Leverage Ratio Requirement and Credit Allocation under Basel III

December 9, 2010

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Abstract

We show in a theoretical model that the introduction of the leverage ratio requirement, when it interacts with the risk-based IRB capital requirements, might lead to less lending to low-risk customers and to increased lending to high-risk customers. If such allocational effects are counter-productive to financial stability, then they may pose a trade-off against the alleged positive financial stability effects of the leverage ratio requirement.

JEL Classification: D41, D82, G14, G21, G28
Keywords: Bank regulation, Basel III, capital requirements, credit risk, leverage ratio

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1. Introduction

The new Basel III framework contains a leverage ratio requirement, which has been added to the earlier Basel II framework to supplement risk-based minimum capital requirements for banks. According to the leverage ratio requirement, banks must have a minimum of three percent of capital of non-risk-weighted total assets, including off-balance sheet items (see Basel Committee on Banking Supervision, 2009).[1]

The Basel Committee on Banking Supervision (2009) argues that the leverage ratio requirement would "help contain the build up of excessive leverage in the banking system, introduce additional safeguards against attempts to game the risk based requirements, and help address model risk". The global financial crisis has indeed shown that many items on banks' trading books and off-balance sheet received very low risk-weights under Basel II but turned out to have substantial risk in the crisis (see e.g. Acharya et al., 2009). Such an outcome may have been a manifestation both of "gaming" the risk-based capital requirements by shifting assets from the banking book to the trading book or off-balance sheet, and of "model risk" embedded in the theory-based risk-weights of Basel II. The leverage ratio requirement would hence set an all-encompassing "floor" to minimum capital requirements, which would limit the potential erosive effects of gaming and model risk on capital against true risks.

The leverage ratio requirement has also been criticized for interfering with the basic idea of risk-sensitive capital requirements, which is to align minimum capital requirements with banks' true asset risks and hence promote efficient credit allocation. According to this argument, an additional leverage ratio requirement would make the effective capital requirement on low-risk assets too high. This could lead to risk-shifting from low-risk to higher-risk assets which could be a perverse outcome with an eye to the very aim of capital regulation to safeguard financial stability.[2]

The purpose of this paper is to study the effects of the combination of a leverage ratio requirement and risk-based IRB (internal ratings based) capital requirements, already introduced in Basel II, on loan pricing and loan allocation.

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[1]It might be more logical to talk about a capital to assets ratio requirement or an inverse of a leverage ratio requirement. For simplicity, however, we henceforth use the term leverage ratio requirement keeping in mind that it in actuality it is imposed in terms of a minimum capital to assets ratio.

[2]A very different view is provided by Hellwig (2010) who argues for a leverage ratio requirement which would set banks' capital at well beyond ten percent of non-risk-weighted total assets; perhaps even to the 20 to 30 percent range. Such a leverage ratio requirement should replace risk-based capital requirements which in themselves according to Hellwig (2010) spur capital arbitrage which can further spur high leverage and excessive risk-taking by banks. A sufficiently high capital to assets ratio would in contrast provide a robust buffer against even very high losses and promote good corporate governance in banks by raising the stake of bank shareholders sufficiently high. Hellwig's (2010) view on limiting gaming the risk-based capital requirements with the leverage ratio requirement appears to be in line with the view of the Basel Committee (2009), but his policy conclusion regarding the level of the leverage ratio requirement is much more extreme and categorical than the one currently opted for by the Basel Committee.

With the exception of Blum (2008), previous literature has not, to the best of our knowledge, considered their joint effect.[3] We use the framework of Repullo and Suarez (2004) who study the loan pricing and loan allocation effects of the Basel II reform. Basel II introduced two options to banks for determining their capital requirements against loan assets: the IRB approach and the standardized approach. In the case of unrated customers the latter option effectively reduces to a leverage ratio type of requirement. While in the analysis of Repullo and Suarez (2004) banks choose which option to follow, in our version of their model, motivated by Basel III, banks are simultaneously subject to both the IRB requirement and a leverage ratio requirement.

We do not try to incorporate in our model any of the possible arguments for rationalizing the leverage ratio requirement, discussed above. Analogously with Repullo and Suarez (2004), who do not model any rationale for the regulator's offering the two options to calculate minimum capital requirements, we take it as given that banks face both the IRB requirement and the leverage ratio requirement. In other words, our contribution is simply to look at how the joint requirements affect loan pricing and loan allocation across different risk categories of loans. Following Repullo and Suarez (2004), the model is consistent with the credit portfolio theory underlying the IRB capital requirements but is highly stylized in the sense that only two loan categories, low-risk loans and high-risk loans, are considered.[4] Our key results comprise comparisons of loan rates and amounts of low-risk and high-risk loans under Basel III with respect to the benchmark of Basel II.[5]

As Repullo and Suarez (2004) state, when the IRB requirements are the only capital requirements in the model, banks have an incentive to specialize in either low-risk or high-risk lending. We introduce the leverage ratio requirement and find three different cases (equilibria) of primary interest depending on where the leverage ratio requirement is located in between the low-risk loan's capital requirement and the high-risk loan's capital requirement.[6] We only consider

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[3] Blum (2008) presents a model in which a leverage ratio requirement can restore banks' incentives to "truth-telling" in setting the internal ratings which form the basis for risk-based capital requirements. This type of rationale might be generally used to motivate the gaming and model risk based arguments for the leverage ratio requirement, stated by the Basel Committee.

[4] Blum (2008) who does rationalize the additional leverage ratio requirement only considers loans of one risk-type.

[5] We note that our results critically rely on the assumption that equity is **a** more expensive form of finance for banks than deposits which in the current model are the other source of finance for banks. This assumption is quite standard in the banking literature and the reasons for the extra premium on banks' equity are discussed e.g. by Repullo and Suarez (2004). However, recently Hellwig (2010) and Admati et al. (2010) have analyzed the reasons why this extra cost should not be exaggerated and why it is critical to make a clear distinction between the private and social costs (or benefits) of bank capital. Nonetheless, as Admati et al. (2010) point out, demand deposits can be understood as being part of a bank's "production function" and hence deposits, as opposed to equity, have a relative advantage as a form of finance for banks.

[6] In addition, for some parameter specifications (but not, we believe, for the economically most relevant ones), the model has an equilibrium in which all banks are still specialized to low-risk or high-risk loans, and the only effect of the leverage ratio requirement is to increase

banks which under Basel II would have chosen the IRB approach to determine their capital requirements. We believe this is the most relevant case in practice because most of the large and sophisticated banks are likely to follow the IRB approach, not least because of supervisory expectations to do so.[7]

We shall label the three possible types of equilibria A, B, and C. We prove that, subject to plausible restrictions on the parameter values, one of the three kinds of equilibria exists for each value of the leverage ratio requirement between the capital requirements for the low-risk and high-risk loans. It turns out that an equilibrium of type A exists when the leverage ratio requirement is above but sufficiently close to the low-risk loan's capital requirement. Similarly, an equilibrium of type C exists when the leverage ratio requirement is below but sufficiently close to the high-risk loan's capital requirement, and there must also be a range in the middle, in which an equilibrium of type B exists.

In equilibrium A, there are specialized high-risk loan banks just like in the absence of the leverage ratio requirement, and also the high-risk interest rate remains unchanged. However, the banks which under Basel II are specialized in low-risk lending become now mixed portfolio banks.[8] Given that high-risk loans are profitable to a bank, even when it is subject to a capital requirement which is larger than the leverage ratio requirement, the low-risk loan bank will cope with the leverage ratio requirement by including some high-risk loans in its portfolio. In equilibrium, the interest rate on low-risk loans increases from the Basel II world and hence reduces the demand for low-risk loans. However, these changes can be expected to be small.

The equilibria of type B are symmetric equilibria in which all banks follow an identical mixed portfolio strategy. We conjecture that these equilibria would in

the amount of capital of the specialized low-risk loan banks. This case is shortly considered in Section 4.

There are also two obvious cases where the leverage ratio requirement is outside the interval between the low-risk and the high-risk requirement. If the leverage ratio requirement is lower than the low-risk loan's capital requirement, then it is redundant in the context of the current model and the loan market equilibrium is determined by the IRB capital requirements. Similarly, if the leverage ratio requirement is above the high-risk loan's capital requirement, then it dominates and the loan market equilibrium is solely determined by the leverage ratio requirement which in the current context corresponds to the type of equilibrium which obtains under (the flat) capital requirements based on the Basel II's standardized approach or Basel I. This latter case could actually be used to analyze Hellwig's (2010) suggestion that a high leverage ratio requirement should replace risk-based capital requirements; see footnote 2 above.

[7]One idea in Basel II was an "evolutionary" approach to determining capital requirements which apparently also involved moving from the standardized approach; if not immediately so at least over time to the more sophisticated IRB approach. Incentives to this were provided by calibrating the average capital requirement in the IRB approach lower than in the standardized approach.

[8]It should be observed that since both the original Repullo-Suarez model and the current model describe games in which only a single round is played, one cannot strictly speaking claim that in the model a bank would change its strategy when the leverage ratio requirement is introduced. Nevertheless, one may intuitively think that, e.g. the mixed-portfolio banks of equilibrium A have resulted from low-risk loan banks, which have included some high-risk loans in their portfolio. We shall use this intuitive way of talking also in the discussion of other equilibria below.

a calibrated version of the model be possible only for a relatively narrow range of values of the leverage ratio requirement. In an equilibrium of type B, interest rates for high-risk loans are decreased and interest rates for low-risk loans are increased, whereas the demands for the two types of loans move in the opposite direction.

In case C, the leverage ratio requirement is below but relatively close to the high-risk capital requirement. In this equilibrium there are banks which specialize in low-risk lending. As a result, the low-risk interest rate raises to the level which corresponds to the leverage ratio requirement (rather than the Basel II capital requirement for low-risk loans). Given this increased interest rate, it turns out to be profitable for a high-risk loan bank to include some low-risk loans in its portfolio. This possibility to increase profits leads to a reduction of the interest rate for high-risk loans because the banking sector is competitive. As a result, the demand for low-risk loans is smaller and the demand for high-risk loans is larger than in the Basel II world. It turns out that the demand for high-risk loans obtains its largest value just on the border of cases B and C.

To summarize our main results, cases A and B are probably the most relevant in practice, given the current plan of having a three-percent leverage ratio requirement under Basel III. Hence we may conjecture that the introduction of the leverage ratio requirement could reduce low-risk lending, although if case A prevails, the effect is probably quite small.[9] Further, if case B or C should prevail, the leverage ratio requirement could lead to an increase in high-risk lending.

As we have not pursued a welfare analysis in the current version of the paper, we cannot make any statements about the social optimality of the above analyzed shifts in lending. We may only speculate, perhaps for reasons outside our model, that if less low-risk lending and more high-risk lending are undesirable from the viewpoint of financial stability, then the allocational effects of adding the leverage ratio requirement appear counter-productive. Hence the allocational effects may pose a trade-off against the alleged positive financial stability effects of the leverage ratio requirement, envisaged in the Basel III reform.

The rest of the paper is organized as follows. We first recapitulate the main features of the Repullo-Suarez (2004) model in Section 2, and in Section 3 we discuss the effects of a leverage ratio requirement in the context of the model in general terms. In the subsequent four sections, we discuss the different kinds of equilibria of the model. Section 8 concludes.

2. The Logical Structure of the Model

In the Repullo-Suarez (2004) model there is a banking sector which finances two kinds of firms, which we label low-risk (L) firms and high-risk (H) firms.

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[9]There is some evidence that parts of the financial industry, e.g. municipality finance in Europe, have been concerned about the effects of the leverage ratio requirement on their low-risk lending.

Both kinds of firms need investments of the same size $\bar{R}$ for their projects, to which we shall refer as low-risk and high-risk projects. The portfolio of a bank may be characterized by naming the share of the high-risk projects among all the projects that it finances, and below we shall say that a bank has portfolio α when this share is α.

Each bank finances the loans that it grants partially by capital and partially by deposits. The amount of capital per loan that the bank owns will below be denoted by k. The interest rate on deposits will be normalized to zero. The capital of the banks would earn the riskless interest rate δ elsewhere in the economy. The banking sector is competitive in the sense that the expected profits of the banks are zero.

The banks are subject to a risk-based capital requirement which states that a part b_η of each loan of the category η ($\eta = L, H$) must be funded by capital. Since we are considering loans of size $\bar{R}$, this means that the capital requirement k_L for low-risk loans is $k_L = b_L \bar{R}$, and the capital requirement k_H for high-risk loans is $k_H = b_H \bar{R}$. Denoting the amount of capital that the bank owns per loan by k, the capital requirement of a bank of a unit size with portfolio α may be formulated as

$$k \geq \kappa(\alpha) \tag{1}$$

where

$$\kappa(\alpha) = (1 - \alpha) k_L + \alpha k_H = ((1 - \alpha) b_L + \alpha b_H) \bar{R} \tag{2}$$

The demand n_η for loans of each category η ($\eta = L, H$) is identical with the number of the firms of category η which choose to make an investment, and it is a decreasing function of the interest rate r_η for the loans of category η. In other words,

$$\frac{\partial}{\partial r_\eta} n_\eta(r_\eta) < 0 \tag{3}$$

The projects chosen by the firms can either succeed of fail. If a firm succeeds in a project of type η, it will give the bank the sum $(1 + r_\eta) \bar{R}$ (i.e. principal plus interest) as a repayment for the loan. If a firm fails, it will default. In this case, the bank will get only $(1 - \lambda) \bar{R}$ as a repayment for the loan. Here the parameter λ characterizes the loss given default of the bank.

The success probability of the project of a firm i is characterized the random variable i which is defined by

$$x_i = \mu_i + \sqrt{\rho} z + \sqrt{1 - \rho} \varepsilon_i \tag{4}$$

and the project defaults if $x_i > 0$. Here $z \sim N(0, 1)$ is the systematic risk factor, and the random variables $\varepsilon_i \sim N(0, 1)$ are independent of each other and of z.

The value of μ_i is equal with the constant μ_L for low-risk projects, and with the constant μ_H for the high-risk projects.

Clearly, the unconditional default probability $\overline{p}_\eta$ of the projects of type η ($\eta = L, H$) is given by

$$\overline{p}_\eta = \Phi\left(\mu_\eta\right) \tag{5}$$

Consider now the success probabilities of the projects when the systematic risk factor z has been realized. The above assumptions imply that for a given value of z the default probability p_η of a project i of type η ($\eta = L, H$) is

$$p_\eta\left(z\right) = P\left(\mu_\eta + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon_i > 0\right)$$

This is equivalent with

$$p_\eta\left(z\right) = P\left(\varepsilon_i > -\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) \tag{6}$$

The repayment that a bank with the portfolio α receives per loan for a given value of z is given by

$$(1-\alpha)\left(\left(1-p_L\left(z\right)\right)\left(1+r_L\right)\bar{R} + p_L\left(z\right)\left(1-\lambda\right)\bar{R}\right)$$
$$+\alpha\left(\left(1-p_H\left(z\right)\right)\left(1+r_H\right)\bar{R} + p_H\left(z\right)\left(1-\lambda\right)\bar{R}\right)$$
$$= \bar{R} + \rho\left(\alpha, r_L, r_H; z\right)\bar{R}$$

where the function

$$\rho\left(\alpha, r_L, r_H; z\right) = \left(1-\alpha\right)\left(r_L - p_L\left(z\right)\left(\lambda + r_L\right)\right) + \alpha\left(r_H - p_H\left(z\right)\left(\lambda + r_H\right)\right) \tag{7}$$

may be viewed as a measure of the average interest rate that the bank earns for its loans.

When the amount of capital that the bank has per loan has the value k, the average profit that the bank earns per loan is given by

$$\bar{R} + \rho\left(\alpha, r_L, r_H; z\right)\bar{R} - \left(\bar{R} - k\right)$$

i.e. by

$$\pi\left(k, \alpha, r_L, r_H; z\right) = k + \rho\left(\alpha, r_L, r_H; z\right)\bar{R} \tag{8}$$

Following Repullo-Suarez (2004), we shall formulate the equilibrium conditions of the model for a bank of a unit size. The expected profit of bank of a unit size with the capital k and with the portfolio α is, in general,

$$\Pi\left(k, \alpha, r_L, r_H\right) = \int_{-\infty}^{\widehat{z}_\alpha} \pi\left(k, \alpha, r_L, r_H; z\right) d\Phi\left(z\right) \tag{9}$$

where $\widehat{z}_\alpha$ is the value of z for which integrand becomes zero. Intuitively, if $z > \widehat{z}_\alpha$, the losses of the bank are larger than the invested capital, and the bank will go bankrupt and yield a zero profit (rather than a negative profit) for its owners.

Given that the interest rate for bank capital is δ, the expected value of the considered bank is

$$V(k, \alpha, r_L, r_H) = -k + \frac{1}{1+\delta}\Pi(k, \alpha, r_L, r_H) \tag{10}$$

Since the banking sector is competitive, the equilibrium conditions of the model state that the value of V must be zero for each of the choices of k and α that banks make. In addition, in equilibrium there cannot be any other legitimate choices of k and α that would yield a positive profit.

In the Repullo-Suarez model, each bank will specialize in either low-risk or high-risk loans. Repullo and Suarez also demonstrate (ibid., p. 502) that it is optimal for the banks to choose the minimum amount of capital which is allowed by the capital requirement (which is k_L in the case of a low-risk loan bank, and k_H in the case of a high-risk loan bank).

As we shall shortly see, the specialized banks are important for the analysis of most of the equilibria that we investigate in the next section. Accordingly, it will turn out to be handy to have notations also for the functions which correspond to the functions ρ, π, Π, and V in the case in which a bank has only loans of single type η ($\eta = L, H$) in its portfolio. The definitions of such functions can be obtained from the definitions (7), (8), (9), and (10) by putting $\alpha = 0$ and $\alpha = 1$. We define the function ρ_η, which expresses the average interest rate earned by a bank of type η ($\eta = L, H$) as

$$\rho_\eta(r_\eta, z) = r_\eta - p_\eta(z)(\lambda + r_\eta) \tag{11}$$

and the function π_η, which expresses the average profit that the bank earns per loan for a given value of z (when its amount of capital per loan is k) by

$$\pi_\eta(k, r_\eta, z) = k + \rho_\eta(r_\eta, z)\bar{R} \tag{12}$$

The expected profit of bank of a unit size with the capital k and with only loans of type η ($\eta = L, H$) in its portfolio is given by

$$\Pi_\eta(k, r_\eta) = \int_{-\infty}^{\hat{z}_\eta} \pi_\eta(k, r_\eta, z)\, d\Phi(z) \tag{13}$$

where $\hat{z}_\eta$ is the value of z for which integrand becomes zero and the bank earns zero profits, and the expected value of the considered bank is

$$V_\eta(k, r_\eta) = -k + \frac{1}{1+\delta}\Pi_\eta(k, r_\eta) \tag{14}$$

The equilibrium conditions of the Repullo-Suarez model may now be formulated by stating that the value of the functions V_L and V_H is zero when the low-risk loan banks and the high-risk loan banks choose the smallest legitimate amount of capital. These conditions determine the equilibrium interest rates in the absence of a leverage ratio requirement. We shall denote these interest rates by $\bar{r}_L$ and $\bar{r}_H$. In other words, the interest rates $\bar{r}_L$ and $\bar{r}_H$ are determined by the equilibrium conditions

$$\begin{cases} V_L\left(k_L, \bar{r}_L\right) = 0 \\ V_H\left(k_H, \bar{r}_H\right) = 0 \end{cases} \tag{15}$$

We also introduce the notations $\bar{n}_L$ and $\bar{n}_H$ for the demand for low-risk and high-risk loans in the absence of the leverage ratio requirement. In other words, $\bar{n}_\eta$ (where $\eta = L, H$) is given by

$$\bar{n}_\eta = n_\eta\left(\bar{r}_\eta\right) \tag{16}$$

3. Introducing a Leverage Ratio Requirement

We now consider the effects of introducing a leverage ratio requirement to the considered economy. More specifically, we postulate that in addition to (2) the banks are also subject to the requirement which states that a part b of each loan (independently of its type) must be funded by capital. Remembering that k refers to the amount of capital that a bank has ber loan, this requirement can in the context of our model be formulated as

$$k \geq b\bar{R} \tag{17}$$

and, putting $k_{lev} = b\bar{R}$, the requirement that applies to a bank of a unit size with portfolio α may be formulated as

$$k \geq \max\left\{\kappa\left(\alpha\right), k_{lev}\right\} \tag{18}$$

In what follows we shall view k_L and k_H, where $k_L < k_H$, as given and consider the nature of the equibrium for different values of k_{lev}. Trivially, the equilibrium will not be affected by the leverage ratio requirement if $k_{lev} \leq k_L$, and if $k_{lev} \geq k_H$, the equilibrium will correspond to a constant capital requirement of size k_{lev}, so that in this case the analysis of Repullo and Suarez is applicable as such if one puts $k_L = k_H = k_{lev}$ in it. The next three subsections will be considered with the non-trivial case in which $k_L < k_{lev} < k_H$.

In this case there is a value of the share α of high-risk projects, whih turns *both* the leverage ratio requirement *and* the risk-based capital requirements into binding constraints when the bank chooses the smallest legitimate amount of capital. This will be the case for the portfolio α_{lev} which satisfies the condition

$$k_{lev} = \left(1 - \alpha_{lev}\right) k_L + \alpha_{lev} k_H \tag{19}$$

This condition is equivalent with

$$\alpha_{lev} = \frac{k_{lev} - k_L}{k_H - k_L} \tag{20}$$

Clearly, when $\alpha < \alpha_{lev}$, the amount of capital which is required by the leverage ratio requirement is larger than the amount of capital required by the risk-based requirement, but the opposite is the case when $\alpha > \alpha_{lev}$.

As we just stated, Repullo-Suarez (2004) point out that in the currently considered model each bank chooses the smallest allowed amount of capital. In our setting this can be proved by concluding from (10), (9), and (8) that

$$\frac{\partial}{\partial k} V(k, \alpha, r_L, r_H) = \partial \left(-k + \frac{1}{1+\delta} \Pi(k, \alpha, r_L, r_H) \right)$$

$$= -1 + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}_\alpha} d\Phi(z) < -1 + \frac{1}{1+\delta} < 0$$

Hence, for each portfolio α, it is in the interest of the banks to have the minimum amount of capital allowed by the capital requirements.

Repullo and Suarez's conclusion that under the Basel II regime banks will specialize in either high-risk or low-risk loans is based on a lemma (ibid., p. 503 and p. 519) which states that the value of a mixed-portfolio bank can never be larger than the value of both a low-risk loan bank and a high-risk loan bank of the same size, and that the value of the mixed-portfolio bank is almost always smaller, with the exception of a very special choice of parameter values.[10] This result is not, as such applicable to our more general setting, because in our model the banks are not subject to a capital requirement of the form (1), but rather of the form (18). In the current setting it turns out to be useful to formulate a somewhat stronger version of Repullo and Suarez's lemma. This version states, intuitively, that if the banks are subject to a risk-based capital requirement and the portfolios α_1 and α_3 (where $\alpha_1 < \alpha_3$) do not correspond to banks with a positive value, none of the portfolios between them (i.e. none of the portfolios α_2 for which $\alpha_1 < \alpha_2 < \alpha_3$) can do any better (i.e., also their value is zero or negative). The following lemma is even more general.

Lemma 1. Consider banks which are subject to a risk-based capital requirement which states that each bank should have finance the part k_η of each loan of type η ($\eta = L, H$) with capital, and suppose that $k_L \leq k_H$. Consider banks with portfolios α_1, α_2, and α_3, where $0 \leq \alpha_1 < \alpha_2 < \alpha_3 \leq 1$. The value of a bank with portfolio α_2 is less than or equal with a weighted average of the values of the banks with portfolios α_1 and α_3,

$V(\kappa(\alpha_2), \alpha_2, r_L, r_H) \leq \beta V(\kappa(\alpha_1), \alpha_1, r_L, r_H) + (1 - \beta) V(\kappa(\alpha_3), \alpha_3, r_L, r_H)$

where β is given by

$\beta = \frac{\alpha_3 - \alpha_2}{\alpha_3 - \alpha_1}$

Lemma 1 simplifies our analysis essentially, because it decreases the number of the portfolios that we must consider. Consider first a portfolio α for which

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[10]It should be observed that the value of the mixed portfolio bank is not necessarily smaller if the number $\hat{z}$ which appears in (6) in Repullo-Suarez (2004), 502, is identical for the low-risk loan bank and the high-risk loan bank.

$\alpha_{lev} < \alpha < 1$, and assume that some banks had this portfolio in equilibrium. In this case it would have to be the case that

$V(\kappa(\alpha), \alpha, r_L, r_H) = 0$

Since the capital requirement which applies to each of the portfolios is α_{lev}, α, 1 is equal with the risk based capital requirement which is given by the function κ, we may now apply Lemma 1 to the portfolios $\alpha_1 = \alpha_{lev}$, $\alpha_2 = \alpha$, and $\alpha_3 = 1$. Since there can be no banks with a positive value in equilibrium, we may conclude from Lemma 1 that a high-risk loan bank and bank with portfolio α_{lev} must also be of zero value in the considered case, and that this is possible only in the very special case in which the condition in Lemma 1 is valid with identity. In this special case the model must have also another equilibrium, in which there are no banks with portfolio α, and in which their loans are owned high-risk loan banks and banks with portfolio α_{lev}.[11]

The above argument justifies our practice of not considering portfolios α between α_{lev} and 1. The practice of not considering portfolios between 0 and α_{lev} may be justified similarly, making use of an analogous lemma which applies to banks that are subject to a constant capital requirement.

Lemma 2. Consider banks which are subject to a constant capital requirement which states that each bank should have finance the part k_{lev} of each of its loan with capital and consider banks with portfolios α_1, α_2, and α_3, where $0 \le \alpha_1 < \alpha_2 < \alpha_3 \le 1$. The value of a bank with portfolio α_2 is less than or equal with a weighted average of the values of the banks with portfolios α_1 and α_3,

$V(k_{lev}, \alpha_2, r_L, r_H) \le \beta V(k_{lev}, \alpha_1, r_L, r_H) + (1 - \beta) V(k_{lev}, \alpha_3, r_L, r_H)$

where β is given by

$\beta = \frac{\alpha_3 - \alpha_2}{\alpha_3 - \alpha_1}$

One may conclude from Lemma 2 that if an equilibrium with banks with portfolio α existed, in this equilibrium the value of the banks with portfolios 0 and α_{lev} would have to be zero, and there would be an equilibrium in which the banks with portfolio α were replaced by banks with portfolios 0 and α_{lev}.[12]

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[11] More rigorously, if the model has an equilibrium (call it equilibrium E_1) in which m loans are owned by banks with portfolio α, these banks must own $(1-\alpha)m$ low-risk loans and αm high-risk loans. In this case one may conclude from Lemma 1 that the model must have another equilibrium which is similar with equilibrium E_1 except for the following differences:

1) There are no banks with portfolio α.

2) In addition to the banks which exist in equilibrium E_1, there are also $((1-\alpha)/(1-\alpha_{lev}))m$ banks of unit size with portfolio α_{lev}, and $\alpha m - (1-\alpha)(\alpha_{lev}/(1-\alpha_{lev}))m$ high-risk banks.

This is because the banks mentioned in 2) above supply the same loans with the banks with portfolio α, and according to Lemma 1, also these banks must be of zero value.

[12] More rigorously, if $0 < \alpha < \alpha_{lev}$ and the model has an equilibrium (call it equilibrium E_2) in which m loans are owned by banks with portfolio α, it must once more be the case that these banks own $(1-\alpha)m$ low-risk loans and αm high-risk loans. This time one may conclude from Lemma 2 that the model must have another equilibrium which is similar with equilibrium E_2 except for the following differences:

On the basis of these considerations, we may from now on restrict attention to just three possible portfolios in our analysis. These are the portfolios 0, α_{lev}, and 1, i.e. a portfolio which consists of only low-risk loans (and for which the only binding constraint is the leverage ratio requirement), the portfolio for which both constraints are binding, and the portfolio which consists of only high-risk loans (and for which the only binding constraint is the risk-based requirement). For short, we shall refer to these banks as low-risk loan banks, mixed portfolio banks, and high-risk loan banks, respectively.

If an equilibrium contains specialized high-risk loan banks, the equilibrium condition

$V_H (k_H, r_H) = 0$

must be valid for them, implying that the high-risk interest rate must have its Basel II value $r_H = \bar{r}_H$, which appears in (15). On the other hand, the capital requirement which applies to the specialized low-risk banks is the leverage ratio requirement k_{lev}. We shall denote the low-risk interest rate with which such banks are faced in equilibrium by $r_{L,lev}$. In other words, the interest rate $r_{L,lev}$ is characterized by the equilibrium condition

$$V_L (k_{lev}, r_{L,lev}) = 0 \tag{21}$$

The equilibrium condition which applies to a bank with the portfolio α_{lev} is

$$V (k_{lev}, \alpha_{lev}, r_L, r_H) = 0 \tag{22}$$

The equilibrium condition (22) differs from the conditions (15) and (21) in so far that by itself it does not suffice to determine either of the interest rates r_L and r_H. Rather, it suffices only for determining r_L as a function of r_H, and vice versa. Below we shall repeatedly make use of this fact in our analysis, and for this reason we introduce the notation $r_{L,E} (r_H, k_{lev}, \alpha_{lev})$ for the value of r_L which satisfies (22) for given values of r_H, k_{lev}, and α_{lev}, and the notation $r_{H,E} (r_L, k_{lev}, \alpha_{lev})$ for the value of r_H which satisfies (22) for given values of r_L, k_{lev}, and α_{lev}. In other words, the functions $r_{L,E}$ and $r_{H,E}$ are characterized by the conditions

$$V (k, \alpha, r_{L,E} (r_H, k, \alpha), r_H) = 0 \tag{23}$$

and

$$V (k, \alpha, r_L, r_{H,E} (r_L, k, \alpha)) = 0 \tag{24}$$

It is clear from the definitions (7)-(10) that $r_{L,E} (r_H, k, \alpha)$ and $r_{H,E} (r_L, k, \alpha)$ are decreasing functions of r_H and r_L.

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1) There are no banks with with portfolio α.

2) In addition to the banks which exist in equilibrium E_2, there are also $(\alpha/\alpha_{lev}) m$ banks of unit size with portfolio α_{lev}, and $(1 - \alpha) m - (1 - \alpha_{lev}) (\alpha/\alpha_{lev}) m$ low-risk banks.

Again, this is because the banks mentioned in 2) above supply the same loans with the banks with portfolio α, and they must be of zero value according to Lemma 2.

4. The Equilibrium with Specialized Banks

In the current version of our paper, we shall not present a detailed analysis of the equilibria in which the specialized low-risk loan banks and the specialized high-risk banks co-exist. The aim of the current section is to motivate this omission.

If an equilibrium contains both low-risk loan banks and high-risk loan banks, the high-risk interest rate must have the value $\bar{r}_H$ that it would have also in the absence of the leverage ratio requirement, and the low-risk interest rate must have the value $r_{L,lev}$ that corresponds to a constant capital requirement of size k_{lev}. Intuitively, if $r_L = r_{L,lev}$ and $r_H = \bar{r}_H$, the low-risk loan banks react to the leverage requirement by increasing their amount of capital to the level required by it, but the business model of the high-risk banks is not affected by the leverage ratio requirement.

However, in this case a low-risk loan bank has also another obvious way of coping with the leverage ratio requirement. It might include high-risk loans in its portfolio to such an extent that the leverage ratio requirement becomes valid also when the bank has only the amount of capital which corresponds to the Basel II requirement (i.e., k_L for each low-risk loan, and k_H for each high-risk loan). The latter strategy decreases the expected value of the bank, even when a high-risk loan bank with capital k_H per loan has zero (rather than negative) expected value, because of the possibility that high-risk loan banks go bankrupt when low-risk loan banks do not, or vice versa. When this happens, a mixed portfolio bank will have to use some of the profits that it earns from low-risk loans for covering the losses from high-risk loans (or vice versa), but this situation is not possible when the loans have been granted by specialized banks. A low-risk bank will prefer the strategy of becoming a mixed-portfolio bank to the strategy of increasing the level of capital and staying specialized whenever the expected loss from the shifts in bankruptcy probabilities are smaller than the losses from extra capital, but not otherwise.

The risk-based Basel II capital requirements have been chosen so that the probability of bankruptcy is small (at most 0.001) for each of the considered portfolios. Hence, for realistically calibrated parameter values the negative effects of having a mixed porfolio may be expected to be small. More specifically, the failure probabilities of low-risk loan banks and high-risk loan banks under the Basel II regime are (using the notation of (13)) given by $1 - \Phi\left(\widehat{z}_L\right)$ and $1 - \Phi\left(\widehat{z}_H\right)$, and if these numbers are close to each other, also the effects that the different failure probabilities of low-risk and high-risk banks have on bank value must according to (13) and (14) be small. Our analysis will be based on the assumption that such effects are sufficiently small to make the mixed portfolio strategy preferable to the strategy of sticking to a low-risk profile, and paying the costs of extra capital.

In the current paper, we shall not present a general answer to the question when the difference between $\Phi\left(\widehat{z}_L\right)$ and $\Phi\left(\widehat{z}_H\right)$ is sufficiently small to make it impossible for the specialized low-risk loan and high-risk loan banks to co-

exist. Rather, we shall rest content with presenting sufficient (but by no means necessary) conditions for eliminating this possibility.

The next section is concerned with an equilibrium in which the high-risk interest rate does not adjust, and the low-risk loans are offered by mixed portfolio banks. It turns out that the mixed-portfolio strategy of this equilibrium is preferable to the strategy of specializing in low-risk loans whenever

$$\max\left\{1 - \Phi\left(\widehat{z}_L\right), 1 - \Phi\left(\widehat{z}_H\right)\right\} < \delta\left(1 - \bar{\alpha}\right)\min\left\{\frac{b_H - b_L}{\lambda - b_H}, \frac{b_H - b_L}{b_H + r_H}\right\} \quad (25)$$

Here $\bar{\alpha}$ is the share of high-risk loans among all granted loans (rather than in the portfolio of some particular bank).

Similarly, Section 7 will be concerned with an equlibrium in which there are low-risk loan banks and mixed portfolio banks. It turns out that for the interest rates that occur in this equilibrium, the strategy of being a mixed-portfolio bank is preferable to the strategy of being a specialized high-risk bank whenever

$$\max\left\{1 - \Phi\left(\widehat{z}_L\right), 1 - \Phi\left(\widehat{z}_H\right)\right\} < \delta\bar{\alpha}^2\frac{b_H - b_L}{b_H + r_H} \quad (26)$$

5. The Equilibrium with Mixed-portfolio Banks and High-risk Banks

The easiest way to understand intuitively the equilibrium that we consider next is, perhaps, to consider the case in which the leverage ratio requirement is quite close to k_L. Intuitively, one may think that in this case the the leverage ratio requirement is irrelevant for the business model of the high-risk loan bank, and if the high-risk loan banks stick to financing high-risk loans only, also the high-risk interest rate r_H must retain the value $\bar{r}_H$ which it would have in the absence of the leverage ratio requirement. Given that the high-risk loan banks are of zero (rather than negative) value, although they are subject to a higher capital requirement than the low-risk loan banks, the low-risk loan banks react to the leverage ratio requirement by becoming mixed-portfolio banks and adding high-risk projects to their portfolio until the risk-based requirement becomes a binding constraint for them.[13]

When $r_H = \bar{r}_H$ has been given, the low-risk interest rate r_L is determined by the condition (22), according to which the value of the mixed-portfolio bank must be zero. In other words, the interest rates r_L and r_H are determined by the equilibrium conditions

$$\begin{cases} V\left(k_{lev}, \alpha_{lev}, r_L, r_H\right) = 0 \\ V_H\left(k_H, r_H\right) = 0 \end{cases} \quad (27)$$

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[13]Cf. footnote 8 above.

Using the notations introduced in (15) and (23), the interest rates which solve these equilibrium conditions are given by

$$\begin{cases} r_L = r_{L,E} \left(\bar{r}_H, k_{lev}, \alpha_{lev} \right) \\ \quad r_H = \bar{r}_H \end{cases} \tag{28}$$

Given these interest rates, the two kinds of banks (i.e. the mixed-portfolio banks, and the high-risk loan banks) will be able to follow the strategies that we just described if the demand for high-risk loans exceeds their supply by the mixed-portfolio banks, which is fixed by the condition that the share of low-risk loans in their portfolio is $1 - \alpha_{lev}$, implying that the total amount of loans that they grant is $n_L \left(r_L \right) / \left(1 - \alpha_{lev} \right)$, and further that the number of high-risk loans that they grant is $\left(\alpha_{lev} / \left(1 - \alpha_{lev} \right) \right) n_L \left(r_L \right)$. Hence, the considered strategies are possible if

$n_H \left(\bar{r}_H \right) \geq \frac{\alpha_{lev}}{1 - \alpha_{lev}} n_L \left(r_{L,E} \left(\bar{r}_H, k_{lev}, \alpha_{lev} \right) \right)$

This is equivalent with

$$\alpha_{lev} \leq f_1 \left(k_{lev} \right) \tag{29}$$

where the function f_1 is given by

$$f_1 \left(k_{lev} \right) = \frac{n_H \left(\bar{r}_H \right)}{n_H \left(\bar{r}_H \right) + n_L \left(r_{L,E} \left(\bar{r}_H, k_{lev}, \alpha_{lev} \right) \right)} \tag{30}$$

The condition (29) has a simple intuitive interpretation. The value $f_1 \left(k_{lev} \right)$ is the share of high-risk loans among all the granted loans in the loan market when the banks follow the strategies that we just described, and the condition states this share is larger than or equal with the share of high-risk loans in the portfolios of the mixed-portfolio banks. This statement must, obviously, be valid if the only banks that there are on the market in addition to the mixed-portfolio banks are high-risk loan banks which specialize in high-risk loans.

We now wish to demonstrate that when (29) is valid, the strategies that we have just described constitute an equilibrium of the model. The assumption (27) immediately implies that in the considered case both the high-risk banks and the banks with the mixed portfolio have zero value. It remains to be shown that there is no other strategy choice for the banks that would yield a positive value for the bank, i.e., that a bank which chooses a loan portfolio with a share α of high-risk projects for which $\alpha \neq \alpha_{lev}$ and $\alpha \neq 1$, cannot have a positive value.

Beginning with the case in which $\alpha_{lev} < \alpha < 1$, it is observed that for a bank with the portfolio α the only binding constraint is the risk-based capital requirement. In this case Lemma 1 immediately implies that the value of the bank with portfolio α cannot be positive.

Consider now a bank with the portfolio α, where $0 \leq \alpha < \alpha_{lev}$. For this portfolio, the only binding constraint is the leverage ratio requirement. In the

analysis of this case, it turns out to be practical to depart from our earlier practice of considering banks of a unit size, and to consider a bank which has a unit of amount of *low-risk loans* in its portfolio. Clearly, a bank of this kind will have a share α of high-risk loans in its portfolio if the number of high-risk loans that it possesses is given by $\alpha/\left(1-\alpha\right)$. The size of the bank must be

$1+\frac{\alpha}{1-\alpha}=\frac{1}{1-\alpha}$

and - given that the leverage ratio requirement is a binding constraint for it - its amount of capital must be given by $k_{lev}/\left(1-\alpha\right)$.

We shall denote the value of a bank of this kind by $V_{LH}\left(\alpha\right)$. Clearly, $V_{LH}\left(\alpha\right)$ and the value $V\left(k,\alpha_k,r_L,\bar{r}_H\right)$ of the corresponding bank of a unit size are related by

$$V_{LH}\left(\alpha\right)=\frac{1}{1-\alpha}V\left(k_{lev},\alpha,r_L,\bar{r}_H\right) \tag{31}$$

The following lemma characterizes the dependence of $V_{LH}\left(\alpha\right)$ on α.

Lemma 3. If (25) is valid and r_L is given by (28), it must be the case that $\frac{\partial V_{LH}(\alpha)}{\partial \alpha}>0$.

Hence, whenever (29) and the equilibrium conditions (27) are valid, the value $V_{LH}\left(\alpha\right)$ of a mixed-portfolio bank is negative whenever $\alpha<\alpha_{lev}$, from which one may, of course conclude that also the value $V\left(k_{lev},\alpha,r_L,\bar{r}_H\right)$ of the corresponding bank with unit size is negative. In other words, whenever the banks are able to choose the strategies that we just described, choosing them will constitute an equilibrium. We summarize the results that we have obtained so far as the following theorem.

Theorem 1. Whenever the leverage ratio requirement k_{lev} lies in the range in which (29) is valid, there is an equilibrium in which there are high-risk banks specializing in high-risk loans only, and mixed-portfolio banks with the portfolio α_{lev}. The result (28) characterizes the interest rates as a function of k_{lev} for these equilibria.

It is also easy to characterize the comparative statics of the equilibrium that we have just found. Just like before the symbols $\bar{r}_L$ and $\bar{r}_H$ refer to the equilibrium interest rates in the absence of the leverage ratio requirements, and the demands for loans that correspond to these interest rates are denoted by $\bar{n}_L=n_L\left(\bar{r}_L\right)$ and $\bar{n}_H=n_H\left(\bar{r}_H\right)$.

Theorem 2. The following statements are valid when the values of k_{lev} lies in the range in which in which (29) is valid:

a) The high-risk interest rate r_H and the demand for high-risk loans n_H are constants, and have the values $\bar{r}_H$ and $\bar{n}_H$ that they would have in the absence of the leverage ratio requirement. However, the number of high-risk loans financed by the specialized high-risk loan banks is decreased.

b) The low-risk interest rate r_L satisfies $r_L \geq \bar{r}_L$, implying that the demand for low-risk loans n_L satisfies $n_L \leq \bar{n}_L$. The low-risk interest rate r_L is a non-decreasing function of k_{lev}, implying that n_L is a non-increasing function of k_{lev}.

c) The share of high-risk loans among all granted loans is larger than or equal with their share in the absence of the leverage ratio requirement, and a non-decreasing function of k_{lev}.

6. The Symmetric Equilibrium with Mixed-portfolio Banks

We now investigate the circumstances under which the model has a symmetric equilibrium in which all the banks have the mixed portfolio α_{lev}. The results of the previous section indicate that an equilibrium of this kind exists at least when

$f_1(k_{lev}) = \alpha_{lev}$

As already explained, in this case the model has an equilibrium in which high-risk interest rate r_H has the value $\bar{r}_H$ that it would have in the absence of the leverage ratio requirement, and the banks financing low-risk loans react to the leverage ratio requirement by including high-risk loans in their portfolio. However, if $f_1(k_{lev}) = \alpha_{lev}$, the mixed-portfolio banks will end up financing all the high-risk projects, and no high-risk projects are left over to the specialized high-risk banks.

When

$\alpha_{lev} > f_1(k_{lev})$

the kind of equilibrium that we considered in the previous section is not possible: if it is the case that $\alpha_{lev} > f_1(k_{lev})$, that $r_H = \bar{r}_H$, and that the banks follow the strategies that we just described, the mixed-portfolio banks will run out of high-risk loans. Intuitively, one may expect that such excess supply of high-risk loans would reduce the interest rates on high-risk loans, and this may be expected to increase the interest rates on low-risk loans (because the reduction in $\bar{r}_H$ makes the mixed portfolio less attractive). Each of these effects tends to increase the share of high-risk loans in the market, and a symmetric equilibrium should be possible if the two interest rates shift to an extent which yields the value α_{lev} for the share of high-risk loans in the loan market. More specifically, one would expect that a symmetric equilibrium should be possible when a moderate increase in r_L suffices to produce the value α_{lev} for the share of high-risk loans; if, however, the necessary raise in r_L is so large that it makes the specialization to low-risk loans preferable to a mixed portfolio, the considered equilibrium will not be possible.

Next we shall investigate rigorously the kind of equilibria to which the above intuitive explanation applies, and we shall begin by giving a rigorous formulation to the last of the points that were made above. We recall that $r_{L,lev}$ denotes

the low-risk interest rate when there are specialized low-risk banks, and that $r_{L,E}\left(\bar{r}_H, k_{lev}, \alpha_{lev}\right)$ denotes the low-risk interest rate in the equilibium that was considered in the previous section. One may conclude from Lemma 3 that

$$r_{L,lev} > r_{L,E}\left(\bar{r}_H, k_{lev}, \alpha_{lev}\right) \tag{32}$$

(because if it were the case that $r_{L,lev} \leq r_{L,E}\left(\bar{r}_H, k_{lev}, \alpha_{lev}\right)$, the interest rate $r_{L,E}\left(\bar{r}_H, k_{lev}, \alpha_{lev}\right)$ would be sufficient to yield a non-negative value for the specialized low-risk bank, contradicting Lemma 3).

It is clear that a symmetric equilibrium cannot exist if the low-risk interest rate would have to raise above $r_{L,lev}$ before the share of high-risk projects could be α_{lev} (because in this situation the strategy of specializing to low-risk projects would be preferable to the mixed-portfolio strategy that we just described). If the low-risk interest rate was $r_{L,lev}$ and the high-risk interest rate had the value $r_{H,E}\left(r_{L,lev}, k_{lev}, \alpha_{lev}\right)$ that corresponds to $r_{L,lev}$ in equilibrium, the share of high-risk loans in the whole loan market would be

$$f_2\left(k_{lev}\right) = \frac{n_H\left(r_{H,E}\left(r_{L,lev}, k_{lev}, \alpha_{lev}\right)\right)}{n_H\left(r_{H,E}\left(r_{L,lev}, k_{lev}, \alpha_{lev}\right)\right) + n_L\left(r_{L,lev}\right)} \tag{33}$$

If $f_2\left(k_{lev}\right) < \alpha_{lev}$, even the interest rate $r_{L,lev}$ is too low to yield the symmetric equilibrium that we considered above. As our next step, we present the result that the symmetric equilibrium exists whenever

$$f_1\left(k_{lev}\right) < \alpha_{lev} < f_2\left(k_{lev}\right) \tag{34}$$

Theorem 3. If (34) is valid, the model has a symmetric equilibrium in which each bank has the portfolio α_{lev}.

It should be observed that, given that the demands n_L and n_H are decreasing functions of the interest rates, one may conclude from (32), (30), and (33) that

$$f_1\left(k\right) < f_2\left(k\right) \tag{35}$$

When k_{lev} ranges from k_L to k_H, the value of $\alpha_{lev} = \left(k_{lev} - k_L\right)/\left(k_H - k_L\right)$ ranges from 0 to 1 but the values of f_1 and f_2 stay positive and smaller than 1. Hence, one may now conclude that there must be values of k_{lev} between k_L and k_H for which α_{lev} is between $f_1\left(k_{lev}\right)$ and $f_2\left(k_{lev}\right)$, i.e. for which (34) is valid.

If all banks have the same portfolio α_{lev} in an equilibrium, the number α_{lev} must be equal with the share of high-risk loans in the whole market. Hence in this case the interest rates r_L and r_H must be such that

$\alpha_{lev} = \frac{n_H(r_H)}{n_H(r_H) + n_L(r_L)}$

Together with the fact that the high-risk interest rate is a decreasing function of the low-risk interest rate, this leads to the following results concerning the comparative statics of the symmetric equilibrium.[14]

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[14]Observe that although the following theorem states that r_H is below $\bar{r}_H$ in the whole

Theorem 4. The following statements are valid when the values of k_{lev} lies in the range in which in which (34) is valid:

a) The high-risk interest rate is smaller and the demand for high-risk loans is larger than in the absence of the leverage ratio requirement.

b) The low-risk interest rate is larger and the demand for low-risk loans is smaller than in the absence of the leverage ratio requirement. The low-risk interest rate is an increasing function of k_{lev}.

c) The share α of high-risk loans is equal with α_{lev}, which is smaller than their share in the absence of the leverage ratio requirement, and an increasing function of k_{lev}.

7. The Equilibrium with Low-risk Loan Banks and Mixed-portfolio Banks

For reasons that were explained in Section 3, we have restricted attention to the equilibria in which each bank has one of the portfolios 0, α_{lev}, and 1. Among such equilibria, there is just one that we have not yet considered, i.e. the equilibrium in which there are specialized low-risk loan banks and mixed-portfolio banks with the portfolio α_{lev}. Analogously with the equilibria of Section 5, the equilibrium conditions for the two kinds of banks may in this case be formulated as

$$\begin{cases} V_L\left(k_{lev}, r_L\right) = 0 \\ V\left(k_{lev}, \alpha_{lev}, r_L, r_H\right) = 0 \end{cases} \tag{36}$$

Clearly, these conditions suffice to determine the interest rates r_L and r_H, and they are solved by

$$\begin{cases} r_L = r_{L,lev} \\ r_H = r_{H,E}\left(r_{L,lev}, k_{lev}, \alpha_{lev}\right) \end{cases} \tag{37}$$

In order to understand intuitively the nature of the equilibria to which (36) and (37) apply, it is helpful to think of a situation in which the value of k_{lev} is only slightly smaller than k_H. For such values the model has an equilibrium in which there are two kinds of banks. Some banks finance low-risk projects only, and for these banks the leverage ratio requirement is the binding constraint, and the interest rate with which these banks are faced by $r_{L,lev}$. As it was explained above, the increase in the required amount of capital tends to decrease the value of the banks, and accordingly,

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range in which (34) is valid, it does not state that r_H is *locally* a decreasing function of k_{lev} for all values of k_{lev} in this range. This is because an increase in k_{lev} increases the capital costs of the symmetric equilibrium bank, and there seems to be no easy way of proving that r_L and r_H cannot *both* increase when k_{lev} is increased. (However, if they did, the increase in r_L would have to be sufficiently large to make the share of high-risk loans among all financed loans increase.)

$$\frac{\partial r_{L,lev}}{\partial k} > 0 \tag{38}$$

All the high-risk projects are financed by the other group of banks, but they finance also some low-risk projects. Their motives for having a mixed portfolio can be understood intuitively as follows. If a bank which finances high-risk loans adds some low-risk loans to its portfolio, the amount of capital that it needs for financing them is not k_{lev} per loan, but k_L per loan because the leverage ratio requirement is not a binding constraint for it. However, the interest rate for low-risk projects has risen to the value which corresponds to k_{lev} and which according to (38) must be larger than the value that corresponds to k_L. Hence, adding low-risk projects to the portfolio will be profitable for the bank until the leverage ratio requirement has become a binding constraint for it, i.e. until the share of high-risk projects has sunk to α_{lev}.[15] However, when the banking sector is competitive, the practice of including low-risk loans in the portfolios of the banks financing high-risk loans tends to lower the interest rates for high-risk loans.

Since the mixed-portfolio banks finance all the high-risk loans in the currently considered situation, the stategies that we just described are possible if the mixed-portfolio banks do not run out of low-risk loans. The mixed-portfolio bank needs $(1 - \alpha_{lev})/\alpha_{lev}$ low-risk loans for each high-risk loan that it finances and, hence, the strategies are possible if and only if

$n_L(r_{L,lev}) \geq ((1 - \alpha_{lev})/\alpha_{lev}) n_H(r_{H,E}(r_{L,lev}, k_{lev}, \alpha_{lev}))$

Using the definition (33), this is seen to be equivalent with

$$\alpha_{lev} \geq f_2(k_{lev}) \tag{39}$$

According to our next theorem, the the currently considered equilibrium exists whenever (39) is valid.

Theorem 5. In the range in which (39) is valid, there is an equilibrium in which there are low-risk loan banks specializing in low-risk loans only, and mixed-portfolio banks with the portfolio α_{lev}. In this equilibrium the interest rates are given by (37).

The following theorem summarizes the basic facts concerning the comparative statics of the currently considered equilibrium. The statement that we make concerning the share of the high-risk loans ((d) below) is quite weak. It seems that a more interesting statement would require some restrictive assumptions on the demand functions n_L and n_H of the two types of loans.

Theorem 6. The following statements are valid when the values of k_{lev} lies in the range in which in which (39) is valid:

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[15]More precisely, this will be the case provided that the increased low-risk interest rate affects the profits of the bank more strongly than the negative effects from a mixed portfolio, but as it was explained in Section 4, the latter effect should be small in a realistically calibrated version of the model.

(a) The interest rate for high-risk loans r_H is lower and the demand for them is higher than in the absence of the leverage ratio requirement.

(b) The interest rate for low-risk loans r_L is higher and the demand for them is lower than in the absence of the leverage ratio requirement.

(c) Both interest rates are increasing, and both demands are decreasing fuctions of k_{lev}.

(d) The share α of high-risk loans among all granted loans is larger than their share in the absence of the leverage ratio requirement.

8. Concluding Remarks

We have shown that the introduction of the leverage ratio requirement, when it interacts with the risk-based IRB capital requirements, might lead to less lending to low-risk customers and to increased lending to high-risk customers. If such allocational effects are counter-productive to financial stability, then they may pose a trade-off against the alleged positive financial stability effects of the leverage ratio requirement.

Our results are based on a theoretical model and are hence qualitative in nature. A natural extension would be to provide numerical examples with the calibrated model of changes in loan prices and quantities after the introduction of a leverage ratio requirement. Another potential extension would be to analyze the trade-off between the potentially negative allocational effects and the positive financial stability effects of the the leverage ratio requirement. In order to do this, one would have to 1) provide a model for why less low-risk lending and more high-risk lending might be socially undesirable, and to 2) model one (or some) of the alleged positive financial stability effects of the leverage ratio requirement.

Appendix. Proofs of Theorems and Lemmata

Proof of Lemma 1. By elementary algebra, it is seen that α_2 may be expressed in the form

$\alpha_2 = \frac{\alpha_3 - \alpha_2}{\alpha_3 - \alpha_1} \alpha_1 + \frac{\alpha_2 - \alpha_1}{\alpha_3 - \alpha_1} \alpha_3 = \beta \alpha_1 + (1 - \beta) \alpha_3$

We may now conclude from (2) that

$\kappa(\alpha_2) = \beta \kappa(\alpha_1) + (1 - \beta) \kappa(\alpha_3)$

and from (7) that

$\rho(\alpha_2, r_L, r_H; z) = \beta \rho(\alpha_1, r_L, r_H; z) + (1 - \beta) \rho(\alpha_2, r_L, r_H; z)$

Hence, one may conclude from (8) and (9) that

$\Pi(k, \alpha_2, r_L, r_H) = \int_{-\infty}^{\widehat{z}_\alpha} \left(\kappa(\alpha_2) + \rho(\alpha_2, r_L, r_H; z) \bar{R} \right) d\Phi(z)$

$$= \int_{-\infty}^{\infty} \min\left\{0, \beta\left(\kappa\left(\alpha_1\right) + \rho\left(\alpha_1, r_L, r_H; z\right) \bar{R}\right)\right.$$
$$\left. + (1-\beta)\left(\kappa\left(\alpha_3\right) + \rho\left(\alpha_3, r_L, r_H; z\right) \bar{R}\right)\right\} d\Phi(z)$$
$$\leq \beta \int_{-\infty}^{\infty} \min\left\{0, \kappa\left(\alpha_1\right) + \rho\left(\alpha_1, r_L, r_H; z\right) \bar{R}\right\} d\Phi(z)$$
$$+ (1-\beta)\min\left\{0, \kappa\left(\alpha_3\right) + \rho\left(\alpha_3, r_L, r_H; z\right) \bar{R}\right\} d\Phi(z)$$
$$= \beta\Pi\left(\kappa\left(\alpha_1\right), \alpha_1, r_L, r_H\right) + (1-\beta)\Pi\left(\kappa\left(\alpha_3\right), \alpha_3, r_L, r_H\right)$$

and from (10) that

$$V\left(\kappa\left(\alpha_2\right), \alpha_2, r_L, r_H\right) = -\kappa\left(\alpha_2\right) + \tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha_2\right), \alpha_2, r_L, r_H\right)$$
$$\leq \beta\left(-\kappa\left(\alpha_1\right) + \tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha_1\right), \alpha_1, r_L, r_H\right)\right)$$
$$+ (1-\beta)\left(-\kappa\left(\alpha_3\right) + \tfrac{1}{1+\delta}\Pi\left(\kappa\left(\alpha_3\right), \alpha_3, r_L, r_H\right)\right)$$
$$= \beta V\left(\kappa\left(\alpha_1\right), \alpha_1, r_L, r_H\right) + (1-\beta)V\left(\kappa\left(\alpha_3\right), \alpha_3, r_L, r_H\right).$$

Proof of Lemma 2. Lemma 2 follows from Lemma 1 by putting
$k_L = k_H = k$.

Proof of Lemma 3. Combining (31) with (7)-(10), it is observed that

$$V_{LH}(\alpha) = -\frac{k_{lev}}{1-\alpha} + \frac{1}{1+\delta}\int_{-\infty}^{\widehat{z}_\alpha}\left(\frac{k_{lev}}{1-\alpha} + \rho_L\left(z, r_L\right)\bar{R} + \frac{\alpha}{1-\alpha}\rho_H\left(z, \bar{r}_H\right)\bar{R}\right)d\Phi(z) \tag{40}$$

Remembering that
$$k_{lev} = \left(1 - \alpha_{lev}\right)k_L + \alpha_{lev}k_H ,$$
the amount of capital of the considered bank, $k_{lev}/(1-\alpha)$, may be expressed in the form
$$\tfrac{k_{lev}}{1-\alpha} = k_L + \tfrac{\alpha}{1-\alpha}k_H + \left(\tfrac{k_{lev}}{1-\alpha} - k_L - \tfrac{\alpha}{1-\alpha}k_H\right)$$
$$= k_L + \tfrac{\alpha}{1-\alpha}k_H + \tfrac{\alpha_{lev}-\alpha}{1-\alpha}\left(k_H - k_L\right)$$
Applying this result to (40), one may conclude that
$$V_{LH}(\alpha) = -k_L + \tfrac{1}{1+\delta}\int_{-\infty}^{\widehat{z}_\alpha}\left(k_L + \rho_L\left(z, r_L\right)\bar{R}\right)d\Phi(z)$$
$$- \tfrac{\alpha}{1-\alpha}k_H + \tfrac{1}{1+\delta}\left(\tfrac{\alpha}{1-\alpha}\right)\int_{-\infty}^{\widehat{z}_\alpha}\left(k_H + \rho_H\left(z, \bar{r}_H\right)\bar{R}\right)d\Phi(z)$$
$$- \tfrac{\alpha_{lev}-\alpha}{1-\alpha}\left(k_H - k_L\right) + \tfrac{1}{1+\delta}\tfrac{\alpha_{lev}-\alpha}{1-\alpha}\int_{-\infty}^{\widehat{z}_\alpha}\left(k_H - k_L\right)d\Phi(z)$$
According to the equilibrium conditions (15) which are valid in the absence of the leverage ratio requirement, the interest rates $\bar{r}_L$ and $\bar{r}_H$ satisfy the conditions
$$V_L\left(k_L, \bar{r}_L\right) = -k_L + \tfrac{1}{1+\delta}\int_{-\infty}^{\widehat{z}_L}\left(k_L + \rho_L\left(z, r_L\right)\bar{R}\right)d\Phi(z) = 0$$
and
$$V_H\left(k_H, \bar{r}_H\right) = -k_H + \tfrac{1}{1+\delta}\int_{-\infty}^{\widehat{z}_H}\left(k_H + \rho_H\left(z, \bar{r}_H\right)\bar{R}\right)d\Phi(z) = 0$$
This allows us to conclude that
$$V_{LH}(\alpha) = V_{LH}(\alpha) - V_L\left(k_L, \bar{r}_L\right) - \tfrac{\alpha}{1-\alpha}V_H\left(k_H, \bar{r}_H\right)$$
$$= \tfrac{1}{1+\delta}\int_{-\infty}^{\widehat{z}_L}\left(\rho_L\left(z, r_L\right) - \rho_L\left(z, \bar{r}_L\right)\right)\bar{R}d\Phi(z)$$
$$+ \tfrac{1}{1+\delta}\int_{\widehat{z}_L}^{\widehat{z}_\alpha}\left(k_L + \rho_L\left(z, r_L\right)\bar{R}\right)d\Phi(z)$$
$$+ \tfrac{\alpha}{1-\alpha}\left(\tfrac{1}{1+\delta}\right)\left(\int_{\widehat{z}_H}^{\widehat{z}_\alpha}\left(k_H + \rho_H\left(z, \bar{r}_H\right)\bar{R}\right)d\Phi(z)\right)$$
$$- \tfrac{\alpha_{lev}-\alpha}{1-\alpha}\left(k_H - k_L\right)\left(1 - \tfrac{1}{1+\delta}\Phi\left(\widehat{z}_\alpha\right)\right)$$

Forming the derivative of $V_{LH}(\alpha)$ with respect to α, it is observed that also $\widehat{z}_\alpha$ depends on α, but this dependence does not affect the value of $\partial V_{LH}/\partial \alpha$, because

$$\frac{\partial V_{LH}}{\partial \widehat{z}_\alpha} = \frac{\varphi(\widehat{z}_\alpha)}{1+\delta}\left(k_L + \rho_L(\widehat{z}_\alpha, r_L)\bar{R} + \frac{\alpha}{1-\alpha}\left(k_H + \rho_H(\widehat{z}_\alpha, \bar{r}_H)\bar{R}\right) + \frac{\alpha_{lev}-\alpha}{1-\alpha}(k_H - k_L)\right)$$

$$= \frac{\varphi(\widehat{z}_\alpha)}{1+\delta}\left(k_{lev} + \rho_L(\widehat{z}_\alpha, r_L)\bar{R} + \frac{\alpha}{1-\alpha}\rho_H(\widehat{z}_\alpha, \bar{r}_H)\bar{R}\right)$$

and the number in parantheses is the profit of the bank when $z = \widehat{z}_\alpha$, and according to the definition of $\widehat{z}_\alpha$ this profit equals zero. Hence,

$$\frac{\partial V_{LH}}{\partial \widehat{z}_\alpha} = 0$$

Since

$$\frac{\partial}{\partial \alpha}\left(\frac{\alpha}{1-\alpha}\right) = \frac{1}{(1-\alpha)^2} \text{ and } \frac{\partial}{\partial \alpha}\left(\frac{\alpha_{lev}-\alpha}{1-\alpha}\right) = -\frac{1-\alpha_{lev}}{(1-\alpha)^2}$$

we may now conclude that

$$\frac{\partial V_{LH}}{\partial \alpha} = \frac{1}{(1-\alpha)^2(1+\delta)}\left(\int_{\widehat{z}_H}^{\widehat{z}_\alpha}\left(k_H + \rho_H(z, \bar{r}_H)\bar{R}\right)d\Phi(z)\right)$$

$$+ \frac{1-\alpha_{lev}}{(1-\alpha)^2}(k_H - k_L)\left(1 - \frac{1}{1+\delta}\Phi(\widehat{z}_\alpha)\right)$$

Let $\bar{\alpha}$ denote the share of high-risk loans among all granted loans. Given that there are also banks which specialize in high-risk loans, it must be the case that $\alpha_{lev} \leq \bar{\alpha}$, and - remembering that $\Phi(\widehat{z}_\alpha) < 1$ - we may conclude that

$$\begin{align}
(1+\delta)(1-\alpha)^2\,(\partial V_{LH}/\partial \alpha)& \\
> \left(\int_{\widehat{z}_H}^{\widehat{z}_\alpha}\left(k_H + \rho_H(z, \bar{r}_H)\bar{R}\right)d\Phi(z)\right) &+ (1-\bar{\alpha})\delta(k_H - k_L)
\end{align} \tag{41}$$

Next, we observe that the currently considered mixed portfolio bank cannot fail for those values of z for which neither a high-risk loan bank nor a low-risk bank would fail in the absence of the leverage ratio requirement, and hence, $\widehat{z}_\alpha \geq \min\{\widehat{z}_L, \widehat{z}_H\}$. We now consider separately the cases in which $\widehat{z}_\alpha \geq \widehat{z}_H$ and $\widehat{z}_L \leq \widehat{z}_\alpha < \widehat{z}_H$.

Beginning with the case in which $\widehat{z}_H \leq \widehat{z}_\alpha$, it is observed that the integral in the above formula will be negative in this case, since in this case the the values of z between $\widehat{z}_H$ and $\widehat{z}_\alpha$ correspond to cases in which a high-risk project bank would fail in the absence of the leverage ratio requirement. In this case we use (11) to arrive at the approximation

$$(1+\delta)(1-\alpha)^2\,(\partial V_{LH}/\partial \alpha) >$$
$$\inf_z\left(k_H + \rho_H(z, \bar{r}_H)\right)(1 - \Phi(\widehat{z}_H)) + \delta(1-\bar{\alpha})(k_H - k_L)$$
$$= \delta(1-\bar{\alpha})(b_H - b_L)\bar{R} - (\lambda - b_H)(1 - \Phi(\widehat{z}_H))\bar{R}$$

This number is positive whenever (25) is valid .

However, it is also conceivable to that $\widehat{z}_L \leq \widehat{z}_\alpha < \widehat{z}_H$. Intuitively, in this case the capital requirement for high-risk projects is so high that a bank financing only them has a *smaller* failure probability than a bank financing only low-risk projects. In this case the integrand in (41) will be positive but the integral is nevertheless negative because its lower bound is larger than its upper bound, and using (11) we arrive at the approximation

$$(1+\delta)(1-\alpha)^2\,(\partial V_{LH}/\partial \alpha)$$
$$> \delta(1-\bar{\alpha})(k_H - k_L) - \sup_z\left\{k_H + \rho_H(z, \bar{r}_H)\bar{R}\right\}(1 - \Phi(\widehat{z}_L))$$
$$= \delta(1-\bar{\alpha})(b_H - b_L)\bar{R} - (b_H + r_H)(1 - \Phi(\widehat{z}_L))\bar{R}$$

Also this number is positive when the assumption (25) is valid and we can now conclude that in either case,

$\frac{\partial V_{LH}}{\partial \alpha} > 0$.

This completes the proof.

Proof of Theorem 2. Part (a) of this theorem follows from the definition of the considered equilibrium, and (c) is a trivial consequence of (a) and (b). Hence, it suffices to prove (b). Given that the low-risk interest rate r_L is determined by the equilibrium condition

$V(k_{lev}, \alpha_{lev}, r_L, \bar{r}_H) = 0$,

it is not, in general, independent of k_{lev}. To see why r_L cannot decrease as a function of k_{lev}, let k_1 and k_2, $k_1 < k_2$, be two values of k_{lev} for which (29) is valid. If r_{L1} and r_{L2} are the low-risk interest rates that correspond to k_1 and k_2, and if the values of α_{lev} which correspond to $k_{lev} = k_1$ and $k_{lev} = k_2$ in accordance with (20)are denoted by to α_1 and α_2, it must be the case that

$V(k_1, \alpha_1, r_{L1}, \bar{r}_H) = 0$,

and - since also the high-risk banks have zero value in equilibrium-

$V(k_H, 1, r_{L1}, \bar{r}_H) = 0$,

and one may conclude from Lemma 1 that

$V(k_2, \alpha_2, r_{L1}, \bar{r}_H) \leq 0$.

Now it cannot be the case that $r_{L2} < r_{L1}$ because V is an increasing function of r_L, and if it were the case that $r_{L2} < r_{L1}$, one could conclude that

$V(k_2, \alpha_2, r_{L2}, \bar{r}_H) < V(k_2, \alpha_2, r_{L1}, \bar{r}_H) \leq 0$.

Proof of Theorem 3. Assume that

$$f_1(k_{lev}) \leq \alpha_{lev} \leq f_2(k_{lev}) \tag{42}$$

and define r_{H2} by $r_{H2} = r_{H,E}(r_{L,lev}, k_{lev}, \alpha_{lev})$. Clearly, the definitions (23) and (24) of the functions $r_{H,E}$ and $r_{L,E}$ imply that

$r_{L,lev} = r_{L,E}(r_{H2}, k_{lev}, \alpha_{lev})$

so that the condition (42) may be put into the form

$$g(\bar{r}_H) \leq \alpha_{lev} \leq g(\bar{r}_{H2}) \tag{43}$$

where the function g is defined by

$g(r_H) = \frac{n_H(r_H)}{n_H(r_H) + n_L(r_{L,E}(r_H, k_{lev}, \alpha_{lev}))}$

One can now conclude from (3) and the fact that $r_{L,E}$ is a decreasing and continuous function that g is an decreasing and continuous function of r_H. Hence there must be a value of r_H between r_{H2} and $\bar{r}_H$ for which

$$g(r_H) = \alpha_{lev} \tag{44}$$

Consider now the case in which the high-risk interest rate has the value defined by (44), and the low-risk interest rate is

$r_L = r_{L,E}(r_H, k_{lev}, \alpha_{lev})$

i.e. the low-risk interest rate has the value which makes the equilibrium condition (22) valid for r_H. Now the equilibrium condition (22) is by construction valid for the given r_H and r_L, and according to (44) also the supply of loans by the banks matches their demand if all the banks have the portfolio α_{lev}. In addition, the interest rate for high-risk loans is lower that $\bar{r}_H$, which is the interest rate that would be needed for giving the specialized high-risk loan banks a non-negative value, and the interest rate for low-risk loans is lower than $r_{L,lev}$, i.e the interest rate which would be needed for giving the specialized low-risk loan banks a non-negative value. Hence the considered situation is an equilibrium.

Proof of Theorem 4. The part (c) of this theorem follows immediately from the definition of the considered equilibrium. It was demonstrated within the proof of Theorem 3 that $r_H \leq \bar{r}_H$, which implies that

$r_L \geq r_{L,E}\left(\bar{r}_H, k_{lev}, \alpha_{lev}\right) > \bar{r}_L$

In other words, the high-risk interest rate is lower and low-risk interest rate is higher than in the absence of the leverage ratio requirement. It still should be proved that the low-risk interest rate is an increasing function of k_{lev}.

To see this, consider two values $k_{lev} = k_1$ and $k_{lev} = k_2$ of k_{lev} for which the symmetric equilibrium exists, and for which $k_1 < k_2$. At least one of the two interest rates r_L and r_H must be larger when $k_{lev} = k_2$ than when $k_{lev} = k_1$ because the mixed portfolio banks are subject to a larger capital requirement when $k_{lev} = k_2$. It is now observed that it cannot be the case that r_L does not increase but r_H increases if the leverage ratio requirement shifts from $k_{lev} = k_1$ to $k_{lev} = k_2$, because in this case also the market share of high-risk projects would have to decrease. Hence, the low-risk interest rate is an increasing function of k_{lev}.

Proof of Theorem 5. We have already shown that the strategies of the two kinds banks are possible whenever (39) is valid. The condition (36) immediately implies that the loan portfolios of the two kinds of banks, i.e. the portfolio $\alpha = 0$ and the portfolio $\alpha = \alpha_{lev}$, correspond to banks with zero value. It remains to be shown that none of the other loan portfolios, i.e. a portfolio α for which either $0 < \alpha < \alpha_{lev}$ or $\alpha_{lev} < \alpha \leq 1$, can lead to a positive value of the bank in the considered case. To see this, it is first observed that if a bank had a portfolio α for which $0 < \alpha < \alpha_{lev}$, for it the only binding constraint would be the leverage ratio constraint, and one could conclude by applying Lemma 2 with $\alpha_1 = 0$ and $\alpha_3 = \alpha_{lev}$ that the value of the bank was at most zero.

Our proof that none of the portfolios α for which $\alpha_{lev} < \alpha \leq 1$ corresponds to a bank with a positive value will be based on a more general result which is concerned with the dependence of the equilibrium interest rate r_H on the size of the of leverage ratio requirement in the region in which the currently considered strategies are possible. We wish to show that in this region

$\frac{dr_H}{dk_{lev}} > 0$

To show this, consider now the behaviour of the high-risk interest rate r_H, which is determined by the condition

$V\left(k_{lev}, \alpha_{lev}, r_{L,lev}, r_H\right)$

$$= -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}_{lev}} \left(k_{lev} + (1-\alpha_{lev}) \rho_L (z, r_{L,lev}) \bar{R} + \alpha_{lev} \rho_H (z, r_H) \bar{R} \right) d\Phi(z)$$
$$= 0$$

in which $\widehat{z}_{lev}$ is the value of z for which the integrand is zero.

Below it turns out to be practical not to consider a bank of the unit size, but a bank which has a unit amount of *high-risk loans* in its loan portfolio. For a bank of this kind, the share of high-risk loans among all the granted loans will be α_{lev} if the size of the bank is $1/\alpha_{lev}$, and in this case the bank has $(1-\alpha_{lev})/\alpha_{lev}$ low-risk loans. We shall denote the value of a bank of this kind by $V_{HL}(k_{lev}, r_H)$. It is clear that $V_{HL}(k_{lev}, r_H)$ must satisfy

$$V_{HL}(k_{lev}, r_H) = \frac{1}{\alpha_{lev}} V(k_{lev}, \alpha_{lev}, r_{L.lev}, r_H) \tag{45}$$

This is equivalent with

$V_{HL}(k_{lev}, r_H)$
$= -\frac{k_{lev}}{\alpha_{lev}} + \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}_{lev}} \left(\frac{k_{lev}}{\alpha_{lev}} + \frac{1-\alpha_{lev}}{\alpha_{lev}} \rho_L(z, r_{L,lev}) \bar{R} + \rho_H(z, r_H) \bar{R} \right) d\Phi(z)$

On the other hand, the value of the low-risk loan bank is given by

$V_L(k_{lev}, r_{L,lev}) = -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}_{L,lev}} \left(k_{lev} + \rho_L(z, r_{L,lev}) \bar{R} \right) d\Phi(z) = 0$

where $\widehat{z}_{L,lev}$ is the value of z for which the integrand is zero. Combining the las two results one gets

$V_{HL}(k_{lev}, r_H) = V_{HL}(k_{lev}, r_H) - \frac{1-\alpha_{lev}}{\alpha_{lev}} V_L(k_{lev}, r_{L,lev})$
$\quad = \frac{1}{1+\delta} \left(\frac{1-\alpha_{lev}}{\alpha_{lev}} \right) \int_{\widehat{z}_{L,lev}}^{\widehat{z}_{lev}} \left(k_{lev} + \rho_L(z, r_{L,lev}) \bar{R} \right) d\Phi(z)$
$\quad\quad -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}_{lev}} \left(k_{lev} + \rho_H(z, r_H) \bar{R} \right) d\Phi(z)$

The mixed-strategy bank and the low-risk loan bank have the same amount of capital per loan althhough the loans of the mixed-strategy bank are partially riskier, and hence, the failure probability of the mixed-strategy bank is larger, so that

$$\widehat{z}_{lev} < \widehat{z}_{L,lev} \tag{46}$$

According to (20),
$\frac{1-\alpha_{lev}}{\alpha_{lev}} = \frac{k_H - k_{lev}}{k_{lev} - k_L}$
Hence, the value of V_{HL} can more naturally be expressed in the form

$V_{HL}(k_{lev}, r_H) = -\left(\frac{k_H - k_{lev}}{k_{lev} - k_L} \right) \left(\frac{1}{1+\delta} \right) \int_{\widehat{z}_{lev}}^{\widehat{z}_{L,lev}} \left(k_{lev} + \rho_L(z, r_{L,lev}) \bar{R} \right) d\Phi(z)$
$\quad -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}_{lev}} \left(k_{lev} + \rho_H(z, r_H) \bar{R} \right) d\Phi(z)$

As the next step, we form an estimate the partial derivative of V_{HL} with respect to k_{lev}. Remembering (46), we conclude that

$\frac{\partial}{\partial k_{lev}} V_{HL}(k_{lev}, r_H)$
$\quad < \frac{k_H - k_L}{(k_{lev} - k_L)^2} \left(\frac{1}{1+\delta} \right) \int_{\widehat{z}_{lev}}^{\widehat{z}_{L,lev}} \left(k_{lev} + \rho_L(z, r_{L,lev}) \bar{R} \right) d\Phi(z)$
$\quad\quad - \left(1 - \frac{1}{1+\delta} \Phi(\widehat{z}_{lev}) \right)$

Since $\widehat{z}_{lev} < \widehat{z}_{L,lev}$, the values of z in the integrand are values for which a low-risk loan bank with capital k_{lev} does *not* fail, and we now arrive at the approximation

$\frac{\partial}{\partial k_{lev}} V_{HL}\left(k_{lev}, r_H\right)$

$\quad < -\frac{\delta}{1+\delta} + \frac{k_H - k_L}{\left(k_{lev} - k_L\right)^2}\left(\frac{1}{1+\delta}\right)\left(k_{lev} + r_{L,lev}\bar{R}\right)\left(\Phi\left(\widehat{z}_{L,lev}\right) - \Phi\left(\widehat{z}_{lev}\right)\right)$

Here $k_{lev} < k_H$ and $r_{L,lev} < \bar{r}_H$. We observe that

$\quad \Phi\left(\widehat{z}_{lev}\right) > \min\left\{\Phi\left(\widehat{z}_L\right), \Phi\left(\widehat{z}_H\right)\right\},$

Since α_{lev} is given by (20), one may also conclude that

$\quad \frac{\partial}{\partial k_{lev}} V_{HL}\left(k_{lev}, r_H\right)$

$\quad\quad < \frac{1}{1+\delta}\left(-\delta + \frac{1}{\alpha_{lev}^2}\frac{b_H + r_H}{b_H - b_L}\max\left\{1 - \Phi\left(\widehat{z}_L\right), 1 - \Phi\left(\widehat{z}_H\right)\right\}\right)$

Since banks that do not have the portfolio α_{lev} are specializing in low-risk loans, the value α_{lev} must be at least as large as the share $\bar{\alpha}$ of the high-risk projects among all financed projects. Hence, the above condition is valid at least when

$\quad \max\left\{1 - \Phi\left(\widehat{z}_L\right), 1 - \Phi\left(\widehat{z}_H\right)\right\} < \delta\bar{\alpha}^2\frac{b_H - b_L}{b_H + r_H}$

i.e. when the assumption (26) is valid. In other words, the condition (26) implies that

$\quad \frac{\partial}{\partial k_{lev}} V_{HL}\left(k_{lev}, r_H\right) < 0$

On the other hand,

$\quad \frac{\partial}{\partial r_H} V_{HL}\left(k_{lev}, r_H\right) > 0$

and when the equilibrium interest rate r_H is viewed as a function $r_H\left(k_{lev}\right)$ of k_{lev}, it must satisfy

$\quad 0 = \frac{dV_{HL}\left(k_{lev}, r_H\left(k_{lev}\right)\right)}{dk_{lev}} = \frac{\partial}{\partial k_{lev}} V_{HL}\left(k_{lev}, r_H\right) + \frac{dr_H}{dk_{lev}}\frac{\partial}{\partial r_H} V_{HL}\left(k_{lev}, r_H\right)$

and this can only be the case if

$$\frac{dr_H}{dk_{lev}} > 0 \tag{47}$$

In the limit in which k_{lev} approaches k_H the equilibrium of the model approaches the equilibrium of a model with the flat-rate capital requirement k_H. Hence, in this limit r_H must approach $\bar{r}_H$, and we may now conclude from (47) that in the situation which is considered in this section

$$r_H < \bar{r}_H \tag{48}$$

We now conclude the profile 1 - i.e. the profile of specialized high-risk loan bank - must correspond to a bank with a negative value in the currently conisdered situation. Utilizing Lemma 1, we further conclude that each profile α for which $\alpha_{lev} < \alpha \leq 1$ corresponds to a bank with a negative value. This completes the proof.

Proof of Theorem 6. The claims concerning low-risk interest rates are trivial consequences of the fact that the amount of capital of a specialized low-risk bank larger than the amount of capital k_L in the absence of the leverage ratio requirement, and equal with k_{lev}. The facts that $r_H < \bar{r}_H$ and that r_H is an increasing function of k_{lev} have already been presented as results (48) and (47) within the proof of Theorem 5. Finally, the statement (d) follows trivially from (a) and (b).

References

[1] Acharya, V.V., T. Cooley, M. Richardson, I. Walter, 2009. Manufacturing Tail Risk: A Perspective on the Financial Crisis of 2007–2009. Foundations and Trends in Finance 4, 247–325.

[2] Admati, A.R., DeMarzo, P.M., Hellwig, M.F., Pfleiderer, P., 2010. Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is *not* expensive? Standford GSB Research Paper No. 2063.

[3] Basel Committee on Banking Supervision, 2009, Strengthening the resilience of the banking sector - consultative document (Bank for International Settlements, December 2009).

[4] Blum, J.M., 2007. Why 'Basel II' may need a leverage ratio restriction? Journal of Banking and Finance 32:8, 1699-1707.

[5] Hellwig, M., 2010. Capital regulation after the crisis: Business as usual? Preprints fo the Max Planck Institute for Research on Collective Goods, Bonn 2010/31.

[6] Repullo, R., Suarez, J., 2004. Loan pricing under Basel capital requirements. Journal of Financial Intermediation 13, 496-521.