EXTRAORDINARY DIMENSION OF MAPS

A. CHIGOGIDZE AND V. VALOV

Abstract. We establish a characterization of the extraordinary dimension of perfect maps between metrizable spaces.

1. Introduction

The paper deals with extensional dimension of maps, specially, with the extraordinary dimension introduced recently by Ščepin [10] and studied by the first author in [1]. If L is a CW-complex and X a metrizable space, we write e-dim$X \leq L$ provided L is an absolute extensor for X (in such a case we say that the extensional dimension of X is $\leq L$, see [3], [4]). The extraordinary dimension of X generated by a complex L, notation $\dim_L X$, is the smallest integer n such that e-dim$X \leq \Sigma^n L$, where $\Sigma^n L$ is the n-th iterated suspension of L (by $\Sigma^0 L$ we always denote the complex L itself). If L is the 0-dimensional sphere S^0, then $\dim_L$ coincides with the covering dimension dim. We also write $\dim_L f \leq n$, where $f\colon X \to Y$ is a given map, provided $\dim_L f^{-1}(y) \leq n$ for every $y \in Y$. Next is our main result.

Theorem 1.1. *Let $f\colon X \to Y$ be a σ-perfect map of metrizable spaces, let L be a CW-complex and $n \geq 1$. Consider the following properties:*

- (1) $\dim_L f \leq n$;
- (2) *There exists an F_σ subset A of X such that $\dim_L A \leq n-1$ and the restriction map $f|(X\backslash A)$ is of dimension $\dim_L f|(X\backslash A) = 0$;*
- (3) *There exists a dense and G_δ subset $\mathcal{G}$ of $C(X, \mathbb{I}^n)$ with the source limitation topology such that $\dim_L(f \times g) = 0$ for every $g \in \mathcal{G}$;*
- (3′) *There exists a map $g\colon X \to \mathbb{I}^n$ is such that $\dim_L(f \times g) = 0$.*

Then (3) ⇒ (3′) ⇒ (1) and (3′) ⇒ (2). Moreover, (1) ⇒ (3) provided Y is a C-space and L is countable.

Here, $f\colon X \to Y$ is σ-perfect if X is the union of countably many closed sets X_i such that $f(X_i) \subset Y$ are closed and the restriction maps $f|X_i$ are perfect.

Theorem 1.1 is inspired by the following result of M. Levin and W. Lewis [7, Theorem 1.8]: If X and Y are metrizable compacta then (3) ⇒ (3′) ⇒ (1) and

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(3) $\Rightarrow$ (2′) $\Rightarrow$ (1), where (2′) is obtained from our condition (2) by replacing $\dim_L f|(X\backslash A) \le 0$ with $\dim f|(X\backslash A) \le 0$. Moreover, the implication (1) $\Rightarrow$ (3) was also established in [7] for a finite-dimensional compactum Y and a countable CW-complex L.

Therefore, we have the following characterization of extraordinary dimension of perfect maps between metrizable spaces:

Corollary 1.2. *Let $f: X \to Y$ be a perfect surjection between metrizable spaces with Y being a C-space. If L is a countable CW-complex, then the following conditions are equivalent:*

(1) $\dim_L f \le n$;
(2) *There exists a dense and G_δ subset $\mathcal{G}$ of $C(X, \mathbb{I}^n)$ with the source limitation topology such that $\dim_L(f \times g) \le 0$ for every $g \in \mathcal{G}$;*
(3) *There exists a map $g: X \to \mathbb{I}^n$ is such that $\dim_L(f \times g) \le 0$.*

If, in addition, X is compact, then each of the above three conditions is equivalent to the following one:

(4) *There exists an F_σ set $A \subset X$ such that $\dim_L A \le n-1$ and the restriction map $f|(X\backslash A)$ is of dimension $\dim f|(X\backslash A) \le 0$.*

The equivalence of the first three conditions follow from Theorem 1.1. More precisely, by Theorem 1.1 we have the following implications: (2) $\Rightarrow$ (3) $\Rightarrow$ (1) $\Rightarrow$ (2). When X is compact, the result of Levin-Lewis which was mentioned above yields that (2) $\Rightarrow$ (4) $\Rightarrow$ (1). Therefore, combining the last two chains of implications, we can obtain the compact version of Corollary 1.2.

Corollary 1.2 is a parametric version of [1, Theorem 4.9]. For the covering dimension dim such a characterization was obtained by Pasynkov [9] and Toruńczyk [11] in the realm of finite-dimensional compact metric spaces and extended in [12] to perfect maps between metrizable C-spaces. Since the class of C-spaces contains the class of finite-dimensional ones as a proper subclass (see [5]), the compact version of Corollary 1.2 is more general than the Levin-Lewis result [7, Theorem 1.8]. It is interesting to know whether all the conditions (1)-(4) in Corollary 1.2 remain equivalent without the compactness requirement on X and Y.

The source limitation topology on $C(X, M)$, where (M, d) is a metric space, can be described as follows: a subset $U \subset C(X, M)$ is open if for every $g \in U$ there exists a continuous function $\alpha: X \to (0, \infty)$ such that $\overline{B}(g, \alpha) \subset U$. Here, $\overline{B}(g, \alpha)$ denotes the set $\{h \in C(X, M) : d(g(x), h(x)) \le \alpha(x) \text{ for each } x \in X\}$. The source limitation topology doesn't depend on the metric d if X is paracompact and $C(X, M)$ with this topology has the Baire property provided (M, d) is a complete metric space. Moreover, if X is compact, then the source limitation topology coincides with the uniform convergence topology generated by d.

All function spaces in this paper, if not explicitely stated otherwise, are equipped with the source limitation topology.

2. Some preliminary results

Throughout this section K is a closed and convex subset of a given Banach space E and $f\colon X \to Y$ a perfect map with X and Y paracompact spaces. Suppose that for every $y \in Y$ we are given a property $\mathcal{P}(y)$ of maps $h\colon f^{-1}(y) \to K$ and let $\mathcal{P} = \{\mathcal{P}(y) : y \in Y\}$. By $C_{\mathcal{P}}(X|H, K)$ we denote the set of all bounded maps $g\colon X \to K$ such that $g|f^{-1}(y)$ has the property $\mathcal{P}(y)$ for every $y \in H$, where $H \subset Y$. We also consider the set-valued map $\psi_{\mathcal{P}}\colon Y \to 2^{C^*(X,K)}$, defined by the formula $\psi_{\mathcal{P}}(y) = C^*(X,K)\backslash C_{\mathcal{P}}(X|\{y\}, K)$, where $C^*(X,K)$ is the space of bounded maps from X into K.

Lemma 2.1. *Suppose that $\mathcal{P} = \{\mathcal{P}(y)\}_{y\in Y}$ is a family of properties satisfying the following conditions:*

(a) *$C_{\mathcal{P}}(X|H, K)$ is open in $C^*(X,K)$ with respect to the source limitation topology for every closed $H \subset Y$;*

(b) *$g \in C_{\mathcal{P}}(X|\{y\}, K)$ implies $g \in C_{\mathcal{P}}(X|U, K)$ for some neighborhood U of y in Y.*

Then the map $\psi_{\mathcal{P}}$ has a closed graph provided $C^*(X,K)$ is equipped with the uniform convergence topology.

Proof. The proof of this lemma follows the arguments from the proof of [12, Lemma 2.6]. □

Recall that a closed subset F of the metrizable apace M is said to be a Z_m-set in M, if the set $C(\mathbb{I}^m, M\backslash F)$ is dense in $C(\mathbb{I}^m, M)$ with respect to the uniform convergence topology, where $\mathbb{I}^m$ is the m-dimensional cube. If F is a Z_m-set in M for every $m \in \mathbb{N}$, we say that F is a Z-set in M.

Lemma 2.2. *Suppose $y \in Y$ and $\mathcal{P}(y)$ satisfy the following condition:*

- *For every $m \in \mathbb{N}$ the set of all maps $h \in C(\mathbb{I}^m \times f^{-1}(y), K)$ with each $h|(\{z\} \times f^{-1}(y))$, $z \in \mathbb{I}^m$, having the property $\mathcal{P}(y)$ (as a map from $f^{-1}(y)$ into K) is dense in $C(\mathbb{I}^m \times f^{-1}(y), K)$ with respect to the uniform convergence topology.*

Then, for every $\alpha\colon X \to (0,\infty)$ and $g \in C^*(X,K)$, $\psi_{\mathcal{P}}(y) \cap \overline{B}(g,\alpha)$ is a Z-set in $\overline{B}(g,\alpha)$ provided $\overline{B}(g,\alpha)$ is considered as subset of $C^*(X,K)$ equipped with the uniform convergence topology and $\psi_{\mathcal{P}}(y) \subset C^*(X,K)$ is closed.

Proof. See the proof of [12, Lemma 2.8] □

Proposition 2.3. *Let Y be a C-space and $\mathcal{P} = \{\mathcal{P}(y)\}_{y\in Y}$ such that:*

(a) *the map $\psi_{\mathcal{P}}$ has a closed graph;*

(b) *$\psi_{\mathcal{P}}(y)\cap\overline{B}(g,\alpha)$ is a Z-set in $\overline{B}(g,\alpha)$ for any continuous function $\alpha\colon X\to(0,\infty)$, $y\in Y$ and $g\in C^*(X,K)$, where $\overline{B}(g,\alpha)$ is considered as a subspace of $C^*(X,K)$ with the uniform convergence topology.*

Then the set $\{g\in C^*(X,K) : g\in C_{\mathcal{P}}(X|\{y\},K)$ for every $y\in Y\}$ is dense in $C^*(X,K)$ with respect to the source limitation topology.

Proof. Let $G=\{g\in C^*(X,K) : g\in C_{\mathcal{P}}(X|\{y\},K)$ for every $y\in Y\}$. It suffices to show that, for fixed $g_0\in C^*(X,K)$ and a positive continuous function $\alpha\colon X\to(0,\infty)$, there exists $g\in\overline{B}(g_0,\alpha)\cap G$. We equip $C^*(X,K)$ with the uniform convergence topology and consider the constant (and hence, lower semi-continuous) convex-valued map $\phi\colon Y\to 2^{C^*(X,K)}$, $\phi(y)=\overline{B}(g_0,\alpha_1)$, where $\alpha_1(x)=\min\{\alpha(x),1\}$. Because of the conditions (a) and (b), we can apply the selection theorem [6, Theorem 1.1] to obtain a continuous map $h\colon Y\to C^*(X,K)$ such that $h(y)\in\phi(y)\backslash\psi_{\mathcal{P}}(y)$ for every $y\in Y$. Observe that h is a map from Y into $\overline{B}(g_0,\alpha_1)$ such that $h(y)\in C_{\mathcal{P}}(X|\{y\},K)$ for every $y\in Y$. Then $g(x)=h(f(x))(x)$, $x\in X$, defines a bounded map $g\in\overline{B}(g_0,\alpha)$ such that $g|f^{-1}(y)=h(y)|f^{-1}(y)$, $y\in Y$. Therefore, $g\in C_{\mathcal{P}}(X|\{y\},K)$ for all $y\in Y$, i.e., $g\in\overline{B}(g_0,\alpha)\cap G$. □

3. Proof of Theorem 1.1

(**1**) ⇒ (**3**) Suppose that L is countable and Y is a C-space. Let X_i be closed subsets of X such that each $f_i=f|X_i\colon X_i\to Y_i=f(X_i)$ is a perfect map and Y_i is closed in Y. Then all Y_i's are C-spaces, and since the restriction maps $\pi_i\colon C(X,\mathbb{I}^n)\to C(X_i,\mathbb{I}^n)$, $\pi_i(g)=g|X_i$, are open, the proof of this implication is reduced to the case when f is a perfect map. Consequently, we may assume that f is perfect.

By [13, Theorem 1.1] (see also [8]), there exists a map q from X into the Hilbert cube Q such that $f\times q\colon X\to Y\times Q$ is an embedding. Let $\{W_i\}_{i\in\mathbb{N}}$ be a countable finitely-additive base for Q. For every i we choose a sequence of mappings $h_{ij}\colon\overline{W_i}\to L$, representing all the homotopy classes of mappings from $\overline{W_i}$ to L (this is possible because L is a countable CW-complex). Following the notations from Section 2, for fixed i, j and $y\in Y$ we say that a map $g\in C(X,\mathbb{I}^n)$ has the property $\mathcal{P}_{ij}(y)$ provided

the map $h_{ij}\circ q\colon q^{-1}(\overline{W_i})\to L$ can be continuously extended to a map over the set $q^{-1}(\overline{W_i})\cup\big(f^{-1}(y)\cap g^{-1}(t)\big)$ for every $t\in g(f^{-1}(y))$.

Let $\mathcal{P}_{ij}=\{\mathcal{P}_{ij}(y) : y\in Y\}$ and for every $H\subset Y$ we denote $C_{\mathcal{P}_{ij}}(X|H,\mathbb{I}^n)$ by $C_{ij}(X|H,\mathbb{I}^n)$. Hence, $C_{ij}(X|H,\mathbb{I}^n)$ consists of all $g\in C(X,\mathbb{I}^n)$ having the property $C_{ij}(y)$ for every $y\in H$. Let $\psi_{ij}\colon Y\to 2^{C(X,\mathbb{I}^n)}$ be the set-valued map $\psi_{ij}(y)=C(X,\mathbb{I}^n)\backslash C_{ij}(X|\{y\},\mathbb{I}^n)$.

Lemma 3.1. *Let $g \in C_{ij}(X|\{y\}, \mathbb{I}^n)$. Then, there exist a neighborhood U_y of y in Y and a neighbourhood $V_t \subset \mathbb{I}^n$ of each $t \in g(f^{-1}(y))$ such that $h_{ij} \circ q$ can be extended to a map from $q^{-1}(\overline{W_i}) \cup \big(f^{-1}(U_y) \cap g^{-1}(V_t)\big)$ into L.*

Proof. Since $g \in C_{ij}(X|\{y\}, \mathbb{I}^n)$, $h_{ij} \circ q$ can be extended to a map from $q^{-1}(\overline{W_i}) \cup \big(f^{-1}(y) \cap g^{-1}(t)\big)$ into L for every $t \in g(f^{-1}(y))$. Because L is an absolute neighborhood extensor for X, there exists and open set $G_t \subset X$ containing $f^{-1}(y) \cap g^{-1}(t)$ and a map $h_t\colon q^{-1}(\overline{W_i}) \cup G_t \to L$ extending $h_{ij} \circ q$. Using that $f \times g$ is a closed map, we can find a neighborhood $U_y^t \times V_t$ of (y,t) in $Y \times \mathbb{I}^n$ such that $S_t = (f \times g)^{-1}(U_y^t \times V_t) \subset G_t$. Next, choose finitely many points $t(k)$, $k = 1, 2, ., m$, with $f^{-1}(y) \subset \bigcup_{k=1}^{k=m} S_{t(k)}$ and a neighbothood U_y of y in Y such that $U_y \subset \bigcap_{k=1}^{k=m} U_y^{t(k)}$ and $f^{-1}(U_y) \subset \bigcup_{k=1}^{k=m} S_{t(k)}$ (this can be done since f is perfect). If $t \in g(f^{-1}(y))$, then $t \in V_{t(k)}$ for some k and $f^{-1}(U_y) \cap g^{-1}(V_{t(k)}) \subset S_{t(k)}$. Since , $S_{t(k)} \subset G_{t(k)}$, the map $h_{t(k)}$ is an extension of $h_{ij} \circ q$ over the set $q^{-1}(\overline{W_i}) \cup \big(f^{-1}(U_y) \cap g^{-1}(V_{t(k)})\big)$ □

Lemma 3.2. *The set $C_{ij}(X|H, \mathbb{I}^n)$ is open in $C(X, \mathbb{I}^n)$ for any i, j and closed $H \subset Y$.*

Proof. We follow the proof of [12, Lemma 2.5]. For a fixed $g_0 \in C_{ij}(X|H, \mathbb{I}^n)$ we are going to find a function $\alpha\colon X \to (0, \infty)$ such that $\overline{B}(g_0, \alpha) \subset C_{ij}(X|H, \mathbb{I}^n)$. By Lemma 3.1, for every $z = (y,t) \in (f \times g_0)((f^{-1}(H))$ there exists a neighborhood U_z in $Y \times \mathbb{I}^n$ such that

(1) $h_{ij} \circ q$ can be extended to a map from $q^{-1}(\overline{W_i}) \cup (f \times g_0)^{-1}(U_z)$ into L.

Obviously, $K = (f \times g_0)((f^{-1}(H))$ is closed in $Y \times \mathbb{I}^n$, so there exists open $G \subset Y \times \mathbb{I}^n$ with $K \subset G \subset \overline{G} \subset U = \bigcup\{U_z : z \in K\}$. Then $\nu = \{U_z : z \in K\} \cup \{(Y \times \mathbb{I}^n)\backslash\overline{G}\}$ is an open cover of $Y \times \mathbb{I}^n$. Let γ be an open locally finite cover of $Y \times \mathbb{I}^n$ such that the family

(2) $\{St(W, \gamma) : W \in \gamma\}$ refines ν and $St(W, \gamma) \subset G$ provided $W \cap K \neq \emptyset$.

Consider the metric $\rho = d + d_1$ on $Y \times \mathbb{I}^n$, where d is a metric on Y and d_1 the usual metric on $\mathbb{I}^n$, and define the function $\alpha\colon X \to (0, \infty)$ by $\alpha(x) = 2^{-1}\sup\{\rho\big((f \times g_0)(x), (Y \times \mathbb{I}^n)\backslash W\big) : W \in \gamma\}$. Let show that $\overline{B}(g_0, \alpha) \subset C_{ij}(X|H, \mathbb{I}^n)$. Take $g \in \overline{B}(g_0, \alpha)$, $y \in H$ and $t \in g(f^{-1}(y))$. Then, $(y,t) = (f \times g)(x)$ for some $x \in f^{-1}(y)$. Since g is α-close to g_0, there exists $W \in \gamma$ such that $W \cap K \neq \emptyset$ and W contains both $(f \times g)(x)$ and $(f \times g_0)(x)$. It follows from (2) that $(f \times g)^{-1}(W) \subset (f \times g_0)^{-1}(U_z)$ for some $z \in K$. In particular, $f^{-1}(y) \cap g^{-1}(t) \subset (f \times g_0)^{-1}(U_z)$. Consequently, by (1), $h_{ij} \circ q$ is extendable to a map from $q^{-1}(\overline{W_i}) \cup \big(f^{-1}(y) \cap g^{-1}(t)\big)$ into L. Therefore, $\overline{B}(g_0, \alpha) \subset C_{ij}(X|\{y\}, \mathbb{I}^n)$ for every $y \in H$ which completes the proof. □

Because of Lemma 3.1 and Lemma 3.2, we can apply Lemma 2.1 to obtain the following corollary.

Corollary 3.3. *For any i and j the map ψ_{ij} has a closed graph.*

Lemma 3.4. *Let $g \in C(X, \mathbb{I}^n)$, $\alpha\colon X \to (0,\infty)$ and $y \in Y$. Then, for any i, j, $\psi_{ij}(y) \cap \overline{B}(g,\alpha)$ is a Z-set in $\overline{B}(g,\alpha)$ provided $\overline{B}(g,\alpha)$ is considered as a subset of $C(X, \mathbb{I}^n)$ with the uniform convergence topology.*

Proof. It follows from [7, Theorem 1.8, (1) $\Rightarrow$ (3)] that if $m \in \mathbb{N}$, then all maps $g\colon \mathbb{I}^m \times f^{-1}(y) \to \mathbb{I}^n$ such that e-dim$\big((\{z\} \times f^{-1}(y)) \cap g^{-1}(t)\big) \leq L$ for every $z \in \mathbb{I}^m$ and $t \in \mathbb{I}^n$, form a dense subset G of $C(\mathbb{I}^m \times f^{-1}(y))$ with the uniform convergence topology. It is clear that, for every $g \in G$ and $z \in \mathbb{I}^m$, the restriction $g|\big(\{z\} \times f^{-1}(y)\big)$, considered as a map on $f^{-1}(y)$, has the following property: $h_{ij} \circ q$ can be extended to a map from $q^{-1}(\overline{W_i}) \cup (f^{-1}(y) \cap g^{-1}(t))$ into L for any $t \in \mathbb{I}^n$. Hence, we can apply Lemma 2.2 to conclude that $\psi_{ij}(y) \cap \overline{B}(g,\alpha)$ is a Z-set in $\overline{B}(g,\alpha)$. □

Now, we can finish the proof of this implication. Because of Corollary 3.3 and Lemma 3.4, we can apply Proposition 2.3 to obtain that the set $C_{ij} = C_{ij}(X|Y, \mathbb{I}^n)$ is dense in $C(X, \mathbb{I}^n)$ for every i, j. Since, by Lemma 3.2, all C_{ij} are also open, their intersection $\mathcal{G}$ is dense and G_δ in $C(X, \mathbb{I}^n)$. Let show that $\dim_L(f \times g) \leq 0$ for every $g \in \mathcal{G}$, i.e., e-dim$(f \times g) \leq L$. We fix $y \in Y$ and $t \in \mathbb{I}^n$ and consider the fiber $(f \times g)^{-1}(y,t) = f^{-1}(y) \cap g^{-1}(t)$. Take a closed set $A \subset f^{-1}(y) \cap g^{-1}(t)$ and a map $h\colon A \to L$. Because the map $q_y = q|f^{-1}(y)$ is a homeomorphism, $h' = h \circ q_y^{-1}\colon q(A) \to L$ is well defined. Next, extend h' to a map from a neighborhood W of $q(A)$ (in Q) into L and find W_i with $q(A) \subset W_i \subset \overline{W_i} \subset W$. Therefore, there exists a map $h''\colon \overline{W_i} \to L$ extending h'. Then h'' is homotopy equivalent to some h_{ij}, so are $h'' \circ q$ and $h_{ij} \circ q$ (considered as maps from $q^{-1}(\overline{W_i})$ into L). Since $h_{ij} \circ q$ can be extended to a map from $q^{-1}(\overline{W_i}) \cup \big(f^{-1}(y) \cap g^{-1}(t)\big)$ into L, by the Homotopy Extension Theorem, there exists a map $\bar{h}\colon q^{-1}(\overline{W_i}) \cup \big(f^{-1}(y) \cap g^{-1}(t)\big) \to L$ extending $h'' \circ q$. Obviously, $\bar{h}|\big(f^{-1}(y) \cap g^{-1}(t)\big)$ extends h. Hence, e-dim$\big(f^{-1}(y) \cap g^{-1}(t)\big) \leq L$.

(3) $\Rightarrow$ (3′) $\Rightarrow$ (1) The implication (3) $\Rightarrow$ (3′) is trivial. It is easily seen that in the proof of (3′) $\Rightarrow$ (1) we can assume f is perfect. Let $g\colon X \to \mathbb{I}^n$ be such that $\dim_L(f \times g) \leq 0$ and $y \in Y$. Then the restriction $g|f^{-1}(y)\colon f^{-1}(y) \to \mathbb{I}^n$ is a pefect map with all of its fibers having extensional dimension e-dim $\leq L$. Hence, by [2, Corollary], e-dim$f^{-1}(y) \leq \Sigma^n L$, i.e, $\dim_L f \leq n$.

(3′) $\Rightarrow$ (2) Because of the countable sum theorem, we can suppose that f is perfect. We fix a map $g\colon X \to \mathbb{I}^n$ such that $\dim_L(f \times g) \leq 0$. According to [12, Lemma 4.1], there exists an F_σ subset $B \subset Y \times \mathbb{I}^n$ such that $\dim B \leq n-1$ and

$\dim(\{y\} \times \mathbb{I}^n) \setminus B \leq 0$ for every $y \in Y$. Then, applying again [2, Corollary], we conclude that the set $A = (f \times g)^{-1}(B)$ is as required.

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Department of Mathematical Sciences, University of North Carolina at Greensboro, P.O. Box 26170, Greensboro, NC 27402-6170, U.S.A.

E-mail address: chigogidze@uncg.edu

Department of Mathematics, Nipissing University, 100 College Drive, P.O. Box 5200, North Bay, ON, P1B 8L7, Canada

E-mail address: veskov@nipissingu.ca