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Bank Run Risk and Optimal Reserves

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Abstract: We characterize the optimal reserves, and the generated probability of a bank run, as a function of the penalty imposed by the central bank, the probability of depositors' liquidity needs, and the return on outside investment opportunities.

Keywords: Bank Runs, Optimal Reserve Ratio, Deposit Insurance

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1. Introduction

The literature on demand deposits has focused on bank runs generated by expectation-driven panics, but it has not offered any general method for calculating the probability of runs generated by a realization of liquidity needs by a large, but finite, number of depositors. Therefore, in this note we propose a method for calculating the probability of bank runs and banks' optimal reserve ratio assuming that depositors face *real* liquidity needs and not rumors or panics concerning bank runs.

The existing banking literature views the depository institutions as "pools of liquidity" providing consumers with insurance against idiosyncratic liquidity shocks. In the influential model by Diamond and Dybvig (1983) banks provide liquidity to depositors who are, *ex ante*, uncertain about their intertemporal preferences with respect to consumption sequences. They demonstrate how deposit contracts offer insurance to consumers and how such contracts can support a Pareto efficient allocation of risk. However, as they show, there exists a second, inefficient Nash equilibrium where the interaction between pessimistic depositor expectations generates bank runs. Such bank runs confronting individual banks may trigger socially costly bank panics.

Against this background, most countries apply explicit or implicit deposit insurance policies as a mechanism for the elimination of inefficient Nash equilibria driven by pessimistic expectations. Despite the indisputable insurance benefits, empirical observations as well as theoretical research convincingly demonstrate how federal deposit insurance will encourage banks to engage in excessive risk taking (see, for example, Cooper and Ross, 1998) and to keep lower levels of liquid reserves than what would be socially optimal. Consequently, researchers have systematically investigated mechanisms other than deposit insurance as instruments for reducing the instability of the banking system. Bhattacharya et al. (1998) categorize those regulatory measures. In addition, all policy commitments relative to distressed financial institutions face a time-consistency problem as governments and central banks seem to have an incentive of bailing out distressed financial institutions with the intention of eliminating potential contagion problems (e.g. Chen, 1999). For an analysis of the issues involved in a characterization of optimal bail-out policies see, for example, Freixas (1999).

A meaningful evaluation of all policy measures directed towards the banking industry rely on knowledge of how *ex-ante* uncertain liquidity needs translate into probabilities of bank runs and of how the characteristics of this transmission mechanism interacts with

banks' optimal allocation of their portfolios between liquid low-yield assets and illiquid high-yield investments.

2. The Model

Consider a three-period economy with one representative commercial bank and n depositors with known bounded withdrawal distributions. Each depositor has d_i dollars to deposit. Therefore, the total amount of money deposited in the bank is $D = d_1 + \dots + d_n$ dollars.

2.1 Timing

The economy operates in periods, $t = 0, 1, 2$. In period 0 consumer i , $i = 1, \dots, n$, makes a deposit of $\$d_i$, followed by the bank's decision of which reserve ratio to maintain and thereby which proportion of the accumulated deposits to allocate to an illiquid outside investment project. In period 1 depositors face uncertain liquidity needs, which may generate a bank run. In period 2 the bank collects the return on the outside investment project and pays a penalty to the central bank in the event that a run on the bank occurred in period 1.

2.2 The commercial bank

Let r , $0 \leq r \leq 1$, be the reserve ratio which is decided by the commercial bank. The bank keeps rD as reserves. The remaining amount, $(1 - r)D$, is then invested into an outside investment project which bears a safe return of $R > 0$. This investment project cannot be liquidated until period 2.

We assume that the central bank maintains a deposit insurance system where the central bank is committed to bail out the bank in case of a bank run. However, the central bank imposes a penalty of γ for every dollar it lends to the bank during the run. The bank has to compensate the central bank for this loan in period 2 after it collects the return from the outside investment project. This penalty serves as a general policy instrument which can be given several interpretations. The most natural interpretation is that of an insurance premium which the bank is forced to accept in order to qualify to be covered by the bail-out policy. Another interpretation would be to think of γ as a partial deposit insurance with the intention of reducing the bank's incentives to exploit the option value

incorporated in the deposit insurance system. Still, we could also view γ as capturing the probability of the bank losing its charter in case of a bank run. All of these interpretations represent policy measures to mitigate the agency costs generated through moral hazard.

Let X ($0 \leq X \leq 1$) be the random withdrawal rate of the bank's deposits, with an associated strictly increasing and absolutely continuous distribution function F . In the presence of a bail-out policy the expected profit of the bank, is given by

$$\begin{aligned} E\Pi &= (1-r)DR - E(X-r)^+\gamma D \\ &= (1-r)DR - \gamma D \int_r^1 (y-r)dF(y). \end{aligned} \quad (1)$$

The first term measures the bank's profit generated by investing $(1-r)D$ in the illiquid investment project. The second term, $\gamma DE(X-r)^+ = \gamma D \int_r^1 (y-r)dF(y)$, measures the expected penalty imposed on the bank.

The bank chooses a reserve ratio, r , to maximize its profit given in (1). By applying Leibniz' rule, the necessary and sufficient conditions for the optimal reserves are found to be given by⁴

$$\frac{1}{D} \cdot \frac{\partial E\Pi}{\partial r} = -R + \gamma(1 - F(r)) = 0 \quad (2)$$

meaning that the probability of a bank run is given by

$$1 - F(r^*) = \frac{R}{\gamma} \quad (3)$$

Reformulating (3) we find the optimal reserve ratio to satisfy

$$r^* = F^{-1} \left(\frac{\gamma - R}{\gamma} \right), \quad (4)$$

where F^{-1} is the inverse distribution function. Equation (3) implies that the commercial bank sets its reserve ratio so that the probability of a bank run equals R/γ , which is the ratio between the return on the investment project and the penalty rate. In order to induce the commercial bank to hold any reserves, the penalty rate γ must exceed the rate of return on the bank's investment project, R . We summarize the results in the following proposition.

⁴Sufficiency follows from the fact that F is increasing in r .

Proposition 1

The probability of a bank run depends only on the penalty rate, γ , and the return on the investment project, R . The bank's optimal reserve ratio is a decreasing (increasing) function of R (γ).

The main message of the Proposition 1 is that the commercial bank always adjusts its reserves so that an increase in the probability of liquidity needs will not affect the probability of a run on the bank. Hence, the probability of a bank run is invariant to the ex ante probability of a representative depositor facing a liquidity shock.

2.3 The aggregate distribution of withdrawals.

Let $X_i \in [0, 1]$ denote the stochastic fraction of the customer i 's liquidity need. Depositor i faces an uncertain liquidity need, $d_i X_i$, in period 1. In the general case the distribution of F is a convolution which has to be identified by the bank. However, it is easily seen that

$$DEX = \sum_{i=1}^n d_i EX_i \quad \text{and} \quad D^2 \sigma_X^2 = \sum_{i,j=1}^n \rho_{i,j} d_i d_j \sigma_{X_i} \sigma_{X_j} \quad (5)$$

where, $\sigma_X^2 = \text{Var}X$ and $\rho_{ij} = \text{Corr}(i, j)$. Under fairly mild conditions, the distribution of the aggregate withdrawal rate X can be approximated by a normally distributed random variable⁵

$$X \sim_{\text{approx}} \mathbf{N}(EX, \sigma_X^2). \quad (6)$$

The probability of a bank run (3) and the optimal reserve ratio (4) then simplify to

$$1 - \Phi\left(\frac{r^* - EX}{\sigma_X}\right) = \frac{R}{\gamma}, \quad (7)$$

$$\text{and} \quad r^* = EX + \sigma_X \cdot \Phi^{-1}\left(\frac{\gamma - R}{\gamma}\right). \quad (8)$$

The implications of equation (8) are summarized in the following Proposition.

Proposition 2

If X is normally distributed, the optimal reserve ratio r^ is*

⁵A normal approximation is justified e.g. if n is large and if the $d_i X_i$'s are identically distributed and $0 < \rho_{ij} < 1$, when $i \neq j$, or if $X_i \sim \mathbf{N}(EX_i, \text{Var}X_i)$. The approximation also applies to an aggregation over a heterogenous customer pool as long as there are sufficiently many customers within each pool in order to justify a normal approximation within the group.

- (a) *linearly increasing in the depositors expected liquidity need,*
- (b) *linearly increasing (decreasing) in the standard deviation of the depositors aggregated liquidity need σ_X , if $\gamma \geq 2R$ ($\gamma \leq 2R$).*

Under the assumption of mutually identical, possibly correlated depositors, it is seen that the variance of total withdrawals can be rewritten in the following form

$$D^2 \sigma_X^2 = n^2 d^2 \sigma_X^2 = d^2 \sum_{i,j=1}^n \rho_{i,j} \sigma_{X_i} \sigma_{X_j} = n d^2 (1 + (n-1)\rho) \sigma_{X_i}^2, \quad (9)$$

where $\rho = \rho_{ij}$, $0 < \rho < 1$ when $i \neq j$. Therefore, it holds that

$$\sigma_X^2 = (\rho + (1-\rho)/n) \sigma_{X_i}^2 \quad (10)$$

This variance is decreasing in the number of customers and it is approaching

$$\lim_{n \rightarrow \infty} \sigma_X^2 = \rho \sigma_{X_i}^2 \quad (11)$$

Substituting (10) into (8) we can summarize our findings in the following proposition

Proposition 3

Under the assumption of mutually identical and dependent depositors, the optimal reserve ratio r^ is*

- (a) *linearly increasing (decreasing) in the correlation between the depositors liquidity needs ρ , if $\gamma \geq 2R$ ($\gamma \leq 2R$),*
- (b) *asymptotically decreasing (increasing) in the number of depositors, if $\gamma \geq 2R$ ($\gamma \leq 2R$).*

The normal approximation is applicable in an environment with a large number of depositors. With many customers the systematic component $\rho \sigma_{X_i}^2$ becomes more important for the determination of the optimal reserves, which emphasizes the importance of collective withdrawal behavior.

3. Concluding Remarks

This paper develops a method for calculating the profit-maximizing amount of reserves of a bank facing uncertain liquidity shocks and it characterizes the associated probability of a bank run. We show that the only information needed to know the probability of a bank run is the cost of maintaining reserves and the penalty rate charged to a bank facing a run.

Our analysis, has strong policy implications. By adjusting the policy instrument, the penalty rate, the central bank can induce a profit maximizing bank to apply a reserve ratio generating a probability of a bank run which is invariant to the ex-ante probability of a representative consumer facing a liquidity shock. Also, our model predicts that the probability of a bank run is invariant to changes in the correlation between liquidity shocks facing consumers. Consequently, with an unchanged policy, the penalty rate applied, fluctuations in the probability of a bank run can be generated only by changes in the return of the investment projects available to the bank. As long as it is easier for the policy maker to detect changes the returns of the investment projects available to banks than to observe fluctuations in the pattern of liquidity shocks this feature makes the informational requirements facing the policy maker less demanding. Of course, in this present paper we have not explicitly addressed the issue of how to design optimal policy, i.e. the socially optimal penalty rate. However, the model is applicable to this important issue.

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