MEDDELANDEN FRÅN
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SWEDISH SCHOOL OF ECONOMICS
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WORKING PAPERS

453

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INTRADAY AND WEEKEND VOLATILITY PATTERNS
- IMPLICATIONS FOR OPTION PRICING

DECEMBER 2000

Key words: Option pricing, Intraday volatility patterns, Weekend volatility,
Continuous time

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SHS intressebyrå IB (Oy Casa Security Ab), Helsingfors2000

ISBN 951-555-680-5
ISSN 0357-4598

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Abstract

This study examines the intraday and weekend volatility on the German DAX. The intraday volatility is partitioned into smaller intervals and compared to a whole day's volatility. The estimated intraday variance is U-shaped and the weekend variance is estimated to 19 % of a normal trading day. The patterns in the intraday and weekend volatility are used to develop an extension to the Black and Scholes formula to form a new time basis. Calendar or trading days are commonly used for measuring time in option pricing. The Continuous Time using Discrete Approximations model (CTDA) developed in this study uses a measure of time with smaller intervals, approaching continuous time. The model presented accounts for the lapse of time during trading only. Arbitrage pricing suggests that the option price equals the expected cost of hedging volatility during the option's remaining life. In this model, time is allowed to lapse as volatility occurs on an intraday basis. The measure of time is modified in CTDA to correct for the non-constant volatility and to account for the patterns in volatility.

Keywords: Option pricing, Intraday volatility patterns, Weekend volatility, Continuous time.

Comments by Kenneth Högholm, Johan Knif, Eva Liljeblom, Lars Nordén and Seppo Pynnönen are gratefully acknowledged. This paper has also benefited from discussions with friends, colleagues and employees at Estlander & Rönnlund Financial Products Ltd and also from comments by participants at the Computational Finance seminar 2000. This study was conducted as a joint work between Mikael Vikström and Kim Sundkvist.

1 Introduction

A debated issue in option pricing is how to account for the lapse of time. A major question is whether an option buyer should pay for the decrease in time value over a weekend or not. The time value left can be seen as the volatility expected to occur during the option's remaining life. The argument for not paying any time value over the weekend is that price discovery is made only when markets are open, which gives no value to holidays and weekends in option pricing. Under this argument, only the time value caused by interest rates calculated in calendar time should be accounted for.

On the other hand, the argument for paying time value over the weekend is that floods, draughts and other pertinent news do happen during weekends, too. This means that on Monday the market would be reacting to three days of information released over the weekend. This gives us unobserved holiday volatility, as assets on most exchanges do not trade during the weekend. Under the calendar time hypothesis one would expect the mean and variance to be three times as high from Friday close to Monday close. Under this assumption, we expect to have holiday volatility from Friday close to Monday open. Fama (1965) found that the variance from Friday close to Monday close was only 22 % higher than between two subsequent trading days. French (1980) found the variance to increase by only 19 % in a similar test on more recent data. The findings in Sundkvist and Vikström (2000) reveal that the weekend variance on the German DAX was 54 % during 1990-1999, but only 17 % during the subperiod 1995-1999. French and Roll (1986) provide possible explanations and testing of the phenomenon that prices are more volatile during exchange trading hours.

The need for taking the time-basis argument one step further is most evident in the pricing of shorter-term options. Pricing of shorter term options raises the question whether days that elapse, let it be trading or calendar days, is a sufficiently small time interval. Shorter maturity options are often more actively traded and the bid-ask spreads are smaller, thus pricing is believed to be more exact. The time decay is also greater for shorter maturity options, making the discrete time interval of whole days even more prominent. Stoll and Whaley (1990) reported that intraday volatility is higher than the overnight volatility on NYSE. It is also a fact that volatility intraday is not realized linearly, but is higher at open and close producing a U-shaped function of volatility. Several authors have documented that the volatility and volume is higher at open and close, for which Admati and Pfleiderer (1988) provide possible explanations. This phenomenon seems to exist on all equity markets and through time, as it has been documented on NYSE by Wood et al. (1985) and Chan et al. (1991), and by Shiyun and Guan (1999) on NIKKEI. Berry and Howe (1994) provided an explanation for the patterns of intraday volatility by measuring the information flow. Public information arrival was shown to be non-constant, displaying distinct intraday patterns.[1] Chang, Chang and Lim (1998) models time as stochastic according to information arrival. Given the phenomena in intraday volatility, at the extreme one can model the option price in continuous time to account for the seemingly predictable patterns of volatility intradaily.

To correct for the unrealistic use of a constant volatility in option pricing, using a model with stochastic volatility should be considered. However, such models are somewhat complex. To get a reasonably simple model from a computational standpoint, we attack the problem by changing the time unit. The change in the time unit should capture the patterns in volatility. This is the central theme of this study. The resulting model will then automatically adjust for the patterns of varying volatility that can be fairly well predicted. This leaves the trader with one task less to concentrate on – the trader now needs to adjust the volatility parameter only when there is some fundamental reason shifting the level of expected volatility.

This paper continues in section two by presenting the Black and Scholes (1973) formula and the two most common methods for counting days in option pricing; trading days and calendar days. Section three develops a methodology in option pricing for accounting for the patterns in volatility, which is accomplished by adjusting the passage of time. Section four provides the results from the estimation of the model on the German market. First overnight and intraday volatility is estimated in relation to each other on a longer period. This is done as comparison for the intraday intervals estimated on a shorter period and to check the robustness of the relationship between overnight and intraday volatilities. The volatility during a holiday is also estimated. Section five demonstrates the impact of choosing the new method instead of one of the old methods. Option prices are compared for shorter and longer maturities with the three different methods of time lapse. Perhaps this is the most interesting section where it is shown

[1] However, no significant relationship was found between public information arrival and volatility. This is argued by Berry and Howe (1994) to be expected as public information is available to the whole market and does not require trading to impact prices, whereas private information affects prices only through trading.

how the model works and how prices compare in trading time, CTDA time and calendar time. Section six summarizes the paper.

2 Time in option pricing

Time to maturity is critical in estimating the theoretical price of an option. The option premium is higher for longer maturities[2], which corresponds to the higher possibility of the option ending in-the-money with longer maturities.

The Black and Scholes formula is dependent on time as shown in equations (1, 2 and 3) and accounts for time to expiry in the parameter *t*. The price of a European call option is estimated as

$$C = SN(d_1) - Xe^{-rt}N(d_2), \tag{1}$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}, \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}, \tag{3}$$

and C is the price of the call option with strike X and underlying S, r is the interest rate, σ is the volatility and t is time to maturity.

In option pricing, it is common practice to use either calendar or trading days to count the days to maturity of an option. Normally there are 365 calendar days during a year and t is given as a fraction of a year like

$$t = \frac{cd}{365}, \tag{4}$$

where cd is calendar days left. The other commonly used measure is trading days, of which there are about 252 during a year and thus we would use

$$t = \frac{td}{252}, \tag{5}$$

[2] This holds for assets that do not pay any dividend before the maturity of the option.

where *td* is trading days left. However, the number of trading days is not necessarily proportional to the number of calendar days left over time, as there are more holidays during some seasons of the year. Therefore, one should consider using a different denominator during different times of the year, at least for pricing options with time to expiry less than a year.

When pricing options with trading days, an inconsistency becomes apparent. The volatility is generated over trading time during approximately 252 days of the year, but interest rate is accumulated over calendar time 360 or 365 days of the year. Obviously, we now have two different time bases. French (1984) suggests using the Black and Scholes formula on a composite-time basis. His model allows for different time units for the interest rate and volatility. As this study is aiming at developing a model with a time basis different from calendar days, we use the idea in French's study and adjust the Black and Scholes formula in equations (2) and (3) as demonstrated in equations (6) and (7) to have two different time bases.

$$d_1 = \frac{\ln(S/X) + rt + \sigma_\tau^2 \tau / 2}{\sigma_\tau \sqrt{\tau}}, \tag{6}$$

$$d_2 = d_1 - \sigma_\tau \sqrt{\tau}, \tag{7}$$

where τ is time to expiration on a new time basis that has yet to be estimated. French used in his study 252 days per year corresponding to volatility only during trading time. French still used t as calendar days in equation (1) in order to count interest rate on a daily basis.

One of the empirical findings of French's study (1984) was that options seem to be priced under the trading day hypothesis but with interest rate accrued during calendar days. This means that holidays would cause no extra volatility, if we were to believe the pricing of options on the market. However, as shown in various articles already mentioned, holidays do cause an unobserved volatility.

With longer maturity options, the difference between using trading and calendar days is minimal. Being consistent in the choice of basis of time usually yields acceptable results. However, with shorter maturity options the difference is increasingly important. The extreme example would be an option with four days left and during three of those days; the market is closed because of a three-day weekend. The option priced on the basis of calendar days has 4/365 days left, but only 1/252 days left priced on the basis of trading days. The time remaining with trading days is 2.76 times that of calendar days in this example.

Now assume that there is some volatility during holidays, but that this volatility is less than during trading days. Using calendar days would make the options too cheap on Mondays and too expensive on Fridays. To adjust for this in practice, an options trader using calendar days as time basis can gradually decrease the volatility in the pricing model during the week just to increase it on Monday morning. The volatility of actual option prices on Friday close could then be observed at a level corresponding to a point in calendar time that may be somewhere between Saturday and Sunday. Changing the volatility or time like this seems like a somewhat cumbersome rule of thumb.

With trading days, it is the other way around - volatility has to be increased on Friday and decreased on Monday morning. This kind of manipulation with the volatility is necessary in order to adjust for the imperfect assumptions of the option-pricing model. The holiday volatility is the cause of this inconsistency with implied volatility in both trading time and calendar time. Hence the need to develop a third measure of time. Furthermore, one assumption of the Black and Scholes formula is that trading is continuous, but it is a fact that markets are closed most of the time. Therefore, we need to implement discrete time in pricing as well. Discrete time is in a way already implemented in Black and Scholes if we let time decrease by a whole day at a time, but a jump of a whole day at a time would be justifiable only with an intraday volatility of zero.

It should also be noted that the implied volatility is dependent on the time basis used. Hence, adapting a time basis with a variable lapse can eliminate the patterns in implied volatility. This implies that the time is ticking faster under higher volatility and that time stands still during non-trading. This leaves the volatility parameter in option pricing to be altered only for fundamental changes in the expected volatility. The time parameter substitutes the trader’s rule of thumb that is used with the original Black and Scholes formula. This way any patterns in implied volatility should diminish.

Patterns in the variation of the implied volatility can be seen as a problem in the specification of the model. An extension to the Black and Scholes to correct for the assumption of continuous trading and constant volatility is developed in the next section. This is done in order to get around the need for systematic adjustments to the implied volatility because of time lapsing. Volatility should change only when the expected volatility during the option’s remaining life changes. First some preliminary tests on the market studied.

3 Methodology

In this section, a model is developed to obtain a theoretical price accounting for the volatility of the underlying on an intraday basis. The model should also take care of the weekend effect in volatility, which has been reported by French (1980), Gibbons and Hess (1981), Lakonishok and Levi (1982), French (1984), Dubois and Louvet (1986) and on the DAX by Sundkvist and Vikström (2000). Given rules of no arbitrage, we expect the time value of an option to equal the expected cost of hedging the volatility during the option’s remaining life. Thus, the model will produce option prices under the assumption that volatility is expected to be realized in the same pattern as historically during different hours of the day. The model developed below uses this argument and this implies the assumption that news causing volatility arrives at a similar rate also in the future. Under this assumption, volatility is expected in a similar pattern during all days of the week. For simplicity, variance is used instead of volatility in the development of the methodology. The variance left is expressed in days. How to use this new extension in option pricing is best demonstrated using examples as will be done at the end of next section. First, the weekend and intraday volatilities are estimated.

3.1 Preliminary tests

The no-arbitrage pricing rule suggests that the option premium equals the expected cost of hedging volatility during the option's remaining life. The cost of hedging is proportional to the variance. In a trending market, the variance estimated with a mean is lower than the variance without a mean. As any trends should be unpredictable, the expected mean should be used in the estimation of the variance and not the realized mean.

Let us begin by dividing variance into intraday and overnight variance. As a first test, compare the intraday portion of variance with the overnight portion of variance. This will result in an observed variance during trading and an unobserved variance during non-trading. These partial volatilities defined as intraday variance *IV* and overnight variance *OV* are calculated as

$$IV = \frac{1}{N}\sum_{t=1}^{N}(c_t - o_t)^2, \tag{8}$$

and

$$OV = \frac{1}{N}\sum_{t=1}^{N}(o_t - c_{t-1} - \mu)^2, \tag{9}$$

where o and c are logarithms of the open and close prices respectively. The overnight returns obtained should first be subtracted by the expected return μ before taking the power of two. As the settlement system allows all trades during one day to be settled on the same day, we see no reason for expecting an intraday return as there is no foregone interest for holding an asset from open to close.

To get a strictly overnight variance, only observations with a single night are considered in equations (8) and (9). Holidays are left out and estimated separately below. The inclusion of holidays in the estimation of equation (9) would lead to an unobserved variance with more than overnight variance in it. The portion of variance of the total variance during twenty-four hours that passes intradaily *PIV* is then

$$PIV = \frac{IV}{IV + OV}, \tag{10}$$

and the portion of overnight variance of total variance is

$$POV = 1 - PIV = \frac{OV}{IV + OV}. \tag{11}$$

Because the portions of volatility can be very different from day to day, a moving average will be used for presenting the portions of intraday and overnight volatility over time.

3.2 Modeling intraday and weekday volatility

Under the trading time hypotheses, no volatility exists during holidays. Under the calendar time hypotheses volatility is just as high during non-trading days as on trading days and is accumulated just to be realized at the open the next trading day. Taking the results of Sundkvist and Vikström (2000) for the DAX during 1990 – 1994 and 1995-1999, Mondays had a variance that was respectively 54.50 % and 16.65 % higher than for the other weekdays. This implies that a model in trading time is not pricing options optimally, neither is a model in calendar time. Therefore, the holiday volatility is to be accounted for.

The variance accumulated during the holiday is expected to be realized when the market opens. In this study the excess variance for a holiday EHV_h of h days of non-trading is estimated as

$$EHV_h = \frac{OV_h - OV}{IV + OV}, \tag{12}$$

where OV_h is estimated as OV but with h days of non-trading between c_{t-1} and o_t. This excess variance is added to the variance of the day succeeding the holiday. EHV is a measure of how much additional variance is realized after a non-trading period of h days. This additional variance is measured in proportion to the total variance ($IV + OV$) for subsequent trading days. Consequently, a trading day that is preceded by another trading day is expected to have variance 1 or 100 %, but a trading day after a holiday of h days is expected to have variance 1 + EHV_h. The total overnight volatility to be realized on Monday morning is thus $EHV + OV$.

Once we have estimated the portions of variance that is realized intradaily and overnight, we can take a closer look at the intraday variance. The largest portion of variance is intraday variance (Lockwood and Linn [1990]). The intraday variance is observable as it is subject to continuous trading. This observed variance is realized during a longer time span than the unobserved overnight variance, which is to be realized at the open of the market.

To get a measure of the magnitude of intraday variance, a measure that is easy to grasp, 30-minute intervals of returns are used to calculate variance. This means that the continuous time is approximated using 30-minute discrete time intervals. Let us look at an example where the market opens at 9:00 AM. The variance for the first half-hour is calculated in the same way as IV and OV from the return from the previous close to 9:30 AM. The variance for the second interval is the squared return from 9:30 AM to 10:00 AM, thus using a zero mean. The average variance for the respective time intervals during the estimated period are the parameters for how fast “time” is ticking during the trading, measured in discrete time intervals of 30 minutes each. The variance for each interval is then presented in proportion to the variance for the whole day, i.e. the close-close variance. Thus, in the CTDA model, time appears to be ticking faster during intervals of higher volatility.

3.3 The data

The data used in this study consists of daily open, close and intraday returns for the future on the German DAX-index. The returns were calculated as differences in log prices, $R_t = 100*(ln(P_t)–ln(P_{t-1}))$ where P_t denotes the price at time t. Intraday data is available for two periods of 65 and 57 trading days respectively. During the first period of intraday data, the future on the DAX was traded until 5:00 PM. For the second period the market closed 30 minutes later. Daily data is available for 993 days. Descriptive measures for the data are presented in Table 1. The daily return is on average 0.10 % and the intraday return 0.02 %, although 0 % was expected intradaily.

Table 1
Descriptive data

1/1/1996-12/30/1999	N	Mean	Variance	Skewness	Kurtosis
Close-close returns	961	0.0993	2.1044	-0.3625	2.3066
Close-open returns	961	0.0862	0.8273	-0.9596	11.1538
Open-close returns	977	0.0161	1.6134	-0.4514	2.1104
10/1-12/30/1998	N	Mean	Variance	Skewness	Kurtosis
30-minute returns	991	0.0078	0.3623	-0.3645	5.9324
9/1-11/19/1999	N	Mean	Variance	Skewness	Kurtosis
30-minute returns	969	0.0088	0.0725	-0.9079	7.2907

Close-close are daily returns, close-open are overnight returns and open-close are intraday returns. Returns are in percentage and were calculated as differences in log prices, $R_t = 100*(ln(P_t)–ln(P_{t-1}))$ where P_t denotes the price at time t.

The descriptive data from the first half of Table 1 implies that the sum of overnight variance (0.82727) and intraday variance (1.61344) is not the same as the close-close variance (2.10439). The correlation between overnight and intraday variance is then –0.146. The fact that the sum of variances in smaller intervals is higher than a daily variance supports the pricing-error hypothesis proposed by French and Roll (1986), but can also be due to negative autocorrelation.

The future expires once every three months, so from the data have been excluded returns involving the expiration day. During the four years of data, there are 16 expiration days for the future. This excludes 32 observations for every return spanning two days, and 16 observations for intraday returns. For the most recent sample of 30-minute intervals there was no expiration of the future, but one futures expiration for the less recent period.

The problem of lagging observations and thus positive autocorrelation arises when using indexes as the underlying. When using the future as the underlying, negative autocorrelation may be present in the data due to the bid-ask bounce. However, the bid-ask spread in the DAX future is minimal, normally a fraction of a percentage. Thus, the impact of the bid-ask bounce is also minimal, but might vary during the day under different levels of liquidity. The effect can however be canceled out if the lower liquidity leads to less volatility and thus smaller bid-ask spreads, which is in fact observed during the middle of the trading day. The autocorrelation in the 30 minute returns is –0.04.

4 Results

As discussed in the beginning of the previous section, the cost of hedging is proportional to the variance around the expected mean. Thus, estimation of the variance calls for estimation of the expected mean. We estimated the annual mean return on the DAX index to 13.64 % during the years 1988-1999. This translates into a daily mean return of 0.06 % which is included in the estimation of the overnight variance. In the estimation of the intraday variance, a mean of zero is used throughout the estimation in this study as there should be no opportunity cost on an intraday basis. The intraday variance *IV* was estimated using equation (12). The overnight variance *OV* for consecutive trading days was estimated using equation (13). The results are presented in Table 2. The portion of variance that was realised during continuous trading, i.e. *PIV*, was 69.30 % of the total variance during twenty-four hours. *POV* was on average 30.70 %[3]. However, the partial volatilities are not constant and *PIV* varies between

[3] Remember that the total variance is the sum of overnight and intraday variance.

44.02 % and 81.29 % in a moving average with 100 observations in it.[4] This means that in general the intraday variance stands for the major part of variance during a twenty-four hour period, but is occasionally less than 50 %.

Table 2
Portions of intraday and overnight volatility

	IV	OV
Mean	1.6696	0.7450
Minimum	0.2579	0.0982
Maximum	5.0326	1.7816
	PIV	POV
Mean	69.30 %	30.70 %
Minimum	44.02 %	18.71 %
Maximum	81.29 %	55.98 %

IV is estimated as intraday (open-close) and OV as overnight (close-open) variance for consecutive trading days on percentage returns 1/1/1996 – 12/30/1999.

The mean variance ratio between IV and OV in Table 2 is 2.24.

By comparing the results in Table 1 that includes all observations and Table 2 that includes observations only between subsequent trading days, we find that there is some variance missing. The *IV* of 1.6696 in Table 2 is a bit higher than the intraday variance in Table 1 of 1.6134, implying that the variance during trading on the day following a holiday was a bit lower than on subsequent trading days. The difference is however small and hereby only stated. The difference between *OV* in Table 2 and the overnight variance in Table 1 is a bit greater and caused by the holiday volatility. This will now be taken into account.

The excess variances EHV for holidays of different lengths are estimated using equation (12) and presented in Table 3. For a one-day holiday, the excess variance turns out to be 20.27 %, but the number of observations is only 15. This means that a one-day holiday should be followed by a day that is 1.2027 days "long" as far as variance is concerned. A two-day holiday leads to a trading day that is 1.1689 days long, which is very close to the 1.1665 estimated by Sundkvist and Vikström (2000). We have small samples for one-day, three-day and four-day holidays, but using an average of all holidays gives us a larger sample. Taking the average excess volatility for a holiday of any length gives us an EHV of 19.26 %, which will be used in the final model. This means that a holiday of any length is expected to be followed by a trading

[4] *PIV* and *POV* are of course dependent on the length of the moving average window as these ratios varies a lot from day to day.

day that has a variance 19.26 % higher than subsequent trading days. However, Lockwood and McInish (1990) show that the variances of overnight and intraday returns are different during bull and bear markets. Thus, the weekend variance, which is also one form of overnight volatility, is not necessarily stable either.

Table 3
Excess holiday volatility

Type of holiday	N	Excess holiday variance
One-day holiday	15	20.27 %
Weekend (two days)	193	16.89 %
Three-day weekend	10	20.73 %
Four-day weekend	6	84.48 %
Average (2.03 days)	224	19.26 %

EHV or excess holiday variance is measured in relation to the total one-day variance for subsequent trading days.

The variance that was realized intradaily is U-shaped and is presented in 30-minute intervals in Table 4. This is the variance realized for a trading day proceeded by another trading day. However, it should be noted that the pattern of intraday volatility is not stable over time and the results presented here should be used on another period with due consideration. The estimated variances are presented to make the idea of the model clear. The overnight variance *OV* was estimated to 30.70 % on the period with four years of data. This is in line with that reported in Table 4 at 9:00 where the overnight variances for the intraday periods tested were 30.0 % and 29.6 % respectively.

The U-shaped intraday volatility is quite evident in this sample in line with the findings in Högnäsbacka et al. (2000) on the same market. However, the U-shaped pattern is perhaps not as evident as for the S&P 500 studied by Chan et al. (1991). The intraday volatilities for this short period are very sensitive to outliers. In addition, the intraday volatilities probably change over time and may need re-estimation at the discretion of the user. However, the results in Andersen and Bollerslev (1996) and Anderson et al. (1998) indicate that the intraday patterns for currencies are reasonably stable.

Table 4
Intraday variance in 30-minute intervals

	10/1-12/30/1998	9/1-11/19/1999
9:00 AM	30.0 %	29.6 %
9:30 AM	9.9 %	6.9 %
10:00 AM	6.2 %	4.8 %
10:30 AM	5.3 %	3.4 %
11:00 AM	4.2 %	3.7 %
11:30 AM	2.1 %	2.6 %
12:00 PM	4.8 %	2.2 %
12:30 PM	3.5 %	3.9 %
1:00 PM	2.0 %	3.5 %
1:30 PM	4.5 %	1.7 %
2:00 PM	5.1 %	6.5 %
2:30 PM	3.8 %	9.2 %
3:00 PM	1.6 %	3.8 %
3:30 PM	4.2 %	4.2 %
4:00 PM	5.4 %	4.1 %
4:30 PM	7.4 %	3.1 %
5:00 PM	-	6.8 %

The realized intraday variance in 30-minute intervals was estimated as squared returns. The number in each cell represents the equivalent amount of time lapsing to correspond to the variance in the respective interval.

The intraday seasonal patterns in volatility from Table 4 are visualized in Figure 1. During the more recent period, trading hours where 30 minutes longer as seen in the figure.

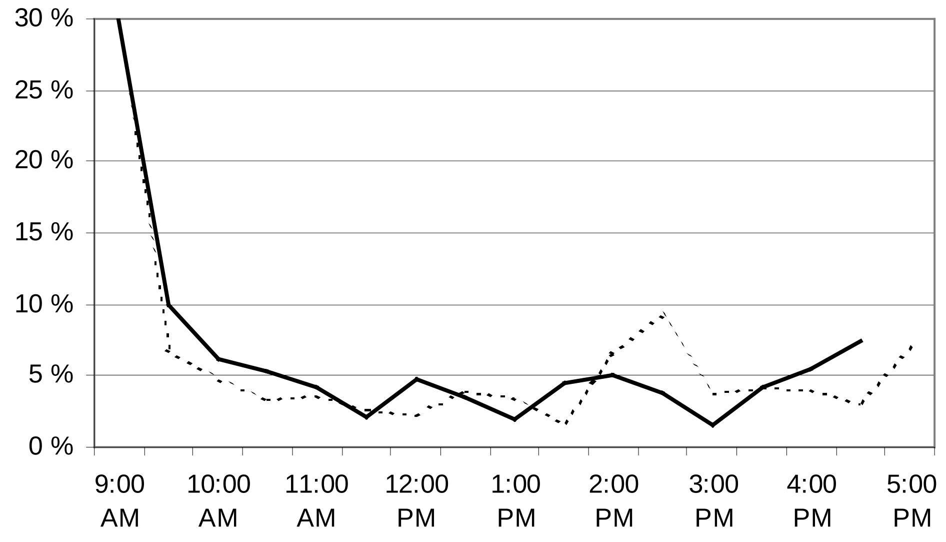

Figure 1
U-smile in intraday volatility
The intraday volatility is higher around open and close. The bold line shows the intraday volatilty for the period 10/1-12/30/1998 with the market closing 4:30 PM and the dotted line shows the intraday volatility for the period 9/1-11/19/1999 with the market closing 5:00 PM.

There is an obvious spike in volatility around 2:30 PM during the more recent period. This is when some financial markets open in the USA, and the volatility is perhaps due to uncertainty where the market will go the minutes before and after opening. Another possible and related cause for volatility in these two intervals is the release of significant economic reports at the same time. Such news announcements have been found to affect volatility by Burghardt and Hanweck (1993). The volatility spike around 2:30 PM appeared also in the study by Andersen and Bollerslev (1996).

The estimation of the model is now complete. Henceforth we will be using the estimated variances for the more recent period in the discussion of the model. The time left at different points is shown in Table 5 for the last few days before expiration.

Table 5
Time left during the last few days before expiration

	Friday	Monday	Tuesday	Wednesday	Thursday	Friday
Calendar days:	7.00	4.00	3.00	2.00	1.00	0.00
Trading days:	5.00	4.00	3.00	2.00	1.00	0.00
CTDA-days:						
9:00 AM	6.19	5.19	4.00	3.00	2.00	1.00
9:30 AM	5.90	4.70	3.70	2.70	1.70	0.70
10:00 AM	5.83	4.64	3.64	2.64	1.64	0.64
10:30 AM	5.78	4.59	3.59	2.59	1.59	0.59
11:00 AM	5.75	4.55	3.55	2.55	1.55	0.55
11:30 AM	5.71	4.52	3.52	2.52	1.52	0.52
12:00 PM	5.68	4.49	3.49	2.49	1.49	0.49
12:30 PM	5.66	4.47	3.47	2.47	1.47	0.47
1:00 PM	5.62	4.43	3.43	2.43	1.43	0.43
1:30 PM	5.59	4.39	3.39	2.39	1.39	0.39
2:00 PM	5.57	4.38	3.38	2.38	1.38	0.38
2:30 PM	5.50	4.31	3.31	2.31	1.31	0.31
3:00 PM	5.41	4.22	3.22	2.22	1.22	0.22
3:30 PM	5.37	4.18	3.18	2.18	1.18	0.18
4:00 PM	5.33	4.14	3.14	2.14	1.14	0.14
4:30 PM	5.29	4.10	3.10	2.10	1.10	0.10
5:00 PM	5.26	4.07	3.07	2.07	1.07	0.07

The figures are number of days left using different time bases. For CTDA-days there are 30-minute intervals representing time to expiration at different moments of the day.

For pricing an option in calendar time, we would use the first row and divide it by 365 to arrive at the t parameter for equation 1. Using trading time we choose the second row and divide it by, let us say, 251 to get the time parameter. Using the CTDA-time model we obtain a time parameter that is changing for every half-hour of trading in this example. In one year, there are about 262 CTDA-days on the German market, depending on the number of holidays. To arrive at the number of CTDA-days, take the number of trading days and add the EHV for every holiday.[5]

The number of CTDA-days in Table 5 should be interpreted as time to expiration prior to this point in time. This means that at Monday morning time to maturity is 5.19/262, which is reduced to 4.70/262 as soon as the market opens in order to account for the excess holiday volatility and the normal overnight volatility. Both of which are to be realized at open. Using discrete intervals of 30 minutes the time is reduced by 0.49/262 at the opening on Monday morning. As the German DAX future normally expires on Fridays at 1:00 PM, we would like to set the time to expiration to zero at 1:00 PM on Friday. This could be accomplished by subtracting 0.43 days from all the values prior to that point in Table 5.

Also, note that when using calendar or trading days, we have stated that there is zero time left on the last Friday. This can obviously cause a problem in trying to obtain a theoretical price. With no time left the option price is only the intrinsic value. At open the last trading day, the theoretical prices will not reflect any time value, whereas there surely is going to be volatility during the few hours left until expiration. To price options on the expiration day using one day left also causes a problem – option premiums are then too high. This is where the CTDA-model might come in handy.

5 The effect on option pricing

We now demonstrate the differences in option prices using the three time units; calendar time, trading time and the continuous time discrete approximation (CTDA) presented in this paper. The option prices are calculated using the different time measures τ for the volatility component, but t is still calendar days as in the Black and Scholes formula in order to count interest rate on a daily basis in calendar time. We use options on the DAX for three different strikes and show differences between the models in simulated prices as time to maturity decreases. Table 6 presents the prices for a given volatility. The time to maturity starts at 106 days in calendar time, 70 days in trading time and 73.85 days using CTDA.

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[5] Please note that several subsequent days of non-trading are treated as only one holiday, which is justified by the results reported in Table 3.

Table 6
Simulated call option prices using the three different measures of time

	In-the-money			At-the-money			Out-of-the-money		
Date	A	B	C	A	B	C	A	B	C
4/6/2000	798.72	793.75	795.37	381.18	373.58	376.07	146.09	139.56	141.70
4/13/2000	788.08	782.78	784.43	367.39	359.09	361.69	135.03	128.05	130.23
4/20/2000	777.31	771.68	773.35	353.17	344.11	346.83	123.85	116.38	118.62
4/27/2000	766.41	763.68	765.26	338.48	333.99	336.60	112.53	108.91	111.01
5/4/2000	755.39	754.00	755.54	323.24	320.88	323.50	101.09	99.24	101.29
5/11/2000	744.26	742.59	744.14	307.40	304.43	307.20	89.55	87.30	89.39
5/18/2000	733.05	731.10	732.66	290.86	287.21	290.15	77.92	75.27	77.40
5/25/2000	721.78	719.57	721.14	273.49	269.07	272.21	66.25	63.20	65.37
6/1/2000	710.52	709.68	710.88	255.15	253.34	255.91	54.60	53.43	55.10
6/8/2000	699.35	698.28	699.45	235.61	233.03	235.84	43.08	41.55	43.21
6/15/2000	688.43	688.65	689.72	214.59	215.18	218.07	31.85	32.17	33.72
6/22/2000	677.99	679.31	680.00	191.61	196.01	198.20	21.23	23.22	24.24
6/29/2000	668.43	669.32	669.89	165.91	170.09	172.64	11.74	13.17	14.07
7/6/2000	660.39	660.83	661.21	136.04	140.05	143.15	4.32	5.11	5.78
7/13/2000	654.62	654.70	654.80	98.45	102.50	106.69	0.43	0.61	0.86
7/14/2000	653.99	654.01	654.07	91.94	93.48	98.39	0.22	0.26	0.43
7/15/2000	*653.39*	*653.45*	*653.49*	*84.97*	*93.17*	*96.46*	*0.10*	*0.26*	*0.37*
7/16/2000	*652.81*	*652.89*	*652.93*	*77.42*	*92.86*	*96.15*	*0.03*	*0.26*	*0.36*
7/17/2000	652.24	652.26	652.28	69.10	82.91	86.94	0.01	0.08	0.13
7/18/2000	651.68	651.68	651.69	59.70	71.66	76.71	0.00	0.01	0.03
7/19/2000	651.12	651.12	651.12	48.61	58.37	64.95	0.00	0.00	0.00
7/20/2000	650.56	650.56	650.56	34.24	41.15	50.65	0.00	0.00	0.00
7/21/2000	-	-	650.00	-	-	30.50	-	-	0.00

Simulated call option prices using the three different measures of time; calendar days (A), trading days (B) and continuous time discrete approximation (C). The underlying is at 6500, volatility is 25.0 % and constant across strikes for comparability, interest rate is 3.5 % and always accrued in calendar time. The strike for the in-the-money call is 5850 and the strike for the out-of-the-money call is 7150, i.e. approximately 10 % away from at-the-money. Dates in italic are holidays. Expiration date is 7/21/2000. All three models accrue interest in calendar time, hence the change in option prices also for (B) and (C) during the weekend, shown in italics. On the expiration date there is zero time left using the calendar and trading time bases, hence there is no price available.

Table 6 gives an indication of the comparative properties of alternative option pricing formulas. However, the simulated prices are just a few arbitrary examples. Option pricing has many dimensions and the pricing dynamics cannot justifiably be described in a two-dimensional table. Changing any parameter also changes the differences using the three different time units, but the pattern is evident. With options being priced according to realized volatility, using either calendar or trading days as time basis may not produce prices under non-arbitrage rules. The major issue is that there is a jump in prices whenever there is a holiday, a jump that is not properly caught in prices in either calendar or trading time. Using CTDA in pricing options we should arrive at the theoretical non-arbitrage price, but using calendar days will produce a price that is too high prior to a holiday and too low after a holiday. When using trading days as time basis the sign of the bias is the opposite and also smaller. In the leap year 2000 there are 366 calendar days, 252 trading days and 262.59 CTDA days for the DAX. Thus, the choice of time basis will indeed affect prices.

The differences in prices shown in Table 6 are in practice avoided by changing the implied volatility, possibly according to a rule of thumb such as described in section two. The CTDA-time model provides a systematic way of accounting for the patterns in volatility, which can easily be implemented in the pricing software.

In-the-money options in Table 6 has an intrinsic value of 650, the rest is time value. These options have a vega close to zero, which implies that the differences in prices are small under different implied volatilities. At-the-money options have a larger sensitivity to volatility and the differences are clear. The two rows in italic in Table 6 is a holiday where the option premium changes by only the effect of the interest in trading and CTDA time. Furthermore, it is obvious that the volatility has to be set higher for the calendar-time model on the following Monday because time left (4/365) is significantly less than for the two other models (4/252 and 5.19/262 respectively). In percentage, the prices for out-of-the-money options are most sensitive to a change in volatility.

When time to maturity is less, we will also want to look at intraday time intervals. The price of at-the-money options with only a few days to maturity is shown in Table 7 priced on the three different time bases. The Appendix shows two similar tables for a 5 % out-of-the-money put and call respectively. The differences are somewhat more accentuated in percentage with out-of-the-money options and also for higher levels of implied volatility.

Tables 6 and 7 show relevant differences in the pricing of options depending on the choice of time unit. This suggests that also the partial derivatives of the option price are different using the three methods of measuring time. It is therefore also important to choose the correct measure of time when watching the Greeks as time passes. This is relevant in the surveillance of a derivatives portfolio, i.e. in risk management.

Table 5 and Table 7 show only one price for the trading and calendar time models for each day, while the CTDA-time model provides updated prices every half hour. This may not be a fair comparison, because in practice the trader is likely to adjust either the time parameter or implied volatility during the day. In order to demonstrate how this is possibly done, Figure 1 reports the implied volatilities iterated from CTDA-prices in Table 7 but using trading and calendar days. This demonstrates the systematic changes to implied volatility during the day with time specified as in equations (1), (2) and (3) for calendar days and as in equations (1), (6) and (7) for trading days.

Table 7
Simulated at-the-money call option prices close to expiration

	Friday	Monday	Tuesday	Wednesday	Thursday	Friday
Calendar days:	91.94	69.10	59.70	48.61	34.24	-
Trading days:	93.48	82.91	71.66	58.37	41.15	-
CTDA-days:						
9:00 AM	98.31	88.65	76.61	64.84	50.51	30.26
9:30 AM	95.81	84.04	73.41	61.03	45.53	21.00
10:00 AM	95.22	83.37	72.64	60.10	44.29	18.18
10:30 AM	94.80	82.90	72.10	59.45	43.41	15.92
11:00 AM	94.51	82.57	71.72	58.99	42.77	14.10
11:30 AM	94.19	82.20	71.30	58.48	42.07	11.81
12:00 PM	93.96	81.94	71.00	58.12	41.56	9.89
12:30 PM	93.77	81.73	70.75	57.81	41.13	7.91
1:00 PM	93.43	81.34	70.30	57.26	40.36	-
1:30 PM	93.12	80.99	69.89	56.76	39.65	-
2:00 PM	92.97	80.81	69.69	56.52	39.31	-
2:30 PM	92.39	80.16	68.93	55.58	37.95	-
3:00 PM	91.57	79.22	67.84	54.22	35.93	-
3:30 PM	91.23	78.82	67.38	53.65	35.07	-
4:00 PM	90.85	78.39	66.87	53.01	34.08	-
4:30 PM	90.48	77.96	66.37	52.38	33.09	-
5:00 PM	90.20	77.64	65.99	51.89	32.33	-

The simulated prices during the last six days before expiration are at-the-money options with the underlying at 6500. Volatility is 25.0 %, interest rate is 3.5 % and always accrued in calendar time. Saturday and Sunday are the only non-trading days in this example. Time to expiration is as in Table 5, but with 0.43 subtracted from the CTDA-days to adjust for the expiration of the DAX options at 1:00 PM instead of at 5:30 PM.

On the Friday with one week until maturity in Figure 2, calendar and trading time yields approximately the same results (5/252 ≈ 7/365). The implied volatility is also close to 25 % at the end of this Friday, but prior to this day and especially after this day the three models diverge. Starting at open on Monday, the implied volatility is 27.1 % for the trading-time model and 23.8 % at close. The differences increase until expiration.

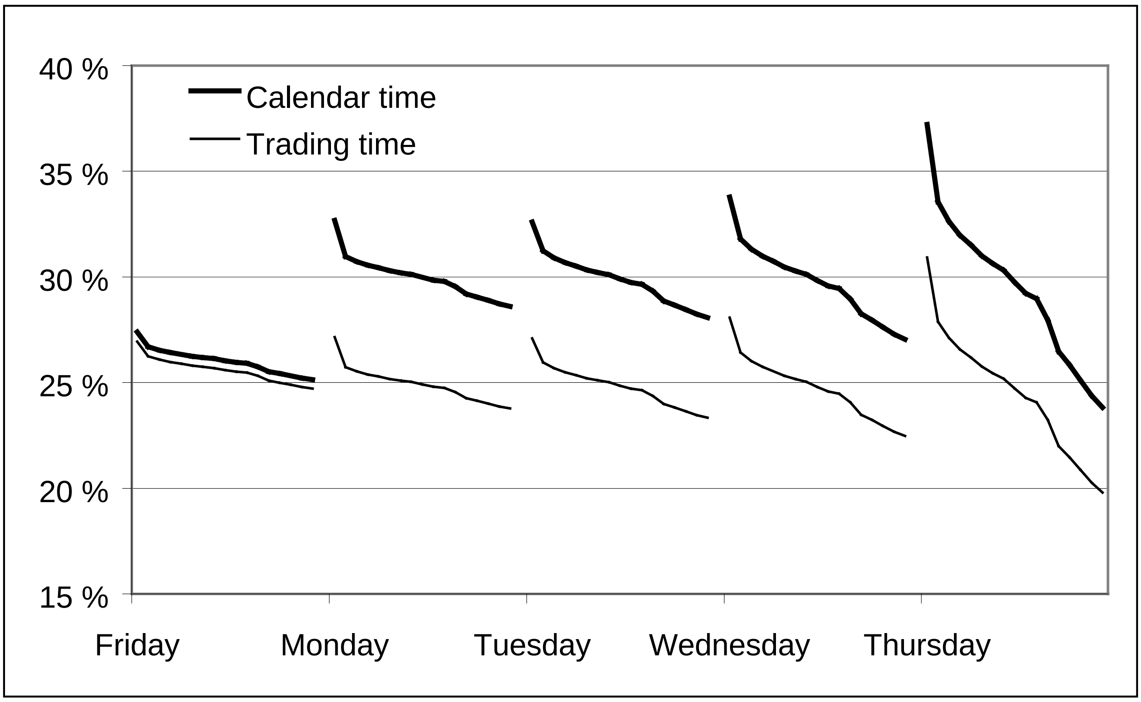

Figure 2
Simulated at-the-money call option implied volatilities close to expiration
The prices are taken from the CTDA-time model in Table 7 and implied volatilities are iterated for trading and calendar days. The underlying is at 6500. Volatility is 25.0 % for the CTDA-time model, interest rate is 3.5 % and always accrued in calendar time. Saturday and Sunday are the only non-trading days in this example. The last Friday is left out, as we in this case have zero time left on the expiration day. Time to expiration is as in Table 5, but with 0.43 subtracted from the CTDA-days to adjust for the expiration of the DAX options at 1:00 PM instead of at 5:30 PM.

In this study we demonstrate the model using 30-minute intervals. This corresponds to a trader with the opportunity to hedge his position in 30-minute intervals. The model can be used with other intervals as well and the results should not change significantly. However, with very short intervals the variance estimates will probably be affected by autocorrelation in the data. Another possible modification to the model is to account for the different days of the week, adjusting for any possible differences in volatility between weekdays as well. Approaching a methodology for predicting volatility, we could also insert dummies for news (mainly statistical reports) to be announced at pre-specified points in time.

6 Summary

This paper discusses briefly the fact that volatility is non-constant during the day and over weekdays. As presented in numerous studies, volatility is higher during trading, especially at the beginning, and at the end of the trading day. This implies that volatility is lower during holidays, but not necessarily negligible. Furthermore, the intraday volatility exhibit clear patterns. These two phenomena of volatility should be considered in option pricing as the cost of the option should be equal to the expected cost of hedging it. The expected cost of hedging in turn is proportional to the expected volatility.

The weekend volatility in excess of a normal night's volatility was estimated in section four. The reported holiday volatility is approximately 19 % of a whole trading day, which is of the same magnitude as in earlier studies. Furthermore, the intraday volatility was estimated in 30-minute intervals. As expected, the intraday volatility follows a U-shaped pattern with a spike around the point of time when important statistics in general are announced.

Observing the patterns in volatility, a brief discussion is provided on the implications in option pricing. The implications are demonstrated using a model that accounts for the reported patterns of volatility intradaily and daily. The correction for the patterns in volatility is conducted by adjusting the rate at which time is ticking. Time is allowed to tick faster at the beginning and end of the day, while time is ticking very slowly during weekends and holidays.

The model developed in this paper should eliminate the weekly patterns observed in the implied volatility when using calendar or trading days as time basis. Furthermore, the model adjusts for the intraday seasonal volatility, obviously altering the time value but also the other greeks. Whether options are traded in accordance with the model remains to be tested on transactions data. French (1984) found that options are traded on a trading time basis with interest accrued on a calendar time basis. If this still holds, one would expect this model not to correspond to market prices. However, the market efficiency may have improved during the last two decades and actual option prices may well correspond to volatility intradaily and over days.

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Appendix

Table A1
Simulated out-of-the-money call option prices close to expiration

	Friday	Monday	Tuesday	Wednesday	Thursday	Friday
Calendar days:	14.13	4.24	1.92	0.46	0.01	-
Trading days:	15.01	9.71	5.15	1.71	0.11	-
CTDA-days:						
9:00 AM	17.93	12.63	7.02	3.14	0.65	0.00
9:30 AM	16.39	10.26	5.78	2.23	0.28	0.00
10:00 AM	16.04	9.94	5.50	2.04	0.22	0.00
10:30 AM	15.79	9.71	5.31	1.91	0.18	0.00
11:00 AM	15.62	9.55	5.17	1.82	0.16	0.00
11:30 AM	15.43	9.38	5.03	1.73	0.13	0.00
12:00 PM	15.29	9.26	4.93	1.66	0.12	0.00
12:30 PM	15.18	9.16	4.84	1.61	0.11	0.00
1:00 PM	14.98	8.98	4.69	1.51	0.09	-
1:30 PM	14.80	8.82	4.56	1.43	0.07	-
2:00 PM	14.72	8.74	4.50	1.39	0.06	-
2:30 PM	14.39	8.44	4.26	1.24	0.04	-
3:00 PM	13.92	8.03	3.92	1.05	0.02	-
3:30 PM	13.73	7.86	3.79	0.98	0.02	-
4:00 PM	13.52	7.67	3.64	0.90	0.01	-
4:30 PM	13.31	7.49	3.50	0.82	0.01	-
5:00 PM	13.16	7.36	3.39	0.77	0.01	-

The simulated prices during the last six days before expiration are out-of-the-money call options. The strike is 5 % out-of-the-money with the underlying at 6500. The strike is rounded to the nearest multiple of 50 and is 6850 for this call. Volatility is 30.0 %, interest rate is 3.5 % and always accrued in calendar time. Saturday and Sunday are the only non-trading days in this example. Time to expiration is as in Table 5, but with 0.43 subtracted from the CTDA-days to adjust for the expiration of the DAX options at 1:00 PM instead of at 5:30 PM.

Table A2
Simulated out-of-the-money put option prices close to expiration

	Friday	Monday	Tuesday	Wednesday	Thursday	Friday
Calendar days:	16,26	5,62	2,81	0,82	0,04	-
Trading days:	17,19	11,87	6,78	2,58	0,24	-
CTDA-days:						
9:00 AM	20.27	15.08	8.95	4.42	1.13	0.01
9:30 AM	18.65	12.48	7.51	3.27	0.54	0.00
10:00 AM	18.28	12.12	7.19	3.02	0.44	0.00
10:30 AM	18.02	11.87	6.96	2.85	0.37	0.00
11:00 AM	17.83	11.69	6.80	2.73	0.33	0.00
11:30 AM	17.63	11.50	6.63	2.60	0.29	0.00
12:00 PM	17.49	11.37	6.51	2.52	0.26	0.00
12:30 PM	17.37	11.25	6.41	2.44	0.24	0.00
1:00 PM	17.16	11.05	6.24	2.32	0.20	-
1:30 PM	16.97	10.87	6.08	2.20	0.17	-
2:00 PM	16.88	10.79	6.00	2.15	0.16	-
2:30 PM	16.53	10.46	5.71	1.95	0.11	-
3:00 PM	16.03	9.99	5.31	1.68	0.06	-
3:30 PM	15.83	9.80	5.15	1.58	0.05	-
4:00 PM	15.61	9.59	4.97	1.46	0.03	-
4:30 PM	15.39	9.38	4.80	1.36	0.02	-
5:00 PM	15.22	9.23	4.67	1.28	0.02	-

The simulated prices during the last six days before expiration are out-of-the-money put options. The strike is 5 % out-of-the-money with the underlying at 6500. The strike is rounded to the nearest multiple of 50 and is 6200 for this put. Volatility is 30.0 %, interest rate is 3.5 % and always accrued in calendar time. Saturday and Sunday are the only non-trading days in this example. Time to expiration is as in Table 5, but with 0.43 subtracted from the CTDA-days to adjust for the expiration of the DAX options at 1:00 PM instead of at 5:30 PM.