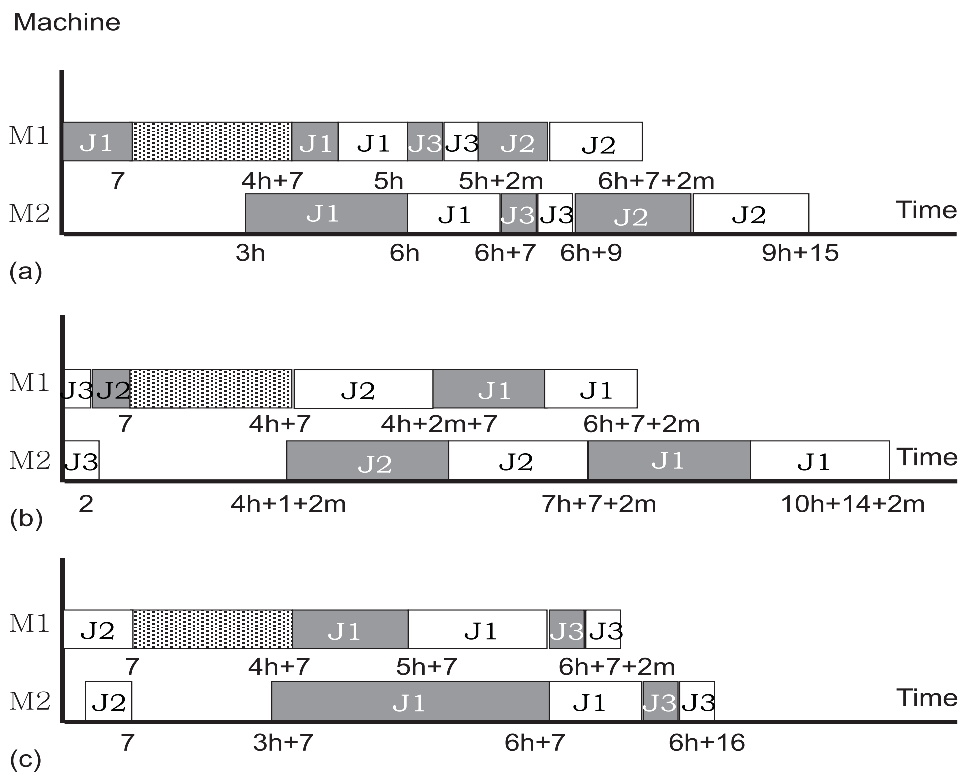

Figure 13: (a)Solution of order π_1, π_3; (b)Solution of order π_2; (c)Solution of order $\pi^\star$.

In this section we present a heuristic for the problem $F2/setup, r-a(M_2)/C_{\max}$ and evaluate its worst-case error bound by Wang and Cheng [12].

4.1 YHA algorithm (π_1)

Equvalent to heuristic $H1$'s π_1. For example let s_1^1 =2, $a_1 = 3$, s_2^1=4, $a_2 = 2$, s_3^1=8,$a_3 = 3$, s_1^2=4, $b_1 = 2$, s_2^2=3, $b_2 = 5$, s_3^2=6, $b_3 = 4$, $s_2 = 15$, and $t_2 = 20$.

Job number	Set A	Set B
1	$s_1^1 + a_1 - s_1^2$=2+3-4=1<2	None
2	$s_1^2 + a_2 - s_2^2$=4+2-3=3<5	None
3	None	$s_1^3 + a_3 - s_2^3$=8+3-6=5>4

Table 4: Values considerer in π_1

Then, J_1, J_2 ∈A, and J_3 ∈B, because $s_1^2 + a_2 - s_2^2 > s_1^1 + a_1 - s_2^1$, then the order in set A will be $\{J_1, J_2\}$ (nondecreasing order) followed by the job in set B. Finally the order will be $\{J_1, J_2, J_3\}$, this is π_1. See Figure 14(a).

4.2 Decreasing ratio (π_2)

Similar to heuristic H1's π_2. Sequence the jobs in nonincreasing order of $(s_i^2 + b_i)/(s_i^1 + a_i)$.

Job number	$(s_i^2 + b_i)/(s_i^1 + a_i)$
1	$(s_1^2 + b_1)/(s_1^1 + a_1) = 6/5$
2	$(s_2^2 + b_2)/(s_2^2 + a_2) = 8/6$
3	$(s_3^2 + b_3)/(s_3^1 + a_3) = 10/11$

Table 5: Values considerer in π_2

Then we get $(s_2^2 + b_2)/(s_1^2 + a_2) > (s_1^2 + b_1)/(s_1^1 + a_1) > (s_3^2 + b_3)/(s_3^1 + a_3)$.Finally the order will be $\{J_2, J_1, J_3\}$, this is π_2. See Figure 14(b).

4.3 Largest job q on machine 1 (π_3)

Next we need find Find job J_q such that

$$s_q^1 + a_q \geq \max\{s_i^1 + a_i | J_i \in \mathrm{S} \setminus \{J_q\}.$$

Job number	$s_i^1 + a_i$
1	$s_1^1 + a_1$=2+3=5
2	$s_2^1 + a_2$=4+2=6
3	$s_3^1 + a_3$=8+3=11

Table 6: Values considerer in π_3

Then we find $q = 3$, and we know the order of π_1, we just need to move job J_p in the last position and keep the other $n - 1$ jobs in the same positions as those in π_1,then the order will be $\{J_3, J_1, J_2\}$, this is π_3. See Figure 14(c).

4.4 Largest job p on machine 2 (π_4)

Next we need find Find job J_p such that

$$s_P^2 + b_p \geq \max\{s_i^2 + b_i | J_i \in \mathrm{S} \setminus \{J_p\}.$$

Job number	$s_i^2 + b_i$
1	$s_1^2 + b_1$=4+2=6
2	$s_2^2 + b_2$=3+5=8
3	$s_3^2 + b_3$=6+4=10

Table 7: Values considerer in π_4

Then we find $p = 3$, and we know the order of π_2, just need to move job J_p in the first position and keep the other $n - 1$ jobs in the same positions as those in π_2,then the order will be $\{J_3, J_2, J_1\}$, this is π_4. See Figure 14(d).

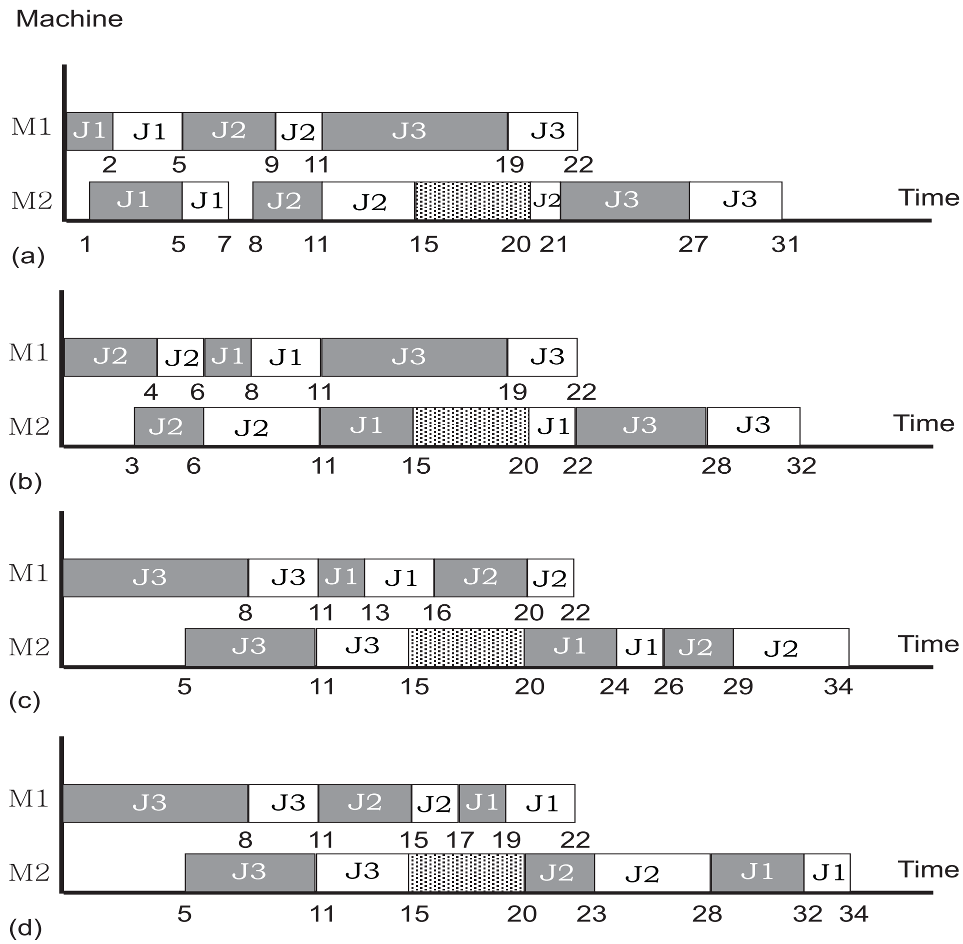

Figure 14: (a)Solution of order π_1; (b)Solution of order π_2; (c)Solution of order π_3; (d)Solution of order π_4

4.5 Heuristic H2:

(1) Find jobs J_p and J_q such that

$$s_P^2 + b_p \geq \max\{s_i^2 + b_i | J_i \in S \backslash \{J_p\}$$

and

$$s_q^1 + a_q \geq \max\{s_i^1 + a_i | J_i \in S \backslash \{J_q\}.$$

(2) Sequence the jobs by YHA. Let the corresponding schedule be π_1 and the corresponding makespan be $C_{\max}(\pi_1)$.

(3) Sequence the jobs in nonincreasing order of $(s_i^2 + b_i)/(s_i^2 + a_i)$. Let the corresponding schedule be π_2 and the corresponding makespan be $C_{\max}(\pi_2)$.

(4) Sequence job J_q in the last position and sequence the remaining $n-1$ jobs by YHA, Let the corresponding schedule be π_3 and the corresponding makespan be $C_{\max}(\pi_3)$.

(5) Sequence job J_P in the first position and sequence the remaining $n-1$ jobs in the same positions as those in Step (3), Let the corresponding schedule be π_4 and the corresponding makespan be $C_{\max}(\pi_4)$.

(6) Select the schedule with the minimum makespan from the above four schedules. Let $C_{H2} = min\{C_{\max}(\pi_1), C_{\max}(\pi_2), C_{\max}(\pi_3), C_{\max}(\pi_4)\}$.

For the problem $F2/setup, r - a(M_2)/C_{\max}$, since an unavailable period exists on M_2, we assume that all the jobs must be processed on M_1 and M_2 as early as possible, and, for a given π, define again the critical job $J_{\pi(k)}$ as the last job in π such that its starting time on M_2 is equal to its finishing time on M_1 or the job in π before which the last idle time on M_2 occurs.

Lemma 2 *For schedule π_2 defined in Step (3) of Heuristic H2, we assume that the completion time of the critical job $J_{\pi_2(k)}$ on M_1 is t, and let π be a given scedule.*

(i) *If $t \leq s_2$ or $t > t_2$,let $J_{\pi(v)}$ be the last job that finishes no later than time t on M_1 in π, then $C_{\max}(\pi_2) \leq C_{\max}(\pi) + b_{\pi_2(k)} + s^2_{\pi(v+1)}$.*

(ii) *If $s_2 < t \leq t_2$, let $J_{\pi_2(h)}$ be the job that finishes just before time s_2 on M_1 in π_2, and $J_{\pi(u)}$ the last job that finishes no later than $J_{\pi_2(h)}$ on M_1 in π, then $C_{\max}(\pi_2) \leq C_{\max}(\pi) + (s^1_{\pi_2(h+1)} + a_{\pi_2(h+1)}) + (s^1_{\pi(u+1)} + a_{\pi(u+1)})$*

Proof. (i) Similar to the proof of Lemma 1.

(ii) Let I_{π_2} be the total idle time on M_2 in π_2. Under the assumption that $J_{\pi_2(k)}$ finishes just before time s_2 on M_1 in π_2, we have $I_{\pi_2} \leq s_2 - \sum_{j=1}^{h}(s^2_{\pi_2(j)} + b_{\pi_2(j)})$. See figure 15.

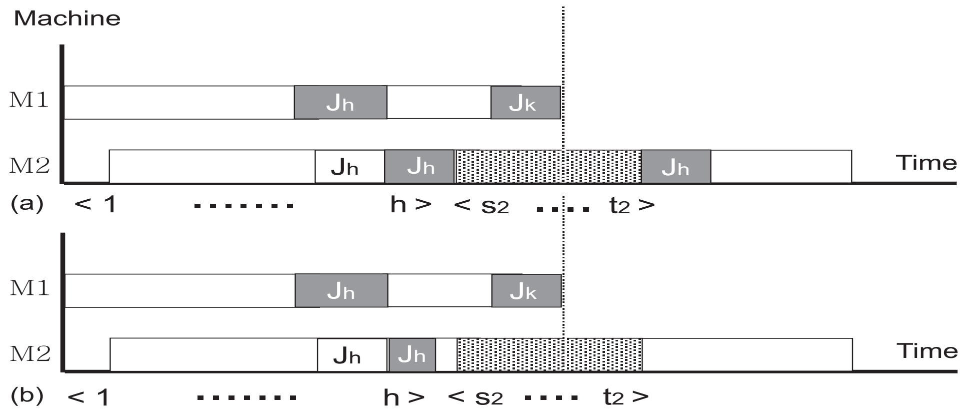

Figure 15: (a)J_h finished after s_2 on M_2; (a)J_h finished before s_2 on M_2;

Let I_π be the total idle time on M_2 in π. So clearly,

$$\begin{aligned} I_\pi &\geq s_2 - \sum_{j=1}^{u}(s^2_{\pi(j)} + b_{\pi(j)}) \\ &\geq s_2 - \sum_{j=1}^{u}(s^2_{\pi(j)} + b_{\pi(j)}) - (s^1_{\pi_2(h+1)} + a_{\pi_2(h+1)}) - (s^1_{\pi(u+1)} + a_{\pi(u+1)}) \end{aligned}$$

Notice that since $\sum_{j=1}^{u}(s^1_{\pi(j)} + a_{\pi(j)}) \leq \sum_{j=1}^{h}(s^1_{\pi_2(j)} + a_{\pi_2(j)})$ and all the jobs are sequenced in nonincreasing order of $(s^2_i + b_i)/(s^2_i + a_i)$ in π_2, it is not difficult to prove that $\sum_{j=h+1}^{n}(s^2_{\pi_2(j)} + b_{\pi_2(j)}) \leq \sum_{j=u+1}^{n}(s^2_{\pi(j)} + b_{\pi(j)})$. We know that $C_{\max}(\pi_2) = \sum_{j=1}^{n}(s^2_{\pi_2(j)} + b_{\pi_2(j)}) + \Delta_2 + I_{\pi_2}$ and $C_{\max}(\pi) = \sum_{j=1}^{n}(s^2_{\pi(j)} + b_{\pi(j)}) + \Delta_2 + I_\pi$. Hence,

$$
\begin{aligned}
C_{\max}(\pi_2) &\leq \sum_{j=h+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}) + \Delta_2 \\
&\leq \sum_{j=u+1}^{n} (s^2_{\pi_2(j)} + b_{\pi_2(j)}) + \Delta_2 \\
&= C_{\max}(\pi) + (s_2 - \sum_{j=1}^{u}(s^2_{\pi(j)} + b_{\pi(j)}) - I_\pi) \\
&\leq C_{\max}(\pi) + (s^1_{\pi_2(h+1)} + a_{\pi_2(h+1)}) + (s^1_{\pi(u+1)} + a_{\pi(u+1)})
\end{aligned}
$$

This completes the proof. □

The following theorem establishes the worst-case error bound of Heuristic H2 for the resumable case.

Theorem 2 *For the problem $F2/setup, r-a(M_2)/C_{\max}, (C_{H2} - C^\star)/C^\star \leq 2/3$.*

Proof. We know that YHA can produce an optimal solution for $F2/permu, setup/C_{\max}$. Since when $t_2 = 0$, $F2/setup, r{-}a(M_2)/C_{\max}$ is equivalent to $F2/permu, setup/C_{\max}$, it is obvious that $C_{\max}(\pi_1) - C^\star \leq t_2$. If $t_2 \leq 2C^\star/3$, then we are done. So, in the following, we focus on the case where $t_2 > 2C^\star/3$.

Let $S' = \{J_i | s^1_i + a_i > C^\star/3, i = 1, 2, ..., n\}$ and $S'' = \{J_i | s^2_i + b_i > C^\star/3, i = 1, 2, ..., n\}$. We can easily show that $\mid S' \mid \leq 2$ and $\mid S'' \mid \leq 2$ from the lower bound $\max\{\sum_{i=1}^{n}(s^1_i + a_i) + \sum_{i=1}^{n}(s^2_i + b_i)\} \leq C^\star$. When $\mid S' \mid = 0$ and $\mid S'' \mid = 0$, from (i) and (ii) of Lemma 2, we have $C_{\max}(\pi_2) \leq 5C^\star/3$. Hence, in the remainder of proof, we only need to consider the following two situations.

Case 1: $\mid S'' \mid = 0$ and $\mid S' \mid > 0$

In this case, we consider schedule π_3. If no critical job exists in π_3, this means no idle time on M_2, then $C_{\max}(\pi_3) = \sum_{i=1}^{n}(s^2_{\pi_3(i)} + b_{\pi_3(i)}) + \Delta_2 = C^\star$. Next, we assume that there exists a critical job in π_3. Let J_q be the critical job, see figure 16. then

$C_{\max}(\pi_3) \leq \max\{\sum_{i=1}^{n}(s^1_{\pi_3(i)} + a_{\pi_3(i)}), t_2\} + b_q \leq C^\star + C^\star/3 = 4C^\star/3.$

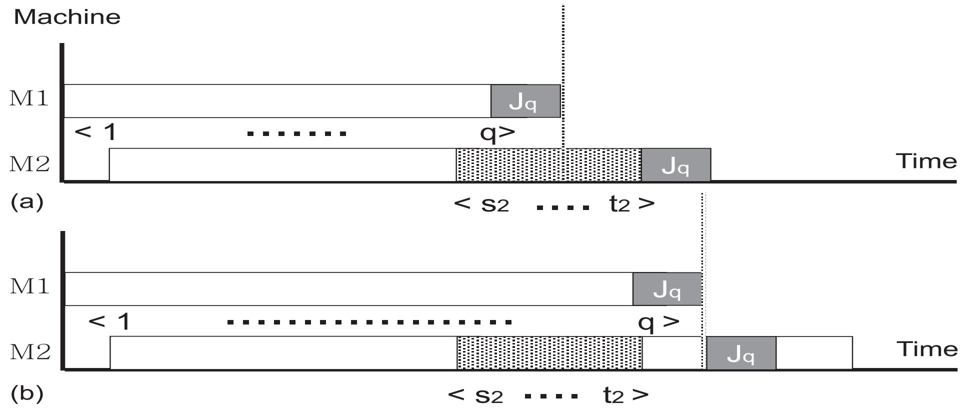

Figure 16: (a)J_q finished brfore t_2 on M_1; (a)J_q finished after t_2 on M_1;

Otherwise, let $J_{\pi_3(k)}$ $(k < n)$ is the critical job, see figure 17. then we have

Figure 17: $\Delta_2 = t_2 - s_2$

$$
\begin{aligned}
C_{\max}(\pi_3) &\leq \sum_{i=1}^{k}(s^1_{\pi_3(i)} + a_{\pi_3(j)}) + (\Delta_2 + b_{\pi_3(k)} + \sum_{i=k+1}^{n}(s^2_{\pi_3(i)} + b_{\pi_3(i)})) \\
&\leq C^\star + 2C^\star/3 = 5C^\star/3.
\end{aligned}
$$

Case 2: $\mid S'' \mid \geq 1$

We check schedule π_4 obtained in Step (5) of Heuristic H2. If no critical job exists in π_4, then $C_{\max}(\pi_4) = \sum_{i=1}^{n}(s^2_{\pi_4(i)} + b_{\pi_4(i)}) + \Delta_2 = C^\star$. In the following, we assume that there exists a critical job in π_4. Since $\mid S'' \mid \geq 1$, we assume that $s^2_p + b_p > C^\star/3$ for J_p, see figure 18. If

$\sum_{i=1}^{n}(s^1_i + a_i) \geq \max\{\max\{s^1_p + a_p, s^2_p\} - \max\{s^1_p + a_p - s_2, 0\}, s^2_P\} + b_p + \alpha\Delta_2,$

where $\alpha = 1$ if $s_2 < \max\{s_p^1 + a_p, s_p^2\} + b_p$; otherwise, $\alpha = 0$. See figure 18. Then, we have

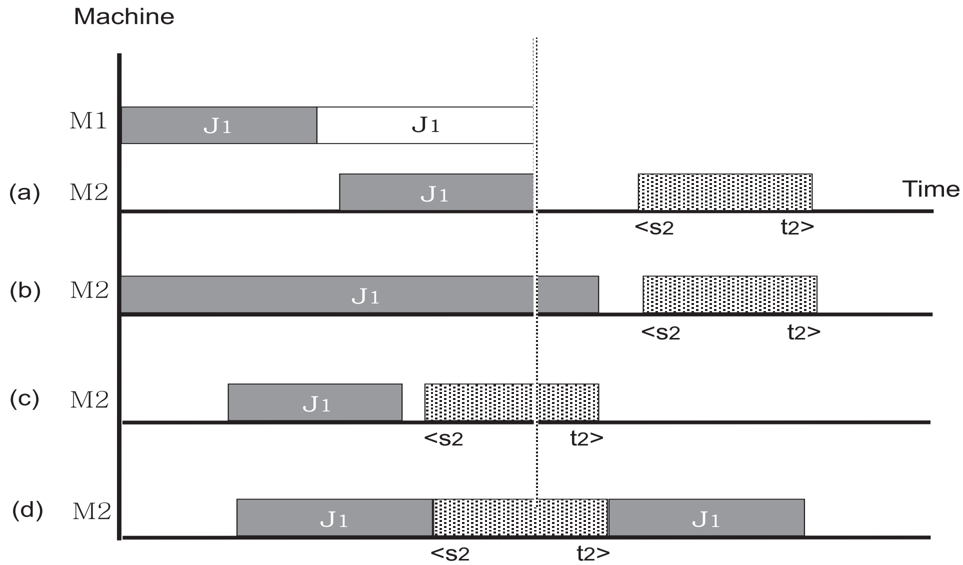

Figure 18: Here has four conditions a,b,c,d.

Condition	$\max\{\max\{s_p^1 + a_p, s_p^2\} - \max\{s_p^1 + a_p - s_2, 0\}, s_P^2\} + b_p + \alpha\Delta_2$
a	$\max\{s_p^1 + a_p - 0, s_P^2\} + b_p + \alpha\Delta_2 = s_p^1 + a_p + b_p + \alpha\Delta_2$
b	$\max\{s_P^2 - 0, s_P^2\} + b_p + \alpha\Delta_2 = s_P^2 + b_p + \alpha\Delta_2$
c	$\max\{s_p^1 + a_p - (s_p^1 + a_p - s_2), s_P^2\} + b_p + \alpha\Delta_2 = s_2 + b_p + \alpha\Delta_2$
d	$s_P^2 + b_p + \alpha\Delta_2$

$$\begin{aligned} C_{\max}(\pi_4) &\leq \sum_{i=n}^{n}(s_i^1 + a_i) + (1-\alpha)\Delta_2 + \sum_{J_i \in S\setminus\{J_p\}}(s_i^1 + a_i) \\ &\leq C^\star + 2C^\star/3 = 5C^\star/3. \end{aligned}$$

Otherwise, J_p is the critical job. From $s_p^2 + b_p > C^\star/3$ and $\max\{s_p^1 + a_p, s_p^2\} + b_p < C^\star$, we obtain that $\max\{s_p^1 + a_p - s_p^2, 0\} < 2C^\star/3$; so

$C_{\max}(\pi_4) \leq \max\{s_p^1 + a_p - s_p^2, 0\} + \Delta_2 + \sum_{i=n}^{n}(s_i^2 + b_i) < C^\star + 2C^\star/3 = 5C^\star/3.$

The proof is complete. □

Although we do not know whether the bound is tight or not, the following instance shows that the worst-case error bound of H2 is no less than 1/3 . Consider an instance with $s_1^1 = h$, $a_1 = 2$, $s_1^2 = 2$, $b_1 = 5$, $s_2^1 = 1 + h/2$, $a_2 = 1 + h/2$, $s_2^2 = 1$, $b_2 = h + 2$, $s_3^1 = h - 3$, $a3 = 2$, $s_3^2 = h - 2$, $b_3 = 2$, $s_1 = h$, and $t_1 = 2$, where $h \gg 1$. Applying heuristic H2, we obtain $\pi_1 = [J_2, J_1, J_3]$ with $C_{\max}(\pi_1) = 4h + 9$ (see figure 19(a)), and $\pi_2 = \pi_4 = [J_2, J_3, J_1]$ with $C_{\max}(\pi_2) = C_{\max}(\pi_4) = 4h + 9$ (see figure 19(b)), and $\pi_3 = [J_1, J_3, J_2]$ with $C_{\max}(\pi_3) = 4h + 8$ (see figure 19(c)). Thus $C_{H2} = 4h + 8$. It is easy to check that $\pi^\star = [J_3, J_2, J_1]$ with $C^\star = 3h + 11$ (see figure 19(d)). Hence, we see that $(C_{H2} - C^\star)/C^\star$ approaches 1/3 as h approaches infinity.

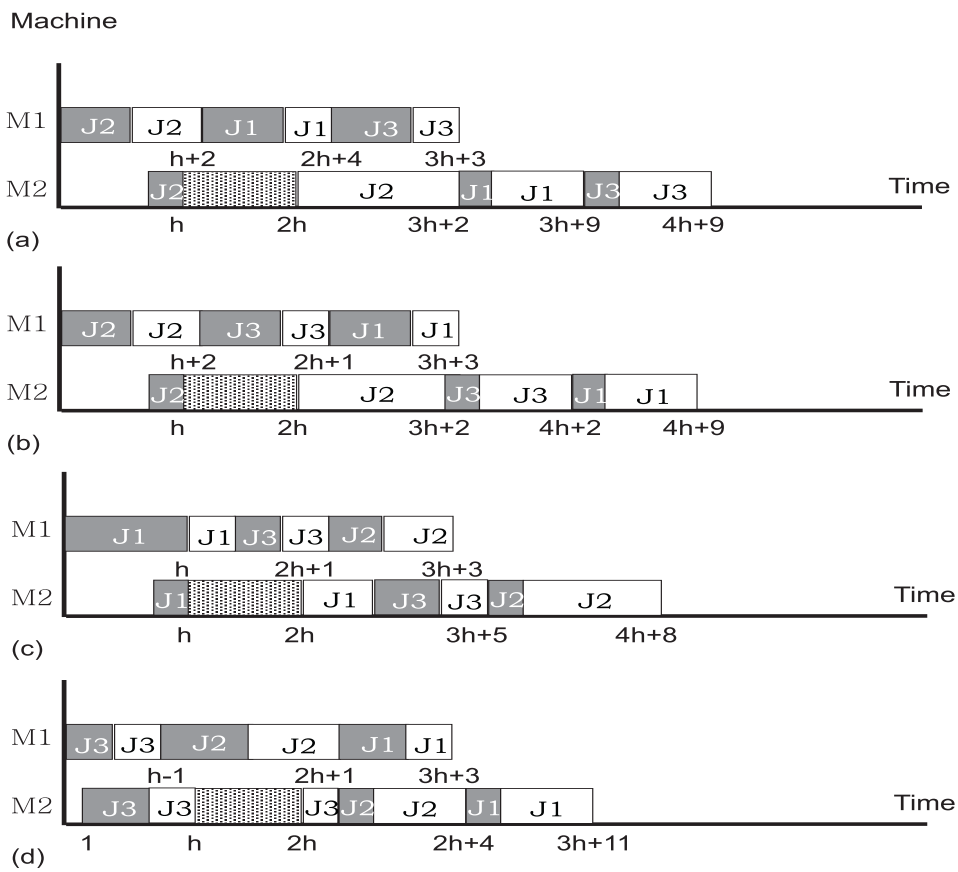

Figure 19: (a)Solution of order π_1; (b)Solution of order π_2, π_4; (c)Solution of order π_3; (d)Solution of order $\pi^\star$.

The heuristic was implemented in Java, on a Pentium-4 PC clocked at 2.7 GHz under the operating system Windows XP. We ran randomly generated job numbers with n=$6, 7, 8, 9, 10, 11, 12$. For each job set we ran 2 different unavailable times on the two machines.

The following table is based on machine 1. The first column is number of jobs, the second column is the number of simulations, Column 3 is the percentage of the simulations that the heuristic yields the optional solution, column 4 is the average error bound and column 5 is the largest error bound.

Based on this program, all jobs' setup times and processing times are taken to be random integer numbers between 1 and 10. The unavailable time is done by choosing a random number, l_1 between the values 0.1 and 0.15, and another random number k_1 between the values 0.2 and 0.25. Then $s_1 = \lfloor l_1 \cdot \sum_{i=n}^{n}(s_i^1 + a_i) \rfloor$; $t_1 = \lfloor k_1 \cdot \sum_{i=n}^{n}(s_i^1 + a_i) \rfloor$.

Job numbers size n	Numbers of simulation	Optimal solution percentage	Average Error bound	Largest Error bound
6	100	77%	0.02512	0.04521
7	100	88%	0.01231	0.05376
8	100	85%	0.02891	0.05427
9	100	84%	0.01929	0.04381
10	100	78%	0.03119	0.04841
11	100	84%	0.02867	0.04639
12	100	75%	0.03243	0.05082

Table 8: Computational results for heuristic 1

The following table is based on machine 2. The first column is number of jobs, the second column is the number of simulations, Column 3 is the percentage of the simulations that the heuristic yields the optional solution, column 4 is the average error bound and column 5 is the largest error bound.

Based on this program, all jobs' setup times and processing times are taken to be random integer numbers between 1 and 10. The unavailable time is done by choosing a random number l_2 between the values 0.1 and 0.15, and another random number, k_2 between the values 0.2 and 0.25. Then $s_2 = \lfloor l_2 \cdot \sum_{i=n}^{n}(s_i^2 + b_i) \rfloor$; $t_2 = \lfloor k_2 \cdot \sum_{i=n}^{n}(s_i^2 + b_i) \rfloor$.

Job numbers size n	Numbers of simulation	Optimal solution percentage	Average Error bound	Largest Error bound
6	100	77%	0.03066	0.04189
7	100	88%	0.03482	0.04641
8	100	85%	0.04482	0.07901
9	100	84%	0.05517	0.06562
10	100	78%	0.02629	0.04671
11	100	84%	0.04227	0.08943
12	100	75%	0.05012	0.07514

Table 9: Computational results for heuristic 2

6 CONCLUSIONS

In this paper we studied the two-machine flowshop scheduling problem with anticipatory setup times and a resumable availability constraint imposed on only one of the machines. Since the problem is NP-hard, we presented two polynomial-time heuristics developed by Wang and Cheng and analyzed their error bounds by simulation. From the computational results, we can see that heuristic 1 is more accurate.

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