Helsinki
Center
of
Economic
Research

Discussion Papers

GMM Estimation with Noncausal Instruments under Rational Expectations

Matthijs Lof
University of Helsinki and HECER

GMM Estimation with Noncausal Instruments under Rational Expectations*

Abstract

There is hope for the generalized method of moments (GMM). Lanne and Saikkonen (2011) show that the GMM estimator is inconsistent, when the instruments are lags of noncausal variables. This paper argues that this inconsistency depends on distributional assumptions, that do not always hold. In particular under rational expectations, the GMM estimator is found to be consistent. This result is derived in a linear context and illustrated by simulation of a nonlinear asset pricing model.

Matthijs Lof

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail: matthijs.lof@helsinki.fi

1 Introduction

In a recent paper, Lanne and Saikkonen (2011a) warn against the use of the generalized method of moments (GMM; Hansen, 1982), when the instrumental variables are lags of noncausal variables. With such instruments, the two-stage least squares (2SLS) estimator is shown to be inconsistent under certain assumptions on the distribution of the error term in the regression model. In this paper, I make no explicit assumptions on this distribution. Instead, the errors are implied by a rational expectations equilibrium and are in fact prediction errors. GMM estimation is in this case consistent even when the instruments are lags of noncausal variables. This result is in line with the nature of rational expectations, as prediction errors are assumed to be uncorrelated with all lagged information.

Lanne and Saikkonen (2011a) consider a linear regression model with a single regressor:

$$y_t = \delta x_t + \eta_t, \tag{1}$$

and evaluate the situation in which x_t is noncausal. A variable is noncausal, when it follows a noncausal autoregressive process, that allows for dependence on both leading and lagging observations. A noncausal AR(r,s) process, as defined by Lanne and Saikkonen (2011b), depends on r past and s future observations:

$$\phi(L)\varphi(L^{-1})x_t = \varepsilon_t, \tag{2}$$

with $\phi(L) = 1 - \phi_1 L - ... - \phi_r L^r$, $\varphi(L^{-1}) = 1 - \varphi_1 L^{-1} - ... - \varphi_r L^{-s}$, $\varepsilon_t \sim i.i.d.(0,\sigma^2)$ and L is a standard lag operator ($L^k y_t = y_{t-k}$). A noncausal AR process has an infinite-order moving average (MA) representation that is both backward- and forward-looking:

$$x_t = \varphi(L^{-1})^{-1}\phi(L)^{-1}\varepsilon_t = \sum_{j=-\infty}^{\infty} \psi_j \varepsilon_{t-j}, \tag{3}$$

in which ψ_j is the coefficient of z^j in the Laurent-series expansion of $\varphi(z^{-1})^{-1}\phi(z)^{-1}$ (Lanne and Saikkonen, 2011b). When x_t is a vector, (2) defines a noncausal VAR(r,s) process (Lanne and Saikkonen, 2009).

Lanne and Saikkonen (2011a) make the following distributional assumption on the errors in (1) and (2):

$$(\varepsilon_t, \eta_t)' \sim i.i.d.(0, \Omega), \tag{4}$$

with nonzero covariance: $\Omega_{12} = E[\varepsilon_t \eta_t] \neq 0$. Since x_t and η_t are correlated, OLS estimation of equation (1) is inconsistent. However, the MA representation (3) reveals that also 2SLS estimation is inconsistent when lags of x_t are used as instruments, since these lags depends on ε_t and are therefore correlated with η_t: $E[x_{t-i}\eta_t] = \psi_{-i}E[\varepsilon_t \eta_t] = \psi_{-i}\Omega_{12}$, which is nonzero if $\varphi_j \neq 0$, for some $j \in \{1,..,s\}$ in equation (2).

The next section shows that inconsistency of the GMM estimator does not hold when η_t is a prediction error. This result is derived for the linear regression model (1), with x_t generated by a Gaussian first-order noncausal (vector) autoregression. In section 3, simulations show that the result is robust to non-Gaussian and higher-order autoregressive specifications of x_t. In section 4, the result is illustrated by simulation of a nonlinear rational expectations model. Section 4 concludes.

2 Prediction errors

In empirical macroeconomics and finance, a regression model like (1) often represents a (linearized) economic model, such as an Euler equation or Philips curve, in which y_t is determined by a rational expectations equilibrium (see, e.g. the survey by Hansen and West, 2002). This implies that the error term η_t has the interpretation of a prediction error . Consider the following example:

$$
\begin{aligned}
y_t &= \delta E_{t-1}[x_t] \\
\eta_t &= -\delta (x_t - E_{t-1}[x_t]).
\end{aligned}
\tag{5}
$$

In this case, all lags of x_t are uncorrelated with η_t and are therefore valid instruments regardless of their dynamic properties:

$$
\begin{aligned}
E\left[x_{t-i}\eta_t\right] &= E\left[x_{t-i}E_{t-1}\left[\eta_t\right]\right] && \{i \geq 1\}\\
&= E\left[x_{t-i}E_{t-1}\left[-\delta\left(x_t - E_{t-1}\left[x_t\right]\right)\right]\right] && (6)\\
&= -\delta E\left[x_{t-i}\left(E_{t-1}\left[x_t\right] - E_{t-1}\left[x_t\right]\right)\right] \;=\; 0.
\end{aligned}
$$

To see how this differs from the result by Lanne and Saikkonen (2011a), assume the regressor x_t to be generated by a Gaussian first-order noncausal autoregressive process, AR(0,1):

$$
\begin{aligned}
x_t &= \alpha x_{t+1} + \varepsilon_t\\
&= \sum_{j=0}^{\infty} \alpha^j \varepsilon_{t+j},
\end{aligned}
\qquad (7)
$$

with $\varepsilon_t \sim N(0,\sigma^2)$. Since x_t is Gaussian, the noncausal process (7) is indistinguishable from a causal AR(1,0) process, and its optimal forecast is identical to the causal case: $E_{t-1}\left[x_t\right] = \alpha x_{t-1}$ (Lanne et al., 2012). The realized prediction error (assuming the true value of α is known) is then:

$$
\begin{aligned}
e_t &= x_t - E_{t-1}\left[x_t\right]\\
&= x_t - \alpha x_{t-1}
\end{aligned}
\qquad (8)
$$

The prediction error e_t is the true 'innovation' in x_t and is, other than in a causal autoregression, not equal to the error term ε_t. In fact, from the MA representation of x_t (7), it is straightforward to see that the forecast error is correlated with lags and leads of ε_t:

$$
\begin{aligned}
E\left[e_t\varepsilon_{t-i}\right] &= E\left[x_t\varepsilon_{t-i}\right] - \alpha E\left[x_{t-1}\varepsilon_{t-i}\right]\\
&= \begin{cases}
0 - \alpha\sigma^2 & = -\alpha\sigma^2 & \{i = 1\}\\
\alpha^i\sigma^2 - \alpha\alpha^{i+1}\sigma^2 & = (1-\alpha^2)\alpha^i\sigma^2 & \{i < 1\}\\
0 - 0 & = 0 & \{i > 1\},
\end{cases}
\end{aligned}
\qquad (9)
$$

Since the implied error term η_t is an exact linear function of the forecast error e_t ($\eta_t = -\delta e_t$), η_t is correlated with leads and lags of ε_t, which contradicts the assumption (4) made by Lanne and

Saikkonen (2011a). The forecast errors e_t and η_t are, however, uncorrelated with lags of x_t :

$$
\begin{aligned}
E\left[e_t x_{t-i}\right] &= E\left[x_t x_{t-i}\right] - \alpha E\left[x_{t-1} x_{t-i}\right] \\
&= \alpha^i E\left[x_t^2\right] - \alpha \alpha^{i-1} E\left[x_t^2\right] = 0 \quad \{i \geq 1\},
\end{aligned} \tag{10}
$$

which means that lags of x_t are valid instruments for estimating (1), regardless of whether x_t is causal or noncausal.

This result can be extended to a multivariate context. Let $\mathbf{x}_t$ be a K-dimensional vector of variables that is generated by a noncausal VAR(0, 1) process:

$$
\mathbf{x}_t = B\mathbf{x}_{t+1} + \varepsilon_t, \tag{11}
$$

with $\varepsilon_t \sim N(\mathbf{0}, \Sigma_B)$, while $\mathbf{x}^*_t$ follows a causal VAR(1, 0) process:

$$
\mathbf{x}^*_t = A\mathbf{x}^*_{t-1} + \varepsilon^*_t, \tag{12}
$$

with $\varepsilon_t \sim N(\mathbf{0}, \Sigma_A)$. The processes $\mathbf{x}_t$ and $\mathbf{x}^*_t$ are identical in first- and second-order moments when:

$$
\begin{aligned}
B &= \Gamma_0^* A' \Gamma_0^{-1} \\
\Sigma_B &= \Gamma_0^* - B\Gamma_0 B',
\end{aligned} \tag{13}
$$

in which the covariance functions are defined by:

$$
\begin{aligned}
\Gamma_0 &= E\left[\mathbf{x}_t \mathbf{x}_t'\right] = B\Gamma_0 B' + \Sigma_B \\
\Gamma_0^* &= E\left[\mathbf{x}^*_t \mathbf{x}^{*\prime}_t\right] = A\Gamma_0^* A' + \Sigma_A.
\end{aligned} \tag{14}
$$

It is straightforward to verify that $\Gamma_0 = \Gamma_0^*$, when (13) holds. Under these conditions, also the autocovariance functions of $\mathbf{x}_t$ and $\mathbf{x}^*_t$ are identical:

$$
\begin{aligned}
\Gamma_{-i} &= E\left[\mathbf{x}_t \mathbf{x}_{t+i}'\right] = B^i \Gamma_0 \\
\Gamma_i^* &= E\left[\mathbf{x}^*_t \mathbf{x}^{*\prime}_{t-i}\right] = A^i \Gamma_0^*.
\end{aligned} \tag{15}
$$

4

Since $\Gamma_{-i} = \Gamma_i'$, the autocovariance function of the causal and noncausal processes are identical if and only if $B^i \Gamma_0 = \Gamma_0^* A'^i$, or equivalently: $B^i = \Gamma_0^* A'^i \Gamma_0^{-1}$, which is satisfied for all i when $B = \Gamma_0^* A' \Gamma_0^{-1}$ and $\Gamma_0 = \Gamma_0^*$.

The equivalence in first- and second-order moments implies that, under Gaussianity, the processes (11) and (12) are indistinguishable, so $E_{t-1}[\mathbf{x}_t] = A\mathbf{x}_t$ is the optimal forecast for both the causal and noncausal process (Lanne et al., 2012). The vector of forecast errors is then, analogues to equation (8), $\mathbf{e}_t = \mathbf{x}_t - A\mathbf{x}_{t-1}$. As in the univariate case (9)-(10) $\mathbf{e}_t$ is correlated with lags and leads of ε_t, but uncorrelated with lags of $\mathbf{x}_t$:

$$
\begin{aligned}
E\left[\mathbf{e}_t \mathbf{x}_{t-i}'\right] &= \Gamma_{-i}' - A^i \Gamma_0 \\
&= \Gamma_0 B'^i - \Gamma_0 B'^i \Gamma_0^{-1} \Gamma_0 = 0 \quad \{i \geq 1\}.
\end{aligned}
\tag{16}
$$

Under the assumption that the error term in a regression equation like (1) is a linear combination of prediction errors: $\eta_t = \gamma' \mathbf{e_t}$, lags of $\mathbf{x}_t$ are uncorrelated with this error term ($E[\eta_t \mathbf{x}_{t-i}] = 0 \ \forall i \geq 1$) and are therefore valid instruments.

3 Non-Gaussian and higher-order processes

As the derivations in the previous section are already rather cumbersome, I use simulations to show robustness of the result to non-Gaussian and higher-order autoregressive specifications of x_t. Consider the linear regression model (1), with x_t generated by an AR(1, 1) process:

$$
(1 - \phi L)(1 - \varphi L^{-1})x_t = \varepsilon_t.
\tag{17}
$$

I consider four different distributions for ε_t and η_t:

$$
\begin{aligned}
(\varepsilon_t, \eta_t)' &\sim N(0, \Omega) & (a) \\
(\varepsilon_t, \eta_t)' &\sim t_3(0, \Omega) & (b) \\
\varepsilon_t &\sim N(0, \sigma^2) & (c) \\
\varepsilon_t &\sim t_3(0, \sigma^2), & (d)
\end{aligned}
\tag{18}
$$

5

in which $\Omega_{11} = \Omega_{22} = \sigma^2$ and t_3 refers to the t-distribution with three degrees of freedom. I generate a sample of random errors according to each distribution (a)-(d) and use them to compute x_t following (17) and y_t following (1). For the last two distributions (c)-(d), no explicit distribution for η_t is formulated, but I assume it is a prediction error: $\eta_t = -\delta (x_t - E_{t-1}[x_t])$. In the Gaussian case (c), the conditional expectation of x_t is, as in section 2, identical to the conditional expectation of a causal process with identical first- and second-order moments (Lanne et al., 2012). It can be verified that the causal AR$(2,0)$ process:

$$(1 - (\phi + \varphi)L + \phi \varphi L^2)x_t^* = \varepsilon_t \tag{19}$$

has identical mean, variance and autocovariance function as (17). The conditional expectation, under Gaussianity (c), of x_t is therefore:

$$E_{t-1}[x_t] = (\phi + \varphi)x_{t-1} - \phi \varphi x_{t-2} \tag{20}$$

For the t-distributed AR$(1,1)$ process (d), I compute the conditional expectation of x_t using the simulation-based forecast method for non-Gaussian, noncausal univariate autoregressions, provided by Lanne et al. (2012). Given these conditional expectations, η_t can be computed for both (c) and (d).

I calibrate $\sigma^2 = 1$, $\Omega_{12} = \Omega_{21} = 0.8$, $\phi = \varphi = 0.5$ and $\delta = 1$, following a simulation exercise by Lanne and Saikkonen (2011a). After computing samples of 50 and 1000 observations of ε_t, x_t, η_t and y_t, according to each distribution in (18), I estimate δ in model (1) by 2SLS, using x_{t-1} as instrument. This process is repeated 10,000 times.

Table 1 shows the average estimates and standard deviations of δ for all four distributional assumptions, which confirm the point made in section 2. Under assumptions (a) and (b), which is the assumption made by Lanne and Saikkonen (2011a), the 2SLS estimator is clearly inconsistent. However, under assumptions (c) and (d), when η_t is a prediction error, the 2SLS estimator is consistent, despite noncausality of x_t.

4 Example: Consumption-based asset pricing

Consumption-based asset pricing was amongst the first applications of GMM (Hansen and Singleton, 1982). The model to estimate is an Euler equation relating financial returns ($R_t = P_{t-1}^{-1}(P_t + D_t)$) to the marginal rate of substitution:

$$E_{t-1}\left[\beta\frac{u'(C_t)}{u'(C_{t-1})}R_t\right] = 1, \tag{21}$$

in which P_t refers to asset prices, D_t to dividends and C_t to consumption. Multiplying the sample equivalent of this optimality condition with a vector of predetermined instruments $\mathbf{z}_{t-1}$ and assuming a constant relative-risk aversion utility function ($u(C_t) = (1-\gamma)^{-1}C_t^{1-\gamma}$) gives the required moment conditions for GMM estimation:

$$\sum_{t=0}^{T}\left(\beta\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}R_t - 1\right)\mathbf{z}_{t-1} = 0. \tag{22}$$

This approach has become leading practice in empirical finance (see e.g. Ludvigson, 2011, for a recent survey). It is illustrative to see that a simple regression model, similar to (1), is obtained after log-linearizing the Euler equation:

$$r_t = \mu + \gamma\triangle c_t + \eta_t, \tag{23}$$

in which $r_t = \log(R_t)$ and $c_t = \log(C_t)$. Yogo (2004) shows that the error term η_t is in this case indeed a linear combination of prediction errors, as assumed in section 2:

$$\eta_t = (r_t - E_{t-1}[r_t]) - \gamma(\triangle c_t - E_{t-1}[\triangle c_t]), \tag{24}$$

I simulate returns and consumption according to (21), to verify that the GMM estimator is consistent even if the instruments are noncausal. The first step is to define log consumption and dividend growth as a first-order VAR process, $(\triangle c_t, \triangle d_t)' = \mathbf{x}_t$, in which $d_t = \log(D_t)$. This process may be causal or noncausal, i.e. is generated by equation (12) or (11). The restrictions (13) apply, so both

specifications are identical in their mean, variance and autocorrelation function. Given a simulated sample of consumption and dividends, I generate returns following the approach of Tauchen and Hussey (1991). Multiplying equation (21) by $\frac{P_{t-1}}{D_{t-1}}$, results in a nonlinear stochastic difference equation describing the dynamics of the price-dividend (PD) ratio:

$$\frac{P_{t-1}}{D_{t-1}} = E_{t-1}\left[\beta\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{D_t}{D_{t-1}}\left(1+\frac{P_t}{D_t}\right)\right], \tag{25}$$

which can be simulated by calibrating a dicrete-valued Markov chain that approximates the conditional distribution of consumption and dividend growth. Details on this approximation for the causal VAR are provided by Tauchen (1986) and this method can be implemented for the noncausal VAR too, as the conditional distributions of the causal and noncausal processes are identical under Gaussianity and the restrictions in (13). Returns are then computed from the simulated dividends and PD ratios.

I consider two different calibrations of the matrices A and Σ_A in (12), which are given in table 2. The first calibration (*i*) of A and Σ_A is following Wright (2003) and is based on actual data on annual consumption and dividend growth. In the second example (*ii*), consumption growth follows a univariate AR(1,0) or AR(0,1) process, which is calibrated to have identical variance and autocorrelation as consumption growth in the first calibration, while dividend growth is set equal to consumption growth. This is an example of a "lucas-tree economy", in which household income consists of dividends alone. It is well known that in this case there exists a no-trade equilibrium in which households consume their entire endowment of dividends (Lucas, 1978).

I use the simulated returns and consumption growth rates to estimate β and γ by two-step efficient GMM, based on the moment conditions (22), using $\mathbf{z}_{t-1} = \left(1,\frac{C_{t-1}}{C_{t-2}},R_{t-1}\right)'$ as instruments, following Hansen and Singleton (1982). I consider 10,000 replications with sample sizes of 50 and 1000 observations.

Table 3 displays the simulation results. The main result is that for both calibrations, noncausality of the instruments seems to have no effect on the finite-sample or asymptotic properties of the GMM estimator. In both cases, the GMM estimates of β and γ are rather poor in small samples, but improve in larger samples. It is clear that the inconsistency of the estimator derived by Lanne

and Saikkonen (2011a), does not hold under the assumptions in this model.

Figure 1 shows plots of the correlation between the Euler-equation errors $u_t = \widehat{\beta} \left(\dfrac{C_t}{C_{t-1}} \right)^{-\widehat{\gamma}} R_t - 1$ and lags and leads of ε_t and $\dfrac{C_t}{C_{t-1}}$. These correlation plots are consistent with the results derived in section 2: When consumption is generated by a causal process, u_t is only correlated with ε_t, but not with its leads and lags. With noncausal consumption, on the other hand, the error term u_t is correlated with lags and leads of ε_t, so assumption (4) does not hold. Despite these intertemporal correlations, the important point to notice is that lags of $\dfrac{C_t}{C_{t-1}}$ are uncorrelated with u_t, which means they are valid instruments.

5   Conclusion

Instead of making explicit distributional assumptions on the error terms in a regression model, I argue that these errors are to be interpreted as prediction errors. This interpretation is consistent with the approach by Hansen and Singleton (1982), amongst others, who base GMM estimation on moment conditions implied by rational-expectations theories. All variables included in the information set on which agents condition to form expectations are in this case valid instruments, whether they are causal or noncausal. This is good news to those who apply GMM, although other caveats, such as weak instruments or misspecified economic theories, are of course still around to complicate the tasks of applied econometricians.

References

Hansen, B. E. and K. D. West: 2002, 'Generalized Method of Moments and Macroeconomics'. *Journal of Business & Economic Statistics* **20**(4), 460–69.

Hansen, L. P.: 1982, 'Large Sample Properties of Generalized Method of Moments Estimators'. *Econometrica* **50**(4), 1029–54.

Hansen, L. P. and K. J. Singleton: 1982, 'Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models'. *Econometrica* **50**(5), 1269–86.

Lanne, M., J. Luoto, and P. Saikkonen: 2012, 'Optimal Forecasting of Noncausal Autoregressive Time Series'. *International Journal of Forecasting* (forthcoming).

Lanne, M. and P. Saikkonen: 2009, 'Noncausal vector autoregression'. Research Discussion Papers 18/2009, Bank of Finland.

Lanne, M. and P. Saikkonen: 2011a, 'GMM Estimation with Noncausal Instruments'. *Oxford Bulletin of Economics and Statistics* **73**(5), 581–592.

Lanne, M. and P. Saikkonen: 2011b, 'Noncausal Autoregressions for Economic Time Series'. *Journal of Time Series Econometrics* **3**(3), Article 2.

Lucas, R. E. J.: 1978, 'Asset Prices in an Exchange Economy'. *Econometrica* **46**(6), 1429–45.

Ludvigson, S. C.: 2011, 'Advances in Consumption-Based Asset Pricing: Empirical Tests'. Working Paper 16810, National Bureau of Economic Research.

Tauchen, G.: 1986, 'Finite state markov-chain approximations to univariate and vector autoregressions'. *Economics Letters* **20**(2), 177–181.

Tauchen, G. and R. Hussey: 1991, 'Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models'. *Econometrica* **59**(2), 371–96.

Wright, J. H.: 2003, 'Detecting Lack of Identification in GMM'. *Econometric Theory* **19**(02), 322–330.

Yogo, M.: 2004, 'Estimating the Elasticity of Intertemporal Substitution when Instruments are Weak'. *The Review of Economics and Statistics* **86**(3), 797–810.

Tables and figures

TABLE 1: Simulation results

Distribution	(a)		(b)		(c)		(d)	
	$(\varepsilon_t, \eta_t)' \sim N(0,\Omega)$		$(\varepsilon_t, \eta_t)' \sim t_3(0,\Omega)$		$\varepsilon_t \sim N(0,\sigma^2)$		$\varepsilon_t \sim t_3(0,\sigma^2)$	
T	50	1000	50	1000	50	1000	50	1000
δ	1.633	1.596	1.630	1.596	1.025	1.001	1.067	0.995
	(0.159)	(0.031)	(0.158)	(0.031)	(0.117)	(0.023)	(0.161)	(0.063)

Notes: Average 2SLS estimates and standard deviations (in parenthesis) of δ, model (1), with instrument x_{t-1}, after 10,000 replications of sample size T. x_t follows a noncausal autoregression (17). The errors ε_t and η_t are either jointly $i.i.d.$ (a)-(b), as in Lanne and Saikkonen (2011a), or ε_t is $i.i.d.$ (c)-(d), with $\eta_t = -\delta(x_t - E_{t-1}[x_t])$. For the Gaussian case (c), $E_{t-1}[x_t]$ is computed by equation (20). For the non-Gaussian case (d), $E_{t-1}[x_t]$ is computed by a simulation-based method for forecasting non-Gaussian noncausal autoregressions (with $M = 50$ and $N = 1000$, see Lanne et al., 2012, for details). Calibration: $\Omega_{11} = \Omega_{22} = \sigma^2 = 1$, $\Omega_{12} = \Omega_{21} = 0.8$, $\phi = \varphi = 0.5$ and $\delta = 1$.

TABLE 2: Calibration

|		A		Σ_A		β	γ
(i)	$(\triangle c_t, \triangle d_t)' \equiv \mathbf{x}_t$	$\begin{pmatrix} -0.161 & 0.017 \\ 0.414 & 0.117 \end{pmatrix}$		$\begin{pmatrix} 0.0012 & 0.0018 \\ 0.0018 & 0.014 \end{pmatrix}$		0.97	1.3
(ii)	$\triangle c_t = \triangle d_t \equiv x_t$	-0.14		0.009		0.97	1.3

Notes: Calibrations of A, Σ_A, β and γ in the Euler equation (21). The first calibration (i) follows Wright (2003). In the second calibration (ii), consumption and dividends are identical as in a Lucas-tree economy (Lucas, 1978). The autoregressive process may be causal or noncausal. The parameter values of the noncausal autoregressive process are derived from A and Σ_A according to equation (13)

TABLE 3: Simulation results

	Causal				Noncausal			
Calibration	(i)		(ii)		(i)		(ii)	
T	50	1000	50	1000	50	1000	50	1000
β	0.965	0.970	0.970	0.970	0.965	0.970	0.970	0.970
	(0.030)	(0.004)	(0.001)	(0.000)	(0.030)	(0.004)	(0.001)	(0.000)
γ	1.742	1.293	1.115	1.285	1.743	1.292	1.114	1.285
	(3.556)	(0.810)	(0.202)	(0.067)	(3.580)	(0.809)	(0.190)	(0.067)

Notes: Average two-step efficient GMM estimates and standard deviations (in parenthesis) of β and γ, model (21), after 10,000 replications of sample size T. Instruments are $\mathbf{z}_{t-1} = \left(1, \dfrac{C_{t-1}}{C_{t-2}}, R_{t-1}\right)'$. Consumption and dividends are generated by a causal or noncausal autoregressive process. Returns are computed following the approach of Tauchen and Hussey (1991). Calibrations of the Euler equation and autoregressive processes are given in Table 2.
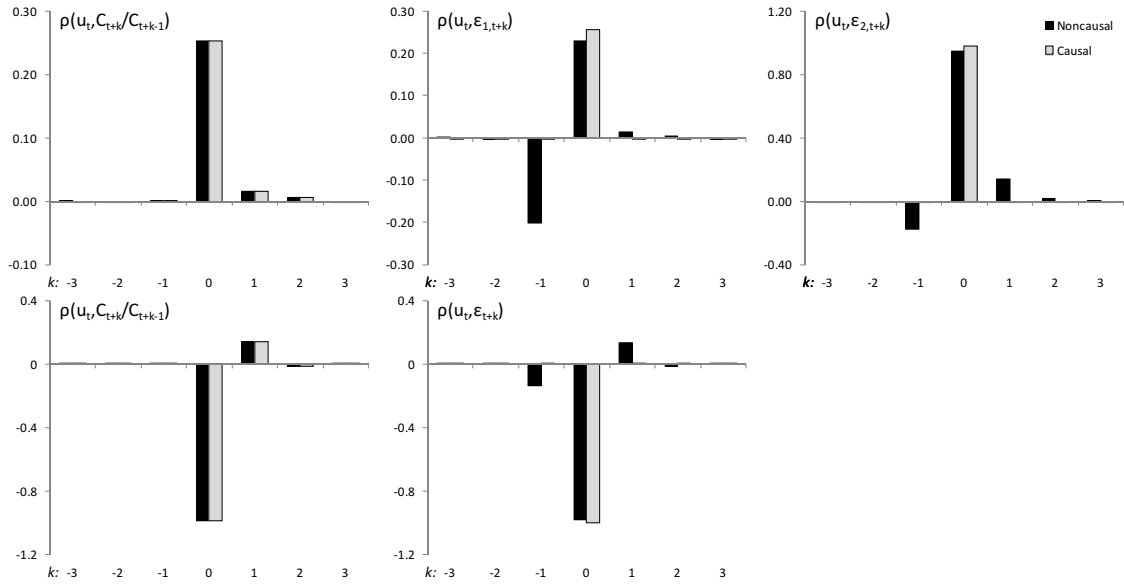
Figure 1: Correlations of errors and instruments: Correlations between residuals from GMM estimates in Table 3: $u_t = \widehat{\beta} \left(\dfrac{C_t}{C_{t-1}} \right)^{-\widehat{\gamma}} R_t - 1$ and lags and leads of ε_t and $\dfrac{C_t}{C_{t-1}}$, for calibration (i), top, and (ii), bottom.