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HEDGING OPTIONS WITH DIFFERENT TIME UNITS IN THE PRICING MODELS

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Key words: Option pricing, Delta hedging, Option Greeks, Intraday option pricing,
Trading time, Calendar time

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Hedging Options with Different Time Units in the Pricing Models

Abstract

This study examined the effects of the Greeks of the options and the trading results of delta hedging strategies, with three different time units or option-pricing models. These time units were calendar time, trading time and continuous time using discrete approximation (CTDA) time. The CTDA time model is a pricing model, that among others accounts for intraday and weekend, patterns in volatility. For the CTDA time model some additional theta measures, which were believed to be usable in trading, were developed. The study appears to verify that there were differences in the Greeks with different time units. It also revealed that these differences influence the delta hedging of options or portfolios. Although it is difficult to say anything about which is the most usable of the different time models, as this much depends on the traders view of the passing of time, different market conditions and different portfolios, the CTDA time model can be viewed as an attractive alternative.

Keywords: Option pricing, Delta hedging, Option Greeks, Intraday option pricing, Trading time, Calendar time.

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1 Introduction

There have been an enormously lot of work in the area of option pricing since Black and Scholes (1973) and Merton (1973) presented their valuation method for European options, but still there are a lot of problems to be solved. One pricing problem comes from the difficulties of how to account for the lapse of time. Calculating the time remaining using calendar days or trading days does not take into consideration patterns in volatility and also does not perhaps account for the weekend properly.

These time unit problems do influence the pricing of options and should therefore be taken into consideration. In empirical studies of these problem Sundkvist and Vikström (2000a) found that the volatility is not constant, but volatility is realized in patterns both intradaily and during the week. In studies of the weekend volatility Fama (1965), French (1980), Gibbons and Hess (1981), Ball et al. (1982) and Lakonishok and Levi (1982) found that the variances for returns are inconsistent with both the calendar- and trading-time hypotheses. This was also confirmed by Sundkvist and Vikström (2000b) in a more recent study. Volatility has also been observed to exhibit a U-shaped function intradaily. The U-shaped volatility pattern is a well-documented phenomena and a number of studies have empirically examined this issue, for example those of Wood et al. (1985), Chan et al. (1991) and Shiyun and Guan (1999).

A pricing model that takes these volatility patterns into consideration was developed by Sundkvist and Vikström (2000a). The Continuous Time using Discrete Approximations (CTDA) model was presented as a model that accounts for the known weekend and intraday volatility in option pricing. Sundkvist (2000) tested the CTDA model on market prices. The results indicated that the CTDA model did not fit market prices best, but rather the trading-time model gave the best fit to market prices. Sundkvist concluded that the historical patterns in volatility are not fully accounted for by the market, rather the market prices options closer to trading time.

In option trading it is important to understand how sensitive an option or portfolio is to each variable that affects the option price. These measures of exposure are often referred to as the Greeks. Because the basis for calculating these sensitivity measures is the pricing model, the time unit problem naturally also influences the Greeks and also the hedging of options. The most common way of hedging options is through delta hedging. Delta hedging means that the portfolio is kept delta neutral through buying or selling the underlying stock. The deltas of the portfolio will change as the underlying stock moves up or down and also as time passes.

Delta hedging strategies play a central role in the theory of derivatives, in particular delta hedging strategies are recipes for replicating the payoff of a complex security by sophisticated dynamic trading or simpler securities. There is a problem though, due to the fact that pricing model usually is continuous time based and for practical purpose trading takes place at discrete intervals. A discrete time implementation of continuous time delta hedging strategy cannot perfectly replicate an option's payoff. The tracking error that arises from implementing a continuos time hedging strategy in discrete time has been addressed in a number of studies, for example those of Boyle and Emanuel (1980) and Bertsimas et al. (2000). Also, among others Leland (1985) and Clewlow and Hodges (1997) have studied similar discrete time delta hedging strategies, motivated by the presence of transaction costs. These studies provide compelling economic motivation for discrete delta hedging, because trading continuously would generate infinite transaction costs.

Using the Black and Scholes model, Söderman (2000) showed that the sensitivity of the delta exposure to changes in time could be measured. This delta-theta or theta-delta measure implies that the delta exposure decrease with an increase in maturity and accordingly that the theta exposure decrease with an increase in the underlying asset, and vice versa. These results indicate that the use of different time units in the pricing models has the largest effect for at-the-money options with shorter maturities. It could therefore be of interest to examine the effect of delta hedging, close to expiration using different time unit models.

The objective of this study is to examine the effects of the Greeks of the options and also to examine the trading results of delta hedging strategies, with three different time units or option-pricing models. The study tries to answer the question if the different time models are just different ways of spreading the option premium over the remaining time or does the different time models also influence trading and trading results. The study does not test the profitability of any specific model instead it tries to establish differences between the models. It also does not address the problem with the tracking errors that arises from implementing a continuos time hedging strategy in discrete time. One of the models in the study is the Black and Scholes formula. The two other models are basically the same as Black and Scholes with modified measures of time. One model is based on trading time for volatility but calendar time for interest rate. The third model is the Continuous Time using Discrete Approximations (CTDA) model. For the CTDA time model some additional theta measures are also developed. These theta measures are believed to be usable for practical purpose as they help the trader to better keep track of the time decay in the option prices.

The study showed that there were differences in the Greeks with different time units. It also revealed that these differences influence the delta hedging of options or portfolios. Although it is difficult to say anything about which is the most usable of the different time models, as this much depends on the traders view of the passing of time, different market conditions and different portfolios, the CTDA time model can be viewed as an attractive alternative. This paper continues in section two by presenting the option pricing models used in the study. Section three presents the Greeks with the different time units and also develops the additional theta measures for the CTDA time model. In sections four and five the methodology and the data of the delta hedging examples are presented. Section six presents the results of the delta hedging strategy. The paper ends in section seven with a summary.

2 Option models

This paper examined the effects of the Greeks of the options and also examined the trading results of delta hedging strategies, with three different time units or option-pricing models. In this section the different pricing models are presented briefly. The three models differ only in the specification of time lapse.

Normally there are 365 calendar days during a year and the calendar time model in this study uses the original Black and Scholes as the option-pricing model. The Black and Scholes (1973) formula gives the price of a European call option as

$$C = SN(d_1) - Xe^{-rt}N(d_2), \tag{1}$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}, \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}, \tag{3}$$

where C is the price of the call option with strike X and underlying S, r is the interest rate, σ is the volatility and t is time to maturity, N is the cumulative univariate normal distribution and ln is the natural logarithm.

Using trading days as time unit, there usually is about 252 trading days during a year. However, the number of trading days is not necessarily equal all years, as there are more holidays during some years. The trading time model in this paper uses the French (1984) composite-time model. This model adjust the Black and Scholes formula in (2) and (3) as demonstrated in (4) and (5) to have two different time basis

$$d_1 = \frac{\ln(S/X) + rt + \sigma_\tau^2 \tau / 2}{\sigma\sqrt{\tau}}, \tag{4}$$

$$d_2 = d_1 - \sigma_\tau \sqrt{\tau}, \tag{5}$$

where τ is time to expiration on a new time basis that has yet to be estimated. French used in his study 252 days per year corresponding to volatility only during trading time. French still used t as calendar days in (1) in order to count interest rate on a daily basis.

The last time unit model used in the study was the CTDA-time model. The CTDA is based on non-arbitrage restrictions and it is considered that the option price should equal the expected cost of hedging volatility until the expiration of the option. This implies that after passing a point in time with expected higher volatility, the time decay of the option should be greater than allowed by ordinary time measures. The most basic consequence of this is that the option price should decrease slowly during non-trading and more rapidly during trading.

The CTDA-time model is based on both the Black and Scholes formula and the French composite-time model. CTDA also uses calendar days to accumulate interest. For the volatility, the model uses a time basis based on trading days and adds "days" for the weekend volatility. The CTDA model accounts for the weekend volatility by including it in the first interval during the next trading day. This excess variance for a holiday EHV_h of h days of non-trading is estimated as

$$EHV_h = \frac{OV_h - OV}{IV + OV}, \tag{4}$$

$$IV = \frac{1}{N}\sum_{t=1}^{N}(c_t - o_t)^2, \tag{5}$$

$$OV = \frac{1}{N}\sum_{t=1}^{N}(o_t - c_{t-1} - \mu)^2, \tag{6}$$

and c and o are logarithms of close and open prices respectively and μ is the expected mean return. *IV* is the intraday variance and *OV* is the overnight variance between subsequent trading days. Note that the expected intraday return is zero as argued in Sundkvist and Vikström (2000a). *OVh* is the same as *OV* but with h days of non-trading between c_{t-1} and o_t. This excess variance is added to the variance of the day succeeding the holiday. *EHV* is a measure of how much additional variance is realized after a non-trading period of h days. This additional variance is measured in proportion to the total one-day variance for subsequent trading days and will lengthen the following trading day by the same amount.

Sundkvist and Vikström (2000a) estimated the average holiday variance to 19.26 %, i.e. the first trading day following a holiday is 1.1926 days “long”. Adding up the holiday variances and the trading days results in approximately 262 CTDA days a year on the German DAX. Of course the number of CTDA days may vary from year to year and also may not be the same for other markets. This is because there are different number of trading day in different markets, but also because the holiday variance may differ between markets. The holiday variance may also change with time.

The CTDA model also divides the trading day into 30 minute intervals. These intervals are given a weight in proportion to a whole trading day’s variance. The variance is estimated on historical intraday data as squared returns, thus the expected return in each intraday interval is zero. An example of the intraday intervals is presented in Appendix 1. The intervals and the holiday variance should be updated regularly as the intraday and weekend pattern may vary over time.

Lockwood and McInish (1990) tested the robustness of overnight and intraday variance. They found that the proportion of intraday and overnight variance was dependent on the market momentum, i.e. the variances are different during bull and bear markets. Consequently, the estimated CTDA-time model can be correct only if expectations on volatility patterns are the same as the historical patterns. A more detailed presentation of the CTDA-time model is given in the paper by Sundkvist and Vikström (2000a).

3 The Greeks with different time units

The measure of exposure of an option or portfolio, for each variable that affects the option price, is often referred to as the Greeks[1]. In option trading it is extremely important to understand how the Greeks change when the underlying moves, what happens to them over time, and why they are an indispensable tool for a trader.

The basis for calculating all of the options sensitivity measures is of course the option-pricing model. The Greeks measure the effect of one pricing variable, if all the others remain constant. Mathematically, each of the Greeks is the partial derivative of the model with respect to one of the four variables. The most common way to do this is to take the partial derivatives of the Black and Sholes model. In this presentation the Greeks are described in a numerical, perhaps little more general way[2].

The delta value of the option is defined as the change in theoretical value if the price of the underlying instrument increase by one currency unit, i.e.

$$\Delta = \frac{T(S+h) - T(S)}{h} \tag{7}$$

where T is the theoretical value from the pricing model, S is the price of underlying and h is the differentiation step (0.001 * S = 0.1 percent of price of underlying). The gamma value is defined as the change in delta if the price of the underlying instrument change by one currency unit. The gamma value of an option is often multiplied by 100 and can be written as,

$$\Gamma = \left[\frac{(T(S+h) - 2*T(S) + T(S-h))}{h^2} \right] *100\,. \tag{8}$$

[1] Since these exposures have Greeks names, it is common to refer to them collectively as “the Greeks.”
[2] This setup is based on the Optas application manual, version 2.2, by Front Capital Systems AB.

The eta or vega value of the option is defined as the change in theoretical value if the volatility increase by one percent unit, i.e.

$$\Lambda = T(v + 0.01) - T(v) \tag{9}$$

where v is the volatility in the form 0.1 = 10 percent. The rho is the value of the option defined as the change in theoretical value if the risk free interest rate increase by one percent unit, i.e.

$$P = T(r + 0.01) - T(r) \tag{10}$$

where r is the risk free rate in the form 0.1 = 10 percent. The rho value differs from the other Greeks here because it will be the same for all the different time models. This comes from the fact that all the presented models are based on the French (1984) approach, which counts the interest rate using calendar days. The theta value of the option is defined as the change in theoretical value if the time to expiration is shortened by for example one day, i.e.

$$\Theta = T\left[t - \frac{1}{365}\right] - T(t) \tag{11}$$

where t is the time to expiration in decimal years. Using trading days (11), normally uses 252 days in stead of 365 days. For the CTDA time model this traditional theta measure may perhaps not be optimal to use in trading. This because the CTDA model also divides the day into intraday intervals and the traditional theta counts only whole days. Therefore some additional theta measures, that could be usable for the CTDA time model, are presented next.

3.1 Theta with the CTDA model

With the CTDA time model it could be interesting to know how much time-value that disappear from the option price, during the next 30-minutes. This theta 30 min. can be defined as

$$\Theta_{30-\min} = T\left[t - \frac{(1/i_{Day})}{262}\right] - T(t) \qquad (12)$$

where t is the time to expiration in decimal years and i_{Day} is the number of intervals the day is divided into, with the CTDA time model. In this paper the CTDA time model is divided into 17 interval during one day, but this number naturally depends on how long the trading day is. The 262 CTDA days in a year may as mentioned of course also vary from year to year.

It could also be of interest to know how much time-value that will disappear during the time left in the trading day. This theta day can be defined as,

$$\Theta_{Day} = T\left[\frac{t_{Next-morning}}{262}\right] - T(t) \qquad (13)$$

where t is the time to expiration in decimal years and $t_{Next\text{-}morning}$ is the time to expiration at the beginning of the following trading day. In this way the trader will know how much the theoretical value of the option will change during the trading day.

Another theta value that could be of interest is the theta night value. Theta night can be defined as the theta from the first half-hour of trading the next day. This is because the drop in theoretical value is to be realised as soon as the market opens, to account for the excess holiday volatility and the normal overnight volatility. This can be written as

$$\Theta_{Night} = T\left[\frac{t_{Next-morning}}{262} - \frac{(1/i_{Day})}{262}\right] - T\left[\frac{t_{Next-morning}}{262}\right] \qquad (14)$$

where $t_{Next\text{-}morning}$ is the time to expiration in decimal years, at the beginning of the following trading day and i_{Day} is the number of intervals the day is divided into, with the CTDA time model (in this paper 17 intervals). To examine how the different time units affect these exposure measurements, the Greeks for an at-the-money option close to expiration is presented in Figure 1[3]. The dynamics of the CTDA model and differences between different time units in pricing models is most interesting during the last weeks or days before expiration and for at-the-money options. The Greeks were calculated so that the option price originally was the same for all the different time units. This also means that the implied volatility of the pricing model may vary with different time unit, although the option price is the same. The Greeks for an in-the-money option and for an out-of-the-money option close to expiration are presented in Appendix 2.

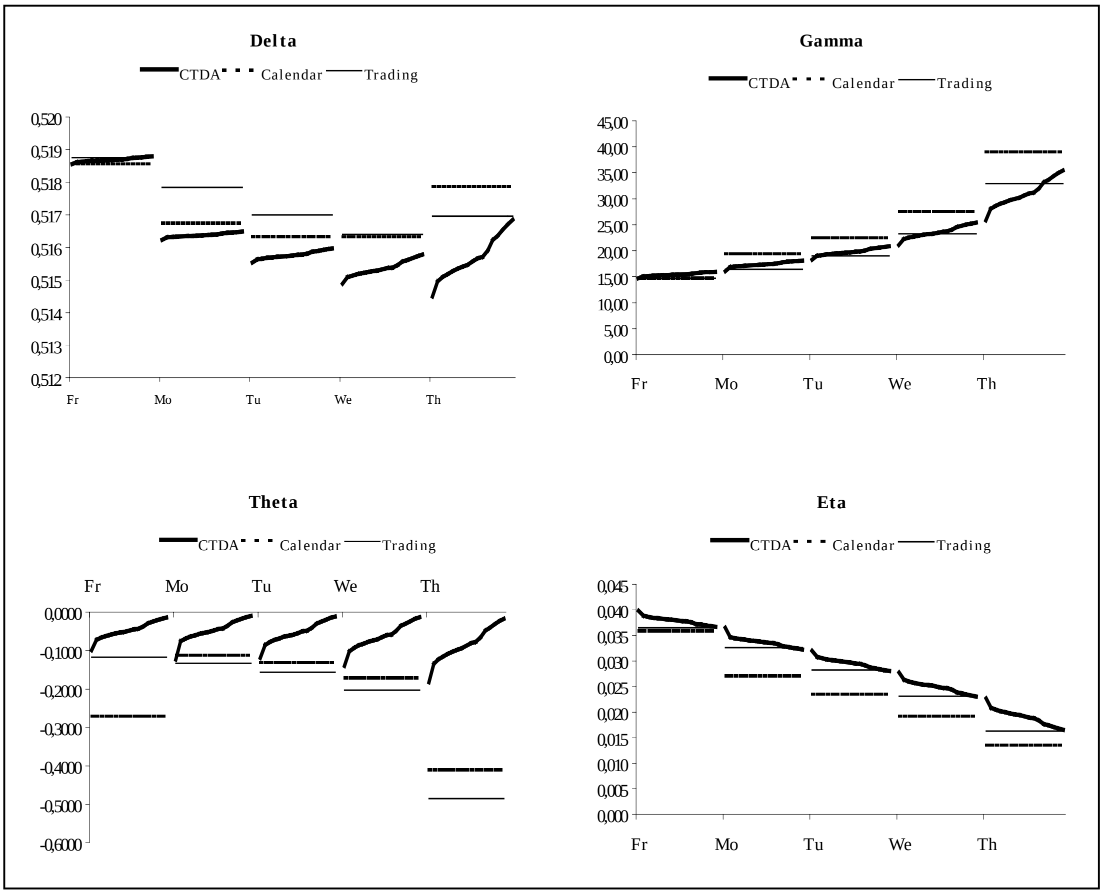

Figure 1
The Greeks for an at-the-money option close to expiration

The simulated values during the last five days before expiration was at-the-money options with the underlying at 65. The Greeks were calculated so that the option price was the same at the beginning of trading on Friday. The theta for CTDA was the theta day parameter. The theta for Friday and for calendar time was calculated as a sum of the theta for Friday, Saturday and Sunday. The implied volatility with CTDA time was 27.0 %, with calendar time 30.0 % and with trading time 29.5 %. The strike price was at 65. The interest rate was 3.0 % and always accrued in calendar time. The intraday intervals with the CTDA model can be seen in Appendix 1 and the excess variance used for a holiday was 19.26 %. Saturday and Sunday are the only non-trading days in this example. The last Friday was left out, as we in this case have zero time left on the expiration day, for calendar days and trading days.

[3] The most interesting Greeks in trading are delta, gamma, theta and eta, which are presented in the figure. The rho measure will not be presented, because it will be the same for all the different time models.

As can be seen from the figure there are differences between the different time units. The largest difference comes from the fact that the CTDA model divides the trading period into smaller intervals, which naturally shows in smoother curves and not any large jumps. How these differences will affect the results of delta hedging portfolios, will be examined in the next section. But first an example of a portfolio with the CTDA model and the presented additional theta measures, on an intraday basis, is presented in Table 1.

Table 1
Example of intraday Greeks for an at-the-money option

Time	Delta			Theta					Gamma			Eta		
	CTDA	C	T	CTDA			C	T	CTDA	C	T	CTDA	C	T
				30-min	Day	Night								
17:00	0.519	0.519	0.519	-0.015	-0.015	-0.052	-0.271	-0.122	14.71	14.71	14.71	0.037	0.036	0.037
9:00	0.516	0.517	0.518	-0.052	-0.134	-0.036	-0.112	-0.132	14.80	19.45	16.43	0.036	0.027	0.033
9:30	0.516	0.517	0.518	-0.008	-0.082	-0.036	-0.112	-0.132	15.54	19.45	16.43	0.035	0.027	0.033
10:00	0.516	0.517	0.518	-0.005	-0.075	-0.036	-0.112	-0.132	15.66	19.45	16.43	0.034	0.027	0.033
10:30	0.516	0.517	0.518	-0.004	-0.070	-0.036	-0.112	-0.132	15.74	19.45	16.43	0.034	0.027	0.033
11:00	0.516	0.517	0.518	-0.004	-0.066	-0.036	-0.112	-0.132	15.80	19.45	16.43	0.034	0.027	0.033
11:30	0.516	0.517	0.518	-0.003	-0.062	-0.036	-0.112	-0.132	15.86	19.45	16.43	0.034	0.027	0.033
12:00	0.516	0.517	0.518	-0.002	-0.059	-0.036	-0.112	-0.132	15.91	19.45	16.43	0.034	0.027	0.033
12:30	0.516	0.517	0.518	-0.004	-0.056	-0.036	-0.112	-0.132	15.95	19.45	16.43	0.034	0.027	0.033
13:00	0.516	0.517	0.518	-0.004	-0.052	-0.036	-0.112	-0.132	16.02	19.45	16.43	0.034	0.027	0.033
13:30	0.516	0.517	0.518	-0.002	-0.048	-0.036	-0.112	-0.132	16.08	19.45	16.43	0.034	0.027	0.033
14:00	0.516	0.517	0.518	-0.007	-0.046	-0.036	-0.112	-0.132	16.11	19.45	16.43	0.034	0.027	0.033
14:30	0.516	0.517	0.518	-0.011	-0.039	-0.036	-0.112	-0.132	16.23	19.45	16.43	0.033	0.027	0.033
15:00	0.516	0.517	0.518	-0.004	-0.028	-0.036	-0.112	-0.132	16.41	19.45	16.43	0.033	0.027	0.033
15:30	0.516	0.517	0.518	-0.005	-0.024	-0.036	-0.112	-0.132	16.48	19.45	16.43	0.033	0.027	0.033
16:00	0.516	0.517	0.518	-0.005	-0.019	-0.036	-0.112	-0.132	16.57	19.45	16.43	0.033	0.027	0.033
16:30	0.516	0.517	0.518	-0.004	-0.014	-0.036	-0.112	-0.132	16.65	19.45	16.43	0.032	0.027	0.033
17:00	0.516	0.517	0.518	-0.011	-0.011	-0.036	-0.112	-0.132	16.71	19.45	16.43	0.032	0.027	0.033
9:00	0.515	0.516	0.517	-0.036	-0.129	-0.041	-0.132	-0.156	16.85	22.46	18.97	0.032	0.024	0.028

The Greeks were calculated for Friday evening, a week before expiration, to Tuesday morning, the expiration week. The Greeks were calculated so that the option price was the same at the end of trading on Friday. The theta for Friday and for calendar time was calculated as a sum of the theta for Friday, Saturday and Sunday. The implied volatility with CTDA time was 29.3 %, with calendar time 30.0 % and with trading time 29.5 %. The underlying and the strike price at 65. The interest rate was 3.0 % and always accrued in calendar time. The intraday intervals with the CTDA model can be seen in Appendix 1 and the excess variance used for a holiday was 19.26 %.

Next the paper presents an methodology to investigate differences in hedging between different time units in the option pricing models.

4 Methodology

To investigate differences in hedging between different time units in the option pricing models, two portfolios were built and delta hedged. The purpose of this study was to illustrate differences between different time units and not to test the profitability of any specific model. The study used tick-data and for each new observation new option prices and deltas were calculated. The portfolios were a bull call spread and a long butterfly. The positions were hedged according to specified delta levels, if for example the level was set to 500 deltas then the portfolio was hedged through buying or selling 500 stocks, as soon as the portfolio was either 500 deltas short or long. The chosen delta levels were 500 deltas, 1000 deltas and 2000 deltas.

The portfolios were also hedged delta-neutral to the next day, this means that no delta positions were held over night. Here the minimum size was set to 100 deltas or stocks, which is the normal minimum lot-size available at Xetra [4]. The study used normal Xetra transaction cost of 0.7 ‰ per trade, with a minimum of €1.5 per trade and a maximum of €17.5 per trade. An example of this could be 500*65*0.00007 = €2.275. The study did not account for any bid-ask spreads of the hedging and it was also assumed that the whole delta position could be hedged in one transaction.

The different time units examined were the CTDA time, the calendar time and the trading time. All these different time units in the pricing models are basically priced according to the original Black and Scholes (1973) pricing model and then adjusted according to the composite-time model by French (1984). Traded options are usually American options on dividend-paying stocks and for these options the standard Black and Scholes model cannot be used. For call options it is common to use the Compound Option Model by Roll (1977), Geske (1979) and Whaley (1981) and for put options the Quadratic Approximation Method by MacMillan (1986) with a dividend correction by either Blomeyer (1986) or Barone-Adesi and Whaley (1988). For index options it is common to use the model by Black (1976). The study examined only American call options with no dividend payments during the maturity and therefore the standard Black and Sholes model could be used. Still, the time unit

[4] This hold for stocks included in the DAX and during continuous trading.

problem is the same also for these other mentioned models. In the next section the data of the study is presented.

5 Data

The study used two different approaches with respect to data. One illustrative approach with real market prices and one confirmatory approach with simulated prices. With the simulation approach many different market conditions could be examined and possible differences between the models might in this way be statically confirmed. The illustrative study examined tick-data stock prices of Deutsche Bank[5] for the duration of August 19, 1999 through August 27, 1999. Data came from the database of Estlander & Rönnlund Ltd. The chosen time period included one weekend. The study came up to a total of 5511 observations. During the examined period the average stock price was €65.90 with a low of €62.11 and a high of €67.5. The options were assumed to expire on Friday the 27 of August 1999[6].

The confirmatory study was built in the same way as the illustrative study except that it used simulated tick-data prices. The time interval and the maturity of the options were the same and the time period also included one weekend. The time series were simulated from a random walk, but with the same start value as the in the illustrative study (€63.1). The study used 500 different time series with 1180 observations in each, which gives a total of 590 000 observations. The ticks were assumed to take place with the same time intervals between each other[7]. The random walk was simulated as

$$P_t = P_{t-1} + a_t \tag{15}$$

where P_t is the price observed at the beginning of time t and a_t is an error term which has zero mean and whose values are independent of each other. To be able to simulate

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[5] It was no special reason for examining the stock Deutsche Bank. However a stock was chosen instead of an index or future because the CTDA model has not been demonstrated on stock-options.
[6] This is in fact not a normal expiration-day, but was assumed because tick-data for the underlying was available for this period. This assumption does not influence the conclusion of the study in any way.
[7] This means that each day had 170 observations, except the first day, which only had 160 observations due to a shorter day.

different volatile markets, the range of the error term was set differently depending on which market conditions that was assumed. For a high volatile market the range of the error term was set to

$$-0.50 \leq a_t \leq 0.50. \tag{16}$$

On the other hand for a low volatile market the range of the error term was set to

$$-0.25 \leq a_t \leq 0.25. \tag{17}$$

The error term could take any of the values within the range independent of the previous selection. The study examined 250 time series of each market condition.

As previously mentioned the investigation built two different portfolios. The first portfolio contained a bull call spread with 100 options long of the strike €62.5 and 100 option short of the strike €67.5. The second portfolio contained a long butterfly with 100 call options long of the strike €62.5, 200 call options short of the strike €65 and 100 call options long of the strike €67.5. The prices of the options, at the opening of the portfolio were €1.46 for the €62.5 strike, €0.44 for the €65 strike, and €0.08 for the €67.5 strike[8]. All the options had a contract size of 100 shares. In Table 2 a description of the different portfolios can bee seen.

Table 2
Option positions in the portfolios

Position	Strike	Number of contracts	Original option price
	I. Portfolio 1: Bull Call Spread		
Long	€62.5	100	€1.46
Short	€67.5	100	€0.08
	II. Portfolio 2: Long Butterfly		
Long	€62.5	100	€1.46
Short	€65	200	€0.44
Long	€67.5	100	€0.08

The original option prices were based on the underlying price of €63.1, at the opening of the positions. The implied volatility with CTDA time was 27.4 %, with calendar time 30.0 % and with trading time 28.8 %. The interest rate was 3.0 % and always accrued in calendar time. All the options had a contract size of 100 shares.

[8] These option prices were arbitrary chosen and not based on any market prices.

The CTDA model partitions the trading day into 30-minute intervals. These intervals are given a weight in proportion to a whole trading day's variance. The intervals used in the study can be seen in Appendix 1. For the simulation this meant that each 30-minute interval included 10 observations. The excess variance used for a holiday was 19.26 %[9].

It should be noted that the implied volatility is dependent on the time basis used. Hence, the different time models can have different implied volatilities although the option prices are the same. In the study the implied volatilities were held constant trough the maturity according to the levels at the opening of the option positions. For the CTDA time model the implied volatility was 27.4 %, for the calendar time model the implied volatility was 30.0 % and for the trading time model the implied volatility was 28.8 %. The interest rate was assumed to be 3.0 % and always accrued in calendar time. The Greeks of the two portfolios at the opening of positions and at the Thursday evening the day before expiration are presented in Table 4. The reason for not presenting the Greeks for the expiration day was because on the last Friday we have zero time left for calendar days and trading days.

At the opening of the option positions the bull call spread was hedged delta neutral through selling 5300 stocks at €63.10 and the long butterfly positions was hedged through selling 1400 stocks at the same price. In Table 3 the Greeks for the illustrative example can be seen. The bull call spread was gamma positive at the beginning of the holding period, but gamma negative at the end of the period. The long butterfly was always gamma negative but became more and more gamma negative as time went by. For the simulated study the Greeks of the different portfolios varied from case to case and are naturally not presented. However, Table 3 shows that the deltas changed over time and how the delta hedges influenced the trading result of the different time units and for the different approaches are presented in the next section.

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[9] These intraday intervals and the excess holiday variance were taken from Sundkvist and Vikström (2000a) and can in a way be seen as an approximation for the German market. The purpose of this example was to establish differences between different time units and not to test the profitability of any specific model. Therefore this assumption should not influence the conclusions in any way.

Table 3
The Greeks of the different portfolios and for the illustrative study

	Delta	Gamma	Theta				Eta
			30-min	Day	Night		
I. Portfolio 1: Bull Call Spread							
<u>19/8/1999</u>							
CTDA	5330	903.4	-36	-410	-175		254
Calendar	5330	903.4				-502	232
Trading	5339	905.6				-688	242
<u>26/8/1999</u>							
CTDA	8758	-1690.5	26	26	291		-84
Calendar	9013	-1598.8				397	-59
Trading	8608	-1716.3				729	-89
II. Portfolio 2: Long Butterfly							
<u>19/8/1999</u>							
CTDA	1393	-478.4	19	191	87		-136
Calendar	1393	-478.4				237	-125
Trading	1397	-481.0				319	-130
<u>26/8/1999</u>							
CTDA	-5605	-2570.0	142	142	500		-126
Calendar	-6360	-2670.9				1094	-97
Trading	-5202	-2481.1				1652	-127

Portfolios at the opening of the positions the 19 of August 1999 when the underlying was at €63.1 and at the Thursday evening the day before expiration the 26 of August 1999 when the underlying was at €66.1.

6 Results

In this sections the results of the two tests of differences in hedging between different time units in the option pricing models are presented. First differences in deltas with the different time units are presented for the two portfolios and for the illustrative example (Figure 2). The differences are presented as differences from the CTDA time model compared to the calendar and the trading time models. As can be seen from the figure the differences increased with time and were largest during the last trading day. The largest difference for the long butterfly was as large as 8045 deltas for the calendar time and the trading time model compared to the CTDA time model. The reason for the large difference in deltas during the last trading day came from the fact that on the expiration day the calendar and trading time model had zero time left. With zero time left in the pricing models the options have either a delta of zero or one depending on the moneyness of the options. All in-the-money options have deltas of one and all out-of-the-money options have deltas of zero. If an option is precisely at-the-money the call options have a delta of one and the put options have a delta of zero. During the expiration day the pricing dynamics of the CTDA model is very interesting and could perhaps also be very useful.

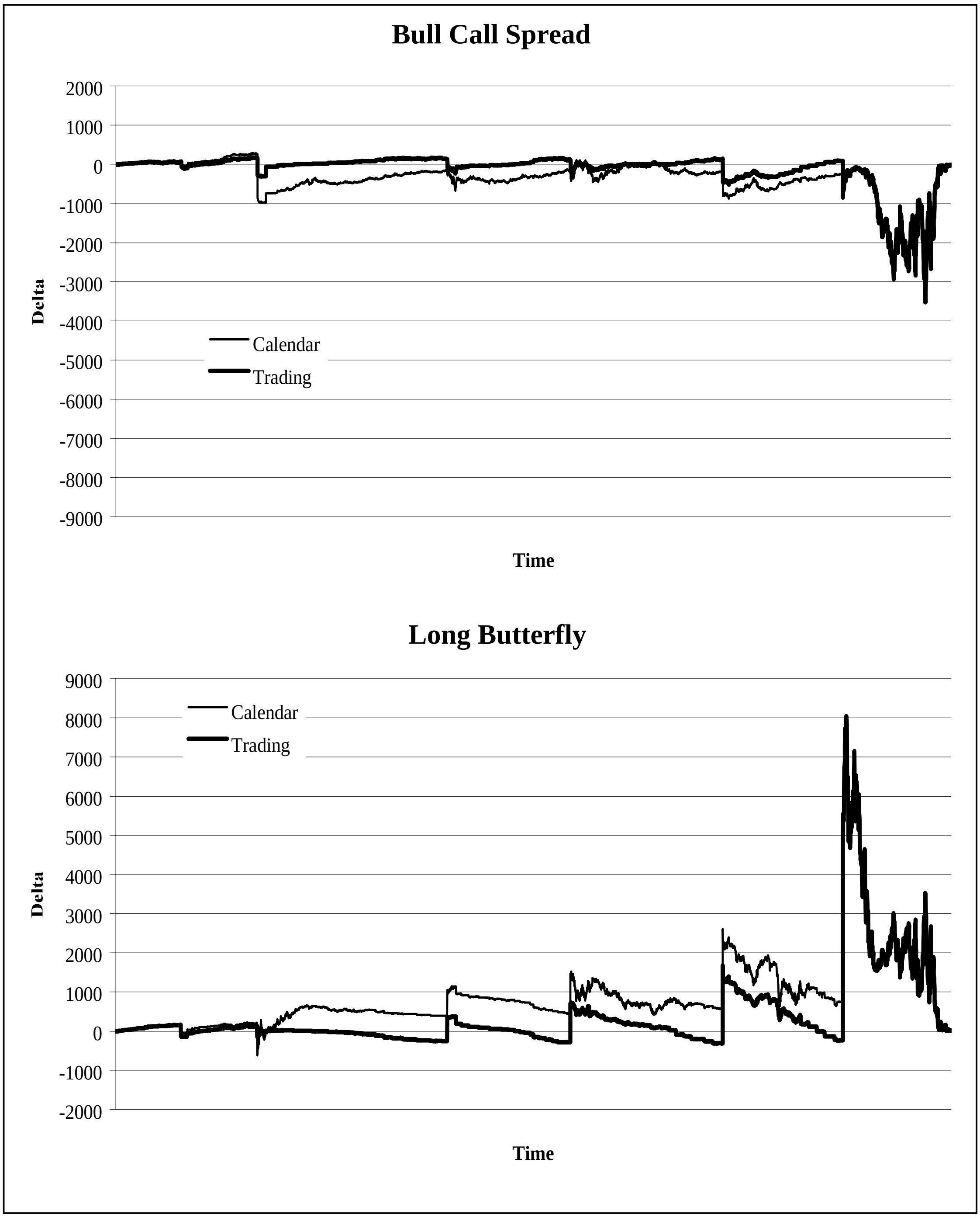

Figure 2
Differences between deltas for the illustrative study
Differences were calculated as (CTDA time – Calendar time) and (CTDA time – Trading time). The portfolios were the bull call spread and the long butterfly. The time period was from the 19 of August 1999 to the 27 of August 27, 1999.

In Table 4 the results of the illustrative study for different hedging strategies and for the different portfolios are presented. The results of the CTDA time model is also presented as they would be if the options had delta one and delta zero during the last trading day. This comes from the fact that during some circumstances it would be desirable to hedge the options without any time-value left in them. It is also common practice for some traders to use this kind of hedging strategy for options during the last day of trade. One reason for this could be that markets usually have a tendency to trade or pin the underlying stock towards strike prices at expiration. With the CTDA time model using deltas one and zero during the last trading day, the differences between the different time models decrease.

Table 4
Results of delta hedging the different portfolios for the illustrative study

	CTDA	CTDA Last day: Δ 0/1	Calendar	Trading
I. Portfolio 1: Bull Call Spread				
Δ= 500				
Number of hedges	105	67	88	73
Profit/Loss	€-1402	€-122	€-1943	€-348
Δ= 1000				
Number of hedges	31	16	28	26
Profit/Loss	€235	€1982	€134	€1090
Δ= 2000				
Number of hedges	8	4	7	7
Profit/Loss	€3397	€3684	€376	€1725
II. Portfolio 2: Long Butterfly				
Δ= 500				
Number of hedges	115	51	66	54
Profit/Loss	€306	€643	€-586	€-1230
Δ= 1000				
Number of hedges	45	19	28	22
Profit/Loss	€343	€716	€-1980	€-959
Δ= 2000				
Number of hedges	17	11	14	13
Profit/Loss	€-1987	€-498	€-2613	€-3051

The study used normal Xetra transaction cost of 0.7 ‰ per trade, with a minimum of €1.5 per trade and a maximum of €17.5 per trade. The study did not account for any bid-ask spreads of the hedging and it was also assumed that the whole delta position could be hedged in one transaction. Δ is the delta at which specified level the positions were hedged.

Examining the hedging results indicate that there were differences in the hedging result and also in the number of hedges. Overall the CTDA time model had more trades than the calendar- and trading-time models. On the other hand the CTDA time model with delta zero or one during the last day had fewer trades. The hedging results vary considerable among the models, but overall the CTDA time model with delta zero or one during the last day seems to have the best hedging result. For the bull call spread the CTDA time model with delta zero or one during the last day had an average result of €1848. This compared to €743 for the original CTDA time model, €-478 for the calendar time model and €822 for the trading time model. For the long butterfly the CTDA time model with delta zero or one during the last day had an average result of €287. This compared to -446 € for the original CTDA time model, €-1726 for the calendar time model and €-1747 for the trading time model.

A reason for the good performance of the CTDA time model with delta zero or one during the last day could be that the portfolios were perhaps more gamma short than gamma long during the examined period. If the portfolio is gamma short often fewer hedges results in a better performance. This on the other hand does not explain the better performance of the original CTDA compared to the other time models as the CTDA model had more hedges than the others.

Although this illustrative example shows good performance of the CTDA model, conclusions should be drawn with care. Overall is difficult to say anything about which model that is the best one as this much depend on the portfolios and the market conditions. During one period one model can perform better, but during another period another model can be better. The simulation study was therefore computed to get a better understanding of at least the different market conditions and also to possibly statically confirm differences between the different time models. In Table 5 the results of the simulation study are presented.

For the Bull Call Spread the simulation gave similar result as the illustrative study with the CTDA time model with delta zero or one during the last day as the best model. For the Bull Call Spread the original CTDA model had the weakest performance, but on the other hand for the Long Butterfly it had the best performance. Any large differences between the performance of the different time models could not be seen. This was also confirmed by the fact that the analysis of variance, where the equality of the results was tested, did not show any statically significant differences. However the standard deviation of the hedging result was considerably lower for the original CDTA than for the other time models. This simulation study shows what was expected, different portfolios respond differently to the time units. Therefore, to get an overall picture of the problem, also many different portfolios would be needed to test. To test also many different portfolios would grow the number of combinations largely and was not computed in this paper, but it could be a proposal for further research in the area.

Table 5
Average simulation results of delta hedging the different portfolios

	CTDA	CTDA Last day: Δ 0/1	Calendar	Trading	F
		I. Portfolio 1: Bull Call Spread			
		Δ= 500			
Number of hedges	97	92	103	92	
Profit/Loss	€1844	€2446	€2308	€2394	0.40
	(9039)	(9614)	(10431)	(9695)	
		Δ= 1000			
Number of hedges	34	34	38	34	
Profit/Loss	€1950	€2566	€2328	€2435	0.39
	(8842)	(9367)	(10286)	(9540)	
		Δ= 2000			
Number of hedges	14	16	17	16	
Profit/Loss	€1930	€2522	€2261	€2370	0.35
	(8784)	(9220)	(10286)	(9572)	
		II. Portfolio 2: Long Butterfly			
		Δ= 500			
Number of hedges	74	75	100	79	
Profit/Loss	€382	€-37	€-62	€-78	0.24
	(7436)	(10290)	(11467)	(10552)	
		Δ= 1000			
Number of hedges	28	31	42	33	
Profit/Loss	€394	€18	€-44	€-86	0.25
	(7187)	(10170)	(11508)	(10577)	
		Δ= 2000			
Number of hedges	13	16	20	17	
Profit/Loss	€415	€38	€64	€-18	0.21
	(7143)	(10288)	(11325)	(10845)	

The study used normal Xetra transaction cost of 0.7 ‰ per trade, with a minimum of €1.5 per trade and a maximum of €17.5 per trade. The study did not account for any bid-ask spreads of the hedging and it was also assumed that the whole delta position could be hedged in one transaction. Δ is the delta at which specified level the positions were hedged. The time series were simulated with a random walk with a start value of €63.1 and used 500 different time series with 1180 observations in each, which gives a total of 590 000 observations. In the parentheses the standard deviations are presented. The F-value comes from an analysis of variance where the equality of the results is tested. The H_0–hypothesis is that the results are equal for the different time model and the H_1-hypothesis is that any result deviates from the others. * indicates significance at 5 % level.

Also, which model to use finally depends on the option trader's view of the passing of time. In studies of different time models to option market prices, for example those of French (1984) and Sundkvist (2000), the trading-time model seems to fit market prices best. To establish which was the best model was never the purpose of this study either. The purpose was to examine if there were differences between different time units in the delta hedging of options and also to see if the different time models influence the trading results. This study clearly showed that different time models are not just different ways of spreading the option premium over the remaining time, although any statistically significant differences could not be observed. How to account for the lapse of time do influence the hedging and therefore also the trading results. Finally, although the purpose of the study was not to test the profitability of different models, the study do show that the CTDA time model can be viewed as an attractive alternative, with some interesting features.

7 Summary

How to account for the lapse of time in option pricing is an important issue. Calculating the time remaining using calendar days or trading days does not take into consideration intraday patterns in volatility and also does not perhaps account for the weekend properly. These problems influence the pricing of options and should therefore be taken into consideration. A pricing model that takes these problems into consideration is The Continuous Time using Discrete Approximations (CTDA) model presented by Sundkvist and Vikström (2000a).

In option trading it is extremely important to understand how sensitive an option or portfolio is to each variable that affects the option price. These measures of exposure are often referred to as the Greeks. Because the basis for calculating these sensitivity measures is the pricing model, the time unit problem naturally also influences the Greeks and also the hedging of options.

This paper investigated the effects of the Greeks of the options and the trading results of delta hedging strategies, with three different time units or option-pricing models. The study did not test the profitability of any specific model instead it tried to establish differences between the models. One of the models in the study was the Black and Scholes formula. The two other models were basically the same as Black and Scholes with modified measures of time. One model is based on trading time for volatility but calendar time for interest rate. The third model was the Continuous Time using Discrete Approximations (CTDA) model. For the CTDA time model some additional theta measures are also developed. These theta measures are believed to be usable for practical purpose as they help the trader to better keep track of the time decay in the option prices.

The study appears to verify that there were differences in the Greeks with different time units. It also revealed that these differences influence the delta hedging of options or portfolios. Although it is difficult to say anything about which is the most usable of the different time models, as this much depends on the traders view of the passing of time, different market conditions and different portfolios, the CTDA time model can be viewed as an attractive alternative.

References

Ball, C. A., Torous, W. N. and Tschoegl, A. E. (1982), "Gold and the weekend effect", *Journal of Futures Markets*, 172-175.

Barone-Adesi, G. and Whaley, R. E. (1988), "On the valuation of American put options on dividend-paying stocks", *Advances in Futures and Options Research*, Vol. 3, 1-13.

Bertsimas, D. and Kogan, L. and Lo, A. W. (2000), "When is time continuous?", *Journal of Financial Economics*, Vol. 55, no. 2, 173-204.

Black, F. (1976), "The Pricing of Commodity Contracts", *Journal of Financial Economics,* Vol. 3, No. 1-2, 167-179.

Black, F. and Scholes, M. (1973), "The Pricing of Options and other Corporate Liabilities", *Journal of Political Economy,* Vol. 81, No. 3, 637-659.

Blomeyer, E. C. (1986), "An analytic approximation for the American put price for options with dividends" *Journal of Financial and Quantitative Analysis*, Vol. 21, No. 2, June, 229-233.

Boyle, P. and Emanuel, D. (1980), " Discretely adjusted option hedges", *Journal of Financial Economics*, Vol. 8, 259-282.

Chan, K., Chan, K. C. and Karolyi, A. (1991), "Intraday Volatility in the Stock Index and Stock Index Futures Markets", *The Review of Financial Studies,* Vol. 4, No. 4, 657-684.

Clewlow, L. and Hodges, S. (1997), "Optimal delta-hedging under transactions costs", *Journal of Economic Dynamics and Control*, Vol. 21, No. 8-9, 1353-1376.

Fama, E. (1965), "The Behaviour of Stock-market Prices", *Journal of Business*, Vol. 38, 34-105.

French, D. (1984), "The Weekend Effect on the Distribution of Stock Prices – Implications for Option Pricing", *Journal of Financial Economics*, Vol. 13, 547-559.

French, K. (1980), "Stock Returns and the Weekend Effect", *Journal of Financial Economics*, Vol. 8, 55-69.

Geske, J. (1979), "The Valuation of Compound Options" *Journal of Financial Economics*, Vol. 7, March, 63-81.

Gibbons, M. and Hess, P. (1981), "Day of the Week Effects and Asset Returns", *Journal of Business,* Vol. 54, No. 4, 579-596.

Lakonishok, J. and Levi, M. (1982), "Weekend Effects on Stock Returns: A Note", *Journal of Finance*, Vol. 37, No. 3, 883-889.

Leland, H. (1985), "Option pricing and replication with transaction costs", *Journal of Finance*, Vol. 40, 1283-1301.

Lockwood, L. and McInish, T. (1990), "Tests of Stability for Variances and Means of Overnight/Intraday Returns During Bull and Bear Markets", *Journal of Banking and Finance*, Vol. 14, No. 6, 1243-1253.

MacMillan, L. W. (1986), "Analytic approximation for the American put option" *Advances In Futures and Option Research*, Vol. 1, 119-139.

Roll, R. (1977), "An analytical formula for unprotected American call options on stock with known dividends" *Journal of Financial Economics,* Vol. 5, November, 251-258.

Shiyun, W. and Guan, L. (1999), “A New Methodology for Studying Intraday Dynamics of Nikkei Index Futures Using Markov Chains”, *Journal of Financial Markets, Institutions and Money*, Vol. 9, No. 3, 247-265.

Sundkvist, K. (2000): “Evaluating Option Pricing Models – Different Ways of Modelling Time”, *Working Paper, Swedish School of Economics and Business Administration.*

Sundkvist, K. and Vikström, M. (2000a): “Intraday and Weekend Volatility Patterns - Implications for Option Pricing”, *Working Paper, Swedish School of Economics and Business Administration.*

Sundkvist, K. and Vikström, M. (2000b): “The day of the Week Effect and Option Pricing – A Study of the German Option Market”, *Working Paper, Swedish School of Economics and Business Administration.*

Söderman, R. (2000): “Risk Monitoring Using the Risk Matrix and the Greeks”, *Working Paper, Swedish School of Economics and Business Administration.*

Whaley, R. E. (1981), “On the Valuation of American Call Options on Stocks With Known Dividends” *Journal of Financial Economics*, Vol. 9, June, 207-211.

Wood, R., McInish, T. and Ord, J. (1985), “An Investigation of Transactions Data for NYSE Stocks”, *Journal of Finance*, Vol. 40, No. 3, 723-739.

Appendix 1

Table A1
Intraday variance in 30-minute intervals

Time	Volatility
9:00 AM	29.6 %
9:30 AM	6.9 %
10:00 AM	4.8 %
10:30 AM	3.4 %
11:00 AM	3.7 %
11:30 PM	2.6 %
12:00 PM	2.2 %
12:30 PM	3.9 %
1:00 PM	3.5 %
1:30 PM	1.7 %
2:00 PM	6.5 %
2:30 PM	9.2 %
3:00 PM	3.8 %
3:30 PM	4.2 %
4:00 PM	4.1 %
4:30 PM	3.1 %
5:00 PM	6.8 %

The realized intraday variance in 30-minute intervals was estimated as squared returns. The number in each cell represents the equivalent amount of time lapsing to correspond to the variance in the respective interval. The variance was estimated on historical intraday data on FDAX, the future on the German DAX-index, during the period 9/1-11/19/1999. During this period the market was open from 9:00 AM to 5:30 PM, which gives 17 intervals. The number of intervals depends of course on the opening hours of the markets and from 2/6/2000 the FDAX trades from 9:00 AM to 8:00 PM, which should give 22 intervals.

Appendix 2

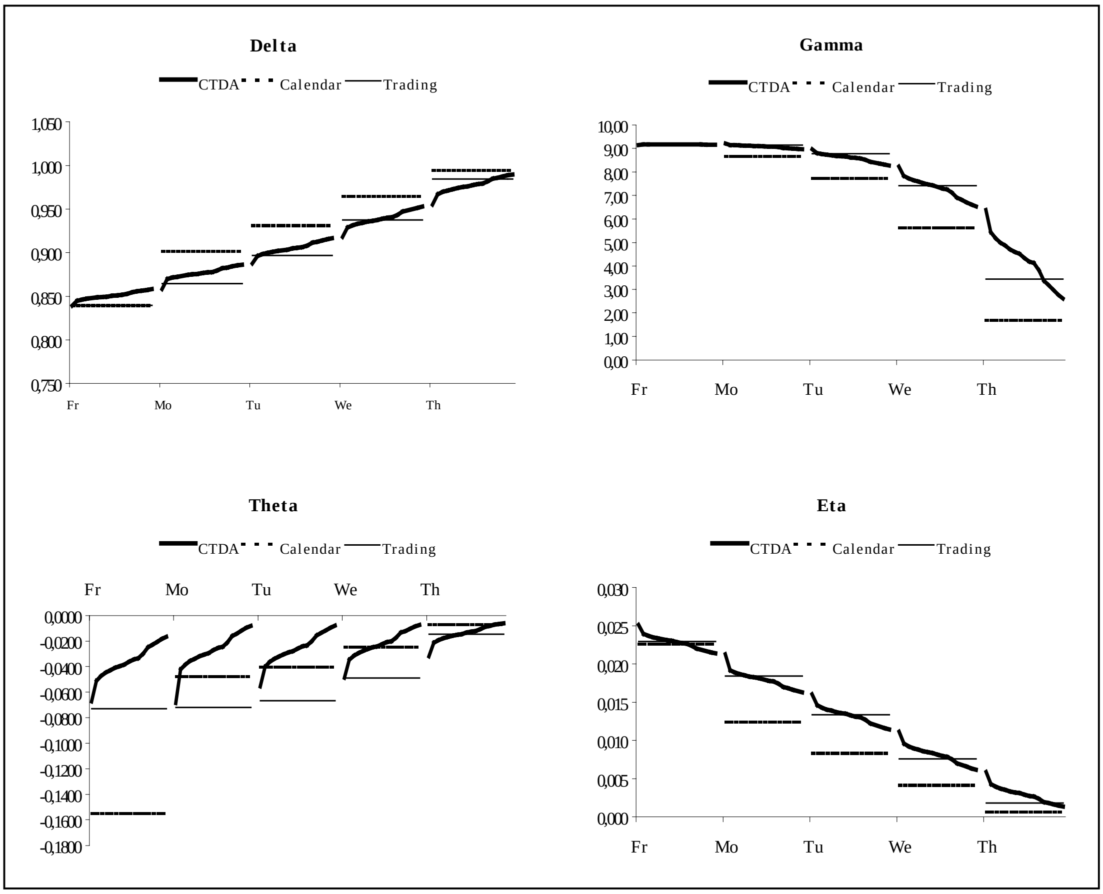

Figure A1
The Greeks for an in-the-money option close to expiration

The simulated values during the last five days before expiration was at-the-money options with the underlying at 65. The Greeks were calculated so that the option price was the same at the beginning of trading on Friday. The theta for CTDA was the theta day parameter. The theta for Friday and for calendar time was calculated as a sum of the theta for Friday, Saturday and Sunday. The implied volatility with CTDA time was 27.0 %, with calendar time 30.0 % and with trading time 29.5 %. The strike price was at 62.5. The interest rate was 3.0 % and always accrued in calendar time. The intraday intervals with the CTDA model can be seen in Appendix 1 and the excess variance used for a holiday was 19.26 %. Saturday and Sunday are the only non-trading days in this example. The last Friday was left out, as we in this case have zero time left on the expiration day, for calendar days and trading days.

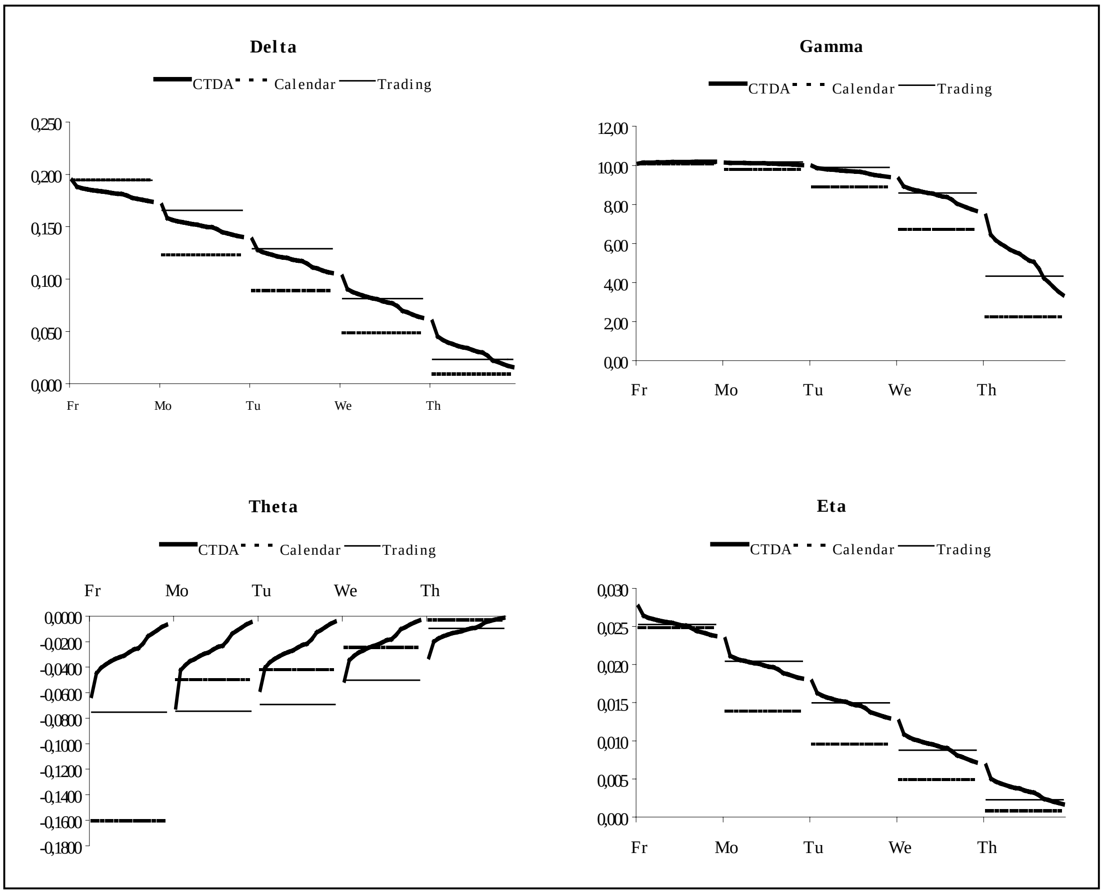

Figure A2
The Greeks for an out-of-the-money option close to expiration

The simulated values during the last five days before expiration was at-the-money options with the underlying at 65. The Greeks were calculated so that the option price was the same at the beginning of trading on Friday. The theta for CTDA was the theta day parameter. The theta for Friday and for calendar time was calculated as a sum of the theta for Friday, Saturday and Sunday. The implied volatility with CTDA time was 27.0 %, with calendar time 30.0 % and with trading time 29.5 %. The strike price was at 67.5. The interest rate was 3.0 % and always accrued in calendar time. The intraday intervals with the CTDA model can be seen in Appendix 1 and the excess variance used for a holiday was 19.26 %. Saturday and Sunday are the only non-trading days in this example. The last Friday was left out, as we in this case have zero time left on the expiration day, for calendar days and trading days.