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SOME TESTS OF PHYSICS BEYOND THE STANDARD MODEL

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Some tests of physics beyond the Standard Model

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Academic dissertation

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Abstract

Quantum field theories are the basis of modern elementary particle physics. Our present understanding of both the smallest structures of matter and the composition of the universe is based on the quantum field theories, that present observable phenomena by describing particles as vibrations of fields. These fields are quantized, *i.e.* they get discrete values. The Standard Model of particle physics is a quantum field theory that combines electromagnetic, weak and strong interactions into a single gauge field theory. The Standard Model describes physics properly only to a certain upper limit of energy scale. This scale is of the order of the masses of the gauge fields ($W, Z^{\pm}$). Beyond this electroweak scale the Standard Model must be modified. For example, the Standard Model cannot explain the quadratic divergences that plaque one of the particles of the theory (the Higgs boson). Because the valid model must be viable to the highest possible energy scale (Planck scale, 10^{19} GeV, which is determined as a scale where the gravity becomes relevant), there must necessarily be some new physics beyond the electroweak scale. In this dissertation I present some viable models that describe physics beyond the Standard Model. These include supersymmetric quantum field theories with specific supersymmetry breaking mechanisms, split supersymmetry and the model with large extra dimensions. I derive limits for parameters of these models and discuss the possibilities of finding evidence of these in future particle accelerators.

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I thank Helsinki Institute of Physics for providing an excellent working environment. I have greatly enjoyed the atmosphere, for which I would like to thank all the personnel of the institute. Also people from SEFO and TFO has been most helpful and crucial for my learning, for which I thank them all. A great thanks goes to the fellow graduate students, with whom I have had pleasant time. Special thanks to my officemates, who have borne my constant desk tapping and random comments (on everything, mostly).

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Finally, I express my deep gratitude to my parents, and my sister and brother, for being the firm tower of strength at all times.

Helsinki, December 2005
Jari Laamanen

Contents

List of accompanying publications

This Ph.D. dissertation consists of two parts. The first is an introduction to the theory of physics beyond the Standard Model – supersymmetry and extra dimensions – and a discussion of related research. The second part is the set of following reprinted articles that have been published in a scientific journal.

I K. Huitu, J. Laamanen and P. N. Pandita, “Sparticle spectrum and constraints in anomaly mediated supersymmetry breaking models,” Phys. Rev. **D65**, 115003 (2002)

II K. Huitu, J. Laamanen and P. N. Pandita, “Upper bounds on the mass of the lightest neutralino,” Phys. Rev. **D67**, 115009 (2003)

III K. Huitu, J. Laamanen, P. Roy and S. Roy, “Infrared fixed point of the top Yukawa coupling in split supersymmetry,” Phys. Rev. **D72**, 055002 (2005)

IV K. Huitu, J. Laamanen, P. N. Pandita and S. Roy, “Phenomenology of non-universal gaugino masses in supersymmetric grand unified theories,” Phys. Rev. **D72**, 055013 (2005)

V A. Datta, K. Huitu, J. Laamanen and B. Mukhopadhyaya, “Linear collider signals of an invisible Higgs boson in theories of large extra dimensions,” Phys. Rev. **D70**, 075003 (2004)

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Author's contribution

Paper I: I calculated independently the sparticle mass sum-rules in anomaly mediated supersymmetry breaking scenarios as well as the focus points that form the major results in the paper. I also calculated and generated the focus point figure and the spectrum figures for different AMSB models by writing an appropriate computer code.

Paper II: I calculated the mass matrix expansion to get the mass formulae of the neutralinos presented in the paper. I also came up with an idea of using the functional form of the expansions to get an upper limit for the neutralino masses. I wrote a computer code in order to plot the figures presented in the paper and also modified an existing one in order to compare different scenarios.

Paper III: I calculated the renormalization group running of the split SUSY parameters. I also inspected the possibility to incorporate vacuum stability bounds to the parameters. These considerations were left, with some other material, to a forthcoming work.

Paper IV: I calculated the renormalization group running of parameters in the case of non-universal gaugino masses. I also modified existing programs to include the non-universal models. I calculated and made the figures in the sections II and IV and the corresponding discussion in the text.

Paper V: I calculated the effective decay width formula presented in the paper. I also calculated the invisible branching ratio of the Higgs boson and the cross section that results from the invisibly decaying Higgs and gravitensor (and the SM contribution) production which was the main result of our paper.

The writing and interpretation of the results were jointly done in all of the publications.

Chapter 1

Introduction

1.1 Background

The Standard Model (SM) of particle physics provides a quantitative description of the elementary particles and forces of nature. This theory joins successfully three of the known four fundamental forces: electromagnetic, weak and strong force.

In modern physics, particles are described by fields. It is not to say that fields are more fundamental than particles, but the effects of quantization are better understood by using field description. In ordinary quantum mechanics a physical state is a ray in Hilbert space, and observables are operators. Thus a spatial displacement $\mathbf{X}$ is an operator, whereas time t is just a parameter. However, Einstein's special relativity requires that the time and displacement are treated similarly. Elevating time to be an operator would lead to a continuous energy spectrum that is not bounded from below. Having discretized energy levels with a minimum energy would then be impossible. Instead, reducing the displacement to be a mere parameter x introduces a dynamical operator type quantity $\psi = \psi(t, x)$, called quantum field. Merging special relativity with quantum mechanics in a field theory results in a most useful tool to describe high energy particles, namely *quantum field theory* (QFT).

Quantum field theory started from the study of relativistic wave equations. It got it's early form in late 1920's when Dirac proposed a theory for quantizing electromagnetic field [6,7] after the work done by Schrödinger and several others. Dirac's method was regarded as a second quantization, since fields that were quantized were wave functions of one-particle quantum mechanics. Nowadays this is known as canonical quantization due to it's close analogy with classical field theory and mechanics.

By using definite time coordinate, one is implicitly choosing an inertial frame while doing the quantization. Therefore the Poincaré invariance is not manifest but must always be checked at the end of the procedure. This can be quite tricky *e.g.* in the case of non-Abelian gauge theories. In 1929 Heisenberg and

Pauli [8, 9] applied the Lagrangian formalism to the field quantization. The field equations are derived from the requirement that the action is stationary under the variation of fields. The idea of using variational methods in classical physics is very old, stating back to Fermat's principle in optics in 1657 and Maupertuis' principle of least action[1] in 1744 (and further developed by the Euler, Lagrange, and Hamilton, to whom the principle is closely related).

In late 1940's Richard Feynman generalized the action principle in his path integral formulation of quantum mechanics [10, 11, 12, 13, 14]. This formalism is manifestly Lorentz invariant at all stages. Applying this method to electromagnetism, one obtains the theory of electromagnetic interactions which is called *quantum electrodynamics* (QED). The same result was derived separately by Schwinger [15, 16, 17] and Tomonaga [18, 19] by using operator formalism. These formalisms were later shown to be equivalent by Freeman Dyson. Dyson also showed that all the infinities arising in the process are of the type that can be removed by renormalization. A theory is said to be renormalizable if all the infinities can be removed by redefinition of a finite number of parameters of the theory. The importance of renormalizability for predictability is apparent; the renormalizable theories can be expressed with help of finite number of constants, and all quantities can be predicted using those.

High energy particle physics experiments are carried out in particle accelerators. Those are nowadays huge enterprises conducted by joint organizations, the most prominent being the Large Hadron Collider (LHC) of CERN (European Organization for Nuclear Research). It is a proton-proton machine planned to collide particle beams at an energy of 14 TeV. Also beams of heavy nuclei, like lead and gold, will be accelerated. After completion, LHC will be the largest particle accelerator in the world. The circumference of the circular beam tube is 27 km and it is placed 100 meters below ground. The first run is scheduled to take place in late 2007. LHC is being built in place of Large Electron-Positron collider (LEP), which ceased to operate in 2000. Other major colliders include Tevatron in Fermilab, DESY in Hamburg and SLAC in Stanford University.

Quantum electrodynamics, as electrodynamics, is based on a $U(1)$ symmetry. A similar *gauge* symmetry can be found also for the weak and strong forces. By combining these, the Standard Model of particle physics is formed. It is evident that symmetries play a very important role in the development of physical theories. Symmetries imply conservation laws. This is known as a Noether's theorem. In a quantum field theory it can be stated: If an action is invariant under some group of transformations, then there exists one or more conserved quantities that are associated to these transformations. Thus, finding a theory that contains some wanted conservation laws is equivalent to finding an action that is invariant under some set of transformations.

[1] More correctly, "principle of *stationary* action" in the view of calculus of variations.

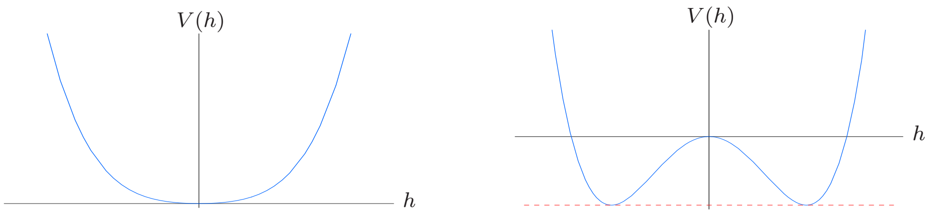

Figure 1.1: Scalar potential of unbroken and spontaneously broken models

1.2 Gauge theories: Standard Model

The mere conservation laws are not enough to form the structure of a proper theory. A physically meaningful particle physics theory must also describe interactions between the particles. In 1961 Salam and Ward [20] proposed a so-called gauge principle which states that the interaction terms of strong, weak and electromagnetic interactions can be generated by making local gauge transformations on the kinetic terms in the free Lagrangian for all particles.

Gauge invariance is a statement that the physics does not change in a certain type of local transformations of the fields. This is based on the fact that in a field theory it is possible to introduce to a model so-called gauge fields with certain transformation properties. The gauge field transformations are such that they exactly cancel the local transformations. In that case the theory is said to be gauge invariant.

The gauge field of the quantum electrodynamics is photon. Following the example of QED, the theory of weak interactions can be formulated using gauge fields that mediate the weak force. This way, *e.g.*, the unitarity violation problem of four-fermion Fermi interaction can be avoided. In contrast to the massless photon of the QED, the intermediate gauge bosons should be massive. The gauge invariance, however, forbids the insertion of mass terms to the Lagrangian for the gauge bosons or the fermions. Therefore some specific mechanism that generates masses, must be introduced. The way to achieve this is to break a local symmetry spontaneously, and this is known as *Higgs mechanism* [21, 22, 23].

The idea of Higgs mechanism is that the minimum of the scalar potential is obtained with non-zero vacuum expectation value (*vev*) of the Higgs field. This results in a lower total energy density than it would be for a zero Higgs field *vev*. This can be illustrated by an energy density $\mathscr{L} \supset -m^2h^2 - \lambda h^4$. If m^2 is negative and λ positive, the minimum of the potential is not at the zero of the Higgs field, but rather at the $h^2 = -4\lambda/m^2$. The requirement of m^2 being negative seems rather unnatural at this point, and actually in the SM it is introduced by hand for this purpose (*ad hoc*). However, in supersymmetric models this condition is naturally achieved as can be seen later.

The Higgs field has couplings with other particles (at the tree level) of the theory, especially with the gauge fields. If the expectation value of the Higgs field is non-zero, the coupled terms do not vanish at the minimum of the potential, but give contributions to the gauge field terms. It turns out that the gauge fields acquire masses. In the same way, the Higgs mechanism generates masses for the fermions. The masses are produced in a renormalizable way, thus preserving the predictability of the model.

The Higgs particle is fundamentally different particle than the other SM particles. First of all, it is the only scalar particle in the SM. It is introduced to the theory in order to trigger the spontaneous electroweak symmetry breaking. When the idea of local gauge invariance is combined with the spontaneous symmetry breaking and the Higgs mechanism, the theory for the electromagnetic and weak interactions can be consistently formulated.

The Standard Model is a gauge theory that combines the electromagnetic, weak and strong interactions to a single model. This model is invariant under a particular set of gauge transformations. The gauge group of the Standard Model is $SU(3)_C \times SU(2)_L \times U(1)_Y$, where the subscript C denotes the color force of strong interactions (quantum chromodynamics, QCD), L denotes the left chiral weak interactions and Y denotes the weak hypercharge. The matter particles of the Standard model are three families of quarks and leptons. These are spin-$\frac{1}{2}$ particles, *i.e.*, fermions. They are divided into $SU(2)_L$ doublets, Q_L containing left handed quarks, and L_L containing left-handed leptons. The right-handed fields are put into $SU(2)_L$ singlets e_R, u_R and d_R (in the Standard Model there are no right-handed neutrinos). After imposing the gauge principle, the intermediate gauge bosons that mediate the particle interactions, emerge. The kinetic energy terms for the three $SU(2)_L$ gauge bosons W^i and one $U(1)_Y$ gauge boson B can be written as [24]

$$\mathscr{L}_{ke} = -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{1.1}$$

where the gauge field strengths are defined as

$$W^i_{\mu\nu} = \partial_\nu W^i_\mu - \partial_\mu W^i_\nu + g\epsilon^{ijk}W^j_\mu W^k_\nu, \tag{1.2}$$

$$B_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu. \tag{1.3}$$

The quantity ϵ^{ijk} is a totally antisymmetric tensor. The kinetic and potential terms for the Higgs boson are

$$\mathscr{L}_H = (D^\mu\phi)^\dagger(D_\mu\phi) - (\mu^2|\phi^\dagger\phi| + \lambda|\phi^\dagger\phi|^2), \tag{1.4}$$

where the covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu + i\frac{g}{2}\boldsymbol{\sigma}\cdot\mathbf{W}_\mu + i\frac{g'}{2}B_\mu Y, \tag{1.5}$$

with $\boldsymbol{\sigma}$ being the Pauli matrices (see Appendix A.1). Fermion mass terms are acquired via the Yukawa interactions of the form

$$\mathscr{L}_f = -\lambda_u\overline{Q}_L\phi^c u_R - \lambda_d\overline{Q}_L\Phi d_R - \lambda_e\overline{L}_L\phi e_R + \text{H.C.}. \tag{1.6}$$

The terms are in family space and the Yukawa couplings are 3×3 matrices, and $\phi^c = i\sigma_2\phi^*$. The kinetic terms of the fermions are of the form $\bar{f}i\gamma^\mu D_\mu f$ for each of the fermion multiplet with appropriate part of the covariant derivative (the $SU(2)$ part is absent for the right-handed fields, and the quantities γ are the Dirac gamma matrices, see *e.g.* [24]). The $SU(3)_C$ interactions can be included similarly. When the Higgs mechanism breaks the electroweak symmetry $SU(2)_L \times U(1)_Y$ spontaneously, the gauge bosons and fermions acquire masses. Then there exist three massive weak gauge bosons: One neutral (Z^0) and two charged ($W^\pm$). After the electroweak symmetry breaking there remains a residual $U(1)_{em}$ symmetry that is observed as an electric charge of the particles. The associated gauge boson, photon, remains massless. While the Higgs mechanism explains the gauge boson masses, the Yukawa couplings are in principle arbitrary parameters, thus providing no fundamental understanding of the origin of the fermion masses. Because the Higgs field is giving mass to three gauge bosons, the simplest way to do this is one complex $SU(2)_L$ doublet of scalar fields. The remaining fourth degree of freedom is the physical Higgs boson.

The Higgs boson is the only Standard Model particle that has not been discovered yet. It is hoped for that the next generation particle colliders, like the presently built CERN's Large Hadron Collider, will be able to discover the Higgs particle. There are several theoretical restrictions on the mass of the Higgs boson. First of all, the interactions between particles must be unitary, which sets bounds to the scattering amplitudes [25, 26, 27, 28]. The unitarity bound for the Standard Model Higgs boson at the tree-level is given as [29, 30]

$$m_H^2 \leq \frac{8\pi\sqrt{2}}{5G_F} \approx (780 \text{ GeV})^2. \tag{1.7}$$

The one-loop calculations push the Higgs boson mass value as low as $m_H \lesssim 350$ GeV [31]. If the unitarity bounds are violated, the perturbation theory is no longer a reliable way to calculate physical processes.

Another bound can be imposed by requiring that the model is not trivial. The Higgs self coupling λ changes with the energy scale as stated by the renormalization group equations (see Section 2.5). A pure ϕ^4 theory is said to be trivial in the sense that as the energy scale increases, the coupling constant eventually blows up (the point of divergence is known as Landau pole). In other words, for a finite (positive) value of the coupling λ at the high scale, the coupling constant vanishes at the low scale, hence suppressing interactions. In the SM there are more terms contributing to the running of the quartic coupling λ, so the requirement of model not to be trivial sets bounds to the involved parameters. If the SM stays perturbative up to the grand unification scale, then the bound obtained for the Higgs boson mass is roughly $m_H \lesssim 200$ GeV [32].

Yet another bound is obtained by the requirement that the scalar potential is bounded from below, or in other words, the coupling λ is positive. In the SM the top Yukawa coupling gives negative contribution to the quartic coupling

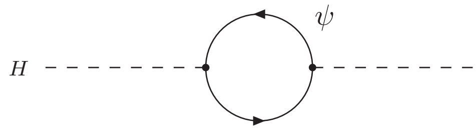

Figure 1.2: Fermion contribution to the self energy of the Higgs boson in the Standard Model.

beta function. For large top mass m_t the coupling λ may become negative. In that case the scalar potential could be negative, $V < 0$, meaning that the vacuum is no longer the minimum of the potential, $V(\lambda) < V(v)$. The requirement that the scalar potential is bounded from below gives a lower bound on the couplings, and thus to the Higgs boson mass. If the SM is valid up to the Planck scale 10^{19} GeV, the vacuum stability requires $m_H > [127.9 + 1.92(m_t - 174) - 4.25(\frac{\alpha_s(m_Z)-0.124}{0.006})]$ GeV [33] (see also [34]), where α_s is the strong coupling constant and the masses are given in units of GeV. If one supposes that some new physics, *e.g.* supersymmetry, is emerging at the scale of 1 TeV, the bound is weakened to $m_H > [52 + 0.64(m_t - 175) - 0.50(\frac{\alpha_s(m_Z)-0.118}{0.006})]$ GeV [35]. The Particle Data Group gives a combined experimental lower limit on the SM Higgs boson mass as $m_H > 114.4$ GeV with 95% confidence level [36, 37].

On one hand, the Standard Model parameters, besides the Higgs boson mass, are measured to a great accuracy, while on the other hand the large number of free parameters that has been introduced to the model by hand is a disadvantage. That would hint towards some more fundamental theory where all the other parameters had a common origin. That has motivated a number of Grand Unified Theories (GUT).

1.2.1 Hierarchy problem

While giving a proper description of particle interactions at the low energy scales, the Standard Model has one important technical problem. The Higgs boson (and also all the other hypothetical fundamental scalars of the model) receives unacceptable large radiative corrections to it's mass term, since there is no such symmetry in the standard model that would issue cancellations to the scalar mass corrections. Avoiding the large corrections would require an extensive fine-tuning of the parameters. This is referred to as a gauge hierarchy problem, since it is related to the two fundamental energy scales that are present in nature. One is the electroweak scale around 100 GeV, and the other is the Planck scale around 10^{19} GeV. The Planck scale M_{P} is the energy scale, where gravity cannot be neglected in particle interactions anymore.

In order to see the hierarchy instability, let us consider a model with a fermion interacting with a massive Higgs boson,

$$\mathscr{L}_\phi = \bar{\psi}(i\partial\!\!\!/)\psi + |\partial_\mu\phi|^2 - m_S^2|\phi|^2 - (\lambda_F\bar{\psi}\psi\phi + \text{H.C.}). \tag{1.8}$$

After the spontaneous symmetry breaking the Higgs field acquires a *vev*, $\langle\phi\rangle = v$, and the original field ϕ is rescaled as $\phi = (H+v)/\sqrt{2}$, where H is the physical Higgs boson. This leads to a fermion mass term $m_F = \lambda_F v/\sqrt{2}$. The self-energy contribution (Fig. 1.2) to the scalar mass can be written as

$$-i\Sigma_S(p^2) = (-i^2)(-i\frac{\lambda_F}{\sqrt{2}})^2 \int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[(\not{k}+m_F)((\not{k}-\not{p})+m_F)]}{(k^2-m_F^2)[(k-p)^2-m_F^2]}. \tag{1.9}$$

The product of two propagators can be simplified in the denominator by using the Feynman parameterization [12]

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[(1-x)a+xb]^2}, \tag{1.10}$$

after which the Eq. (1.9) can be written

$$\Sigma_S(p^2) = -i\frac{2\lambda_F^2}{(2\pi)^4}\int_0^1 dx \int d^4k' \frac{k'^2 + k'p(2x-1)+p^2x(x-1)+m_F^2}{[k'^2-p^2x(x-1)-m_F^2]^2} \tag{1.11}$$

where I have used a momentum shift $k' = k - px$. This is valid for convergent integrals, so some regularization method must be used; cut-off regularization scheme is assumed here.[2] The Eq. (1.11) can be calculated in the Euclidean space by using the Wick rotation [40], after which the integral takes the form

$$\Sigma_S(p^2) = \frac{2\lambda_F^2}{(2\pi)^4}\int_0^1 dx \int d^4k_{\mathrm{E}} \frac{-k_{\mathrm{E}}{}^2 + k_{\mathrm{E}}p(2x-1)+p^2x(x-1)+m_F^2}{[-k_{\mathrm{E}}{}^2-p^2x(x-1)-m_F^2]^2} \tag{1.12}$$

The above integral is divergent, so it must be regularized. This can be done by setting the effective upper limit for integration using the momentum cut-off at the scale Λ. For rotationally symmetric integrands the angular part can be integrated away (see Appendix A.3):

$$\int_{|k_{\mathrm{E}}|<\Lambda} d^4k_{\mathrm{E}} f(k_{\mathrm{E}}^2) = \pi^2 \int_0^{\Lambda^2} dy\, yf(y). \tag{1.13}$$

The Eq. (1.12) is of the required form, since the integration over the odd integrands gives a zero contribution. After applying the Eq. (1.13) and integrating, one gets

$$\Sigma_S(m_S^2) = -\frac{\lambda_F^2}{8\pi^2}\big(\Lambda^2 - \frac{1}{2}(6m_F^2 - m_S^2)\ln\frac{\Lambda^2}{m_F^2} + \mathrm{I}_1(m_S^2, m_F^2) + \mathcal{O}(\frac{1}{\Lambda^2})\big), \tag{1.14}$$

where the integral I_1 is defined as

$$\mathrm{I}_1(m_S^2, m_F^2) = m_F^2 \int_0^1 dx\big(1+\frac{m_S^2}{m_F^2}x(x-1)\big)\big(3\ln\big[1+\frac{m_S^2}{m_F^2}x(x-1)\big]+2\big), \tag{1.15}$$

[2] Also other regularization methods, such as dimensional regularization [38] or Pauli-Villars regularization [39] can be used. Regularization is discussed in some details in the Appendix A.4

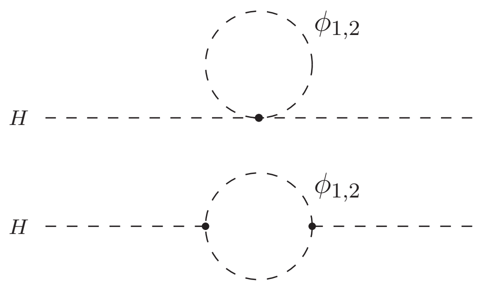

Figure 1.3: Bosonic contributions to the self energy of the Higgs boson in the Standard Model.

which converges if $m_S < 2m_F$, which is the case when the fields ψ are off-shell (see Appendix A.3). The Higgs boson mass is then

$$M_H^2 = M_S^2 + \delta M_H^2, \tag{1.16}$$

where

$$\delta M_H^2 \equiv \Sigma_S(m_S^2). \tag{1.17}$$

From Eq. (1.14) one can see that the mass correction is proportional to the mass of the heaviest particle of the model (the term which is logarithmically divergent). The effect of the correction is clearly seen, if one sets the tree-level mass of the Higgs boson m_S to zero: the self-energy correction is still of the order of the fermion mass. Even more serious than that, the quadratically divergent term in Eq. (1.14) completely wrecks the hope for having the Higgs boson mass around the electroweak scale without enormous fine-tuning. This term can be renormalized away, but that should then be done separately in all orders of perturbation theory, which is quite unattractive. All the other masses and couplings are only logarithmically sensitive to the cut-off. The divergent integral can also be calculated using the dimensional regularization method, in which case the correction is also proportional to the largest mass of the model giving essentially the same problems as the cut-off method. In the Standard Model these divergences appear even if the Higgs boson does not have direct couplings with the heavier particles, since these are generated at the two-loop level via other particles (such as gauge bosons) that couple to both.

There is, however, a way out of this trouble. Consider the same model as in Eq. (1.8), but now including also two additional (spin-0) bosons:

$$\begin{aligned}\mathscr{L} = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 - m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 \\ + \lambda_S\phi(|\phi_1|^2 + |\phi_1|^2) + \mathscr{L}_\phi.\end{aligned} \tag{1.18}$$

After the spontaneous symmetry breaking there are additional two kind of graphs (Fig. 1.3) that contribute to the Higgs boson radiative mass correction. The first one gives the four particle vertex contribution

$$(\delta M_H^2)_{\text{four}} = -\frac{\lambda_S}{16\pi^2}\big(2\Lambda^2 - m_1 \ln\frac{\Lambda^2}{m_1^2} - m_2 \ln\frac{\Lambda^2}{m_2^2} + \mathcal{O}(\frac{1}{\Lambda^2})\big), \tag{1.19}$$

while the second graph gives the three particle vertex contribution

$$\begin{aligned}(\delta M_H^2)_{\text{three}} = \frac{(v\lambda_S)^2}{16\pi^2}\big(2 - \ln\frac{\Lambda^2}{m_1^2} - \ln\frac{\Lambda^2}{m_2^2} \\ + \mathrm{I}_2(m_S^2, m_1^2) + \mathrm{I}_2(m_S^2, m_2^2) + \mathcal{O}(\frac{1}{\Lambda^2})\big),\end{aligned} \tag{1.20}$$

where the integral I_2 is defined as

$$\mathrm{I}_2(m_S^2, m_i^2) = \int_0^1 dx \ln\big[1 + \frac{m_S^2}{m_i^2}x(x-1)\big], \tag{1.21}$$

which again converges if $m_S^2/m_i^2 < 4$. If somehow one can arrange the couplings to be $-\lambda_S = {\lambda_F}^2$, the quadratically divergent terms in Eq. (1.14) and Eq. (1.19) neatly cancel.[3] Furthermore, an interesting result follows, if $m_1^2 = m_2^2 = m_F^2$. Because the *vev* of the Higgs field defines the fermionic mass term as $v^2 = 2m_F^2/\lambda_F^2$, one can write the three particle vertex contribution as

$$(\delta M_H^2)_{\text{three}} = \frac{2m_F^2}{16\pi^2}\big(\frac{\lambda_S}{\lambda_F}\big)^2\big[2 - \ln\frac{\Lambda^2}{m_1^2} - \ln\frac{\Lambda^2}{m_2^2} + \text{finite}\big]. \tag{1.22}$$

After combining all the corrections and requiring that the fermion and boson masses are equal (denoting the common mass as m), and that the couplings are related as $-\lambda_S = {\lambda_F}^2$, one gets

$$\begin{aligned}\delta M_H^2 &= -\frac{\lambda_F^2}{16\pi^2}\big[m_S^2 \ln\frac{\Lambda^2}{m^2} - 6m^2 \ln\frac{\Lambda^2}{m^2} \\ &\qquad + 2m^2 \ln\frac{\Lambda^2}{m^2} + 4m^2 \ln\frac{\Lambda^2}{m^2} + \text{finite}\big] \\ &= -\frac{\lambda_F^2}{16\pi^2}m_S^2 \ln\frac{\Lambda^2}{m^2} + \text{finite}.\end{aligned} \tag{1.23}$$

The contributions proportional to the squared masses of the other than Higgs boson mass have disappeared. The logarithmically divergent term proportional to the Higgs particle mass comes from the fact that the corrections were evaluated at the physical pole mass of the propagator $p^2 = m_S^2$.

The symmetry requirements that were issued in order to derive Eq. (1.23) are indeed present in a symmetry principle called supersymmetry. Supersymmetry solves the technical aspects of the hierarchy problem by assigning to each of the fermions of the Standard Model corresponding supersymmetric partners. In this way the dangerous divergences exactly cancel in all orders of the perturbation theory. The rule $m_1^2 = m_2^2 = m_F^2$ is in fact present in supersymmetric theories.

[3]The requirement that the scalar coupling λ_S is negative is actually a necessity, because otherwise the scalar potential for the Lagrangian (1.18) is not bounded from below.

Chapter 2

Supersymmetry

Supersymmetry (SUSY) is a particular symmetry that relates half integer spin particles (fermions) to the integer spin particles (bosons). The supersymmetry transformations turn fermion fields to boson fields and vice versa. This means that in a supersymmetric theory one can exchange boson fields to fermion fields leaving the equations of motion of the underlying model unchanged. The bosons and fermions have therefore the same couplings to the gauge bosons.

Supersymmetric models are considered to be very attractive models beyond the Standard Model. The technical problem of Higgs boson fine tuning (the gauge hierarchy problem discussed in Section 1.2) is solved in supersymmetric theories due to the systematic cancellation of divergences. Another pleasant aspect is the apparent gauge coupling unification at the scale of 10^{16} GeV that could be taken as a hint in favor of a grand unified theory. Under specific circumstances, supersymmetry also provides a good dark matter candidate.

Originally SUSY was not developed to cure the hierarchy problem. The attempt to add fermions to the bosonic string theory resulted a group algebra that included both bosonic and fermionic operators [41, 42, 43, 44]. This superalgebra was defined on the superstring world sheet. A few years later Wess and Zumino [45] generalized the idea of supersymmetry to the quantum field theories in four spacetime dimensions. They also realized [46] that supersymmetry is a way to circumvent the Coleman–Mandula theorem [47]. The Coleman–Mandula theorem is a no-go theorem that states that the only conserved quantities except for the generators of the Poincaré group in a consistent quantum field theory must always be Lorentz scalars. Independently of the string theory development, in 1971 Gol'fand and Likhtman [48] had extended the Poincaré group to a superalgebra and used that to construct supersymmetric field theories in four spacetime dimensions. Their model contained a massive photon and photino (spin-$\frac{1}{2}$ partner of photon), a charged Dirac spinor and two charged scalars (spin-0 particles). After the work of Gol'fand and Likhtman, Volkov and Akulov tried to associate the massless fermion appearing due to spontaneous supersymmetry breaking with the neutrino [49, 50]. After a year from this, Volkov and Soroka considered the super-Higgs mechanism and gauged the super-Poincaré group [51], which was an early attempt of supergravity.

2.1 Supersymmetry algebra and superfields

The Poincaré group P is a group of rotations, translations and the Lorentz transformations, and whose generators satisfy the relations

$$\begin{aligned} [P_\mu, P_\nu] &= 0 \\ [P_\mu, M_{\nu\rho}] &= i(\eta_{\mu\nu} P_\rho - \eta_{\mu\rho} P_\nu) \\ [M_{\mu\nu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho} M_{\nu\sigma} + \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma}), \end{aligned} \tag{2.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric. If one combines the Poincaré group with an internal symmetry group G with generators

$$[T_r, T_s] = i f_{rst} T_t, \tag{2.2}$$

then the Coleman–Mandula theorem states that any Lie group leading to non-trivial physics must be a direct product of these.[1] In other words, the generators of the Poincaré group and the internal symmetry group commute,

$$[P_\mu, T_s] = 0 = [M_{\mu\nu}, T_s]. \tag{2.3}$$

The restrictions of the Coleman–Mandula theorem can be avoided if one introduces the concept of the graded Lie algebra [52], which contains both the commutation and anti-commutation relations between generators. The supersymmetry algebra is required to have a $\mathbb{Z}_2$ graded structure, which implies that the even (bosonic) and odd (fermionic) generators satisfy

$$\begin{aligned} [\text{even}, \text{even}] &= \text{even} \\ \{\text{odd}, \text{odd}\} &= \text{even} \\ [\text{even}, \text{odd}] &= \text{odd}. \end{aligned} \tag{2.4}$$

Eq. (2.3) is still valid, since the even subgroup must obey the Coleman–Mandula theorem. The simplest choice for the SUSY generators is a two-component Weyl spinor Q and its conjugate $\overline{Q}$. The algebra of these fermionic operators can be written as [53, 54, 55]

$$\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0, \tag{2.5}$$

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu, \tag{2.6}$$

[1] The theorem assumes that there is a mass gap (between vacuum and one-particle states). If there is no mass gap, the combined Lie group could be a direct product of the conformal group with an internal group.

where the indices α and β (and the dotted versions) take values 1 or 2 and $\sigma^\mu = (\mathbf{1}, \sigma_i)$ where σ_i are the Pauli matrices. Similarly

$$[Q_\alpha, P_\mu] = [\overline{Q}_{\dot{\alpha}}, P_\mu] = 0, \tag{2.7}$$

$$[Q_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta, \tag{2.8}$$

$$[\overline{Q}_{\dot{\alpha}}, M_{\mu\nu}] = -\frac{1}{2}(\overline{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} \overline{Q}_{\dot{\beta}}. \tag{2.9}$$

This is known as a simple (or N=1) supersymmetry. In this case the internal symmetry group G of Eq. (2.2) becomes just a rotation with generator R

$$[Q_\alpha, R] = -Q_\alpha, \quad [\overline{Q}_{\dot{\alpha}}, R] = \overline{Q}_{\dot{\alpha}}. \tag{2.10}$$

This is known as the R-symmetry. The internal group G has now non-trivial commutation relations with the fermionic part of the supersymmetry group, hence circumventing the Coleman-Mandula theorem. If there are several supercharges Q^n, then the SUSY algebra is called N-extended supersymmetry, where $N > 1$.

An important consequence of the Eq. (2.6) is that the vacuum energy is always non-negative. The Hamiltonian of a supersymmetric theory can be written as

$$H \equiv P^0 = \frac{1}{4}(Q_1\overline{Q}_{\dot{1}} + \overline{Q}_{\dot{1}}Q_1 + Q_2\overline{Q}_{\dot{2}} + \overline{Q}_{\dot{2}}Q_2), \tag{2.11}$$

which is a sum of perfect squares of hermitean operators, implying non-negative eigenvalues. If the vacuum is supersymmetric, *i.e.* $Q_\alpha|0\rangle = 0 = \overline{Q}_{\dot{\alpha}}|0\rangle$, then the vacuum energy is necessarily zero by Eq. (2.11). If the vacuum breaks supersymmetry, *i.e.*, there is at least one generator that does not annihilate the vacuum, then the vacuum energy is necessarily positive. Thus, global supersymmetry can be broken only if there is a positive vacuum energy, or in other words, there is potential with a positive vacuum expectation value $\langle V\rangle > 0$.

Another consequence of the SUSY algebra is that if the supersymmetry is not broken, the particles in the same supermultiplet have the same mass. The Eq. (2.7) $[Q_\alpha, P_\mu] = 0$ implies that $[Q_\alpha, P_\mu^2] = 0$, where $P_\mu^2 = P_\mu P^\mu$ is the mass operator, thus giving the same mass squared for the particles Q acts on.

When constructing supersymmetric models it is useful to make use of the formalism of the so-called superspace [56]. In such a space, in addition to the usual spatial (bosonic) coordinates there are also four anticommuting (fermionic) coordinate dimensions that are represented with the Grassman variables θ^1, $\overline{\theta}^{\dot{1}}$, θ^2 and $\overline{\theta}^{\dot{2}}$. A superfield is a field that depends on all coordinates of the superspace. Every superfield $S(x, \theta, \overline{\theta})$ can be expanded with respect to the Grassman coordinates since the Grassmannian expansion terminates in the second power ($\theta^\alpha\theta^\alpha = -\theta^\alpha\theta^\alpha = 0$). The coefficients of the expansion are ordinary fields. When imposing constraints on the superfields that are covariant under the supersymmetry algebra, the component fields form supermultiplets. Supermultiplets are irreducible representations of the supersymmetry algebra.

The action of the supersymmetry algebra on a superfield is generated by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu, \quad \overline{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{2.12}$$

When one includes also the operator $P_\mu = i\partial_\mu$ one gets a linear representation of the supersymmetry algebra. The group element of the finite transformation is defined as

$$G(a^\mu, \xi, \overline{\xi}) = e^{i(\xi Q + \bar{\xi}\overline{Q} - a^\mu P_\mu)}. \tag{2.13}$$

In order to find irreducible representations one defines covariant fermionic derivatives as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu, \quad \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} - i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{2.14}$$

These anticommute with the fermionic generators Q_α and $\overline{Q}_{\dot{\alpha}}$. Thus, the covariant derivatives commute with the combination $\xi Q + \bar{\xi}\overline{Q}$ making it possible to apply the covariant condition on a superfield (which is invariant under the supersymmetry group)

$$\overline{D}_{\dot{\alpha}} S = 0. \tag{2.15}$$

A superfield that obeys this condition is called a (left) chiral (or scalar) superfield. Any chiral superfield Φ can be expressed as a function of θ and $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$. Expanding $\Phi(y, \theta)$ gives

$$\Phi(y^\mu, \theta) = \phi(y^\mu) + \sqrt{2}\theta^\alpha\psi_\alpha(y^\mu) + \theta^\alpha\theta^\beta\varepsilon_{\alpha\beta}F(y^\mu). \tag{2.16}$$

The quantity $\varepsilon_{\alpha\beta}$ is the anti-symmetric tensor in two dimensions, ϕ and F are complex scalar fields and ψ is a left-handed Weyl spinor field. This can be written in terms of the original variables:

$$\begin{aligned} \Phi(x^\mu, \theta, \overline{\theta}) = \phi(x^\mu) &+ \sqrt{2}\theta\psi(x^\mu) + \theta\theta F(x^\mu) \\ &+ i\partial_\mu\phi\theta\sigma^\mu\theta - \frac{i}{\sqrt{2}}\theta\theta\partial^\mu\psi\overline{\theta} - \frac{1}{4}\partial_\mu\partial^\mu\phi\theta\theta\bar{\theta}\bar{\theta}, \end{aligned} \tag{2.17}$$

where the spinor indices have been suppressed. The name "left" chiral super field is now obvious, since Eq. (2.17) depends on the left-handed spinor ψ. A superfield obeying a conjugated version of the Eq. (2.15)

$$D_\alpha S^\dagger = 0. \tag{2.18}$$

is called correspondingly a right chiral superfield as it depends on a right-handed spinor $\overline{\psi}$. Chiral superfields contain spin-$\frac{1}{2}$ particles and their superpartners.

Another possible covariant constraint that can be imposed to a superfield is hermitian conjugation. A superfield that is it's own self-conjugate (in other words, real) is said to be a vector superfield,

$$V(x, \theta, \overline{\theta}) \equiv V^\dagger(x, \theta, \overline{\theta}). \tag{2.19}$$

In particular, a product $\Phi^\dagger\Phi$ obeys this condition. The vector superfield V can be represented in a component form as

$$V(x,\theta,\bar{\theta}) = -\theta\sigma_\mu\bar{\theta}A^\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \tag{2.20}$$

where the so-called Wess–Zumino gauge [45, 46, 57] has been used to remove unphysical degrees of freedom. The ordinary gauge freedom is not fixed yet, though, and expanding e^{gV} would lead to a mass term to the gauge field. In fact, the Wess–Zumino gauge is not invariant under the supersymmetry transformations. This can be circumvented by defining a combined transformation which consists of a supersymmetry transformation followed by an extended gauge transformation. This is called de Wit–Freedman transformation [58]. Vector superfields contain gauge bosons and their superpartners.

2.1.1 Lagrangians

The Lagrangian density itself cannot be invariant under supersymmetric transformation, because in the Eq. (2.6) on the right-hand side there is a spacetime derivative, which implies that if $\delta\mathscr{L} = 0$ then $\mathscr{L}$ must be a constant. Despite of that, the action can still be supersymmetric provided that $\delta\mathscr{L}$ is a total divergence that vanishes after the spacetime integration. To construct invariant actions this way, one needs to know supersymmetry transformation properties of chiral and vector superfields. For a chiral superfield Eq. (2.17) the appropriate divergence is found from the transformation of the F-term (*i.e.* the coefficient of the $\theta\theta$-term)

$$\delta_\xi F = -i\sqrt{2}\partial_\mu\psi^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}. \tag{2.21}$$

Because $Q, \overline{Q}$ and $\overline{D}, D$ are linear operators on superspace, it is clear that any polynomial function of left (right) chiral superfields is again a left (right) chiral superfield. Thus, supersymmetric invariant actions can be constructed extracting the F-terms from the so-called superpotential $W(\Phi)$ which contains products of the left chiral superfields. In order to get renormalizable Lagrangian the superpotential can include no higher than products of three superfields.[2] The most general superpotential can then be written as

$$W(\Phi) = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k, \tag{2.22}$$

where m_{ij} and λ_{ijk} are symmetric and real and the tadpole term linear in Φ has been neglected. The linear term can be later used to break supersymmetry. The superpotential $W(\Phi)$ does not contain derivatives, so it is analytic with respect to the chiral superfields Φ.

[2] Product of three superfields has a dimension three. Since the F-term of a superfield has one dimension more, three superfields is the highest number of products which produce dimension four Lagrangian.

Similarly, for the vector superfields of Eq. (2.20), the total divergence is found from the transformation of the D-term (*i.e.* the coefficient of the $\theta\theta\bar{\theta}\bar{\theta}$-term)

$$\delta_{\xi,\bar{\xi}} D = \partial_\mu (\lambda \sigma^\mu \overline{\xi} - \xi \sigma^\mu \overline{\lambda}). \tag{2.23}$$

Thus vector superfields can appear in the Lagrangian via D-terms. An important term that can be added to the Lagrangian this way is the D-term of the combination $\Phi^\dagger\Phi$, which gives the kinetic terms of the component fields. It can be noted that the transformation of the D-term of a general superfield (*e.g.* a superfield constructed by multiplying both left *and* right chiral superfields and their superderivatives) is also a total derivative, but the physically meaningful action can be constructed only from vector superfields.

The superpotential (2.22) contains the scalar potential, which can be written as [59]

$$V = \left| \frac{\partial W(\phi)}{\partial \phi} \right|^2, \tag{2.24}$$

where the superfields are replaced by the corresponding scalar component of the supermultiplet. After calculating the field equations of the Lagrangian containing the F-terms of the superpotential (2.22) and the D-term of the kinetic term $\Phi^\dagger\Phi$, the tree level effective potential becomes

$$V = F_i^\dagger F_i \equiv |F|^2, \tag{2.25}$$

and the field equation for the $F_i^\dagger$-field is

$$F_i^\dagger = -m_{ij}\phi_i - \lambda_{ijk}\phi_j\phi_k = -\frac{\partial W(\phi)}{\partial \phi_i}. \tag{2.26}$$

There are no kinetic terms in the field equation, so the F-field is auxiliary and can be removed from the Lagrangian using the field equation (2.26). The $W(\phi)$ is the superpotential with Φ_i replaced by ϕ_i from Eq. (2.17). The scalar potential of Eq. (2.25) is a square of an absolute value, hence it is positive semi-definite and as such, bounded from below.

The coupling of the gauge superfields to the chiral (matter) superfields is attained by a supersymmetric version of the minimal coupling,

$$\Phi^\dagger\Phi \to \Phi^\dagger e^{2gT_aV^a}\Phi. \tag{2.27}$$

It is possible to construct chiral superfields from a vector superfield by using the derivatives of Eq. (2.14). The superfield obtained this way is called field strength superfield

$$W_\alpha = \overline{D}^2 e^{-gT_aV^a} D_\alpha e^{gT_bV^b}. \tag{2.28}$$

In the case of Abelian gauge symmetry this reduces to $W_\alpha = \overline{D}^2 D_\alpha V$. Clearly $\overline{D}W_\alpha = 0$, so W_α is a left chiral superfield. Because a product of left chiral superfields is again a left chiral superfield, one can construct gauge kinetic terms by including the F-term of the square of the field strength superfield to the Lagrangian.

Collecting all these together one can construct a general version of a (renormalizable) supersymmetric Lagrangian as

$$\mathscr{L} = \int d^2\theta d^2\overline{\theta}\ \Phi_i^\dagger e^{2gT_aV^a}\Phi_i + \Big(\int d^2\theta\big(\frac{1}{64}W^{a\alpha}W_{a\alpha} + W(\Phi_i)\big) + \text{H.C.}\Big). \quad (2.29)$$

One should note that integrating over the whole Grassmannian superspace is equivalent of taking the D-term, and integrating over the $\theta\theta$-part is equivalent of taking the F-term of the integrand. The integrals over a fermionic space are known as Berezin integrals [60]. The Berezin integration has the same effect as taking a derivative, and can be characterized as

$$\int d\theta_\alpha f(\theta) = \partial_{\theta_\alpha} f(\theta). \quad (2.30)$$

In the N=1 supersymmetry one can replace the derivative ∂_{θ_α} with D_α (or $\partial_{\theta_{\dot\alpha}}$ with $\overline{D}_{\dot\alpha}$). This replacement makes the derivation of the component form Lagrangian much easier than expanding the superfields directly.

2.2 MSSM

The minimal supersymmetric Standard Model (MSSM) is a supersymmetric extension of the Standard Model with a minimal particle content. This implies that each Standard Model spin-$\frac{1}{2}$ fermion is accompanied with a pair of spin-0 bosons (called sfermions) and each gauge boson with a corresponding spin-$\frac{1}{2}$ superpartner (photino, wino, bino and gluino). Because none of the SM fermions belong to the adjoint representation of $SU(3) \times SU(2) \times U(1)$ they cannot be identified as superpartners of the SM gauge bosons. Therefore, a new superpartner must be introduced for each SM particle.

Particles in the same supermultiplet transform similarly under the gauge group. This means that the physical left- and right-handed SM fermions can't be in the same supermultiplet, since it is known that the left-handed SM fermions are in $SU(2)_L$ doublets while the right-handed are in singlets. In extended supersymmetry with $N \geq 2$ a supermultiplet contains both left and right handed fermions. Therefore it is believed that the low-energy SUSY has to be N=1 supersymmetry, though high energy models might be constructed using extended supersymmetries. Theories with N greater than eight contain necessarily particles that has spin two or higher, which are difficult to couple to other particles in a consistent way. Therefore the N=8 theory is said to be maximally extended supergravity theory.

In contrast to the SM, in MSSM two chiral Higgs doublets are needed. Firstly, supersymmetry forbids the SM-like $\overline{\psi}\psi H^\dagger$ terms in the Lagrangian, since that would imply mixing left- and right-chiral superfields with each other, as discussed

Table 2.1: Gauge supermultiplets in the MSSM

Supermultiplet	spin $\frac{1}{2}$	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^{\pm}\,\widetilde{W}^0$	$W^{\pm}\,W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$

in the section 2.1. Secondly, the multiplets with different weak hypercharges have different Yukawa couplings for up and down type quarks. Thus both are needed. Also with only one Higgs multiplet there would be a triangle gauge anomaly, since by adding superpartners to the model the anomaly cancellation of the SM is spoiled [61]. By adding two Higgs doublets with opposite hypercharges the anomaly cancellation is restored. The field content of the Higgs doublets is augmented with spin-$\frac{1}{2}$ doublets (higgsinos) to form the Higgs supermultiplets. Even though the left slepton supermultiplet has the same SM quantum numbers as the H_d supermultiplet, they cannot be identified, since that would lead to large violation of experimental limits of the mass of (at least one) neutrino in addition to the lepton number violation and non-cancellation of triangle anomaly [62]. The supermultiplets of MSSM are displayed in the Tables 2.1 and 2.2.

MSSM has the same gauge group structure as the Standard Model which specifies the gauge interactions. Those don't, however, specify the superpotential completely. The most general superpotential of the MSSM is written as

$$W_{\text{MSSM}} = \overline{u}\boldsymbol{y}_u QH_u - \overline{d}\boldsymbol{y}_d QH_d - \overline{e}\boldsymbol{y}_e LH_d + \mu H_u H_d + W_{\not B} + W_{\not L}, \tag{2.31}$$

where the gauge and family indices have been suppressed. The dimensionless couplings $\boldsymbol{y}_u, \boldsymbol{y}_d$ and $\boldsymbol{y}_e$ are the Yukawa coupling matrices of quarks and leptons, and give rise to quark masses and to the mixing angles of the quarks and leptons. The μ-term yields the higgsino mass terms as well as the mass-squared Higgs terms in the scalar potential. Because the scalar potential Eq. (2.25) is non-negative, additional (negative) Higgs mass-squared terms are needed to trigger the electroweak symmetry breaking. These terms also break supersymmetry, as will be discussed in the section 2.4. Finally, $W_{\not B}$ and $W_{\not L}$ are the baryon and lepton number violating parts of the superpotential, respectively. Because B- and L-violating processes have not been observed, it is natural to require that either those terms give extremely small contribution or, they are forbidden by a symmetry.

2.3 R-parity – LSP – Cosmology

In a supersymmetric theory it is possible to include in the superpotential terms that violate B- and L-symmetries, but which are gauge invariant and analytic in

Table 2.2: Chiral supermultiplets in the MSSM

Supermultiplet		spin 0	spin $\frac{1}{2}$	$SU(3)_C, SU(2)_L, U(1)_Y$
(s)quarks	Q	$(\widetilde{u}_L\, \widetilde{d}_L)$	$(u_L\, d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$\overline{u}_R$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$\overline{d}_R$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
(s)leptons	L	$(\widetilde{\nu}\, \widetilde{e}_L)$	$(\nu\, e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$\overline{e}_R$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs(inos)	H_u	$(H^+_u\, H^0_u)$	$(\widetilde{H}^+_u\, \widetilde{H}^0_u)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_d	$(H^0_d\, H^-_d)$	$(\widetilde{H}^0_d\, \widetilde{H}^-_d)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

the chiral superfields. These are

$$W_{\not{B}} = \frac{1}{2}\lambda''^{ijk}\overline{u}_i\overline{d}_i\overline{d}_i, \tag{2.32}$$

$$W_{\not{L}} = \frac{1}{2}\lambda^{ijk}\overline{e}_i L_j L_k + \lambda'^{ijk}\overline{d}_i L_j Q_k + \mu'^i L_i H_u. \tag{2.33}$$

The Eq. (2.32) violate the baryon number by one unit, and the terms in Eq. (2.33) violate the lepton number by one unit. If the terms for λ, λ' and λ'' were all included in the Lagrangian, the proton lifetime would be extremely short, unless the couplings were very small. In the Standard Model the B- and L-conservations are accidental symmetries, since it is not possible to include B- or L-violating terms in the renormalizable Lagrangian.

The presence of $W_{\not{B}}$ and $W_{\not{L}}$ is prevented by introducing a discrete global symmetry called R-parity [63, 64, 65, 66, 67]. All of the sparticles (squarks, sleptons, gauginos and higgsinos) are required to have negative R-parity, while the SM particles are required to have positive R-parity. This can be written in terms of the particle spin J, the baryon number B and the lepton number L as

$$P_{\mathrm{R}} = (-1)^{2J+3(\mathrm{B}-\mathrm{L})}. \tag{2.34}$$

A term in the Lagrangian is allowed only, if the product of the R-parities of the particles is +1, thus excluding the terms of Eq. (2.32) and Eq. (2.33). The consequence is that every interaction vertex contains an even number of particles with $P_{\mathrm{R}} = -1$, *i.e.* sparticles. The remarkable consequence is that the lightest supersymmetric particle (LSP) must be absolutely stable. Every other sparticle eventually decays into a state which contains an odd number of LSPs. In particle colliders sparticles can be produced only in even numbers, since the initial state contains only ordinary particles.

The lightest supersymmetric particle is of great importance, since it always remains in the end of decay chains involving supersymmetric particles. In many

SUSY models the LSP is the lightest neutralino ($\widetilde{\chi}_1^0$). Neutralinos are combinations of neutral gauginos and higgsinos, which get mixed after the electroweak symmetry breaking. The relative amounts of these mixing components define the properties of neutralinos. Because neutralinos don't have electromagnetic interactions (or strong either), they are not detected in colliders. Instead, a missing energy of $2m_{\widetilde{\chi}_1^0}$ is observed as two LSPs leave the detector. Notice: In hadron colliders only the missing energy component associated to the transverse momentum component can be observed.

In the perspective of cosmology, the fact that the lightest neutralino is a weakly interacting massive particle (WIMP) is desirable. The neutral LSP is a good candidate for the cold dark matter. The calculations of the neutralino LSP relic density are in accordance with the range of observed critical density (see *e.g.* [68, 69]). Another possible supersymmetric WIMP candidate, sneutrino, has been ruled out by direct searches.

In this dissertation the R-parity is supposed to be an exactly conserved symmetry. Usually this assumption is also included to the meaning of the word "Minimal" in the MSSM acronym.

2.4 Breaking of Supersymmetry

It is evident that if supersymmetry exists, it must be a broken symmetry. In unbroken supersymmetry the chiral superfields obey the tree level relation [70]

$$\mathrm{STr}M^2 \equiv \sum_J (-1)^{2J}(2J+1)m_J^2 = 0, \tag{2.35}$$

where m_J is the mass associated with the field of spin J and $\mathrm{STr}M^2$ is so-called supertrace, which is a spin-weighted sum taken over the squared mass matrix of the real fields. For example, for the electron supermultiplet $-2m_e^2 + m_{\tilde{e}_1}^2 + m_{\tilde{e}_2}^2 = 0$, which implies that one of the selectrons is lighter than or equal to the mass of the electron. Charged superpartners lighter than corresponding Standard Model fields would have been easy to detect, so the obvious conclusion is that supersymmetry, if it exists, is a broken symmetry. This is in accordance with the fact that particles in the same supermultiplet have the same mass. When this is applied to the Higgs particle mass correction formula Eq. (1.23) one can see how the supersymmetry removes the instability of the scalar mass radiative corrections.

As was discussed in the section 2.1, if supersymmetry is unbroken in the vacuum state, the vacuum has zero energy, and if supersymmetry is broken (spontaneously), the vacuum has positive energy. Thus, if a supersymmetric vacuum state exists as a local minimum of the scalar potential, it is also the global minimum of the potential. This requires that the scalar potential can't have any supersymmetric minima, if the supersymmetry is supposed to be broken.

The global supersymmetry can be spontaneously broken, if some of the component fields of a superfield gets a non-zero expectation value. For chiral superfields, the only possible field that can acquire a *vev* without breaking the Lorentz invariance is the auxiliary scalar field F_i [71, 72]. This kind of SUSY breaking is called F-term (Fayet-O'Raifeartaigh) breaking. When the field F_i gets a *vev* and breaks the global symmetry, a massless Goldstone fermion, goldstino, appears. The Goldstino is a fermionic equivalent of the massless Goldstone boson which necessarily appears via Goldstone mechanism [73], when a global symmetry (with bosonic generators) is broken.

The superpotential of the Eq. (2.22) cannot produce spontaneous SUSY breaking because one can always find a field configuration with a supersymmetric minimum by setting the *vev*s of ϕ_i in the Eq. (2.26) to zero. The Eq. (2.22) can be generalized to have a linear term

$$W(\Phi) = -k_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k, \tag{2.36}$$

which makes it possible to find a field configuration that breaks supersymmetry. In this case the superfields Φ_i that appear in the linear term must be gauge singlets in order not to break gauge symmetries.

In the case of vector superfields the possible field that may get a *vev* is the $D(x)$ field of the Eq. (2.20). This is again stated by the requirement of Lorentz invariance. This method is called D-term (Fayet-Iliopoulos) supersymmetry breaking [74]. This kind of breaking is useful when considering a gauge theory which includes the $U(1)$ gauge group. In this case the *vev* that resides in the vectorial supermultiplet is a gauge singlet with respect to the other gauge groups (in N=1 supersymmetry the $U(1)$ symmetry is R-symmetry, an internal symmetry that has non-trivial transformation properties w.r.t. supersymmetry,[3] as discussed in the Sec. 2.1). In that case one can add a term

$$\mathscr{L}_{\not{D}} = \xi D(x) \tag{2.37}$$

to the Lagrangian [74, 64]. If all the other scalar fields are prevented from having a *vev*, then $D(x)$ must have a *vev* equal to $-\xi$, and supersymmetry is broken.

The supertrace formula (2.35) is actually valid also after a pure F-term supersymmetry breaking. Even though Eq. (2.35) is modified by the radiative corrections in the presence of the supersymmetry breaking, it is not possible to construct a spontaneously broken supersymmetric model with sparticle masses at the tree level, where all the sfermions are heavier than corresponding fermions. Using a D-term breaking this is possible, though, but then a new $U(1)$-symmetry must be introduced, whose D-term gets a *vev* in the minimum of the potential. The condition (2.35) can be circumvented, if the sparticle masses are created

[3] Under an $U(1)$ R-symmetry in the N=1 supersymmetry the fermionic coordinates rotate as $\theta \to e^{i\alpha}\theta$ and a chiral super field transforms as $\Phi(x,\theta,\overline{\theta}) \to e^{-iq\alpha}\Phi(x, e^{iq\alpha}\theta, e^{-iq\alpha}\overline{\theta})$ where q is the charge of the $U(1)$ group.

through radiative corrections (see Sec. 2.6.2). It should be noted that if supersymmetry is not broken at the tree level, then it cannot be broken by perturbative corrections either [75, 76, 77]. This means in particular, that the supersymmetry is broken at some hidden sector, and then communicated to the observable sector by some interactions (gauge, gravitational, *etc.*).

In the MSSM there are no proper candidates for a supersymmetry breaking fields. There are no gauge singlets whose F-term could generate a *vev*, and using the D-term breaking associated with the $U(1)_Y$ leads to unacceptable particle spectrum (and some of the quarks or leptons would get a non-zero *vev*). Thus the supersymmetry breaking must be generated by some yet unknown fields at some much higher mass scale than the electroweak scale inaccessible to experiments. The phenomenology of MSSM can be still studied without knowing the exact process that breaks supersymmetry.

Supersymmetry breaking can be parameterized by adding so-called soft breaking terms to the supersymmetric Lagrangian. The soft terms are terms of dimension three or less with respect to the fields, and they preserve gauge invariance and parity. Therefore the couplings of the soft terms must have positive mass dimensions. Inserting terms that break the symmetry between fermions and sfermions brings back the hierarchy problem. The term "soft" means that these terms do not destroy the cancellation of the divergences, and the main motivation to use supersymmetry is maintained. Also the relationship of the dimensionless couplings must be the same as in unbroken theory. The effective Lagrangian can then be written in the form

$$\mathscr{L} = \mathscr{L}_{\text{SUSY}} + \mathscr{L}_{\text{soft}}, \tag{2.38}$$

where $\mathscr{L}_{\text{soft}}$ violates supersymmetry but contains only mass terms and couplings with positive mass dimension while $\mathscr{L}_{\text{SUSY}}$ is still supersymmetric. Even though the supersymmetry breaking is performed by hand, it is assumed that actual breaking is spontaneous and the original theory is exactly supersymmetric. It can be thought as if there were two different sectors, hidden and visible (or observable). In the hidden sector the theory is fully supersymmetric, while the visible sector contains the soft breaking terms.

The most general soft terms for a renormalizable supersymmetric theory are

$$\begin{aligned} \mathscr{L}_{\text{soft}} = & -\frac{1}{2}\left(M_{\lambda,a}\lambda^a\lambda^a + \text{H.C.}\right) - (m^2)^j_i\phi^{i*}\phi_j \\ & -\big(\frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{H.C.}\big), \end{aligned} \tag{2.39}$$

where M_λ is a gaugino mass for each gauge group, m^2 is a scalar mass term, b^{ij} bilinear (squared mass) term and a^{ijk} is a trilinear coupling term. A softly broken Lagrangian of the form Eq. (2.39) is free of quadratic divergences to all orders of perturbation theory [78].

If there are no chiral supermultiplets that are singlets under the gauge group, then it is also possible to add terms like

$$\mathscr{L}_{\text{soft}} = \frac{1}{2} r_i^{jk} \phi^i \phi_j \phi_k + \frac{1}{2} m_F^{ij} \psi_i \psi_j + m_A^{ia} \psi_i \lambda_a + \text{H.C.} \tag{2.40}$$

to the Lagrangian [79, 80, 81]. The term proportional to m_F^{ij} can be ignored, since this term can be absorbed to the redefinition of the superpotential and to the first term of Eq. (2.40). The last term in Eq. (2.40) is present only if there are some matter fields (other than gauginos) in the adjoint representation of the gauge group [82], which is not the case in the MSSM. When issuing a specific supersymmetry breaking conditions, the terms of Eq. (2.40) are usually ignored.

2.5 Renormalization group evolution and the electroweak symmetry breaking

In the context of the minimal supersymmetric standard model the most general soft supersymmetry breaking terms are [83]

$$\begin{aligned}
\mathscr{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2}(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \widetilde{g}\widetilde{g}) + \text{H.C.} \\
- (\overline{u} \boldsymbol{a}_u Q H_u - \overline{d} \boldsymbol{a}_d Q H_d - \overline{e} \boldsymbol{a}_e L H_d) + \text{H.C.} \\
- Q^\dagger \boldsymbol{m}_Q^2 Q - L^\dagger \boldsymbol{m}_L^2 L - \overline{u} \boldsymbol{m}_{\overline{u}}^2 \overline{u}^\dagger - \overline{d} \boldsymbol{m}_{\overline{d}}^2 \overline{d}^\dagger - \overline{e} \boldsymbol{m}_{\overline{e}}^2 \overline{e}^\dagger \\
- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{H.C.}),
\end{aligned} \tag{2.41}$$

where M_1, M_2, and M_3 are the bino, wino, and gluino mass terms, respectively. The terms $\boldsymbol{a}_u, \boldsymbol{a}_d$ and $\boldsymbol{a}_e$ are complex 3×3 matrices in family space, and the squared mass terms $\boldsymbol{m}_i^2$ are 3×3 mass squared hermitean matrices in family space. Finally, $m_{H_u}^2$ and $m_{H_d}^2$ are supersymmetry breaking contributions to the Higgs potential, and b is the corresponding one for the bilinear mixing term.

The Lagrangian (2.41) is thought to be given at some high energy scale near the Planck scale. Thus, if that is used to calculate the masses and cross-sections at the electroweak scale, the results will include large logarithms coming from loop diagrams. The logarithms can be resummed using the renormalization group equations (RGE) by treating the parameters of the Lagrangian as running parameters. The energy scale dependence of the parameters is governed by the renormalization group (RG) evolution, and the corresponding equations that describe such evolution are called renormalization group equations [84, 85, 86, 87, 88].

In order to get useful predictions of masses and cross-sections one must therefore evaluate the RG-running of the soft and superpotential parameters as well as the gauge couplings down to the electroweak scale. One remarkable result of the RGE-running in the MSSM is the unification of the gauge couplings [89, 90, 91, 92]. The one-loop RGEs for the gauge coupling and the gaugino masses are

$$\frac{d\alpha_i}{dt} = \frac{1}{4\pi} b_i \alpha_i^2, \quad \frac{dM_i}{dt} = 2 b_i \alpha_i M_i, \tag{2.42}$$

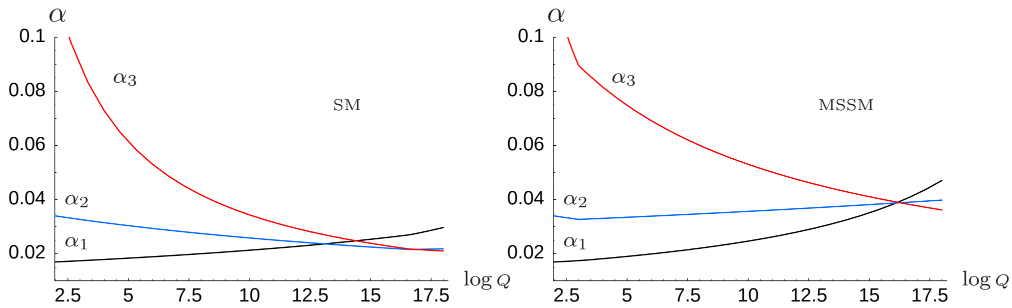

Figure 2.1: Gauge coupling unification in the SM and MSSM. One-loop evolution from the electroweak scale up to the 10^{18} GeV.

where $t = \ln \frac{Q}{M_{\rm GUT}}$ and Q is the renormalization scale, $\alpha_i = \frac{g_i^2}{4\pi}$ and $g_1^2 = (5/3)g'^2$. The β-function coefficients are given by $b = (33/5, 1, -3)$ in the MSSM and $b = (41/10, -19/6, -7)$ in the SM. The running of the gauge couplings is shown in Fig. 2.1. By using the measured initial conditions for the gauge couplings, the coupling constants unify at around the scale 2×10^{16} GeV in the MSSM, whereas in the SM they do not meet in a single point. This may be taken as a hint pointing at some grand unification theory or superstring models, which predict gauge coupling unification at some scale lower than $M_{\rm P}$. The energy scale at which the gauge couplings unify is called the grand unification scale. The unification of the gauge couplings is usually taken as a postulate when studying the supersymmetric models,

$$\alpha_3^G = \alpha_2^G = \alpha_1^G = \alpha^G (\approx 1/25). \tag{2.43}$$

The RG-equations are coupled differential equations. Because of that, it turns out that if the gaugino mass parameters M_λ are non-zero at the GUT-scale, then all the other soft parameters will be generated via RG-evolution.[4] Of a specific interest are the equations for the soft Higgs mass-squared parameters [93, 94], which can be written at one-loop order as

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2, \tag{2.44}$$

$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2. \tag{2.45}$$

In the models, where the trilinear $\boldsymbol{a}$-terms are proportional to the Yukawa couplings, the X_t, X_b, and X_τ are (in the limit, where other but the third family Yukawa couplings are neglected)

$$X_t = 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2, \tag{2.46}$$

$$X_b = 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\bar{d}_3}^2) + 2|a_b|^2, \tag{2.47}$$

$$X_\tau = 2|y_\tau|^2(m_{H_d}^2 + m_{L_3}^2 + m_{\bar{e}_3}^2) + 2|a_\tau|^2. \tag{2.48}$$

[4]The RG-equations are given in the Appendix A.5

These are positive quantities, so their effect is always to decrease the Higgs particle masses as one evolves the RG equations downwards from the M_{GUT}, with X_t giving the largest contribution. This becomes relevant, when one examines the behavior of the scalar potential at the minimum. The classical scalar potential of the MSSM can be written (after using the $SU(2)_L$ gauge freedom to rotate away a possible *vev* of one of the Higgs field components at the minimum)

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 \\ - (b\, H_u^0 H_d^0 + \text{H.C.}) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \quad (2.49)$$

Without loss of generality, the phase of b can be absorbed into a redefinition of the phases of H_u and H_d thus making b real and positive. In Eq. (2.49) the Higgs doublets are written in component form,

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}. \quad (2.50)$$

Now, due to the large value of the top Yukawa coupling, the parameter $m_{H_u}^2$ is pushed to the negative values at the electroweak scale, while the rest of the scalar mass parameters remain positive. This is precisely what is needed for the Higgs mechanism as discussed in the section 1.2. This mechanism is called radiative electroweak symmetry breaking (rEWSB) [93,94,95,96,97,98], since the electroweak symmetry breaking (EWSB) is driven purely by radiative corrections (the resummation of the loop contributions is translated into the RG-evolution).

The tree level minimization condition of the scalar potential of the MSSM can be written as

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m_Z^2, \quad (2.51)$$

where $\tan\beta \equiv \langle H_u^0\rangle/\langle H_d^0\rangle = v_u/v_d$. It is important that the true minimum of the scalar potential is the minimum of the neutral Higgs potential (2.49). Otherwise some of the other scalars could develop a charge and color breaking minimum, which would make the vacuum unstable [99, 98, 100]. The one-loop corrections to the scalar potential make this analysis quite complicated, but the tree level potential can be used provided that the renormalization scale, which minimizes the loop contribution, is chosen [101].

After the EWSB, three of the eight real degrees of freedom of the Higgs doublets become the longitudinal components of the Z^0 and $W^\pm$ gauge bosons, which then acquire mass. The remaining five degrees of freedom are Higgs particle mass eigenstates denoted by h^0, H^0 (scalars), A^0 (pseudoscalar) and $H^\pm$ (charged).

The electroweak symmetry breaking affects naturally also the gaugino and higgsino sectors. In the MSSM there are several sets of particles with same color, charge, baryon and lepton numbers but different $SU(2)\times U(1)$ quantum numbers. When the $SU(2) \times U(1)$ gauge symmetry gets broken, these states mix. The neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$ and $\widetilde{W}^0$) combine in order to form four neutral mass eigenstates called neutralinos. In the basis

$$\psi^0 = \begin{pmatrix} \widetilde{B} \\ \widetilde{W}^0 \\ \widetilde{H}_d^0 \\ \widetilde{H}_u^0 \end{pmatrix}, \tag{2.52}$$

the neutralino mass terms are written

$$-\frac{1}{2}(\psi^0)^T \boldsymbol{M}_{\widetilde{\chi}^0} \psi^0 + \text{H.C.} \tag{2.53}$$

where [102, 83]

$$\boldsymbol{M}_{\widetilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix}, \tag{2.54}$$

where the shorthand notation $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_W = \sin\theta_W$, and $c_W = \cos\theta_W$ has been used. The mass matrix (2.54) can be diagonalized by a unitary transformation, after which one finds the physical masses of the neutralinos. The gaugino mass entries M_1 and M_2 can be chosen to be real by redefining the phases of $\widetilde{B}$ and $\widetilde{W}$. The phase of the μ parameter is a physical parameter and cannot be rotated away. This can lead to possible combined charge conjugation and parity (CP) -violating effects. (The Standard Model is invariant under the CP-transformations except for a phase in the Yukawa matrices and the strong QCD phase.) The properties of neutralinos were studied in the paper II of this dissertation.

Similarly, the charginos are formed from four gauge eigenstates. The charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) mix to form two charged mass eigenstates (actually four, but the positive and negative charged mass eigenstates are degenerate in mass). In the basis $\psi^{\pm T} = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$ the chargino mass terms are

$$-\frac{1}{2}(\psi^\pm)^T \boldsymbol{M}_{\widetilde{\chi}^\pm} \psi^\pm + \text{H.C.} \tag{2.55}$$

where [83]

$$\boldsymbol{M}_{\widetilde{\chi}^\pm} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{X}^T \\ \boldsymbol{X} & \boldsymbol{0} \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} M_2 & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & \mu \end{pmatrix}. \tag{2.56}$$

In contrast to the neutralinos, chargino masses are easily calculated by diagonalizing the 2×2 matrix $\boldsymbol{m}^2 = \boldsymbol{X}^\dagger \boldsymbol{X}$.

In principle, all the up-type squarks of the same color mix together by a 6×6 matrix. The same happens to the down-type squarks as well as the charged sleptons. Also the sneutrinos mix, but by a 3×3 matrix. Usually one supposes that the squarks don't get mixed (notably) in the flavor space, and the same is assumed for the charged sleptons. In addition, only the third generation Yukawa couplings give a substantial contribution through the RG-evolution, so in practice, only the third generation squarks are mixed, as well as the third generation sleptons. The mass mixing matrix for the top squarks in the gauge-eigenstate basis $\phi_{\tilde{t}}^T = (\widetilde{t}_L, \widetilde{t}_R)$ is given by

$$-\phi_{\tilde{t}}^{\dagger} \boldsymbol{m}_{\tilde{t}}^2 \phi_{\tilde{t}} \tag{2.57}$$

where

$$\boldsymbol{m}_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_u & v(a_t s_\beta - \mu y_t c_\beta) \\ v(a_t s_\beta - \mu y_t c_\beta) & m_{\overline{u}_3}^2 + m_t^2 + \Delta_{\overline{u}} \end{pmatrix}. \tag{2.58}$$

In above $\Delta_\phi = m_Z^2 \cos 2\beta (T_3^\phi - Q_{\mathrm{EM}}^\phi \sin^2 \theta_W)$ where T_3^ϕ and Q_{EM}^ϕ are the third component of weak isospin and the electric charge of the chiral supermultiplet to which ϕ belongs. The quantity v is related to the *vevs* of Higgs fields as $v^2 = v_u^2 + v_d^2$. The mass eigenstates are

$$\begin{pmatrix} \widetilde{t}_1 \\ \widetilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & \cos\theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{2.59}$$

where $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$ are the eigenvalues of Eq. (2.58) and $0 \leq \theta_{\tilde{t}} \leq \pi$. The off-diagonal entries will typically induce a significant mixing which reduces the lighter top-squark (mass)^2 eigenvalue. Often the $\widetilde{t}_1$ is the lightest squark. The mass mixing matrix for the bottom squarks in the gauge-eigenstate basis $\phi_{\tilde{b}}^T = (\widetilde{b}_L, \widetilde{b}_R)$ is given by

$$\boldsymbol{m}_{\tilde{b}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_d^2 & v(a_b c_\beta - \mu y_b s_\beta) \\ v(a_b c_\beta - \mu y_b s_\beta) & m_{\overline{d}_3}^2 + \Delta_{\overline{d}} \end{pmatrix}. \tag{2.60}$$

The tau slepton mixing matrix can be obtained from $\boldsymbol{m}_{\tilde{b}}^2$ by changing $d \to e$, $\bar{d} \to \bar{e}$ and $b \to \tau$ in Eq. (2.60). Diagonalizing these will give the mass eigenstates $(\widetilde{b}_1, \widetilde{b}_2)$ and $(\widetilde{\tau}_1, \widetilde{\tau}_1)$.

The mass eigenstates of the first two generations are nearly the same as the gauge eigenstates. Because the RGEs are highly coupled, it is convenient to solve them only numerically.

2.6 Supergravity – Models for SUSY breaking

It is natural to think that supersymmetry, as an extension of the Poincaré symmetry, is a local symmetry. Since the supersymmetry is a spacetime symmetry by nature (though the residual R-symmetry makes it also a type of an internal symmetry), one may expect that the local supersymmetry is a theory of local coordinate transformations, *i.e.* theory of gravity. Most of the symmetries in particle physics are local symmetries rather than mere global symmetries, which also gives a hint that supersymmetry is a local symmetry. In locally supersymmetric theory the parameters of SUSY transformation of Eq. (2.13) depend on the spacetime point explicitly. The locally supersymmetric theory is known as *supergravity* [103, 104, 105, 106].

The supersymmetry breaking is supposed to be generated spontaneously. The breaking of the global supersymmetry introduces a massless Weyl fermion, goldstino. The spin-2 graviton has a spin-$\frac{3}{2}$ superpartner called gravitino. In principle, the superpartner of graviton could also be a spin-$\frac{5}{2}$ fermion, but it would be hard to couple that to other fields, so the spin-$\frac{3}{2}$ superpartner is chosen. When local supersymmetry is spontaneously broken, the gravitino acquires mass via super-Higgs mechanism [107, 106], analogous to the usual Higgs mechanism. It turns out that the contributions from supergravity are of the same order of magnitude as the non-gravitational ones, implying that the gravitational effects cannot be neglected even when considering only the low-energy theory [108, 109]. In supergravity the supertrace formula Eq. (2.35) receives radiative corrections thus making the superpartner masses heavier, which removes the mass degeneracy of a supermultiplet. This is due to the non-renormalizability of the supergravity.

To derive a locally invariant supersymmetric Lagrangian one starts with a generalized globally supersymmetric Lagrangian [110]

$$\mathscr{L}_{\text{GLOBAL}} = \int d^4\theta K(\Phi^\dagger e^{2gV}, \Phi) + \int d^2\theta (W(\Phi) + \text{H.C.}) + \int d^2\theta (f_{ab}(\Phi) W_a^\alpha W_{\alpha b} + \text{H.C.}). \quad (2.61)$$

The first term of Eq. (2.29) is generalized to be a general function of superfields $K(\Phi^\dagger, \Phi)$, because a theory involving gravity is not required to be renormalizable.[5] Coupling to the vector multiplets is arranged with the minimal coupling e^{2gV}. The function coefficient f_{ab} of the gauge field strength superfields W_a (see Eq. (2.28)) is called gauge kinetic function and it is an arbitrary function of the superfields Φ_i. In a similar way, $W(\Phi)$ may contain any number of products of superfields. The only restriction is that the resulting non-renormalizable terms must contain gravitational coupling constant in such a way that the theory will be renormalizable again when gravity decouples (in the limit $M_\text{P} \to \infty$). The function $K(\Phi^\dagger, \Phi)$ can be reformulated as

$$K(\Phi^\dagger, \Phi) = -3|W(\Phi)|^{\frac{2}{3}} e^{-\frac{G(\phi^*, \phi)}{3}}. \quad (2.62)$$

The dimensionality of $K(\Phi^\dagger, \Phi)$ is two, $[K] = 2$. The function G in Eq. (2.62)

[5] In fact, any interacting theory containing a spin-2 particle is non-renormalizable.

can be expressed with the help of K and W as

$$G(\phi^*,\phi) = J(\phi^*,\phi) + \ln \frac{|W|^2}{M_{\rm P}^6}, \tag{2.63}$$

$$J(\phi^*,\phi) = -3\ln \frac{-K(\phi^*,\phi)}{3M_{\rm P}^2}, \tag{2.64}$$

where $G(\phi^*,\phi)$ is a real analytic function of scalar fields called Kähler potential with dimension zero ($[G]=0$; here $M_{\rm P}$ is the reduced Planck mass $8\pi G_{\rm N} M_{\rm P}^2 \equiv 1$). The same K is obtained for different choices of J because G is invariant under the Kähler transformations [110, 111]

$$\begin{aligned} J \to& J + h(\phi) + h^*(\phi^*) \\ W \to& e^{-h} W. \end{aligned} \tag{2.65}$$

where h is an arbitrary holomorphic complex function.

From the global supersymmetric action one can construct a locally invariant action systematically by using the Noether procedure [103, 112, 54]. First, a local transformation is performed to the globally symmetric Lagrangian. The original Lagrangian is not invariant anymore, but there exists a remainder term. That term is explicitly cancelled by subtracting it from the original Lagrangian, and modifying the transformation rule accordingly. The modified Lagrangian becomes schematically now

$$\mathscr{L}_1 = \mathscr{L}_0 - \delta\mathscr{L}_0. \tag{2.66}$$

The new local transformation is then performed to $\mathscr{L}_1$ and the iterative process is continued until the Lagrangian is invariant under the newly constructed transformation. In the process new gauge fields have to be introduced in order to be able to cancel the remainder. The gauge field of gravity is gravitino in this sense. If terms to the transformation law has been added, the closure of supersymmetry algebra must be confirmed at each step. After quite tedious calculations [112, 106] the locally supersymmetric Lagrangian is obtained.

The scalar potential (*i.e.* the part of the Lagrangian that does not contain fermions or derivatives) becomes [102]

$$\begin{aligned} V = M_{\rm P}^4 e^G \big(G^i (G^{-1})_i^j G_j - 3\big) \\ + \frac{g^2 M_{\rm P}^4}{2} {\rm Re} f_{ab}^{-1} G^i (T^a)_{ij}\phi_j G^k (T^b)_{kl}\phi_l, \end{aligned} \tag{2.67}$$

where $G^i \equiv \partial G/\partial\phi_i^*$ and $\big(G^{-1}\big)_i^j$ is the matrix of second derivatives of G^{-1}. The matrix G_i^j is called Kähler metric. The first term of Eq. (2.67) comes from F-terms and the second from D-terms. If the supersymmetry breaking is thought to be induced via F-term breaking, then the D-term contributions can usually be ignored.

The Kähler potential determines the conditions for the supersymmetry breaking. Supersymmetry is broken spontaneously via F-term, if

$$\langle G_i \rangle \equiv \langle \partial G/\partial \phi_i \rangle \neq 0 \tag{2.68}$$

for some field ϕ_i. For the D-term breaking the condition is

$$G^i (T^a)_{ij} \phi_j \neq 0. \tag{2.69}$$

From Eq. (2.67) one can see that if supersymmetry is not broken (*i.e.* the conditions of the Eq. (2.68) and Eq. (2.69) evaluate to zero) the potential Eq. (2.67) becomes $V = -3M_{\mathrm{P}}^4 e^G$. This is negative semidefinite, which is quite an opposite to the global supersymmetry (see Eq. (2.11) on page 13). If local supersymmetry is broken, the potential can have any value: positive, negative or zero in the minimum (in global supersymmetry the vanishing vacuum energy was a condition for unbroken supersymmetry). The vanishing of the potential in a broken phase enables one to fine-tune the cosmological constant to an appropriate value $\langle V \rangle \approx 0$.

After the super-Higgs mechanism (the D-terms neglected) the gravitino absorbs the massless goldstino and gains a mass [102, 83]

$$m_{3/2}^2 = \frac{1}{3} \langle G_j^i F_i F^{*j} \rangle = M_{\mathrm{P}}^2 \, e^{\langle G \rangle}, \tag{2.70}$$

where $F_i = -M_{\mathrm{P}}^2 e^{G/2} (G^{-1})_i^j G_j$ is the order parameter for the supersymmetry breaking. The last equality follows if $\langle V \rangle = 0$.

It is assumed that the superfields of the theory can be divided into two sectors: observable and hidden. The observable sector fields are the superfields of the MSSM. The hidden sector fields include gauge superfields of an asymptotically free gauge interaction that becomes strong at some intermediate scale Λ_S between the weak and Planck scales. These non-perturbative interactions are weak at the Planck scale but become strong at the scale Λ_S. There are also chiral superfields Z_i that can feel this gauge interaction. The observable sector superfields obviously don't feel the hidden sector gauge interactions, because otherwise those would have already been discovered.

After the generation of the soft breaking terms, there are 124 independent parameters in the MSSM [113, 114]. Of these, 18 parameters correspond to Standard Model parameters and one corresponds to one of the Higgs sector masses. The remaining 105 are genuinely new parameters of supersymmetric origin: 21 squark and slepton masses, 36 real mixing angles to define the squark and slepton mass eigenstates, five real parameters and three CP-violating phases in the gauge/gaugino/Higgs/higgsino sector, and 40 new CP-violating phases that can appear in squark and slepton interactions.

2.6.1 mSUGRA

As discussed above, the soft supersymmetry breaking terms derived from spontaneous hidden sector SUSY breaking are highly diverse. In general there are over hundred new parameters introduced to the model, which makes studying the phenomenology of the model practically impossible. Fortunately, there are scenarios that reduce the number of free parameters.

In gravity mediated (or minimal supergravity inspired) supersymmetry breaking scenario mSUGRA the information of the breakdown of supersymmetry at the hidden sector is transmitted to the visible sector via gravitational interactions. It is assumed that there are no Fayet-Iliopoulos (D) terms and the actual breaking is due to the *vev*s of the auxiliary superfields in the F-terms. Also the Kähler metric (see p. 29) is flat, which means that the Kähler potential is only linear in $\phi^*\phi$ and receives the value $(G^{-1})^i_j = \delta^i_j$.

The hidden sector superfields are completely neutral with respect to the Standard Model gauge group. If the observable sector superfield is denoted by Y, and the hidden sector superfield by Z, then the superpotential can be written schematically as

$$W(Z_i, Y_r) = \overline{W}(Z_i) + \widetilde{W}(Y_r) \tag{2.71}$$

This way any other than the gravitational effects are absent between the two sectors.

In this scenario, in the leading order, one may assume the minimal form of the kinetic terms, in which case the Kähler potential is

$$G = M_{\mathrm{P}}^{-2}(z^{*i}z_i + y^{*r}y_r) + \ln\frac{|W|}{M_{\mathrm{P}}^6}, \tag{2.72}$$

where z and y are scalars corresponding to the superfields Z and Y, respectively. Supersymmetry is assumed to be broken by the hidden sector vacuum expectation values

$$\langle z_i\rangle = d_i M_{\mathrm{P}}, \qquad \langle\widetilde{W}_i\rangle = \langle\partial\widetilde{W}/\partial z_i\rangle = c_i\mu M_{\mathrm{P}}, \qquad \langle\widetilde{W}\rangle = \mu M_{\mathrm{P}}^2, \tag{2.73}$$

with zero energy vacuum expectation value. The low energy effective potential is obtained by replacing $z_i, \widetilde{W}_i$ and $\widetilde{W}$ by their expectation values and keeping only those terms that do not vanish in the flat-limit $M_{\mathrm{P}} \to \infty$ (but keeping the gravitino mass fixed). The gravitino mass in this case is

$$m_{3/2} = |\mu|\exp(|d_i|^2/2). \tag{2.74}$$

Using the rescaled visible sector superpotential

$$\widehat{W} = \widetilde{W}\exp(|d_i|^2/2) \tag{2.75}$$

the effective potential becomes [115, 116]

$$V = |\widehat{W}_r|^2 + m_{3/2}^2|y_r|^2 + m_{3/2}(y_r\widehat{W}_r + (A-3)\widehat{W} + \text{H.C.}) + (D\text{-terms}) \tag{2.76}$$

where $A = d_i^*(c_i^* + d_i)$. If one expands the visible sector superpotential like [117]

$$\widehat{W} = m_i^2 y_i + b_{ij} y_i y_j + a_{ijk} y_i y_j y_k + \dots \tag{2.77}$$

the potential (2.76) can be written as

$$V \simeq |\widehat{W}_r|^2 + m_{3/2}^2 |y_r|^2 + \\ m_{3/2}\big[(A-2)m_i^2 y_i + (A-1)b_{ij} y_i y_j + A a_{ijk} y_i y_j y_k + \text{H.C.}\big]. \tag{2.78}$$

The first term is scalar potential of the unbroken global supersymmetry (see Eqs. (2.24) – (2.26)), the second term is a common mass term for all the scalars in the observable sector and following terms give linear, bilinear and trilinear couplings between the scalars. The Eq. (2.78) is of the form of the softly broken globally supersymmetric model discussed in Sec. 2.4 on page 22.

The derivation of the gaugino mass terms requires a non-minimal gauge kinetic function f_{ab}. If the gauge kinetic function can be expanded in powers of $1/M_{\text{P}}$ as

$$f_{ab} = \eta_{ab}\Big[\frac{1}{g_a^2} + \frac{1}{M_{\text{P}}} f_a^i \Phi_i + \dots\Big], \tag{2.79}$$

then the gaugino masses are generated by supersymmetry breaking as

$$m_{\lambda^a} = \frac{1}{2M_{\text{P}}} \text{Re}(f_a^i)\langle F_i\rangle. \tag{2.80}$$

If one assumes the equality of $\text{Re}(f_a^i)$ for each gaugino, then also the gaugino masses are universal at the GUT scale. This is, however, not motivated by the gravity mediated SUSY breaking scenario, but rather only by the request for simplicity.[6]

In the context of the MSSM, the resulting parameter space has shrunk into only four independent SUSY breaking parameters: The common scalar mass $m_0 = m_{3/2}$, the common gaugino mass $m_{1/2} = \frac{1}{2M_{\text{P}}}\text{Re}(f^i)\langle F_i\rangle$, the common trilinear coupling $A_0 = m_{3/2}Aa$ and the Higgs doublet mixing parameter $B_0 = (A-1)b$, which is usually written as $B_0\mu$, where μ is the supersymmetric Higgs mass parameter, which can be considered as a fifth input parameter. After the electroweak symmetry breaking and requirement that the Z-boson obtains its measured value, the μ and another input parameter B_0 can be written in terms of the ratio of the Higgs doublet *vevs* $\tan\beta$ and the electroweak symmetry breaking scale v. The sign of μ remains as a free parameter.

The gravity mediated supersymmetry breaking as considered above doesn't explain, why the soft breaking terms in the squark sector should be almost degenerate in flavor in order to avoid constraints from flavor-changing neutral current (FCNC) processes [118]. In the Standard Model the accidental symmetries suppress the flavor violation (the so-called GIM mechanism by Glashow, Iliopoulos

[6]One can construct string and GUT models, which suggest universality of gaugino masses.

and Maiani [119]), while in the MSSM in the presence of new scalar particles this mechanism is replaced by the super-GIM mechanism [120,121,122,123]. One solution to the flavor problem is provided by the so-called gauge mediated supersymmetry breaking (GMSB) model [124,125,126,127,128,129], where some new chiral supermultiplets, called messengers, mediate the supersymmetry breaking from the hidden sector to the visible sector. There is, however, a built-in solution to the flavor problem in the gravity mediated model already, provided that the tree level SUSY breaking contributions coming from the gravity mediation are suppressed. It is called the *anomaly mediated supersymmetry breaking*.

2.6.2 Anomaly mediated SUSY breaking

The soft supersymmetry breaking terms in the gravity mediated supersymmetry breaking mechanism has contributions originating from the super-Weyl anomaly via loop effects [130]. In order to get the supergravity Lagrangian kinetic terms into the canonical form, one must rescale the metric by the Weyl transformation

$$g_{\mu\nu} \to e^{K/3M_{\rm P}^2} g_{\mu\nu}. \tag{2.81}$$

This is justified by the fact that the supergravity Lagrangian is invariant under the Kähler transformations of Eq. (2.65) provided that an auxiliary chiral superfield is "compensating" the transformations [106, 110],

$$\varphi \to e^{h/3}\varphi. \tag{2.82}$$

Note that the field φ, called Weyl compensator, is not physical and is rotated away through a Weyl rescaling, until its scalar component receives a *vev*. In that case, the compensator superfield φ is usually written

$$\varphi = 1 + \theta^2 m_{3/2}. \tag{2.83}$$

However, in the quantum level the Lagrangian is not invariant under the Weyl transformation (2.81). Due to this breakdown of the superconformal Weyl invariance, the symmetry can be violated at the loop level giving rise to the anomalous contributions to the soft Lagrangian. Thus the soft masses are expected to be generated at the loop level, and this effect is present in all hidden sector models.

The anomalous contributions are usually suppressed, because the tree level couplings give the dominant contribution. If the tree level contributions, however, are somehow absent or very suppressed, the anomaly mediated contributions can dominate, as may happen, *e.g.*, in brane models [131]. (The brane models will be discussed in more detail in the context of extra dimensions in Section 3.1.) This kind of mechanism of supersymmetry breaking is referred to as the anomaly mediated supersymmetry breaking (AMSB) [131, 132, 133]. Anomaly mediation is a predictive framework for supersymmetry breaking in which the breaking of scale invariance mediates the supersymmetry breaking between the hidden and visible sectors.

The soft supersymmetry breaking parameters can be written in terms of the beta functions of the RG-equations and anomalous dimensions.[7] In the MSSM the pure anomaly mediated contributions to the soft supersymmetry breaking parameters m_λ (gaugino mass), m_i^2 (soft scalar mass squared), and A_y (the trilinear supersymmetry breaking coupling, where y refers to the Yukawa coupling) can be written as

$$m_\lambda = \frac{\beta_g}{g} m_{3/2}, \tag{2.84}$$

$$m_i^2 = -\frac{1}{4}\left(\frac{\partial \gamma_i}{\partial g}\beta_g + \frac{\partial \gamma_i}{\partial y}\beta_y\right) m_{3/2}^2, \tag{2.85}$$

$$A_y = -\frac{\beta_y}{y} m_{3/2}, \tag{2.86}$$

where $m_{3/2}$ is the gravitino mass, quantities β are the relevant beta functions, and quantities γ are the anomalous dimensions of the chiral superfields. An immediate consequence of these relations is that supersymmetry breaking terms are renormalization group invariant to all orders. Also the flavor violation effects are proportional to the Yukawa couplings, thus avoiding large effects in FCNC processes. In this way the gaugino masses are proportional to their corresponding gauge group β-functions with the lightest supersymmetric particle being mainly an $SU(2)$ gaugino wino (in a scenario, where the breaking is mainly due to the gravitational interactions at the tree level, the LSP is usually a $U(1)$ gaugino bino). Analogously, the scalar masses and trilinear couplings are functions of gauge and Yukawa coupling β-functions.

However, since the beta functions for $SU(2)$ and $U(1)$ are both positive, it turns out that the pure scalar mass-squared anomaly contribution for sleptons is negative giving rise to tachyons in spectrum [131]. There are a number of proposals for fixing the problem of tachyonic slepton masses [134, 135, 136, 137, 138, 139, 140, 141]. Additional contributions to the slepton masses can arise in a number of ways, but some of the solutions will spoil the most attractive feature of the anomaly mediated models, *i.e.*, the renormalization group invariance of the soft terms and the consequent ultraviolet insensitivity of the mass spectrum. Nevertheless, there are various ways to cure this problem without re-introducing the supersymmetric flavor problem [131,134]. The simplest option is to introduce a common mass parameter m_0 to all of the squared scalar masses [142]. This parameterizes the non-anomaly mediated contributions to the slepton masses, so as to cure their tachyonic spectrum. This addition does not re-introduce the supersymmetric flavor problem. This model is called a minimal anomaly mediated supersymmetry breaking model (mAMSB).

Another possible way of resolving the tachyonic slepton mass problem is the so-called gaugino assisted AMSB model ($\tilde{g}$AMSB). In the gaugino assisted anomaly

[7] Anomalous dimensions are defined as a derivative of the wave function renormalization w.r.t. the renormalization scale, $\gamma = \partial \ln Z / \partial \ln \mu$.

mediated model it is assumed that the gauge and gaugino fields reside in the bulk of the extra dimension [143]. There are no singlets in the hidden sector thus suppressing the tree level contributions and leaving the AMSB to be the leading breaking effect. The scalar soft masses receive additional contributions proportional to the eigenvalues of the quadratic Casimir operators of the relevant gauge group, thus removing the tachyonic sleptons.

The third possible solution to the tachyonic slepton problem is to add an extra $U(1)$ gauge group to the model in addition to the usual MSSM gauge symmetry structure [141]. The D-term contributions to the scalar masses are of opposite sign for the left- and right-handed particles. If now both the left- and right-handed particles have a positive charge under the new $U(1)$-group, then there can be positive contributions to the squared masses of the scalars.

In the article I of this dissertation the properties of these different anomaly mediated supersymmetry breaking scenarios were considered. The sum rules for sparticle masses were derived in order to distinguish different AMSB models, as well as rules to distinguish SUGRA type models from AMSB models. The sparticle spectrum were calculated for each model in order to compare the effects of the tachyonic mass fixing. Also, in the paper II the properties of the lightest neutralino in the AMSB was considered. General upper limits for the lightest neutralino mass were derived for both the AMSB and SUGRA type models.

In the AMSB scenario the gaugino masses are non-universal at the GUT-scale. This could be true also in other breaking scenarios. The possibility of non-universal gaugino masses originating from the specific GUT-scenarios ($SU(5)$) is discussed in the Section 2.8.

2.7 Split supersymmetry

The main motivation for supersymmetry, as discussed so far, has been its ability to give a solution to gauge hierarchy problem, as discussed in the Section 1.2. This is known as a naturalness criterion for a model. The naturalness in the MSSM is not perfect, though. The most pressing issue is the even more severe fine-tuning problem associated with the cosmological constant, which seems to be tuned to one part in 10^{120}.

During the past year an idea not to treat the hierarchy problem as a real problem has emerged. In the so-called split supersymmetry models [144,145,146] most scalars have very high masses whereas the fermions are kept light. In this framework it is assumed that the smallness of the cosmological constant could be explained by an anthropic principle [147] stating that the structure formation of the galaxies would require the smallness of the cosmological constant. If the cosmological constant was something else, no-one would be here wondering it. In a similar way, the formation of atoms [148] requires a small enough Higgs *vev*.

Abandoning the requirement of solving the hierarchy problem removes also the need for the theory to be supersymmetric. However, if one requires a proper dark matter candidate and the gauge coupling unification, the particle content of the MSSM is still the most minimal which satisfies these requirements [145].

The major successes of the supersymmetric standard model are the gauge coupling unification (as discussed in the Section 2.5) and the emergence of the viable dark matter candidate, the lightest supersymmetric particle (see Section 2.3). In the split supersymmetry these are still present, but some of the problems of the MSSM are swept away. When the gauginos and higgsinos are kept light (as required by the chiral symmetries and the gauge coupling unification to be still valid), the light neutralino is a good dark matter candidate for an LSP. When all the scalar masses are heavy w.r.t. the electroweak scale (except one Higgs particle which mass is fine-tuned to be at the EW-scale), the FCNC effects are suppressed at the low scale. The same happens for the dimension five interactions that mediate proton decay. Also the non-observation of the supersymmetric particles or the Higgs boson at the LEP or Fermilab can be explained by the heavier SUSY breaking scale.

The split supersymmetry model is described by six parameters: (1) a common mass $\widetilde{m}$ for the heavy scalars, (2) $\tan\beta$, where the angle β defines the combination of neutral $SU(2)$-doublet Higgs fields which remains light, (3) the higgsino mass parameter μ, (4) the gluino mass $m_{\tilde{g}}$, (5) the grand unification scale $M_{\rm GUT}$, and (6) the unified value of the gauge coupling strength α_G at $M_{\rm GUT}$. The last two are more or less fixed by the requirement of consistency with measurements of the three gauge coupling strengths at laboratory energies, thus leaving four relevant parameters.

Above the common heavy scalar mass scale $\widetilde{m}$ the model is equal to the ordinary MSSM. The model below the scale $\widetilde{m}$ is described by the effective theory obtained from the MSSM by removing squarks, sleptons and charged, heavy neutral and pseudoscalar Higgs particles from the particle content. The two Lagrangians are then matched at the boundary at $\widetilde{m}$, likewise the split SUSY Lagrangian which is matched to the SM Lagrangian at the SUSY breaking scale (say, $m_{\rm LSP}$). A notable difference compared to the MSSM is the existence of gaugino couplings below the splitting scale $\widetilde{m}$.

In the paper III of this dissertation the split supersymmetry has been analyzed in the light of the infrared fixed point of the top Yukawa coupling. In general, the observed physical quantities are assumed to be renormalization group evolved values of the parameters of the underlying high energy theory. This suggests that the low-energy values are defined by the actual values at the high scale. However, there are conditions, where the values of the parameters are determined only by the dynamics at the low energy scale. If the value of the parameter at the low energy scale does not depend on the value at the high energy scale, it is said to be on its fixed point value. This can happen *e.g.* for the top Yukawa coupling if the value at the high scale is large enough.

One can relate the Yukawa couplings to the gauge couplings through the RG evolution using the (quasi) infrared fixed points [149, 150]. The ratio of the top Yukawa coupling and the QCD gauge coupling is an example that obeys the (exact) fixed point structure. The related quasi fixed point is gained formally when the RGE of a parameter has a Landau pole at the high renormalization scale (*e.g.* $M_{\rm GUT}$).

In the Standard Model the top Yukawa coupling is larger than the others. The fixed point structure would explain this quite nicely, but the mass of the top quark obtained this way is a bit too large. In the MSSM the fixed point structure relates the mass of the top quark to the value of $\tan\beta$. Numerically the low-energy value of the top Yukawa coupling is indeed insensitive to its high-energy value for a wide range of GUT-scale values $Y_t(\rm GUT) \gtrsim 0.01$ [151]. Thus, if the fixed point is realized, the parameter space can be reduced and the model is more predictive. Since the infrared fixed point behavior depends on the running of the gauge couplings, its behavior is different in the split supersymmetry model than in the general MSSM.

If one supposes that the top quark Yukawa coupling is at its fixed point value, then, also in the split SUSY the top quark mass depends on the value of $\tan\beta$. Because the mass of the top quark is known within limits (1σ) $M_t^{pole} = 178.0 \pm 4.3$ GeV [36], the values of $\tan\beta$ can be limited to a narrow region in $\widetilde{m}$-$\tan\beta$ space. Our conclusion is that the infrared fixed point scenario is strongly disfavored in the case of split supersymmetry, more recently pointed out also by Delgado and Giudice [152].

2.8 Non-universal gaugino masses

Particle spectrum and masses are the quantities that are observed in the collider experiments. The neutralino sector has significant role in these experiments, since the lightest neutralino is in most scenarios of SUGRA type models the lightest supersymmetric particle. If the LSP is supposed to be stable, as is the case when R-parity is conserved, it eventually participates in any process involving SUSY particles, as being the last supersymmetric particle to be produced. Because neutralino mass eigenstates are mixtures of the gaugino and higgsino gauge eigenstates, the properties of neutralinos are determined mainly by the initial conditions of the gaugino and higgsino mass parameters. In the minimal supersymmetric standard model all the gaugino mass parameters are assumed to be equal at the GUT scale. In grand unified theories it is, however, equally possible to have a situation where the gaugino masses are not universal at the GUT scale.

The large number of free parameters in the Standard Model gives motivation for grand unified theories. Furthermore, when supersymmetry is introduced to solve the technical aspects of the gauge hierarchy problem, the number of free parameters is increased with respect to the SM. In order to reduce the number of parameters, it is appealing to think that there exists a unifying theory.

The SU(5) is the simplest model for GUT that is able to contain the SM group

$SU(3)\times SU(2)\times U(1)$ as a subgroup. $SU(5)$ is a simple group and it has rank four, the same as the SM group, which is the minimal requirement for a unifying group. In the global supersymmetric $SU(5)$ theory there are three different degenerate vacua corresponding to the $SU(3)\times SU(2)\times U(1)$, $SU(4)\times U(1)$ and the unbroken $SU(5)$ [59,153,154]. This degeneracy can be lifted by making the supersymmetry to be a local symmetry. If one chooses parameters suitably, the gravitational effects sets a *vev* that gives $SU(3) \times SU(2) \times U(1)$ to be the global minimum [155, 156].

In the local supersymmetric theory the Lagrangian can be written in terms of two fundamental functions, the Kähler potential [106] $G(\Phi_i, \Phi^*{}_i)$ which is a real function of chiral superfields and a singlet under the gauge group, and the gauge kinetic function $f_{ab}(\Phi)$, which transforms as a symmetric product of two adjoint representations. In general, the Φ_i fields are a set of superfields that are relevant to the resulting effective theory. Here one is interested in the gaugino mass terms, that result from the gauge kinetic part of the Lagrangian. The Lagrangian for the coupling of gauge kinetic function to the gauge field strength W^a is written as

$$\mathscr{L}_{gk} \;=\; \int d^2\theta f_{ab}(\Phi) W^a W^b + \text{H.C.}, \tag{2.87}$$

where a and b are gauge group indices, and repeated indices are summed over. The gauge kinetic function can be expanded in terms of the gauge non-singlet fields as

$$f_{ab}(\Phi) \;=\; f_0(\Phi^s)\delta_{ab} + \sum_n f_n(\Phi^s)\frac{\Phi^n_{ab}}{M_\text{P}} + \cdots \tag{2.88}$$

The Φ^s and the Φ^n are the gauge singlet and the gauge non-singlet chiral superfields, respectively. It is actually possible to cut the expansion of the gauge kinetic function after the linear non-singlet term, since the *vev* of the relevant non-singlet fields are assumed to be small [157], provided that the unification scale $M_\text{GUT} \ll M_\text{P}$ [158].

It is assumed that the auxiliary part F_Φ of a chiral superfield Φ gets a *vev* $\langle F_\Phi\rangle$, and breaks supersymmetry. Additionally, it is assumed that the $SU(5)$ gauge symmetry is broken to the SM gauge group $SU(3) \times SU(2) \times U(1)$ by a non-zero *vev*'s of non-singlet scalar fields. Then the gaugino masses arise from the coupling of $f(\Phi)$ with the field strength superfield W^a,

$$\mathscr{L}_{gk} \supset \frac{\langle F_\Phi\rangle_{ab}}{M_\text{P}}\lambda^a\lambda^b + \text{H.C.}, \tag{2.89}$$

where the fields λ^i are the gaugino fields. Since the gauge kinetic function transforms as a symmetric product of two adjoint representations, Φ and F_Φ can belong to any of the (irreducible) representations appearing in the symmetric product of the two **24** dimensional representations of $SU(5)$:

$$(\mathbf{24}\otimes\mathbf{24})_{Symm} = \mathbf{1}\oplus\mathbf{24}\oplus\mathbf{75}\oplus\mathbf{200}. \tag{2.90}$$

Table 2.3: Ratios of the gaugino masses at the GUT scale in the normalization $M_3(\text{GUT}) = 1$, and at the electroweak scale in the normalization $M_3(\text{EW}) = 1$ at the 1-loop level.

F_Φ	M_1^{G}	M_2^{G}	M_3^{G}	M_1^{EW}	M_2^{EW}	M_3^{EW}
1	1	1	1	0.14	0.29	1
24	-0.5	-1.5	1	-0.07	-0.43	1
75	-5	3	1	-0.72	0.87	1
200	10	2	1	1.44	0.58	1

Only the component of F_Φ that leaves the SM group $SU(3) \times SU(2) \times U(1)$ invariant should acquire a *vev*, in which case

$$\langle F_\Phi \rangle_{ab} = c_a \delta_{ab}. \tag{2.91}$$

In general the gauge couplings are not equal at the GUT scale anymore due to the corrections of the order of $\mathcal{O}(M_{\text{GUT}}/M_{\text{P}}) \sim \mathcal{O}(1/100)$. It is, however, adequate here to neglect such small non-universalities in the $SU(3)\times SU(2)\times U(1)$ couplings. In fact, the contributions of non-universality to the gauge couplings have little effect on the phenomenological aspects.

In the minimal case Φ and F_Φ are assumed to be in the singlet representation of $SU(5)$, which implies equal gaugino masses at the GUT scale. However Φ can belong to any of the non-singlet representations **24**, **75**, and **200** of $SU(5)$, in which case these gaugino masses are unequal but related to each another via the representation invariant coefficients c_a.

If one assumes that the dominant component of gaugino masses comes from one of the non-singlet F-component (*i.e.* the possible singlet component *vev* is small enough), then the ratios of resulting gaugino masses can be calculated as in Table 2.3. The coefficients that determine these mass relations are calculated from the generators of the unifying gauge group. For example, for the representation **24** the group theoretical coefficients c_a of the Eq. (2.91) are [157]

$$c_a \delta_{ab} \equiv d_{ab\,24} = 2\text{Tr}\big[\{\lambda_a/2, \lambda_b/2\}\lambda_{24}/2\big], \tag{2.92}$$

where the $\lambda_a/2$ are the generators of $SU(5)$ in the adjoint representation of dimension **24**. The coefficients for other representations are calculated similarly. It should be noted that Φ can actually transform as a linear composition of any of the representations, in which case the ratios of the gaugino masses change.

The RG-running of the gaugino masses M_i and the gauge couplings α_i (see Eq. (2.42)) are closely related at the one-loop level. The ratio of these is the same regardless of the renormalization scale:

$$\frac{M_i(t)}{\alpha_i(t)} = \frac{M_i(\text{GUT})}{\alpha_i(\text{GUT})}. \tag{2.93}$$

Hence the RG-dependence of the gaugino masses can be expressed as

$$M_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_W}\frac{M_1(\textsc{gut})}{\alpha_1(\textsc{gut})}, \tag{2.94}$$

$$M_2 = \frac{\alpha}{\sin^2\theta_W}\frac{M_2(\textsc{gut})}{\alpha_2(\textsc{gut})}, \tag{2.95}$$

$$M_3 = \alpha_3\frac{M_3(\textsc{gut})}{\alpha_3(\textsc{gut})}. \tag{2.96}$$

Using the gaugino mass relations determined at the GUT-scale one can write down the relations between the gaugino masses at any scale in the chosen representation. For the **24** dimensional representation of $SU(5)$ the gaugino mass relations are

$$\frac{M_1}{M_3} = -\frac{1}{2}\Big(\frac{5}{3}\frac{\alpha}{\cos^2\theta_W}\Big)\frac{1}{\alpha_3}, \qquad \frac{M_2}{M_3} = -\frac{3}{2}\Big(\frac{\alpha}{\sin^2\theta_W}\Big)\frac{1}{\alpha_3}. \tag{2.97}$$

Similarly, for the **75** dimensional representation of $SU(5)$ one has

$$\frac{M_1}{M_3} = -5\Big(\frac{5}{3}\frac{\alpha}{\cos^2\theta_W}\Big)\frac{1}{\alpha_3}, \qquad \frac{M_2}{M_3} = 3\Big(\frac{\alpha}{\sin^2\theta_W}\Big)\frac{1}{\alpha_3}, \tag{2.98}$$

and for the **200** dimensional representation of $SU(5)$ the relations are

$$\frac{M_1}{M_3} = 10\Big(\frac{5}{3}\frac{\alpha}{\cos^2\theta_W}\Big)\frac{1}{\alpha_3}, \qquad \frac{M_2}{M_3} = 2\Big(\frac{\alpha}{\sin^2\theta_W}\Big)\frac{1}{\alpha_3}. \tag{2.99}$$

These results are then run down to the electroweak scale by using the relevant renormalization group equations for the gauge couplings. The relations at the electroweak scale are shown in the Table 2.3. These are calculated using one loop RG-equations for the gaugino masses and the gauge couplings. Two-loop effect is to increase the M_1/M_2-ratio slightly.

After the $SU(5)$ and supersymmetry breaking the resulting model can be parameterized by six parameters: The gluino mass $M_3(\textsc{gut})$, ratio of Higgs *vev*'s $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$, the supersymmetric Higgs mixing parameter μ, the universal scalar mass m_0, pseudoscalar Higgs boson mass m_A and the trilinear coupling A_0. After the radiative EWSB the values μ and m_A can be calculated leaving only sign(μ) undetermined.

In paper IV of this dissertation the phenomenology of the non-universality of the gaugino masses was studied. Because of the importance of the LSP in particle interactions as well as in the cosmological dark matter considerations, the properties of the lightest neutralino were analyzed. In this work it was assumed that the lightest neutralino is the LSP. Also two specific types of particle decays involving Higgs bosons and neutralinos were considered in order to study the effects of the non-universality in the Higgs sector phenomenology.

From the trace of the neutralino and chargino matrices, one can calculate the average mass squared difference of the charginos and neutralinos. This mass squared difference depends only on the physical masses, and not on the Higgs(ino) mass parameter μ or the ratio of *vevs*, $\tan\beta$ [159]. For the four different representations of $SU(5)$ which arise in Eq. (2.90), one finds at the tree-level the sum rules

$$\begin{aligned}
M_{sum}^2 &= 2(M^2_{\tilde{\chi}_1^\pm} + M^2_{\tilde{\chi}_2^\pm}) - (M^2_{\tilde{\chi}_1^0} + M^2_{\tilde{\chi}_2^0} + M^2_{\tilde{\chi}_3^0} + M^2_{\tilde{\chi}_4^0}) \\
&= (\alpha_2^2 - \alpha_1^2)\frac{M_{\tilde{g}}^2}{\alpha_3^2} + 4m_W^2 - 2m_Z^2, \qquad \text{for } \mathbf{1}, \qquad (2.100) \\
&= (\frac{9}{4}\alpha_2^2 - \frac{1}{4}\alpha_1^2)\frac{M_{\tilde{g}}^2}{\alpha_3^2} + 4m_W^2 - 2m_Z^2, \qquad \text{for } \mathbf{24}, \qquad (2.101) \\
&= (9\alpha_2^2 - 25\alpha_1^2)\frac{M_{\tilde{g}}^2}{\alpha_3^2} + 4m_W^2 - 2m_Z^2, \qquad \text{for } \mathbf{75}, \qquad (2.102) \\
&= (4\alpha_2^2 - 100\alpha_1^2)\frac{M_{\tilde{g}}^2}{\alpha_3^2} + 4m_W^2 - 2m_Z^2, \qquad \text{for } \mathbf{200}. \qquad (2.103)
\end{aligned}$$

From these sum rules one can see that at the tree-level the average mass squared difference between charginos and neutralinos is positive for the representations **1**, **24** and **75**, whereas for the representation **200** it is negative. In this respect the representation **200** resembles the anomaly mediated supersymmetry breaking scenario, where it was found that the average mass squared difference is negative [1].

If all of the supersymmetric partners are not heavier than the Higgs particles, it is possible that the decays of H^0, A and $H^\pm$ to the supersymmetric particles can be important or even dominant. In paper IV two specific decay chains were considered, namely $pp \to (H^0, A^0) \to \chi_2^0\chi_2^0 \to 4l$ and the cascade decay chain $\tilde{q}, \tilde{g} \to \tilde{\chi}_2^0 + X \to \tilde{\chi}_1^0 h(H^0, A^0) + X \to \tilde{\chi}_1^0 b\bar{b} + X$, both in the LHC context. In the first chain l is an electron or a muon (τ's may decay to pions *etc.* thus giving hadronic signature, which is drowned by the hadronic background). The method of producing Higgs bosons by cascade decays from supersymmetric particles does not depend on the value of $\tan\beta$. This is due to the fact that the Higgs particle final states are produced by the strong production of squarks and gluinos and their subsequent decays to gauginos and higgsinos. The significance of $\tan\beta$ is diminished by the presence of other relevant parameters [160], such as gaugino and higgsino mass parameters, as opposed to the case of direct Higgs boson production, where the parameter $\tan\beta$ sits directly in the interaction vertices. Thus, this method of producing Higgs bosons may help to cover a larger parameter space as compared to the more conventional methods of studying the Higgs sector of supersymmetric models, including also the heavier Higgs bosons.

It was found that depending on the region of the parameter space, the Higgs boson decay $h(H^0, A^0) \to \tilde{\chi}_2^0\tilde{\chi}_2^0$ may be observable in any representation in the Eq. (2.90). However the region in which $\tilde{\chi}_2^0 \to 2l + X$ is large, and it is possible for Higgs bosons to decay to the second lightest neutralinos, is rather limited in any of these models. Interestingly, for the production of the Higgs bosons via the decay chain including $\tilde{\chi}_2^0 \to h(H^0, A^0)\tilde{\chi}_1^0$, in addition to the singlet, a relevant region of the parameter space was found only for the representation **24**. It should be noted that in these two cases the signatures are clearly different, since in the representation **24** the cross section is largest at the lighter values of the gluino mass as opposed to the case of **1** representation. Also the fact that all the neutral Higgs channels are open in the **24** case distinguishes it from the singlet case, where only the light Higgs channel is available.

Neutralinos are combinations of gauginos and higgsinos, so it is evident that their properties vary greatly with respect to the chosen representation. The composition and mass of the neutralinos and charginos will play a key role in the search for supersymmetric particles. These properties determine also the time-scale of their decays. They can play an important role in the decays of Higgs bosons when they are kinematically allowed to decay to the second lightest neutralino pair, which in turn may decay to the lightest neutralinos and two leptons. Generally it is important to realize that the detection modes (*e.g.* of Higgs particles) depend strongly on the parameters of the model. Thus the investigation of non-minimal models is of a great importance in order to gain understanding of the underlying model.

Chapter 3

Extra dimensions

The four-dimensional Minkowski space is the fundamental spacetime structure, within which the laws of physics are formulated. The goal of introducing extra dimensions beyond the standard three spatial and one temporal dimensions is to simplify the structure of a theory. This is not a new idea; the first to introduce the fifth dimension to formulate electromagnetism and theory of scalar gravity into a single unified theory was the Finnish physicist Gunnar Nordström in 1914 [161]. Later, in the 20's, the idea of unification in extra dimensions was discovered again by German mathematician Theodor Kaluza (in 1919, but published only two years later) and Swedish physicist Oskar Klein (1926) [162, 163, 164].

3.1 Idea of compactification

An obvious difficulty in multidimensional theories is finding a mechanism that hides the extra dimensions. Usually the extra dimensions are thought to be compact as opposed to the infinitely ranging four standard dimensions. If all the fields are allowed to occupy the possible extra dimensions, the characteristic size (the compactification radius) of the new dimensions must be smaller than the wavelength of the particle fields, since the extra dimensions are still hidden. This is evident from the fact that the momentum of a field is quantized in the direction of that dimension due to the periodicity of the dimension. In the standard non-compact dimensions this looks like massive excitations of the field. In fact, an infinite tower of massive new particles is seen in the non-compact dimensions. This sets the limit of the size of the extra dimensions to be extremely small (of the order of the inverse weak scale) [165, 166, 167].

If, however, only gravity is allowed to traverse the extra dimensions, the above limits do not apply, and the size of the extra dimensions can be substantially larger, even of the order of the parts of a millimeter. This kind of approach means that ordinary matter is confined to a subspace of the multidimensional space. This is common in string theory, where the consistency requires the existence of non-perturbative soliton-like objects called Dirichlet p branes (Dp-branes) [168, 169]. Particles are localized on the D-branes as endpoints of open strings (with Dirichlet boundary conditions), hence confining matter to the brane. Branes are embedded in the $(p+\delta)$-dimensional bulk space. In the view of low-energy theories, the "world" is confined to a D3-brane (1+3 dimensions) and embedded into a $(3+\delta)$-dimensional bulk, where δ is the number of extra dimensions. It should be noted that in addition to gravity, one may also allow other particles that have no interactions with the SM particles, to occupy the bulk space. One such candidate is clearly the right handed neutrino [170, 171, 172, 173].

The idea of large extra dimensions have a couple appealing features: (1) They may re-interpret the hierarchy problem (and solution) and (2) the gravitational effects could be tested in colliders already in the TeV range. Recently a few models based on the large extra dimensions have been introduced. The first of the recent ones was a model by Arkani–Hamed, Dimopoulos and Dvali (ADD) in 1998 [174, 175]. In their model there are in principle any number of extra dimensions and the ordinary matter is localized on the brane. This model was the subject of the paper V of this dissertation and will be discussed in the Sections 3.2 and 3.3.

Approximately a year later as the ADD model emerged, Randall and Sundrum proposed their model of warped extra dimension (RS) [176]. In this model there is only one extra dimension, but it is strongly curved due to the large negative cosmological constant. This type of space is known as anti de Sitter (AdS) space. The metric of the RS model is written as

$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \tag{3.1}$$

where k is a scale of the order of the M_P, x is the coordinate of the ordinary four-dimensional metric and ϕ is the coordinate of the extra dimension. The coordinate ϕ is bounded to the interval $0 \le \phi \le \pi$ set by r_c (*i.e.* $r_c\pi$). The metric (3.1) is non-factorizable, *i.e.* the four-dimensional metric is not independent of ϕ, but it is multiplied by an exponential warp factor depending on ϕ. The model can be described by two 3-branes, which are situated at the orbifold fixed points $\phi = 0$ and $\phi = \pi$ of the extra dimension (the fifth dimension is an orbifold $S^1/\mathbb{Z}_2$, *i.e.* a circle with opposing points identified as $\phi = -\phi$). One (or both) of the branes contains ordinary four-dimensional field theories. The four-dimensional mass scales are then related to the five-dimensional by the exponential factor of Eq. (3.1). In this setting, the large hierarchy between the Planck scale M_P and the electroweak scale is due to the exponential factor of a small number r_c, which then solves the hierarchy problem. The exponential factor suppresses also the observable cosmological constant from the large value that originally induced the warped metric (3.1). It must be noted, though, that the parameter k has to be large for this to happen, thus introducing a new hierarchy. Therefore some mechanism to stabilize the compactification radius r_c should be found. Randall and Sundrum also considered a model, where the compactification radius was set to infinity [177], in which case only one brane is needed. They found that also this case is not in contradiction with the present observations of four-dimensional gravity.

The supersymmetry is comfortably connected to the concept of extra dimensions in the two-brane model of Randall and Sundrum [131, 178]. This is closely related to the case of anomaly mediated supersymmetry breaking, which was discussed in the Section 2.6.2. The visible fields are located at the $\phi = \pi$ brane, which is called visible brane. The other brane is called hidden brane in the spirit of hidden sector supersymmetry breaking. In the effective low energy point of view the actual SUSY breaking can occur also in the bulk, in which case the hidden sector is named a sequestered sector. The flavor violation is absent, since the couplings that result from the heavy states are exponentially suppressed. If also the gauge and gaugino fields are allowed to live in the bulk, the hidden sector is not so hidden anymore, but the gauginos obtain masses through their direct couplings to the supersymmetry breaking source [179]. The flavor changing neutral currents are still suppressed giving motivation also for this model. Furthermore, the scalar masses are positive solving the tachyonic slepton mass problem of the pure anomaly mediated supersymmetry breaking scenario.

A third extra dimensional model suggested by Appelquist, Cheng and Dobrescu [180] is called universal extra dimensions. In that model all the particles are allowed to travel in the bulk space. The universal extra dimensional models are able to produce a viable dark matter candidate through the Kaluza-Klein decomposition of the fields.

3.2 Large extra dimensions (ADD)

Arkani–Hamed, Dimopoulos and Dvali [174, 175, 170] proposed that the standard model fields live on a brane embedded in 2 to 6 extra dimensions. Since the ordinary matter does not propagate in the bulk, the extremely small limits for the size of the extra dimensions do not apply. With the flat metric, the gravity cannot be restricted to the branes since it is a property of the spacetime. Thus the upper limit of the size of the extra dimensions is set by the gravitational interactions. There is only one fundamental scale M_D in the theory, which is identified with the electroweak scale.

To get an idea of the compactification radius, let's now assume the spacetime to be $\mathbb{R}^4 \times \mathbb{T}^{D-4}$ (where $\mathbb{T}$ is a torus, or, in general, some other $\delta = D - 4$ dimensional compact manifold M_δ, *e.g.* a δ-sphere S^δ) and the metric is thought to be factorizable (*i.e.* it can be divided into a 4-dimensional metric g and into a part, that does not depend on g). When calculating the effective action one can integrate the extra dimensional part away, which then gives a relation between the D-dimensional and the ordinary 4-dimensional Newton constant,

$$\frac{1}{G_N} = \frac{V_{D-4}}{\widehat{G}_N}, \tag{3.2}$$

		Radius of compactified space	
δ	D	$(\text{GeV})^{-1}$	m
1	5	5.93×10^{27}	1.17×10^{12}
2	6	2.44×10^{12}	0.000481
3	7	1.81×10^{7}	3.57×10^{-9}
4	8	49400	9.74×10^{-12}
5	9	1430	2.82×10^{-13}
6	10	135	2.66×10^{-14}

Table 3.1: The compactification radius in the ADD-model in terms of $(\text{GeV})^{-1}$ and meters when the fundamental scale M_D is supposed to be 1 TeV. Here $D = 4 + \delta$.

where G_N is the four-dimensional Newton constant, $\widehat{G}_N$ is the D-dimensional Newton constant and V_{D-4} is the volume of the compact space.[1] The reduced Planck mass in four dimensions is $\overline{M}_{\text{P}}^2 = 1/8\pi G_N$. Analogously, in D dimensions $\overline{M}_D^{-2+D} = 1/8\pi\widehat{G}_N$. Therefore, using the fact that $V_{D-4} = (2\pi R)^{D-4}$ for torus, one gets

$$\overline{M}_{\text{P}} = R^{\frac{D-4}{2}} \left[(2\pi)^{\frac{D-4}{D-2}} \overline{M}_D\right]^{\frac{D-2}{2}} \equiv M_D (RM_D)^{\frac{D-4}{2}}. \tag{3.3}$$

This defines the relation between the reduced D-dimensional Planck mass and the D-dimensional Planck mass (the fundamental scale) as $\overline{M}_D = (2\pi)^{-\delta/(2+\delta)} M_D$. If one now assumes that the fundamental Planck scale is set as $M_D \sim 1\text{TeV}$, the compactification radius R can be calculated as in the Table 3.1. Obviously the case of one extra dimension is already ruled out, since the compactification radius would be of the order of the orbital distance of Saturn from Sun. For distances less than R the Newton's law is modified, since the space is D-dimensional. The smallness of the electroweak scale is explained by the largeness of the extra dimensions and the gravity is weak since it is diluted by the large total volume of the spacetime. But also here there is a hierarchy involved: One needs an explanation for $RM_D \gg 1$ in Eq. (3.3).

To see the effects of extra dimensions in the colliders, one needs to consider the effective model in the four dimensions [181, 182]. In the low energy limit the spectrum includes just Standard Model particles in four dimensions with the graviton in $4 + \delta$ dimensions (and maybe some other light fields related to the brane dynamics as in the reference [181]). Effectively the graviton is described in the four dimensions by the massive Kaluza-Klein (KK) excitations. The couplings between graviton modes and ordinary matter are of the gravitational strength. When considering distances greater than $1/M_D$ from the brane, the gravitons can

[1] Volume of an n-sphere is $V_n = \frac{\pi^{n/2} R^n}{\Gamma(n/2+1)}$ and of an n-torus $V_n = (2\pi R)^n$

be derived from the linearized metric,

$$g_{AB} = \eta_{AB} + 2\frac{h_{AB}}{\overline{M}_D^{1+\delta/2}}, \quad A, B = 1, \ldots, D. \tag{3.4}$$

The metric g is assumed to be factorizable. When the four-dimensional coordinates are denoted as x^μ, $\mu = 0, \ldots, 3$, the extra dimensional coordinates as y_i, $i = 1, \ldots, D-4$, and the periodicity of the translation in the compactified space is required, the perturbed part of the metric (3.4) can be expanded as [182]

$$h_{AB} = \sum_{n_1=-\infty}^{\infty} \cdots \sum_{n_\delta=-\infty}^{\infty} \frac{h_{AB}^{(n)}(x)}{\sqrt{V_\delta}} e^{in^j y_j/R}, \tag{3.5}$$

where $n = (n_1, \ldots, n_\delta)$, V_δ is the volume of the compactified space and the fields $h_{AB}^{(n)}(x)$ are Kaluza-Klein modes which occupy the four-dimensional space. In the limit of weak gravitational field the energy-momentum (EM) tensor can be written

$$T_{AB}(x, y) = \eta_A^\mu \eta_B^\nu T_{\mu\nu}(x)\delta(y), \quad \mu, \nu = 0, \ldots, 3, \tag{3.6}$$

where the delta function ensures that the matter is confined on the brane and all the KK-modes of the EM-tensor are independent of n in the low energy region.

The equations of motion of the low-energy effective theory obey the D-dimensional Einstein equation,

$$\mathcal{G}_{AB} \equiv \mathcal{R}_{AB} - \frac{1}{2} g_{AB}\mathcal{R} = -\frac{T_{AB}}{\overline{M}_D^{2+\delta}}, \tag{3.7}$$

where the $\mathcal{R}_{AB}$ and $\mathcal{R}$ are the Ricci tensor and scalar in D-dimensions, respectively. After substituting the linearized and expanded metric (3.4) and (3.5) to Eq. (3.7), the equations of motion for the graviton fields emerge. The non-propagating degrees of freedom can be eliminated by inspecting the gauge transformation

$$z_A \to z'_A = z_A + \epsilon_A(z), \tag{3.8}$$

which has an effect in the variation of the metric as

$$\delta_\epsilon h_{AB} = -\partial_A \epsilon_B - \partial_B \epsilon_A. \tag{3.9}$$

Some of the fields in expanded Eq. (3.7) are not invariant under the general coordinate transformation of Eq. (3.8) (thus being not physical fields) and can be rotated away. After this, there are four physical fields remaining:

$$G_{\mu\nu}^{(n)}, \; V_{\mu,j}^{(n)}, \; S_{jk}^{(n)} \quad \text{and} \quad H^{(\widehat{n})}, \tag{3.10}$$

The $\hat{n}$ in $H^{(\widehat{n})}$ is defined as $\hat{n} \equiv n/R$ (n is from Eq. (3.5) and below). The Greek indices run as $\mu = 0, \ldots, 3$ and the Latin indices as $j = 4, \ldots, D-1$.

The first one of the fields in (3.10), $G_{\mu\nu}^{(n)}$, is the graviton (also called gravitensor). The choice of the gauge, which eliminated the non-propagating degrees of freedom (*d.o.f.*) results in an emergence of a mass to $G_{\mu\nu}^{(n)}$. The mass is equal to $|n|/R$. Because the mass splitting of subsequent KK-modes is proportional to R^{-1}, the masses are extremely close to each other in the ADD-model (see Table 3.1). In practical calculations the distribution of the graviton KK-modes is considered to be a (quasi) continuum. Gravitensor $G_{\mu\nu}^{(n)}$ has five degrees of freedom (five of the original ten *d.o.f.* turned into the mass of $G_{\mu\nu}^{(n)}$). The second quantity $V_{\mu,j}^{(n)}$ is a set of massive vector fields. $\delta-1$ of them are independent and they are called gravivectors (or graviphotons). As opposed to gravitensors, they do not couple to the EM-tensor (in the weak limit) and therefore they do not participate in the particle processes. The symmetric tensor $S_{jk}^{(n)}$ is a set of massive real scalars called graviscalars. They have $(\delta^2-\delta-2)/2$ independent degrees of freedom and like gravivectors, they don't couple to the EM-tensor. The remaining field $H^{(\widehat{n})}$ is a scalar field, which is coupled only to the trace of the EM-tensor. For conformally invariant theories the trace of the EM-tensor vanishes (on-shell), and therefore the scalar $H^{(\widehat{n})}$ does not participate in the tree-level processes with massless particles. For the massive particles the coupling is proportional to the mass of the particle. The zero-mode of $H^{(\hat{n})}$ is called radion and it should not be massless [170]. The mechanism that stabilizes the radius R is thought also to give a sufficient mass to radion.

The astrophysical bounds (graviton emissions in supernovae and neutron stars) limit the possible observation of graviton-induced processes at future collider experiments in the case of 2 and 3 flat extra dimensions [183,184]. However, modifying the compactification manifold it is possible to weaken these limits (and even for $\delta = 1$) [185].

3.3 Invisible Higgs

If large extra dimensions are realized in Nature, the consequences are seen as the modification of the way that gravity behaves. In colliders, the effects of gravitational strength are usually vanishing when compared to the gauge interactions, if the fundamental scale is not very low (around a few TeV). However, the graviton Kaluza-Klein modes that result from the compactification of the extra dimensions offer a way to get in touch with the extra dimensional space. One way is to look at particles that have the same quantum numbers as some of the graviton KK-excitations. The Higgs field is a simple candidate: it is a scalar with no baryon or lepton number, color or charge. The same applies to the graviscalar field $H^{(\widehat{n})}$. So it is possible that the Higgs boson can decay invisibly by oscillating into a graviscalar Kaluza-Klein tower. Then it is possible that the Higgs boson can have a substantial branching ratio for decay into invisible final states. In ADD-type models as discussed in Sec. 3.2, the projection on the brane leads to a continuum of scalar and tensor graviton states.

The effects of extra dimensions on the Higgs field decay modes depend on the size of the extra dimensions. In the case of the large extra dimensions the major effects are due to the closely spaced KK-levels. The amount of closely spaced Kaluza-Klein states is so huge that even if the probability of transforming into an individual KK-state is small, the total probability can be considerable. This leads to the possibly effective invisible decays of the Higgs boson. In case of the small extra dimensions the effect comes from the mixing of the Higgs boson with the single radion [176, 177].

The methods of detecting the invisible Higgs are usually based on tagging of some other particle than the decay products of the Higgs boson. Mass limits for a Higgs boson decaying dominantly to invisible particles have also been obtained by the LEP experiments [186]. It is assumed that the Higgs boson is produced in association with a Z-boson, which decays either to charged leptons or hadrons. Moreover, a constraint that the invariant mass of the decay products are consistent with the Z mass is also applied. This then leads to the mass limit of $m_H > 114.4$ GeV.

The energy-momentum tensor $T_{\mu\nu}$ can be extended by a term of the type $\xi(\eta_{\mu\nu}\partial_\alpha\partial^\alpha - \partial_\mu\partial_\nu)$ while it is still conserved [187]. Therefore one can add to the Lagrangian a mixing term

$$S = -\xi \int d^4x \sqrt{-g_{ind}}\mathcal{R}(g_{ind})H^\dagger H, \tag{3.11}$$

where H is the Higgs doublet, ξ is a dimensionless mixing parameter, g_{ind} is the induced metric on the brane, and $\mathcal{R}$ is the Ricci scalar. Once electroweak symmetry is broken, the coupling of the trace of the additional part to the graviscalars leads to a mixing between the physical Higgs field (h) and each member of the graviscalar tower. One can parameterize such mixing by the following term in the Lagrangian:

$$\mathscr{L}_{mix} = \frac{1}{M_{\mathrm{P}}} m_{mix}^3 h \sum_n H^{(\widehat{n})}, \tag{3.12}$$

where $m_{mix}^3 = 2\kappa\xi v m_h^2$, M_{P} is the reduced Planck mass, and v is the Higgs field vacuum expectation value. The dimensionless parameter κ is expressed in terms of the number of extra dimensions as

$$\kappa \equiv \sqrt{\frac{3(\delta-1)}{\delta+2}}. \tag{3.13}$$

The specific choice of κ is due to arranging the $H^{(\widehat{n})}$ field to have a canonical normalization.

One proceeds by considering the Higgs propagator in the flavor basis and incorporating all the insertions induced by the mixing term. The effect of having a large number of real intermediate states inserted leads to the development of an imaginary term in the propagator. This imaginary part can be interpreted as an effective decay width entering into the propagator [187]:

$$\Gamma_G = 2\pi\kappa^2\xi^2 v^2 \frac{m_h^{1+\delta}}{M_D^{2+\delta}} \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}, \tag{3.14}$$

where M_D is the $(4+\delta)$-dimensional Planck scale (sometimes also called the string scale). A Higgs boson has a finite probability (proportional to ξ^2) of oscillating into the invisible states corresponding to the graviscalar tower. The transition is favored when masses of the Higgs boson and the corresponding graviscalar are close to each other.

Even though the probability of the Higgs field mixing with a single graviscalar state $H^{(\widehat{n})}$ is suppressed by M_{P}, summing the oscillation probabilities over the huge Kaluza-Klein tower makes the resulting probability for the Higgs field transforming into one of the many graviscalars large. The opposite is not true: the probability of a graviscalar turning into a Higgs boson is practically infinitesimal. Thus, once the Higgs boson is transformed into a graviscalar, it becomes invisible. This is reflected in the "invisible" decay width developed by the propagator.

In the paper V of this dissertation the possibilities of detecting invisibly decaying Higgs boson in a future electron-positron collider were studied. The effective invisible decay width grows as m_h^3 for $\delta = 2$. This implies that even for $m_h < 2m_W$ total Higgs boson decay width can be considerably larger than the Standard Model width. As a consequence, even for a light Higgs boson, the resonance may not be very sharp, hence complicating the reconstruction of recoil invariant Higgs mass peak. The Higgs boson can have a very large invisible branching ratio to invisible states in large extra dimensions.

The process of producing Higgs bosons,

$$e^+e^- \to Z(\to \mu^+\mu^-)h(\to \mathrm{inv}), \tag{3.15}$$

also known as the Higgs-strahlung process, at a linear collider with center of mass energy of 1 TeV was studied. The final state consists of a $\mu^+\mu^-$ pair with missing energy/momentum. The direct graviscalar production have to be taken into account. A similar final state can arise also in the production of a Z-boson with towers of graviton (spin-2). Thus, in addition to the SM processes $e^+e^- \to ZZ/WW \to \mu^+\mu^- + \not{E}$, the gravitensor and graviscalar production together with a Z-boson has to be considered, all leading to identical final states.

To filter out the Higgs boson effects from continuum gravitensor contributions one reconstructs the recoiled invariant mass which peaks at the Higgs mass modulo the Higgs width and detector resolution. The other important factor is the height of the peak against the continuum background. It is determined by the Higgs-graviscalar mixing ξ, the same quantity which also determines the Higgs width, making the width large for $\xi \sim \mathcal{O}(1)$. This causes the invisible decay recoil mass distribution to lose its sharp character even for a Higgs boson mass of the order of 120 GeV.

The Higgs boson mixing with graviscalars causes the Higgs particle to develop an invisible decay width. In the context of a linear e^+e^- collider such invisibility brings in additional problems in reconstructing the Higgs boson as a recoil mass peak against an identified Higgs boson. This is because of (a) the simultaneous presence of graviton continuum production in association with a Z-boson, and (b) the broadening of the Higgs peak due to enhancement of the total effective decay width. It was found that while an angular cut partially softened the difficulty, the broadening of the peak remains a problem, particularly for $m_H > 2m_W$.

In hadron colliders, such as the LHC, the identification of an invisibly decaying Higgs is in general a difficult task. In four dimensions, the associated production of Higgs particles (such as WH) and the gauge boson fusion channel might be of some use [188,189], but in the ADD model the gravitons can be produced by the same mechanism as considered above. Therefore, the search for a Higgs boson and the detailed investigation of its properties in the framework of this kind of a scenario has wider implications than an invisibly decaying Higgs particle arising in most other models.

Chapter 4

Summary

One of the most challenging tasks for all current and future accelerators is to discover the Higgs boson, whether it is the supersymmetric, Standard Model or some other type of Higgs. At LHC one will be able to identify or exclude the Higgs boson of mass below the TeV-scale. After finding the Higgs boson, the next task is to determine its origin. Several clues point towards the physics beyond the standard model, and a few features of conceivable models beyond the SM has been discussed in this dissertation.

The supersymmetry is a beautiful concept that continues the idea of using symmetries as the fundamental basis of physical theories. In the low energy effective theories it nicely removes the gauge hierarchy instability and incorporates a pinpoint gauge coupling unification. However, without the knowledge of the underlying, fundamental theory, the supersymmetry introduces an overwhelming amount of new parameters. All the predictability of the model is drowned by the ignorance of the values of parameters. Some of this uncertainty can be eased by making the supersymmetry a local symmetry. This is thought to be a necessary feature at the higher energy scales and in models that relate general relativity to supersymmetry. Local supersymmetry, *i.e.* supergravity, gives hints of possible ways to eventually break the supersymmetry, which must happen at some point, since the observed world certainly is not supersymmetric. Taking inspiration from the supergravity and the possible underlying grand unified theory, the number of over hundred soft supersymmetric parameters can be reduced to only a few. This minimal SUGRA model is still far from being fixed, but some predictability is restored. There are a few other well-studied means to gain supersymmetry breaking. One of them, the anomaly mediated SUSY breaking was discussed in this dissertation. It is well motivated and somewhat more predictive scenario than the minimal SUGRA breaking scenario, yielding a spectrum clearly different from the other breaking scenarios. Especially the nature of the lightest supersymmetric particle is changed. After masses of some of the supersymmetric particles have been measured, a whole lot more can be said about the mechanism that breaks supersymmetry. Some of the devices to determine the correct model has been presented in this dissertation.

Yet, it is not excluded that Nature has chosen not to take the simplest way of organizing parameters. For theoretical grounds, the unification of certain parameters is only one possibility amongst others. This is the case with the gaugino masses, which can well be non-universal. However, the predictability is not lost, since in the simple case there are no more than four different representations that can be chosen. The boundary conditions of gaugino masses have significant effect on the composition of the neutralino masses. Since the lightest supersymmetric particle in SUGRA and AMSB is in most cases the lightest neutralino, this has also prominent consequences on the observed effects on the collider physics. Also the composition of the neutralino LSP has cosmological significance, since the LSP is a potential dark matter candidate.

If no sign of supersymmetry is shown in experiments, alternative explanations must be sought. One possibility is that supersymmetry is not the explanation for the hierarchy problem but rather for the gauge coupling unification. The interest for abandoning the gauge hierarchy solution of supersymmetry is motivated by the more extensive fine-tuning required by the proper value of the cosmological constant. If the fine-tuning is accepted, then the spectrum can be vastly split and the supersymmetric scalar particles can have very heavy masses. In addition to the one light Higgs particle, only the gauginos and higgsinos are kept light in order to produce a viable dark matter candidate. This split supersymmetry model, however, has some restrictions. For example, the top Yukawa coupling fixed point is nearly ruled out, as discussed in the Section 2.7. In split supersymmetry model the Higgs boson is fine-tuned to be light, so the chances to find it in the LHC are favorable.

A completely different idea in order to solve the hierarchy problem is to extend the spacetime by extra dimensions. The new dimensions can be quite large, or there might be a non-factorizable, highly curved metric. The both scenarios have predictable consequences. This sets the viewpoint with respect to the gauge hierarchy problem in a new position: There is only one fundamental scale, and that can be as low as one TeV. Hence the gauge hierarchy problem is swept away, or to be more precise, transformed into another problem, namely determination of the size of the compactified space or the parameters of the warp factor. The attractive feature in these models is the possibility to be able to probe the Planck scale physics already in near-future colliders, like the LHC. The observation of quantum gravity effects in the TeV range would give a great deal of instructions how to build the Theory of Everything, which would incorporate all the known forces of Nature in a consistent quantum level theory. With respect to the Higgs physics there is a drawback: Higgs boson can be escaped into the extra dimensions via mixing with graviton modes, thus avoiding the detection in colliders.

If the Higgs boson is not found at the LHC, then for sure the perturbative unitarity has broken down and the Standard Model can't describe the physics at that scale. This is an evidence that there must be some new physics just around the corner waiting to be discovered. For a physicist, the times are thrilling!

Appendix A

Notation, integrals and RG-equations

Here is a collection of notation and some of the formulae used in the derivation of results in the text. Also one-loop MSSM renormalization group equations are given.

A.1 Notation

The flat metric used in this dissertation is

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A.1}$$

For this choice, *e.g.* the sign of the fifth dimension in the Randall-Sundrum metric Eq. (3.1) is negative, whereas in the original article it is positive [176].
Units: Reduced Planck constant and the velocity of light are put equal to one, $\hbar = c = 1$.

Weyl spinors: $(\psi_\alpha,\ \alpha = 1, 2) \in$ Lorentz representation $(\frac{1}{2}, 0)$. The spinor components are Grassman variables: $\psi_\alpha \psi'_\beta = -\psi'_\beta \psi_\alpha$
Complex conjugate spinor: $(\bar{\psi}_{\dot{\alpha}} = \psi^*_\alpha,\ \dot{\alpha} = 1, 2) \in$ Lorentz repr. $(0, \frac{1}{2})$.
Raising and lowering of spinor indices:

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta, \tag{A.2}$$

where $\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$, $\varepsilon^{12} = 1$, $\varepsilon_{\alpha\beta} = -\varepsilon^{\alpha\beta}$, $\varepsilon^{\alpha\beta}\varepsilon_{\beta\gamma} = \delta^\alpha_\gamma$, and the same for dotted indices.
Pauli matrices:

$$(\sigma^\mu_{\alpha\dot{\beta}}) = (\sigma^0_{\alpha\dot{\beta}}, \sigma^1_{\alpha\dot{\beta}}, \sigma^2_{\alpha\dot{\beta}}, \sigma^3_{\alpha\dot{\beta}}) \tag{A.3}$$

$$\bar{\sigma}_\mu^{\dot{\alpha}\beta} = \sigma_\mu^{\beta\dot{\alpha}} = \varepsilon^{\beta\alpha}\varepsilon^{\dot{\alpha}\dot{\beta}}\sigma_{\mu\,\alpha\dot{\beta}}, \tag{A.4}$$

$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{2}[\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu]_\alpha{}^\beta, \tag{A.5}$$

$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} = \frac{i}{2}[\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu]^{\dot{\alpha}}{}_{\dot{\beta}}, \tag{A.6}$$

where

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.7}$$

$$\bar{\sigma}^0 = \sigma^0,\ \bar{\sigma}^i = -\sigma^i = \sigma_i, \tag{A.8}$$

$$\sigma^{0i} = -\bar{\sigma}^{0i} = -i\sigma^i,\ \sigma^{ij} = \bar{\sigma}^{ij} = \varepsilon^{ijk}\sigma^k, \quad i,j,k = 1,2,3. \tag{A.9}$$

A.2 Volume of an n-sphere

Volume of n-dimensional sphere is of the form $V_n = c_n r^n$ by dimensional grounds. The surface area is then $S_n = dV/dr = nc_n r^{n-1}$. The constant c_n can be calculated using the n-dimensional Gaussian integral by converting it to a radial integral by integrating the angular part away,

$$\pi^{n/2} = \int d^nR\, e^{-|R|^2} = \int_0^\infty dr e^{-r^2} nc_n r^{n-1}. \tag{A.10}$$

After the familiar change of variables, $x = r^2$, the Eq. (A.10) becomes

$$\pi^{n/2} = \frac{nc_n}{2}\int_0^\infty dx e^{-x} x^{n/2-1}, \tag{A.11}$$

which is just the definition of gamma function $\Gamma(n/2)$. Hence

$$c_n = \frac{\pi^{n/2}}{(n/2)\Gamma(n/2)}. \tag{A.12}$$

A.3 Integrals

Cut-off integral in n Euclidean dimensions is easily calculated using the above result:

$$\begin{aligned}\int_{|k|<\Lambda} d^nk_E\, f(k_E^2) &= \int_0^\Lambda dr S_n(r) f(r^2) \\ &= \int_0^\Lambda dr[n\frac{\pi^{n/2}}{(n/2)\Gamma(n/2)} r^{n-1}] f(r^2) \\ &= \int_0^{\Lambda^2} dy \frac{n}{2}\frac{\pi^{n/2}}{(n/2)!} y^{n/2-1} f(y) \\ &= \frac{\pi^{n/2}}{(n/2-1)!}\int_0^{\Lambda^2} dy y^{n/2-1} f(y),\end{aligned} \tag{A.13}$$

which proves the Eq. (1.13).

Definite integrals in quadratically divergent mass corrections

The finite integrals of Sec. 1.2 can be evaluated and expressed in terms of elementary functions.

$$
\begin{aligned}
\widetilde{\mathrm{I}}_1(a) &= \int_0^1 dx\big(1+a^2(x-1)x\big)\big(3\ln(1+a^2(x-1)x)+2\big) \\
&= \frac{1}{a}(4-a^2)^{3/2}\arctan\frac{a}{\sqrt{4-a^2}} - \frac{1}{2}(4-a^2),
\end{aligned}
\tag{A.14}
$$

which converges if $0 < a < 2$. This is equal to Eq. (1.15) by replacing $a = m_S/m_F$ and multiplying Eq. (A.14) by m_F^2. The Eq. (1.21) is

$$
\begin{aligned}
\mathrm{I}_2(a) &= \int_0^1 dx\ln\big(1+a^2(x-1)x\big) \\
&= \frac{2}{a}\sqrt{4-a^2}\arctan\frac{a}{\sqrt{4-a^2}} - 2,
\end{aligned}
\tag{A.15}
$$

where again $0 < a < 2$. This is equal to 1.21 by replacing $a = m_S/m_i$. It can be noted that integral (A.15) is just part of the integral (A.14).

A.4 Note about regularization

In supersymmetry the renormalization is done as in any field theory: Apply a desired regularization scheme, use Ward-Takahashi/Slavnov-Taylor identities and a renormalization scheme to subtract divergent parts. Dimensional regularization [38] is the most convenient way to regularize divergent integrals. The main advantage in dimensional regularization is that it preserves gauge invariance (and most of the other symmetries), thus there is no need to use Ward-Takahashi or Slavnov-Taylor identities. This method does not, however preserve supersymmetry, because the fermionic and bosonic degrees of freedom are equal only in specific spacetime dimensions, while dimensional regularization continues the integrals to an arbitrary dimensionality. In the case of ultraviolet divergences it is only necessary to continue integrals to lower dimensions. A method called *dimensional reduction* employs this fact [190]. The Lorentz indices of spinors and matrices are kept in usual four-dimensional space but the momentum integrals are calculated in the $d < 4$ dimensional spacetime. For example, N=1 supersymmetry in $d = 4$ dimensions can be regarded as extended N=2 supersymmetry in $d = 3$ dimensions. This preserves the number of bosonic and fermionic degrees of freedom.

In dimensional reduction Kronecker delta-functions resulting from field and matrix operations are treated four-dimensional, and deltas coming from momentum integrals are treated d-dimensional (as in usual dimensional regularization). If the theory contains only scalars and spinors, the dimensional regularization is the same as regularization by dimensional reduction, except for the spinor field normalization. Thus, in the case of Higgs self energy of Sec. 1.2, the corrections can be calculated in the dimensional reduction scheme just like in the dimensional regularization. Therefore the equation Eq. (1.12) can be regularized using dimensional reduction scheme. The trace contribution does not change, and after Wick rotation to the Euclidean space, the d-dimensional integral gives Euler gamma-functions and the self-energy correction becomes

$$\Sigma_S(m_S^2) = \frac{\lambda_F^2}{16\pi^2}(6m_F^2 - m_S^2)(\frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)), \tag{A.16}$$

where $\epsilon = 2 - d/2$. Also in Eq. (A.16) the mass correction is proportional to the mass of the heaviest particle of the model as in Eq. (1.14). The first of the bosonic graphs (Fig. 1.3) contribute to the Higgs radiative mass correction as

$$(\delta M_H^2)_{\text{four}} = \frac{\lambda_S}{16\pi^2}(m_1^2 + m_2^2)(\frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)), \tag{A.17}$$

while the second graph gives the three particle vertex contribution

$$(\delta M_H^2)_{\text{three}} = -\frac{2m_F^2}{16\pi^2}(\frac{\lambda_S}{\lambda_F})^2[2(\frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)) - \mathrm{I}_2(m_S^2, m_1^2) - \mathrm{I}_2(m_S^2, m_2^2))], \tag{A.18}$$

where the integral I_2 is defined in Eq. (1.21) and given in Eq. (A.15). Arranging the couplings to be $-\lambda_S = {\lambda_F}^2$, the divergent terms *exactly* cancel, provided that the mass formula $m_1^2 + m_2^2 - 2m_F^2 = 0$ is valid:

$$\begin{aligned}\delta M_H^2 =& \frac{\lambda_F^2}{16\pi^2}\big[(2m_F^2 - m_1^2 - m_2^2) - m_S^2\big]\big(\frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)\big)\\ &+ \frac{\lambda_F^2 m_F^2}{8\pi^2}\big[\mathrm{I}(m_S^2, m_1^2) + \mathrm{I}(m_S^2, m_2^2)\big]\\ =& -\frac{\lambda_F^2}{16\pi^2}m_S^2(\frac{1}{\epsilon} - \gamma) + \text{finite},\end{aligned} \tag{A.19}$$

The mass trace formula $m_1^2 + m_2^2 - 2m_F^2 = 0$ (see discussion about the supertrace of Eq. (2.35)) is weaker requirement than the equality $m_1^2 = m_2^2 = m_F^2$ that was required to cancel all divergences in the cut-off scheme. There still remains a term proportional to the scalar mass itself, but that can be renormalized in a usual way. The brute momentum cutoffs are not a good choice in QFT since they destroy the Poincaré invariance. However, in the specific case of Higgs self energy calculation, the breakdown of gauge invariance or translation invariance does not give (relevant) contribution, so the cut-off method gives the same result as the dimensional reduction, as has been shown above.

A.5 One-loop RG-equations of MSSM

For completeness, the renormalization group equations for the MSSM at the one-loop order are given here. The two-loop expressions are given in Ref. [191].

Here only the third family Yukawa couplings are taken to be significant. The equations are in $\overline{DR}$ scheme, which is the modified minimal subtraction scheme using the dimensional reduction. The RGEs for the gauge couplings and gaugino masses were already presented in Eq. (2.42). The dimensionless scale is defined as $t = \ln \frac{Q}{M_{\rm GUT}}$. The renormalization group equations for the Yukawa couplings are

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}[6|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2], \tag{A.20}$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}[6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2], \tag{A.21}$$

$$\frac{dy_\tau}{dt} = \frac{y_\tau}{16\pi^2}[4|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2]. \tag{A.22}$$

Equations for the first and second family squark and slepton squared masses are

$$\frac{dm_\phi^2}{dt} = -\frac{1}{16\pi^2}\sum_{i=1,2,3} 8g_i^2 C_i^\phi |M_i|^2, \tag{A.23}$$

where the coefficients C_i^ϕ are the quadratic Casimir group theory invariants for the scalar ϕ for each gauge group. They are defined as $C_i^\phi \delta_a^b = (T^i T^i)_a^b$, where the T^i are the group generators acting on the scalar ϕ. Explicitly, $C_1^\phi = (3/5)Y_\phi^2$, where Y_ϕ is the weak hypercharge, $C_2^\phi = 3/4$ for $\phi = \widetilde{Q}_i, \widetilde{L}_i, H_u, H_d$ and 0 for other scalars, and $C_3^\phi = 4/3$ for $\phi = \widetilde{Q}_i, \widetilde{\overline{u}}_i, \widetilde{\overline{d}}_i$ and 0 for other scalars. Third family squark and slepton (mass)^2 parameters also get contributions which depend on X_t, X_b and X_τ (defined in Eqs. (2.46)-(2.48))

$$\frac{dm_{Q_3}^2}{dt} = \frac{1}{16\pi^2}(X_t + X_b - \frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2), \tag{A.24}$$

$$\frac{dm_{\overline{u}_3}^2}{dt} = \frac{1}{16\pi^2}(2X_t - \frac{32}{3}g_3^2|M_3|^2 - \frac{32}{15}g_1^2|M_1|^2), \tag{A.25}$$

$$\frac{dm_{\overline{d}_3}^2}{dt} = \frac{1}{16\pi^2}(2X_b - \frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2), \tag{A.26}$$

$$\frac{dm_{L_3}^2}{dt} = \frac{1}{16\pi^2}(X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2), \tag{A.27}$$

$$\frac{dm_{\overline{e}_3}^2}{dt} = \frac{1}{16\pi^2}(2X_\tau - \frac{24}{5}g_1^2|M_1|^2). \tag{A.28}$$

In above, the possible contributions proportional to the Tr$[Ym^2]$ are neglected. The equations for $m_{H_u}^2$ and $m_{H_d}^2$ were given in Eq. (2.45).

RGEs for the trilinear soft SUSY breaking terms (in models, where they are proportional to the corresponding Yukawa couplings) are

$$
\begin{aligned}
\frac{da_t}{dt} &= \frac{a_t}{16\pi^2}[18|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2] \\
&+ \frac{1}{16\pi^2}[2a_b y_b^* y_t + y_t(\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1)], \qquad \text{(A.29)} \\
\frac{da_b}{dt} &= \frac{a_b}{16\pi^2}[18|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2] \\
&+ \frac{1}{16\pi^2}[2a_t y_t^* y_b + 2a_\tau y_\tau^* y_b \\
&\quad + y_b(\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1)], \qquad \text{(A.30)} \\
\frac{da_\tau}{dt} &= \frac{a_\tau}{16\pi^2}[12|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2] \\
&+ \frac{1}{16\pi^2}[6a_b y_b^* y_\tau + y_\tau(6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1)]. \qquad \text{(A.31)}
\end{aligned}
$$

The equations for the Higgs doublet mixing parameter b and the supersymmetric mass parameter μ are

$$
\begin{aligned}
\frac{d\mu}{dt} &= \frac{\mu}{16\pi^2}[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2], \qquad \text{(A.32)} \\
\frac{db}{dt} &= \frac{b}{16\pi^2}[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2] \\
&+ \frac{\mu}{16\pi^2}[6a_t y_t^* + 6a_b y_b^* + 2a_\tau y_\tau^* + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1], \qquad \text{(A.33)}
\end{aligned}
$$

which complete the set of RG equations of the MSSM parameters. The two-loop equations for the gauge couplings can be found also in the paper III of this dissertation.

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