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UNREALIZED EXPECTATIONS OF JUMPS IN VOLATILITY: AN EXPLANATION TO THE LOW AND TIME-VARYING PREDICTIVE POWER OF IMPLIED VOLATILITY

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Unrealized expectations of jumps in volatility: An explanation to the low and time-varying predictive power of implied volatility

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Abstract

The low predictive power of implied volatility in forecasting the subsequently realized volatility is a well-documented empirical puzzle. As suggested by e.g. Feinstein (1989), Jackwerth and Rubinstein (1996), and Bates (1997), we test whether unrealized expectations of jumps in volatility could explain this phenomenon. Our findings show that expectations of infrequently occurring jumps in volatility are indeed priced in implied volatility. This has two important consequences. First, implied volatility is actually expected to exceed realized volatility over long periods of time only to be greatly less than realized volatility during infrequently occurring periods of very high volatility. Second, the slope coefficient in the classic forecasting regression of realized volatility on implied volatility is very sensitive to the discrepancy between ex ante expected and ex post realized jump frequencies. If the in-sample frequency of positive volatility jumps is lower than ex ante assessed by the market, the classic regression test tends to reject the hypothesis of informational efficiency even if markets are informationally effective.

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Due to the central role the expected volatility of future returns plays in asset and derivatives pricing, risk management, financial market regulation, and even monetary policy, good estimates of future volatility are important. Since the volatility implicit in options prices is widely recognized as the market's volatility forecast (e.g. Schwert, 1990), we would expect it to be a good forecast of future volatility. Hence, it is rather surprising that the informational efficiency of implied volatility in predicting the subsequent realized volatility has been a subject of great controversy.

In a growing body of literature, the informational efficiency of implied volatility as a forecast of future volatility is tested by regressing the ex post realized volatility on the ex ante volatility implicit in options prices. More formally, as proposed by Fair and Shiller (1989):

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{implied} + \varepsilon_t \tag{1}$$

where $\sigma_t^{realized}$ is the volatility realized in period t, $\sigma_t^{implied}$ is the implied volatility at the beginning of period t for options expiring at the end of period t, and ε_t is an error term with the common zero mean and i.i.d. assumptions. Under the joint hypothesis that options markets are informationally efficient and the test procedures are correct, we would expect the estimates of the constant and the coefficient of the implied volatility not to differ significantly from zero and one, respectively. The extant empirical findings, however, do not confirm this.

In most regression-based studies on the forecasting power of implied volatility, the slope coefficient of implied volatility is estimated to be significantly below the a priori expectation of unity. This empirical result seems to hold across different markets. Chiras and Manaster (1978), Beckers (1981), Christensen and Prabhala (1998), and Fleming (1998) come to this conclusion using stock market data, while Scott and Tucker (1989) and Jorion (1995) reach the same conclusion with data from the foreign exchange markets. Similarly, Amin and Ng (1997) do the same empirical finding in the short-term fixed income markets. For an excellent and more detailed review of the recent literature on the subject, refer to Poon and Granger (2000). Assuming that the test methodologies used are correct, this empirical finding suggests that implied volatility be a biased estimate of future volatility.

Even more surprisingly, the results of Canina and Figlewski (1993) suggest that implied volatility have almost no predictive power in forecasting future volatility while Lamoreux and Lastrapes (1993) find that historical volatility contains predictive information over and above that of implied volatilities and that implied volatility is a biased estimate of realized volatility.

Rather than rejecting the hypothesis that options markets are efficient, most researchers tend to contribute the low predictive power of implied volatility in empirical tests to problems in the test procedures. As all efficiency tests, tests of the predictive power of implied volatility are joint tests of the research hypothesis per se, in this case the informational efficiency of the options markets, and the test methodology.

The possible sources of problems in the empirical testing of the predictive power of implied volatility can be classified into four main groups:

1. The model used to retrieve expectations of future volatility from options prices may be different from the model market participants use to price the options.
2. The price data used in the tests are likely to be measured with error.
3. Due to the mean-reverting nature of volatility, the relatively short periods for which we have data on options prices may induce statistical problems in estimating the relationship between implied and realized volatility.
4. Rational but unrealized expectations of infrequently occurring jumps in volatility may affect the relationship between the ex ante implied and ex post realized volatility causing it, although being completely rational, to *appear* irrational even in relatively large samples.

First, although it may seem conceptually questionable to test the forecast quality of the implied volatility calculated with a model that assumes constant volatility, as the Black-Scholes (1973) and Merton (1973) family models do, it has been shown that the pricing errors caused by valuing options with a Black-Scholes-Merton class model when volatility actually is stochastic are very small for nearby at-the-money options (Lamoreux and Lastrapes, 1993; Fleming, 1998). Since many studies have been conducted using at-the-money options with short maturities, the inability to establish a clear relationship between implied and realized volatility is probably not caused by the choice of a wrong model.

Second, the problems caused by measuring the price data with error are considered to be a more serious empirical problem in explaining the anomalously weak relationship between realized and implied volatility than that of using the wrong pricing model. Even if the options pricing model used in the tests were exactly the same as the model used by the market, it would only give a single theoretically correct options price for any set of given parameter values. However, in practice market frictions, such as the bid-ask spread, make arbitrage costly, which creates bounds around the "true" arbitrage-free price instead of giving a single price (Figlewski, 1989). Since the price quoted in some of the data sets used can be the bid, ask or a price there between, the price that reflects the true market conditions may be measured with an error. Similarly, due to the censoring of price observations which violate options price bounds, option prices which are biased up by noise are more likely to be accepted to the sample than prices that are biased down (Canina and Figlewski, 1993). In addition, the use of non-synchronous data, especially in the case of index options, worsens the measurement error problem. However, in a study of options on currency futures that are traded side-by-side with the underlying futures contracts and accounting for measurement errors of the size of the bid-ask spread and other statistical problems, Jorion (1995) concludes that the bias in the regression of the subsequently realized volatility on implied volatility cannot be explained by measurement errors of reasonable size. In more recent papers, Poteshman (2000) and Blair et al. (2001) show that a part of the bias can be eliminated by estimating the realized volatility from high-frequency data instead of using end-of-day data.

Third, since volatility is a highly autocorrelated process, the slope coefficient may be biased downward in small samples (Dickey and Fuller, 1979). As shown in Jorion (1995), though, such a bias would hardly lower the estimate from the true slope of unity by more than 0.1 to an estimated value lower than 0.9 and could, thus, not explain the much lower slope coefficient estimates in most previous studies.

To the best of our knowledge, this paper presents the first empirical evidence on the fourth hypothesis that the relationship between implied and realized volatility be affected by expectations of a positive jump in volatility. Since the first three sources of problems do not provide a sufficient explanation to the weak empirical relationship between the realized and implied volatilities, the conclusion that can be drawn from the extant body of empirical evidence is that implied volatility seems to be an upward biased estimate of the subsequently realized volatility. In addition, the results seem to be strongly sample-dependent (Poon and Granger, 2000).

In this paper, we present an explanation to this puzzle of a weak and time-varying empirical relationship between the implied and subsequently realized volatility. As suggested by e.g. Feinstein (1989), Jackwerth and Rubinstein (1996), and Bates (1997), we argue that the volatility implicit in options prices is affected by expectations of a positive jump in the level of volatility. If the market's ex ante assessment of the probability of a positive jump in volatility, caused for example by an exogenous shock such as a foreign financial crisis, is higher than the ex post realized frequency over a certain period, then the implied volatility will erroneously appear to be an upward biased estimator of the realized volatility in this period. Since volatility jumps caused by financial crises are rare events, we would actually expect implied volatility to slightly overestimate the subsequently realized volatility during long periods of low realized volatility and dramatically underestimate it in periods when a financial crisis does occur. Under the proposed hypothesis, the ex post forecast errors differ systematically from zero but the unconditional expected forecast error is zero. Thus, implied volatility remains an unbiased estimator of future volatility. In addition, since the volatility jump expectations vary strongly over time, noise is induced to the empirical relationship between the realized and implied volatilities. This novel source of measurement error combined with the low realized frequency of volatility jumps in samples used in many previous studies, in some of which the jumps are even deliberately omitted as outliers, is the likely cause to the slope coefficient being estimated to be below the a priori expectation of unity.

Using a parsimonious two-regime framework, we first show theoretically that implied volatility can be seen as a weighted average of the expected volatilities in two regimes, low and high volatility, with the assessed probabilities of occurrence as weights. Then, with the help of Monte Carlo simulations, we show that time-varying expectations of a jump in volatility affect the relationship between realized and implied volatility so that the slope coefficient in the forecasting regression is, indeed, biased downward. The size of this bias is found to be positively related to the magnitude of the peso problem the sample is subject to, i.e. how low the realized ex post frequency of jumps is compared to the ex ante assessed jump probability. This effect is verified by inducing a peso problem into the forecasting regression on actual financial market data. Then, using a relatively large sample of S&P 500 stock index futures and futures options data, we test our hypothesis empirically by regressing the ex post forecast error conditional on no jump in volatility on a constant and a measure of ex ante expectations of a jump in volatility. The empirical results provide strong support for the proposed hypothesis.

The results presented in this paper have important implications to asset and derivatives pricing, risk management, financial market regulation, and even monetary policy to which it is essential to understand how volatility, as such, and the relationship between implied and realized volatility, in particular, develop over time. More specifically, this paper provides an explanation to the anomalously low and strongly time-varying predictive power of implied volatility found in many previous studies. In addition, since implied volatility has previously been seen as a smoothed expectation of the subsequently realized volatility, the empirical finding that implied volatility is more volatile than the realized volatility in relatively large samples has been considered to be anomalous (Christensen and Prabhala, 1998). Our theoretical model and empirical findings are consistent with and can, thus, also help to explain this phenomenon. Furthermore, our finding that jumps constitute an essential part of the development of volatility over time support the inclusion of jumps in the process used to model the volatility process rather than modeling volatility as a diffusion as e.g. in Hull and White (1987) and Stein and Stein (1991).

The rest of the paper is structured as follows. In section I, we describe the theory behind our hypothesis and come up with testable implications. Our data sample is described in section II. The empirical results are presented in section III. Section IV provides a summary and a conclusion.

I. Hypothesis and Method

A. Theory

With limited upside potential and theoretically unlimited downside risk, writing options is a highly risky business. One source of this risk is the time-varying nature of volatility. The “true” volatility, commonly measured as the realized variability, of financial prices tends to remain on a relatively low level for long periods only to suddenly rise to a much higher level during market turmoil. We would expect rational investors to take the dynamic behavior of volatility into consideration when pricing assets and derivative products the prices of which are sensitive to the level of future volatility.

During periods of relatively low levels of volatility, there exists a small but positive probability that the market will be hit by an exogenous shock such as a foreign financial crisis that will cause the volatility to jump in the nearby future. The fair price of an option should reflect this risk. Such a situation can parsimoniously be modeled with a simple two-regime model [see Evans (1996) for an excellent review of the literature on the effects of unrealized expectations on financial time series]. Assuming market efficiency and applying the Krasker (1980) and Lizondo (1983) framework with two possible future regimes, the expected volatility[2] over a future period can be modeled as follows:

$$E_t\left(\sigma_{t,T}\right) = \left(1-\pi\right)\cdot E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot E_t\left(\sigma_{t,T}^{high}\right), \tag{2}$$

where E_t is the expectations operator conditional on all information available at time t, $\sigma_{t,T}$ is the realized volatility in the period from t through T, π is the ex ante probability of being in the high volatility regime between t and T as assessed by the market at t, $\sigma_{t,T}^{low}$ is the realized volatility in the period from t through T conditional on the occurrence of the low volatility regime, and $\sigma_{t,T}^{high}$ is the realized volatility in the period from t through T conditional on the occurrence of the high volatility regime. Thus, under the assumption of a positive ex ante assessed probability of the high volatility regime occurring, implied

[2] Since options prices are almost linear in volatility, it can, from the perspective of this paper, be considered equivalent to model the pricing process with options prices as such or with the volatilities implied by these

volatility can be interpreted as a weighted average of future volatility in two possible states, high and low volatility, with the regime probabilities as weights. Hamilton (1988) introduces a more elaborate model with regime switches but the underlying idea is the same as in the more parsimonius model presented above.

If we define the volatility in the high-volatility regime to equal the sum of the volatility in the low-volatility regime and a constant[3]

$$\sigma_{t,T}^{high} = \sigma_{t,T}^{low} + \delta \, , \tag{3}$$

where δ is a non-negative constant, then obviously

$$E_t\left(\sigma_{t,T}\right) = \left(1-\pi\right) \cdot E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot E_t\left(\sigma_{t,T}^{low} + \delta\right) \tag{4}$$

$$= E_t\left(\sigma_{t,T}^{low}\right) - \pi \cdot E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot \delta \tag{5}$$

$$\Leftrightarrow$$

$$E_t\left(\sigma_{t,T}\right) = E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot \delta \, . \tag{6}$$

Thus, the unconditional expected future volatility is the sum of the expected volatility in the low-volatility regime and a term that reflects expectations of a volatility jump, or more precisely, the product of the assessed jump probability and the magnitude of the jump.

Assuming that the market uses the unconditional expected volatility in (6) as the "implied" volatility to price options:

$$\sigma_{t,T}^{implied} = E_t\left(\sigma_{t,T}\right), \tag{7}$$

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prices. Since this paper concentrates on the behavior of volatility over time, we choose to model the process with volatilities.

[3] In light of the results of e.g. Eraker, Johannes and Polson (2000), assuming that the jump in the level of volatility to be constant can be seen as a reasonable realistic yet parsimonious way of modelling volatility jumps.

where $\sigma_{t,T}^{implied}$ is the volatility implicit in options prices at time t for options expiring at T, equation (6) can be rewritten as

$$\sigma_{t,T}^{implied} = E_t\left(\sigma_{t,T}^{low}\right) + \pi \cdot \delta \,. \tag{8}$$

If (i) options markets are informationally efficient, (ii) options are priced according to the two-regime model shown above, and (iii) the correctly assessed probability of a volatility jump is positive (π>0), then the relevant ex ante volatility implicit in option prices will systematically deviate from the realized volatility. This is because only one of the two possible regimes can occur in a single period. The implied volatility will systematically exceed the realized volatility in the low-volatility regime, while it will systematically be below the realized volatility in the high-volatility regime.

Since financial crises, by the very definition, are rare events, we would expect the relationship between the implied and subsequently realized volatilities to be characterized by long periods during which implied volatility exceeds the subsequently realized volatility by a relatively small amount and short periods during which the realized volatility greatly exceeds the volatility forecast implied by option prices. Assuming risk neutrality, the implied volatility, however, should equal the future volatility on average so that implied volatility remains an unbiased estimator of realized volatility.

It is the existence of the second product on the right-hand side of (8), $\pi \cdot \delta$, that may cause the suggested peso problem in the time-series behavior of implied volatility. Since this factor is of first order, it will also exist under risk-neutrality. Thus, it is not a common risk premium that could be considered compensation for bearing a more risky future cash flow but rather a risk-neutral adjustment that affects the mean of the expectation directly.

Whether volatility risk also has a second order effect depends on whether it is priced. If it can be hedged, either directly, e.g. in the OTC market for volatility swaps as suggested by Poon and Granger (2000), or indirectly by hedging the regime (Bollen et al., 2000), it is likely not to be priced. However, if volatility risk cannot be diversified away and it correlates with wealth, there may also exist a second order effect in the form of a common risk premium. Green and Figlewski (1999) suggest that writers of options would demand a volatility markup due to model risk, i.e. the uncertainty related to which options pricing model is the one used by the market. Another effect that may further strengthen this possible volatility markup is the empirical finding that underestimating the future volatility seems to lead to a considerable increase in the risk of an options portfolio hedged on a discrete basis whereas an overestimated volatility does not seem to increase hedge risk much at all (Figlewski, 1989). This would suggest that it is less costly to err on the side of overestimating than underestimating volatility for out-of-the-money options. Poteshman (2000) also finds evidence in support of the view that volatility risk is priced. In this paper, however, we concentrate on the first order effect only and leave the analysis of the effects of volatility risk being priced for future study.

B. Simulation

To see how jumps in volatility, both expected and unexpected, actually affect the point estimates of the constant and slope coefficient in the classic forecasting regression, we run Monte Carlo simulations on the empirical relationship between realized and implied volatility over 100 periods. We model the realized volatility as a mixture of an ARMA(1,1) process (Christensen and Prabhala, 1998) and infrequently occurring exogenous jumps in volatility to a pre-determined level. With the exception of the expected and realized jump frequencies, which are let to vary in the simulation, the parameter values are rounded approximations of the values estimated from the true realized volatility of the S&P 500 stock index futures pertaining to the monthly options expiration cycle between February 1991 and March 2000. The true volatility process is assumed to be fully known by market participants so that the implied volatility, when not subject to the peso problem, is an informationally efficient estimate of future realized volatility. However, to make the model more realistic, the assessed jump probability embedded in the implied volatility, as shown in (8), is set to vary randomly. Thus, we incorporate the effect of time-varying volatility jump expectations to the simulation by multiplying the expected jump probability with an error term that is symmetrically distributed around the mean of one. For simplicity and similarly to Jorion (1995), we let the error term be generated randomly from a uniform distribution between zero and one and multiply it by two to get a mean of one. Varying between zero and the double of the "true" probability, this level of variance over time can be considered realistic. See the Appendix for a more detailed description of the simulation procedures.

The simulation results are shown in Table I. From the perspective of this paper, the most interesting results are the coefficient estimates on and below the diagonals.

Table I

Simulation results on the effect of jump expectations on the forecasting regression coefficients

This table shows the effect expectations of a jump in the level of volatility has on the coefficient estimates in the forecasting regression:

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{implied} + \varepsilon_t$$

where $\sigma_t^{realized}$ is the realized volatility, $\sigma_t^{implied}$ is the implied volatility used to price options, and ε_t is the error term with the common i.i.d. assumptions. The values shown are the simulated estimates of the constant (above) and the slope coefficient of implied volatility (below):

$$\alpha$$
$$\beta$$

The realized volatility is simulated to follow a mixture of an ARMA(1,1) process and exogenous jumps to the level of 45%. With the exception of jump frequency (probability), which is let to vary, the parameter values are rounded approximations of the values estimated from the realized annualized monthly volatility of daily first log differences of the S&P 500 futures prices pertaining to the monthly options expiration cycle between February 1991 and March 2000. The true volatility process is assumed to be fully known by market participants so that the implied volatility, when not subject to a peso problem, is an unbiased estimate of future realized volatility. In Panel B, we incorporate the effect of time-varying volatility jump expectations to the simulation by multiplying the expected jump probability with an error term that is uniformly distributed around the mean of one. The results are based on 5000 regressions of simulated paths of realized volatility on implied volatility each consisting of 100 periods. See the Appendix for a more detailed description of the simulation procedures.

Panel A: Constant volatility jump fears

Expected jump frequency	Realized jump frequency						
	0	0.005	0.01	0.02	0.03	0.04	0.05
0	*-0.00405*	-0.00159	0.00080	0.00631	0.01072	0.01743	0.02200
	1.02691	1.02021	1.01436	0.99758	0.98699	0.96354	0.95408
0.005	-0.00638	*-0.00342*	-0.00064	0.00536	0.00963	0.01595	0.02042
	1.03207	*1.02243*	1.01413	0.99483	0.98516	0.96462	0.95451
0.01	-0.00872	-0.00604	*-0.00389*	0.00309	0.00870	0.01296	0.01852
	1.03728	1.02958	*1.02516*	0.99998	0.98275	0.97376	0.95626
0.02	-0.01348	-0.01135	-0.00711	*-0.00186*	0.00299	0.00824	0.01374
	1.04786	1.04380	1.02646	*1.01163*	0.99970	0.98486	0.96965
0.03	-0.01835	-0.01465	-0.0122	-0.00624	*-0.00091*	0.00559	0.00821

	1.05866	1.04491	1.03854	1.02034	*1.00591*	0.98464	0.98574
0.04	-0.02331	-0.02086	-0.0179	-0.0112	-0.00594	*0.00002*	0.00414
	1.06969	1.06383	1.05457	1.03171	1.01828	*0.99980*	0.99167
0.05	-0.02838	-0.02493	-0.02189	-0.01699	-0.01157	-0.00665	*0.00195*
	1.08095	1.06917	1.05969	1.04800	1.03321	1.02172	*0.98824*

Panel B: Time-varying volatility jump fears

Expected jump frequency	Realized jump frequency						
	0	0.005	0.01	0.02	0.03	0.04	0.05
0	*-0.00405*	-0.00125	0.00087	0.00659	0.01253	0.01631	0.02313
	1.02691	1.01787	1.01377	0.99620	0.97692	0.97088	0.94596
0.005	-0.00471	*-0.00211*	0.00032	0.00524	0.01185	0.01589	0.02164
	1.02109	*1.01392*	1.00729	0.99487	0.97086	0.96384	0.94723
0.01	-0.00210	0.00051	*0.00321*	0.00801	0.01224	0.01694	0.02409
	0.99400	0.98674	*0.97892*	0.96664	0.95944	0.94736	0.92210
0.02	0.01130	0.01345	0.01450	*0.01969*	0.02490	0.02918	0.03409
	0.88906	0.88517	0.88793	*0.87452*	0.86127	0.85233	0.84045
0.03	0.03118	0.03240	0.03333	0.03758	*0.04060*	0.04352	0.04730
	0.74718	0.74915	0.75214	0.74603	*0.74576*	0.74723	0.74292
0.04	0.05116	0.05291	0.05344	0.05547	0.05842	*0.06207*	0.06504
	0.61008	0.60872	0.61537	0.62223	0.62284	*0.62000*	0.62314
0.05	0.06900	0.06995	0.07110	0.07286	0.07517	0.07676	*0.07993*
	0.49070	0.49485	0.49709	0.50629	0.51052	0.52024	*0.51947*

On the diagonal, the expected and realized frequencies are equal, i.e. the estimates are informationally efficient, but the estimates below the diagonal are subject to a peso problem, i.e. the realized frequency of jumps in volatility is lower than the ex ante expected frequency. As can be seen in Panel A with results based on constant jump expectations, the higher the magnitude of the peso problem, the more downward biased is the estimate of the constant and the more upward biased is the estimate of the slope coefficient in an OLS regression. From Panel B with time-varying jump expectations, we can see that the opposite is true: the stronger the peso problem, the more the constant is biased upward and the slope coefficient biased downward. The directions of the biases, thus, seem to depend on how strongly the jump expectations vary over time. In light of our empirical results presented below, the level of the market's volatility jump fears seems to be rather strongly time-varying so that we can expect the slope coefficient to be downward biased in a sample on real data which is subject to a peso problem. In section III below, we show that this is, indeed, the case in our sample.

C. Test Methodology

A testable implication of our hypothesis follows from the relationship between the level of volatility jump fears and the ex post forecast error. Rewriting (8), we define the ex post forecast error in the low-volatility regime to be:

$$fe_T^{low} = \sigma_{t,T}^{implied} - E_t\left(\sigma_{t,T}^{low}\right) = \pi \cdot \delta \tag{9}$$

where, fe_T^{low} is the forecast error for the period from t through T conditional on the occurrence of the low-volatility regime and the other variables are as defined earlier. Thus, the ex post realized forecast in the low-volatility regime is simply the difference between the relevant implied and realized volatilities, i.e. the product of the jump probability and the size of the jump. Hence, this conditional forecast error should vary positively with the product of the assessed probability of occurrence and magnitude of a jump in volatility. Since volatility shocks tend to follow negative rather than positive shocks to the value of the underlying asset in the stock market and our sample consists of options on the S&P 500 stock index futures contract, we use the skewness premium introduced by Bates (1991) as a proxy for fears of a jump in volatility:

$$skew_{t,T} \equiv \frac{C_t\left(F_t, T, X_c\right)}{P_t\left(F_t, T, X_p\right)} - 1 \tag{10}$$

where $skew_{t,T}$ is the skewness premium at time t for options maturing at T, C_t and P_t are the prices of call and put options as functions of F_t, the futures price at t, *T*, the maturity of the option, and the strike prices X_c and X_p, respectively. The strike prices of both options are defined to be x percent out-of-the money (x>0) and spaced geometrically around the price of the underlying futures contract in the following way:

$$X_p = \frac{F}{1+x} < F < F(1+x) = X_c \tag{11}$$

As described in Bates (1991), the skewness premium can be used as a diagnostic of the symmetry or skewness in the risk neutral distribution implicit in options prices. Theoretically, negative skewness in the risk neutral distribution reflects either the existence of crash fears *or* that volatility is expected to rise if the market falls. Since unexpected increases in volatility tend to be associated with negative stock returns (French et al., 1987, and Low, 2000)[4], we think this is a good proxy for the markets' expectations of a positive jump in volatility, i.e. the second product on the right-hand side of (8). To test our hypothesis, we regress the ex post forecast errors for periods of low volatility on a constant and Bates' skewness premium:

$$fe_T^{low} = \alpha + \beta \cdot skew_{t,T} + \varepsilon_T \, , \tag{12}$$

where $skew_{t,T}$ is Bates' skewness premium at time t for options maturing at T, and ε_T is an error term with the common zero mean and i.i.d. assumptions. Since the skewness premium is inversely related to the level of volatility jump fears, our working hypothesis suggests that β be negative. To alleviate the possible problem of heteroscedasticity that could be induced if we used the absolute difference between the implied volatility and realized volatility directly, we take the natural logarithm of their ratio which is, of course, equal to taking the first log difference of the variables. As a robustness test and to estimate the relative call/put price elasticity of the conditional forecast error, we also run the regression with the first log difference of the call and put prices, instead of Bates' skewness premium, as the independent variable.

[4] Schwert (1990) actually estimates that stock market volatility tends to increase by a factor of 2.5 after a negative shock compared to after a positive shock.

To calculate the forecast errors conditional on being in the low volatility regime, we need to differentiate between the low and high volatility regimes. One way to do this is to run regression (12) for different samples, starting with the whole sample and then systematically deleting the data for the period with the highest change in realized volatility. This way, we do not have to impose any priors regarding what size of a change in volatility constitutes a jump.

II. Data

We test our hypothesis using daily data, provided by the Futures Industry Association, on the American futures options on the liquid S&P 500 futures contract traded in monthly and quarterly cycles on the Chicago Mercantile Exchange. Since the options are traded side by side with the underlying futures contract in the same market on a cash settlement basis, arbitrage should be relatively easy and cheap in this market compared to e.g. the S&P 100 stock index options market. In addition, since the daily settlement prices of these options are set and checked for clerical errors by the exchange, a neutral counterpart between the trade participants, and the daily marking to market is conducted using these very same prices, we believe that they reflect true market conditions very well. In comparison to the data sets used in several prior studies, with Jorion (1995) among the few exceptions, our sample can be regarded to have a very low level of measurement error.

Since the volatility implicit in option prices is generally viewed as the market's forecast of the future volatility over the remaining life of the relevant option, we match the return horizon with the life of the option. We calculate the realized volatility as the standard deviation of the daily first log differences of the relevant futures price over the relevant period[5]. As suggested by French (1984), we use the square root of the average number of trading days in a year, 252 in our sample, instead of the number of calendar days, to annualize the volatilities. Since June 1987, the quarterly options expire on the third Thursday of the month, while the options of the monthly cycle expire on the third Friday of the month. For the quarterly cycle, we use the implied volatility on the first business day after the expiry of the previous option in the cycle as the market's forecast for the volatility up to the option's maturity. To estimate the variable values for the monthly cycle, which are much more sensitivity to the inclusion or exclusion of one single trading day than the estimates pertaining to the quarterly cycle, we use the period consisting of exactly 18 trading days prior to the expiration of the options. As suggested by Christensen and Prabhala (1998), we use non-overlapping data to avoid the possible problem of less precise and inconsistent estimates obtained from overlapping samples used in previous research.

As suggested in several previous studies, we believe that a Black-Scholes-Merton family option pricing formula, which assumes that volatility be constant, provides a good approximation of the market's subjective implied volatility even when the true volatility follows a stochastic process. We use at-the-money volatilities to estimate the market's implied volatility for two reasons. First, the at-the-money options contain most information about volatility, i.e. they are the most sensitive options to changes in volatility (e.g. Harvey and Whaley, 1991). And second, the bias caused by the non-linearity in volatility of the Black-Scholes-Merton family option pricing formulae is minimized for at-the-money options (Bodie and Merton, 1995). Beckers (1981) actually shows that using only at-the-money volatilities is preferable to various other weighting schemes. In addition, Ederington and Guan (2000) find it of little importance what kind of a weighting scheme is issued in calculating the implied volatility from the nearest-to-the-money options. When no available strike price exactly equals the futures price, we calculate the at-the-money implied volatility as the arithmetic average of the volatilities implied by the two calls and two puts with striking prices closest to but on different sides around the underlying using the Black (1976) formula**.** As shown in Whaley (1986), the pricing errors caused by using the Black (1976) formula for European options on futures to price American options on futures are small for at-the-money and out-of-the-money options.

We use options prices that are 4% out-of-the-money to calculate the skewness premium for quarterly options. For monthly options we use strikes that are 2% out-of-the-money due to a lower number of out-of-the-money strike classes available closer to maturity. Since options exist only for specific strike prices, we interpolate the relevant

[5] The potential Jensen's inequality type bias caused by taking the square root, i.e. making a non-linear transformation, of the variance, which is the Black-Scholes measure of volatility, is considered to be

option prices fitting a constrained cubic spline through observed option price/future price ratios as a function of observed strike price/futures price ratios (see Appendix A in Bates (2000) for detailed information on the calculation of these constrained cubic splines). Similarly to Bates (1991, 1996, 2000), we require that prices exist for at least four call strikes and four put strikes. As noted above, however, we do not use the estimated cubic splines to estimate the implied at-the-money volatilities. This is to avoid inducing a possible spurious empirical relationship between the at-the-money volatility and the skewness premium.

Given the conditions described above, we end up with a sample that covers options expiring in the period from June 1990 through March 2000 for the quarterly cycle and from February 1991 through 2000 for monthly options. The fact that the series are continuous and no periods are missing between the beginning and end of the series is coincidental and likely due to the growth in trading activity over time.

We proxy the risk-free rate by the interest rate paid on U.S. T-bills over the relevant period, i.e. one-month and three-month maturities, provided by the Federal Board of Governors. For missing values, the previous available value is used.

III. Empirical Results

A. Sample Analysis

Since we are studying a sample-dependent phenomenon, it is important to carefully analyze the sample used in the empirical tests. Our sample consists of 40 and 110 non-overlapping observations of each variable for the quarterly and monthly cycles, respectively. As can be seen from Table II, the distributions of both the realized and implied volatilities over both monthly and quarterly intervals are, as expected, positively skewed. Figures 1 and 2 confirm this visually for the realized volatility. The Bera-Jarque test rejects the hypothesis of a normal distribution at a very high confidence. The range of realized volatility is larger for monthly data than for quarterly data. This is expected due to the mean-reverting nature of volatility.

Visual inspection of the time-series behavior of the implied and realized volatilities supports our hypothesis. As shown in Figures 3 and 4, the relevant implied volatility seems to be above the realized volatility during periods of low or downward-

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negligible.

drifting volatility. However, as predicted by our hypothesis, when volatility experiences a positive shock, realized volatility seems to be higher than the relevant implied volatility. Although perhaps clearer on quarterly data, this pattern seems to apply to data on both intervals used. In our sample, implied volatility is on average more than two percentage points higher than realized volatility. We can reject the hypothesis that the forecast error, i.e. the difference between the relevant implied and realized volatilities, is zero in a two-sided *t*-test with *t*-values of 3.68 and 5.14 for quarterly and monthly data, respectively.

Table II

Descriptive statistics

Panel A: Quarterly data
(N=40)

	Realized volatility	Implied volatility	Skewness premium (4%)	Forecast error (implied volatility-realized volatility)
Mean	0.1451	0.1659	-0.3146	0.0208
Median	0.1316	0.1522	-0.3042	0.0275
Minimum	0.0701	0.1054	-0.5645	-0.1015
Maximum	0.3116	0.3372	-0.1067	0.0959
Standard deviation	0.0574	0.0514	0.1225	0.0357
Skewness	0.9121	1.1381	-0.1817	-1.1255
Excess kurtosis	0.5085	1.5168	-0.8197	2.7485
Bera-Jarque	16.43***	11.25***	23.11***	7.61**

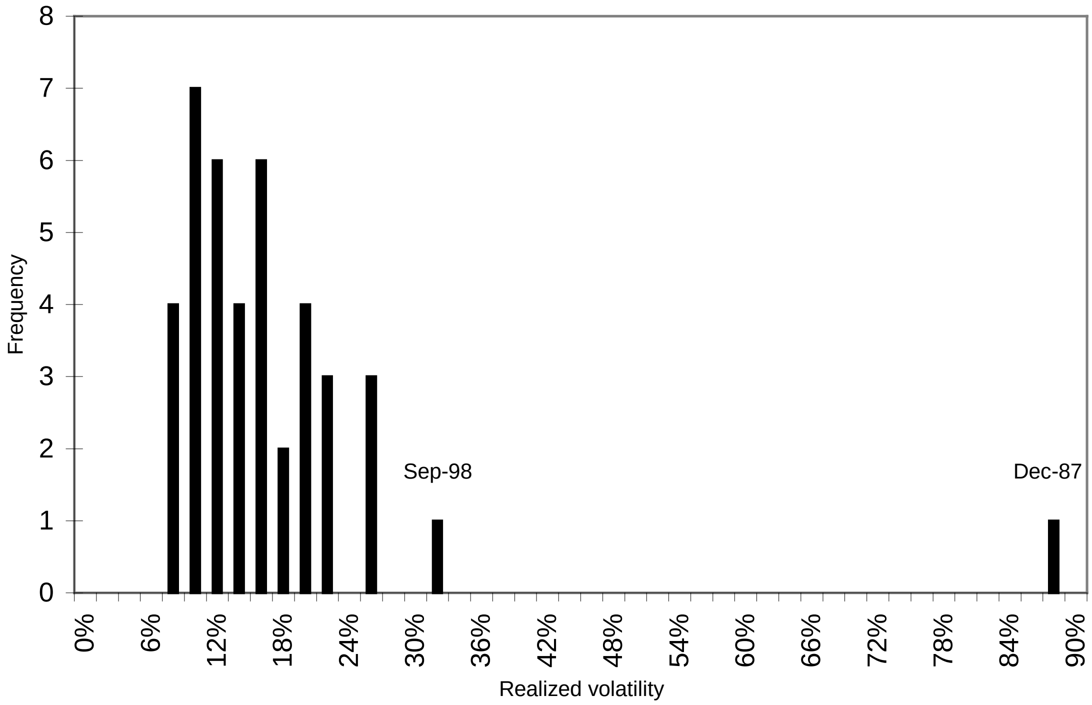
Panel B: Monthy data
(N=110)

	Realized volatility	Implied volatility	Skewness premium (2%)	Forecast error (implied volatility-realized volatility)
Mean	0.1360	0.1592	-0.2760	0.0232
Median	0.1189	0.1477	-0.2675	0.0284
Minimum	0.0453	0.0859	-0.8562	-0.2532
Maximum	0.5206	0.3184	0.6347	0.1338
Standard deviation	0.0681	0.0506	0.2101	0.0473
Skewness	2.2064	0.7974	0.6349	-1.8518
Excess kurtosis	8.9085	0.0133	2.8779	9.6228
Bera-Jarque	200.46***	55.50***	11.71**	234.98***

** and *** denote significance levels of 5 and 1 percent, respectively.

This would imply that either the realized frequency of jumps does not equal the ex ante assessed frequency, in the risk-neutral case, or that volatility risk is, indeed, priced. Since risk aversion would only magnify the effects of the proposed hypothesis, it is, from the perspective of this paper, irrelevant whether volatility risk is priced. As shown in Figures 5 and 6, the forecast error, i.e. the difference between the implied and realized volatilities seems to revert quickly to positive territory when a jump in volatility has caused it to fall below zero. Measured by the standard deviation of the volatility series, realized volatility is more volatile than implied volatility in our sample.

Figure 1. Frequency histogram of the realized quarterly volatility of the S&P 500 stock index futures contract, June 1990 - March 2000 and December 1987. The volatility is measured as the standard deviation of the daily first log differences of the futures price in the quarterly expiration cycle, from the second trading day after the expiration of the options in the previous quarter through the expiration of the options in the quarter in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year.



From Figures 3 and 4 it can also be seen that our sample covers periods of both high and low volatility. To compare our sample with the period immediately around the stock market crash of 1987, the relevant realized volatilities in the last quarter of 1987 are included in Figures 1 and 2. Although the annualized realized volatility of 87.7% in the last quarter of 1987 is clearly higher than the highest realized volatility in our quarterly sample, the quarter ending in September 1998, the magnitude of the financial crisis of 1998 can, when viewed from the perspective of monthly volatility, be compared to that of the 1987 crash.

As shown in Figure 7, the skewness premium is negative for the whole sample period with a few exceptions on monthly data. The 4% skewness premium estimated on quarterly data gets values between –0.5645 and –0.1067. These estimates are very similar to those obtained by Bates (2000). The statistics in Table II and visual inspection of Figure 7 confirm our belief that volatility jump expectations vary strongly over time

Figure 2. Frequency histogram of the realized monthly volatility of the S&P 500 stock index futures contract, February 1991 - March 2000 and October, November and December 1987. The volatility is measured as the standard deviation of the daily first log differences of the futures price from eighteen trading days prior to through the expiration of the options in the month in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year.

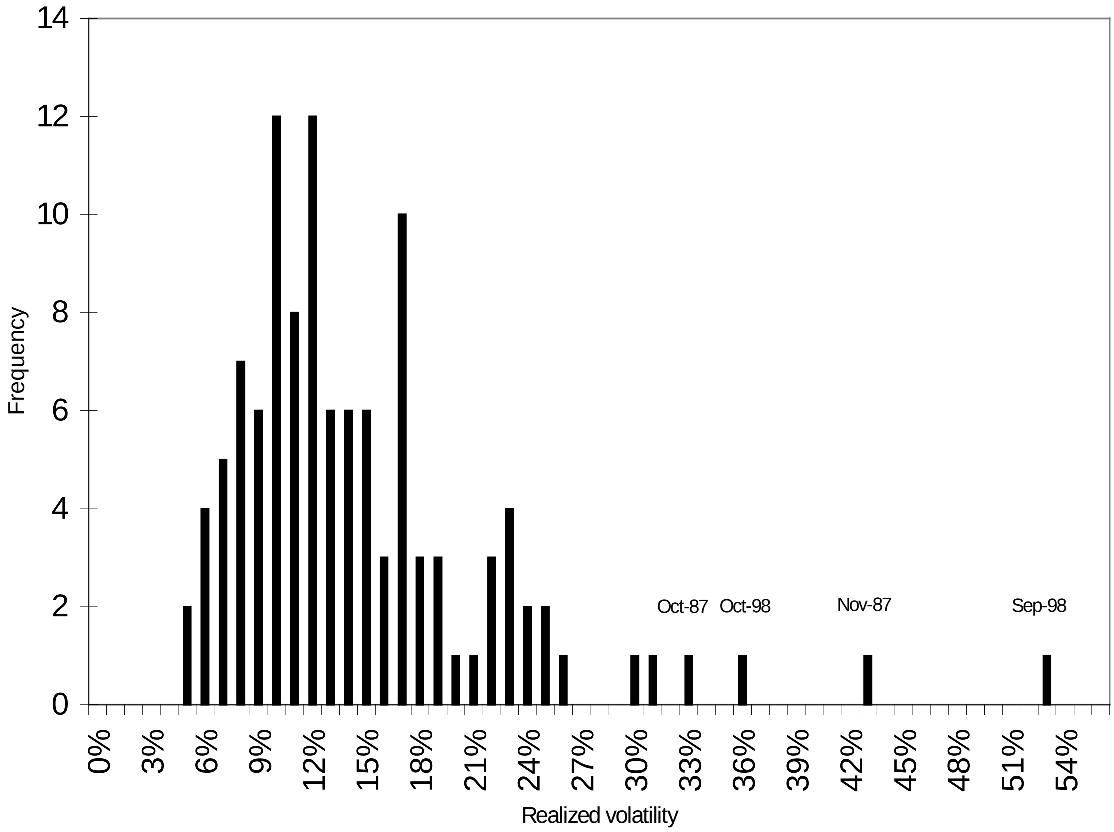


Figure 3. The quarterly implied and realized volatility of the S&P 500 stock index futures contract, June 1990 - March 2000. The implied volatility is estimated as the average volatility implicit in two closest-to-the-money calls and two closest-to-the-money puts on the first trading day after the expiration of the options in the previous quarter using the Black (1976) formula. The realized volatility is measured as the standard deviation of the daily first log differences of the futures price in the quarterly expiration cycle from the second trading day after the expiration of the options in the previous quarter through the expiration of the options in the quarter in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year.

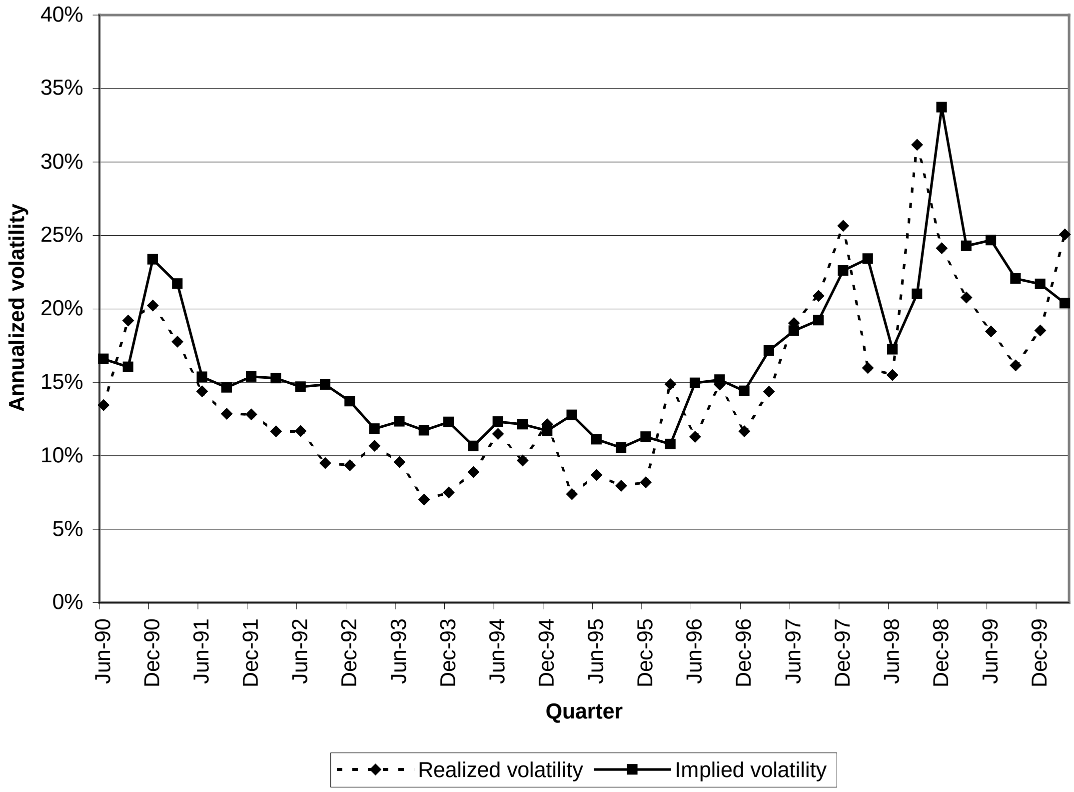


Figure 4. The monthly implied and realized volatility of the S&P 500 stock index futures contract, February 1991 - March 2000. The implied volatility is estimated as the average volatility implicit in two closest-to-the-money calls and two closest-to-the-money puts nineteen trading days prior to the expiration of the options using the Black (1976) formula. The realized volatility is measured as the standard deviation of the daily first log differences of the futures price from eighteen trading days prior to through the expiration of the options in the month in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year.

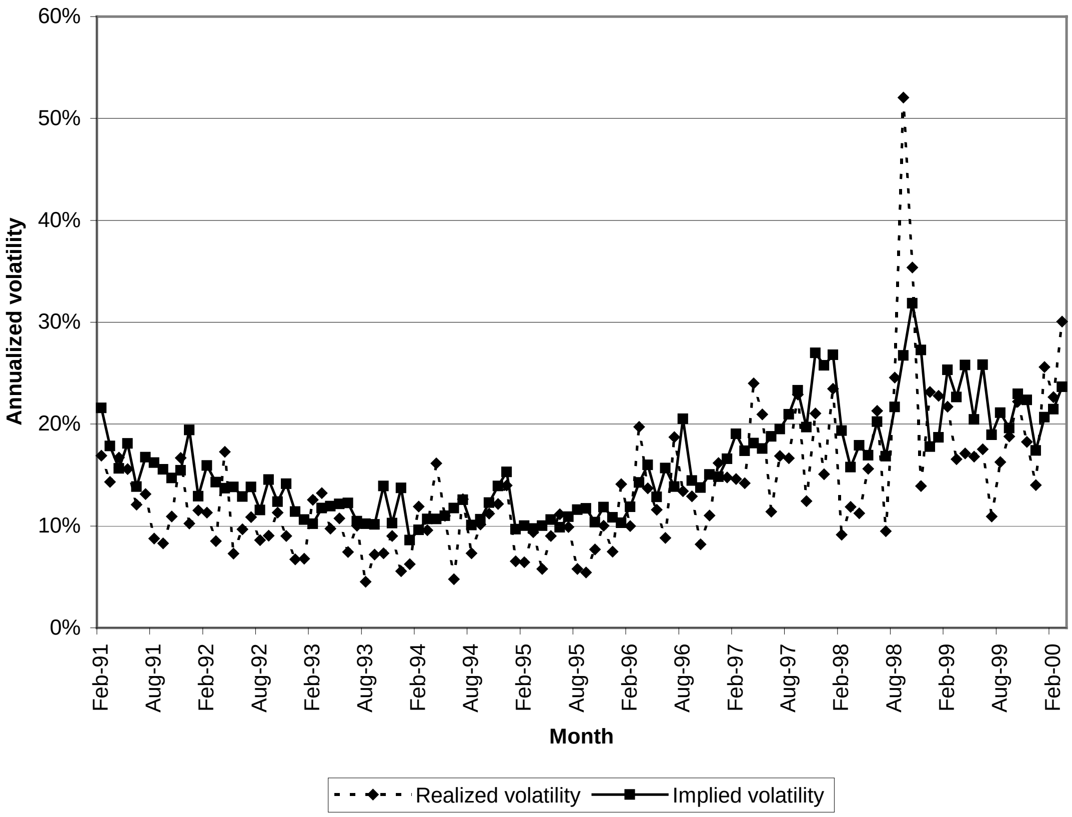


Figure 5. The quarterly forecast error of implied volatility in predicting the realized volatility of the S&P 500 stock index futures contract, June 1990 - March 2000. The forecast error is calculated as the difference between the relevant implied and realized volatilities. The implied volatility is estimated as the average volatility implicit in two closest-to-the-money calls and two closest-to-the-money puts on the first trading day after the expiration of the options in the previous quarter using the Black (1976) formula. The realized volatility is measured as the standard deviation of the daily first log differences of the futures price in the quarterly expiration cycle from the second trading day after the expiration of the options in the previous quarter through the expiration of the options in the quarter in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year.

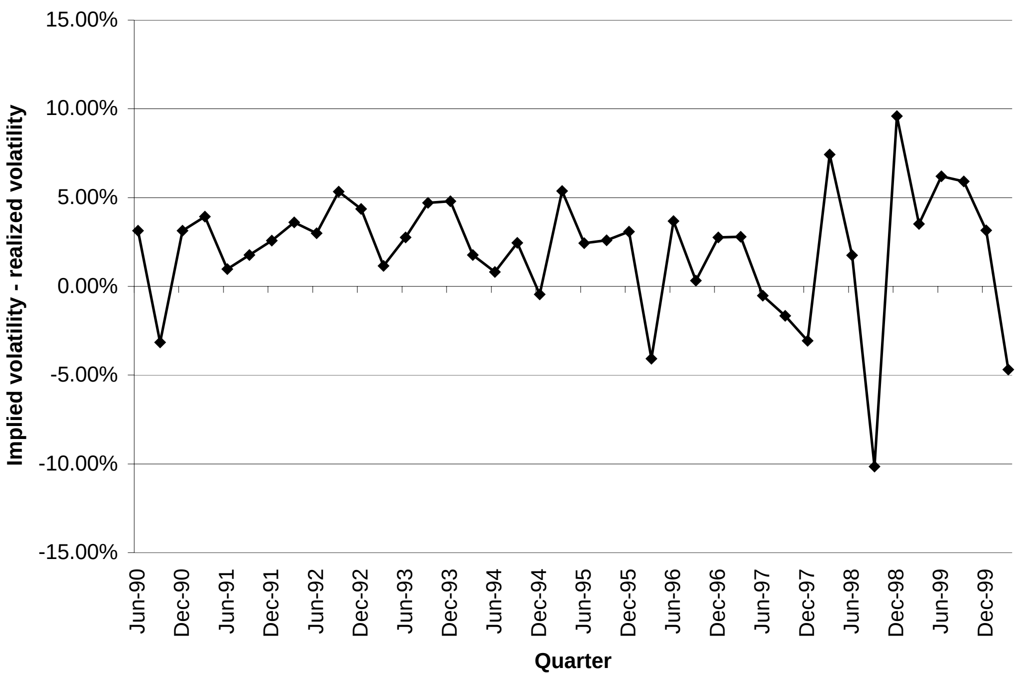


Figure 6. The monthly forecast error of implied volatility in predicting the realized volatility of the S&P 500 stock index futures contract. The forecast error is calculated as the difference between the relevant implied and realized volatilities. The implied volatility is estimated as the average volatility implicit in two closest-to-the-money calls and two closest-to-the-money puts nineteen trading days prior to the expiration of the options using the Black (1976) formula. The realized volatility is measured as the standard deviation of the daily first log differences of the futures price from eighteen trading days prior to through the expiration of the options in the month in question. The volatility is annualized by multiplying the standard deviation with the square root of 252, the average number of trading days in a year. The figure covers the months from February 1991 through March 2000.

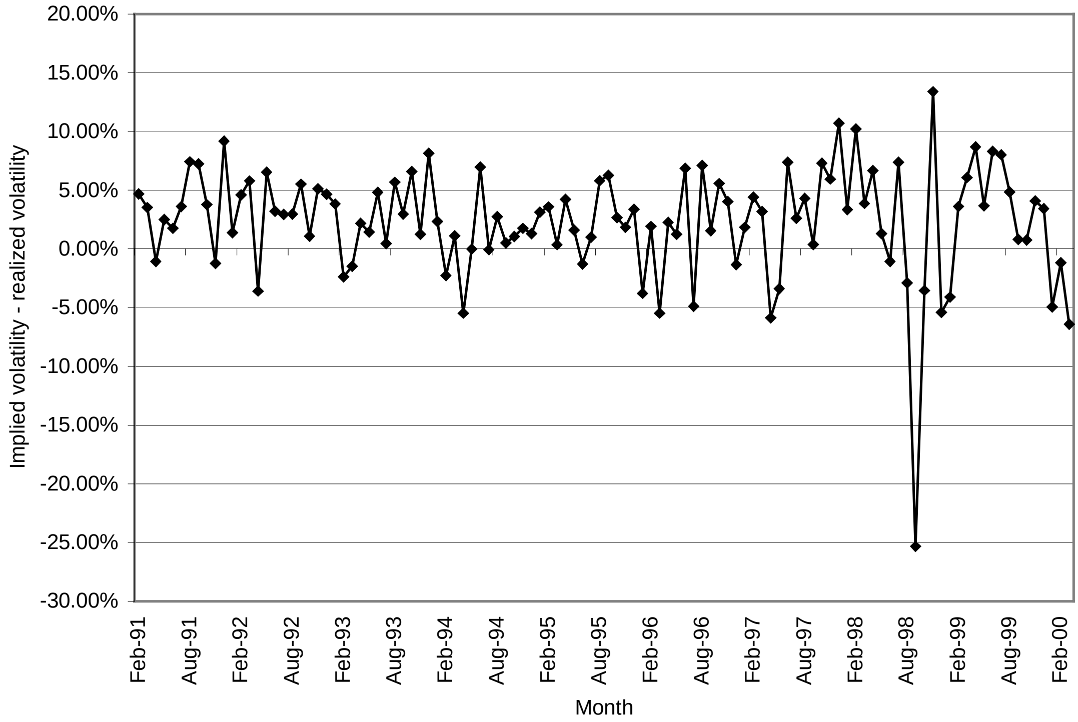
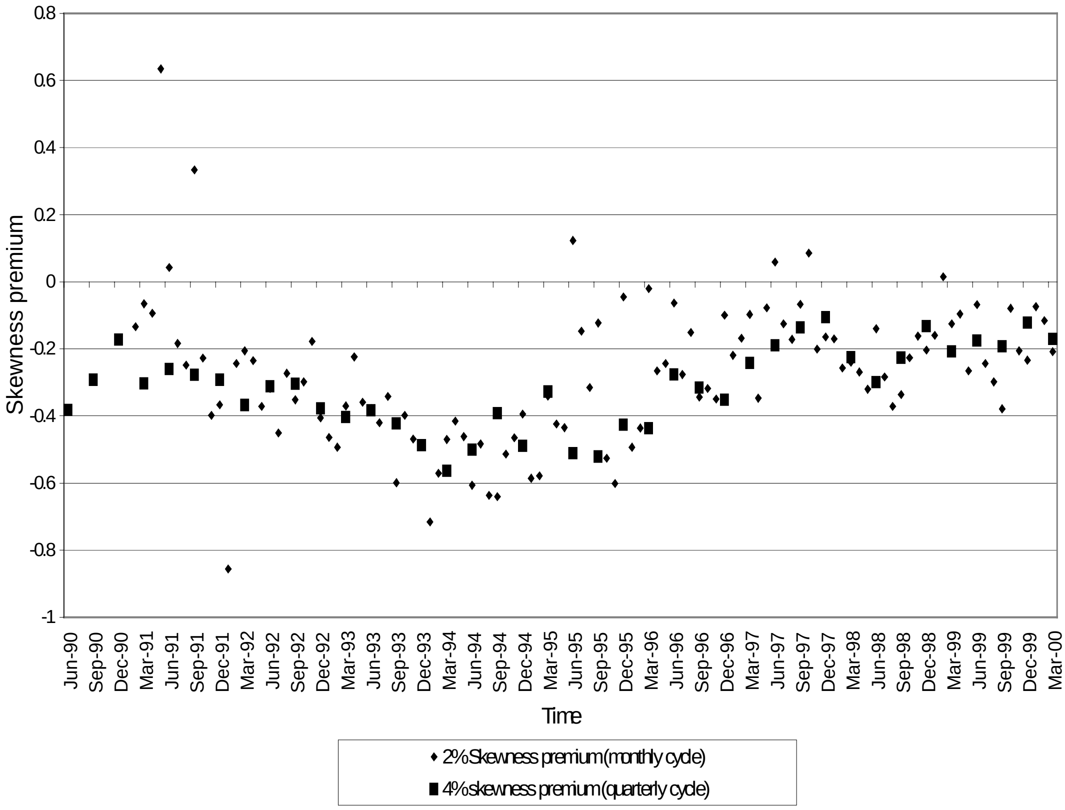


Figure 7. The 4% and 2% skewness premia implicit in quarterly and monthly S&P 500 stock index futures options, from June 1990 and February 1991 through March 2000, respectively. Bates' (1991) skewness premium, i.e. the relative price of out-of-the-money call and put options less one. The lower the skewness premium is, the higher is the probability of a volatility jump/crash in the underlying index. The exact 4% and 2% premia are calculated by fitting a cubic spline through observed strike price/futures prices ratios (Bates, 2000).



B. Forecast Regressions

To see how well implied volatility predicts future realized volatility in our sample, we run the well-known forecasting regression (1) explaining realized volatility on a constant and implied volatility and the so-called encompassing regression, i.e. regressing the realized volatility with both the relevant implied and historical realized volatilities. As shown in Table III, measured both by the significance of the estimated coefficients and the adjusted R-square of the relevant regressions, implied volatility dominates historical volatility as a forecast of future volatility. The coefficient estimate of historical volatility does not differ significantly from zero whereas the coefficient estimate of implied volatility remains significantly different from zero even when historical volatility is added as an explainatory variable. In addition, the hypothesis that the coefficient of implied volatility be one cannot be rejected in any of the forecasting regressions presented in Table III. The lower adjusted R-squares in the regressions on monthly data versus quarterly data are probably due to two reasons. First, each observation of realized volatility in the shorter time period is estimated from only 18 trading days compared to more than sixty trading days in the quarterly data. This induces more uncertainty into the estimates of monthly realized volatility vs. quarterly realized volatility. Second, due to the mean-reverting nature of volatility, implied volatility can be measured more accurately when the forecast horizon increases (Poon and Granger, 2000). The good forecasting performance of implied volatility in our sample is likely due to the fact that the frequency of volatility jumps in our sample approximately equals the market's expectations, i.e. our sample is probably large enough not to be strongly subject to the effects of the proposed peso problem hypothesis. In addition, due to the nature of our data, our sample is also likely not affected by measurement errors to the same degree as previous studies using index options data (e.g. Canina and Figlewski, 1993, Christensen and Prabhala, 1998).

To empirically confirm our simulation results presented earlier, we induce a peso problem into our sample. We run the forecasting regression (1) on different samples by systematically deleting the observation pertaining to the highest positive change in realized volatility. We repeat this procedure for every sample. As can be seen from Tables IV and V, the estimated value of the implied volatility coefficient falls with the size of the maximum (positive) volatility jump in the sample. This relationship is monotonic on quarterly data with the estimates falling from 0.88 to 0.70 when the sample size is lowered from 40 to 10. On monthly data, excluding the four highest jumps in volatility causes the coefficient estimate of implied volatility to fall from approximately 0.97 to circa 0.85. Similarly, the standard errors of the estimated slope coefficients also fall until the sample size is only about half of the original size. Thus, the sample being subject to a peso problem has two effects on the statistical testing of the hypothesis that the slope coefficient be of unit value. First, the point estimate of the slope coefficient is biased down. And second, the estimated standard error is lowered so that the proposed hypothesis can seemingly be rejected with a lower *p*-value. For example, if we omit the largest positive jump in the volatility from the quarterly sample, we can reject the hypothesis that the slope be of unit value on the 10% level. Omitting the five largest jumps, we can reject the hypothesis on the 1% level. The adjusted R-square and F-values also increase systematically when the largest jumps are excluded from the sample. These results are consistent with our hypothesis that if a sample contains a lower frequwncy of volatility jumps than ex ante expected, the slope coefficient in the forecasting regression will be biased downward.

Table III

Forecasting the future realized volatility

The realized volatility of the S&P 500 stock index futures contract in each period is regressed on a constant, the at-the-money volatility implicit in the options on the futures contract with the relevant maturity at the beginning of the period, and the historical realized volatility:

$$\sigma_t^{realized} = \alpha + \beta_1 \cdot \sigma_t^{implied} + \beta_2 \cdot \sigma_{t-1}^{realized} + \varepsilon_t$$

The sample covers the periods from June 1990 and February 1991 through March 2000 for the quarterly and monthly data, respectively. The realized volatility is estimated as the sample standard deviation of the first log differences of daily settlement prices of the relevant futures contract and annualized by multiplying it with the square root of the average number of trading days in a year (252). The implied volatility is estimated from the average of two closest-to-the-money call and two closest-to-the-money put options of relevant maturity using the Black (1976) formula. Standard errors of the estimates are in parentheses below the estimates.

Panel A: Quarterly data

			Coefficient estimates (standard error)		
N	Adjusted R-square (%)	*F*-value	Intercept	Implied volatility	Historical volatility
40	61.40	63.04***	-0.001108 (0.019261)	0.8814*** (0.1110)	
39	61.03	30.75***	-0.006425 (0.020871)	1.121130*** (0.325468)	-0.240003 (0.305607)
39	49.59	38.37***	0.038975** (0.018407)		0.747284*** (0.120632)

Panel B: Monthly data

			Coefficient estimates (standard error)		
N	Adjusted R-square (%)	*F*-value	Intercept	Implied volatility	Historical volatility
110	51.25	115.59***	-0.018016 (0.015029)	0.967439*** (0.089983)	
109	50.88	56.94***	-0.017479 (0.015343)	0.923598*** (0.136981)	0.049220 (0.104066)
109	30.47	48.33***	0.058667*** (0.012356)		0.572891*** (0.082408)

*** denotes a significance level of 1 percent.

Table IV

The effects of inducing a peso problem into the forecasting regression
Quarterly data

The realized volatility of the S&P 500 stock index futures contract in each quarter is regressed on a constant and the at-the-money volatility implicit in the options on the futures contract at the beginning of the period:

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{implied} + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 40 quarters running from the expiration month June 1990 through March 2000. The subsequent samples are otherwise same as the previous samples but the quarter with the highest change in the realized volatility is systematically excluded. The realized volatility is estimated as the sample standard deviation of the first log differences of daily settlement prices of the relevant futures contract and annualized by multiplying with the square root of the average number of trading days in a year (252). The implied volatility is estimated from the average of two closest-to-the-money call and two closest-to-the-money put options of relevant maturity using the Black (1976) formula. Standard errors of the estimates are in parentheses below the estimates.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (standard error)	Implied volatility (standard error)
40	101%	Sep-98	61.40%	63.04	-0.00111	0.88142
					(0.01926)	(0.11101)
39	81%	Mar-96	67.87%	81.28	0.00505	0.82421*
					(0.01577)	(0.09142)
38	43%	Sep-90	71.10%	92.02	-0.00182	0.85687
					(0.01553)	(0.08932)
37	35%	Mar-00	73.72%	101.99	-0.00392	0.86030
					(0.01484)	(0.08519)
36	32%	Jun-97	77.73%	123.17	-0.00089	0.82867**
					(0.01294)	(0.07467)
35	31%	Sep-96	78.76%	127.07	-0.00065	0.82070***
					(0.01259)	(0.07281)
34	29%	Jun-94	79.48%	128.81	-0.00195	0.82417**
					(0.01260)	(0.07262)
33	25%	Dec-94	79.68%	126.50	-0.00361	0.83130**
					(0.01292)	(0.07391)
32	23%	Mar-97	80.67%	130.37	-0.00709	0.84687**
					(0.01308)	(0.07417)
31	23%	Dec-97	80.67%	126.18	-0.00723	0.84666*
					(0.01330)	(0.07537)
30	19%	Mar-94	85.07%	166.24*	-0.00142	0.79713***
					(0.01079)	(0.06182)
29	17%	Jun-95	84.67%	155.67	-0.00227	0.80107***
					(0.01133)	(0.06421)
28	15%	Dec-99	84.11%	143.97	-0.00229	0.80115***
					(0.01191)	(0.06677)
27	14%	Mar-93	83.71%	134.61	-0.00137	0.79262***
					(0.01208)	(0.06832)
26	10%	Sep-97	83.90%	131.30	-0.00360	0.80250***
					(0.01251)	(0.07003)
25	7%	Dec-93	89.13%	197.74	-0.00295	0.78491***
					(0.00994)	(0.05582)
24	5%	Dec-90	89.19%	190.84	-0.00007	0.77263***
					(0.01007)	(0.05593)

23	3%	Dec-95	89.16%	181.98	0.00244	0.75209***
					(0.00988)	(0.05575)
22	0.18%	Jun-92	88.60%	164.22	0.00348	0.74745***
					(0.01048)	(0.05833)
21	0%	Dec-91	88.55%	155.63	0.00306	0.74888***
					(0.01087)	(0.06003)
20	-2%	Dec-92	88.77%	151.22	0.00206	0.75180***
					(0.01115)	(0.06114)
19	-3%	Jun-98	88.68%	141.99	0.00389	0.74483***
					(0.01153)	(0.06251)
18	-8%	Sep-95	89.98%	153.71	0.00246	0.74581***
					(0.01113)	(0.06016)
17	-9%	Mar-92	89.10%	131.80	0.00291	0.74384***
					(0.01223)	(0.06479)
16	-10%	Jun-93	88.93%	121.45	0.00289	0.74390***
					(0.01287)	(0.06750)
15	-11%	Sep-91	88.26%	106.22	0.00258	0.74519***
					(0.01405)	(0.07230)
14	-11%	Jun-99	89.17%	108.04	-0.00115	0.75857***
					(0.01440)	(0.07298)
13	-12%	Mar-91	88.40%	92.40	-0.00137	0.76035***
					(0.01527)	(0.07910)
12	-13%	Sep-99	88.44%	85.12	-0.00058	0.74942***
					(0.01551)	(0.08123)
11	-14%	Mar-99	88.05%	74.66	-0.00084	0.75262***
					(0.01637)	(0.08710)
10	-15%	Jun-90	88.58%	70.79	0.00441	0.70660***
					(0.01525)	(0.08398)

*, **, and *** denote a rejection of the hypothesis that the slope coefficient equal unity at significance levels of 10, 5, and 1 percent, respectively, in a two-sided *t*-test.

Table V

The effects of inducing a peso problem into the forecasting regression
Monthly data

The realized volatility of the S&P 500 stock index futures contract in each month is regressed on a constant and the at-the-money volatility implicit in the options on the futures contract at the beginning of the period:

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{implied} + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 110 months running from the expiration month February 1991 through March 2000. The subsequent samples are otherwise same as the previous samples but the month with the highest change in the realized volatility is systematically excluded. The realized volatility is estimated as the sample standard deviation of the first log differences of daily settlement prices of the most active futures contract and annualized by multiplying by the square root of the average number of trading days in a year (252). The implied volatility is estimated from the average of two closest-to-the-money call and two closest-to-the-money put options of relevant maturity using the Black (1976) formula. Standard errors of the estimates are in parentheses below the estimates.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (standard error)	Implied volatility (standard error)
110	164%	Jul-94	51.25%	115.59	-0.01802	0.96744
					(0.01503)	(0.08998)
109	159%	Aug-98	51.34%	114.96	-0.01867	0.97023
					(0.01514)	(0.09049)
108	112%	Jul-96	50.75%	111.27	-0.01736	0.95884
					(0.01516)	(0.09090)
107	112%	Sep-98	51.55%	113.76	-0.01889	0.96424
					(0.01510)	(0.09040)
106	103%	Apr-92	54.96%	129.14	-0.00300	0.84698**
					(0.01236)	(0.07453)
105	98%	Mar-96	55.80%	132.31	-0.00434	0.85184**
					(0.01230)	(0.07406)
104	90%	Feb-94	57.17%	138.47	-0.00588	0.85672*
					(0.01211)	(0.07280)
103	89%	Jan-96	57.70%	140.15	-0.00793	0.86704*
					(0.01222)	(0.07324)
102	85%	Feb-93	58.78%	145.01	-0.01063	0.88024
					(0.01223)	(0.07310)
101	83%	Jan-00	59.44%	147.57	-0.01282	0.89104
					(0.01231)	(0.07335)
100	69%	Nov-97	59.71%	147.73	-0.01111	0.87508*
					(0.01205)	(0.07200)
90	47%	Dec-96	61.47%	143.00	-0.01520	0.87996
					(0.01215)	(0.07358)
80	33%	Mar-00	60.91%	124.07	-0.01725	0.87552
					(0.01311)	(0.07860)
70	15%	Dec-94	61.87%	112.95	-0.01491	0.83221**
					(0.01310)	(0.07830)
60	4%	Jun-99	64.27%	107.14	-0.02448	0.87295
					(0.01453)	(0.08434)
50	-3%	Mar-97	63.69%	86.96	-0.02715	0.88502
					(0.01601)	(0.09490)
40	-13%	May-97	62.20%	65.18	-0.02300	0.83715
					(0.01735)	(0.10369)

* and ** denote a rejection of the hypothesis that the slope coefficient equal unity at significance levels of 10 and 5 percent, respectively, in a two-sided *t*-test

C. Explaining the Forecast Error

As shown in Tables VI and VIII, the coefficient estimates of the skewness premium variable are negative in all regressions (12), both with quarterly and monthly data. Excluding the observations related to the highest positive changes in realized volatility, the regressions, as predicted by our hypothesis, become statistically more significant. With quarterly data and excluding positive volatility changes of more than 25%, the regression *F*-value is significant at the 5% level and the coefficient estimate of the skewness premium variable is negative with a *p*-value of 2.76%. The adjusted R-square gets its highest value of 13.99% in the regression excluding volatility jumps higher than 10%. Based on these results, the cut-off point for a positive jump in volatility on the quarterly level seems to be somewhere in the range of 10-25%. The intercept does not differ from zero on the quarterly level.

The results based on monthly data are similar to, although slightly less significant than, those on quarterly data. Based on the adjusted R-square values and the *p*-values of the skewness premium variable, the cut-off point for a positive jump in volatility on the monthly level seems to be in the region of 5-10%. Interestingly, the constant is significantly positive in all regressions on monthly data. The reason for this and the lower significance of the skewness premium variable could be due to the higher uncertainty in estimating the realized volatility with only 18 trading days on the monthly level compared to more than 60 on the quarterly level.

As presented in Tables VII and IX, the regressions with the first log difference of the out-of-the-money call and put prices as the independent variable produce very similar results as those with Bates' skewness premium as the independent variable. The elasticity of the conditional forecast error to the relative call/put price is estimated to be in the ranges from –0.09 to –0.40 and from –0.01 to –0.28 for quarterly and monthly data, respectively. The values at the lower end of the ranges correspond to the regressions with the highest jumps excluded from the samples and are, thus, probably closer to the "true" values.

The main results presented in Tables VI through IX strongly support the proposed hypothesis that expectations of volatility jump fears are taken into consideration when options are priced. The higher the level of volatility jump fears is, the higher, ceteris paribus, will the implied volatility be.

Table VI

Explaining the relative forecast error with the skewness premium
Quarterly data

The natural logarithm of the ratio of the implied and realized volatilities of the S&P 500 stock index futures options and futures contracts in each quarter is regressed on a constant and the 4%skewness premium at the beginning of the period:

$$\ln\left(\frac{\sigma_t^{implied}}{\sigma_t^{realized}}\right) = \alpha + \beta \cdot skew_t + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 40 quarters running from the expiration month June 1990 through March 2000. The subsequent samples are otherwise same as the previous samples but excluding the quarter with the highest change in the realized volatility. *P*-values of the estimates are in the parentheses.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (*p*-value)	4% skewness premium (*p*-value)
40	101%	Sep-98	2.75%	2.10	0.03779 (0.6866)	-0.40007 (0.1550)
39	81%	Mar-96	1.40%	1.54	0.07703 (0.3823)	-0.31854 (0.2224)
38	43%	Sep-90	6.32%	3.50*	0.05414 (0.4951)	-0.43654* (0.0696)
37	35%	Mar-00	6.79%	3.62*	0.06832 (0.3682)	-0.42249* (0.0652)
36	32%	Jun-97	3.70%	2.34	0.10827 (0.1467)	-0.32731 (0.1350)
35	31%	Sep-96	1.92%	1.67	0.13002* (0.0882)	-0.27734 (0.2058)
34	29%	Jun-94	1.99%	1.67	0.13652* (0.0723)	-0.27506 (0.2053)
33	25%	Dec-94	4.90%	2.65	0.11833 (0.1172)	-0.35218 (0.1137)
32	23%	Mar-97	12.33%	5.36**	0.08879 (0.2091)	-0.48004** (0.0276)
31	23%	Dec-97	11.87%	5.04**	0.09098 (0.2116)	-0.47583** (0.0325)
30	19%	Mar-94	5.06%	2.54	0.14477* (0.0535)	-0.33592 (0.1218)
29	17%	Jun-95	9.42%	3.91*	0.11542 (0.1341)	-0.44693* (0.0582)
28	15%	Dec-99	11.40%	4.47**	0.09885 (0.2171)	-0.51272** (0.0441)
27	14%	Mar-93	9.65%	3.78*	0.09987 (0.2560)	-0.50993* (0.0633)
26	10%	Sep-97	13.99%	5.07**	0.08522 (0.3202)	-0.58290** (0.0338)
25	7%	Dec-93	5.38%	2.36	0.15455* (0.0803)	-0.39421 (0.1377)
24	5%	Dec-90	-0.22%	0.95	0.18859** (0.0411)	-0.26360 (0.3406)
23	3%	Dec-95	-2.51%	0.46	0.21409** (0.0326)	-0.19475 (0.5046)

22	0%	Jun-92	-3.20%	0.35	0.21765** (0.0377)	-0.18005 (0.5613)
21	0%	Dec-91	-3.43%	0.34	0.21958** (0.0410)	-0.18085 (0.5685)
20	-2%	Dec-92	-3.84%	0.30	0.22670** (0.0393)	-0.17224 (0.5926)
19	-3%	Jun-98	-4.86%	0.17	0.23401** (0.0378)	-0.13257 (0.6886)
18	-8%	Sep-95	-5.22%	0.16	0.24527** (0.0287)	-0.12609 (0.6972)
17	-9%	Mar-92	-5.13%	0.22	0.23030* (0.0722)	-0.18299 (0.6462)
16	-10%	Jun-93	-5.33%	0.24	0.22621* (0.0907)	-0.20325 (0.6314)
15	-11%	Sep-91	-5.08%	0.32	0.21512 (0.1260)	-0.25460 (0.5791)
14	-11%	Jun-99	-5.56%	0.32	0.22822 (0.1055)	-0.24799 (0.5848)
13	-12%	Mar-91	-6.15%	0.30	0.21991 (0.1677)	-0.27174 (0.5920)
12	-13%	Sep-99	-6.54%	0.32	0.22392 (0.1725)	-0.28744 (0.5812)
11	-14%	Mar-99	-6.99%	0.35	0.20769 (0.2653)	-0.33178 (0.5704)
10	-15%	Jun-90	-11.55%	0.07	0.27287 (0.1975)	-0.15901 (0.8006)

* and ** denote a significance level of 10 and 5 percent, respectively.

Table VII

Explaining the relative forecast error with the log of the relative out-of-the-money call/put prices

Quarterly data

The natural logarithm of the ratio of the implied and realized volatilities of the S&P 500 stock index futures options and futures contracts in each quarter is regressed on a constant and the natural log of the ratio of the prices of a 4%-out-of-the-money call and a 4%-out-of-the-money put option at the beginning of the period:

$$\ln\left(\frac{\sigma_t^{implied}}{\sigma_t^{realized}}\right) = \alpha + \beta \cdot \ln\left(\frac{C_t}{P_t}\right) + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 40 quarters running from the expiration month June 1990 through March 2000. The subsequent samples are otherwise same as the previous sample but excluding the quarter with the highest change in the realized volatility. *P*-values of the estimates are in the parentheses.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (*p*-value)	Log of the call/put relative price (*p*-value)
40	101%	Sep-98	2.05%	1.81	0.06726	-0.24450
					(0.3998)	(0.1858)
39	81%	Mar-96	0.68%	1.25	0.10255	-0.18959
					(0.1750)	(0.2690)
38	43%	Sep-90	4.94%	2.92*	0.08749	-0.26351*
					(0.1999)	(0.0958)
37	35%	Mar-00	5.01%	2.90*	0.10241	-0.25023*
					(0.1196)	(0.0974)
36	32%	Jun-97	2.28%	1.82	0.13643**	-0.18996
					(0.0353)	(0.1864)
35	31%	Sep-96	0.71%	1.241	0.15544**	-0.15749
					(0.0192)	(0.2731)
34	29%	Jun-94	0.59%	1.20	0.16283**	-0.15330
					(0.0142)	(0.2819)
33	25%	Dec-94	3.21%	2.06	0.14811**	-0.20562
					(0.0249)	(0.1611)
32	23%	Mar-97	10.10%	4.48**	0.12440**	-0.29262**
					(0.0446)	(0.0426)
31	23%	Dec-97	9.63%	4.20**	0.12646**	-0.28962**
					(0.0481)	(0.0496)
30	19%	Mar-94	3.93%	2.19	0.17038***	-0.20409
					(0.0090)	(0.1503)
29	17%	Jun-95	9.03%	3.78*	0.14063**	-0.29508*
					(0.0362)	(0.0624)
28	15%	Dec-99	11.45%	4.49**	0.12395*	-0.34960**
					(0.0754)	(0.0437)
27	14%	Mar-93	9.73%	3.80*	0.12635*	-0.34451*

					(0.0952)	(0.0625)
26	10%	Sep-97	13.94%	5.05**	0.11621	-0.39176**
					(0.1152)	(0.0341)
25	7%	Dec-93	6.28%	2.61	0.17210**	-0.27432
					(0.0227)	(0.1199)
24	5%	Dec-90	0.07%	1.02	0.20046**	-0.18372
					(0.0118)	(0.3243)
23	3%	Dec-95	-2.16%	0.53	0.2211414**	-0.14029
					(0.0101)	(0.4727)
22	0%	Jun-92	-2.87%	0.41	0.22368**	-0.13121
					(0.0125)	(0.5275)
21	0%	Dec-91	-3.14%	0.39	0.22600**	-0.13068
					(0.0141)	(0.5386)
20	-2%	Dec-92	-3.61%	0.34	0.23329**	-0.1230
					(0.0137)	(0.5686)
19	-3%	Jun-98	-4.63%	0.20	0.23773**	-0.09833
					(0.0140)	(0.6570)
18	-10%	Jun-93	-5.09%	0.18	0.25019***	-0.08969
					(0.0099)	(0.6794)
17	-11%	Sep-91	-4.71%	0.28	0.23209**	-0.14653
					(0.0400)	(0.6044)
16	-11%	Jun-99	-4.87%	0.30	0.22867*	-0.16176
					(0.0520)	(0.5904)
15	-12%	Mar-91	-4.47%	0.40	0.21894*	-0.20116
					(0.0759)	(0.5374)
14	-13%	Sep-99	-5.09%	0.37	0.23351*	-0.19101
					(0.0608)	(0.5540)
13	-14%	Mar-99	-5.65%	0.36	0.22586	-0.20853
					(0.1069)	(0.5614)
12	-15%	Jun-90	-6.13%	0.36	0.23192	-0.21540
					(0.1093)	(0.5593)
11	-14%	Mar-99	-6.53%	0.38728	0.21717	-0.24761
				0.54917	0.18577	0.54917
10	-15%	Jun-90	-11.30%	0.08607	0.27443	-0.12641
				0.77670	0.14351	0.77670

* and ** denote a significance level of 10 and 5 percent, respectively.

Table VIII

Explaining the relative forecast error with the skewness premium
Monthly data

The natural logarithm of the ratio of the implied and realized volatilities of the S&P 500 stock index futures options and futures contracts in each month is regressed on a constant and the 2% skewness premium at the beginning of the period:

$$\ln\left(\frac{\sigma_t^{implied}}{\sigma_t^{realized}}\right) = \alpha + \beta \cdot skew_t + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 110 months running from the expiration month February 1991 through March 2000. The subsequent samples are otherwise same as the previous samples but excluding the quarter with the highest change in the realized volatility. *P*-values of the estimates are in the parentheses.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (*p*-value)	2% skewness premium (*p*-value)
110	164%	Jul-94	-0.91%	0.02	0.20657***	-0.01764
					(0.0000)	(0.8987)
100	69%	Nov-97	-0.67%	0.34	0.23845***	-0.07457
					(0.0000)	(0.5623)
90	47%	Dec-96	-0.33%	0.71	0.25636***	-0.10613
					(0.0000)	(0.4025)
80	33%	Mar-00	-0.27%	0.78	0.27745***	-0.11747
					(0.0000)	(0.3785)
70	15%	Dec-94	-0.96%	0.34	0.32216***	-0.07976
					(0.0000)	(0.5600)
69	12%	Jan-92	-0.83%	0.44	0.32331***	-0.09019
					(0.0000)	(0.5108)
68	12%	Jul-92	0.05%	1.03	0.31372***	-0.14586
					(0.0000)	(0.3137)
67	12%	Jan-94	0.21%	1.14	0.31352***	-0.15526
					(0.0000)	(0.2896)
66	10%	Oct-94	0.45%	1.30	0.31090***	-0.17307
					(0.0000)	(0.2591)
65	10%	May-93	1.19%	1.77	0.30839***	-0.20414
					(0.0000)	(0.1883)
64	9%	Jul-91	1.51%	1.97	0.30967***	-0.21592
					(0.0000)	(0.1656)
63	9%	Nov-94	1.44%	1.91	0.31181***	-0.21405
					(0.0000)	(0.1723)
62	5%	Mar-93	2.12%	2.32	0.31076***	-0.23818
					(0.0000)	(0.1328)
61	5%	Sep-92	3.28%	3.03*	0.31297***	-0.26584*
					(0.0000)	(0.0868)
60	4%	Jun-99	3.11%	2.89*	0.31255***	-0.26250*
					(0.0000)	(0.0943)
59	3%	Apr-99	3.16%	2.89*	0.31080***	-0.26592*
					(0.0000)	(0.0944)
58	2%	Oct-93	3.24%	2.91*	0.30867***	-0.26965*
					(0.0000)	(0.0937)
57	1%	Jan-93	2.71%	2.56	0.30808***	-0.25475
					(0.0000)	(0.1155)
56	-1%	Feb-97	2.51%	2.42	0.30815***	-0.25280

					(0.0000)	(0.1258)
55	-1%	Aug-97	2.42%	2.34	0.31015***	-0.25093
					(0.0000)	(0.1320)
54	-2%	Jan-99	2.32%	2.26	0.31298***	-0.24844
					(0.0000)	(0.1390)
53	-2%	Feb-95	2.17%	2.15	0.32655***	-0.23436
					(0.0000)	(0.1485)
52	-2%	May-99	2.02%	2.05	0.32629***	-0.23720
					(0.0000)	(0.1582)
51	-2%	Feb-92	2.15%	2.10	0.32922***	-0.24108
					(0.0000)	(0.1537)
50	-3%	Mar-97	2.13%	2.07	0.33002***	-0.24158
					(0.0000)	(0.1571)
49	-4%	Sep-96	1.84%	1.90	0.33482***	-0.23384
					(0.0000)	(0.1748)
48	-5%	Feb-99	2.44%	2.17	0.33762***	-0.25004
					(0.0000)	(0.1472)
47	-5%	Sep-91	1.72%	1.80	0.34550***	-0.23176
					(0.0000)	(0.1860)
46	-6%	Apr-98	4.90%	3.32*	0.31393***	-0.33341*
					(0.0000)	(0.0753)
45	-6%	Sep-95	4.81%	3.22*	0.31280***	-0.33227*
					(0.0000)	(0.0796)
44	-7%	May-91	6.13%	3.81*	0.29799***	-0.35536*
					(0.0000)	(0.0578)
43	-9%	Jan-97	4.67%	3.06*	0.28190***	-0.41135*
					(0.0003)	(0.0879)
42	-11%	Jul-95	4.52%	2.94*	0.28991***	-0.40375*
					(0.0003)	(0.0941)
41	-12%	Feb-00	3.80%	2.58	0.30210***	-0.38040
					(0.0002)	(0.1162)
40	-13%	May-97	2.49%	1.99	0.32511***	-0.33078
					(0.0001)	(0.1660)
30	-24%	Mar-99	-2.55%	0.28	0.45097***	-0.13251
					(0.0000)	(0.6015)
20	-31%	Jun-93	-5.16%	0.07	0.51668***	-0.08287
					(0.0004)	(0.7973)

* and *** denote a significance level of 10 and 1 percent, respectively.

Table IX

Explaining the relative forecast error with the log of the relative out-of-the-money call/put prices

Monthly data

The natural logarithm of the ratio of the implied and realized volatilities of the S&P 500 stock index futures options and futures contracts in each month is regressed on a constant and the natural log of the ratio of the prices of a 2%-out-of-the-money call and a 2%-out-of-the-money put option at the beginning of the period:

$$\ln\left(\frac{\sigma_t^{implied}}{\sigma_t^{realized}}\right) = \alpha + \beta \cdot \ln\left(\frac{C_t}{P_t}\right) + \varepsilon_t$$

The first regression is run on the whole data sample consisting of the 110 months running from the expiration month February 1991 through March 2000. The subsequent samples are otherwise same as the previous sample but excluding the quarter with the highest change in the realized volatility. *P*-values of the estimates are in the parentheses.

N	Max. change in realized volatility	Expiration month	Adjusted R-square	*F*-value	Intercept (p-value)	4% skewness premium (p-value)
110	164%	Jul-94	-0.00901	0.026941	0.20596*** (0.0000)	-0.01483 (0.8699)
100	69%	Nov-97	-0.0075	0.26285	0.24310*** (0.0000)	-0.04284 (0.6093)
90	47%	Dec-96	-0.00685	0.394494	0.26601*** (0.0000)	-0.05171 (0.5315)
80	33%	Mar-00	-0.00749	0.412957	0.28829*** (0.0000)	-0.05576 (0.5223)
70	15%	Dec-94	-0.01317	0.102921	0.33282*** (0.0000)	-0.02838 (0.7493)
69	12%	Jan-92	-0.01281	0.139814	0.33490*** (0.0000)	-0.03311 (0.7096)
68	12%	Jul-92	0.002069	1.138921	0.31311*** (0.0000)	-0.11289 (0.2897)
67	12%	Jan-94	0.003824	1.25333	0.31299*** (0.0000)	-0.11983 (0.2670)
66	10%	Oct-94	0.009348	1.61333	0.30630*** (0.0000)	-0.14877 (0.2086)
65	10%	May-93	0.019184	2.251762	0.30244*** (0.0000)	-0.17809 (0.1384)
64	9%	Jul-91	0.022523	2.451623	0.30402*** (0.0000)	-0.1862 (0.1224)
63	9%	Nov-94	0.02145	2.35907	0.30624*** (0.0000)	-0.18409 (0.1297)
62	5%	Mar-93	0.029858	2.87736*	0.30463*** (0.0000)	-0.20522* (0.0950)
61	5%	Sep-92	0.042008	3.63103*	0.30733*** (0.0000)	-0.22499* (0.0615)
60	4%	Jun-99	0.04042	3.48523*	0.30680*** (0.0000)	-0.22267* (0.0669)
59	3%	Apr-99	0.041213	3.49307*	0.30473*** (0.0000)	-0.22598* (0.0667)
58	2%	Oct-93	0.042374	3.52217*	0.30219*** (0.0000)	-0.2297* (0.0657)
57	1%	Jan-93	0.03688	3.14435*	0.30154*** (0.0000)	-0.21861* (0.0817)

56	-1%	Feb-97	0.034878	2.98760*	0.30159*** (0.0000)	-0.2179* (0.0896)
55	-1%	Aug-97	0.033594	2.87712*	0.30358*** (0.0000)	-0.21594* (0.0957)
54	-2%	Jan-99	0.032033	2.753916	0.30650*** (0.0000)	-0.21315 (0.1030)
53	-2%	Feb-95	0.02852	2.526579	0.32127*** (0.0000)	-0.19766 (0.1181)
52	-2%	May-99	0.027758	2.456057	0.32011*** (0.0000)	-0.20511 (0.1234)
51	-2%	Feb-92	0.028451	2.464184	0.32339*** (0.0000)	-0.20642 (0.1229)
50	-3%	Mar-97	0.028052	2.414235	0.32417*** (0.0000)	-0.20639 (0.1268)
49	-4%	Sep-96	0.024449	2.202988	0.32909*** (0.0000)	-0.19945 (0.1444)
48	-5%	Feb-99	0.030126	2.459912	0.33224*** (0.0000)	-0.21046 (0.1236)
47	-5%	Sep-91	0.022506	2.059131	0.34014*** (0.0000)	-0.19604 (0.1582)
46	-6%	Apr-98	0.048142	3.27596*	0.31434*** (0.0000)	-0.25674* (0.0771)
45	-6%	Sep-95	0.04763	3.20051*	0.31282*** (0.0000)	-0.25648* (0.0806)
44	-7%	May-91	0.062999	3.89110*	0.29658*** (0.0000)	-0.27843* (0.0551)
43	-9%	Jan-97	0.045113	2.984256	0.29786*** (0.0000)	-0.27520* (0.0916)
42	-11%	Jul-95	0.042484	2.819108	0.30613*** (0.0000)	-0.26799 (0.1009)
41	-12%	Feb-00	0.035169	2.458032	0.31763*** (0.0000)	-0.25170 (0.1250)
40	-13%	May-97	0.022533	1.899056	0.33872*** (0.0000)	-0.21872 (0.1762)
30	-24%	Mar-99	-0.02597	0.265979	0.45707*** (0.0000)	-0.08601 (0.6100)
20	-31%	Jun-93	-0.05375	0.030785	0.52793*** (0.0001)	-0.03658 (0.8626)

* and *** denote a significance level of 10 and 1 percent, respectively.

IV. Summary and Conclusion

This paper provides an explanation to the anomalously low and sample-dependent predictive power of the volatility implicit in option prices. As suggested by Feinstein (1989), Jackwerth and Rubinstein (1996), and Bates (1997), among others, we argue that rational expectations of infrequently occurring jumps in volatility affect the relationship between realized and implied volatility in two ways. First, volatility will systematically exceed the subsequently realized volatility over long periods only to be strongly below it when a positive jump in volatility does occur. Second, the coefficient estimates in the classic test of forecast efficiency, i.e. regressing the realized volatility on the relevant implied volatility, are likely to be biased. The direction and magnitude of the bias depends on the expected and realized jump frequencies, the jump size, and how strongly the jump expectations vary over time. We show this both through regressions of simulated paths of realized and implied volatilities as well as by inducing a peso problem in the forecast regressions on actual financial market data.

We test the proposed hypothesis empirically by regressing the ex post forecast error on a measure of volatility jump fears. The empirical results provide strong support to the hypothesis that expectations of a jump in volatility are taken into consideration in options pricing. The higher the level of jump fears, the higher will the difference between the implied and the subsequently realized volatility be given that no jump does occur. Thus, the relationship between the implied and realized volatilities is, indeed, affected by volatility jump fears. The time-varying nature of volatility jump fears combined with a discrepancy between the ex ante expected and ex post realized frequencies of volatility jumps is the likely cause to the sample-dependent and seemingly anomalous results presented in earlier work on the forecast efficiency of implied volatility.

Since both implied volatility as such and the relationship between implied and realized volatility are strongly affected by expectations of jumps in volatility, great care should be used when reducing the impact of large forecast errors in volatility analysis. It has previously been argued that relatively more emphasis should been given to the errors in low volatility periods due to their nature as “outliers”, e.g. by removing the data pertaining to the period around the 1987 stock market crash from the sample. Similarly, it has been proposed that implied volatility should be scaled to correct for the apparent “bias” in it as a forecast (Fleming et al., 1995). On the contrary, our results strongly suggest that the effects of both expected and realized jumps in volatility should carefully be included in the empirical analysis of volatility. In addition, since implied volatility has previously been seen as a smoothed expectation of the subsequently realized volatility, the empirical finding that implied volatility is more volatile than the realized volatility in relatively large samples has been considered to be anomalous (Christensen and Prabhala, 1998). Our theoretical model and empirical findings are consistent with and can, thus, also help to explain this phenomenon. Furthermore, our finding that jumps constitute an essential part of the development of volatility over time support the inclusion of jumps in the process used to model the volatility process rather than modeling volatility as a diffusion as e.g. in Hull and White (1987) and Stein and Stein (1991).

The results have important implications to asset and derivatives pricing, risk management, performance measurement, financial market regulation, and even monetary policy to which it is essential to understand how volatility as such and the relationship between implied and realized volatility in particular develop over time. Given that volatility risk is not priced, implied volatility can be seen as an unbiased estimate of future volatility. How, if at all, the results would differ from ours if volatility risk is priced would be a natural extension of our study. In light of our results, the level of volatility jump fears, proxied in this paper by Bates (1991) skewness premium, can be viewed as a forward-looking measure of uncertainty in the market's estimate of future volatility. How the information in this and similar leading indicators that help predict extreme events could be incorporated into volatility forecasting and risk management is an important strand of future research.

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Appendix

Monte Carlo simulations are used to examine the effects of expectations, both correct and incorrect, of a jump in the level of volatility on the coefficient estimates in the classic forecasting regression:

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{implied} + \varepsilon_t \tag{A1}$$

where $\sigma_t^{realized}$ is the realized volatility, $\sigma_t^{implied}$ is the implied volatility used to price options, and ε_t is the error term with the common zero mean and i.i.d. assumptions. The realized volatility is simulated to follow the following mixture of an ARMA(1,1) process (Christensen and Prabhala, 1998) and exogenous jumps.

If a jump in volatility does not occur:

$$\sigma_t^{realized} = \sigma_{t-1}^{realized} - 0.9 \cdot \left(\sigma_{t-1}^{realized} - 0.15\right) + u_t - 0.6 \cdot u_{t-1} \tag{A2}$$

If a jump in volatility does occur:

$$\sigma_t^{realized} = \delta \tag{A3}$$

where u_t is a normally distributed error term, $N\left(0{,}0.025^2\right)$, and δ is the volatility in the high volatility regime which is set to 45%. With the exception of jump frequency (probability), which is let to vary, the parameter values are rounded approximations of the values estimated from realized monthly volatility of the S&P 500 futures contract between February 1991 and March 2000, estimated as the standard deviation of the 18 first log differences of the relevant daily futures settlement prices prior to the expiration date of the monthly options cycle. The true volatility process is assumed to be fully known by market participants so that the implied volatility, when not subject to the peso problem, is an informationally efficient estimate of future realized volatility. However, to make the model more realistic, the assessed jump probability embedded in the implied volatility, as shown in (8), is set to vary randomly. We incorporate the effect of time-varying volatility jump expectations to the simulation by multiplying the expected jump probability with an error term that is symmetrically distributed around a mean of one. For simplicity and similarly to Jorion (1995), we let the error term be generated randomly from a uniform distribution between zero and one and multiply it by two to get an expected value of one. This simulates the effect of a time-varying jump probability. Varying between zero and the double of the "true" probability, this level of variance over time can be considered realistic. The results show the regression coefficients of realized volatility on implied volatility based on 5000 simulated paths each consisting of 100 periods. The simulated realized volatility process is allowed to develop for twenty periods, starting at the long-term average value of 0.15, prior to the 100 periods used in the regressions.