STEIN, CATHERINE C., Ph.D. An Exploration of Elementary Preservice Teachers' Performance and Beliefs When Negotiating Reform-Based Mathematics Education. (2008)
Directed by Dr. Samuel D. Miller. 165 pp.

The purpose of this qualitative research study was to investigate the relationship between eight preservice teachers' participation and beliefs about their role as teachers in a reform-based mathematics methods course and the ways they performed, believed, and imagined themselves as teachers of mathematics in their internships and student teaching. The study was conducted in two schools in an urban setting. Data sources included lesson observations, field notes, interviews, and written reflections. Data analysis performed included domain analysis, grounded theory, and non-parametric statistical analysis. Results of a background study indicated that preservice teachers participated as students during mathematics methods in ways that resisted, acknowledged, embraced, and created the complexity of reformed-based teaching. Scores on a performance observation framework and other qualitative data indicated that the preservice teachers in the four groups from mathematics methods performed as teachers in internships and student teaching in ways that were significantly different from each other. In addition, they perceived their role as teachers of mathematics differently. The preservice teachers imagined themselves differently in relation to the classroom context, but no discernible patterns existed between the four groups and contextual factors. The findings suggest that preservice teachers make their own meanings of their participation in common experiences in mathematics methods and teach in ways that reflect those meanings. The different meanings preservice teachers make can be understood as different entry points

into the practice of reform-based teaching. Knowing the entry points and the paths to which they lead has practical implications for teacher educators as they make instructional decisions in methods courses and policy implications for the structure of teacher education courses. Future research should help identify these paths and the experiences that help preservice teachers move along them.

AN EXPLORATION OF ELEMENTARY PRESERVICE TEACHERS' PERFORMANCE AND BELIEFS WHEN NEGOTIATING REFORM-BASED MATHEMATICS EDUCATION

by

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A Dissertation Submitted to
the Faculty of The Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Greensboro
2008

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To my parents, Neil and Carolyn Stein, my first and best teachers.

APPROVAL PAGE

This dissertation has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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July 31, 2008
Date of Acceptance by Committee

July 31, 2008
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ACKNOWLEDGMENTS

Completion of this project would not have been possible without the help of many people. I am thankful for the help of Dr. Sam Miller who got me started on this path in the first place and continued to push my thinking every step of the way. I am also grateful to the other members of my committee, Dr. Heidi Carlone, Dr. Gerald Duffy, and Dr. Kerri Richardson, who each helped me in very different and very significant ways. I am grateful to my fellow doctoral students who were always there to listen to ideas and provide feedback, particularly to Holt Wilson for his help with the quantitative data analysis. Finally, I must thank my family and friends who supported and encouraged me throughout this process.

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CHAPTER I

STATEMENT OF PROBLEM

Reform-based mathematics calls for teachers to employ instructional practices that help students to participate successfully in mathematical practices, to develop a strong conceptual understanding of mathematics content (NCTM, 2000), and to establish classroom norms where students are the source of mathematical ideas and share the responsibility for learning (Hufferd-Ackles, Fuson, & Sherin, 2004). Such reform often necessitates a change in teachers' beliefs from a narrow view of mathematics as procedural and fixed to a broader, more dynamic view of mathematics which includes participation in practices such as explanation, questioning, listening, representation, reasoning and proof, making connections, and problem-solving. This change may be particularly problematic for preservice teachers who have only experienced traditional instruction because it involves two challenges. First, they need to participate in the practices of reform-based mathematics as students. Second, as teachers, they need to implement reform-based instructional practices in their classrooms. Concurrently they are making the general shift from the role of student in methods courses to the role of teacher in internships and student teaching.

Elementary preservice teachers who enter teacher education programs likely view mathematics in the way that they experienced it as students (Ball, 1988). Since at least the early twentieth century (Hoetker & Ahlbrand, 1969; Tharp & Gallimore 1988), typical traditional lessons involved teacher direction of a new procedure followed by an assignment of seatwork problems to be worked independently during class (Fey, 1979; NACOME Report, 1975). As a result, many students have a view of mathematics as procedural and fixed. Schoenfeld (1989) found that students thought that if you understand the mathematics, problems should take no longer than five minutes to solve. Participation in the mathematical community of practice in the schooling experience of preservice teachers, then, probably entailed sitting quietly and independently practicing a taught procedure. Therefore, in research an approach to shifting these beliefs in preservice teachers has been to have them experience reform-based mathematics themselves as students in mathematics methods and content classes. The assumption is that by changing preservice teachers' beliefs through their experience as students, they will employ reform-based instructional practices as teachers. However, translating beliefs into practice is challenging. Many studies have indeed shown a change in beliefs at the end of such courses, but this change does not necessarily result in reform-based practice as teachers.

Focusing on more than just beliefs, a critical assumption of this study relates to the need for teachers to be able to understand what students experience prior to providing reform-based instruction in their classrooms. How do they participate in reform-based mathematics themselves? What meaning do they make of that participation for their teaching? If they do not participate in and affiliate with the practices of reform-based mathematics as students, I questioned how their orientations towards reform-based mathematics might affect their teaching. As a result, I first put them into a situation where they were participants as students in a reform-based mathematics community and then followed them over a year and a half as they provided instruction to students. The goal of this study was to understand the meaning preservice teachers made of their experiences across contexts as they made the transition from student to teacher. Specifically it addressed two questions: (a) How do elementary preservice teachers' participation and beliefs as students in a reform-based mathematics community indicate the ways they perform, believe, and imagine themselves as teachers in their own mathematics classroom community?; and (b) How do preservice teachers change over time in multiple settings the ways they believe, perform, and imagine themselves to be as teachers of mathematics?

Section one describes relevant research regarding preservice teachers' change in beliefs after participation as students in mathematics methods. I also report the findings of a background study which examined how the different ways preservice teachers participated in a reform-based mathematics community were related to their beliefs about their roles as teachers of mathematics. Next, I discuss studies in which preservice teachers were followed into the classroom; consider some general challenges faced by teachers when implementing reform-based mathematics; and put forward some possible explanations for discrepancies between beliefs and practice. In section three, I propose examining preservice teachers' emerging identities as mathematics teachers as they make the dual shift from traditionally-experienced students seeking to do and understand reform-based mathematics to teachers who have to put reform-based instruction into practice. I then explain the theoretical underpinnings behind this approach. Finally, I explain the research goals of the current study which followed preservice teachers from phase one into their internship and student teaching classrooms.

Background Study

Preservice Teachers as Students

One attempt at shifting beliefs in preservice teachers has been to have them experience reform-based mathematics as students by giving them problem-solving tasks which allow them to construct their own understanding of mathematics. Ball (1988) engaged prospective teachers in an introductory education course in a project where they first explored the topic of permutations as students. Then, after observing the instructor helping a child explore permutations, they were asked to teach someone else (a roommate, friend, etc.) about permutations. She found that engaging preservice teachers in the experience of learning mathematics topics helped them bring their existing knowledge, beliefs, and attitudes to the surface so that those ideas could be built upon, challenged, or changed. Surveys, observations, interviews, and student artifacts from studies of this kind continue to document the value of giving preservice teachers the opportunity to construct their own mathematics understanding as a means to help them change their beliefs about the nature of mathematics, mathematics learning, and mathematics teaching.

Lubinski and Otto (2004) used pre-/post-course written surveys to show changes in perceptions towards standards-based instruction when preservice teachers took a methods course focused on conceptual understanding, covering fewer topics, and the nature of mathematics. Results indicated that even though some preservice teachers initially resisted new ideas about mathematics learning, many changed their beliefs and attitudes by the end of the course. Preservice teachers reported having more positive attitudes towards mathematics, increased focus on conceptual understanding, more persistence during problem-solving, and a sense that authority needed to shift from the teacher to the student.

Steele and Widman (1997) found from artifacts, observations, and interviews of five preservice teachers participating in a methods course based on Cognitively Guided Instruction (Carpenter & Fennema, 1991) and the Professsional Standards for Teaching Mathematics (NCTM, 1991) that preservice teachers changed their views about mathematics and teaching mathematics. Specifically, they understood mathematics to be more than the memorization of rules; they were willing to take more risks while justifying their solutions; and they represented their thinking with manipulatives and diagrams. When asked during interviews at the end of the course what point their beliefs began to change, several cited experiences with problem-solving in which they understood something conceptually about which they had only known procedurally. Recognizing conceptual versus procedural understanding is a key component in reform-based mathematics, but this important "turning point" does not necessarily indicate that these preservice teachers will strive for the same understanding for their students.

Liljedahl, Rolka, and Rosken (2007) followed preservice teachers as they engaged in problem-solving during a semester of their mathematics methods class. Participants kept problem-solving and reflective journals throughout the course of the semester. Data from the first and last week of preservice teachers' reflective journal entries showed a movement in their beliefs about mathematics and about learning and teaching mathematics towards a *process aspect* (Torner & Grigutsch, 1994 as cited in Liljedah et al., 2007), a view that mathematics involves “creative steps, such as generating rules and formulas, thereby inventing or reinventing the mathematics” (p. 321). They changed their views of the nature of mathematics from a focus on solutions to a focus on the process of doing mathematics.

These studies provide important insights into the change in beliefs that occurs as preservice teachers experience reform-based mathematics as students. However, there is no indication of how these new found beliefs influence preservice teachers when they transition into practice. Furthermore, the studies do not indicate how the beliefs preservice teachers hold about their role as a mathematics teacher may be related to the different ways they participated in the opportunities provided. In the background study, the goal was to understand the different ways that preservice teachers participated in the same tasks in the same mathematics methods course, and how those differences were related to their beliefs. The current study followed the same preservice teachers into the classroom to examine their instructional practice in relationship to their participation and beliefs in methods.

Background Study Findings

The goal of the background study was to better understand how preservice teachers participated in a reform-based mathematics methods course and how that participation related to their beliefs about their role as a teacher of mathematics. In other words, how was what they thought they should do as teachers embodied in their role as students? In this study, 26 preservice elementary teachers participated in a reform-based mathematics methods class, in which they solved open-ended problems. Work on the problems was followed by whole class discussions in which students explained and discussed their solutions. Participants were encouraged, but not required to work in groups. Detailed lesson plans of the mathematical territory of the class were recorded each week and were developed based on the observations of the previous week's work. Consequently, problems were sometimes carried across multiple class periods.

Data from reflective journals and coursework indicated that preservice teachers participated in a reform-based mathematics community in very different ways and that their participation related to beliefs about their role as a teacher of mathematics. Preservice teachers participated in ways that resisted, acknowledged, embraced, or created complexity in the activity of the community (see Appendix A).

Resisting complexity. Preservice teachers *resisting* complexity (n=3) continued to see the role of the teacher as the authority in the classroom who presented mathematical procedures for students to learn. When working on problems, preservice teachers in this category were either unsure of the mathematics and therefore, got frustrated and stopped, or were fluent in the skills and procedures needed to get the right answer, but got frustrated when asked to explain in more depth. They resisted complex tasks, stopping when they reached a point during problem-solving when they did not find a solution quickly. For example, in a lesson on using numbers in different bases, two students in this category ceased working all together until the end of the lesson when a connection was made to base ten place value, which they did understand. One explained, "I think that I participated in everything except the discussion about the other countries' base numbers. I read the information, but didn't understand how to use it creating my own number." When faced with a challenge, the response of these preservice teachers was to not participate, rather than to think it through, collaborate, or ask for help from another.

When asked what they learned about teaching from participating in the activities during mathematics methods class, the preservice teachers rarely reported learning anything about teaching or the learning reported was that they do not want to give their students problem-solving tasks. When students did report learning something, it was often on a task in which there was no frustration at all for them. They still saw mathematics teaching as presentation of a procedure which students should imitate, rather than exploring the concepts behind the procedures.

> I feel like these worksheets are good practice for me, but so far, I can't relate this to how I would teach math. Maybe it is because we are thinking so much about "Why?" in everything we do.

Not only did these preservice teachers place an emphasis on procedures over conceptual understanding, they also resisted engaging in mathematical practices such as argumentation and explanation. In one activity, the class was discussing the relationship between perimeter and area. The preservice teachers were responding to a student's assertion that as the area of a figure increases, so does the perimeter. After working with their groups with some square tiles, they were asked to take a position and justify their reasoning in the discussion. One resisting preservice teacher listening to the class discussion was even more convinced that this was not the way to teach:

> I understand the concepts of area and perimeter from previous math classes. As far as the stuff we did today, I didn't learn anything new about how to teach these concepts to children. Doing the garden problem made me really confused and I didn't really understand the examples given to support or dispute the correct answer. I like the way that I learned area and perimeter by being given a shape with numbers giving the length of each side and must use addition/multiplication to find area and perimeter. That was much easier than the method we used in class.

In this case, the preservice teachers' *personal identities* (Cobb, Gresalfi, & Hodge, in press) did not change as they engaged in a community that held different norms than they did for the meaning of mathematics.

Acknowledging complexity. Another group of preservice teachers (n=16) participated in ways that *acknowledged* new ways of thinking about and doing mathematics. They attributed much of their new understandings to collaborative group work, the use of manipulatives and the eliciting of multiple solutions, and consequently saw providing these opportunities in their classroom as part of their role as a teacher. Preservice teachers in this group often reported being unsure of the subject matter and being frustrated while working problems. They nevertheless persisted, sometimes feeling uncertain throughout the entire process while at other times feeling good about the result even if it was a struggle. They reported feeling good about developing an understanding of something they were uncertain about before the activity.

> My work on the square problem has been good. Even though I did not come up with a perfect solution, I worked on getting one. I am feeling good about my work. The more I started understanding the problem, the more I started to enjoy it and felt that I actually understood it.

Preservice teachers in this group commonly reported learning two things about teaching mathematics. First, they asserted that their own growth in understanding will improve their teaching.

> Now seeing my mistakes, like today, I can understand the foundations of fractions better, therefore be able to teach the understanding better.
>
> After doing these problems and working through them I feel I have a better understanding of how to relate perimeter and area than I have had before. . . . If I understand perimeter and area better using the tiles then I feel my students will probably have a better understanding through using them also.

Second, preservice teachers in this group were mindful of the ways they participated in terms of the tools provided to them. They recognized the importance of manipulatives and visualization, the value of group work, and seeing the multiple strategies used by their classmates in their own learning during the activities in class and therefore in their future teaching.

> After working with my group I have found that when teaching a problem such as this one it helps to place students in a group to help them find a solution and figure out a different or better strategy of solving the problem. Some of my group members used a different strategy than I originally had and their way of thinking made it easier to understand how to begin to find the trapezoid numbers in a triangle.
>
> If I were teaching, I would definitely use them [the tiles] to help children understand and visually see how perimeter and area are related. If I understand perimeter and area better using the tiles then I feel my students will probably have a better understanding through using them also.

In a few cases, however, there was a desire for their students to use these tools because they themselves were helped by them, and therefore, their students would find the same tools useful in the same way. They were still viewing teaching as "telling" (Chazan & Ball, 1995).

> This [work with manipulatives] will help me to teach my students different strategies to do things. I can now show them how to find patterns and look at problems in different perspectives.

They also tended to see the manipulatives and group work as something to make the math more engaging rather than as a vital part of the mathematics itself.

> Teaching is something that should be interesting and fun. Using manipulatives and games will help the students to learn while having fun.

This statement is reminiscent of the findings of Moyer (2001) about the misunderstanding of the purpose of manipulatives as tools for engagement rather than representations of the mathematics.

The preservice teachers in this group were *acknowledging* new ways of learning and teaching mathematics. The word "acknowledge" means "to recognize." In this sense, these students were re-cognizing (Davis, 1996) or re-forming (Chapman, 2002) their view of mathematics, finding it to be much more than they thought it was.

Embracing complexity. Preservice teachers *embracing* complexity (n=4) often worked alone first even when sitting in a group, but were also eager to engage in discussions about their ideas with a small group or the whole class. They were so absorbed in the task that to talk to them about their work, you first needed to pull them away from it. Even when the task was challenging, they reported being excited and engaged about what they were learning, especially about developing a deeper understanding of something they already knew.

> It [the problem] was hard! This was fun to work with, but it was frustrating! I feel GREAT now. Very hard!

They often referred to persistence as an important aspect of their own problem-solving or something that needs to be developed in their students.

> I am learning to be more perseverant with my work.
>
> You have to keep pushing and truly press your students even though it's tough sometimes.

Like those in the acknowledging group, preservice teachers in this group also reported wanting to use manipulatives and work in groups, but they saw it as a tool for understanding mathematics rather than for fun.

> But I think another thing that helped me a lot was the manipulatives and actually getting hands on experience. It made more and more sense to me after discussing with my group because we were able to talk through our problems as well as show each other different ideas and see many perspectives on how to solve it. It really helped me make sense of a challenging problem.

Preservice teachers also recognized their need to find their own understanding in order for the learning to be meaningful.

> I couldn't understand another's point of view or explanation but when I tried it myself and tried to figure it out, I understood. It was an interesting problem.

Therefore, they valued the importance of exploration so that students can find their own meaning and unique ways to solve.

> This [work on the problem] tells me that I need to try to help my students develop their own reasoning skills. I should encourage them to learn to think for themselves that way they can really and fully understand something because they have that experience with it. If I had not come to my own understanding before hearing from the class, I would have not completely understood anything!

The striking feature of the preservice teachers in this group, then, is their recognition of themselves as the source of mathematical ideas, and therefore the need to give students time to develop their own ideas. The responsibility for learning has shifted away from the teacher as the sole source of ideas.

Creating complexity. Finally, some preservice teachers (n=3) *created* more complexity for themselves and others by adjusting the task through problem-posing (Brown & Walter, 2005) and trying to solve in multiple ways. They came into mathematics methods with a solid conceptual understanding of mathematics although their work on the problems helped them make some new connections.

> I drew pictures, charts, formulas, and tried to understand why and how everything worked. I'm feeling good about the problem because I was able to solve it algebraically and spatially. This gives me a greater understanding of the problem to start with. While the algebra, etc. is important, seeing something in a picture or on the table is a clearer way to show relationships altogether. I hadn't been taught that in school; I learned from experience.

They often created complexity by making the problems more challenging and complex.

> I actually made my own problem up and worked on it.

When asked about how their work informs their teaching, these preservice teachers noted the connections they are making among different strands of mathematics, asserting that by understanding these connections and by listening to the strategies their classmates are using, it will help them better understand all of the different ways students might think about a problem so they can respond accordingly.

> I feel that I have learned more about geometric relationships and how to translate these relationships into numeric/algebraic forms. . . . This insight I believe will greatly help in my teaching because I see the various ways people see/visualize and interpret problems. Most importantly, I am learning different ways to work problems, which I can use in the classroom to stimulate students to think differently.

Moreover, these preservice teachers began to think about the role they play as a teacher in setting up the climate in which students work. For instance, one preservice teacher realized the importance of choosing an appropriate task.

> It tells me that the proper activity . . . is a POWERFUL tool in developing mathematical reasoning. Fifteen or 20 minutes of actively participating in this activity is far more effective than <u>hours</u> of listening to a teacher lecture/demonstrate.

The preservice teachers in this group were not only accepting of the idea that students need to construct understanding; they began to think about the different components involved in supporting that understanding.

All the preservice teachers were working on the same tasks in the same mathematical community during methods class. They all wrote about their participation and beliefs in the moment of their practice in this community, yet they participated in different ways by resisting, acknowledging, embracing, or creating complexity. These differences were reflected in the meaning they made of their mathematical work in terms of their beliefs about the role of the teacher in a mathematics classroom. As Chapman (1997) suggests, the beliefs of preservice teachers in this study were embodied in their actions as they participated in a reform-based mathematical community.

Current Study: Making the Transition to Teaching Mathematics

Research indicating preservice teachers' change in beliefs and attitudes toward doing, learning, and teaching mathematics is promising, yet there is evidence that these changes do not always manifest themselves in the classroom as teachers try to implement reform-based teaching. Explanations for this discrepancy can be found both in the general challenges all teachers face when implementing reform-based mathematics and in methodological issues that arise when measuring beliefs.

In a multiple case study, Raymond (1997) found that new teachers' beliefs about teaching mathematics were less traditional than their actual teaching practices, and that beliefs about the nature of mathematics were more traditional than beliefs about pedagogy. She suggests as one possible interpretation that beliefs about mathematics content may be more influential on actual teaching practice than beliefs about pedagogy. As a result, it seems the ways preservice teachers participated in the problem-solving in phase one, mathematics methods, would indicate the ways they implemented reform-based practice as teachers in the classroom. Studies of practicing teachers in California found that only one-third of the teachers reported using reform-based methods (Cohen & Hill, 1998) and that teachers in general over report the changes they make toward reform-based teaching (Stigler & Hiebert, 1997). Similarly, a case study of a preservice teacher (Eisenhart et al., 1993) revealed the complexity of reform-based teaching, showing that even though the mathematics methods course focused on teaching for conceptual understanding and this was the expressed goal of both the teachers in the placement schools and the student teacher, it was difficult for the teacher educators, classroom teachers, and preservice teacher to move beyond emphasizing procedural knowledge.

A second issue is the role of autonomy. Student autonomy is a key goal in reform-based mathematics (Yackel & Cobb, 1996) and, according to Piaget (1948/1973), is the goal of education in general. Kamii (1989) distinguishes between students who are intellectually autonomous, using relevant factors to judge mathematical ideas, and students who are heteronomous, depending on an authority to determine what is appropriate or correct. In a discussion of an inquiry-based mathematics classroom in which students are autonomous, Yackel and Cobb (1996) assert that "the teacher guides the development of a community of validators and thus discourages devolution of responsibility" (p. 473). An assumption can then be made that in a classroom which promotes heteronomy, the teacher would not distribute the responsibility with students. In addition to student autonomy, however, teachers must believe in their own freedom and ability to make instructional decisions (Cooney & Shealy, 1997) rather than relying solely on the authority of the text. In a study of seven beginning teachers, Warfield, Wood, and Lehman (2005) found that teachers who did not implement reform-based practices did not believe their students or themselves to be autonomous. It would then be reasonable to assume that the preservice teachers who were autonomous in their work on the problems in mathematics methods might better employ reform-based practice in their teaching.

Another problem is a misunderstanding of the role of the teacher and instructional practices in reform-based mathematics. Reform initiatives clearly call for "worthwhile mathematical tasks" (NCTM, 1991), tasks that include important mathematical ideas, a high level of challenge, and can often be solved in varying ways (NCTM, 2000). High level tasks are complex and longer in duration than most classroom activities, resulting in increased student learning (Doyle, 1983) and motivation (Miller & Meece, 1999). However, Chazan and Ball (1995) suggest that reform-based mathematics often gets misinterpreted as "Don't tell the answer." The assumption is that since the students are given autonomy in the classroom, the teacher simply presents the task and stands back while students work. Henningsen and Stein (1997) explored ways that high level tasks were implemented in four middle school classrooms, describing high level tasks as those with cognitive demands "that involved doing mathematics or the use of formulas, algorithms, or procedures with connection to concepts, understanding, or meaning" (p. 532). In their analysis, they noted three types of decline in the level of challenge during the implementation of high level tasks in the classroom: (a) decline into using procedures without connections to concepts, meaning, and understanding, resulting in lower cognitive demands; (b) decline into unsystematic exploration; and (c) decline into no mathematical activity.

Similarly, another assumption is that manipulatives are simply a pedagogical tool rather than a representation of the mathematics itself. Some teachers think the purpose of manipulatives is to insert “fun” into the mathematics classroom rather than as a legitimate tool for understanding mathematics (Moyer, 2001) or that by exploring with manipulatives students will naturally make the intended connections to the content objective (Ball, 1992). Others teach a rote use of manipulatives that is no more effective than rote instruction in a paper/pencil format (Hiebert & Wearne, 1992). Thus, while it is encouraging that preservice teachers broaden their views of mathematics learning and teaching as a result of experiencing reform-based mathematics, changes in those views do not ensure changes in future practice. Furthermore, it is not indicative of why some preservice teachers resist the change, why some report changing beliefs, but then do not act on them, and why some do embrace the reform-based practices in their classrooms.

Chapman (1997) argues for the need to understand what teachers bring to the table from a wider perspective than just professed beliefs. “The focus has generally been on the teachers’ theorized or espoused beliefs compared to their actions as opposed to their beliefs embodied in their actions and the personal context underlying these beliefs” (p. 204). In recent years, researchers have examined beliefs in the social context within which they occur (De Corte, Op’t Eynde, & Verschaffel, 2002), focusing on the perceived role of the teachers and the students in a mathematics classroom community and the norms that are established within the mathematical activity. The word norm is derived from the Latin word *norma*, which can be literally translated as “carpenter’s square,” a tool to measure something against a standard. Norms refer to the expectations for the teacher and students in a mathematics community. While norms are often negotiated as the teacher and students participate in mathematical activity together, the teacher is the key factor in establishing the mathematics classroom culture. The nature of the norms in a classroom ultimately determines the types of opportunities have to engage in reform-based mathematics.

Speer (2005) asserts that a lack of shared understanding of what is meant by terms such as “problem-solving” may result in inconsistencies in teachers’ reported beliefs and their observed practice. She notes that beliefs are situated in people’s experiences, and therefore, should be measured within the context of practice in which they seek to explain rather than as a separate entity. Several studies have sought to examine preservice teachers’ beliefs as they are engaging in practice. Liljedahl (2005) asked students in an undergraduate mathematics course for preservice teachers to write about an “AHA! Experience,” finding that those experiences had a role in changing the beliefs of students, who reported having more confidence and different views about what it meant to do mathematics. That is, they changed from the view of quick learning, the idea of getting the answer fast as an indication of mathematical understanding, to valuing the process of doing mathematics.

Llinares (2002) asserts that preservice teachers’ knowledge and beliefs serve two roles. They are both the lens through which they view learning and the focus of that learning. This juxtaposition was explored as four preservice teachers engaged in a community of practice in which they analyzed a case study of elementary students struggling with fraction concepts. As the preservice teachers discussed the students’ thinking they were at the same time renegotiating their own meanings of fractions and revealing their beliefs about what it means to do mathematics and their role as teachers of mathematics. Llinares found that only when beliefs were challenged during the discourse were they open to being reified. When beliefs were not challenged through the conversation, there was no need for negotiation. Thus, they may have become fuller participants in the community through their practice, but their underlying beliefs remained unchanged. Those students in phase one who resisted complexity may not have changed their beliefs because they disengaged in the challenge of problem-solving.

In a case study of a preservice secondary mathematics teacher, Lloyd (2005) found that "Todd," who expressed views in line with reform-based teaching during mathematics methods, became more focused on the role of the teacher in encouraging mathematical processes over the course of his student teaching. At first he was simply making traditional activities more student-centered by adding in group work or the use of manipulative, but then tended to remove himself from the discussion except to facilitate student presentations with a "Good job." Later in the semester, he began to change his view of his role as teacher as he became more involved in supporting student discourse about the mathematics by asking questions. Lloyd suggests having preservice teachers' engage in activities that help them reflect on the role the teacher and students play in relationship to the learning of mathematics.

These studies call for consideration of beliefs as they are related to actions. That is, research needs to consider preservice teachers' beliefs about mathematics and their role as teachers of mathematics in the context of their participation in mathematical activity, first as students in mathematics methods and later as teachers in their internship classrooms. Studies that span multiple semesters are needed to understand how preservice teachers identify with the norms of a reform-based mathematics classroom in mathematics methods and see how the meanings they make of that experience inform the norms they establish in their own classrooms.

Viewing preservice teachers' change in beliefs over time in the moment of their practice is essential to understanding their growth as teachers of mathematics, but it is not enough. If preservice teachers are asked to establish classroom communities in which their students participate in reform-based mathematics practices, then research should also explore the ways that they participate in, or perform, these practices as they learn mathematics in a reform-based community themselves and then as teachers in their internship classrooms. Noting that the construct of beliefs places emphasis on mental constructs rather than the contexts in which actions occur, some researchers use the term identity instead (Philipp, 2007). Tonso (2006) defines the identities of engineering students as "*thinking about oneself as an engineer*, *performing an engineer self*, and ultimately *being thought of as an engineer*" (pp. 273-274). Similarly, we need to understand not only what preservice teachers believe about their role as a teacher of mathematics, but how they perform themselves as teachers in the different contexts in which they find themselves over the course of their teacher education program. What norms are established in their classroom? What tasks are assigned and how are they implemented? How is mathematics content viewed?

In addition to preservice teachers' beliefs and performance, the role of the context must be taken into account. Preservice teachers are in the unique situation of being a teacher and a student in a classroom at the same time. They themselves perform as teachers, but within the boundaries of the classroom norms of the cooperating teacher. Often then, preservice teachers imagine themselves as being either similar to or different from their cooperating teachers and imagine their future trajectories as mathematics teachers when they have their own classroom. Wenger (1998) asserts that through imagination, people “envision possible futures” (p. 178), and that, unlike the common use of the word, which often implies the realm of fantasy, imagination is seeing images of self that go beyond engagement in the current activity or context. He provides the example of two stone cutters describing their work. One explains that he is cutting the slab of stone into a perfect square. The other explains that he is building a cathedral. The stonecutters are performing the same function, but make different meanings of their work. Thus, two people who perform similarly in the same context, mathematics methods, for example, may experience it differently and, as a result, imagine their current and future selves differently.

Wenger (1998) proposes that every person is a member of multiple communities of practice, has a “nexus of multimemberships” (p. 158). When the practices of these different communities conflict, the person must reconcile them all as a part of his identity. Given preservice teachers’ rapid acceleration of participation as they make the transition from student to teacher and from traditional to reform-based mathematics, understanding the ways they make meanings of their experiences and the perceived affordances and constraints of the different contexts is critical.

While beliefs are one indication of identity as a mathematics teacher, the way a teacher participates, or performs, in a mathematics classroom community and the way they imagine the trajectory of their participation (Wenger, 1998) also gives insight into their emerging identities. Thus, for this study, identity is defined as the way preservice teachers believe, perform, and imagine themselves as teachers of mathematics (see Figure 1). The goal of this study is to follow preservice teachers across the multiple semesters and contexts of their teacher education program in order to understand their emerging identities as mathematics teachers.

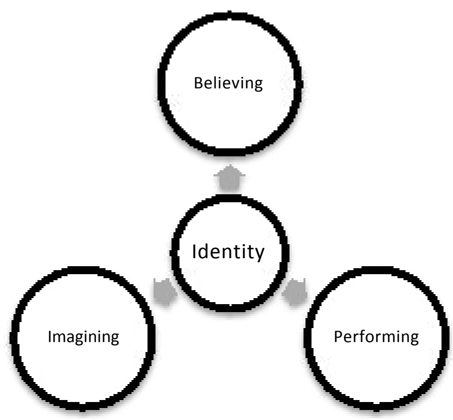


Figure 1. ***Conceptualizing Identity***

Theoretical Framework

Preservice teachers enter teacher education holding beliefs about what mathematics teachers do based largely on experiences in traditional classroom where the primary goal was to memorize procedures in order to acquire answers as quickly as possible. During methods courses preservice teachers participate in reform-based communities as learners with the expectation that they will later create reform-based communities as teachers in their internship and student-teaching classrooms. Clearly, both the individual preservice teacher and the different contexts in which they find themselves need to be considered. Research from several theoretical perspectives has focused primarily on the individual or the context. The goal of this study is to consider the individual within the context.

I rely primarily on a situative perspective (Lave & Wenger, 1991) in which knowing means participating in a community of practice; that is, people who are mutually engaged in a joint enterprise using shared repertoires (Wenger, 1998). Researchers who espouse a *participation metaphor* see learning as the action of *knowing* rather than as *possessing knowledge* (Sfard, 1998). Rather than mathematics understanding being just an endpoint to be achieved, learning is about becoming a member of a mathematical community as students work, often very publicly, toward a shared knowledge. Reform-based mathematics signals a new way of teaching and learning for most preservice teachers, which focuses on the processes of doing mathematics in addition to conceptual understanding (NCTM, 2000). Learning is dynamic and contextual (Pirie & Kieren, 1994; Boaler & Greeno, 2000). Thus, participation in mathematical practices is as much a part of the content as developing a conceptual understanding of the mathematics itself (Lampert & Cobb, 2003; Van Oers, 2001), so in addition to beliefs, preservice teachers' participation, or how they perform themselves as students in methods courses and as teachers in internship classes, is a part of their identity as mathematics teachers.

A central assumption of the situative perspective, grounded in Vygotsky's (1934/1986) work on social processes, is that a relationship exists between students' participation and conceptual understanding; that the two build upon each other. As students participate in a mathematical community, they learn by interacting with others and using material and psychological tools. According to Vygotsky, learning of a concept occurs not by memorization of knowledge, but by understanding the processes behind the concept, first through interaction (*interpsychological*) and then internally (*intrapsychological*).

Cobb, Stephan, McClain, and Gravemeijer (2001) describe the relationship between the social and psychological as reflexive, a theoretical framework which they refer to as an *emergent perspective* (see Table 1). From the sociocultural point of view, they are concerned only with the group norms and meanings made of mathematics within the classroom setting, not the political and historical associations with the discipline of mathematics which are often considered when examining the practices of a community. From a psychological point of view, they do not refer to individual students' internalization of concepts, but to individual reasoning and use of tools within the activity.

Table 1

Framework for Analyzing Collective and Individual Learning

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical interpretations and reasoning

Source: Cobb et al., 2001, p. 119

When observing a series of activities about measurement within a first grade classroom, the researchers' goal was to at the same time to document the collective understanding of the group and the reasoning of individual students as they participated in the activities (Cobb et al., 2001). The unit of analysis for this study was both the classroom mathematical practice and students' different ways of participating in that practice. The implication, then, for studying preservice teachers is that there may be collective growth among teachers, but individual teachers may participate in that growth in different ways. Therefore, learning trajectories are not so much concepts to be acquired, but trajectories of participation as members engage (or do not engage) in the practices of the multiple communities to which they belong (Wenger, 1998).

Lave and Wenger (1991) provide insight about how new members are apprenticed into a community of practice. They begin participating *legitimately* and *peripherally* by completing smaller, but necessary, tasks, and gradually come to *full participation* as they come to understand the norms and functions of the community. Newcomers to the community learn through gradually assuming more responsibility in the practice of the community. Through this participation, sometimes the culture of the community is reproduced as the newcomers participate in ways that are similar to the oldtimers, and other times, it is transformed.

Yackel and Cobb (1996) address the idea of communities of practice specifically in terms of mathematics learning. In a year-long teaching experiment in an inquiry-based second grade classroom, they described the norms negotiated in the classroom for doing mathematics, what they coined *sociomathematical norms.* These norms were established and continually changed, becoming more nuanced as the teacher and students negotiated such things as what counts as a sophisticated solution or what counts as an acceptable explanation. Through the practice of this community, students were making meaning not only of the mathematics content at hand, but of the nature of mathematics itself. They were becoming enculturated into a mathematics community of practice, what Van Oers (2001) refers to as "initiation into mathematical culture" (p. 59).

It is important to note that Lave and Wenger's theory of apprenticeship is a theory about learning, not about schooling. Yet when applied to schooling, one assumes that the teacher is the old timer, or expert, and the students are the newcomers, or novices. If they have only been exposed to traditional instruction before entering a teacher education program, preservice teachers must move quickly from the role of a newcomer in the community during mathematics methods to the role of teacher in internships and student teaching where they are expected to be the old timer or expert in the very practices in which they have recently been peripheral participants. Preservice teachers are constantly re-negotiating what it means to participate through their practice as they move among communities and roles.

Certainly, even though the norms are negotiated by all the members of the community, the teacher as the full participant has an inordinate amount of power in terms of what norms are established. As Yackel and Cobb (1996) conclude,

> The analysis of sociomathematical norms indicates that the teacher plays a central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity. It further highlights the significance of the teacher's own personal mathematical beliefs and values and their own mathematical knowledge and understanding. In this way, the

> critical and central role of the teacher as a representative of the mathematical community is underscored. (p. 475)

Teachers play such a central role in the community that studying the teachers' views and practices of doing, learning, and teaching mathematics is critical. This is especially true for preservice teachers who play such a central role after so newly joining the community. However, if knowing in communities is distributed and thus situated within activity rather than within ourselves, then studying the individual teacher from a situative perspective seems problematic. Moreover, the notion of teacher change implies learning across time and across settings. Sfard (1998), pointing to Lave's (1988) assertion that transfer is "seriously misconceived" (p. 39), proposes that from a purely situative perspective, the idea of transfer cannot exist because what is known resides in the action of doing in a particular context, not in the individual knower.

Martin, Towers, and Pirie (2006) refer to an analogy offered by Sawyer (2003) of the improvisation of a musical ensemble. Each member of the ensemble contributes creatively to the music by playing off the creativity of the other musicians playing other instruments. The resulting music emerges from the collective action of all the musicians together, but when the piece is finished, there is no end product that can be reproduced. That is, nothing that can be transferred to a new setting. In this instance, studying the actions of the individuals does not indicate the true nature of the performance, and similarly, as Martin et al. (2006) argue, it does not adequately capture the growth in mathematical understanding as it occurs collectively.

When considering this analogy, however, improvisation seems to be the key factor. Conceivably these musicians could reproduce such music if there were notes or sheet music or if they continued to repeat the sequence until it was memorized. Even in the case of improvisation, to carry on the analogy, each of the musicians is also bringing the skills of playing the instrument, the understanding of the genre of music being improvised, and the practice of listening to others and playing within the emerging group structure. In other words, past experiences and practices are coming to bear on present participation.

Therefore, despite a common perception that the individual is not considered in situative theory, the individual is not discounted. After all, a community is made up of individual members. Wells and Claxton (2002) maintain that participants in a shared activity have different goals and values and that these different goals are often the sources of transformations of communities as they are negotiated. They stress that an individual's action incorporates the whole person rather than just the mind. The individual is not viewed as entirely independent of the context as in radical constructivism in which all knowledge is situated internally (von Glasersfeld, 1995). Rather the individual is always acting, is being, within a social context, what Wenger (1998) terms *identity.*

The term identity has gained popularity in the last few years, and is often used in educational research without clear definition (Sfard & Prusack, 2005). In some instances, the term identity is used in a way that remains primarily cognitive in nature. For instance, Collopy (2003) defined identity as "the constellation of interconnected beliefs and knowledge about subject matter, teaching, and learning as well as personal self-efficacy and orientation toward work and change" (p. 289). Beliefs are an important part of identity, but they are not the only part. Wenger (1998) asserts that identity is the intersection between the individual and the social, allowing each to be examined in relation to the other. Identity is influenced by the communities in which we practice and the nature of participation in those communities. From this perspective, individuals can never truly be separated from the social because the contexts in which they find themselves afford and constrain their practice and the meanings they make of their experiences.

Consider the experience of preservice teachers over the course of their teacher education experience. Whether or not they can promote the practices of reform-based mathematics depends not only on the meaning they made of their participation in mathematics methods and their prior mathematics experiences as a student, but also on the factors in the school setting that may help or hinder such practice. For example, are they required to strictly follow a textbook? Does the cooperating teacher use traditional methods of teaching and expect the same from her interns? In this way, the preservice teacher's actions are inevitably tied both to her own meanings and to the context, not to one or the other. Preservice teachers are constantly re-negotiating what it means to participate through their practice as they move among communities and roles.

Wenger (1998) discusses both participation and non-participation citing a difference between *peripherality* and *marginality.* By virtue of being newcomers, preservice teachers may be operating on the periphery and therefore performing themselves with a mix of participation and non-participation. The assumption is that eventually they will move towards *full participation*, or at least continue with *legitimate peripheral participation*. However, in some cases, the non-participation restricts rather than supports future participation, and the member is marginalized or leaves the community. For example, after participating in non-routine problem-solving tasks in mathematics methods, a preservice teacher who still holds onto an absolutist view of mathematics as a fixed set of knowledge that does not require critical thinking is not likely to later create a classroom community that promotes reform-based mathematics.

Cobb et al. (in press) propose the constructs of *normative identity* and *personal identity*. They propose viewing identity in terms of normative identity and personal identity. Normative identity is what it means to be a legitimate participant, a "math person," based on the socio-mathematical norms (Yackel & Cobb, 1996) in a particular classroom community. In contrast, personal identity refers to the way that an individual interacts within the community, the degree to which students resist, cooperate, or affiliate with the normative identity of the community. Cobb et al (in press) followed 11 eighth-grade students who took two different mathematics classes at the same time, one on statistical data analysis which was a classroom design experiment taught by the researchers, and an algebra class taught by their regular mathematics teacher. The normative identities of each classroom and personal identities of the students were documented through observations and video-recording of classes and student interviews. Students reported differences in the ways of doing mathematics in each of the classrooms. While they merely cooperated in the algebra classroom, most developed a sense of affiliation in the design experiment class in which they felt a sense of conceptual agency and shared authority.

The constructs of normative and personal identity offer promise in studying preservice teachers across contexts because it allows the researchers to understand how they are participating in the multiple communities in which they find themselves. For example, a preservice teacher who resisted the complexity of mathematical practices in mathematics methods might very well embrace and reify the traditional context of a cooperating teacher's classroom. Two preservice teachers may leave mathematics methods with the same beliefs and intentions when they start teaching, but one may acquiesce when presented with constraints while another perseveres. Part of a preservice teacher's identity, then, is the performance of self given within and against the normative identities of the different communities to which they belong across their teacher education program. In addition, because preservice teachers are implementing the norms of their cooperating teacher's classroom, how they view themselves as teachers of mathematics both in relation the current context and in their future classrooms, how they imagine themselves as teachers of mathematics, may be different than their performance of self.

In sum, elementary preservice teachers must make two shifts, from student to teacher and from traditional to reform-based mathematics teaching. If they have only been exposed to traditional mathematics instruction, they enter as *legitimate peripheral participants* (Lave & Wenger, 1991) in both the teaching and reform based mathematics communities. As they engage in the use of material and psychological tools and interact with others in methods courses and internships, their identities as teachers of mathematics emerge. That is, the ways they believe, perform, and imagine themselves as teachers of mathematics change.

To understand how preservice teachers might identify with the practices of reform-based mathematics, I examined preservice teachers' trajectory of participation as they engaged in multiple contexts across time. The current study was the second phase of a two-year study of a group of preservice teachers as they moved through their teacher education program. Phase one examined the relationship between preservice teachers' participation in mathematics methods and their beliefs about the role of mathematics teachers. The current study explored how preservice teachers' participation and beliefs in methods were related to their performance and imagination of self during their internships and student teaching.

Dissertation Study

Research Goals

One goal of mathematics education research is to understand how to help teachers implement reform-based mathematics set forth in the Principles and Standards for School Mathematics (NCTM, 2000), particularly when they often experienced more traditional mathematics instruction as students. Many studies have reported shifts in preservice teachers' beliefs about doing, learning, and teaching mathematics during methods and content courses. Yet this change in beliefs does not always manifest itself during instruction as preservice teachers move to internships and student teaching. Thus, research needs to consider participation in the different contexts over the multiple semesters of a teacher education program to understand how preservice teachers' identities emerge in terms of the ways they believe, perform, and imagine themselves as teachers of mathematics. This dissertation study built on the results from the background study.

The purpose of this study was two-fold: (a) to explore the ways preservice teachers' participation and beliefs in a reform-based community (mathematics methods) related to their beliefs about their role as teachers and their instructional practice, and (b) to understand how their mathematical identities change over time and how those changes were manifested in and influenced by the different contexts in which they find themselves. Specifically it addressed the following research questions:

1. How do elementary preservice teachers' participation and beliefs as students in a reform-based mathematics community indicate the ways they perform, believe, and imagine themselves as teachers in their own mathematics classroom community?; and

2. How do preservice teachers change over time in multiple settings the ways they believe, perform, and imagine themselves to be as teachers of mathematics?

Better understanding the role that participation in different contexts plays in preservice teachers' emerging identities is significant for teacher educators, who may be able to better facilitate experiences that help preservice teachers identify with reform-based teaching, and for policy makers who can influence larger issues of context to create more supportive environments for teacher change.

Definition of Terms

community --a group of people who are mutually engaged in a joint activity with a shared repertoire (Wenger, 1998)

mathematical practice	--any process associated with the doing or learning of mathematics, including, but not limited to explanation, questioning, listening, representing, reasoning and proof, making and testing conjectures, making connections, problem-solving, and mathematical argumentation
participation	--acting in a mathematical community; in the background study, the way preservice teachers engaged in mathematics methods (resisting, acknowledging, embracing, creating)
identity	--how preservice teachers believe, perform, and imagine themselves as teachers of mathematics
beliefs	--understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior (Schoenfeld, 1992, p. 358) and the teaching of mathematics
performance	--classroom instructional practice as measured by ratings on the performance observation framework
imagination	--how a participant sees him or herself in relation to the context (in this study to the cooperating teacher, the curriculum, and the discipline of mathematics)
context	--the situation surrounding the action of teaching including, but not limited to the tools, norms, participants (students and adults), and practices of a particular setting; in this study, particular features of the context emerged from the qualitative data using domain analysis including the norms and practices of the cooperating teacher, the curriculum, and the nature of mathematics promoted.

CHAPTER II

METHODOLOGY

This study examines how preservice teachers' participation and beliefs as students in a reform-based mathematics community related to the ways they performed, believed, and imagined themselves as teachers in their own mathematics classroom community, and how their performance, beliefs, and imagination as mathematics teachers changed over time in multiple settings (mathematics methods and internships).

Research Sites

The participants completed their internships and student-teaching in the same classroom at one of two schools. Six participants were at Hunter Elementary, a Title I Professional Development School. Hunter Elementary teachers were in their fourth year of implementation of the Everyday Mathematics program. The Everyday Mathematics Program was implemented school-wide, and the curriculum was strictly followed. Developed through a National Science Foundation grant, Everyday Mathematics emphasizes problem-solving, communications, hands-on activities, and multiple methods for computing (Isaacs, Carroll, & Bell, 2001). However, from informal observations and conversations with a visiting Everyday Mathematics consultant, it was clear that at Hunter Elementary School, the program was being implemented school-wide in a traditional direct instruction format. This is in line with findings that teachers enact the

Everyday Mathematics curriculum in ways that are different from the intended curriculum (C. Ditto, personal communication, 2007).

Two participants were at Jones Elementary. The teachers at Jones Elementary used common lesson plans created by a teacher in the district and distributed among the elementary schools. Teachers at Jones implemented these lessons in varying ways, some following them strictly and others using them as a resource. The two cooperating teachers in this study reported using them as a resource for activities and as a guide to keep on pace with the other teachers. One teacher employed more reform-based strategies while the other was more traditionally oriented.

Participants

The participants were eight preservice teachers (7 female, 1 male) from phases one, the mathematics methods course. I used *purposeful sampling* (Patton, 1990, p. 169) to ensure that two preservice teachers were chosen from each of the identities from phase one: resisting, acknowledging, embracing, and creating. A variety of grade levels were represented including one preservice teacher in kindergarten, one preservice teacher in second, three in third grade, and two in fourth grade.

Measures

To understand how the preservice teachers performed and imagined themselves over the course of their internships and student teaching and how their beliefs changed, observations, field notes, interviews and written reflections were collected and analyzed. All observations and interviews were audio-taped. The observations were the primary measure of performance. The other data sources were used to triangulate conclusions. The interviews and written reflections were also used to measure imagination.

Observations

Observations were measure using the performance observation framework (See Appendix B). The performance observation framework has multiple continua in three broader categories: the norms promoted in the classroom, the structure and implementation of the tasks, and the nature of the mathematics. Specifically in terms of the mathematics classroom, the norms answer the question, “What does it mean to be a good mathematics student in this classroom?” The continua in the norms category include how learning is promoted, the goals for students, the reasons for doing mathematics, the level of independence, and what counts as explanation. The second category on the performance observation framework is task structure and implementation. More simply, what are the students being asked to do and how are they being asked to do it? The continua in this category include task structure, duration, grouping, participation, and level of difficulty. The third category on the framework is focused on mathematics. How is mathematics viewed and practiced? The continua in the mathematics category describe the nature of mathematics, the use of representations, and practices of doing mathematics.

The performance observation framework was developed to examine the specific components of the lesson and the instructional practices that preservice teachers employed. For each continuum, a four point rating scale was created ranging from traditional, procedure-driven instruction (1) to reform-based instruction (4). The descriptions for each rating were based on existing research and on conjectures from early data by focusing on areas where differences in lessons among preservice teachers were particularly noticeable. The differences observed were then examined in relation to the relevant literature on grouping and collaboration to create the four points on the grouping continua. This process was followed for each of the continua. To explicate this process, I will describe in detail the development of the *level of independence* continua on the framework.

The level of independence continuum focused on autonomy. Student autonomy is a key goal in reform-based mathematics (NCTM, 2000; Yackel & Cobb, 1996) and, according to Piaget (1948/1973), is the goal of education in general. Autonomy is the idea that students should be self-regulated, that they have control over their own learning. Students who feel autonomous tend to exhibit more intrinsic motivation, curiosity, and preference for challenge in the classroom (Deci, Schwartz, Scheinman, & Ryan, 1981). Webster's Dictionary defines autonomy as self-governance, so it may seem strange to propose that students should be encouraged to be autonomous in order to increase their participation in a community. Yet in self determination theory, Deci and Ryan (2000) argue that a when students' basic needs for autonomy, relatedness, and competence are met, they are then able to *internalize* (Deci & Ryan, 2000) the values of a community into the self. Being autonomous allows students the freedom to engage in mathematics in a meaningful way.

Kamii (1989) distinguishes between students who are intellectually autonomous, using relevant factors to judge mathematical ideas, and students who are heteronomous, depending on an authority to determine what is appropriate or correct. In a discussion of an inquiry-based mathematics classroom in which students are autonomous, Yackel and Cobb (1996) assert that "the teacher guides the development of a community of validators and thus discourages devolution of responsibility" (p. 473). An assumption can then be made that in a classroom which promotes heteronomy, the teacher would not distribute the responsibility with students.

Hufferd-Ackles et al. (2004) address the idea of autonomy in their levels of math talk framework. One component of the framework is *Source of Mathematics Ideas* which describes the change among levels from the teacher being the source of ideas raised in class to a climate in which student ideas are valued and explored. Just as noted in their framework, early lessons were observed in which the teacher was the only source of ideas, others in which student examples were used, and still others where ideas were taken up by the whole class. For example, when one of the preservice teachers taught volume, she was the source of ideas, having students copy definitions from the board and then using the given formula to calculate volume.

Another preservice teacher used student ideas and incorporated them into his own activity. Students in the fourth grade class were doing a partner activity in which they add fractions with a denominator of ten. The teacher told each partner to pick a number between 1-10 to use as the numerators for the addends and then find the sum. As students were choosing numbers, some partners realized quickly that if they chose lower numbers, they would not have to deal with mixed numbers. The teacher told these groups they had to pick numbers above 5. He allowed more student autonomy by incorporating their ideas and adjusting his planned activity accordingly.

Finally, preservice teachers sometimes used student ideas to launch whole group discussions. In a kindergarten class in which students were instructed to practice making the numbers 0-9 with play dough. A student at one table decided to make two-digit numbers. First she made 18 and then 10. The student next to her noticed and said to the classroom teacher, “How do you make ten?” Both students continued to make two digit numbers. In the sharing after the activity, the teacher had the students share the number 10 and led a whole class discussion on the meaning of 10. An early version of the framework, then included three descriptors (See Figure 2).

Teacher is the only source of ideas.	Teacher gives up some authority and student examples are used.	Students raise ideas that are taken up by the whole class.

Figure 2. ***Early Version of the Levels of Independence Continuum on the Performance Observation Framework***

As lesson observations progress, constant comparative analysis (Miles & Huberman, 1994) was used to refine codes. One nuance observed in subsequent lessons was the source of ideas for the preservice teachers. Within the descriptor which labeled the teacher as the only source of ideas, some preservice stayed strictly with the text while others used other resources and their own ideas. This observation is in line with research about teacher autonomy being important in addition to student autonomy. Teachers must believe in their own freedom and ability to make instructional decisions (Cooney & Shealy, 1997) rather than relying solely on the authority of the text. In a study of seven beginning teachers, Warfield et al. (2005) suggested teachers who did not implement reform-based practices do not believe their students or themselves to be autonomous. Thus, the first descriptor was separated into two different descriptors (See Figure 3).

Teacher stays strictly with the problems in the text.	Teacher may introduce problems from sources other than text, but is still the only determiner of what will be discussed.	Teacher gives up some authority and student examples are used.	Students raise ideas that are taken up by the whole class.

Figure 3. ***Levels of Independence Continuum Modified***

Another nuance observed was the recognition that teachers gave or did not give when student ideas were solicited. Sometime even when student examples were solicited, they were not used. Only the examples that were aligned with the teachers' goals were accepted. For example, in one lesson, second grade students were asked to write some doubles facts they knew on their paper. The teacher then had students share their doubles fact as she wrote them on the board. All the student facts were recorded on the board except for 10,000 + 10,000 = 20,000 which was relegated as "too big," and not written down. In this case, the teacher only recognized the examples that supported her plan. She was still the only determiner of what would be discussed. The students, then were another possible source for ideas other than the text, but still were not given authority. To account for this nuance, the phrase "including students" was added in parenthesis in the second descriptor. After all the revisions were made, all the lessons were re-coded with the final version of the continuum.

The importance of autonomy, both for students and teachers, is clear in reform-based mathematics. The continuum for *levels of independence* on the performance observation framework ranges from an external source, such as the text, as the judge of what is relevant on one end to the students as the source of ideas on the other end.

Across the internship lessons, continua were adjusted until a saturation point was reached (Glaser & Strauss, 1967). All lessons were then re-coded with the final framework. Lessons were coded by independent researchers with an interrater reliability of 90% over 5 lessons. In theory, preservice teachers' ratings on the performance observation framework would correspond to their orientations at the end of mathematics methods, resisting scoring the lowest and creating the highest. Continua were grouped into three like categories, norms, task structure, and mathematics.

Classroom norms refer to the expectations of students and the teacher. The first continuum describes the ways learning is promoted from a reflexive view, requiring quick answers, to a reflective view that requires students to reflect on their work and persist through difficulties. The goals for students range from imitation of procedures presented by the teacher to student responsibility for discussions and thinking. On the next continuum, the teacher may not give students a reason for doing mathematics or they may give extrinsic or intrinsic reasons, or both. The level of independence refers to the amount of autonomy given to students as the source of ideas in the classroom. Norms for what counts as sufficient explanation vary from students claiming understanding with yes or no answers to students explaining their own thinking and commenting on others' work.

Task structure and implementation is the second category on the performance observation framework. Task structure refers to whether the task is closed, an exercise with a single solution, or open, with multiple solutions and mathematics connections. Duration refers to the length of time given for the task. Grouping examines the ways students are or are not working with each other and whether there is follow up with the whole class just to check answers or for a consensus of ideas. Participation is analyzed based how much the teacher expects and encourages students to engage. For example, the teacher may only call on volunteers or may foster active participation from most students in whole group discussion. A teacher might even support students during individual and partner work so they can share later in the whole group. Finally, the level of difficulty describes the amount of challenge for students, ranging from tasks that include material the students already know to tasks that challenge students to push further even when they get the correction solution.

The last category on the performance observation framework includes the continua related to content. The nature of mathematics continuum describes views of the nature of mathematics from fixed and procedural to dynamic and conceptual. The use of representations ranges from one or more representations focused only on procedure to multiple representations that inform conceptual understanding. The practices of mathematics refer to what doing mathematics means in the classroom. In some classrooms, the practice of doing mathematics is simply completing exercises. In other classrooms it involves making and testing conjectures, argumentation, and problem-solving.

Field Notes

Field notes taken during each observation included a detailed account of what the preservice teacher, students, and if applicable, the cooperating teacher said during the lesson, what was written on the board, the problems presented to students, and the tools used (See Appendix C for sample) the field notes were used in two ways. First, the field notes were used in the post-lesson interviews with preservice teachers to ask about preservice teachers' beliefs and decision-making. For example, when Allie was introducing a lesson one of her third grade students blurted out, "Oh, we did that in first grade." He then proceeded to explain the whole activity. Indeed, most students in the class were quickly able to solve the problems. During the interview, the episode was used to ask Allie about the level of challenge she wanted to provide her students and how she saw this lesson in relation to that level. In this instance, Allie was satisfied with this activity as an appropriate challenge for her students.

Secondly, the field notes were used to confirm ratings on the performance observation framework. For example, in a lesson on order of operations, Andres gave each table three dominoes and asked them to figure out the total number of dots. Groups were asked to record their thinking on the board. In this case, records of what was written on the board and how students were using manipulatives informed the use of representations during the lesson.

Interviews

Preservice teachers completed two types of interviews: a post lesson interview after each observation and a final interview at the end of student teaching. Cooperating teachers completed one interview during the student teaching semester.

Post-lesson interviews. Post-lesson interviews were conducted with the preservice teachers for each lesson observed (See Appendix D).The goal of the interviews was to understand the thinking, beliefs, and contextual influences behind decisions made by the preservice teachers before and during the lessons and to examine how teaching the lesson influenced current beliefs about teaching mathematics. These semi-structured interviews began with a general inquiry into how the preservice teacher felt the lesson went with follow-up questions relating to any issues raised. Next, specific instances were pulled from the field notes and shared with the preservice teacher with questions regarding planning before the lesson and/or decision-making during the lesson. These instances, which often were used to confirm ratings on the performance observation framework, were chosen for a variety of reasons including, but not excluded to, cases in which the preservice made a change from past teaching, seemed unsure, made a change to the lesson midstream, or had an interesting exchange with a student. The interview ended by asking preservice teachers how teaching the lesson changed or confirmed their beliefs about teaching mathematics.

Final interviews. Each preservice teacher also completed a final interview at the end of student teaching (See Appendix E). They were asked the same questions from their final assignment in the methods course, attending to the preservice teachers' definition of mathematics, beliefs about how students learn mathematics and their role as teachers of mathematics. A second set of questions addressed the preservice teachers' participation in the classroom in which they were student teaching including whether or not they felt their teaching matched the beliefs they had espoused in response to earlier questions, how their beliefs changed over the course of the two years and what factors influenced the change, and the aspects in the context they felt afforded or constrained their ideal view of mathematics teaching. Finally, preservice teachers were asked about future goals for their mathematics teaching.

Cooperating teacher interviews. In addition, cooperating teachers were interviewed about their student teacher's mathematics teaching (See Appendix F). They were asked what feedback they gave to given to their student teacher about a formally observed mathematics lesson and about their mathematics teaching in general. In the course of the discussion, cooperating teachers were also asked about their own mathematics teaching including views of the curriculum, student learning, and a typical lesson. These questions were asked to better understand the norms of the classroom in which the preservice teacher was student-teaching and to confirm preservice teachers' perceptions of their student-teaching classroom.

Written Reflections

All preservice teachers completed written reflections (See Appendix G) for every mathematics lesson taught during the internships. The goals of these reflections were the same as the goals of the post-lesson interview and served as another source of data to confirm findings. They were completed before post-lesson interviews. In the reflection, preservice teachers were asked to explain whether or not the lesson was successful and why, what critical decisions were made during the lesson, if the students met the goals and how they knew, students' and their own participation in discussions, and how their beliefs were being confirmed or changed.

Procedures

Observations

Each preservice teacher was observed six to eight times over the course of the internships and student teaching. Observations were audio-taped and coded according to the performance observation framework. Observations ranged in length from thirty minutes to an hour and a half, depending on the grade level. The performing framework was coded by the observer at the end of the lesson. When multiple observers were present, they coded independently.

Field Notes, Interviews, and Written Reflections

Field notes, interviews, and written reflections were used to support the ratings given on the performance observation framework. Field notes taken during the lesson were approximately five to eight pages in length, typed, single-spaced. They were read the end of the lesson and marked for instances to share with preservice teachers during the post-lesson interviews. In a second reading, field notes were coded as descriptive evidence for the ratings given on the performance observation framework.

Preservice teachers completed written reflections on Blackboard the day of the lesson. The average reflection was two to four pages in length. Interviews were audio-taped and transcribed. Post-lesson interviews were approximately 20 minutes in length.

Final interviews were approximately 45 minutes in length. In addition to providing descriptive data and confirming the ratings on the performance observation framework, the data was also used to address how preservice teachers imagined themselves as teachers of mathematics.

Like the performance measure, the way preservice teachers imagined themselves as teachers of mathematics was measured in terms of norms, tasks, and mathematics. However, in this instance, I examined the norms, tasks, and mathematics related to the context. Specifically, I investigated how preservice teachers perceived themselves in relation to the norms of the cooperating teacher and the tasks presented in the curriculum. I also examined their beliefs about mathematics and about their role as teachers of mathematics.

The influence of the cooperating teacher and the curriculum were measured in terms of whether the preservice teachers perceived them as an affordance or constraint to practicing their ideal view of mathematics teaching. The goal was two-fold: to understand how the preservice teachers' identity affiliated with or were different than the norms of the student teaching classroom and to determine factors that influenced change. Interview and written reflections from both preservice and cooperating teachers were coded using a domain analysis (Spradley, 1980) with strict inclusion (X is a kind of Y) as the semantic relationship and "affordance" and "constraint" as the cover terms. For example, the multiple strategies included in the text were a kind of affordance. The pressure from a cooperating teacher to teach to the standardized test was a kind of constraint. Codes were refined throughout the semester using constant comparative analysis (Miles & Huberman, 1994). Coding was completed until saturation (Glaser & Strauss, 1967) was reached.

The second research question explores how preservice teachers' beliefs about mathematics and their role as teachers of mathematics changed across the multiple settings (mathematics methods, internships and student teaching). Reported beliefs were coded as resisting, acknowledging, embracing, or creating, using the taxonomy developed in phase one (See Appendix A). Beliefs from the final interviews were compared to beliefs at the end of mathematics methods using this coding taxonomy.

Validity

In this study I was a *complete participant* (Spradley, 1980), meaning that I researched a situation in which I was already an ordinary participant. During phase one, I was the instructor for the mathematics methods course, and in phase two, I was the student teaching supervisor for each of the preservice teachers. While this has the advantage of allowing me to see the whole picture, my role as an insider presented several threats to validity. First, preservice teachers may have been reluctant to express dissenting views in the mathematics methods course since my bias was clearly towards promoting reform-based mathematics instruction. Several measures were taken in the pilot studies to help counteract this problem. Preservice teachers were explicitly encouraged to write their true thoughts in journal entries, even if they disagreed with the ideas in the course, and the journals were given only a completion grade. Given the response from the students in the resisting category that they were not learning how to teach math in class and liked the way they learned instead, it seems that students did not feel pressure to state what they thought was expected. In addition, students answered questions on the final assignment twice, once from the perspective they felt was being advocated by the class and once from their own perspective. During the internships and student-teaching, outside researchers served as observers and coded data to ensure consistency and counteract bias on my part. The observations served as the main source of data, and conclusions were confirmed through a variety of other sources including the interviews, field notes, and interviews were used to confirm the ratings on the performance framework. Finally, member checking (Lincoln & Guba, 1985) was used to verify interpretations of data.

CHAPTER III

RESULTS

The purpose of this study was two-fold: (a) to understand how preservice teachers' performed, believed, and imagined themselves as teachers of mathematics related to their participation and beliefs in mathematics methods, and (b) to explore how preservice teachers changed over time in terms of the ways they believe, perform, and imagine themselves to be as mathematics teachers. I first share results from statistical analysis of preservice teachers' scores on the performance observation framework. Next, I use data from the observations, interviews, and written reflections to provide a description of how the teachers in the different orientations performed themselves through their instructional actions and the beliefs associated with them and how they imagined themselves as mathematics teachers in relation to the context of their placements.

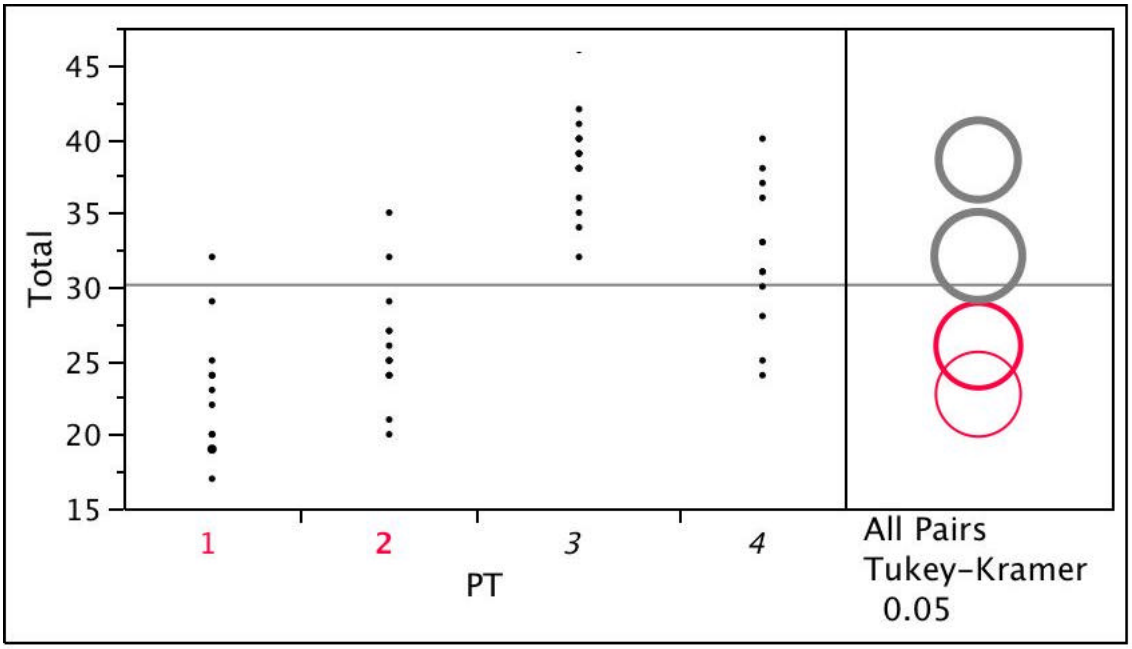
Statistical Analysis

Question 1

The first research question examined how preservice teachers' participation and beliefs in a reform-based mathematics methods course related to the ways they performed, believed, and imagined themselves as teachers during internships and student teaching. Findings indicated that the preservice teachers performed in ways that related to their participation and beliefs from methods. They changed their beliefs about their role as teacher in patterns that were reminiscent of the beliefs expressed in the methods course. The preservice teachers imagined themselves in relation to the context in different ways, but there was no discernible pattern among the four orientations.

I used qualitative data to examine preservice teachers' beliefs about teaching and the ways they imagined themselves within the context. This data will be discussed in the next section. When investigating their performance, I used the scores from the performance observation framework to determine if there were differences among the four orientations, resisting, acknowledging, embracing, and creating. First, I found a total score for each observed lesson by adding the scores assigned for each continuum on the framework. Because the small sample size does not permit enough information to investigate the population distribution to determine if normality assumptions are warranted, I used the Kruskal-Wallis Test, a nonparametric version of a one-way ANOVA. This test assumes that the means of the four groups are equal and compares group mean scores to the total mean. The p value was less than 0.001, indicating that a significant difference exists in the performance of at least one of the groups (See Figure 4). Therefore, I ran a multiple comparison of all pairs with a Tukey-Kramer Test, with significance indicated at the 0.05 level. These post hoc comparisons assuming normality indicated that the performance of preservice teachers in the resisting and acknowledging groups (1 and 2) did not differ significantly from each other. Preservice teachers in the embracing and creating groups (3 and 4) performed significantly different from each other and from the acknowledging and resisting groups.

One-way Analysis of Total by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level				Mean
3	A			38.562500
4		B		32.076923
2			C	26.000000
1			C	22.714286

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	15.84821	11.9252	19.77126	
3	2	12.56250	8.6395	16.48555	
4	1	9.36264	5.2337	13.49153	
3	4	6.48558	2.4829	10.48829	
4	2	6.07692	1.9480	10.20581	
2	1	3.28571	-0.7660	7.33742	

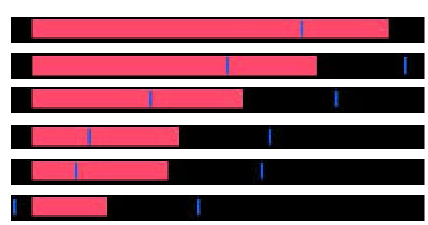


Figure 4. ***Results of One-way Analysis of Total Scores by Orientations***

In regards to research question one, the performance of preservice teachers was related to their participation and beliefs in mathematics methods.

To further understand the nature of the differences among the groups, the same process was used to analyze group differences for each continuum (See Appendix H). The scores of the preservice teachers in the embracing orientation (group 1) were significantly higher than those in the acknowledging and resisting orientations (groups 2 and 1, respectively) on every continuum. However, the scores for the creating orientation (group 4) varied in the way they differed from the other groups. The mean scores (see Figure 5) for the creating group fell in between the embracing group and the other two groups on all the continua except reasons for doing mathematics and duration, an anomaly that will be addressed later in the discussion.

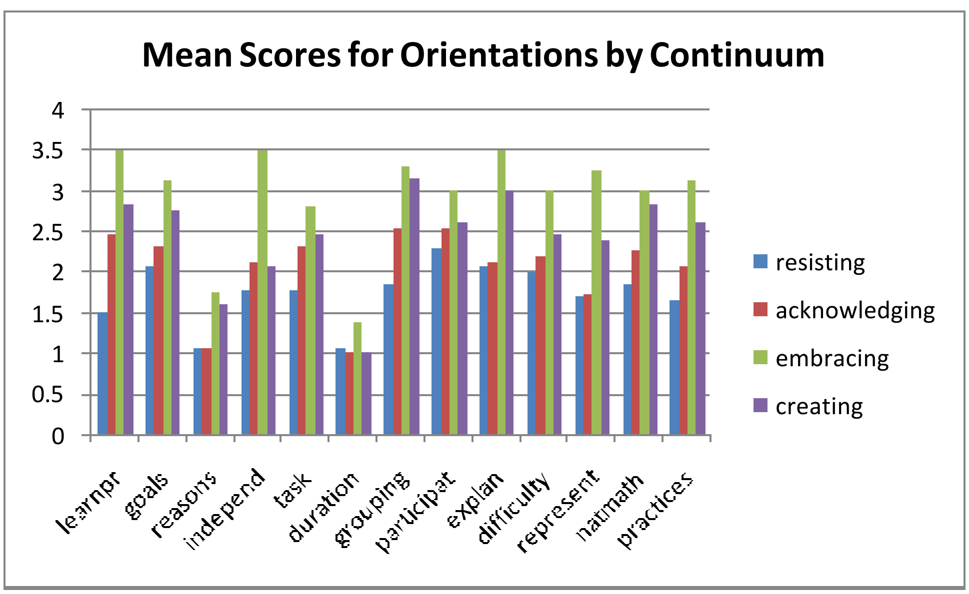


Figure 5. ***Mean Scores for Orientations by Continuum***

The analysis by continuum indicated the ways the creating group's scores differed from the other groups. On the continuum that addressed how learning was promoted, the creating group and the acknowledging group were not significantly different from each other, but were significantly lower than the embracing group and higher than the resisting group. On the tasks and difficulty continua, the creating group lay in between the other groups with no significant difference. For example, on the tasks continuum, the creating group was not significantly different from the embracing or the acknowledging groups, although the two groups were significantly different from each other. Only the acknowledging group was not significantly different from the resisting group. Similarly, on the level of difficulty continuum the creating group was not significantly different from any of the other groups.

In two instances the creating group was significantly different than the embracing group only. On both the level of independence and participation continua, the creating, acknowledging, and resisting groups were not significantly different from each other, but were significantly lower than the embracing group.

Finally, in six of the continua, the embracing and creating groups were not significantly different from each other, but were significantly different from the acknowledging and resisting groups, which did not differ. These continua were goals for students, grouping, explanation, representations, the nature of mathematics, and the practices of doing mathematics. It is important to note that this includes all the continua in the mathematics category of the framework.

In sum, the orientations of preservice teachers from phase one differed significantly from each other on the performance observation framework that was used to measure instructional practice. According to the analysis based on the total scores, the embracing orientation scored significantly higher than all the other groups and the resisting and acknowledging orientations scored significantly lower than the other groups with the creating orientation in the middle, differing significantly from the other groups. Then, analysis based on each continuum indicated that the differences among groups for each continuum were similar to those found for the total score for those in the embracing, acknowledging, and resisting groups. The analysis by continuum did provide further information about the way the preservice teachers in the creating orientation varied in their performance. The nature of these differences will be further elaborated during the discussion of the qualitative data.

Question 2

The second research question investigated how preservice teachers changed their performance, beliefs, and imagination as mathematics teachers over time. Interviews, field notes, and written reflections were used in addition to the scores from the performance observation framework to explore this question. To examine change over time on the scores on the performance observation rubric for each orientation, I used Kendall's Tau, the nonparametric equivalent to correlation. Like the Wilcoxon test, it uses ranks rather than values, so it is appropriate for ordinal data, and it makes very few distributional assumptions. For each orientation group, the mean score for each continuum was correlated with the lesson observation number (See Appendix I). The assumption was that if there was no change over time, then the correlation would not differ significantly from 0. The only group with significant correlations indicating change over time was the creating complexity orientation ($p<0.05$ for tasks, level of independence, explanation, and practices of doing mathematics).These changes, however, do not indicate an upward trend, but rather inconsistency in performance. This inconsistency will be discussed as I report findings from the qualitative data. Although the scores on framework did not indicate a change over time in the ways the preservice teachers performed themselves, evidence from the qualitative data suggests that they did change in the ways they imagined themselves as teachers of mathematics. These changes will be discussed in the next section.

Qualitative Data

A theme has been selected for each of the four orientations to capture the overall sense of the data. In this section I will first tell why the theme was selected. Then, I will describe for each orientation how the preservice teachers performed, imagined, and believed themselves to be during their internships and student teaching. Performing included the classroom norms established, structure and implementation of tasks, and the nature of the mathematics. Imagining included how the preservice teachers related to the norms of the cooperating teachers, how they worked with the curriculum provided, and what they believed about mathematics and their role as a mathematics teacher. I will also give examples of how the preservice teachers related to their participation and beliefs in mathematics methods. Rather than discuss the two research questions separately, evidence of change will be imbedded into the discussion for convenience.

When looking at mean scores for each continuum by orientation (See Figure 4), it is clear that all the groups scored low on two continua, reasons for doing mathematics and duration of the task. On the reasons for doing mathematics continuum, preservice teachers scored a one if they did not explicitly give a reason, and the majority of the time, this was the case. Also, due to the expectation in both schools that the curriculum would be followed closely with a lesson per day, rarely did the duration of tasks last more than one day. Therefore, these continua will not be discussed separately for each of the four orientations.

Dead Monkeys Smell Bad: Resisting Complexity

Preservice teachers in this orientation resisted complexity in mathematics methods, disengaging in problem-solving activity when they encountered challenge and continuing to see the role of the teacher as the authority in the classroom who presents mathematical procedures for students to learn. An archetypal example was Lisa's instruction of long division in which she ignored the hands-on activity focused on developing a conceptual understanding of division and instead presented the phrase "dead monkeys smell bad" as a way to remember the steps for the algorithm (divide, multiply, subtract, bring down). This focus on mathematics as a set of procedures to be presented by the teacher and imitated by the students continued to be prevalent in the classroom teaching both preservice teachers.

Performing

As noted in Figure 6, mean scores on the performance observation framework ranged from 1.5 to 2.09 for preservice teachers in this category.

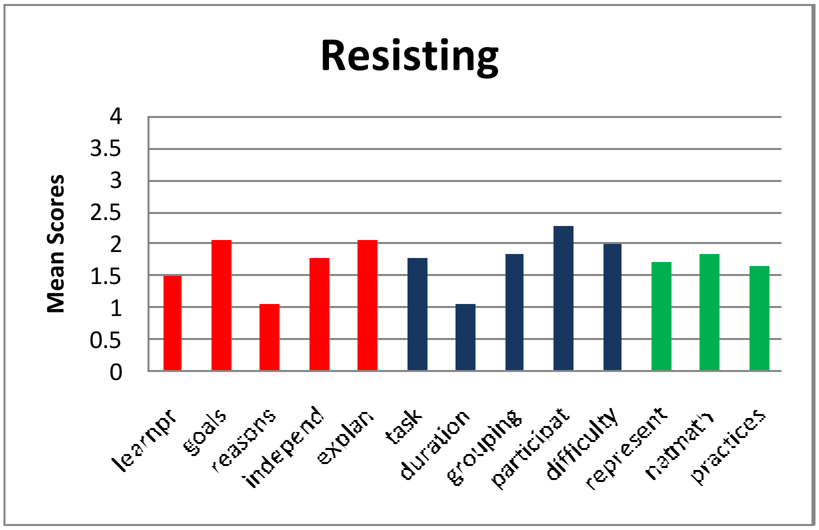


Figure 6. ***Mean Scores for the Resisting Orientation***

Norms. The norms refer to the accepted ways of learning mathematics in the classroom. The norms of the classroom matched the traditional classroom that both preservice teachers advocated during mathematics methods.

Learning was promoted as a reflexive endeavor focused on getting the right answer to problems (1.5). The level of independence for students was low (1.79) with the teacher and the text as the sources of ideas. This level of independence was intentional. In response to a prompt on the written reflection asking the preservice teachers to describe their own participation and that of their students, Rachel responded, “I was the supplier of information while students gave little feedback.” Rachel and Lisa mainly introduced problems from the text, but used other resources occasionally. When students were asked to offer ideas, they were only recognized if they fit into the teacher’s plan. For example, Rachel asked second grade students to write some doubles facts they knew on their papers. After they listed the facts, she asked students to share while she recorded their doubles on the board. Several facts had already been written on the board, including 80 + 80 = 160, 13 + 13 = 26, and 100 + 100 = 200, when a student offered a large fact, 10,000 +10,000 = 20,000. Rachel responded, “Good. I’m not going to write that because it’s really big.” She continued writing facts on the board as the next student offered 0 + 0 = 0.

The goal for students (2.09) was usually to imitate the procedure presented by the teacher, and as such, most discussion was limited to clearing up misunderstandings. However, both Rachel and Lisa did ask students for explanations (2.09), although they did not usually follow up to clarify student thinking. They focused on explanation of procedure without linking it to the conceptual understanding of the mathematics. In particular lessons, however, the explanation was more prevalent than others. Several times throughout Lisa’s first observed lesson, she asked students to explain what they had done. She commented in the post-lesson interview, “That’s what you used to do to us!” Rachel not only asked students for explanations in her second lesson during the fall internship, she followed up several times to clarify student thinking. When asked in the post lesson interview why she was spending more time on explanations in this lesson than she had in the past, she responded with two reasons. First, she noticed that students could give a correct solution without understanding. Secondly, the Everyday Mathematics lesson was focused on students being able to describe what was happening in addition number stories using a graphic organizer. The lesson itself was focused on process.

> I think it’s just that I’m realizing that kids can sometimes—they’ll say answer but they don’t really know how they got it. The important thing for this lesson, the goal was for them to know how they got the answer, so I wanted to make sure knew how they got the answer.

Throughout her student teaching, Rachel's cooperating teacher also encouraged her to question students more about their thinking. Interestingly, Rachel and Lisa both cited external reasons for asking students for explanations. Lisa was modeling after what she had seen in methods and Rachel was trying to achieve the goal expressed by both the curriculum and her cooperating teacher. Neither considered explanation a means of assessing student understanding in order to guide instruction. Rather, an authority, whether it be a teacher or the text, claimed it was important. Also, while they changed how they sought explanations, the purpose for their actions remained the same. They wanted to check to see if students knew the procedure to get the expected answer.

The norms promoted by Rachel and Lisa were consistent with the beliefs they espoused in mathematics methods. The text and the teacher are the source of ideas taken up in the classroom. The goal of learning is to imitate procedures as presented by the teacher, asking questions or explaining thinking only to clear up misunderstanding.

Task structure and implementation. The tasks in the classroom were examined in terms of structure (1.79), duration (1.07), grouping (1.86), participation (2.29), and level of difficulty (2.00).

The tasks in the classrooms were presented predominantly as closed tasks (1.79), with a single solution or process. Lisa's tasks were also often presented only symbolically. For example, in one lesson she put a group of division problems on the board and explained step-by-step how to complete them while students copied the examples. The tasks in Rachel's lessons usually included more than just symbolic representations, but often she showed students a single solution or process for solving.

There were two exceptions, two lessons in which Rachel presented open-ended tasks that were intended to give students a chance to "explore and see" before coming back together to tell her more. These lessons were different, Rachel explained, because on Wednesdays they did not do lessons from the Everyday Mathematics curriculum. Her cooperating teacher asked her to do a lesson in her fall internship on probability since it was not covered in the curriculum but was an objective from the state. Students were put into groups to flip coins and pull different colored slips of paper from a bag. They were to guess how the coin would land or what color paper would be pulled and then record their results. The goal of the lesson was for students to see that they had a fair chance with the coin, but not with the bag. Rachel decided to play these games because she thought it would thought it would help students connect the skill or concept, "practice it in a different way than just doing a worksheet." So even though the task was open-ended so that students could explore, she still viewed it largely as more engaging repetitive practice instead of an opportunity to construct new understanding.

Lisa and Rachel differed from each other in their approach to grouping students. The mean score for grouping was 1.86, but in this case the mean does not tell the whole story (see Figure 7).

Lisa consistently expected students to work alone, with only occasional chances to talk if a student needed help. Some students sat in groups, but others desks were pulled away from the groups and stood alone. When asked after the first lesson about her expectations of students in regards to grouping Lisa was unsure.

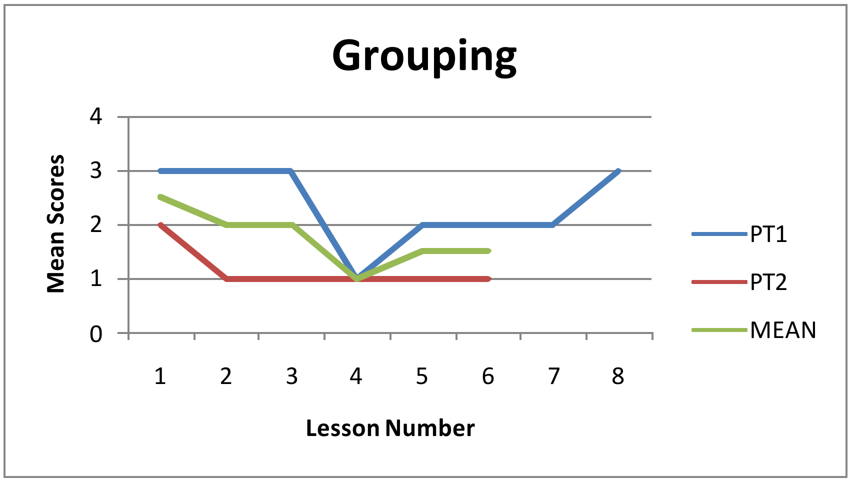


Figure 7. ***Grouping Scores for Resisting Preservice Teachers***

> I don't think I gave directions, um, sometimes I will tell them that their work is individual. A lot of times we'll have some students that like to help others so they'll just take it up themselves. Sometimes I correct it and sometimes I don't, so I guess that's inconsistency again, but sometimes I tell them. . . . If I know that they are talking about math or whatever we're doing as long as we're staying on task and not everywhere, but if they're not then I don't like that. [pause] Oh goodness, that's a lot to think about.

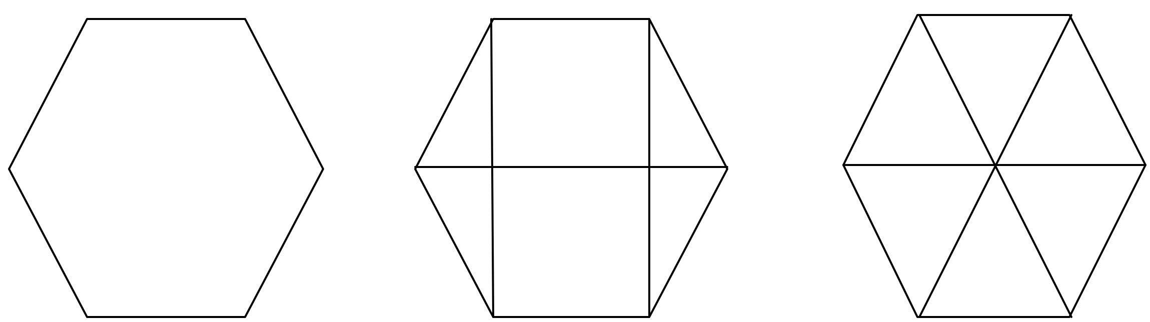
Despite Lisa's assertion that sometimes talking was okay as long as students are on task, in the lessons that followed, talking was clearly discouraged. A frequent question from her while students were working was, "Why are you talking?" In one lesson, she passed out stickers to those who were working quietly. In contrast, in most of Rachel's lessons, students sat in groups at their tables and on the carpet. They often worked in pairs during work time, sometimes coming back together to share answers at the end of the lesson. Even though the focus was on procedure, Rachel's students gained some experience in communicating their mathematical ideas.

Participation (2.29) was similar in the two classrooms. Both Lisa and Rachel called primarily on volunteers, but did include other students during whole group discussion. Rachel usually taught only half the class at a time so she was often able to make sure most students were able to speak and were actively listening. Her cooperating teacher strongly encouraged her to find ways to have multiple students respond simultaneously so that Rachel could assess students more easily.

The mean score for the level of difficulty of the tasks presented was 2.00; the material was familiar with only one or two difficult problems. Many times, tasks were presented that students already knew how to do, or could do with little help. In these instances, Rachel reported that students could work independently, that she “didn’t need to teach.” Already knowing the material seemed to be a prerequisite for allowing more student input. Rachel reflected on the lesson in which students already knew how to find doubles facts, “This lesson was very student centered. I had not intended for it to be that way, although they knew the material I was going to teach.” In other words, challenge for students was not desired. When the material was new for students, she felt she needed to be teacher-directed.

When material was challenging for students, the response of both preservice teachers was to decrease the demands of the work rather than supporting students in their struggle. Lisa postponed a planned test on combinations when students were confused during a lesson. At her cooperating teacher’s direction, she passed out the exact test questions and showed students how to work through each question step-by-step. The test was given the following day. Rachel taught the same lesson twice each day, to half of the class each time. In a lesson on fractions during student teaching, the first group of students struggled with a problem on their journal page that asked them to split the given hexagon into six equal parts. Students were splitting the hexagon into six unequal parts (See Figure 8). When the second group came to the same problem, Rachel drew it on the board for them to copy into their journal so they could continue with the second part of the task which involved shading different fractional parts.

Task: Spit the hexagon into six equal parts.



Hexagon on the journal page from the text | Student's attempt at splitting the hexagon | Rachel's presentation of the task for the second group

Figure 8. *Lowering Task Demands*

During the interview Rachel commented on the change, "I think that's the benefit of doing the two different groups because I can teach one and know what I need to kind of change for the second." Just like Lisa, Rachel's response to confusion was to present an easier task.

The ways Lisa and Rachel structured and implemented tasks were similar to the tendencies they exhibited in their participation in mathematics methods. When faced with a challenging problem in methods, their response was to disengage rather than struggle with the task. Similarly, when a task was challenging for their students, they either became more teacher directed or altered the task to take away the struggle.

Mathematics. The mean scores for representations, the nature of mathematics, and practices of doing mathematics were 1.71, 1.86, and 1.64, respectively. Because of the tendency to decrease the cognitive demands of tasks, very little problem-solving occurred that gave students a chance to engage in the practices of doing mathematics. Although the preservice teachers did question and ask for explanations, students mostly just completed exercises. Mathematics was presented in the classrooms as a set of procedures to be memorized, which may or may not be related. Lisa began her instruction of long division with her fourth grade class by teaching them the mnemonic device "Dead Monkeys Smell Bad" to remember the steps in order (divide, multiply, subtract, bring down). During the observation, her cooperating teacher said to the observer that Lisa had skipped the introductory hands-on part of the lesson provided by the county. The activity asked student to dividing varying numbers of cubes into equal groups to address the meaning of division. For example, students were to split 24 cubes into equal groups in as many ways as possible. Lisa sat down for the post-lesson interview and declared, "It was a disaster. . . . because I don't feel like—I don't feel like I taught it effectively. Like I don't feel like the things that I said made sense to them." When asked why the steps to long division that she presented to the class work conceptually, she responded after a long pause, "I don't know." She then explained that she had decided to skip the cube activity and go straight to the numbers because she did not understand what to do. The interviewer worked through the activity with Lisa, and she used the activity with students the following day. She was observed in a lesson later in the year pulling them out to help a student who was struggling with a division problem.

Despite this experience the focus in the classroom continued to be procedural. After explaining to the class how to multiply by counting the number of zeros and adding them to the end of the number, she stated to the class, “That’s just a rule you’re going to have to remember.” On two subsequent occasions, when a mistake had been made in class and an answer to a problem did not make sense, Lisa decided to skip the problem, telling students, “Okay, just forget about that one.” Again, this decision is reminiscent of her pattern of disengagement in methods when unsure about a problem.

Because of the focus on procedure, the representations in Lisa’s classroom were almost always symbolic. Rachel also primarily used symbolic representations. She did use others, but only if they were included in the Everyday Mathematics lesson plan. For example, in a fractions lesson, students folded and shaded pieces of paper into halves, fourths, and eighths. Rachel called on a student to come show 1/2 on the board. The student began to draw a rectangle, but was stopped as Rachel directed, “Just write the fraction. Don’t draw the picture.” Later in the lesson a student came to the board to show 1/8 and again began to draw a picture. “No, the fraction, what’s the fraction? Not a picture.” The student’s incorrect answer of 1/7 was changed to 1/8 with no discussion of why the student might have thought 1/7 was correct, even though this is a well-documented misconception in students’ early fraction work. Rather the procedure of counting the total number of squares to find the denominator was simply reinforced. Even in the few times that other representations were used and connected to a concept, the presentation was very procedural.

In mathematics methods Lisa and Rachel advanced very traditional beliefs about their role as teachers of mathematics. Mathematics is the knowledge of a set of related procedures. The teacher is the giver of that knowledge, and the students are the receivers. They performed themselves as teachers in the same ways they participated in mathematics methods. They resisted the complexity of reform-based mathematics.

Imagining

In this section, I examine how the preservice teachers in the resisting category imagined themselves as teachers of mathematics. Imagining includes how they related to the norms of the cooperating teachers, how they worked with the curriculum provided, and what they believed about mathematics and their role as a teacher of mathematics. Because the context of each student teaching placement was different, each preservice teacher will be discussed separately.

***Rachel*.** Rachel identified strongly with her cooperating teacher, explaining that she plans to organize her classroom and her lessons in the same way during her first year of teaching. She couldn't think of anything she would do differently. However, if given the choice, she would choose a different curriculum because she did not like the spiraling nature of the curriculum. She felt the curriculum constrained her mathematics teaching. "They kind of introduce a concept and then you may or may not practice that concept that day . . . and then they come back to it later. . . . I don't like that set up so I would look for something different." Her cooperating teacher also believed that Everyday Math was constraining Rachel's teaching, but not for the same reasons cited by Rachel. When discussing an observation of a lesson Rachel taught on measurement, she expressed a concern that Rachel would not deviate at all from the lesson plans.

> Sometimes it [Everyday Math] hinders her in that maybe she won't do something that she thinks might be good for kids, but because it doesn't say it in Everyday Math, she shies away. . . . We talked about letting them measure, giving each of them a ruler, and it probably didn't say, "Give each student a ruler."

Later in the year, Rachel chose some of the "extra activities" provided in the Everyday Mathematics curriculum to use for her Wednesday exploration. She stated that she would like to do more to push students to think outside the box, but when asked if she could incorporate some of the activities into her daily lessons, she said no. It doesn't work." While in some ways the text did constrain Rachel's teaching, in other ways, it afforded her movement towards reform-based teaching. She began asking students to explain more when the goal of the Everyday Mathematics lesson was to focus on the process of addition number stories. She also stated that she was learning new strategies to solve problems through the curriculum. In this instance, strictly adhering to a reform-based curriculum was helpful.

Rachel reported change over the course of her teacher education program. While these changes are not as evident in how she performed herself in the classroom based on the performance observation framework, changes did occur in how she imagined her role as a teacher of mathematics.

Rachel continued to focus on mathematics procedures in her instructional practice, but she did become more focused over time on the use of multiple strategies. In the fall internship, she wrote that the lesson on addition number stories demonstrated the importance of "showing students multiple ways to solve a problem." Over the course of the year, the use of multiple strategies is linked to three different influences. First, as in the example above, in relation to new strategies she learned from the Everyday Mathematics book. In a later lesson, she reports showing different strategies because the students were struggling and she was trying to find a way to show the mathematics to help students understand. Finally, K-12 experience is still playing a role in the way she believes she should be as a teacher. Rachel explained that her favorite mathematics teacher in high school was so good because she "explained how to do it in several different ways up on the board and then she let you practice it like a set routine." This willingness to consider multiple strategies is a shift in her beliefs and practice, but when the use of multiple strategies was mentioned in written reflections and interviews and observed in lessons, it was in the context of showing students multiple strategies, rather than incorporating students' strategies. However, in the interview for a lesson taught towards the end of student teaching, she refers to students offering up strategies, commenting that, "sometimes they say things that I'm like, 'Oh yeah, that works, too,' and then I let them share and then maybe that will help somebody else in the group, too. "

Rachel also began asking students for more explanations over the course of the year, although the expectation from students for what counts as a good explanation was still focused on explaining procedure, rather than understanding student thinking. While she did not mention it in her interviews except when asked directly, just as with the multiple strategies, she cited the curriculum and the students. Sometimes the Everyday Mathematics lesson was focused on process. Other times she was not sure that the students understood even when they got the right answer, so she asked how they got it. The cooperating teacher reported encouraging Rachel to ask students questions more and "tell them" less, which may also have had an influence even though Rachel never mentioned it. Small evidence of a shift towards a classroom norm of explanation occurred in a lesson at the end of her student teaching as student interrupted Rachel to challenge another student asking, "But how do you know?"

Perhaps the most marked change in Rachel is the way she sees herself as a teacher. In final interview, she imagines herself as "not really an authoritative figure in the classroom but almost equal, um, or a facilitator to students," a vastly different view than the first lesson in which she described herself as the "supplier of information." She believes her role as a teacher of mathematics is to introduce concepts to students and explain them in multiple ways. After explaining her job is to "step back and let them explore it for themselves and kind of be more like a facilitator type persons, letting them explore and practice on their own. " She also wants to incorporate group work and manipulatives.

Despite her budding view of herself as a facilitator and her new emphasis on reform-based pedagogical practices such as using manipulatives and multiple strategies, it's important to note that she still sees her job as a teacher to explain first and then let students explore. In addition, Rachel still proposes a narrow view of mathematics during the final interview, defining it as "using numbers to count, or um, I guess just using numbers in a variety of ways because it's addition, subtraction, multiplication, division, and then it's also counting like money and fractions so—I guess it all revolves around numbers, all different kinds. " She still believed mathematics to be a set of procedures used to get a solution. The purpose of using tools like multiple strategies was so students could choose which procedure worked best for them in the quest for the right answer.

Lisa. Unlike Rachel, Lisa imagined herself differently from her cooperating teacher. She often expressed in interviews a desire to do more hands on activities, describing the lessons provided to her by the county as "review" and "kind of boring." She indicated that the expectation from her cooperating teacher was that she follow the lessons. "I just feel like things are more standard, you know, like sit at your desk and work out of a book. . . . It hasn't been horrible; I would just do a few things differently to make it more fun for the kids." Yet when her cooperating teacher was asked about the kinds of feedback she was giving Lisa on her math teaching, she responded that she repeatedly told Lisa that she needed to make sure that the students were "doing something, not just watching and listening." As she noted, "There's a lot of that." She was also concerned because Lisa would often leave out activities in the lesson plans that required manipulatives.

> I think it kind scares her to get that kind of stuff out so if there is a way to do it without using them [manipulatives]. . . . I'm seeing it in the way of, "It says use manipulatives. I'm not going to because I don't want to get them out," rather than coming up with a different way to do it. She is coming up with a different way, but maybe not the best way.

Lisa's different way to do it she said was to stand at the board and tell. This is in direct contrast to Lisa's claim about how her beliefs have changed. She notes, "At first you think that's your job to tell them, tell them, tell them, but then you realize that they can do." Like Rachel, Lisa also expressed a desire to do more hands-on activities and to make mathematics more fun. She believes her role as the teacher is "to model how they [students] would do things with manipulatives or whatever it is, to guide their thinking, but still let them do the thinking." There is no evidence, however, either from the scores or the descriptive data that she was moving toward implementing these beliefs in the classroom.

With the exception of one other preservice teacher whose concentration was mathematics, Lisa has more mathematics background than any of the other preservice teachers, having taken calculus and functions/analytic geometry in high school. Interestingly, when asked during her final interview how she has changed, she focused more on her own need for understanding than on teachings practices.

> Just being in the classroom and just seeing how you can have the whole class that understands and one student that doesn't understand, and you know, you just have to change. You almost have to review math yourself to teach math. You have to constantly be learning things. I feel like because sometimes, I mean, I really, I'll know how to teach the lesson, but I won't sometimes know exactly what I'm talking about. . . . I'm learning more math with every lesson and then how prepared you have to be, how you have to understand the point of a certain area.

This realization that she did not always understand the mathematics she was teaching may indicate the beginning of a shift in her beliefs about what mathematics is, but when asked for a definition of mathematics during the final interview, the focus was still procedural. Lisa responded “a set way to solve things using numbers.” At this point it is unclear if Lisa is simply claiming beliefs about her role as a teacher because she thinks she is supposed to believe in hands-on activities and student thinking or if she really holds these beliefs but is so uncomfortable with the mathematics that she does not implement them in her classroom teaching.

Summary

The instructional practice of the resisting preservice teachers was similar to their participation and beliefs in mathematics methods. They continued to perform in a traditional manner as teachers of mathematics, promoting mathematics as procedural and teaching as telling. They did imagine themselves differently at the end of student teaching, claiming affiliating with some of the practices of reform-based mathematics such as the use of manipulatives, student explanation, and multiple strategies to solve problems, but their views of mathematics itself did not change.

Big Men on Top: Acknowledging Complexity, Illusions of Reform

The preservice teachers in the acknowledging category recognized over the course of the methods class in phase one that mathematics was more than they thought it was. They worked collaborative in groups during problem-solving, used manipulatives, and found multiple strategies to solve problems. Excited about their new found understanding of mathematics concepts, they believed their role as a teacher was to use the same tools to help their students. Throughout their internships and student teaching, Allie and Jenna taught in a way that matched their beliefs as stated in mathematics methods. They used manipulatives, multiple strategies, and asked students to explain their work. A typical example was Jenna's lesson in which she engaged the students with real data to find the difference between the high and low temperatures in different cities. Students worked in partners and came back together to discuss as she asked the students to explain how they found their answers. The discussion quickly turned into a review of "how we subtract" with Jenna consistently leading the students to recite chorally, "If the big man's not on top, just go next door and knock," as a way to remember when to regroup. The acknowledging orientation's results on the performance observation framework were not significantly different from those teachers in the resisting group (See Figure 9) because, even though they used some of the tools of reform-based mathematics, they continued to focus on procedure.

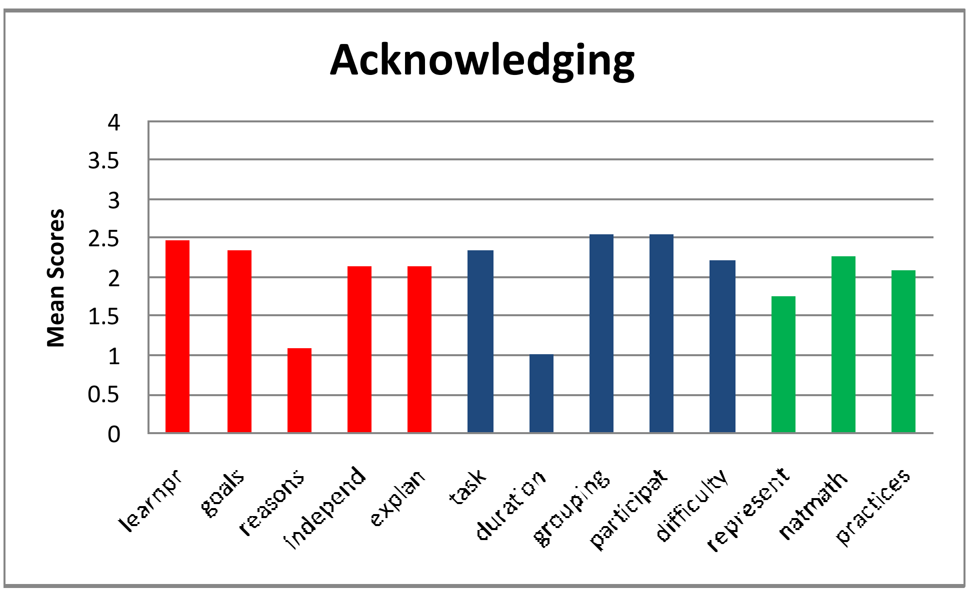


Figure 9. ***Mean Scores for Acknowledging Orientation***

Performing

Norms. The norms examined in the classroom included how learning was promoted (2.47), the goals for students (2.33), the reasons for doing mathematics (1.07), the levels of independence (2.13), and what counts as explanation (2.13). Learning was promoted as an activity involving longer problems, sometimes requiring reflection, but rarely a need to persist through difficulties. When students did struggle, the teachers would move on to another student. For example, in Allie's classroom third grade students were given index cards with a place name written on them. Nine lines were drawn on the board separated by commas. Students came to the board one at time to put their place value card under the appropriate line. The student with the card that read "hundred thousands" placed her card on the board under the line for "ten millions."

Allie: Do you think she is right?

Other students, chorally: No

Allie: Someone else come up. You have to explain it first.

Rather than asking the first student to explain and revise, she was asked to sit down and the task was given to another student. Another common response of both teachers when a student made a mistake was to re-voice the solution correctly themselves, often under the guise of the student's thinking. Jenna asked a student to share her solution for how many cubes it would take to build a wall that is 5 cubes long, 3 cubes wide and 2 cubes high.

Isabelle: I figured out the volume by doing 5 x 3 and 3 x 2 and I got my answer.

Jenna: So you did 5 x 3 x 2. What's that strategy called? How did you get 5 x 3 x 2? What did you know?

Isabelle: [pause] I divided them up by—[pause]

Jenna: Five is our what, Isabelle? In our problem it tells us it's what?

Isabelle: 5 cubes

Jenna: Five cubes what? Long. Long is our . . .

Isabelle: length

Jenna: Length. Now it tells me it is three cubes wide. Three is my what? What Christopher?

Christopher: width

Jenna: The width. What's two? It says it's two cubes . . .

Another Student: high

Jenna: high, so that's my . . .

Several students together: height

Jenna: So Isabelle used length times width times height, 5 x 3 is

Students: 15

Jenna: 15 x 2 is . . .

Students: 30.

Jenna: 30 [Isabelle erases the answer on her paper, which was not 30. The class moves to the next activity.]

In fact, Isabelle did not use 5 x 3 x 2 to get 30. She added 5 x 3 and 3 x 2 to get her solution, 21. Jenna did not pursue Isabelle's thinking and require Isabelle to persist.

She instead re-voiced Isabelle's ideas as she wanted them to be. The goal was for students to imitate what the teacher did with limited discussion to clear up misunderstandings. Both teachers asked for students to share explanations on a regular basis. "How did you know?" and "Explain your thinking" were common phrases. However the expectation for those explanations was either repetition of a taught procedure or use of previously taught strategies. In the excerpt above, Jenna asked Isabelle what strategy she used. Two strategies, learned the previous day, were acceptable answers during that lesson, the "length x width x height" strategy and the "area of the base x height" strategy.

In interviews and written reflections, both preservice teachers insisted that it was important for students to explain their thinking and be autonomous. In spite of this, Jenna and Allie mainly determined what was discussed, using the text as well as other sources for ideas. They sometimes did solicit ideas and examples from students, but often the ideas were then never used in the lesson, as with Isabelle's thinking. Toward the end of student teaching, Allie began writing student ideas on the board with their name beside them. For example, after fifteen minutes of copying definitions from the book, Allie listed students' descriptions of a rectangular prism she was holding, the shape introduced in that lesson. "I'm just going to write some students' thoughts". Vicki said that it was three-dimensional. Matt said it was hollow. Cathy said that it looked like a double square. Deanna said it was made up of rectangles and squares. After several more students gave ideas, she announced that they would next learn the word base and instructed the students to copy the definition from the book. The lesson never returned to the ideas listed on the board.

When students did raise questions that could have been taken up by the whole class, they were answered quickly with no discussion or follow-up. For example, in Allie's lesson on place value, as students were placing their index cards on the board, Jeff asserted that if they had four places in the millions instead of three that it would equal a "thousand millions" (See Figure 10). After pausing to think, Allie responded, "Yes, but we don't need to worry about that right now."

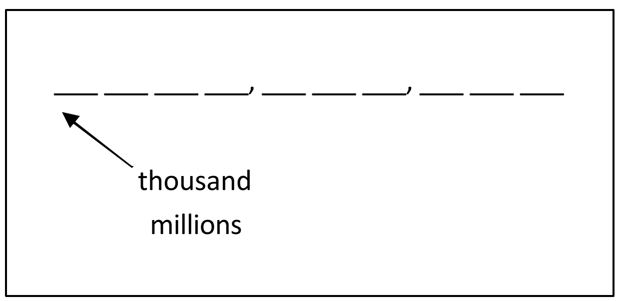


Figure 10. ***Jeff's Assertion***

Similarly, when students were listing fractions less than one half, a student in Jenna's room challenged the notion that 0/0 was less than one half. It must be one, she reasoned, because the same number was on the top and bottom. Jenna assured her it was correctly placed (which it was not), and moved on to the next fraction. The students' argument was not addressed in any way other than to say that it was wrong.

Jenna and Allie performed themselves in the ways they thought would promote the ideals they espoused in mathematics methods—asking questions, having students share ideas and explanations, using strategies for discussion. Yet, the use of these teaching strategies was surface level. Ultimately, they still tightly focused on learning as imitating procedure for the purpose of getting the right answer.

Task structure and implementation. The tasks were examined in terms of structure (2.33), duration (1.00), grouping (2.53), participation (2.53), and level of difficulty (2.2).The acknowledging preservice teachers perceived that their tasks were open-ended and challenging and that they promoted group work and active participation.

The tasks used in the classrooms usually had a context beyond just symbolic representations and were presented as open-ended, but when enacted in the classroom, students were given step-by-step directions about how to complete them. For example, in a lesson on volume, Allie gave students centimeter grid paper and students made four different rectangular prisms. Students then estimated the number of cubes they thought were in the box. At this point in the lesson, rather than provide cubes and any other tools they needed to check their estimates for the boxes and look for patterns, Allie said, “You are going to multiply the area of the base times the height to get the volume. I’m going to put the formula on the board if you need it.” Students then used the formula to fill in a chart with the volume of each box (See Figure 11).

Volume Chart

Rectangular Prism	Area of Base	Height	Volume
A			
B			
C			
D			

Figure 11. ***Volume Chart in Student Journals***

When asked why she went straight to the formula, Allie responded that she did not understand how students would come up with the formula without being told.

> We cut out the cubes, the A, B, C, and D cubes and they were all different. Um, they weren't cubes, they were rectangular prisms. And so they were able to figure out the formulas for the volume. And what I found—when I looked earlier in the lesson and decided what I wanted to talk about . . . I realized I knew the formula in my head, but it never actually said that in the lesson. . . . I looked at it and I thought, how are they going to know where to get these answers? . . . It's never ever stated, so in my mind, I thought, is this something they are going to be able to figure out or something that they need to be told and then work with it? And from my understanding, I couldn't just assume that you just multiply those two [area of base and height]. I mean for all they knew you added them. . . . I looked at that lesson. It was very strange to me. I didn't see that formula and I feel like, yeah, you shouldn't sit there and memorize everything for math. You shouldn't memorize every formula, but some things you just have to know.

In another exploration, as Allie often called them, students were trying to find all the possible combinations that could be worn if you had four pairs of pants and four pairs of socks. When students rotated to this station, Allie showed them how to use different colored crayons for each pair and how to use *p* for pant and *s* for socks and started them on making at list. Students simply had to finish the list. Allie explained in the post-lesson interview that the text had pictures of pants and socks that students could cut out and use to make combinations, but she thought that would be too easy.

Jenna also expressed a desire to do hands-on activities, and she did let students work for a longer period of time before directing students in the procedure. In the same volume lesson Allie taught, Jenna gave students time to figure out how many cubes were in box A, B, C, and D, which most of them did by filling the boxes with centimeter cubes and counting them. Then at the end of the lesson she introduced the two strategies they would use the next day, “length x width x height” and “area of the base x the height.” Jenna explained, “I kind of wanted to give them food for thought, like an experiment, and then the actual volume lesson . . . was a lot more focused on taking notes.” For both preservice teachers, terms like hands-on and exploration held meanings that were more associated with engagement than mathematics. The “actual lesson” was the instruction of procedure, while the “exploration” served as a way to engage the students’ thinking initially and make mathematics more fun rather than as a part of the mathematics.

The level of challenge of the tasks presented was low. Students knew most of the material with only one or two difficult problems. When the material was perceived as difficult, the tendency was to provide more direct instruction with less student input. For example, Allie introduced a lesson by doing four problems with the whole class before they worked with their groups. The students gave ideas and explained how to solve for the first two problems, but for the second two, Allie began with a think aloud, demonstrating how she would solve the problems. When asked about the shift from student input to teacher think aloud, she said that word problems were difficult for the students so she “wanted to show them how I would think about it.”

Most students were active participants in the classes and both teachers encouraged that participation, saying things like, “I want to want to see more hands. Someone I haven’t spoken to.” Group work was very prevalent in the lessons. Students often worked as partners or in groups. Jenna, in particular, decided to make student talk one of her goals during the course of student teaching. In addition to partner and group work during activities, she regularly asked students to turn to a partner and share their ideas during whole class discussion. Allie, on the other hand, viewed group work as a means of addressing a wide range of student needs. Students, who knew how to do the task at hand, were expected to help the struggling students in their groups. Lessons in both classrooms often ended with the group and partner work. When the class did come back together, the purpose was to check for right answers, rather than to have a whole group discussion to achieve consensus of ideas.

Allie and Jenna encouraged collaboration and tried to provide open-ended tasks that could be solved using multiple strategies and allowed for student exploration. However, the level of challenge was low as students were give specific directions for how to complete "explorations", especially when the teachers were uncertain of the content themselves. Both teachers emphasized the use of multiple strategies, but rather than students finding these strategies, they were presented as a laundry list of procedures from which to choose. This contradiction was also seen in mathematics methods where the preservice teachers reported being excited about all the strategies they had found with their group because that meant they knew several different ways to show their students, the optimal word being "show" in their journals then, and in their performance during internships and student teaching.

Mathematics. The mean scores for the nature of mathematics, representations, and the practices of doing mathematics were 2.26, 1.73, and 2.07, respectively. Despite the stated emphasis on explanation and exploration, hands-on activities and group work, both Jenna and Allie continued to focus primarily on procedure. In a lesson in which students were finding the difference between high and low temperatures for the day on a

United States map, Jenna reminded students how to know if they need to regroup. “If the big man’s not on top, just go next door and knock.” This rhyme and the procedure were used on all the problems, including 14-8, one that most students knew in their head or counted up to solve.

Different representations were used frequently in both classrooms, which is not surprising given the emphasis on hands-on activities and the use of manipulatives. However, connections among representations to inform conceptual understanding of the mathematics were not made. Instead, after the initial “exploration”, material was presented primarily symbolically. Both preservice teachers taught a fractions lesson in which students were to sort their fraction cards in to two piles—more than one half and less that one half. The cards were shaded to represent the different fractions, and yet this shading was only mentioned in passing and was not used when discussing the task. Instead, students were taught to find the number that was half of the denominator and see if the numerator was more or less than that number. Certainly, the shading on the fraction cards may have informally supported conceptual understanding simply through student use, but there was no discussion to help bring those understandings to the forefront. Students were asked to explain their ideas, because explorations became teacher directed, they did not engage in other practices of doing mathematics such as making and testing conjectures, argumentation, or problem-solving. Rather they completed exercises with some explanation and questioning. While Allie and Jenna worked hard to use the reform-based pedagogical practices they learned in mathematics methods. However, their view of the nature of mathematics itself was still very traditional.

Imagining

Allie. Allie had a unique experience as a preservice teacher. In January, during the student teaching semester, one of the teachers in the school left unexpectedly. Allie was offered the job and became the classroom teacher in that room. The classroom was in the same grade level as the classroom in which she interned, and she continued to plan regularly with her cooperating teacher. Her observations, however, were done by a variety of school personnel. Consequently, she had more freedom than the others to do what she wanted, because she was teaching in her own classroom.

Allie identified strongly with her cooperating teacher. They worked well together during the first internship junior year and both requested to be placed together during the senior internship and student teaching. Several times during post-lesson interviews, when asked about something she had done, Allie would say, "Salem taught me that." When asked how she had changed things when she got her own room, she explained that most things were organized in the same way, but she was able to do more explorations, something her cooperating teacher had done in other subjects, but not necessarily math. When she was in the internship classes, she felt like she needed to do the math review boxes provided in the student journals every day. Now that she had her own classroom, she was only doing them some days. The other days she was using the time for her hands-on activities. In the first interview after she switched classrooms, Allie stated, "Each day this week has been something hands-on, and for me that's more important [than worksheets]. That's definitely changed."

The curriculum served as a source of both help and frustration for Allie. “It’s hard because it’s helpful in a lot of ways, but in other ways it limits you.” She continued to explain that she needed to see the strategies explained, and the text also showed what was important and had good assessment questions. She felt like this was especially important for someone who was not good in math. Everyday Mathematics showed her different ways to do problems, and for Allie, that was “most helpful.” On the other hand, she felt limited by the amount of time she had to teach the concepts. She perceived the curriculum map given to her by the school as a similar constraint. “It’s very hard when your time constraints get in the way and these students need maybe an extra few days on this concept, but you know you’ve got to get to a certain point.” Again she had mixed feelings. “I think that the curriculum map itself is hindering me, but it’s also helping me to stay on track, and you do need to see what you teach, especially since I’ve been here my first year in third grade.”

When asked how her beliefs have changed over the course of her two years in teacher education, rather than talked about teaching mathematics, she talked instead about the change in her own relationship with mathematics. Mathematics in her school years was “nerve-wracking.”

> Math was not my field. I was so nervous with it. It would take extra work for me to understand it. . . . I just could not stand it. . . . It was very hard. Very rarely did I make an A. College I was told to take—I needed one requirement for it. I took the easiest class and that was it. I wanted to avoid it in general. . . . I think that was a big, big fear, even before I got to the school of ed., I was thinking, “Now am I really going to go in there and go in to teach math when I’m so horrible at it. I think I’m just going to have to teach kindergarten and first grade. One plus one, I can do.”

Allie reported feeling much more comfortable with mathematics at the end of her student teaching. She explained that she now has different strategies to do things, and therefore, she can respond to students using different strategies where before she would just tell them that that was not the way to do it.

Allie's confidence in her mathematics ability may have changed, but her beliefs about mathematics and her role as a teacher of mathematics have not changed. Allie believes her role as a teacher of mathematics is to provide manipulatives and hands-on activities, teach multiple strategies, and use math journals. Mathematics "encompasses manipulating numbers and just in general the mechanics of it, the idea of quantity." She explained, "Math is the one subject I can say that you definitely need prior knowledge . . . so definitely scaffolding and making sure you repeat the concepts and ideas in a lesson. You need the basics. You constantly need a good foundation in the basics before you get higher level concepts." Allie recognizes, or *acknowledges*, that conceptual understanding it important, but it comes after learning the procedures, the "basics."

Jenna. Unlike Allie, Jenna imagined herself differently than her cooperating teacher. She felt that her cooperating teacher was more focused on direct instruction, citing examples of times when she was told that her lesson plans needed more "teacher time". Jenna on the other hand, emphasized throughout student teaching that she wanted students to talk more and become more independent. She also felt pressured to teach to the end-of-grade standardized testing. In fact, the last two lessons observed with Jenna teaching involved students completing multiple choice test preparation questions for the state standardized testing. These worksheets, Jenna explained, were the result of a workshop that cooperating teachers had attended where they were encouraged to incorporate practice test questions into every topic throughout the year.

Jenna also felt disconnected with the curriculum. Like Rachel, she did not like the spiraling nature of the curriculum. She would rather have a larger concept and spend four weeks teaching it gradually. In fact, she felt strongly enough about spiraling curricula that she planned to ask if the schools use them when she goes for her job interviews to be sure she does not have to use one the next year. Jenna also did not like the "weird, oddball ways they go about teaching stuff." When asked to give an example, she talked about some lessons on mean, median, and range.

> Well, when we did a mean activity, I did my own lesson just because they didn't add all the numbers up and divide by how many they were, which is the simpler way. They know how to add, they know how to divide, but even if they can't divide large numbers, they still could draw a picture of it. You know. They wanted them to figure out how to put them into equal groups. So, for instance, they had a stack of pancakes—it's been a while, but—they tried to figure out, they had like different size pancakes [stacks of pancakes], and they wanted to put them all in these equal groups. And I think that's a lot like multiplication, and when I looked in the SRB [student resource book] it just didn't make sense for me to teach it that way.

Jenna went on to explain that after they had done an activity in which they used calculators to add up the numbers of each color of M&Ms and divide by the number of groups, they then discussed where the number comes from and that if everyone had the same amount, that number is how much they would have. Like Allie, her approach was to show the procedure first and then explain the concept.

At the end of student teaching, Jenna believed mathematics to be problem-solving and higher order thinking with numbers. She doesn't "think it necessarily all revolves around numbers." She still believed in the importance of manipulatives, hands-on activities, and using multiple strategies, but also emphasized fostering student independence, perhaps because she felt the classroom she was in was so focused on test-taking.

> Independence—that's really, especially at this age, that's really what I want to push for because you know, at the end of the year, they are going to be taking a test and I'm not going to be sitting there with them. I can't help them and I think instead of teaching to the test, if you can get them that independence. It's not so much—the content will fall in its place. You just have to create that student in order to be able to do it—to think independently.

Jenna saw this focus on independence and critical thinking as a change in her beliefs about teaching. At the end of student teaching she imagined her role as a teacher was to be a facilitator, encouraging students to share strategies through student discussion. She made it her goal during student teaching to get students to talk more with each other. She believed students should be allowed to experiment and then to share their own strategies. When asked about what influenced these beliefs, she explained that the change began in the methods course because she did and saw mathematics in a whole new way and she was more comfortable with it than she had been in school, where her experiences were mixed. However, she also stated that in spite of the change there, she began teaching in her internships more in the way that she thought about teaching mathematics before methods, as "not the dictator, but this is how it is, this is how you're going to solve it." The change happened through the experience of teaching and reflecting on those lessons. She also cited her work on project-based instruction and self-regulated learning that was advocated in her literacy methods courses.

Jenna clearly incorporated more student talk as the semester progressed. Her scores on the performance framework for participation, explanation, and levels of independence were at their highest at the end of student teaching (3.00).Even though students were completing test practice worksheets, students were asked to explain and justify their choices. However, as exhibited in her views on teaching about the concept of mean, Jenna's mathematical focus remained primarily on procedure.

Summary

Allie and Jenna performed in a way that matched their original beliefs in mathematics methods, but not in the ways they imagined themselves at the end of student teaching. In addition to beliefs espoused in methods, the importance of manipulatives, multiple strategies, and group work, they also felt a need for students to be autonomous and share their thinking. They worked hard to put their beliefs into practice, yet their scores on the performing framework were not significantly different from those preservice teachers in the resisting framework because, though they did talk about conceptual understanding, they both still focused on procedural mathematics. In several instances, they were confused themselves about the concepts (Allie with volume; Jenna with the mean, for example). They did the hands-on activities, but did not make connections between the representations being used and the mathematical concepts. They showed students multiple strategies, but the only student choice was which of those taught strategies to use. Thus, they used reform-based pedagogical practices to teach traditional mathematics.

To Market, to Market to Buy a Fat Pig: Embracing Complexity in the Real World

Data indicated that Andres and Tara were in the embracing complexity orientation during the mathematics methods course. Deeply engaged in the problem-solving in methods, Andres and Tara both took their own need to explore as a sign that their students also need to explore when learning mathematics, to figure things out for themselves. Thus as Tara and Andres entered their internships, they were intent on providing their students with challenging tasks that require persistence. They believed their role as a teacher was to encourage conceptual understanding, student thinking, and explanation. The theme of this section is from a story Andres recounted a story about going to the market with his uncle to buy pigs. His uncle who had never been to school was able to calculate prices in his head that Andres could not do without pencil and paper. It was at this point that he realized that mathematics was useful in life. For Tara and Andres, the ultimate goal of their teaching was sense-making of mathematics so that it could be used in real world contexts. Their mean scores on the performance observation framework reflected their beliefs from mathematics lessons and were significantly higher than the scores from the other groups (See Figure 12).

Performing

Norms. Throughout their internships and student teaching, Andres and Tara's goal was for students (3.13) to explain their ideas to demonstrate a conceptual understanding of mathematics. They gave particular attention in their lessons to how learning was promoted (3.5), level of independence (3.5), and student explanation (3.5), their three highest mean scores.

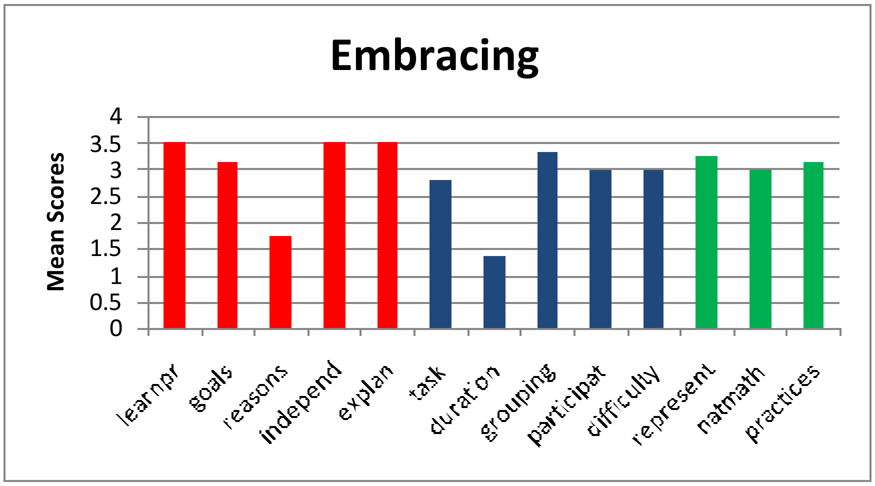


Figure 12. ***Mean Scores of the Embracing Orientation***

In both classrooms, learning was promoted as a reflective endeavor. Students were expected to reflect on their work and persist through difficulties. During the first lesson in the fall internship, a lesson on the order of operations, Andres asked students to simplify the expression (14 - 6) x 6 + 40 ÷ 2. While four students came to the board to show their solutions, Andres circulated the room looking at each student's work. All four students at the board got 68 as an answer; however, Andres decided to ask other students to share their answers, which he recorded on the board as they called them out. Other answers included 44, 62, 72, 25, and 56. The students at the board began to explain their work, and the class concluded that the correct solution was in fact 68. Each student who got a different answer was asked to explain where he or she made a mistake and how it affected the answer. Andres placed an emphasis on students learning from mistakes and persisting until they understood a concept. The focus was as much on process as product.

In the post-lesson interview, Andres stated that he wanted to be sure they understood if they had a different answer, why they had that answer. He also was not convinced that the four students at the board really got the same answer. "Nobody wanted to be wrong. The good thing is that the kids that had those different answers were brave enough to give different answers. That was good. . . . That was a learning moment. That was crucial for those kids." Thus learning was promoted as a process of thinking, exploring, discussing, and revising, rather than transmission of information. This view of learning matches the beliefs expressed by both Andres and Tara during mathematics methods about the importance of exploration, and Andres clearly attributes the student thinking during this lesson to such exploration. "They need more practice in thinking because I get the feeling that what they are doing is a different way to do math without time to figure out things. Today they had to think." Tara also noted a change in her classroom in the way students expected to engage in mathematics.

> I think they are starting to catch on, on like what I'm going to ask—not necessarily the exact questions I'm going to ask, but they're not going to be told exactly how to do it, and a lot of them I see like talking to each other at the tables. It's really cool to see them go from, you know, "I can't do it," to their trying to do it themselves. . . . I love it. I love seeing the kids like do all the questioning and find out what they're figuring out and the higher level stuff.

Both preservice teachers expected a high level of independence from their students during student teaching. They not only solicited ideas from their students, they consistently used ideas raised by students to launch whole class discussions. For example, early in the year in a kindergarten lesson focused on having students practice writing numbers, Tara instructed the students to practice making the numbers 0-9 with play dough. A student at one table decided to make two-digit numbers. First she made 18 and then 10. The student next to her noticed and exclaimed, “I want to do ten!” He asked the classroom teacher for help. Both students continued to make two digit numbers. In the whole class sharing after the activity, Tara asked the two students to share the number 10 and led a whole class discussion on the meaning of 10. She noted in the interview that she was surprised that they had gone on to the two-digit numbers because they had been focusing so much on the digits 0-9. Her plan had been to tell students what numbers to make with the play dough, but she changed her plans in the middle of the lesson. She thought if she let them pick their own numbers she would be able to “gauge who was able to do what.”

Another example of willingness to allow students to be the source of ideas and to follow them into unexpected conversations was observed when a student in Andres’ fourth grade class challenged the notion that the decimal 0.7 could be represented with coins in two different ways. What was intended as a quick warm-up before the lesson turned into a twenty-minute discussion of different representations of decimals. The willingness to adapt lesson plans to follow student thinking was a salient feature in the lessons of both preservice teachers.

In order to make these adaptations, Andres and Tara asked for and listened to student explanations. For example, a kindergartner in Tara’s class split a group of six counters into two groups with four counters in one group two in the other, claiming that the counters were split in half. Tara not only asked for an explanation, when it was established that the two groups did not represent half because they were not equal, she followed up with the question, “What do we need to move to make them equal?” She first pushed students to re-think the meaning of half and then immediately asked them to apply their new understanding in order to fix the mistake. In addition, in both classrooms, students were expected to listen and comment on each other’s work. Andres recounted in his written reflections a second grade lesson from his spring internship.

> One student came up front to participate in an activity about sharing an apple. I cut the apple in two unequal parts and I gave the student the small piece. Most of the students said that it was not fair because I got the big piece and the student got the little piece. I took advantage of the situation to talk about sharing. We talked about equal parts and the discussion became very interesting because they thought I should cut the apple in two equal parts. We talked about the small piece that I gave to the student. One student said that it was one sixth of the apple. I asked him why, and he said because it was so small. Many students agreed with the idea. The student had the concept that one sixth was smaller than a half part but the problem was they did not have any way to prove it.

The students were expected to make sense of the mathematics and contribute and communicate ideas. The ways Andres and Tara performed are reflective of their participation in mathematics methods. Just as they persisted in their own problem-solving to make sense of the mathematics, they expected students to reflect on their own learning and persist through challenges.

Task structure and implementation. The tasks in the classroom were examined in terms of structure (2.81), duration (1.38), grouping (3.31), participation (3.0), and level of challenge (3.0). As demonstrated in earlier examples provided in the discussion of norms, students in Andres and Tara's classrooms actively participated in individual and group work often followed by whole group debriefings.

The tasks presented were open-ended. That is, they couched in real world contexts, had multiple solution paths, and offered opportunities to make connections among different content strands of mathematics. For example, to meet the goals of a lesson on directional words (e.g. right, left), Tara had each group of students make their own "treasure" map of the class room. This task involved a discussion of what objects in the room should be included on the map and where they should be positioned. Students took turns using the map and giving directions to fellow students to find the hidden "treasure."

While the mean score for the task category of the performing framework is higher for the embracing orientation than the others, it was still the lowest of all the continua within the group (2.81). Both held fast to their assertions that students need to be engaged in open-ended tasks in order to make sense of the mathematics for themselves, yet implementation of such tasks was sometimes difficult to negotiate in practice. Andres and Tara presented challenging tasks to their students. The dilemma was deciding how and when the task should be scaffolded. When students struggled, both preservice teachers leaned toward continuing to let students explore, which sometimes resulted in unproductive exploration of the mathematics or even digressing from the mathematics all together. In a lesson on attributes and shapes, Tara put students in pairs to play an "I spy" game in which they were to describe objects around the classroom using the attributes of color and shape (e.g. I spy with my little eye a white circle) for their partner to guess.

Students easily used colors, but struggled to describe shapes. Consequently, although students continued to play the game enthusiastically, they were not engaging with the mathematics goals addressing the attributes of shapes. Instead the clues become non-mathematical (e.g. I spy with my little eye something fuzzy on the shelf). In reflecting afterward, Tara spoke of needing to model the game more with students in whole group in order to launch the task successfully. The very next lesson, she overcompensated. Various pattern blocks and other shapes were spread on the floor. Students were to ask questions to help them guess which of the shapes she had chosen. Consider the following exchange from the lesson.

Student: Is it red?

Tara: No, so take out all the things that are red.

Student: Does it have blue?

Tara: No, can you come take out everything that is blue?

Student: Is it green?

Tara: It is green, can you come take out everything that is not green?

When she answered the students' questions, she also told them what to do next based on the answer, rather than having students do the thinking. This fluctuation in the implementation of open-ended tasks between almost no guidance and doing the thinking for students continued throughout the lessons during the student teaching semester.

Andres struggled with task implementation as well. During the fall internship, Andres felt like students were confused about a domino activity he presented. He wondered during the interview if he should have shown them more first or let them explore. He believed exploration to be important, but the mechanics of implementing the task so that students were successful in making sense of the activity was still “a mystery”. This issue also manifested itself in the amount of time spent on particular activities. He reported that his cooperating teacher helped him by observing and talking with him about when to stay with a topic and when to move onto the next thing. The struggle with pacing improved, but did not completely dissipate over the course of the year as Andres tried to find a balance. In the post interview for a lesson near the end of student teaching, he was feeling more comfortable, but was still concerned. “Sometimes I try to explain too much because I want to be sure everybody understands.”

Preservice teachers in the embracing category often described the tasks they worked on in mathematics methods as both hard and fun. So it is not surprising that in spite of the struggle with task implementation, both preservice teachers continued to present their students with challenging, open-ended tasks and encouraged students to participate both individually and collaboratively.

Mathematics. The mean scores for representations, the nature of mathematics, and the practices of doing mathematics were 3.25, 3.00, and 3.13, respectively. In addition to focusing on a conceptual understand of mathematics, Andres also gave attention to the practices of doing mathematics, such as argumentation and proof. Consider the example given earlier about students cutting an apple into two pieces. One student thought the small piece was 1/6, and many of the students agreed. But as Andres recounted, “They did not have any way to prove it.”

The mean score for representation (3.25) fell in the middle of the range for the embracing orientation, but it was distinctly higher than mean score for representation in the other three orientations (almost one point greater than the next highest mean score). Both preservice teachers consistently made connections among representations and often asked students to explicitly state those connections. Tara frequently took students on walks around the classroom and school looking for numbers. In the first walk, some students were finding written number symbols (room numbers, numbers on the clock) while other students were counting objects (five doors in the hallway) that led to a class discussion about finding numbers in different forms. In another example, Andres posed the problem 6/10 +7/10 for students to complete independently in their journals. During the class discussion, two rectangles were drawn on the board (see Figure 13).

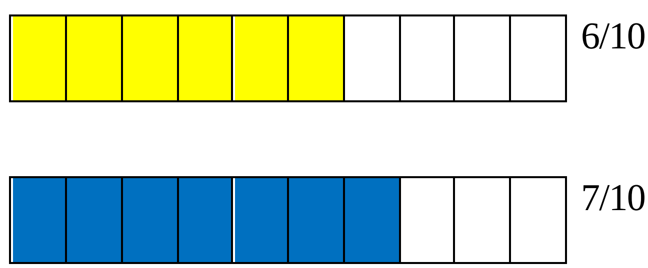


Figure 13. ***Drawing of Fractions on the Board***

A student came to the board to explain that he split 7/10 into 4/10 plus 3/10 so that he could fill in the first rectangle. The total he said was 13/10. Another student reminded the class that 13/10 was improper and that class worked together to make the fraction a mixed number, 1 3/10. The answer to the problem had been found and the solution explained, but Andres went a step further by next asking students, “Which parts do these represent in our picture? What is the 1 and what is the 3/10?" The discussion of the problem was not complete until the figures on the board had been labeled to represent the solution.

Both preservice teachers emphasized conceptual understanding and provided opportunities for students to engage in problem-solving using a variety of representations.

Imagining

Tara. Tara imagined herself as being different than her cooperating teacher. Tara felt she pushed students to think more. In fact, her cooperating teacher expressed that Tara's questioning of students was one of her strengths. Even though she thought of herself differently, Tara very much felt supported by her cooperating teacher. "She gave me a whole lot of freedom to do pretty much what I wanted….[before] math kind of got skipped over a lot of times, and I think that I was focused a lot more on it, trying to get it in everyday, and even multiple times a day if I can." Tara could not think of anything in the classroom that had constrained, or "gotten in the way", of her teaching mathematic exactly the way she wanted, but she did relay a story relating to the very strict school-wide climate. The kindergarten students were walking in line between buildings at the school. She asked the students to count the poles aloud as they walked past each one. The students reminded her that the school rule was to be silent in the halls and walkways. She informed them that it was okay because she was the teacher, and she gave them permission. Tara had no qualms about establishing her own rules if she did not agree with the norms of the community.

Tara liked the Everyday Mathematics curriculum so much that if given a choice she would continue to use it. She explained that “it’s really flexible,” a view not held by many other cooperating and preservice teachers in the building, including her own cooperating teacher. “Like I’ve jumped ahead three lessons because it relates better to what we were doing at that time, and then I’ve gone back and correlated it into the past lessons, and I think it’s really—it gives you freedom.” She did not seem constrained as other the other preservice did by the need to follow the curriculum lesson by lesson, one day at a time. She often enjoyed the extra projects provided in the curriculum, like the treasure map activity, and incorporated them into her every day lessons.

When asked how her beliefs about mathematics and her role as a teaching had changed Tara spoke mostly about her feelings towards mathematics. She hated mathematics prior to starting in the teacher education program and was excited that her students see mathematics as purposeful and fun, the way she does now. “I mean I love math and when I couldn’t get to it yesterday, the kids hated it. I hate it. It’s just one of the subjects they love….It’s good because I wasn’t a math person.” When asked if she is a math person now, she replied, “Actually, more so.” Tara emphasized connecting mathematics to the real world in almost every lesson and interview, and while it may not be a change in beliefs, it was mentioned only once in passing in all of her writings during mathematics methods. Making this connection to the real world was important for Tara because she remembers her experience as a high school student question why she had to learn the mathematics and how it applied to her life. Her beliefs about her role as a teacher have not changed, but her view of herself in relation to mathematics did change.

Andres. Andres also felt supported by his cooperating teacher, describing his student teaching experience as “wonderful”. He reported learning a lot from her, but still imagined himself differently when it came to teaching mathematics.

> I will give students a little bit more freedom to discover things in the classroom. It doesn’t bother me if the students are talking or whatever while doing activities in the classroom because that’s what I want. I want them to be able to discuss things in the group and talk. I love when there is talking, but I feel the pressure that they need to be quiet, and then, um, that creates a little disagreement between the student and the teacher.

When asked what he would do if he ended up at a school where the expectations of teachers were different than his beliefs, he smiled and said, “Probably I might get into a little trouble.” He then continued to speak of finding a balance without giving up what you believe to be right. In short, it is all about negotiation.

Andres used the curriculum, in this case the lesson plans provided by the county, but felt free to pull from other resources to address the objectives. He saw the lesson plans as simply another source for ideas rather than a document that dictated his instruction.

Andres reported changing over the course of his teacher education program, beginning with the mathematics methods course. In his final interview, he spoke of his continued belief in the importance of challenge and sense-making.

> Well, the first math that I took in the university about two years ago, with you I guess, I remember trying to figure out the things . . . but I didn’t really understand because it was kind of fun and kind of challenging, but at the end we were doing something that becomes something very exciting—the answer, all the data we were collecting, the different ways to do one problem. Not only one way, there are different ways to do it. In all the learning, teaching that I have been exposed

> to, it changed my mind, the way I was thinking before about math. It changed it because now when I'm teaching I'm trying [to get students] to make sense of what they are learning.

Interestingly, Andres, like Tara, underlined the making connections to real world, again something that was not mentioned much in methods writings. Andres spoke of an experience as a boy in fifth or sixth grade when he went to the market with his uncle to buy pigs. His uncle, who never went to school, asked him how much they would have to pay for the pigs. When he requested a paper and pencil to figure it out, his uncle was in disbelief that he had been to school so many years and couldn't figure it out in his head. His uncle then reasoned out loud to come up with the answer. For Andres, connecting mathematics to the real world is not only important; it is the goal of mathematics. As he explained in his final interview, "I have the feeling that most students think math is only related to tests. My goal is to make math practical, usable and linked with reality as much as possible."

Summary

Andres and Tara performed in ways that aligned with their beliefs and participation in mathematics methods. The provided challenging, open-ended tasks and expected students to make sense of the mathematics and persist through difficulties. In addition, they felt a sense of autonomy in relationship to the contexts in which they found themselves, adjusting curriculum and classroom routines to teach in the ways they felt best for their students.

Leaping Backwards? Creating Complexity

Preservice teachers in the creating complexity orientation in mathematics methods often made the problems they encountered more difficult and solved them in multiple ways, looking for connections between them. They believed that students should reason through problems and that this work was helping them understand the different ways students in their own classrooms would approach tasks. Kristen and Helen attempted to implement their beliefs in the classroom, allowing students to explore and reason through their working. In mathematics methods, Kristen and Helen were able to make the connections among different solutions and between concepts and procedures. When their students were exploring, however, they did not always make the intended connections. Kristen explained that she did not know how to help the students "make the leap." The students were not the only ones doing the leaping. Helen described her experience as "working backwards," taking apart the mathematics she learned as a student in order to teach it. Both preservice teachers worked hard to put reform-based mathematics instruction into practice, but they had difficulties helping their students leap forward while they leapt backwards. Thus, the mean scores for the preservice teachers in this group (see Figure 14) were significantly different than the other three groups, higher than the resisting and acknowledging groups, but lower than the scores of the preservice teachers in the embracing complexity group.

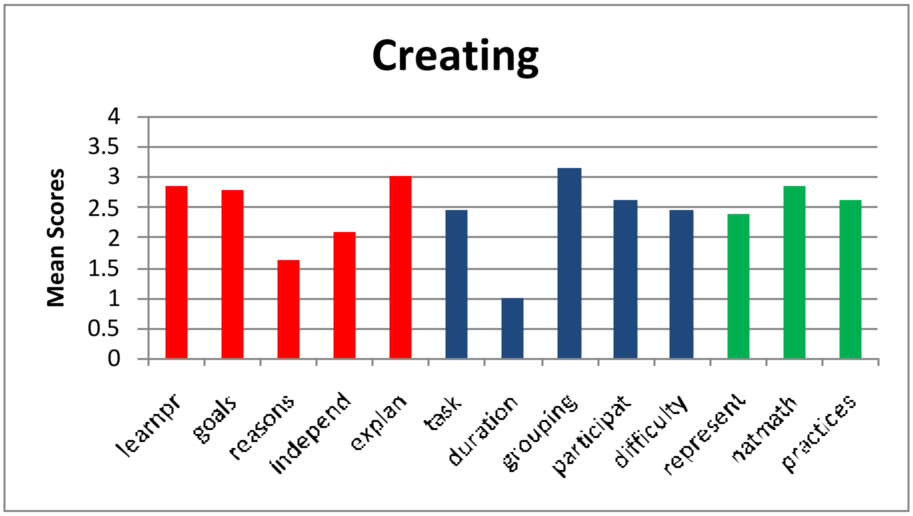


Figure 14. ***Mean Scores for the Creating Orientation***

Performing

Norms. The norms were examined in terms of how learning was promoted (2.85), the goals for students (2.77), the reasons for doing mathematics (1.62), the level of independence (2.08), and what counts as explanation (3.00).

How learning was promoted often involved reflection on work and required some persistence, but at other times the goal for students was to imitate procedure. In a third grade lesson on volume, each group used cubes to find the volume of the same 3x3x3 cube they had assembled from a net. When each of the four groups came up with a different answer, Helen recorded these answers on the board. She told the students that only one group had the correct number of cubes, but that she would not say which group. Instead, each group should think again and try a different strategy. She wanted them to reflect on their answers and persist. She circulated and asked the different groups to explain their thinking to her. For example, one group arrived at 54 because there were nine cubes on the bottom and six sides, and nine times six is 54. When the whole class came back together, Helen recorded the revised answers on the board, all of which were now incorrect, as each group called out their number. However, rather than have students explain their strategies and come up with a consensus together, Helen immediately gave students the formula of base times height. The expectation for learning changed as student gave short, quick answers, telling Helen when prompted, the base, and then the height, and then the solution to 9x3, base x height. She then instructed them to fill in the chart (See Figure 10) correctly for Box B, C, and D. Students used the base x height formula for the rest of the lesson, never reflecting on or revising their previous work for box A.

Both teachers gave students autonomy to raise ideas with the class, and expected students to explain their thinking, and to occasionally comment on other's thinking, asking them whether they agreed or disagreed with a classmate's comment and why. Yet, the shift from these reform-based expectations to more traditional expectations of students continued to occur for both preservice teachers. Sometimes this shift was prompted by the students and other times by the mathematics itself. Kristen expressed some frustration during a post-lesson interview when students during the lesson kept asking her to show them exactly how to get the fill in graphic organizers for some word problems. She wanted them to think about it on their own and could not figure out how to give guidance without telling.

> Of course I want to avoid that [telling exactly how to do the problem].That's not what it's about at all. But, yeah, that's where I started to really--my mind was going a thousand directions with those boxes. I couldn't give them any specific,

> not instruction, but guidelines to how they should begin thinking about it, how should they put the word problems into one of those boxes. How do they make that leap? That's what I couldn't do on the fly.

In a different lesson on place value, however, Kristen was deliberately more directive, having students copy as they filled in decimal places on a number line. When asked why the lesson was so different from others in which she focus more on student ideas and explanation, she replied that she was teaching a system that was already in place. "It seemed not to lend itself to their own discovery, more so. They had to be told things."

Helen and Kristen saw the value of student reasoning in mathematics methods and believed their role was to support such reasoning. They performed in ways that indicated that they were trying to establish the norms of student reflection, persistence, and explanation, but when students struggled, they reverted back to a traditional focus on teaching as telling.

Task structure and implementation. The tasks were examined in terms of structure (2.46), duration (1.00), grouping (3.15), participation (2.62), and level of difficulty (2.46). As shown in Helen's lesson on volume, the structure of the tasks given to students were generally intended to be open-ended, but were not always implemented in that way. Both preservice teachers had students work on tasks in partners and groups. In addition, they consistently brought the whole class back together at the end of each lesson at the least to review answers, and sometimes to discuss and get a group consensus of ideas.

In general most students were actively engaged, and Helen, in particular, encouraged students to participate. When a student tried to share a new strategy before another students' idea had been explained, she said, "Hold your thought," and continued to lead the discussion about the first student's idea. In another instance, a student who reluctantly mumbled her answer was told to "speak up. Be proud of what you are saying." However, sometimes, both teachers became engaged with a small group of volunteers or even a single student in a discussion about a problem. They would become absorbed in an extended conversation about the mathematics while the rest of the class was clearly disengaged.

The level of difficulty varied in the same way as the task structure and norms. Sometimes students knew the material, and other times they were challenged. Kristen was particularly concerned with finding the right amount of challenge for her students explaining, "Sometimes I'm surprised, not just in math, but all subjects. If I give them something that is too hard and then I realize it after—like a non-fiction book—they eat it up." A recurring theme in her reflections and interview was her struggle with finding ways to provide remediation and extensions for her students.

Kristen asserted in the methods class that the mathematical activity the teacher chooses is of vital importance for both engagement and understanding, yet just like the norms category, the tasks presented and their implementation varied throughout the year for both teachers.

Mathematics. The mean scores for representation, the nature of mathematics, and the practices of doing mathematics were 2.38, 2.84, and 2.62 respectively. Both preservice teachers largely focused on conceptual understanding. For example, in a lesson on division, Kristen led a discussion on the meaning of the remainder in different problems. The first problem asked how many sticks of gum each of three students would get if there were 13 sticks of gum. Students concluded that each student would get four and the remaining stick would be split into three pieces; each student would get 4 1/3 pieces. Several students tried the same method on a subsequent problem involving cars until one person realized that you couldn't split a car into pieces like the gum if you still wanted to drive it. More situations involving bugs, fruit bars, notebooks, and cupcakes gave students an opportunity to reason through how to interpret the remainders in varying situations.

When the lessons did focus more on procedure, often Helen and Kristen attributed the switch to factors in the context. Helen planned a "quick, how to do basic two digit multiplication" right before the district's benchmark testing because they had not covered it yet. When asked whether she thought of multiplication as requiring conceptual work or memorization, she explained, ". . . I don't think it's memorization, but we have a lot of stuff to teach them, and like, the quickest way to teach it to kids is to have them memorize. I don't want to, but . . ." Time and testing pressure influenced her to focus solely on the procedure in that situation.

Both preservice teachers used multiple representations to inform conceptual understanding in each lesson. After students collected and graphed data on the number of raisins in the individual-sized boxes, Kristen drew a line around the data on the tally chart (See Figure 15) and asked students what they saw in the line. Responses from students included words like hill, curve, snake, mountain, and roller coaster. They then discussed what the shape of the data told them about what was typical, tying it back to the mode of 76, which they had found earlier. Kristen ended the discussion by asking students how they might use the shape of the data to make predictions about future data.

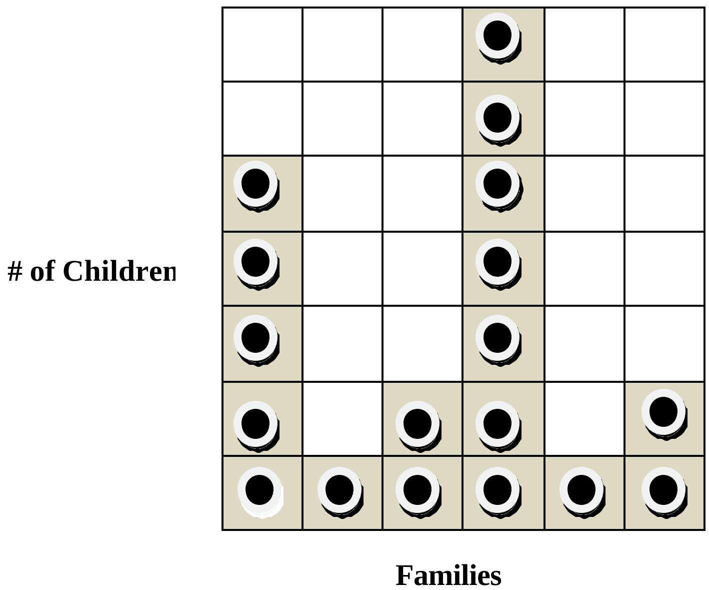
Tens	Ones
72	ll
73	l
74	l
75	lll
76	llll
77	
78	
79	
80	l
81	l
82	l
83	l
84	ll
85	
86	ll
87	
88	
89	
90	
91	ll
92	
93	l
94	
95	
96	
97	
98	
99	
100	l

Figure 15. ***Tally Chart of the Number of Raisins in Each Box***

Although multiple representations were used, students did not always see how those representations were connected. In a lesson on finding the mean, Helen began by having students put counters on a bar graph showing the number of children in different families. They were to put one counter on each cell that was shaded and then make the groups equal (See Figure 16). Students were able to successfully move the counters to end up with three counters in each group, the mean number of children in each family. Next, Helen asked students how the mean could be found without using the counters. One student began to give the procedure for median and was stopped. When no other students had ideas, Helen showed them that they could add up the numbers and divide by six since there were six families. On subsequent problems, students continued to use the procedure. Multiple times Helen asked the question, “What is the mean?” The accepted answer each time was, “the average.” Similar to the volume lesson described previously, Helen started focusing on conceptual understanding then skipped to procedure without, as Kristen said, making “the leap.”

The ways the preservice teachers performed themselves were only somewhat consistent with the expectation based on data from mathematics methods. At times they employed reform-based instructional practices and at other times they used traditional instructional practices. Kristen and Helen attempted to practice in ways that matched their beliefs by giving students opportunities to reasons, share strategies, and explain their work, and by providing tasks that focused on conceptual understanding and the use of multiple strategies. Yet many times these lessons would digress into traditional lessons, resulting in inconsistent scores on the performance observation framework.

Original Graph



Graph with Counters Put Into Equal Groups to Show that the Mean Equals Three

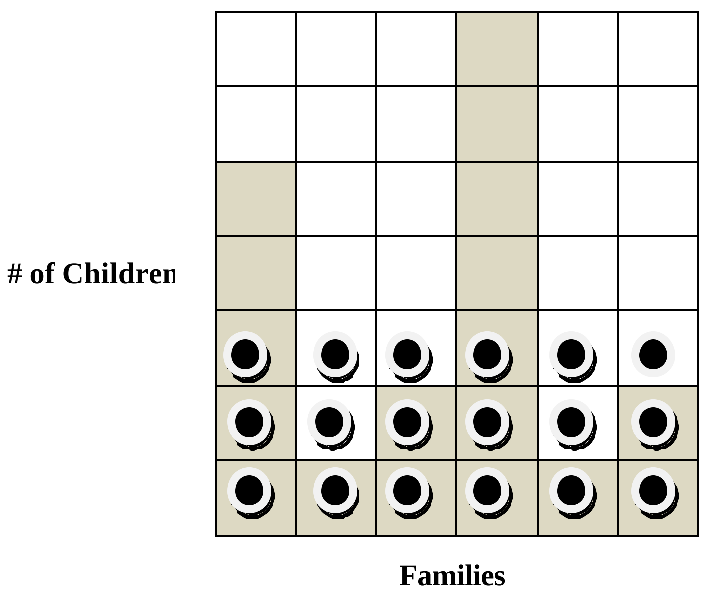


Figure 16. ***Finding the Mean***

Imagining

Helen. Helen identified strongly with her cooperating teacher, although she felt that she might “push them [students] more to explain the strategy.” Her cooperating teacher agreed, citing her teaching of mathematic, in general, and her questioning of students, specifically, as strengths. Other than that, however, Helen felt their teaching styles were similar. This similarity was evident in some of the comments made by the cooperating teacher during lessons. While helping individual students during work time, she said to a student, “Think like a mathematician. Try before you ask me.” On another occasion, she partnered up with a student who was shy and helped her explain her thinking to the rest of the class; encouragement that permeated Helen’s teaching as well.

Like other preservice teachers, Helen liked the guidance and strategies provided by the Everyday Mathematics curriculum. “It’s been really supportive. It helps you set it up and structure it in a way that you might think about and you can see different ideas.” She did not like, however, what she perceived as a focus on smaller goals that build up to a bigger concept. She explained, that “sometimes learning the bigger picture and then breaking it down is a better way to teach for me.” Instead of doing one strategy per day, she would rather teach a group of strategies and then work on them for a longer period of time.

Each time Helen is asked about how her beliefs have changed, she focuses on the mathematics instead of her role as a teacher, not surprising since her second concentration is mathematics. She referred to new strategies she has learned that are different from the way she learned mathematics. “I’m always learning new things. New ideas come up; new strategies that I didn't even know. I mean, I enjoy it." Helen recounted how at first she saw herself teaching with formulas. In her final interview, she explained that the formulas were still there, "but it's like you teach different strategies to get to that formula, so it's like you are doing it backwards." When asked to clarify what she means by doing mathematics backwards, Helen replied that she was now breaking everything down and then putting it all together again, which is not how she's "used to doing it. Like it's backwards from everything I ever learned in math, like starting left to right instead of right to left and breaking partial products. I've never done any of that, so it's all new to me, and to me I feel like that's working backwards, like breaking it down step-to-step and then putting it all together again." In methods, Helen explained that what she was learning from her work as a student solving the problems was the different ways to solve problems. This is what she also reports learning from her work as a teacher.

Kristen. Due to circumstances unrelated to this study, Kristen changed cooperating teachers in the middle of student teaching. She did not identify with either one of her cooperating teachers. The first cooperating teacher preferred students to work independently and quietly, but Kristen felt like she had the freedom to try new things and like she was the expert in mathematics in the room. She got the opposite impression when she went to the second cooperating teacher who insisted that math was the last subject Kristen took over because it was always the hardest for student teachers to learn.

The change in classrooms influenced Kristen's performance, but not her beliefs. The last two observed lessons took place in the second classroom in which Kristen felt she had little autonomy to make decisions. Kristen was teaching second grade students how to play a game with money in which students were to put the number rolled on a die on a chart in their work book (See Figure 17). Once the number was placed, it could not be moved, and the player with the most money at the end won the game. Students were to add the values of the dollars and coins with a calculator so they could practice entering the numbers in decimal form. When Kristen suggested to students that they write the decimal form above the number of coins to help them, the cooperating teacher jumped into the lesson saying, “I don’t believe they’re supposed to change their book. They just do the calculators. They are supposed to leave their book with those numbers.” Kristen erased what she had written on the board and went on with the lesson.

Kristen’s writing which
was later erased (in bold).

Pennies	Nickels	Dimes	Quarters	Dollars	Total
		2 **(.10)**			

Figure 17. ***Chart on the Board***

After the game, students completed some review problems in which they were given numbers and asked to find the median. Although Kristen did ask some students to share how they had done the problem, they focused primarily on the procedure of crossing out the numbers, a very different focus that the fourth grade lesson described earlier when students collected data on the number of raisins in a box and analyzed it. During the lesson the cooperating teacher made notes on Kristen’s lesson. Two pieces of feedback were written, “way too slow and too much” and “NO need to go over the problem. Just ask for the answer and move on.” Kristen stated at the beginning of the post-lesson interview that her lesson was a “train wreck.” When asked why she thought so, she said because her cooperating teacher told her it was. She talked later in the interview about how they had very different styles.

In spite of some frustration, Kristen also described how the same cooperating teacher was helping her focus on the whole class during a lesson, noting that she paid too much attention to individuals and would lose track of the group. Kristen also noted that the cooperating teacher knew exactly where her students were. “It’s not just hit and miss, what they like. She makes sure they have the right level so it’s not frustrating or too easy. They are constantly being challenged and moved along.” She was also the only teacher that Kristen ever heard say she liked Everyday Mathematics. Like Helen, Kristen liked the Everyday Mathematics curriculum. She explained, “Everyday Math is growing on me because it builds them up little by little. It really addresses their understanding of what they are doing.” So although Kristen felt less freedom to teach the way she wanted, her strategy was to do what her cooperating teacher asked and to glean the things she did like. As she said, “Six weeks is nothing.”

Kristen stayed the most consistent of any of the preservice teachers when asked about her beliefs about teaching mathematics. Throughout her interviews and written reflections, she continued to reiterate that students needed to internally conceptualize mathematical concepts and use mathematical reasoning. Her job was to provide hands-on activities and visual representations and anticipate how students would respond to new ways of doing things so that she could scaffold.

Summary

Although Kristen and Helen believed students should reason through problems to develop conceptual understanding, and these beliefs did not change over the course of the two years, the ways they performed as teachers were not consistent. Instead they tended to start lessons using reform-based practices and then switch to traditional telling of both concepts and procedure.

Conclusion

For the most part, the preservice teachers in this study performed in ways that were similar to their participation and beliefs in mathematics methods. Furthermore, the embracing group performed significantly higher than the other three groups. They promoted the learning of mathematics as an endeavor that required autonomy, reflection, and conceptual understanding. The resisting and acknowledging groups had the lowest scores and were not significantly different from each other. The preservice teachers in these groups promoted a narrow view of mathematics and some struggled with their conceptual understanding of the mathematics. The preservice teachers in the creating complexity orientation varied in their performance. They had consistently high scores in the mathematics category of the framework, but varied on other continua.

Evidence from the quantitative and qualitative data suggests that the preservice teachers in three resisting, acknowledging, and embracing orientations did not change their instructional practice over time and the change in the creating complexity group was a result of fluctuating scores rather than a trend in a particular direction.

In contrast, some preservice teachers did change the ways they imagined themselves in terms of their beliefs about their role as a mathematics teacher, and these changes did seem to follow a pattern. The beliefs held by Lisa and Rachel at the end of student teaching about the importance of manipulatives, multiple solutions, and student explanation matched the beliefs held by the acknowledging group in mathematics methods. Similarly, the acknowledging preservice teachers emphasize student thinking and autonomy in their final interviews, beliefs held by the embracing category in methods. The preservice teachers in the embracing and creating orientations did not change their beliefs about their role as teachers, and instead reported the changes in their own understanding of mathematics and of pedagogy.

Preservice teachers reported a number of influences on their teaching over the course of the two years including the mathematics methods class, their cooperating teachers, the curriculum, and interactions with students. However, these influences do not seem to be related to the different orientation groups.

CHAPTER IV

DISCUSSION

Preservice teachers who enter teacher education programs often have only experienced traditional mathematics instruction. Therefore they must make two transitions, from the role of student to teacher and from a traditional to a reform-based view of mathematics. In this study I followed preservice teachers across the two years of their teacher education program to see how their emerging identities as mathematics teachers in internships and student teaching related to their participation and beliefs as students in mathematics methods. I characterized identity as the way the preservice teachers performed, imagined, and believed themselves as teachers of mathematics. In phase one, the preservice teachers participated as students in ways that resisted, acknowledged, embraced or created the complexity of reform-based mathematics. In phase two, the preservice teachers in the four orientations performed differently from each other in internships and student teaching. They did not change their instructional practice much over time, but they did change the ways they imagined their role as mathematics teachers. All the preservice teachers had difficulty implementing open-ended tasks in the classroom, but the nature of their struggles and their resulting teaching decisions differed. A major implication for this study is that, just as with open-ended tasks in the classroom, there are multiple entry points to the task of teaching reform-based mathematics. Future research should seek to understand these entry points and to explore the paths taken by preservice teachers as they continue in their profession.

Performance as Teachers of Mathematics

Preservice teachers' instructional practice was related to their participation in mathematics methods. Preservice teachers' performance in the embracing and creating groups differed significantly from each other and from the resisting and acknowledging groups. Although the preservice teachers in the resisting group performed slightly lower than the acknowledging group on the performance observation framework there was not a significant difference in the scores.

Resisting and Acknowledging

All the preservice teachers in the resisting and acknowledging groups used teaching practices associated with reform-based mathematics to varying degrees, but they still continued to focus on mathematics as procedure. For example, they did ask student for explanations, but the discourse was *low press* (Kazemi & Stipek, 2001) meaning that the teachers never followed-up to ask students to connect procedures to the concepts behind them. The norms for explanation, that is, the expectation for the students' responses, indicated that they were not seeking to gain a deeper understanding of the student's thinking. Instead they were expecting a repetition of taught procedures. When multiple strategies were employed to solve problems, they were not students' ideas. Rather, the different strategies were presented as a laundry list of procedures from which students could choose. Similarly, echoing previous research findings, the manipulatives were often used to engage students in a topic (Moyer, 2001) or as a rote part of doing a procedure (Hiebert & Wearne, 1992), but were not used as a valuable tool in developing a conceptual understanding of the mathematics itself. The mathematics itself was still procedural.

This continuation of the focus on procedure is reminiscent of Raymond's (1997) findings that novice teachers' instructional practice was more in line with their beliefs about mathematics than their beliefs about mathematics pedagogy. The preservice teachers' own understanding of mathematics may have come into play in addition to their views of the nature of the discipline. Lisa, Allie, and Jenna all expressed that they sometimes did not understand what they were teaching, and consequently reverted to telling procedures in the ways they learned as K-12 students.

Embracing

The teachers in the embracing category performed did focus on conceptual understanding. They wanted students to be challenged and struggle, and therefore, they gave students open-ended tasks and let them construct their own understanding. In contrast to the first two groups, their discourse was *high press* (Kazemi & Stipek, 2001), that is, discourse that pushes students to link the strategies and procedures used to the concepts behind them and helps students learn from mistakes. In addition, Andres and Tara believed in giving their students autonomy and allowing them to be the source of ideas in the classroom. They saw listening (Davis, 1996) as one of their major functions in the classroom. Then often then built their lessons on student ideas, which required them to make changes to lessons on the spot, what Duffy (2005) calls "thoughtful adaptation" (p. 300).

Creating

Preservice teachers in the creating category, like the embracing, believed students should reason to solve problems, but they fluctuated in the implementation of these beliefs. In some ways, their teaching was very reminiscent of their work in methods. When a student had an interesting solution or had made a mistake in a way that interested the teacher, Helen and Kristen would spend a lengthy amount of time delving into the problem with the student or small group while other students sat idle and disengaged. They would examine it from every angle just as they did when solving problems in methods. Another facet of the creating teachers' experience was their emphasis on the mathematics itself. Their concern was the need to the need to break apart mathematics they could do simply and then build it back up again, or as Helen said, work backwards. This idea relates to the assertion by Ball, Hill, and Bass (2005) that the mathematics knowledge required for teaching is different than that required of mathematicians; teachers need to unpack the mathematics to make it explicit for students while mathematicians spend their careers trying to make it more simple and elegant.

Summary

As described above, preservice teachers' instructional practice differed in significant ways that related to their participation as students in mathematics methods and their beliefs about their role as teachers of mathematics. In the next section, to further illustrate how these differences manifested themselves in the classroom, I provide an example of how the different groups handled an undertaking in reform-based teaching, the implementation of open-ended tasks.

An Example of Difference: Implementing Open-ended Tasks

Preservice teachers in all the orientations struggled to implement open-ended tasks, but dealt with their difficulties in different ways. The dilemma for the teachers occurred at different points. For the preservice teachers in the resisting and acknowledging groups, the point of decision occurred as soon as students began to struggle with a problem, or even during planning, if they anticipated that students would struggle. Their decision was either to eliminate the open-end part of the task all together or to lower the cognitive demands of the task. The embracing and creating teachers' point of decision came later, during the task. Both groups would launch the task and give students time to work, explain, and reason through mistakes. At times, though, the students did not make the intended connections to mathematics concepts. Ball (1992) suggests that often teachers just assume that by using manipulatives students will make the intended connections to the mathematics concept being taught. The teachers in the creating group would continue having students work and share to point, but then in frustration, or maybe desperation, would switch to telling. To use Kristen's phrase, they didn't know how to "make the leap." In contrast, Andres and Tara, in the embracing group, would err in the opposite direction. They would continue to give little or no guidance resulting in unsystematic exploration that was ultimately unproductive in terms of making the desired connections.

Henningsen and Stein (1997) discuss several ways that tasks that are intended to have high cognitive demand fall short, including declining from doing mathematics to procedures without connection to concepts and declining from doing mathematics into unsystematic exploration or into no mathematical activity at all. What is significant about the current study is the relationship between the struggles with implementation and the different orientations of preservice teachers. Rachel and Lisa resisted; they disengaged when met with difficult problems in methods. Similarly, when met with a challenging task in their teaching, they chose to eliminate the struggle both for themselves and for their students. Alternatively, Tara and Andres as students relished in challenging problems, describing them at the same time as hard and fun. When students were struggling with open-ended tasks, their reaction was to stand back and let them continue to work.

One implication of this study is that there are different entry points to reform-based teaching for preservice teachers. Just as they approached mathematical tasks from different directions as students in mathematics methods, the preservice teachers approached the task of teaching differently, working from their strengths. Those in the acknowledging profile did not feel confident in mathematics, but general classroom pedagogy was their strength. After all, Allie was the preservice teacher chosen to take over when a teacher left unexpectedly—recognized as one of the strongest student teachers in the school. Allie and Jenna were earnest in their attempt to employ reform-based teaching strategies, but they sometimes had trouble understanding the mathematics. Likewise, Kristen and Helen worked from their strength. In this case, their strength was the mathematics, and they struggled with the pedagogy. Because the preservice teachers are at different starting points, movement toward fuller reform-based instruction will require them to take different paths, follow different trajectories of participation (Wenger, 1998). Teacher educators need to recognize the different entry points and possible paths in order to help scaffold preservice teachers' movement. In addition, further research is needed to explore the paths that these and other preservice teachers take as they begin teaching.

Imagination as Teachers of Mathematics

While there was not significant movement over time in the preservice teachers' instructional practice as measured by the performance observation framework, they did imagine themselves differently. Imagination refers to the ways preservice teachers saw themselves as teachers in the context of their practice. This includes their beliefs about their role as teacher of mathematics and whether or not they identified with the curriculum used and their cooperating teachers.

Beliefs

The teachers in the embracing and creating categories remained firm in their beliefs about teaching mathematics. For Andres, Helen, and Kristen, the reports of change were not about pedagogy or the nature of mathematics, but that they were learning more and different strategies to teach particular concepts. For Tara, the change was how she saw herself in relation to mathematics as she started to like something that before she had hated.

Conversely, the preservice teachers in the resisting and acknowledging group did report changes in their beliefs. In methods, the preservice teachers in the resisting group strenuously objected to reform-based practices, preferring the traditional instruction they experienced in their K-12 schooling. Yet they both expressed in their final interview a belief that students needed to use manipulatives, to find multiple strategies, and to explain their solutions. These beliefs were the same as those held by the teachers in the acknowledging group at the end of mathematics methods. Allie and Jenna, in the acknowledging category, reported that they had begun to realize the importance of focusing on student thinking and autonomy, beliefs that match those of the preservice teachers in the embracing category. This change for both groups is encouraging, but it did not manifest itself in a substantial change in practice. Ambrose (2004) proposes that mismatches between preservice teachers' beliefs and practice could be viewed not as conflicting beliefs, but as "evidence of evolving beliefs" (p. 117). Given that preservice teachers' newfound beliefs about their role as mathematics teachers seem to follow a pattern from resisting to acknowledging to embracing, it will be important to follow the preservice teachers further to see if the ways they perform and imagine themselves continue to evolve and eventually coincide.

Curriculum

The curricula used in the classrooms did influence the preservice teachers. Regardless of mixed opinions, one commonality among all the preservice teachers who used Everyday Mathematics was that they liked that they were given the different strategies for each concept. They felt like they learned more mathematics through the text. The preservice teachers in the embracing group seemed the only ones willing to make major adjustments to the lessons in either curriculum. The others made minor adjustments, particularly when they did not understand something, but felt the need at the very least to "stay on the right day". Conversely, Tara did lessons in a different order than prescribed or substituted the program's enrichment projects in place of lessons. Andres used the lessons provided to him only as a resource, picking and choosing what he wanted. If the lessons were not playing out as planned Andres and Tara would change them on the spot. Cooney and Shealey (1997) proposed that when teachers transition from procedural to reform-based teaching, the first shift that needs to be made is a shift from the text as an authority to themselves and the mathematics as the authority. The embracing group, more than any of the others, felt autonomous to deviate from the curriculum. Once again, the embracing teachers were willing to thoughtfully adapt (Duffy, 2003), not only in their planning, but in the moment.

Cooperating Teachers

The cooperating teachers were certainly influential for each preservice teacher, but no discernible patterns existed among groups. Some identified with their cooperating teacher and others did not. Perhaps the most interesting observation from the data was that the cooperating teachers very seldom gave the preservice teachers advice specific to mathematics. Almost all the feedback provided to student teachers revolved around general teaching issues such as classroom management and planning. The preservice teachers were newcomers to reform-based mathematics as students, and then immediately thrust into the role of expert as teachers. They received support in their internships to make the shift from student to teacher, but lacked the same domain-specific support to help them continue the transition for reform-based teaching. Instead, they were expected to "make the intended connections." One implication of this lack of content specific support is that teacher education programs need to work with cooperating teachers on ways to support student teachers in the teaching of mathematics in addition to general teaching practices. This support for cooperating teachers is particularly important in elementary schools, where teachers are often more focused and hold more expertise in literacy.

Summary

Findings indicate that the preservice teachers change the ways they imagined themselves as mathematics teachers. Their beliefs about what they should do as mathematics teachers began to evolve, but this shift was not yet evident in their practice. There was no pattern in the ways that the different groups either affiliated with or contested against the norms of the cooperating teacher and the curriculum, except that the preservice teachers in the embracing category demonstrated a greater sense of autonomy. Although not the original goal of the study, an unexpected finding was the lack of feedback from the cooperating teachers that related specifically to the teaching of mathematics instead of general pedagogy.

Policy Implications

In addition to the practical implications for teacher educators that have already been discussed, this research has policy implications for teacher education programs. Results from this study indicated that preservice teachers' participation while doing reform-based mathematics as students is related to their practice as teachers. Yet once preservice teachers leave mathematics methods, the opportunity to engage in mathematics problem-solving as students, that is, to become fuller participants in a reform-based community, diminishes. Teacher education programs need to be structured in such a way that preservice teachers continue to have opportunities to increase their own understanding of mathematics and to participate in the practices of doing mathematics throughout their course of study. Similarly, as the preservice teachers do make the shift to teaching, they need more domain-specific pedagogical feedback throughout their internships and student teaching. Finally, a sense of autonomy, and the willingness to thoughtfully adapt (Duffy, 2003) as a result, was a key feature of the instruction of the creating, and particularly, the embracing groups. This orientation is important in multiple domains and should be fostered in all teacher education courses.

Future Studies

Several opportunities for future studies exist in relation to this research. First, this particular group of preservice teachers should, and will, be followed into their first year of teaching to investigate how they continue to evolve in yet another context, particularly since they will presumably have more freedom to teach in the way that they would like. Second, more studies are need to examine the ways that preservice teachers are participating in reform-based mathematics as students to elaborate and confirm the four orientations presented as a result of phase one of this study. Lastly, this research suggests that learning to teach reform-based mathematics is not a linear, upward bound trajectory, but a circuitous route with multiple entry points and multiple possible paths. Research needs to explore these entry points and routes to better understand how to help teachers navigate them.

Limitations

The nature of this study presents several limitations. First, this is a study of one group of preservice teachers from one methods class at one university. The intention was to understand the different ways that preservice teachers participated in a common mathematics methods class and how that difference influenced the meaning they made of their participation and their teaching in multiple settings across time. Due to the situated nature of the study, findings may not generalize to larger populations. Secondly, since this is a qualitative study in which I was both teacher and researcher, my judgments and values were present in every step of the research. Steps were taken to mitigate biases including member checking, multiple sources of data, and coding by other researchers.

Conclusion

Learning to teach is a complex endeavor. Learning to view a discipline in a new way is a complex endeavor. We are asking preservice teachers to learn both at the same time. Preservice teachers make their own meaning of their participation in common experiences in mathematics methods and teach in ways that reflect those meanings. The different meanings preservice teachers make can be understood as different entry points into the practice of reform-based teaching. Knowing the entry points and the paths to which they lead has practical implications for teacher educators as they make instructional decisions in methods courses. In addition, teacher education programs need to consider structuring field experiences in such a way that preservice teachers continue to receive feedback specific to the domain of mathematics. While teachers enter at different points and are situated in different classrooms, those with a common entry point may also follow common paths. Future research should help identify these paths and the experiences that help preservice teachers move along them.

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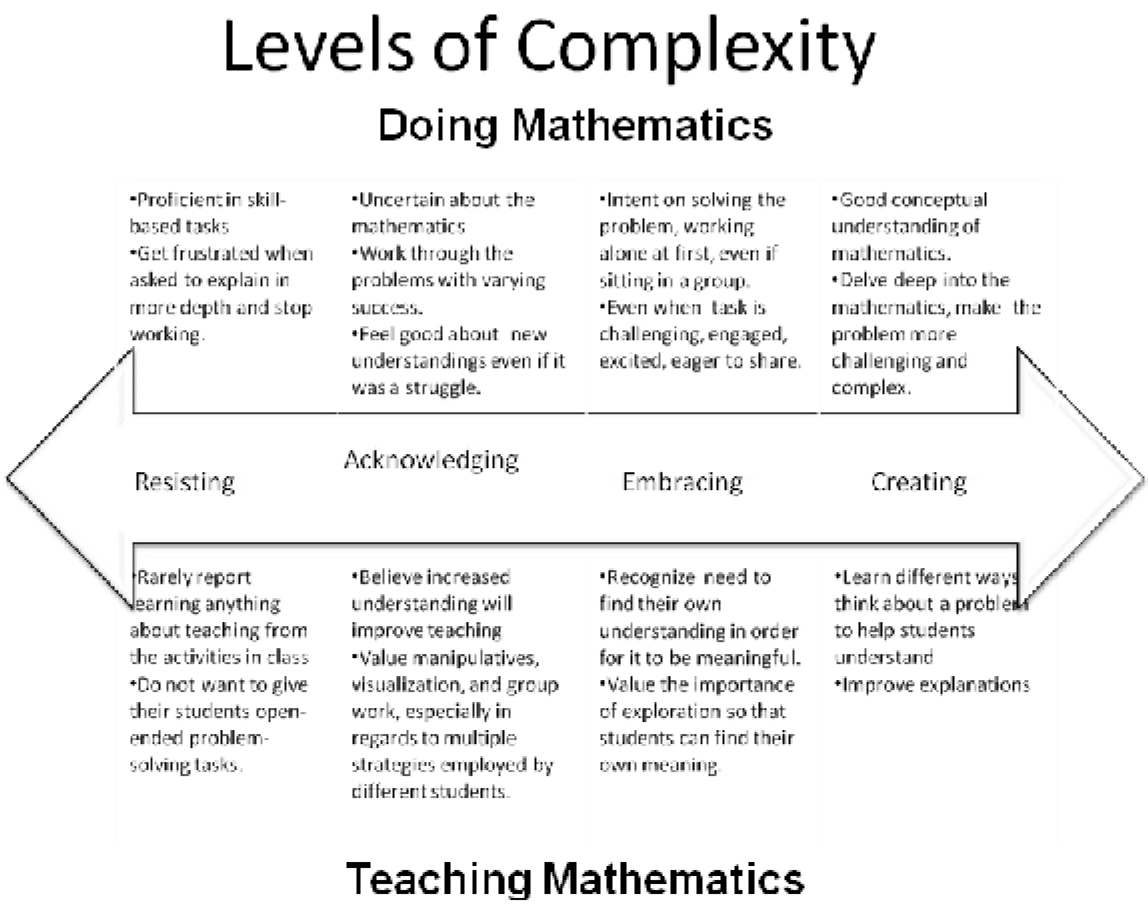
Appendix A

Levels of Complexity



Appendix B

Performance Observation Framework

Norms					
	1	2	3	4	
HOW LEARNING IS PROMOTED Reflexive (quick learning, immediate response)	Right answers only. Short wait time.	Right answers only, longer problems, minimal reflection.	Reflect on work,, but persistence not needed.	Need to reflect on work and persist through difficulties.	HOW LEARNING IS PROMOTED Reflective (requires persistence, transformation model)
GOAL FOR STUDENTS To give students info (transmission model)	To imitate what the teacher did without any discussion	To imitate what teacher did with limited discussion to clear up any misunderstandings.	Teacher has explicit plan & emphasizes need for student to offer explanations.	Teacher places responsibility on the student(s) for discussions and thinking.	GOAL FOR STUDENTS To help students understand (transformation model)
REASONS FOR DOING MATH (extrinsic, accountability based)	No reason given.	For rewards, accountability, or because you are supposed to.	The stated emphasis is on learning and accountability.	To think like a mathematician or apply mathematics to the real world (learning).	REASONS FOR DOING MATH (intrinsic, learning)
LEVEL OF INDEPENDENCE HETERONOMY (students look to teacher for ideas)	Teacher stays strictly with the problems in the text.	Teacher may introduce problems from sources other than text, but is still the only determiner of what will be discussed.	Teacher gives up some authority and student examples are used.	Students raise ideas that are taken up by the whole class.	REASONS FOR DOING MATH (intrinsic, learning)
WHAT COUNTS AS EXPLANATION CLAIMING (telling)	Asks if students understand. Yes or no is an acceptable answer.	Teachers asks for explanation but does not follow-up to clarify.	Teacher asks students to explain and follows up to get more information or clarity.	After explanation or follow up, teacher asks other students to comment on the work.	WHAT COUNTS AS EXPLANATION DISPLAYING (modeling)

Tasks					
	1	2	3	4	
TASK STRUCTURE CLOSED (single answer or process)	Single solution. Task presented as exercise, not problem, usually presented symbolically only.	Right answers only, longer problems, minimal reflection.	Texts have a context and multiple solutions or processes. Makes implicit connections among different strands of mathematics.	Tasks have a context and multiple solutions or processes. Makes explicit connections among different strands of mathematics.	TASK STRUCTURE OPEN (multiple answers or multiple processes)
DURATION (short)	1 day	2 days	3-5 days	more than a week	DURATION (long)
GROUPING INDEPENDENT (students work alone)	Students work alone. Talking is discouraged, but sometimes allowed for help-seeking.	Students work in partners or in groups with no follow up at the end of the lesson, but may seek help from others.	Students work with partners or groups and briefly com back together to report back to the whole class.	Students work with partners or groups and then come together as a whole class for discussion and consensus of ideas.	GROUPING COLLABORATIVE (whole class works together based on previous pair or independent work)
QUANTITY OF STUDENT PARTICIPATION	Teacher only calls on volunteers (same small group of students who "get it"). Other students are disengaged.	Teacher still calls primarily on volunteers but does include some others.	Most students speak or are actively listening in whole group discussions.	Most students speak or are actively listening in whole group discussions. Teacher supports students during individual and partner work to share in whole group.	QUANTITY OF STUDENTS PARTICIPATION
LEVEL OF DIFFICULTY EASY (students already understand the material)	Students already know how to do most of the material.	Material is familiar. Only one or two problems are difficult.	Problems are higher level, but there is no 'what if' (problem-posing)	Students are challenged even when they get the problem right, 'what if' pusher further.	LEVEL OF DIFFICULTY CHALLENGING (requires some struggle)

Mathematics					
	1	2	3	4	
QUANTITY/QUALITY OF REPRESENTATIONS (single, isolated)	One or more representations, focuses only on procedure.	One or more representations inform conceptual understanding; no connections are made among representations	Multiple, representations inform conceptual understanding; Connections among the representations are implicit	Multiple representations inform conceptual understanding; Connections among the representations are made explicit	REPRESENTATIONS (multiple, connected)
NATURE OF MATHEMATICS	procedural, fixed, and isolated, numbers only	procedures are related and build on each other sequentially	conceptual—fixed relationships and patterns; formalization only	conceptual—relationships and patterns viewed as emerging rather than fixed, dynamic; formalization plus inventising	NATURE OF MATHEMATICS
PRACTICES OF DOING MATHEMATICS (exercises)	completing exercises with minimal discussion	completes exercises with some explanation, questioning	Problem-solving with explanation and questioning	Making and testing conjectures, argumentation, problem-solving	PRACTICES OF DOING MATHEMATICS

Appendix C

Sample Field Notes

The following excerpt includes pages 4-6 of the field notes for Andres' first lesson. In the notes, the letters T and S are used to indicate who is speaking. T represents the teacher; S represents student; Ss indicates that multiple students are answering at once, chorally.
NS Lesson 1
10-01-07
Grade 4, 10:20-11:30

11:00 Okay. Activity with dominoes.
T explains that each domino is split into two parts and asks: "How many in each group (holding up double 6)?
S: 6 in top, 6 in bottom
T holds up two more 6/5.
How many all together?
How could we figure it out?
S: Need to add
T: We are going to use the order of operations to represent what we are doing?
Passes out three or four dominoes to each group and asks them to find the sum of the dots on all the dominoes together.
When you are ready, send one person from your table to write answer on the board.

Group	What student writes on the board
Group one (written vertically)	12 + 11 + 10 = 33 goes back up at prompting of teachers and writes 6 + 6 = 12; 5 + 6 = 11; 5 + 5 = 10 (problems are written vertically) Then draws pictures of dominoes.
Group two	9 + 8 = 17 (vertically)
Group three	5 + 3 + 5 + 0 + 3+ 2 + 3 + 1 8 + 5 + 6 + 4 13 + 10 = 23
Group four	6 x 6 + 4 / 4 -4 = ?

Okay when you finish,
draws dominos on board, look at numbers?
11:08

T: Only one hand raised is a problem in mathematics. Okay maybe we don't understand. We are going to try to make this into one equation. Look at the different representations – picture, numbers, symbol (pointing to group four's inclusion of ? for answer)
Student comes up from group one to explain
Back table not listening…students are quiet, but maybe not attending
T: What strategy did they use?
S: Only addition.
T (to class): What other operation could they use?
Ss: multiplication
T: What would that look like?
5 x 2
5 x 6
6 x 2
Everyone look at these dominoes. Draws them larger on board—drawings have actual dots.

5 dots	5 dots	5 dots	6 dots	6 dots	6 dots

T: Group one said 5 x2, why Jorge?
Jorge: 5 x 2 because on that domino there are two groups of five
T: okay so we know that one so let's put it aside. (left with double six and 6/5 dominos). So does 5 x 6 work with what is left?
Group 1 is engaged. Others have lost track although they come back in as T walks to other side of room and asks sum of two dominoes
S: multiply
T: What would you multiply?
S: 6 x 6
Why 6 x 6
I mean 6 x 2
To another student: do you think 6 x 6
S: No
T: Why?
original Student: oh there are not six dominoes
S: 6 x 3
T: Why?
S: Because there are 3 groups of 6
T: What is 6 x3?
S: 18
T: What is left?
S: 5
T Holds up first domino with double 5
T: What can we do?

S: 5 x 3
T: Why
S: b/c… inaudible-check tape
T: When we finish multiplying now what?
Two students give incorrect answers ---inaudible-check tape
S: you have to add
S: 15 + 18
T: How much?
S: You can use multiplication again.
T: Explain how
S: 15 x 2 + 3
T writes on board
15 + 18
15 x 2= 30 + 3 = 33
T: Why might you want to do it that way? Use mult.?
pause…
S: It's faster.
T: It's faster!!
11: 22
T: I'm sorry, but class has to end so we can't finish. Are there questions?
S: When is your next class?
T: Please write these on the board in your notebook for next time.

Appendix D

Post-lesson Interview Protocol

Preservice Teacher:
Time of Interview:
Date:
Place:

1. Tell me about your lesson today.
   As student is discussing, ask about any events/decisions from the lesson that seem relevant. Questions in this portion will vary. Examples:
   - What influenced your decision to_____________?
   - When a student said __________, why did you respond _____________?
2. If you teach were teaching this lesson in your own classroom next year, how would you do it?
3. How does your experience teaching this lesson confirm or change your beliefs about teaching mathematics?

Appendix E

Final Interview Protocol

Beliefs (first five are the same questions from the end of Math Methods.)

1. What is mathematics?
2. How does a student learn mathematics?
3. What is your role as a teacher of mathematics?
4. How would you design your classroom to make it possible for students to learn mathematics?
5. How would you evaluate a student's mathematics learning?
6. How have your beliefs changed over the course of the last two years and what influenced the change?
7. Do you think your teaching now matches the beliefs you've just stated? Why or why not? (may lead naturally into discussion of context and goals)

Context

1. What factors have supported you in the way you would like to teach mathematics?
2. What factors have hindered you in the way you would like to teach mathematics?
3. What curriculum is used in the classroom in which you are currently teaching? If you could choose any curriculum for your own classroom would you choose it? Why or why not?

Performance

Rate yourself on each of the following dimensions (performance observation framework). Explain your ratings.

Goals (Imagining)

1. What will you continue to do in your mathematics teaching next year? What will you do differently? Explain.
2. If you are put in a situation in which the teachers in the school in which you work teach differently than you believe, what will you do? What concerns do you have?
3. Where do you see your mathematics teaching in five years?

Appendix F

Interview Protocol for Cooperating Teachers

Cooperating Teacher:
Time of Interview:
Date:
Place:
This interview will be structured as an informal conversation. Here are the questions/topics to be addressed.

1. Tell me about the feedback that you gave to ________.
2. What is a typical day in your mathematics classroom?
3. How do you use the textbook? outside resources?
4. How do you know if a student is successful in mathematics?
5. What things would you like to do in the future/change?

Appendix G

Questions for Written Reflections

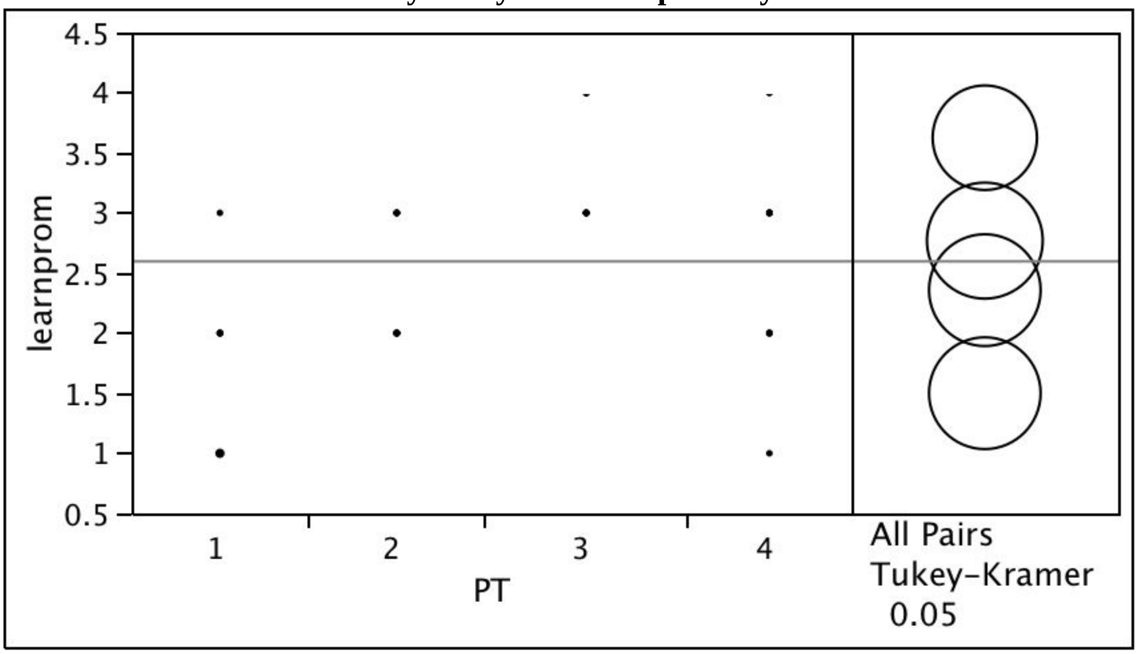
1. Do you think your lesson was successful? Why or why not?
2. What critical decisions did you have to make during the lesson? How did those decisions affect the lesson?
3. Did the students meet the goals of the lesson? How do you know? (If multiple strategies were allowed) What were the most common strategies students used? Were there any you had not seen before? When a student succeeded quickly or struggled, how did you change the situation to meet his or her learning needs?
4. Describe your participation and your students' participation during the math lesson. Describe the "math talk". (The talk may include representations drawn or written on the board. It includes anything that was shared knowledge in the classroom.)
5. How does the lesson you taught change or confirm your beliefs about doing, learning, or teaching mathematics?

Appendix H

One-way Analysis of Each Continuum by Orientation Group

How Learning is Promoted

Oneway Analysis of learnprom by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

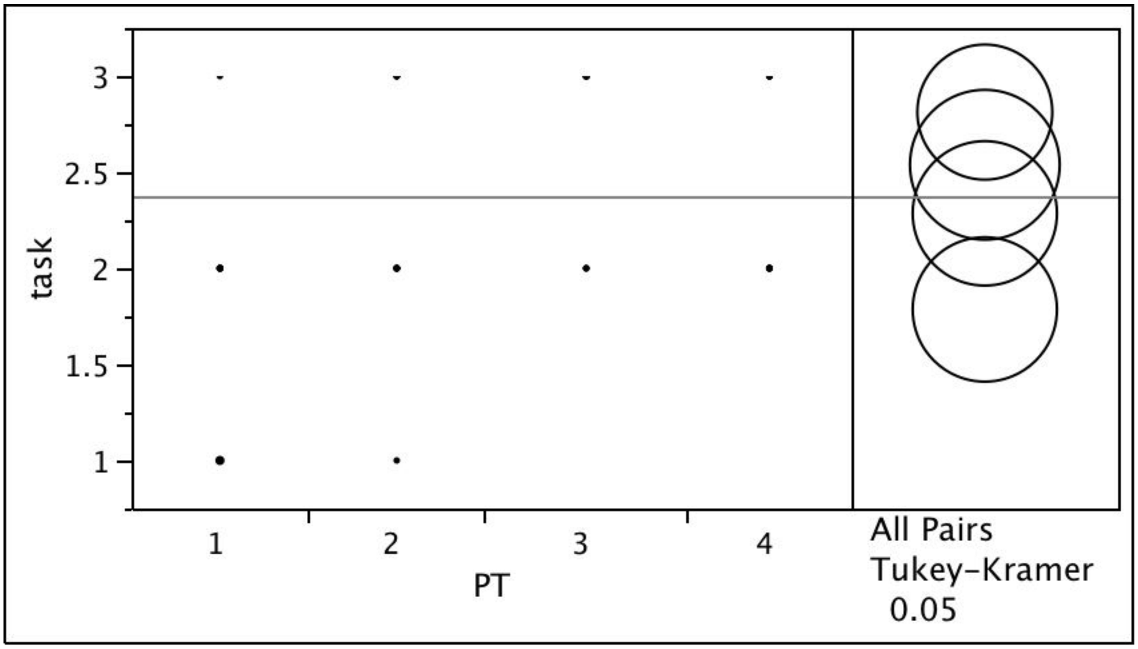
Level				Mean
3	A			3.6250000
4		B		2.7692308
2		B		2.3571429
1			C	1.5000000

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	2.125000	1.48874	2.761262	
4	1	1.269231	0.59958	1.938877	
3	2	1.267857	0.63160	1.904119	
2	1	0.857143	0.20001	1.514271	
3	4	0.855769	0.20659	1.504952	
4	2	0.412088	-0.25756	1.081734	

Task Structure

One-way Analysis of Task by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

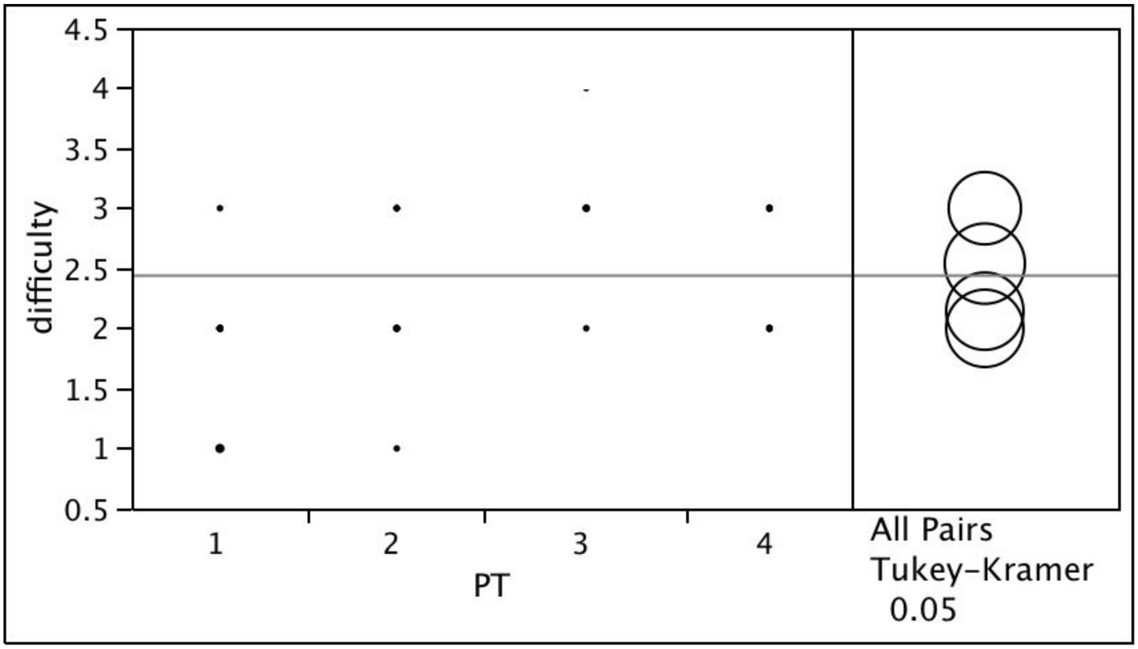
Level				Mean
3	A			2.8125000
4	A	B		2.5384615
2		B	C	2.2857143
1			C	1.7857143

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	1.026786	0.512417	1.541154	
4	1	0.752747	0.211390	1.294104	
3	2	0.526786	0.012417	1.041154	
2	1	0.500000	-0.031238	1.031238	
3	4	0.274038	-0.250775	0.798852	
4	2	0.252747	-0.288610	0.794104	

Level of Difficulty

One-way Analysis of Difficulty by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

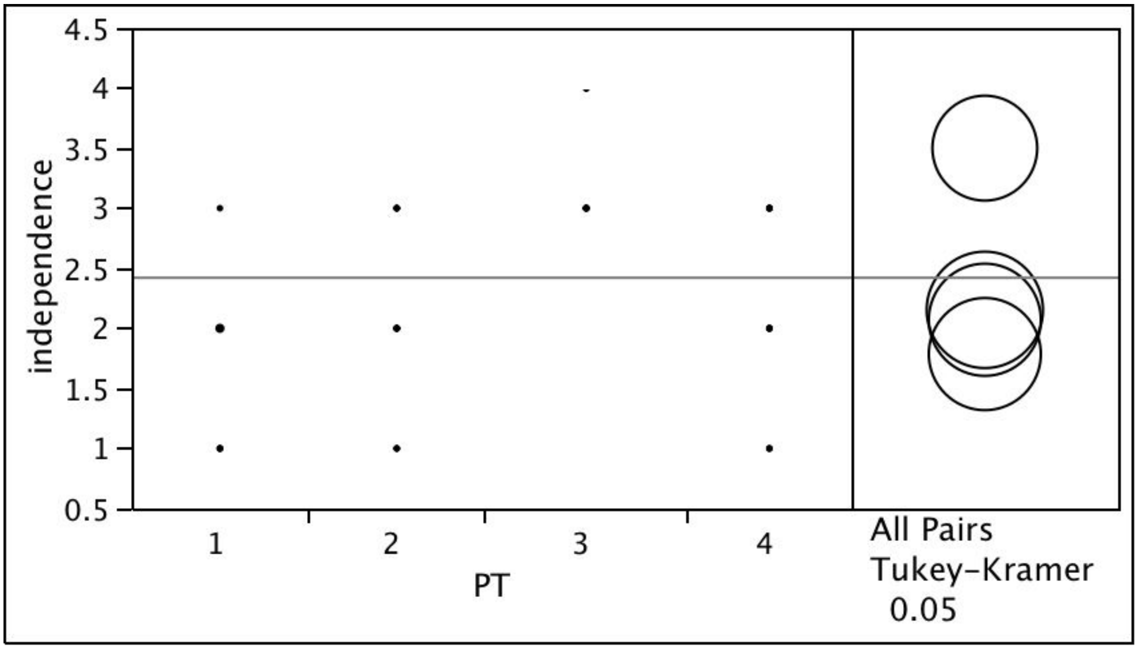
Level				**Mean**
3	A			3.0000000
4		B		2.5384615
2		B	C	2.1428571
1			C	2.0000000

Levels not connected by same letter are significantly different.

Level	**- Level**	**Difference**	**Lower CL**	**Upper CL**	**Difference**
3	1	1.000000	0.558892	1.441108	
3	2	0.857143	0.416035	1.298251	
4	1	0.538462	0.074209	1.002714	
3	4	0.461538	0.011473	0.911604	
4	2	0.395604	-0.068648	0.859857	
2	1	0.142857	-0.312717	0.598431	

Level of Independence

One-way Analysis of Independence by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

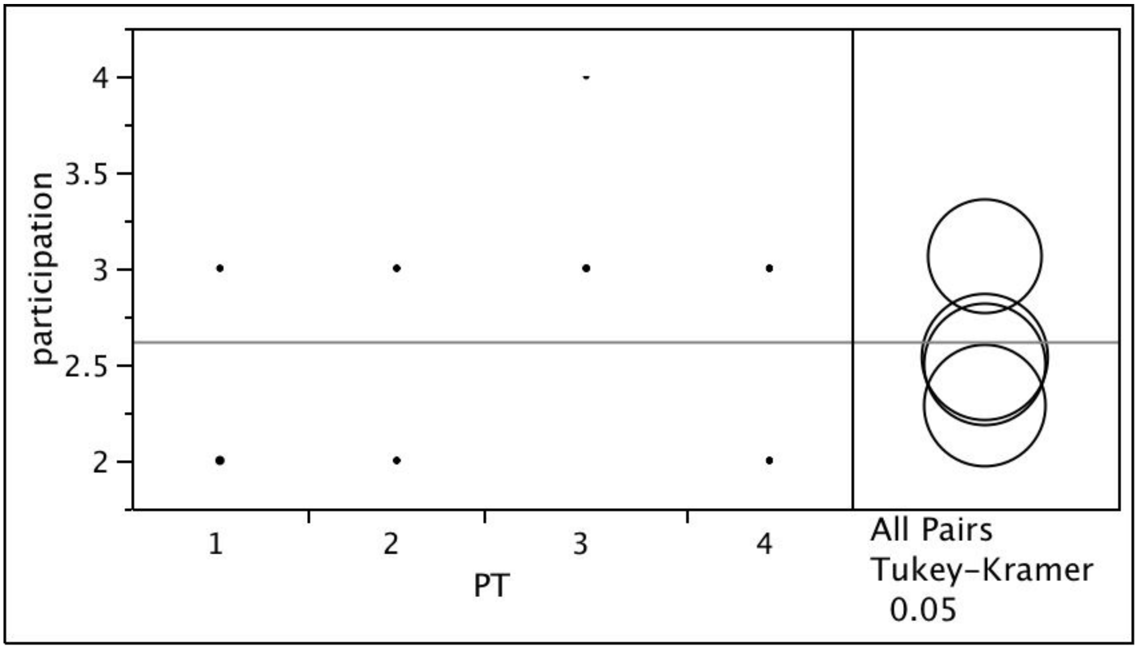
Level			Mean
3	A		3.5000000
4		B	2.1538462
2		B	2.0714286
1		B	1.7857143

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	1.714286	1.07515	2.353420	
3	2	1.428571	0.78944	2.067705	
3	4	1.346154	0.69404	1.998267	
4	1	0.368132	-0.30454	1.040801	
2	1	0.285714	-0.37438	0.945809	
4	2	0.082418	-0.59025	0.755087	

Participation

Oneway Analysis of Participation by PT

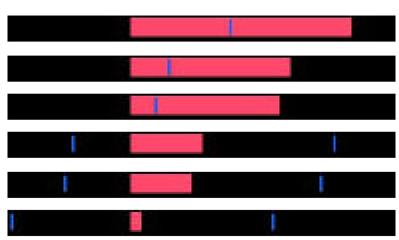


Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.0625000
4		B	2.5384615
2		B	2.5000000
1		B	2.2857143

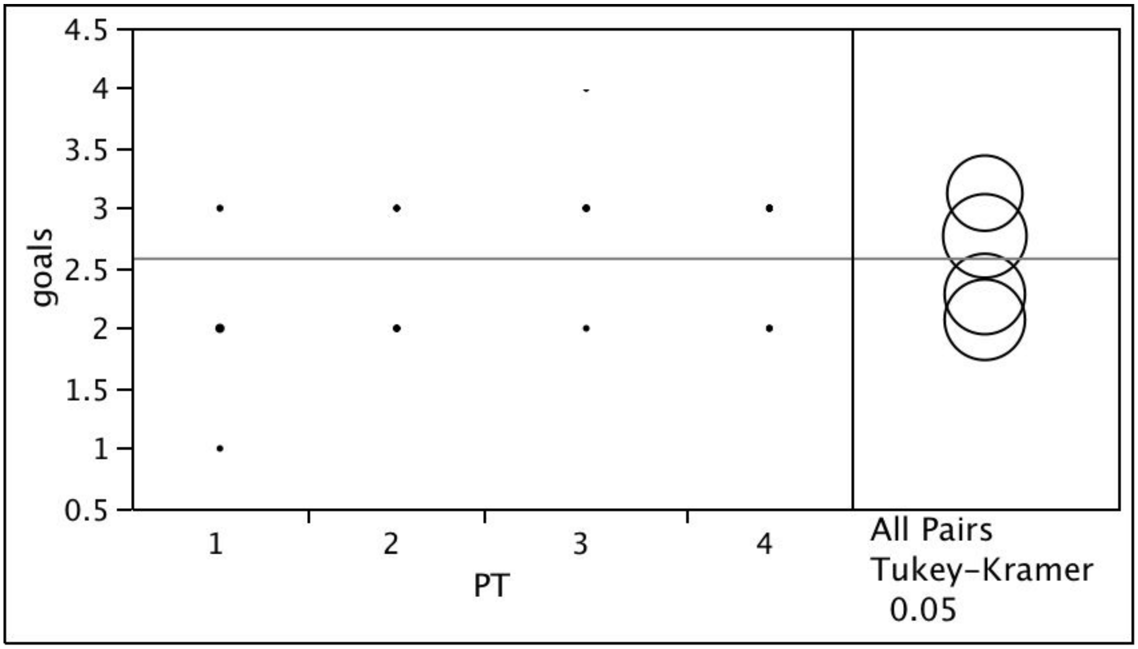
Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	0.7767857	0.344217	1.209355	
3	2	0.5625000	0.129931	0.995069	
3	4	0.5240385	0.082686	0.965391	
4	1	0.2527473	-0.202518	0.708013	
2	1	0.2142857	-0.232469	0.661041	
4	2	0.0384615	-0.416804	0.493727	



Goals for Students

One-way Analysis of Goals by PT

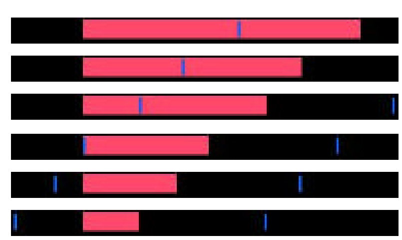


Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.1250000
4	A		2.7692308
2		B	2.2857143
1		B	2.0714286

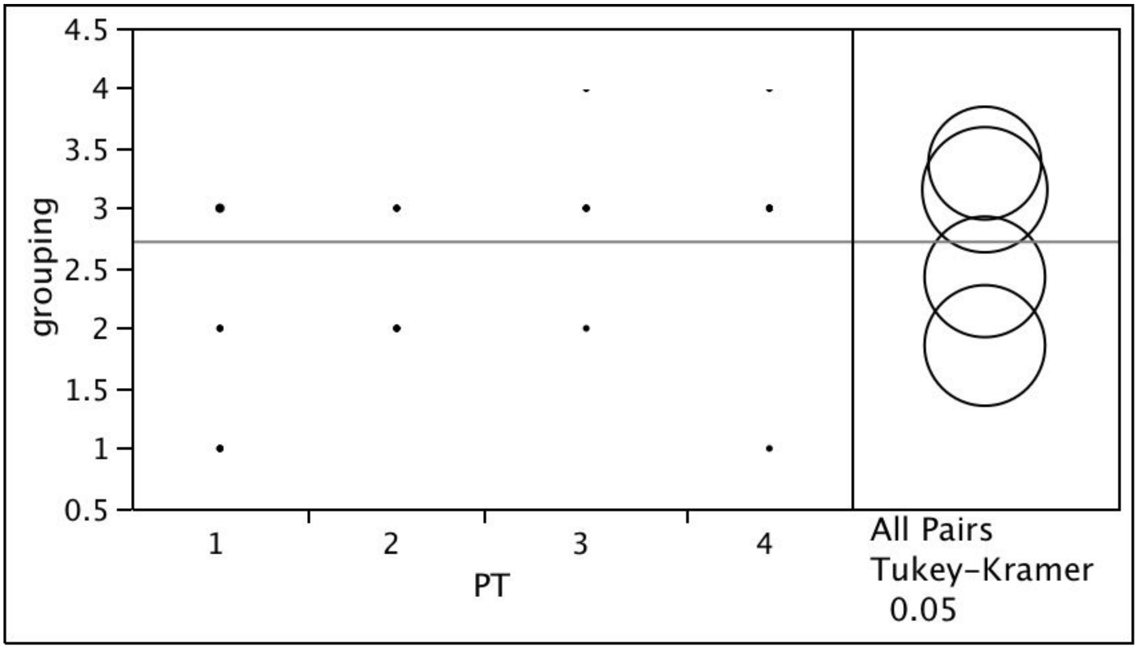
Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	1.053571	0.594718	1.512425	
3	2	0.839286	0.380432	1.298139	
4	1	0.697802	0.214873	1.180731	
4	2	0.483516	0.000587	0.966446	
3	4	0.355769	-0.112402	0.823940	
2	1	0.214286	-0.259616	0.688187	



Grouping

Oneway Analysis of Grouping by PT

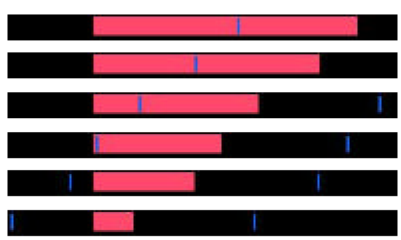


Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.3750000
4	A		3.1538462
2		B	2.4285714
1		B	1.8571429

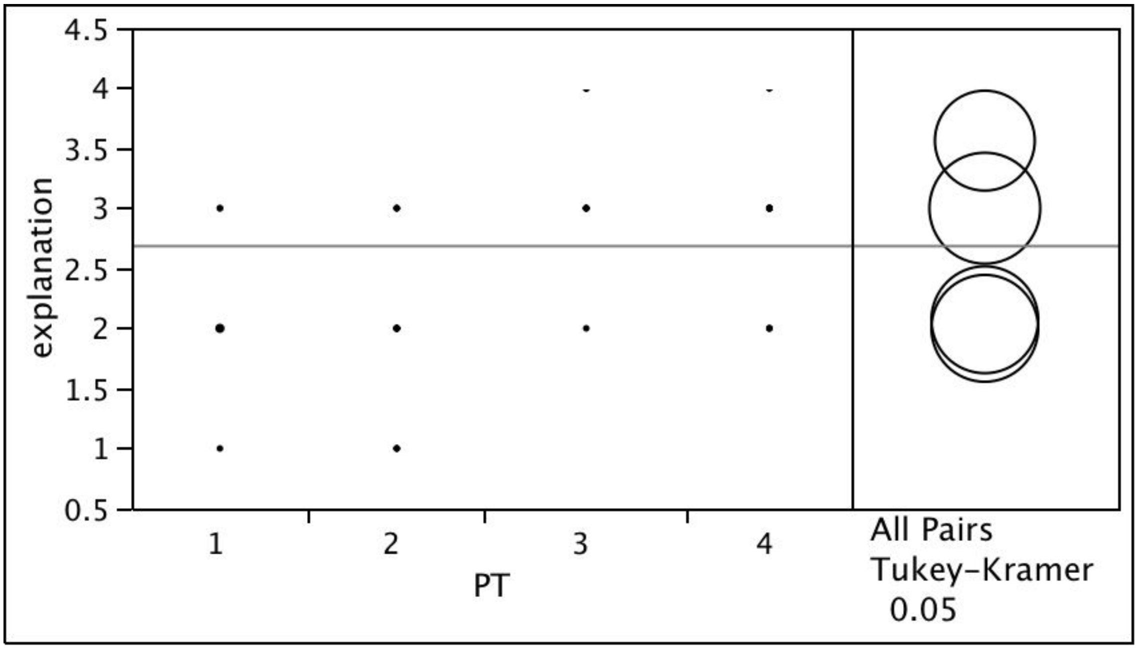
Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	1.517857	0.830385	2.205330	
4	1	1.296703	0.573159	2.020247	
3	2	0.946429	0.258956	1.633901	
4	2	0.725275	0.001731	1.448819	
2	1	0.571429	-0.138590	1.281447	
3	4	0.221154	-0.480279	0.922587	



Explanation

One-way Analysis of Explanation by PT

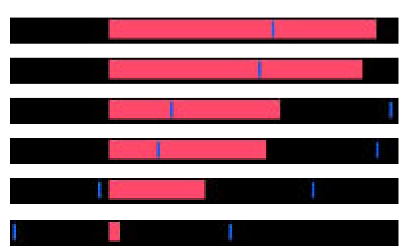


Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.5625000
4	A		3.0000000
1		B	2.0714286
2		B	2.0000000

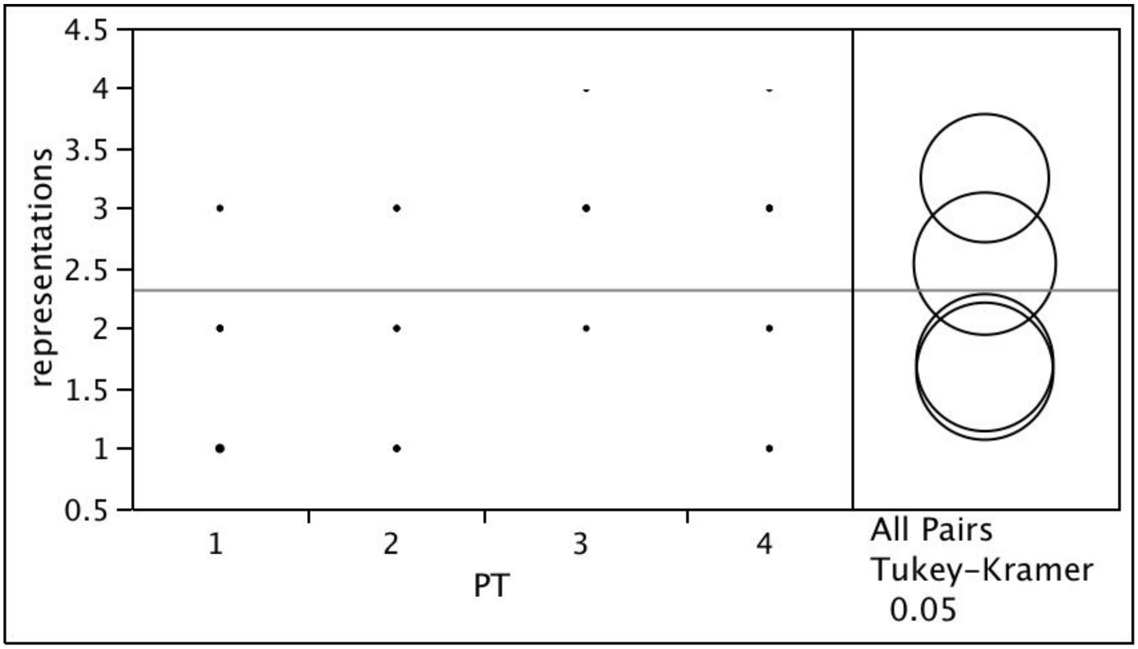
Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL
3	2	1.562500	0.953446	2.171554
3	1	1.491071	0.882017	2.100126
4	2	1.000000	0.358989	1.641011
4	1	0.928571	0.287560	1.569583
3	4	0.562500	-0.058922	1.183922
1	2	0.071429	-0.557600	0.700457



Representations

One-way Analysis of Representations by PT



Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

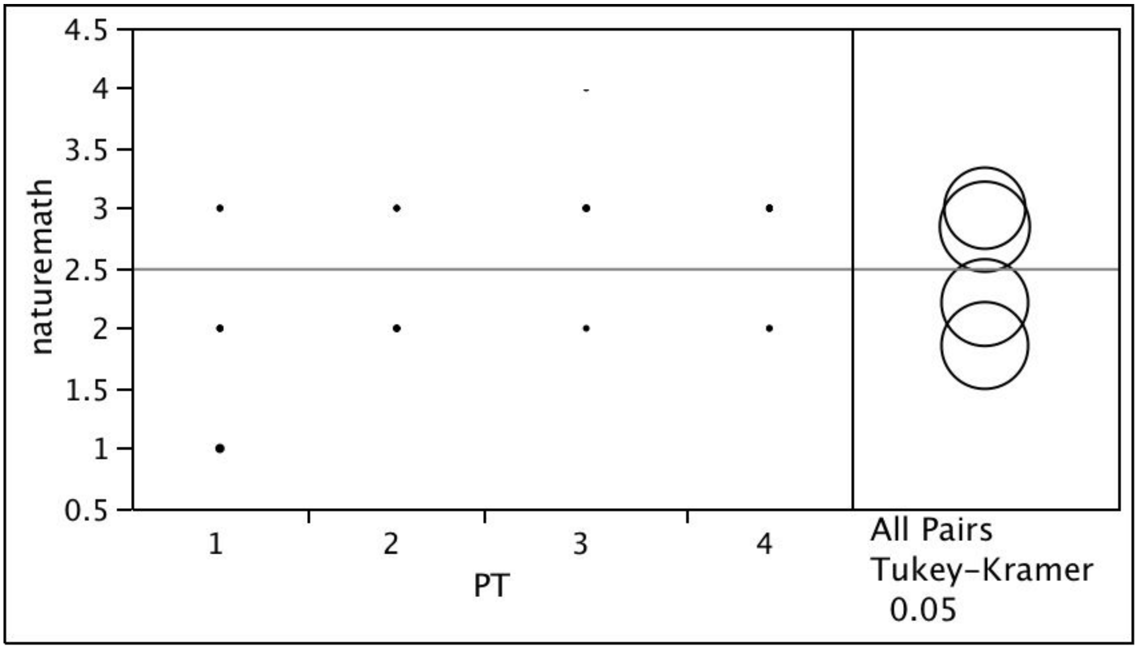
Level			Mean
3	A		3.2500000
4	A		2.5384615
1		B	1.7142857
2		B	1.6428571

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	2	1.607143	0.826241	2.388045	
3	1	1.535714	0.754812	2.316616	
4	2	0.895604	0.073729	1.717480	
4	1	0.824176	0.002300	1.646052	
3	4	0.711538	-0.085221	1.508298	
1	2	0.071429	-0.735084	0.877941	

Nature of Mathematics

One-way Analysis of Naturemath by PT



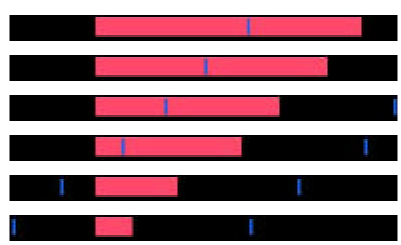
Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.0000000
4	A		2.8461538
2		B	2.2142857
1		B	1.8571429

Levels not connected by same letter are significantly different.

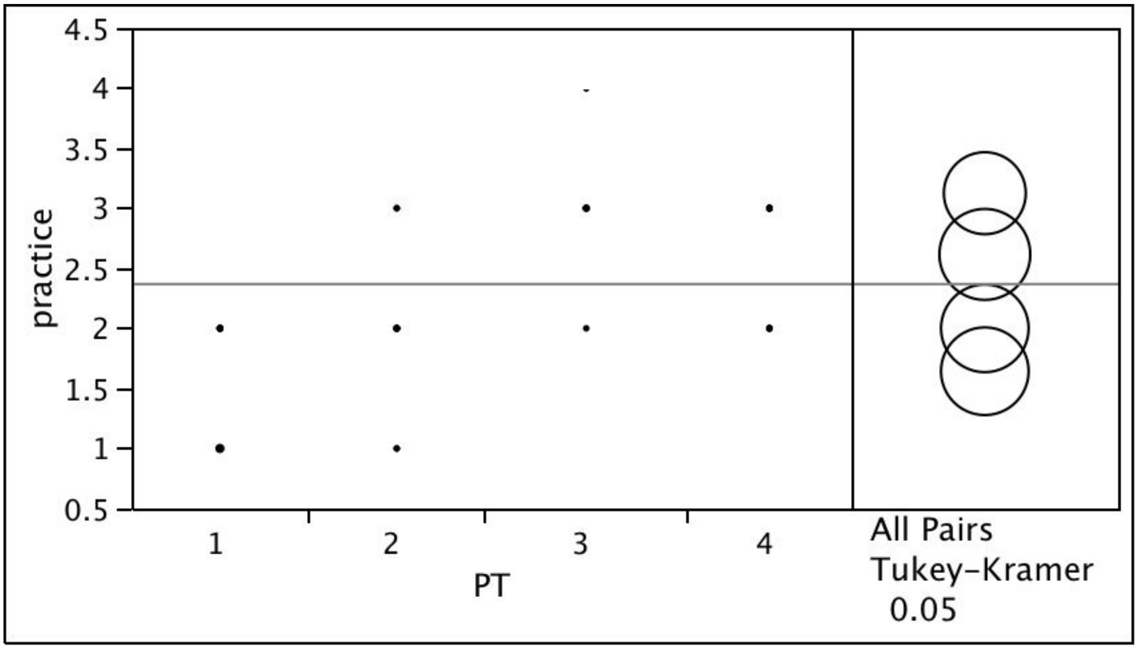
Level	- Level	Difference	Lower CL	Upper CL
3	1	1.142857	0.648201	1.637514
4	1	0.989011	0.468400	1.509622
3	2	0.785714	0.291058	1.280371
4	2	0.631868	0.111257	1.152479
2	1	0.357143	-0.153736	0.868022
3	4	0.153846	-0.350855	0.658548

Difference



Practices of Doing Mathematics

One-way Analysis of Practice by PT

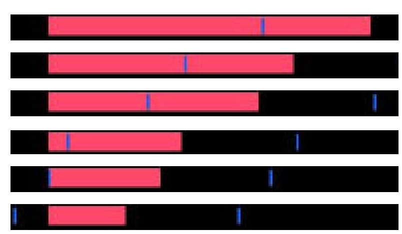


Group 1—Resisting; Group 2—Acknowledging; Group 3—Embracing; Group 4—Creating

Level			Mean
3	A		3.1250000
4	A		2.6153846
2		B	2.0000000
1		B	1.6428571

Levels not connected by same letter are significantly different.

Level	- Level	Difference	Lower CL	Upper CL	Difference
3	1	1.482143	0.982525	1.981761	
3	2	1.125000	0.625382	1.624618	
4	1	0.972527	0.446695	1.498360	
4	2	0.615385	0.089552	1.141217	
3	4	0.509615	-0.000148	1.019379	
2	1	0.357143	-0.158860	0.873146	



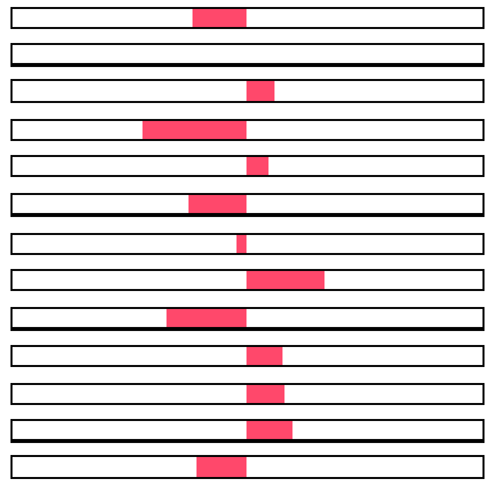
Appendix I

Correlations of Observation Number by Continuum to Examine Change over Time

Resisting Complexity Correlations

Nonparametric: Kendall's t

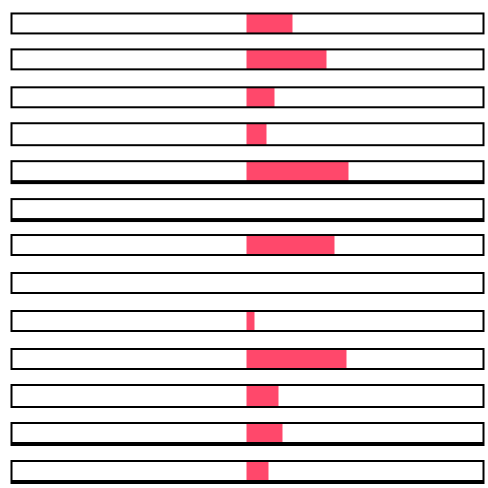
Variable	by Variable	Kendall t	Prob>|t|	Plot
Observation #	learnprom	-0.2235	0.3410	
Observation #	goals	0.0000	1.0000	
Observation #	reasonsmath	0.1203	0.6175	
Observation #	independence	-0.4339	0.0647	
Observation #	task	0.0930	0.6922	
Observation #	duration	-0.2407	0.3179	
Observation #	grouping	-0.0407	0.8587	
Observation #	participation	0.3258	0.1762	
Observation #	explanation	-0.3300	0.1616	
Observation #	difficulty	0.1519	0.5217	
Observation #	representations	0.1680	0.4669	
Observation #	naturemath	0.1913	0.4035	
Observation #	practice	-0.2102	0.3830	



Acknowledging Complexity Correlations

Nonparametric: Kendall's t

Variable	by Variable	Kendall t	Prob>|t|	Plot
Observation #	learnprom	0.1952	0.4201	
Observation #	goals	0.3450	0.1541	
Observation #	reasonsmath	0.1210	0.6171	
Observation #	independence	0.0838	0.7162	
Observation #	task	0.4346	0.0653	
Observation #	duration	.	1.0000	
Observation #	grouping	0.3780	0.1185	
Observation #	participation	0.0000	1.0000	
Observation #	explanation	0.0434	0.8514	
Observation #	difficulty	0.4160	0.0784	
Observation #	representations	0.1445	0.5325	
Observation #	naturemath	0.1519	0.5302	
Observation #	practice	0.0987	0.6739	



Embracing Complexity Correlations

Nonparametric: Kendall's t

Variable	**by Variable**	**Kendall** t	**Prob>|t|**	**Plot**
Observation #	learnprom	0.1464	0.5127	
Observation #	goals	0.2779	0.2042	
Observation #	reasonsmath	-0.0546	0.8073	
Observation #	independence	0.3307	0.1391	
Observation #	task	-0.3934	0.0785	
Observation #	duration	0.3937	0.0717	
Observation #	grouping	-0.2019	0.3551	
Observation #	participation	-0.1464	0.5127	
Observation #	explanation	-0.1524	0.4855	
Observation #	difficulty	-0.0526	0.8112	
Observation #	representations	0.2227	0.3080	
Observation #	naturemath	-0.1228	0.5773	
Observation #	practice	0.2911	0.1835	

Creating Complexity Correlations

Nonparametric: Kendall's t

Variable	**by Variable**	**Kendall** t	**Prob>|t|**	**Plot**
Observation #	learnprom	-0.2133	0.3620	
Observation #	goals	-0.4487	0.0739	
Observation #	reasonsmath	-0.3104	0.2059	
Observation #	independence	-0.5839	0.0143	
Observation #	task	-0.5237	0.0370	
Observation #	duration	.	1.0000	
Observation #	grouping	-0.1059	0.6649	
Observation #	participation	-0.0361	0.8856	
Observation #	explanation	-0.6228	0.0093	
Observation #	difficulty	-0.4515	0.0722	
Observation #	representations	-0.2305	0.3236	
Observation #	naturemath	-0.4492	0.0736	
Observation #	practice	-0.5182	0.0391	