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Consequences of Supersymmetry in the Early Universe

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Abstract

Currently, we live in an era characterized by the completion and first runs of the LHC accelerator at CERN, which is hoped to provide the first experimental hints of what lies beyond the Standard Model of particle physics. In addition, the last decade has witnessed a new dawn of cosmology, where it has truly emerged as a precision science. Largely due to the WMAP measurements of the cosmic microwave background, we now believe to have quantitative control of much of the history of our universe.

These two experimental windows offer us not only an unprecedented view of the smallest and largest structures of the universe, but also a glimpse at the very first moments in its history. At the same time, they require the theorists to focus on the fundamental challenges awaiting at the boundary of high energy particle physics and cosmology. What were the contents and properties of matter in the early universe? How is one to describe its interactions? What kind of implications do the various models of physics beyond the Standard Model have on the subsequent evolution of the universe?

In this thesis, we explore the connection between, in particular, supersymmetric theories and the evolution of the early universe. We begin by providing the reader with a general introduction to modern day particle cosmology from two angles: first, by reviewing our current knowledge of the history of the early universe, and then, by introducing the basics of supersymmetry and its derivatives. Subsequently, with the help of the developed tools, we direct the attention to the specific questions addressed in the three original articles that form the main scientific contents of the thesis. Each of these papers concerns a distinct cosmological problem, ranging from the generation of the matter-antimatter asymmetry to inflation, and finally to the origin or very early stage of the universe. Nevertheless, they share a common factor in their use of the machinery of supersymmetric theories to address open questions in the corresponding cosmological models.

List of publications

The content of this thesis is based on the following research articles [1, 2, 3]:

I **Supersymmetric Leptogenesis and the Gravitino Bound,**
Gian Giudice, Lotta Mether, Antonio Riotto and Francesco Riva,
Phys. Lett. **B664** (2008) 21, [arXiv:0804.0166].

II **Supergravity origin of the MSSM inflation,**
Kari Enqvist, Lotta Mether and Sami Nurmi,
JCAP **0711** (2007) 014, [arXiv:0706.2355].

III **On the origin of thermal string gas,**
Kari Enqvist, Niko Jokela, Esko Keski-Vakkuri and Lotta Mether,
JCAP **0710** (2007) 001, [arXiv:0706.2294].

In all publications authors are listed alphabetically, according to the particle physics convention. The present author's contribution to the research is detailed below.

Author's contribution

I The idea that flat directions could help alleviate the conflict with the gravitino bound in supersymmetric leptogenesis originates in discussions with Antonio Riotto and Gian Giudice. The detailed model was subsequently developed through discussions between all authors, and numerical estimates were performed jointly. The draft was written mainly by Antonio Riotto and Francesco Riva, and molded into its final form by all authors.

II Kari Enqvist suggested to look for a supergravity model that could alleviate the fine-tuning of the MSSM inflation model, and offered some ideas for directions of research. Calculations and the identification of the presented model were performed in close collaboration between Sami Nurmi and the present author. The paper was written through joint efforts, the present author contributing mainly to Sections 2 and 5.

III The question of the origin of a thermal string gas in the early universe was posed by Kari Enqvist, whereas Esko Keski-Vakkuri suggested to address it through D-brane decay. The idea was then elaborated on through discussions between all authors. Calculations were performed jointly with Niko Jokela, and graphical presentations mainly by the present author. All authors took part in writing the paper, the contribution of the present author being Sections 2 and 4.

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Chapter 1

Introduction

Many of the most intriguing open problems in modern high energy physics are involved in a subtle interplay between the very smallest and the very largest scales in our universe: at one end of the spectrum there is the world of subatomic particles and, at the other end, the cosmic length scales that describe the structure of the universe itself. The connection between these two seemingly disjoint worlds traces back some 13.7 billion years to the very early stages of our universe [4], where, essentially, they become one and the same. To fully understand the one at its most fundamental level, it seems an equal understanding of the other is required.

The world of particles is of course known to be represented very well by the Standard Model (SM) of particle physics [5] up to the highest energies tested in collider experiments so far. It is also known on very general grounds that the Standard Model cannot be the most fundamental theory of nature, but only a low energy effective description thereof. The two most popular, and perhaps theoretically best motivated suggestions for physics reaching beyond the SM are string theory and supersymmetry. The former is based on the idea that the fundamental description of nature on its very smallest length scales would be given not in terms of point particles but of one-dimensional strings and their higher-dimensional generalizations, branes.

For consistency, string theory requires an additional local symmetry to be realized in nature, namely supersymmetry [6]. Supersymmetry relates fundamental fermions and bosons to their so-called superpartners, respectively of bosonic and fermionic nature. In addition to being motivated by theoretical considerations, supersymmetry has several vital consequences for low-energy particle physics phenomenology, most importantly the solution it provides to the long-standing hierarchy problem. Regardless of whether it is eventually superseded by string theory at high energies, low-energy supersymmetry has emerged as the strongest candidate for physics beyond the Standard Model. Today research in this field is more topical than perhaps ever before, due to the recent startup of the Large Hadron Collider (LHC) at CERN, which is hoped to provide the first experimental hints of supersymmetry [7].

In contrast to experimental particle physics, where the last decade has been mainly a time of preparation and anticipation of results to come, cosmological observations have seen nothing short of an explosion of new data within the same time period. The unprecedented accuracy, as well as the variety, of the cosmological data made available have given rise to a standard model of cosmology: the ΛCDM, or concordance model [8]. The result of this concordance, however, is somewhat surprising: matter as we known it in the Standard Model makes up but five percent of the total energy density of the universe. The remaining 95% is attributed to two exotic forms of energy density: dark matter and dark energy. Whereas the nature of dark energy remains as much a mystery as ever, several possible candidates for dark matter have been identified in particle theories, most notably perhaps the lightest stable superpartner of low-energy supersymmetry [9].

In addition, our explorations of the cosmos have raised a number of more subtle questions that remain to be answered. Observations of standard baryonic matter in the universe, for example, show an asymmetry in the amounts of matter and antimatter, which finds no explanation within the Standard Model. Moreover, the very special initial conditions that our observable universe can be traced back to can most naturally be achieved by a period of accelerated expansion in the very early universe. Observations seem consistent with this period of inflation being driven by a scalar field, but the possible identity of such a field is yet unknown.

Whichever exotic forms of matter turn out to be the winners, it is clear that the fields of cosmology and high energy particle physics are inherently intertwined in the quest for the true extensions to the Standard Model. Where cosmologists look to the world of particles for new types of matter that might explain their observations, these very same observations in return offer the particle physics community a peek at energy scales that one could currently only dream of probing in terrestrial facilities.

The work at hand is a PhD thesis based on three original articles [1, 2, 3] written by the author and various collaborators. Each article addresses a distinct cosmological problem, using different theoretical tools; however, they all have a common denominator in the desire to understand the interplay between the early stages of the universe and modern day particle physics. More precisely, they are all based on the assumption that physics beyond the Standard Model exhibits supersymmetry, and explore possible consequences of this assumption at various stages in the early universe.

Ref. [1] addresses the generation of the matter-antimatter asymmetry through the mechanism of leptogenesis. A possible solution to a long-standing naturalness problem in supersymmetric leptogenesis is presented, which unlike other proposed solutions does not require a significant alteration of the scenario. Ref. [2] considers a particular model of inflation, in which the expansion is driven by a flat direction of the Minimal Supersymmetric Standard Model (MSSM). The model requires an extremely flat potential, which a priori can only be guaranteed by a high degree of fine-tuning. In Ref. [2] a class of supergravity models which identically satisfy the flatness requirement is identified, in order to alleviate the severity of the fine-tuning. Finally, Ref. [3] addresses a string theory based model for the very early universe, which provides an example of how the initial singularity plaguing the standard cosmological model can be avoided through string theory. It is argued that the string gas cosmology model, whose origin is usually not explained, can be the result of the decay of an initial configuration of unstable branes, which furthermore allows one to control the properties of the model through various physical properties of the brane setup.

To bring the above topics into a broader context, the articles included in the thesis are preceded by an introductory part, which is organized as follows. In Chapter 2, we review the most important concepts of modern day cosmology, discussing also the observational data that our current understanding of the evolution of the universe is based on. Chapter 3, on the other hand, provides an introduction to supersymmetric theories, placing particular emphasis on the concepts and topics relevant in the early universe. Chapters 4–6 are then devoted to deeper analyses of the subjects treated in each of the research papers [1, 2, 3], proceeding in this order. Finally, in Chapter 7, we conclude by providing a brief summary of the thesis.

Notation

The natural units, with $c \equiv 1$, $\hbar \equiv 1$ and $k_B \equiv 1$, are used throughout the thesis. The Planck mass is defined as $M_P = (8\pi G)^{-1/2}$, where G is Newton's constant. The signature of the metric is chosen to be $(-,+,+,+)$. Greek letters $\mu, \nu, \ldots$ run over all the spacetime coordinates and Latin letters $i, j, \ldots$ over the spatial coordinates. Summation over repeated indices is understood. The covariant derivative is denoted by $\nabla\mu$ and is defined using the standard Christoffel connection. The notation used for the Kähler metric and the associated indices in supergravity is explained in the relevant context. For the Fourier transformation we use the normalization

$$f(\mathbf{x}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3k \, f(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}.$$

Any other introduced notation is explained in the text. The notation used in the introductory part of the thesis differs partially from that used in the enclosed research papers.

Preliminaries

Chapter 2

The early universe

In this introductory chapter we present the standard picture of cosmology that emerges when the assumption of a homogenous and isotropic, expanding universe is combined with our understanding of particle physics at various energy scales. The result is a brief review of standard cosmology, aimed at setting the stage for the treatment of the three included research papers, in Chapters 4-6, respectively.

We begin with a presentation of the Friedmann-Robertson-Walker universe and the hot big bang model, with emphasis on the baryon asymmetry, in order to prepare for the treatment of leptogenesis in Chapter 4. Subsequently, in Section 2.3, we discuss the limitations of this model, and introduce the inflationary paradigm. We review the general properties of scalar field inflation, and how it can be constrained by observations, paving the way for the discussion on the MSSM inflation model in Chapter 5. Having presented the main features of scalar field inflation, in the final section we briefly address some of its shortcomings and discuss the potential of seeing beyond inflation, heading towards the introduction of the string gas cosmology scenario in Chapter 6. More detailed presentations on the subjects treated here can be found, for example, in Refs. [10, 11, 12].

2.1 The Friedmann-Robertson-Walker universe

Einstein's theory of general relativity allows us to study the universe as a deterministic dynamical system, whose evolution is determined by its previous conditions. With this powerful tool at hand, the modern cosmological picture has emerged from but a few groundbreaking discoveries. First, in 1929 Hubble made the observation that distant objects in every direction on the sky appear to be receding from us, the faster the further away they are [13]. The obvious conclusion to be drawn was that the universe is expanding. If this were indeed the case, our universe should have been both denser and hotter in the past.

Another cornerstone of modern cosmology is the observation, first performed by Penzias and Wilson in 1965 [14], of the redshifted relic radiation from the time when the universe was only a few hundred thousand years old: the cosmic microwave background (CMB). The CMB radiation exhibits a perfect blackbody spectrum with a temperature of 2.726 K, with anisotropies only of the order of 10^{-5} [15]. While these anisotropies make up the seeds for the structure observed in the universe today, mappings of the large scale structure, such as the Sloan Digital Sky Survey [16] and the 2dF survey [17], confirm the observed isotropy on scales larger than about 100 Mpc. Together with the Copernican principle, which states that the Earth is not in a central, or specially favored position in the universe, observations thus lead us to conclude that we live in an expanding universe that is to a first approximation homogenous as well as isotropic.

The Robertson-Walker metric

A time evolving, spatially homogeneous and isotropic spacetime is described most generally by the Robertson-Walker (RW) metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right), \tag{2.1}$$

where r, θ and ϕ are the polar comoving coordinates, defining the cosmic rest frame, and t is the time measured by a comoving observer. The scale factor $a(t)$ determines the physical size of spatial coordinate distances as a function of time, while κ parameterizes the curvature of spatial surfaces.

The RW metric Eq. (2.1) describes a spatially homogenous and isotropic spacetime without any restrictions on the functional form of the scale factor. The dynamical behaviour of the scale factor depends on the matter and energy contents of the universe through the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_P^2}T_{\mu\nu}. \tag{2.2}$$

The left hand side of the equation is a function of the metric – in this case the scale factor – only, while the energy-momentum tensor $T_{\mu\nu}$ derives from the matter action. To be consistent with the assumed homogeneity and isotropy, the energy momentum tensor should satisfy the same symmetries as the RW metric. This constrains the matter and energy contents of the universe to be of perfect fluid form, with the energy-momentum tensor

$$T_{\mu\nu} = (\rho + p)\,u_\mu u_\nu + pg_{\mu\nu}, \tag{2.3}$$

where ρ and p are the energy density and pressure, and $u^\mu = (1, 0, 0, 0)$ is the four-velocity of the isotropic fluid in comoving coordinates.

The Friedmann equations

Given the energy-momentum tensor, the Einstein equation Eq. (2.2) determines the evolution of the scale factor in terms of the energy density and the pressure of the cosmic fluid. The 00-component yields the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_P^2} - \frac{\kappa}{a^2}, \tag{2.4}$$

where the dot indicates derivatives with respect to the cosmic time. The spatial components all give rise to the same equation, which can be put in the form of the second Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6M_P^2}. \tag{2.5}$$

The left hand side of the Friedmann equations is often written in terms of the Hubble parameter

$$H = \frac{\dot{a}}{a}, \tag{2.6}$$

whose current value is measured to be $H_0 \simeq 70\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ [4]. Using the Friedmann equation Eq. (2.4), we can define the critical density

$$\rho_c \equiv 3M_P^2 H^2, \tag{2.7}$$

such that the curvature term κ in the equation vanishes. The critical density, on the other hand, is used to define the dimensionless density parameter $\Omega = \frac{\rho}{\rho_c}$, in terms of which the Friedmann equation can be rewritten as

$$\Omega - 1 = \frac{\kappa}{(aH)^2}. \tag{2.8}$$

The best fit values for current observations give $\Omega = 1 \pm 0.03$, implying that the universe is very nearly flat and $\kappa \simeq 0$.

The cosmic fluid

A convenient tool for studying the evolution of the cosmic fluid is provided by the continuity equation

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1 + w) = 0, \tag{2.9}$$

which can be derived either from the two Friedmann equations or from the conservation of energy-momentum. We have defined the equation of state parameter w, such that $p = w\rho$. In particular if w is constant, Eq. (2.9) can be integrated to yield the energy density as a function of the scale factor

$$\rho \propto a^{-3(1+w)}. \tag{2.10}$$

Plugging this into the Friedmann equation Eq. (2.4), we can solve for the evolution of the scale factor in a universe dominated by a fluid with the equation of state parameter $w \neq -1$

$$a \propto t^{2/3(1+w)}. \tag{2.11}$$

The cosmic fluid is assumed to be composed of three separately evolving fluids: matter, radiation and a vacuum energy like component. Matter accounts for any non-relativistic species of particles, with essentially vanishing pressure. Thus $w_m \simeq 0$, and the energy density behaves as $\rho_m \propto a^{-3}$. According to the best fit model to recent observations, the matter component accounts for about 28% of the total energy density, with 5% made up of baryonic matter and the remaining 23% presumably dark matter [4]. For radiation, which refers to any highly relativistic particle species or actual electromagnetic radiation, $w_r = 1/3$ and the energy density evolves as $\rho_r \propto a^{-4}$. The energy density in the radiation component thus decreases with a factor of $1/a$ compared to matter, and only accounts for a very small fraction of the total energy density in the universe today. A constant energy density, however, which by definition has $w_\Lambda = -1$, will inevitably become dominant in the late universe. Recent observations of distant supernovae [18] indeed seem to imply that a dark energy component, with $w \simeq -1$, is present and currently accounts for about 72% of the total energy density.

2.2 The hot big bang

The time evolution of the different energy components suggests that the early universe should be dominated by relativistic forms of matter. Starting with this presumption, the hot big bang model describes the subsequent evolution, as the cosmic fluid gradually cools down with the expansion of the universe. Since particle interactions would be frequent in the hot and dense early universe, the cosmic fluid is expected to have been largely in thermal equilibrium, in accordance with the near perfect blackbody spectrum of the CMB. Nevertheless, the events that have left the clearest imprints in the cosmic background have taken place due to the decoupling of some component of the fluid from the thermal bath.

Our understanding of the evolution of the early universe relies on our knowledge of physics at high energies, based on experiments and theoretical considerations. Any attempts to describe the state of the universe at early times, such as the Planck time, are therefore at most speculative. At lower energies, however, we can make fairly robust predictions. Below we briefly overview the thermal history of the universe, and point out the most relevant events that have taken place. A more detailed review can be found, for example, in Ref. [19].

2.2.1 Thermal history

Assuming that the hot big bang stage begins at some higher energy scale, the electroweak phase transition is expected to take place at an energy of about 100 GeV. It is followed by the quark-hadron phase transition at around 100 MeV, after which the newly formed nucleons and anti-nucleons annihilate each other. Some nucleons nevertheless remain, indicating an asymmetry in the abundances of matter and antimatter. At this point the thermal bath consists mainly of photons, neutrinos, electrons, positrons, neutrons and protons. When the temperature reaches about 1 MeV, the neutrinos decouple from the other relativistic species, and shortly after, once the temperature drops below the electron rest mass, electrons and positrons annihilate one another, leaving only a small excess of electrons. Around the same temperatures nuclear reactions become efficient, and free protons and neutrons combine into helium and other light elements in the process of big bang nucleosynthesis (BBN) [20, 21].

Nucleosynthesis

As the neutrinos decouple from the thermal bath, the weak interactions that keep the neutron-to-proton ratio in equilibrium become inefficient, with the consequence that the neutrons freeze out and start decaying into protons. Before all neutrons have decayed, the remaining ones bind with protons into deuterium nuclei, which subsequently form heavier nuclei through secondary reactions. These processes, however, only become efficient once the deuterium abundance reaches its equilibrium value. Although the binding energy of deuterium is 2.2 MeV, the large photon-to-baryon number delays the event until a temperature of about 0.06 MeV. After this, heavier nuclei are rapidly produced, but most isotopes cannot reach their equilibrium abundance before the Coulomb barrier shuts the reactions off at a temperature of 30 keV.

Most neutrons end up in ^{4}He isotopes, which have the highest binding energy per nucleon, and only small amounts of the other light isotopes ^{2}H, ^{3}H, ^{3}He, ^{7}Li and ^{7}Be are produced. The amount of ^{4}He produced crucially depends on the number of available neutrons. This in turn depends on two quantities: the number of relativistic particle species, which affects the temperature of neutron freeze-out, and the total number of baryons, characterized by the baryon-to-photon ratio η. In the Standard Model of particle physics, the former is well constrained, and the abundances of the light elements predicted by nucleosynthesis essentially depend on η alone. As illustrated in Fig. 2.1, observations of light element abundances are in agreement with the predictions from nucleosynthesis (at 95% CL) given that [20]

$$\eta^{\mathrm{BBN}} = \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (5.6 \pm 0.9) \times 10^{-10}, \tag{2.12}$$

where n_B, $n_{\bar{B}}$ and n_γ are the number densities of, respectively, baryons, anti-baryons and photons, and the subscript 0 refers to the value today. The fact that there is a range of η which is consistent with all measured abundances is one of the most convincing pieces of evidence in support of the hot big bang cosmological model.

Recombination

After nucleosynthesis the cosmic fluid consists mainly of photons, electrons, protons and ionized helium nuclei. Due to the abundance of free charges, the universe is opaque to electromagnetic radiation. When the universe is 380 000 years old and the temperature has come down to 0.3 eV, electrons and nuclei combine to form neutral atoms in the process of recombination. As recombination proceeds, the number of free electrons falls, and matter and radiation decouple. Consequently, the photons can stream freely through the universe, and constitute the CMB sky we observe today. Matter on the other hand is free from the damping interactions with radiation and begins to form the large scale structure from the initial perturbations observed in the CMB.

The CMB temperature anisotropy, as measured by the WMAP satellite, offers an independent test of the value of the baryon-to-photon ratio inferred from nucleosynthesis. A thorough introduction to the physics of the CMB and the estimation of cosmological parameters therefrom can be found in Ref. [22]; for a shorter review see Ref. [23]. The latest analysis of the WMAP data [4] estimates the baryon-to-photon

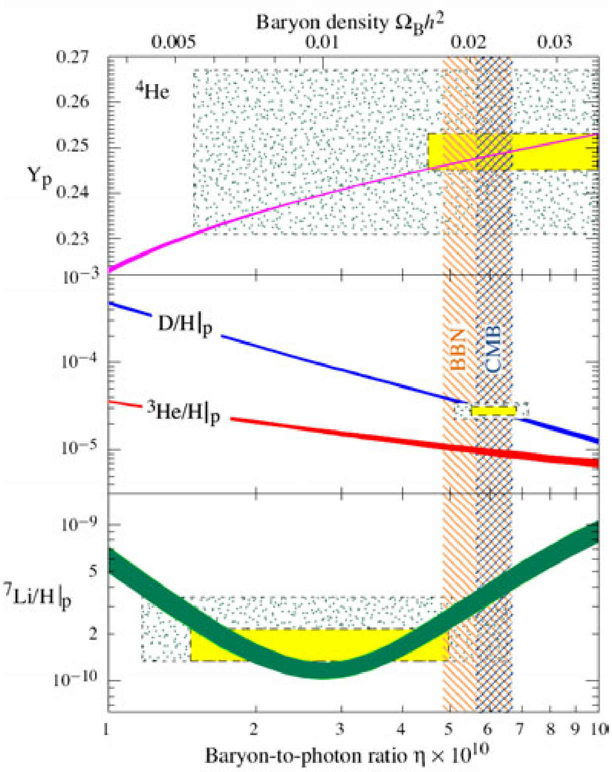

Figure 2.1: The observed abundances of ^{4}He, D, ^{3}He and ^{7}Li compared to the standard BBN predictions [20]. The boxes indicate the observed light element abundances (smaller boxes correspond to 2σ statistical errors, larger boxes to $\pm 2\sigma$ statistical and systematic errors). The narrow vertical band indicates the CMB measure of the baryon-to-photon ratio, while the wider band indicates the BBN concordance range (both at 95% CL).

ratio (at 68% CL, assuming ΛCDM cosmology with a scale-invariant power spectrum)

$$\eta^{\mathrm{CMB}} = (6.225 \pm 0.170) \times 10^{-10}. \tag{2.13}$$

The constraints on η derived from nucleosynthesis and from the CMB, respectively, are illustrated in Fig. 2.1. The consistency between the constraints from these two probes, which stem from events that took place some 380 000 years apart, is yet another triumph of the standard cosmological model.

2.2.2 The baryon asymmetry

From a particle physics perspective, the observed baryon-to-photon ratio, which quantifies the asymmetry in matter and antimatter, is something of a puzzle. Since there is no evidence of primary forms of antimatter in the universe, at least up to the scale of galaxy clusters, the asymmetry is unlikely to be a local effect, but rather a fundamental property of the universe. In addition, there is good reason to believe that it does not simply reflect an initial condition; since a primordial phase of inflation (see the following section) would quickly dilute away any initial asymmetry, the baryon asymmetry must be dynamically generated at some point before the onset of nucleosynthesis. However, as discussed below, the Standard Model alone is not able to explain its origin.

Baryogenesis

The criteria for a dynamical process of baryogenesis to be possible were considered by Sakharov in 1967 [24], shortly after the first quantitative predictions from nucleosynthesis had been compared to astrophysical observations. Sakharov identified three necessary conditions for baryogenesis to take place:

1. Baryon number violation

2. C and CP violation

3. Departure from thermal equilibrium[1]

All of these conditions could, in principle, be fulfilled within the Standard Model, during the electro-weak phase transition. If the transition was of first order, it would provide the required departure from equilibrium [26]. Furthermore, the weak interactions maximally violate C, whereas the complex phase in the quark mixing matrix violates CP through the Kobayashi-Maskawa mechanism [27]. Finally, baryon number B, as well as lepton number L, are violated at the quantum level due to the chiral anomaly of the weak interactions [28].

The anomaly gives rise to degenerate vacua differing by their B and L contents, which are separated by potential barriers of the height of the electro-weak scale. Non-perturbative field configurations between adjacent vacua violate B and L by units of three each, but keep $B - L$ conserved. At zero temperature, these transitions are instanton solutions, which have an exponentially suppressed rate and hence no observable effect. At finite temperatures, however, the transitions can take place through thermal fluctuations over the barrier, commonly referred to as sphaleron processes [29]. Above the critical temperature of the electro-weak phase transition these sphaleron transitions become unsuppressed, leading to rapid $B + L$ violation [30].

This scenario of electro-weak baryogenesis nevertheless fails on two of these three accounts. Computations of the thermal Higgs potential show that the electro-weak phase transition is of first order only for a Higgs mass $m_H \lesssim 70$ GeV [31], which is in apparent conflict with the experimental bound $m_H \gtrsim 115$ GeV [32]. In addition, the CP-violating phase in the Standard Model is simply too small to generate an asymmetry of the observed magnitude [33]. Hence, physics beyond the Standard Model is called for. In Chapter 4 we present some models of baryogenesis, and discuss in more detail the model of thermal leptogenesis.

[1] While the first two conditions are necessary to generate a baryon-antibaryon asymmetry in general, the third condition is a consequence of CPT invariance. For completeness, let us mention that also models of baryogenesis with CPT violation in the early universe have been considered [25], although we shall not discuss this possibility further.

2.3 A period of inflation

Despite its remarkable agreement with observations, the hot big bang model with a radiation dominated primordial universe is not a very successful theory if extrapolated arbitrarily far back in time. It provides no dynamical explanation for the primordial fluctuations observed in the CMB, which make up the seeds for the large scale structure seen in the universe today. Nor can it explain, except by fine-tuning, the observed flatness and homogeneity, which as such imply that parts of the universe which have never been in causal contact have exactly the same conditions.

Both the flatness and homogeneity can be explained successfully by assuming that the early universe underwent a period of inflation, i.e. accelerated expansion, defined by $\ddot{a} > 0$ [34]. From the equivalent definition

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \tag{2.14}$$

one straightforwardly sees how inflation solves the aforementioned problems. Firstly, Eq. (2.8) readily tells us that inflation drives the universe towards flatness. Furthermore, Eq. (2.14) implies that the comoving horizon, roughly corresponding to the distance over which one can have causal interaction on cosmological timescales, decreases with time. Consequently, the observable universe actually becomes smaller during inflation, and the observable universe today could have been well within the horizon and causally connected at the onset of inflation, provided that the amount of expansion was sufficient.

2.3.1 Scalar field inflation

By the second Friedmann equation Eq. (2.5), the inflationary condition $\ddot{a} > 0$ implies that the pressure and density of the energy component driving inflation must satisfy

$$p < -\frac{1}{3}\rho. \tag{2.15}$$

This excludes both ordinary matter and radiation, but suggests that a vacuum energy like component could be a plausible candidate. However, it is hard to fathom how a non-dynamical vacuum energy could dominate the universe, and then suddenly give way to the standard hot big bang evolution. Although several other dynamical sources of inflation can be imagined, the simplest and most common assumption is that this early era of expansion is caused by the large potential energy of a scalar field, the inflaton. In the work at hand we follow this assumption and consider inflation driven by a single scalar field. For the remainder of this introductory chapter we focus on exploring various aspects of this paradigm. Similar treatments of the topic, but with more detail can be found in Refs. [11, 12].

For a scalar field ϕ, with a canonically normalized kinetic term and the potential $V(\phi)$ the energy-momentum tensor takes the form

$$T_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\nabla_\rho\phi\nabla_\sigma\phi + V(\phi)\right). \tag{2.16}$$

In the RW metric the energy-momentum tensor Eq. (2.16) matches that of a perfect fluid with energy density and pressure

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{(\nabla\phi)^2}{2a^2} + V(\phi), \tag{2.17}$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{(\nabla\phi)^2}{6a^2} - V(\phi). \tag{2.18}$$

Since the universe is assumed to be homogeneous, the inflaton, whose energy density dominates the universe, must to the first approximation be homogeneous as well. In reality the field will, nevertheless, have small quantum fluctuations, which can source the primordial density fluctuations, but we shall postpone the discussion of these until a later section. For a homogeneous field, the spatial gradient terms in the energy density Eq. (2.17) and pressure Eq. (2.18) are negligible, and the condition Eq. (2.15) for inflation is satisfied for $\dot{\phi}^2 < V(\phi)$. Furthermore, the inflaton equation of motion can then be written in the form

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{2.19}$$

Slow roll inflation

The required condition $\dot{\phi}^2 < V(\phi)$ is difficult to maintain for a sufficient period of time, unless the potential energy is properly dominant over the kinetic energy and the evolution of ϕ is slow. The standard way of analyzing the dynamics of inflation, is to make these assumptions in the form of the slow-roll approximation

$$\dot{\phi}^2 \ll V(\phi), \tag{2.20}$$

$$\ddot{\phi} \ll 3H\dot{\phi}. \tag{2.21}$$

The Friedmann equation Eq. (2.4) and the equation of motion Eq. (2.19), which govern the dynamics, then simplify to the following set of equations

$$H^2 \simeq \frac{V(\phi)}{3M_P^2}, \tag{2.22}$$

$$3H\dot{\phi} \simeq -V'(\phi). \tag{2.23}$$

A priori, it is not obvious that this approximation is able to represent generic inflationary solutions, given that they reduce the order of the full equations. The solutions to the full equations, however, all approach the same asymptotic attractor solution regardless of the initial conditions, as long as $\dot{\phi}$ is monotonic. The slow-roll solution turns out to be a good approximation to this attractor, which validates its use in analyzing the inflationary dynamics [35].

In the context of slow-roll inflation, it is useful to introduce the two dimensionless slow-roll parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \tag{2.24}$$

$$\eta \equiv M_P^2\frac{V''}{V}. \tag{2.25}$$

Whenever the slow-roll approximation is valid, these parameters satisfy the slow-roll conditions

$$\epsilon \ll 1, \quad |\eta| \ll 1. \tag{2.26}$$

These conditions are necessary but not sufficient, since they only constrain the form of the potential while $\dot{\phi}$ can break the slow-roll approximation. The end of inflation is usually taken to occur when one of the slow-roll conditions are violated, i.e. when either $\epsilon \simeq 1$, or $|\eta| \simeq 1$. Whereas this is neither a necessary nor a sufficient condition, the amount of inflation taking place when the slow-roll conditions are violated is usually small. The violation of the slow-roll conditions occurs when the inflaton field ϕ reaches a steeper region of the potential. In typical inflationary models, the inflaton then descends towards the absolute minimum of the potential and begins to oscillate about it.

2.3.2 Recovering the hot big bang

During inflation the universe expands enormously. Consequently, the number densities of any initially present particles are diluted and the energy density of radiation is redshifted, leaving the universe cold and practically empty. The energy density of the universe is, nevertheless, stored in the scalar field potential, in the form of the oscillating inflaton field. To connect the inflationary period to the hot big bang era, this energy density must be transferred into relativistic particles.

Reheating

For the inflaton energy to be transferred into relativistic particles, the inflaton must be coupled to some matter fields. In this case, the coherent inflaton oscillations at the end of inflation are damped by quantum mechanical particle creation, as vacuum energy is converted into energy of particles in the process of reheating [36]. The decay of the inflaton in reheating can be described phenomenologically by adding an effective friction term to its equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0, \tag{2.27}$$

whereΓ_ϕ denotes the total decay rate of the inflaton. Let us assume that the inflaton couples to bosons χ and fermions ψ through the interaction terms $\mathcal{L}_I = -\frac{1}{2}g^2\phi^2\chi^2$ and $\mathcal{L}_I = -h\bar{\psi}\psi\phi$, respectively. The inflaton decay rates are then expressed as

$$\Gamma(\phi \to \chi\chi) = \frac{g^4\langle\phi^2\rangle}{8\pi m}, \qquad \Gamma(\phi \to \psi\psi) = \frac{h^2 m}{8\pi}, \tag{2.28}$$

assuming that the fermion and boson masses are negligible in comparison to the inflaton mass m, and the total decay rate is then the sum of the above.

While the inflaton field is oscillating, the energy density of the universe evolves as if it was matter dominated. As an approximation, we can assume that when$\Gamma_\phi = H$, the inflaton suddenly decays into relativistic particles. Requiring that the critical energy density $\rho = 3M_P^2H^2$ thus corresponds to the energy density of the relativistic thermal bath $\rho \propto g_* T_{\rm RH}^4$, we obtain an upper limit on the reheating temperature of the universe

$$T_{\rm RH} = \frac{1.7}{g_*^{1/4}} \sqrt{M_P \Gamma_\phi}, \tag{2.29}$$

where g_* is the number of relativistic degrees of freedom, which in the Standard Model takes the value 106.75. The actual value of the reheating temperature naturally depends on the model of inflation. From direct observations we know that the temperature must be at least somewhat higher than the temperature for nucleosynthesis, but low enough to avoid the production of monopoles, which sets the temperature in the range 10^{-2} GeV $\leq T_{\rm RH} \leq 10^{16}$ GeV. For specific models, also several other considerations impose constraints on the reheating temperature, some of which we will discuss elsewhere in this thesis, leading to a much more stringent overall constraint on the temperature.

Parametric resonance

When the inflaton is coupled to bosons with the interaction term $\mathcal{L}_I = -\frac{1}{2} g^2 \phi^2 \chi^2$, there is a possibility that the oscillating inflaton field decays non-perturbatively through parametric resonance [37, 38]. This non-perturbative decay of the inflaton, termed preheating, is extremely rapid compared to the perturbative decay of reheating. The decay is induced by the time-dependent mass

$$m_\chi^2(t) = \bar{m}_\chi^2 + g^2 \phi^2(t), \tag{2.30}$$

which the coupling to the inflaton generates for the χ field, where $\bar{m}_\chi^2$ is the bare mass of the field. Quanta of the field χ will be created due to this time-varying background, taking their energy from the inflaton condensate, gradually damping the inflaton oscillations.

The particle production occurs as a result of the violation of adiabaticity and hence takes place only when

$$|\dot{m}_\chi(t)| \gtrsim m_\chi^2(t). \tag{2.31}$$

Since the change in m_χ is maximal when the inflaton condensate passes through the origin, while its absolute value is minimal, adiabaticity is maximally violated and particle production efficient in a region around $\phi = 0$. Neglecting the bare mass $\bar{m}_\chi$, the condition for violation of adiabaticity Eq. (2.31) implies

$$g|\dot{\phi}| \gtrsim g^2 |\phi|^2. \tag{2.32}$$

For a harmonic oscillator of mass m, the velocity of the field in the minimum of the effective potential can be written as $|\dot{\phi}| = m\phi_0$, where ϕ_0 is the initial amplitude of the field. Particle production thus occurs within the region

$$|\phi| \lesssim |\phi_*| \equiv \sqrt{\frac{m\phi_0}{g}}, \tag{2.33}$$

where we have defined ϕ_* as the value of the inflaton condensate when particle production begins. In general this interval is very short, and the particle production occurs nearly instantaneously, within the time

$$\Delta t \sim \frac{|\phi_*|}{|\dot{\phi}|} \sim (gm\phi_0)^{-1/2}\,. \tag{2.34}$$

The uncertainty principle then implies that the particles will typically be created with momenta

$$k \sim \Delta t^{-1} \sim (gm\phi_0)^{1/2}\,. \tag{2.35}$$

The particles are estimated to be produced with an occupation number [38]

$$n_k \simeq \exp\left(-\pi\frac{k^2+\bar{m}_\chi^2}{g|\dot{\phi}|}\right), \tag{2.36}$$

which is valid also for a vanishing bare mass $\bar{m}_\chi$. An integration over the momentum yields the number density of particles produced during one oscillation

$$n_\chi \simeq \frac{1}{2\pi^2}\int \mathrm{d}k\,k^2 n_k \sim \frac{(g\dot{\phi})^{3/2}}{8\pi^3}\exp\left(-\frac{\pi\bar{m}_\chi^2}{g|\dot{\phi}|}\right). \tag{2.37}$$

In the case that $\bar{m}_\chi^2 \lesssim g|\dot{\phi}|$, there is no exponential suppression, and χ particles are created with a large number density. Depending on the mass and couplings of the given inflationary model, the mass of these particles can be quite large, even a few orders of magnitude above the inflaton mass. In general the produced particles are far away from thermal equilibrium, and hence decay further immediately after preheating. Eventually, the decay products thermalize and the hot big bang era takes off.

Instant preheating

Let us suppose now that the boson χ couples to another field, which we here take to be a fermionic field ψ, through the interaction

$$\mathcal{L}_I = -h\bar{\psi}\psi\chi. \tag{2.38}$$

As the inflaton starts its oscillations, at the bottom of the first oscillation the effective mass of χ reaches its minimum and field quanta are copiously produced. As the inflaton climbs up its potential again, the mass of the produced χ particles grows and, finally, when the mass is large enough the field decays into ψ particles. This process damps the inflaton oscillations very efficiently, and preheating may take place nearly instantaneously, during just a single inflaton oscillation [39]. Furthermore, this instant preheating process allows for production of particles with masses even two orders of magnitude greater than those produced by the usual preheating mechanism.

While here we have discussed the non-perturbative production of particles through parametric resonance of the oscillating inflaton field, similar processes can of course take place for any oscillating scalar field with interactions. In Section 3.3 we shall return to the discussion of this production mechanism, but in the context of supersymmetric flat directions.

2.3.3 The primordial perturbations

So far, we have considered only the homogeneous part of the inflaton. During inflation, however, any scalar field including the inflaton will experience quantum fluctuations [40, 41]. Since the inflaton dominates the energy density of the universe during inflation, its fluctuations generate perturbations also in the energy density and the background metric. After inflation, when the universe becomes matter dominated, these metric perturbations induce matter perturbations, which may eventually be seen as the temperature fluctuations in the CMB and as the large scale structure of the universe.

Whereas the inflationary paradigm was originally proposed on other grounds, the remarkable fact that it is able to provide such a simple and elegant explanation for the origin of the primordial perturbations is nowadays considered its main virtue. Here we very briefly survey the generation of primordial metric perturbations during inflation, in order to understand what constraints the CMB observations can place on inflationary models. For more detailed analyses, see for example Refs. [12, 42]. Since statistical information is extracted from the observations in terms of correlation functions of the temperature fluctuations, our primary goal shall be to identify the correlation functions of the primordial perturbations.

The generation of perturbations during inflation

Perturbations of the metric can in general be decomposed into scalar, vector and tensor degrees of freedom, which evolve independently from each other at the linear level [43, 44]. During inflation, only scalar and tensor perturbations are produced. We focus on the scalar perturbations, which couple to density and pressure perturbations, and are thus the most relevant ones for the growth of structure. First order scalar perturbations around the flat FRW solution can be represented by the metric

$$\mathrm{d}s^2 = -(1+2\phi)\mathrm{d}t^2 + 2a\partial_i B \mathrm{d}t\mathrm{d}x^i + a^2(t)\big((1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\big)\mathrm{d}x^i\mathrm{d}x^j. \tag{2.39}$$

The generation of perturbations during inflation is convenient to treat in the spatially flat gauge, in which the spatial part of the metric reads simply $g_{ij} = a^2\delta_{ij}$. The equation of motion for the Fourier modes $\delta\phi_k(t)$ of the inflaton perturbation $\delta\phi(x)$, derived from Eq. (2.19), is then solved in the slow-roll approximation by [45]

$$\delta\phi_k(t) = \frac{iH}{\sqrt{2k^3}}\left(1 - \frac{ik}{aH}\right)e^{\frac{ik}{aH}} \tag{2.40}$$

and its complex conjugate $\delta\phi_k^*(t)$, where it has been assumed that $m \ll H$ and H is treated as a constant. Upon quantization these solutions become the mode functions of the inflaton perturbation operator $\delta\hat{\phi}_{\mathbf{k}}(t) = \delta\phi_k(t)\hat{a}_{\mathbf{k}} + \delta\phi_k^*(t)\hat{a}^\dagger_{-\mathbf{k}}$, and the two-point function can be written as [46]

$$\langle\delta\hat{\phi}_{\mathbf{k}}(t)\delta\hat{\phi}_{\mathbf{k}'}(t)\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')|\delta\phi_k(t)|^2. \tag{2.41}$$

For wavelengths well within the horizon, $k \gg aH$, the perturbations are oscillatory, corresponding to standard vacuum fluctuations. After horizon crossing, however, the perturbations eventually freeze as the solution, Eq. (2.40), approaches a constant value for $k \ll aH$. Consequently also the two-point function freezes to a constant value

$$\langle \delta\hat{\phi}_{\mathbf{k}} \delta\hat{\phi}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \frac{H^2}{2k^3}, \tag{2.42}$$

and the perturbations essentially become classical quantities.

The curvature perturbation

Throughout the previous calculation, we have treated the Hubble parameter as a constant. Consequently, all results, including the constancy of the super-horizon fluctuations holds only for a given value of H. During inflation, H changes slowly enough that it can with reason be approximated to a constant during the evolution of a given scale through the horizon. For following the evolution of the perturbations after horizon exit, however, and for relating them to the CMB observations, it is convenient to interpret them in terms of metric perturbations.

To this end, a useful quantity is the metric perturbation ψ of Eq. (2.39), also referred to as the curvature perturbation, which determines the perturbation in the spatial curvature scalar ${}^{(3)}R$ induced by the metric perturbations,

$${}^{(3)}R = \frac{4}{a^2} \nabla^2 \psi. \tag{2.43}$$

The primordial perturbations generated during inflation are commonly expressed in terms of the curvature perturbation evaluated in the uniform energy density gauge [47]

$$\zeta = \psi|_{\delta\rho=0} = \psi + \frac{H}{\dot{\rho}_0} \delta\rho, \tag{2.44}$$

where the right hand side corresponds to the quantity in any given gauge. The virtue of this definition is that, for adiabatic perturbations, such as are created during slow-roll inflation, it remains constant on super-horizon scales. Another commonly used quantity is the curvature perturbation evaluated in the comoving gauge

$$\mathcal{R} = \psi|_{\delta\phi=0} = \psi + \frac{H}{\dot{\phi}} \delta\phi, \tag{2.45}$$

which is given here for a scalar field dominated universe.

On large scales, these two quantities approximately coincide, and may be used interchangeably. From $\mathcal{R}$, which is directly related to the inflaton perturbation, it is easy to find the translation between the two. In the spatially flat gauge $\mathcal{R} = \frac{H}{\dot{\phi}} \delta\phi$, and we obtain the two-point function for the curvature perturbations

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = \frac{H^2}{\dot{\phi}^2} \langle \delta\hat{\phi}_{\mathbf{k}} \delta\hat{\phi}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \frac{H^2}{\dot{\phi}^2} \frac{H^2}{2k^3}. \tag{2.46}$$

Cosmological density fields provide an example of the ergodic property, which implies that averages over a large volume tend to the same answer as averages over a statistical ensemble. Thanks to this property of statistical random fields, these ensemble averages predicted by slow-roll inflation can rightfully be compared to the spatial averages observed in the CMB.

2.4 Inflationary potentials: observables and constraints

In the previous section, we have discussed how inflation driven by a scalar field successfully explains aspects of the observed universe, which cannot be accounted for within the hot big bang model. In our treatment, however, we have considered only a general scalar field, without any mention of the form of the potential. Let us now take a step further, and discuss how inflationary potentials can be constrained with observational data, in order to single out potentials that can indeed lead to a successful period of inflation in light of these constraints.

We shall continue to make the assumption that inflation takes place within the slow-roll regime, which already in itself poses some constraints on the potential. In particular when it comes to identifying inflationary models within some well-motivated particle physics theory, potentials that satisfy the slow-roll conditions for a sufficient period of time are non-trivial to come by. Here we will, nevertheless, consider the inflationary potential from a purely phenomenological perspective and postpone the discussion of embedding inflation into particle theory to Chapter 5.

2.4.1 The amount of expansion

To explain the observed flatness and homogeneity, the inflationary expansion must have lasted sufficiently long for an initial region of space inside the causal horizon to grow at least to the size of the observable universe today. The amount of expansion during inflation is conveniently expressed through the number of e-folds

$$N(t) = \ln \frac{a(t_{\rm end})}{a(t)}, \tag{2.47}$$

which measures the factor by which the scale factor grows between some time t during inflation and the end of inflation at $t_{\rm end}$ [11]. For slow roll inflation, this can be written as

$$N(t) = \int_t^{t_{\rm end}} H \mathrm{d}t \simeq -\frac{1}{M_P^2} \int_\phi^{\phi_{\rm end}} \frac{V}{V'} \mathrm{d}\phi, \tag{2.48}$$

where $\phi_{\rm end}$ is the value of the inflaton at the end of inflation, defined by the violation of the slow-roll conditions Eq. (2.26). In typical models of inflation, the scales corresponding to the current size of the universe have grown larger than the horizon about 50-70 e-folds before the end of inflation [48], which is thus the minimum required number. In most single-field models, however, the total number of e-folds is much greater.

Although the total number of e-folds during inflation is a highly model dependent quantity, it is not an observable, and hence cannot be used to distinguish between otherwise viable potentials. The cosmological data, including the CMB observations, only probes the first 10 or so of the last 60 e-folds before the end of inflation [8]. The best, if not only, means of testing inflationary potentials is thus to compare theoretical predictions of the properties of the primordial perturbations with the perturbations observed in the CMB.

2.4.2 Predictions from the primordial spectrum

In the previous section, we derived an expression for the two-point correlation function Eq. (2.46) of the primordial curvature perturbation produced in slow-roll inflation. From the correlation function we define the power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ of the curvature perturbation as follows

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k). \tag{2.49}$$

The shape of the primordial power spectrum is a very powerful and convenient tool for constraining inflaton potentials. By Eq. (2.46), single field slow-roll inflation generates primordial perturbations with the power spectrum [11]

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 \simeq \frac{1}{24\pi^2 M_P^4} \frac{V}{\epsilon}, \tag{2.50}$$

where H, V and ϵ are to be determined at the horizon exit of the scale k. Since the amplitude of the primordial perturbations is controlled by the quantity V/ϵ, measurements of this amplitude provide information about the energy scale which dominates the universe during inflation.

Another important observable is the spectral index n_s, which measures the dependence of the power spectrum on the wave number k. The scale dependence of the curvature power spectrum is given by the scalar spectral index, which is defined as

$$n_s(k) - 1 \equiv \frac{\mathrm{d} \ln \mathcal{P}_{\mathcal{R}}(k)}{\mathrm{d} \ln k} \simeq 2\eta - 6\epsilon, \tag{2.51}$$

where the latter expression is valid to first order in the slow-roll parameters. Also the scale dependence, i.e. running, of the scalar spectral index $\mathrm{d}n_s(k)/\mathrm{d}\ln k$ is an observationally relevant quantity, which is of second order in the slow-roll parameters [49].

Similarly to the spectrum of curvature perturbations, one can calculate the spectrum of gravitational waves $\mathcal{P}_h(k)$ from the two-point correlation function of the tensor perturbations [44]. For slow-roll inflation the spectrum takes the form

$$\mathcal{P}_h(k) \simeq \frac{8}{M_P} \left(\frac{H}{2\pi}\right)^2 \simeq \frac{2}{3\pi^2} \frac{V}{M_P^4}. \tag{2.52}$$

An analogous expression to Eq. (2.51) can be written for the tensor part of the primordial spectrum. By custom, however, the tensor spectral index is defined as

$$n_t(k) \equiv \frac{\mathrm{d} \ln \mathcal{P}_h(k)}{\mathrm{d} \ln k} \simeq -2\epsilon. \tag{2.53}$$

Constraints from observations

The overall amplitude of the primordial curvature perturbations can be determined from the CMB observations. The best-fit value from the latest measurements reads [4]

$$\mathcal{P}_{\mathcal{R}}(k_0) = 2.41 \times 10^{-9}, \tag{2.54}$$

evaluated at the scale $k_0 = 0.002\text{Mpc}^{-1}$. Requiring that the spectrum, Eq. (2.50), predicted by slow-roll inflation fits this value imposes an important constraint on the inflationary energy scale

$$\left(\frac{V}{\epsilon}\right)^{1/4} \simeq 0.027 M_P \simeq 6.6 \times 10^{16} \text{ GeV}, \tag{2.55}$$

with V and ϵ assumed to be evaluated at the horizon crossing of the scale k_0. Since the slow-roll conditions require $\epsilon < 1$, the constraint implies that the inflationary energy scale must be at least a couple of orders of magnitude below the Planck scale, although it can be much smaller for smaller ϵ. Observations of the tensor part of the spectrum, which in slow-roll inflation is proportional to the potential only, could help determine the value of the inflationary potential and the slow-roll parameter ϵ separately.

Unfortunately, current data is not able to directly detect any tensor perturbations, but some bounds on their magnitude can be inferred. Observational constraints on the tensor spectrum are usually expressed in terms of the relation between the tensor and the scalar power spectrum [50]

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_\mathcal{R}(k)} \simeq 16\epsilon \simeq -8n_t, \tag{2.56}$$

which is proportional to the tensor spectral index in the slow-roll approximation. In fact, this relation is often termed the consistency equation for slow-roll inflation, since a measurement of the two amplitudes involved could disprove the framework, if they were found not to satisfy the proportionality relation. The tensor spectral index, nevertheless, remains completely unconstrained by observations and is usually not even considered a free parameter in cosmological data analysis, but its value is enforced by the consistency condition Eq. (2.56).

Observations of both the tensor-to-scalar ratio and the scalar spectral index provide further constraints on specific inflationary models. Due to degeneracies in the parameter space, these cannot be determined independently of one another. Assuming that the running of the scalar spectral index is negligible, the tensor-to-scalar ratio is constrained to $r < 0.20$ [8]. In this case the best-fit value for the scalar spectral index reads

$$n_s = 0.968 \pm 0.015, \tag{2.57}$$

indicating a preference for a slightly tilted spectrum, consistent with slow-roll inflation. If significant running of the spectral index is allowed for, however, the bounds are considerably relaxed. At present, the data shows no preference for a running spectral index, but the possibility cannot be ruled out.

Another important observable is the statistical nature of the primordial perturbations [51]. Since inflation requires such flat potentials, the fluctuations in the inflaton field are very weakly coupled to one another. This implies that the primordial density fluctuations obey gaussian statistics. To date no non-gaussian correlations have been seen in the CMB, but the existence of small non-gaussianities cannot be excluded [8].

2.4.3 Models of inflation

Single-field inflationary models can be classified into two general categories: large-field and small-field models, according to the change in the vacuum expectation value (vev) of the inflaton during inflation. Also a third category, hybrid inflation, is closely related to the single-field models, even though it requires the presence of two scalar fields. Here we very briefly outline the main features of each category; more details can be found in Refs. [11, 48].

Large-field models

In large-field models the inflaton field is initially displaced far from its minimum, usually even at values larger than the Planck scale, from where it rolls towards its minimum at the origin. The typical example is chaotic inflation [41], referring to models with monomial potentials $V(\phi) \propto \phi^p$. These models are characterized by chaotic initial conditions, which presumably correspond to some sort of quantum gravity or string state. Within this chaotic quantum gravity state, it can generically be assumed, without detailed knowledge of its nature, that a large enough region eventually becomes dominated by some scalar field and starts to inflate. This explanation for the initial conditions is a big advantage of the chaotic model. The required large field values of the inflaton, on the other hand, are a potential source of problems, although it can be argued that they pose no real problem if the self-coupling of the inflation is small so that the value of the potential stays well below M_P^4.

In the most recent CMB observations, the precision is high enough that some of the chaotic inflation models can be excluded by the data [8]. This is the case in particular for the $p = 4$ case, which previously was considered a viable inflationary model on theoretical grounds. The simplest case with a quadratic potential, however, still remains in agreement with data.

Small-field models

Sub-Planckian field values can be achieved in models with polynomial potentials. In such models, the inflationary energy scale is typically smaller than for large field models, and consequently tighter constraints on the flatness of the potential are usually implied, with the risk of requiring fine-tuning. For example, in small-field inflation the initial value of the inflaton is often required to be specified very precisely. Due to the low inflationary scale, these models can usually not be constrained by observations. On the other hand, small-field models are in general easier to connect to some known physics, since non-renormalizable extensions of the Standard Model can be trusted at least to some extent. The inflationary scenario discussed in Chapter 5 provides an example of a model within this category.

Hybrid models

In hybrid models of inflation [52], the expansion occurs through the interplay of two scalar fields. The dynamics during inflation is that of a single field in slow-roll, whereas the end of inflation is induced by the second field settling in its true minimum. For this reason, hybrid models can usually be constructed with sub-Planckian vevs and have a close connection to particle physics [53]. Nevertheless, the parameter space of the simplest hybrid models is strongly constrained by CMB observations, unless significant running of the spectral index is allowed for [8].

2.5 Beyond inflation

From a phenomenological perspective, the fact that a model as simple as single-field slow-roll inflation is able to solve several fundamental cosmological problems renders the inflationary scenario a very attractive possibility. The additional feature that even the most elementary models, such as chaotic inflation with a quadratic potential, can account for the primordial spectrum of temperature fluctuations, seemingly in perfect agreement with cosmological observations, only further adds to its appeal. However simple and compelling, scalar field inflation is, nevertheless, not entirely satisfactory from a theoretical point of view. Some of its problems can merely be accounted issues of naturalness and fine-tuning, but others are of a more conceptual nature. Here we review a few of the most pressing of these issues in short and discuss their implications for our understanding of the very early universe.

Shortcomings of inflation

As already alluded to above, the success of many inflationary models relies on having specific initial conditions for the inflaton field. Given that the main motivation for inflation is to provide a physical explanation for the otherwise highly fine-tuned initial conditions required for the success of the hot big bang, one might question whether there is really a call for inflation after all. Although one can reasonably argue that the fine-tuning of initial conditions required for inflation is less severe than that for the hot big bang, a resolution to this issue would certainly place the motivation for inflation on more solid ground.

Another important problem is the question of trans-Planckian effects in the primordial perturbations, see e.g. [54]. In scalar field inflation, the generation of primordial perturbations relies on microscopic quantum fluctuations during inflation being magnified by the enormous expansion of space into cosmic scale perturbations. The initial state of these quantum fluctuations is typically assumed to be given by the Bunch-Davies vacuum of de Sitter space. The choice of vacuum, however, can only be affirmed by assuming that a given fluctuation mode can be followed to arbitrarily small scales, since in general the notion of a vacuum in an expanding spacetime is not unique. Nonetheless, the calculation of the perturbations is performed in the semi-classical theory, where scalar fields are treated quantum mechanically, while the background geometry is treated classically in Einstein's theory of General Relativity. It is obvious that such a framework cannot in general be expected to be valid at length scales smaller than the Planck scale. Hence, the most attractive feature of the inflationary paradigm, the prediction of the origin of the primordial perturbations, is based on calculations whose validity cannot be guaranteed.

While this might give reason to question the validity of inflationary predictions altogether, it also opens up the possibility of probing physics beyond the Planck scale through inflation. Several approaches towards estimating the possible magnitude and nature of such trans-Planckian corrections have been suggested. Although the question is not entirely settled, the magnitude of the effect is believed to be fairly small, but might, in fact, be observable in the near future [55].

Looking beyond inflation

Apart from the problems discussed above, there are a number of issues that were originally considered problems of the hot big bang model, which inflation completely fails to address, leaving them fully unresolved. One such issue, which we will discuss further in Chapter 6, is the problem of the initial singularity, which cannot be avoided in a cosmology based on Einstein's field equations if the matter source obeys the weak energy condition. The singularity theorem has also been generalized to apply to any manifold on which the local Hubble parameter measured by an observer on a null or time-like geodesic is bounded from below by a positive constant in the past [56]. This implies that even in scalar field-driven inflationary cosmology, a past singularity at some point in space is inevitable. A consistent model of the primordial universe, on the other hand, would either need to be non-singular or have a controlled singularity that does not lie outside its domain of validity.

One might note that the problems discussed above can all be associated with our ignorance about the physics which governs the energy scales around the Planck scale, which inflation comes very close to probing. Some of these problems could thus be naturally resolved by whatever theory of quantum gravity that resolves the ultraviolet problems of the Standard Model. This quantum gravity theory may well lead to a period of cosmological inflation, making the primordial perturbations a testing ground for fundamental physics. However, the possibility of alternative scenarios for the early universe should perhaps not be forgotten. Single-field slow-roll inflation is after all only one possible way of making inflation theoretically possible, but inflation does not necessarily require a scalar field at all. Moreover, the flatness and horizon problems and even the generation of primordial perturbations may eventually be explained without any period of inflation whatsoever.

Chapter 3

Supersymmetry and friends

In the previous chapter we have highlighted several cosmological problems that cannot be satisfactorily addressed with the physics of the Standard Model. In the enclosed research papers, to be discussed in Chapters 4-6, we consider a number of these issues, assuming that physics beyond the Standard Model exhibits supersymmetry. To facilitate the coming presentations, we here briefly review the basics of the supersymmetric framework. As the main focus of this thesis is on the cosmological questions, we refrain from arguing the case for supersymmetry and by no means attempt to provide a complete introduction to this vast topic. We simply present, in a rather concise form, the tools and concepts that are required for following the treatment in the subsequent chapters.

Since the early universe corresponds to high energy scales, supersymmetry in this context implies supergravity, and possibly also string theory, see e.g. [57]. The former two are reviewed in Sections 3.1 and 3.2, respectively, with emphasis on the scalar sector of the theories. More detailed accounts of the subjects covered can be found in several excellent reviews on low-energy supersymmetry [58, 59]. Section 3.3 is then devoted to supersymmetric flat directions, which play a particularly important role in the early universe [60]. Although we will on several occasions further on in this thesis touch upon the subject of string theory, we choose not to discuss it here, since even a very compact introduction to the relevant topics would take up a disproportionate part of this work. Instead, we refer the reader to some basic texts on string theory [61].

3.1 Supersymmetry

Supersymmetry can be realized either as a global or a local symmetry. We will first consider the case of global supersymmetry, and make the generalization to local supersymmetry below. In order to make the connection to low-energy physics, we focus on $N = 1$ supersymmetry in four dimensions. The supersymmetry algebra then has two irreducible representations containing fields of spin less than or equal to one: chiral and vector supermultiplets.

The chiral superfield$\Phi(\theta, \bar{\theta}, x)$ carries the field contents of the chiral supermultiplet: a Weyl spinor $\psi_\alpha(x)$ and a complex scalar $\phi(x)$. In addition, it contains an auxiliary complex scalar field $F(x)$, which encodes off-shell degrees of freedom.

The vector superfield $V(\theta, \bar{\theta}, x)$ contains a massless gauge field $A_\mu(x)$ and its superpartner, a Weyl spinor $\lambda_\alpha(x)$, along with a real auxiliary scalar field $D(x)$.

To write down the action for a set of chiral superfieldsΦ_n, transforming in some representation of a gauge group G, a vector superfield V^a is introduced for each gauge generator [62]. In the representation defined by the scalar fields (excluding the possible Fayet-Iliopoulos term) the most general renormalizable Lagrangian is then

$$\mathcal{L} = \sum_n \int d^4\theta \Phi_n^\dagger e^V \Phi_n + \frac{1}{4g^2} \int d^2\theta W_\alpha^2 + \text{h.c.} + \int d^2\theta W(\Phi_n) + \text{h.c.} \tag{3.1}$$

Here $V = T^a V_a$, where T^a are the hermitian generators of the gauge group G and W_α is the superspace analogue of the gauge invariant field strength $F_{\mu\nu}$. The superpotential $W(\Phi)$ is a holomorphic function of the chiral superfields, which determines the interactions between fields in the chiral multiplet. In a renormalizable theory the superpotential can be at most cubic in the fields, corresponding to a quartic potential.

The scalar potential

Of particular relevance for the topics treated in this thesis, is the potential for the scalar components of the chiral multiplet, which can be identified from the Lagrangian Eq. (3.1) as

$$V(\phi_n, \phi_n^*) = \sum_n |F_n|^2 + \frac{1}{2} \sum_a D_a D^a. \tag{3.2}$$

The F- and D-terms are obtained from the equations of motion of the auxiliary fields in the chiral and vector multiplets, respectively

$$F_n^* = \frac{\partial W}{\partial \phi_n}, \quad D^a = \sum_{m,n} \left(g^a \phi_m^* T_{mn}^a \phi_n \right). \tag{3.3}$$

In supersymmetry, unlike in the Standard Model, the scalar potential is evidently strongly constrained by gauge symmetries and supersymmetry. An important consequence of this is the existence of a number of directions in the space spanned by the scalar fields, known as flat directions, along which the potential is exactly zero. We will return to these in more detail in Section 3.3.

3.1.1 The Minimal Supersymmetric Standard Model

In the minimal supersymmetric extension of the Standard Model (MSSM), the gauge symmetry is still $SU(3)\times SU(2)\times U(1)$, but every particle has a supersymmetric partner (sparticle) with the same quantum numbers. For each fermion, a chiral superfield with a spin zero sfermion is introduced. Likewise, every gauge field is partnered with a spin 1/2 gaugino into a vector superfield. In addition, two chiral Higgs doublets of opposite hypercharge, each containing a Higgs boson and a spin 1/2 higgsino, need to be introduced to couple to up- and down-type quarks respectively.

The superpotential of the chiral superfields in the MSSM can be written in the form

$$W = h_u \bar{u} Q H_u - h_d \bar{d} Q H_d - h_e \bar{e} L H_d + \mu H_u H_d, \tag{3.4}$$

where all gauge and flavor indices are suppressed. The quark and lepton superfields are denoted by Q, L, $\bar{u}$, $\bar{d}$ and $\bar{e}$, with barred fields representing $SU(2)$ singlets, while H_u and H_d are the Higgs doublets. Finally h_u, h_d, h_e are the Yukawa coupling matrices in flavor space, and the μ-term is a supersymmetric mass term for the Higgs fields. Although the term is a supersymmetry preserving quantity, it is forced to be of similar magnitude, $\mu \sim 100$ GeV, as quantities that break supersymmetry, in order for the electro-weak symmetry breaking to be phenomenologically viable [63]. The scalar components of the Higgs bosons have the vacuum expectation values $\langle H_u \rangle \equiv v_u = v \sin\beta$ and $\langle H_d \rangle \equiv v_d = v \cos\beta$, where $v = 174$ GeV is the electroweak breaking scale. Their ratio is thus given by

$$\tan\beta \equiv \frac{v_u}{v_d}. \tag{3.5}$$

The superpotential Eq. (3.4) is the minimal one required for a phenomenologically viable model. In addition, there is a set of dimension four terms permitted by the gauge symmetries. These terms, however, violate baryon and lepton number and hence produce proton decay, unless couplings are highly fine-tuned. A simple solution to the problem with proton stability, is to impose a discrete Z_2 symmetry, known as R-parity [64]. Under this symmetry, all ordinary particles are even, while their superpartners are odd. Under exact R-parity, no mixing between particles and sparticles is allowed, which eliminates all of the dangerous operators. Apart from solving the immediate problem, R-parity has several phenomenological consequences. Most importantly, it predicts that the lightest supersymmetric particle (LSP) is stable and weakly interacting, making it an excellent dark matter candidate [65].

3.1.2 Supersymmetry breaking

The supersymmetry algebra implies that particles in the same supermultiplet have equal masses, but so far no supersymmetric partners of any Standard Model particles have been observed. Consequently, if supersymmetry is indeed a symmetry of nature, it must be broken at low energies.

Global supersymmetry can be broken either spontaneously or explicitly. From a theoretical point of view, it is expected to be an exact symmetry that is spontaneously broken by a non-invariant vacuum state, $Q_{\bar{a}}|0\rangle \neq 0$, where $Q_{\bar{a}}$ is the supersymmetry generator. The expectation value of the total energy

$$\langle H \rangle = \frac{1}{4}\left(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2 \right) \geq 0 \tag{3.6}$$

works as an order parameter for spontaneous symmetry breaking. The vacuum energy is zero only when supersymmetry is unbroken. Consequently, supersymmetry is spontaneously broken if at least one of the F- and D-terms in the scalar potential Eq. (3.2) has a non-vanishing vev

$$\langle F_n \rangle \neq 0 \quad \text{or} \quad \langle D^a \rangle \neq 0. \tag{3.7}$$

Spontaneous symmetry breaking in general implies the existence of a massless Nambu-Goldstone particle with the same quantum numbers as the symmetry generator. In supersymmetry this is the goldstino: the fermionic component of the supermultiplet in which the auxiliary field whose vev breaks supersymmetry resides.

In the absence of a well defined ultraviolet completion of the MSSM, numerous phenomenological models for spontaneous symmetry breaking have been proposed. The standard procedure amounts to extending the theory with some set of fields, the hidden sector, which are responsible for the symmetry breaking [66]. The models are classified as gravity-mediated if the interaction between the standard fields and the hidden sector is of gravitational strength. In this case the scale of the symmetry breaking particles is intermediate between the Planck and the weak scale. Alternatively, in the gauge-mediated models, the breaking takes place at some much lower energy with gauge interactions serving as the messenger fields [67]. In the following section, we shall discuss the gravity mediated scenario in more detail, in the context of supergravity.

Soft breaking

At low energies, however, most of the supersymmetry breaking models simply give rise to an effective Lagrangian that is supersymmetric, apart from explicit supersymmetry breaking terms. From the low energy perspective, it is hence practical to ignore the details of supersymmetry breaking and simply parameterize its effect by introducing explicit supersymmetry breaking interactions.

Let us write the effective Lagrangian as $\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$, where $\mathcal{L}_{\rm soft}$ is the supersymmetry violating part. For such a theory to maintain its solution to the hierarchy problem, the explicit supersymmetry breaking terms must be soft, i.e. have couplings with positive mass dimension. The most general renormalizable soft supersymmetry breaking Lagrangian for the MSSM can be written as [68]

$$\mathcal{L}_{\rm soft} = -\frac{1}{2}\left(M_a \lambda^a \lambda^a + \frac{1}{3} A^{ijk} \phi_i \phi_j \phi_k + b^{ij} \phi_i \phi_j\right) + \text{h.c.} - (m^2)^{ij} \phi_i^* \phi_j, \tag{3.8}$$

where M_a and m^2 stand for gaugino and scalar masses respectively, and A^{ijk} and b^{ij} are trilinear and bilinear scalar couplings. With $\mathcal{L}_{\rm soft}$ of the form above, quantum corrections to the Higgs mass from the supersymmetry breaking terms will be proportional to $m_{\rm soft}$, the largest scale associated with $\mathcal{L}_{\rm soft}$. To naturally produce the right value for the Higgs vev, $m_{\rm soft} \lesssim 1$ TeV is required.

3.2 Supergravity

So far we have considered only global supersymmetry, with the additional assumption of renormalizability. If supersymmetry is not merely an accidental symmetry, however, it must be a local one. When the supersymmetry is made local, invariance under general coordinate transformations is necessarily imposed, and hence the theory automatically includes gravity. The theory of supergravity, like other theories of gravity, is in general thought to be non-renormalizable.[1] The field content of supergravity thus includes an additional representation: the gravity supermultiplet, with a massless graviton $h_{\mu\nu}$ and its superpartner the spin 3/2 gravitinoΨ_{μ}. The chiral and vector supermultiplets are

[1] However, some recent studies suggest that supergravity with eight sets of supersymmetry generators could be finite in the ultraviolet limit [69].

defined as in supersymmetry, but their interactions are generalized as the requirement of renormalizability is relaxed.

The full supergravity Lagrangian, which can be found e.g. in Ref. [70], will not be necessary for our purposes. Here we focus on the effective Lagrangian for the chiral and vector superfields, which reduces to the one of global supersymmetry in the limit $M_P \to \infty$. Including terms of up to two derivatives, the Lagrangian

$$\mathcal{L} = \int d^4\theta \sum_n K(\Phi_n^*, \Phi_n) + \int d^2\theta \left(\sum_{ab} f_{ab}(\Phi_n) W_\alpha^a W^{\alpha b} + W(\Phi_n) + \text{h.c.} \right), \quad (3.9)$$

which contains within it the most general non-renormalizable supersymmetric theory, is fully specified by three functions as follows. The Kähler potential $K(\Phi_n^*, \Phi_n)$ is a real, gauge invariant function of both the chiral fields and their conjugates; it determines the kinetic terms of the scalar fields. The gauge kinetic function $f_{ab}(\Phi_n)$ is a dimensionless holomorphic function of the fields, symmetric in indices a and b, and gives the couplings between fields in the gauge multiplet. The superpotential $W(\Phi_n)$ is defined as above but can now include any holomorphic and gauge invariant combination of the fields such that its dimension is $[\text{mass}]^3$.

The scalar potential

Again, we are particularly interested in the Lagrangian of the scalar components of the chiral superfields. Since the Kähler potential is an arbitrary real function of the fields, the scalar kinetic term is in general non-canonical. Using the convention where lower indices m and $\bar{m}$ refer to derivation with respect to ϕ_m and ϕ_m^*, the kinetic term reads

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \sum_{m,n} K_{\bar{n}m} \partial_\mu \phi_{\bar{n}}^* \partial^\mu \phi_m. \quad (3.10)$$

The scalar potential, which is a generalization of the potential in global supersymmetry, takes the form

$$V(\phi_n^*, \phi_n) = e^G \left(K^{m\bar{n}} G_m G_{\bar{n}} - 3 \right) + \frac{1}{2} \left(\text{Re} f_{ab}^{-1} \right) D^a D^b. \quad (3.11)$$

Here $K^{m\bar{n}}$ is the inverse of the Kähler metric $K_{\bar{n}m}$ and is used to raise lower indices. The potential has been written in terms of the function $G = K + \ln(|W|^2)$, which yields the supergravity generalization of the F-term

$$F^m = e^{G/2} K^{m\bar{n}} G_{\bar{n}}. \quad (3.12)$$

The contribution from the gauge sector is again encoded in the D-term, which here takes the form

$$D^a = -g \sum_{m,n} G_m T_{mn}^a \phi_n. \quad (3.13)$$

3.2.1 Supersymmetry breaking in supergravity

Similarly to global supersymmetry, supergravity is spontaneously broken when at least one of the F- or D-terms obtains a non-zero vev

$$\langle F^m \rangle \neq 0 \quad \text{or} \quad \langle D^a \rangle \neq 0. \tag{3.14}$$

Unlike in supersymmetry, however, this does not necessarily imply that the scalar potential Eq. (3.11) and the energy have positive vevs. This allows for the vacuum energy to be arbitrarily small, which is an attractive feature in light of the very small value for the vacuum energy density measured by cosmological observations.

Supergravity provides a natural framework for gravity mediated models of supersymmetry breaking. In these models, local supersymmetry is assumed to be spontaneously broken in a hidden sector and mediated to the visible sector through non-renormalizable Planck-suppressed couplings between the two sectors, see Ref. [59]. At tree-level, the soft supersymmetry breaking parameters can be derived straight from the Lagrangian Eq. (3.9). The details of the resulting soft terms thus depend on the particular supergravity model, i.e. on the form of the functions K, f_{ab} and W. Given a more fundamental theory, such as string theory, which yields supergravity as a low energy limit, these functions can in principle be derived by integrating out massive modes. In the absence of such a theory, however, one simply needs to treat them on phenomenological grounds.

F-term supergravity breaking

Here we consider supersymmetry breaking taking place in some hidden sector with gauge singlet fields, so that supersymmetry is broken by non-vanishing F-terms in the hidden sector. The scalar potential is then made up only by the first term in Eq. (3.11). The assumption of a hidden sector implies that the superpotential takes the form

$$W = \hat{W}(h_a) + I(h_a, \Phi_i), \tag{3.15}$$

where h_a denotes the hidden sector fields and Φ_i the visible sector MSSM fields. We use the hat to denote quantities such as $\hat{W}(h_a)$, which are independent of Φ_i, but are functions of the hidden sector fields. Given the assumptions above, the scalar potential Eq. (3.11) can be written as

$$V = |\hat{W}|^2 f + \Big(\hat{W} I^* g + \text{h.c.}\Big) + |I|^2 k, \tag{3.16}$$

where

$$f = e^K \Big(K^{m\bar{n}} \Big(K_m K_{\bar{n}} + \frac{\hat{W}_m \hat{W}^*_{\bar{n}}}{|\hat{W}|^2} + \frac{K_m \hat{W}^*_{\bar{n}}}{\hat{W}^*} + \frac{K_{\bar{m}} \hat{W}_n}{\hat{W}} \Big) - 3 \Big), \tag{3.17}$$

$$g = e^K \Big(K^{m\bar{n}} \Big(K_m K_{\bar{n}} + \frac{I_m \hat{W}^*_{\bar{n}}}{I \hat{W}^*} + \frac{K_m \hat{W}^*_{\bar{n}}}{\hat{W}^*} + \frac{K_{\bar{m}} I_n}{I} \Big) - 3 \Big), \tag{3.18}$$

$$k = e^K \Big(K^{m\bar{n}} \Big(K_m K_{\bar{n}} + \frac{I_m I^*_{\bar{n}}}{|I|^2} + \frac{K_m I^*_{\bar{n}}}{I^*} + \frac{K_{\bar{m}} I_n}{I} \Big) - 3 \Big). \tag{3.19}$$

To find the explicit expression for the potential, the form of the Kähler potential as well as the superpotential must be specified, so that the functions f, g and k can be expanded in powers ofΦ_i. In general there are three different kinds of contribution to the potential for the visible sector fields. The first term in Eq. (3.16), which arises from the Kähler potential and the hidden sector superpotential, gives rise to the soft mass terms. The second term in Eq. (3.16), which mixes the hidden sector and the visible sector superpotentials, contributes to the trilinear soft terms. The last term yields the visible sector potential as in global supersymmetry but also contains higher order contributions, which are absent in the global case.

To get a rough idea of the structure of the low energy theory in F-term supergravity breaking, let us suppose that $W(\Phi_i) = 0$, and that the Kähler potential takes the minimal form $K = \delta_{ij}$. The potential for the visible sector scalars then reads

$$V(\phi_i^*, \phi_i) = e^{\hat{K}} |\hat{W}|^2 \sum |\phi_i|^2, \tag{3.20}$$

and the scalars all have a common soft mass

$$m = \left\langle e^{\hat{K}/2} \frac{|\hat{W}|}{M_P^2} \right\rangle = m_{3/2}, \tag{3.21}$$

the scale of which is set by the mass of the gravitino $m_{3/2}$. In supergravity breaking, the gravitino becomes massive by eating the degrees of freedom of the Nambu-Goldstone boson of broken supersymmetry, the goldstino, through the super-Higgs mechanism.

3.2.2 The gravitino problem

Although the gravitino is a very weakly interacting particle, it plays an important role in the early universe. During reheating, when the energy density stored in the inflaton is converted into radiation, large amounts of gravitinos can be produced through scattering in the thermal bath. If gravitinos are overproduced during this period, they can cause serious problems in the later cosmological evolution [71, 72].

If the gravitino is heavy and unstable, as in gravity-mediated scenarios, it poses a threat to the success of nucleosynthesis. Since gravitinos couple to Standard Model particles with gravitational strength, they are long-lived and can decay after the onset of BBN. Energetic particles produced in their decays may subsequently dissociate light elements produced in nucleosynthesis, while other elements could be overproduced, thus spoiling the agreement with observations. Since the gravitino production rate is proportional to the temperature, avoiding the gravitino problem leads to constraints on the reheating temperature. In the minimal supergravity framework, depending on the values of the supersymmetric parameters, the reheating temperature must satisfy [73, 72]

$$T_{\rm RH} \lesssim 10^5 - 10^6 \text{ GeV}, \tag{3.22}$$

unless the gravitino mass is larger than about 10 TeV, in which case the upper bound is relaxed to $10^9 - 10^{10}$ GeV.

In gauge-mediated models, on the other hand, the gravitino is generically light and stable, and constitutes the dark matter candidate. In this case the gravitino abundance must not exceed the dark matter abundance, since this would over-close the universe. Furthermore, the weakness of the gravitino coupling may cause the next-to-lightest supersymmetric particle (NLSP) to be long-lived, causing similar problems with nucleosynthesis as above. These considerations again lead to a strict upper bound on the reheat temperature which, depending on the nature of the NLSP, is constrained to [74]

$$T_{\rm RH} \lesssim 10^7 - 10^9 \text{ GeV}. \tag{3.23}$$

In some cases the constraints are so stringent that gravitino masses larger than 10 GeV can be entirely excluded [72]. The strict bound on the reheating temperature implied by the gravitino problem creates a challenge for many scenarios of the early universe. In Chapter 4 we discuss the difficulties that the bound imposes on the model of thermal leptogenesis.

3.3 Supersymmetric flat directions

Let us return for a moment to the case of renormalizable globally supersymmetric theories. These theories generically possess a large vacuum degeneracy, which is manifested in the existence of a number of directions in the space of scalar fields along which the potential identically vanishes [75]. As mentioned above, these flat directions, collectively called the moduli space, are a consequence of supersymmetry and gauge symmetries, which together constrain the terms allowed in the Lagrangian far more than gauge symmetries alone. As we shall see some examples of in the following two chapters, flat directions have important consequences for cosmology, where the behavior of the theory at large field strengths is relevant, see [60] for an extensive review.

3.3.1 Identifying the flat directions

In principle, a flat direction is identified by the requirement that the scalar potential Eq. (3.2) vanishes along it, amounting to the F- and D-flatness conditions

$$F_n^* = \frac{\partial W}{\partial \phi_n} = 0, \quad D^a = \sum_n \left(g^a \phi_m^* T^a_{mn} \phi_n \right) = 0. \tag{3.24}$$

A more convenient means of identification, however, is provided by the correspondence between flat directions and gauge-invariant, holomorphic polynomials of chiral superfields [76]. Any D-flat direction can be parameterized by such a gauge-invariant polynomial, $X_m = \Phi_1 \Phi_2 \ldots \Phi_m$, which obeys a finite set of redundancy relations. The condition of F-flatness then imposes additional constraints on the operator X_m, determined by the form of the superpotential.

Let us study as an example a flat direction of the MSSM. The full set of monomials invariant under the MSSM gauge group, which parameterize the moduli space of the MSSM, has been identified and classified in Ref. [77]. An example of a D- and F-flat direction is provided by the operator $X_3 = L_i L_j \bar{e}_k$. The D-flatness condition is satisfied along directions whose scalar components take the form

$$L_i = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L_j = \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad e_k = \phi, \tag{3.25}$$

given that $i \neq j$. The complex field ϕ parameterizes the vevs of the fields along the flat direction. It relates to the scalar component of the invariant operator as $X_3 = c\phi^3$, where c is a constant.

The MSSM superpotential Eq. (3.4) gives rise to one F-term that contains fields in the flat direction $LL\bar{e}$, which yields the F-flatness constraint

$$F_{H_d}^{\alpha} = h_e^{ij} L_i^{\alpha} \bar{e}_j = 0. \tag{3.26}$$

The D-flatness condition Eq. (3.25) allows for nine family combinations of $LL\bar{e}$, out of which five are linearly independent, leaving in total three flat directions of this type after imposing the constraint from F-flatness.

3.3.2 Lifting the flatness

The non-renormalization theorem of global supersymmetry [78] guarantees that the vacuum degeneracy along a flat direction will not be broken by perturbative quantum corrections. The flatness of the potential can, nevertheless, be lifted by higher order non-renormalizable operators in the superpotential as well as by explicit supersymmetry breaking effects.

Non-renormalizable superpotential terms formally arise from some more fundamental theory, such as string theory, when massive degrees of freedom are integrated out. In the absence of a specified fundamental theory, one generally includes any terms allowed by the symmetries of the low-energy model. Flat directions are then classified by the mass dimension of the lowest order allowed non-renormalizable term which lifts their potential. A flat direction of dimension n can be lifted by superpotential terms of the form [79, 80]

$$W \supset \frac{\lambda}{nM^{n-3}}\phi^n, \tag{3.27}$$

$$W \supset \frac{\lambda}{M^{n-3}}\psi\phi^{n-1}, \tag{3.28}$$

where λ is an effective coupling constant. As above, ϕ is the flat direction vev, which is related to the corresponding invariant operator as $X_m = c\phi^m$, and ψ is a visible sector scalar field that is not included in the flat direction. The scale M denotes the cutoff where new physics is expected to become important.

The non-renormalizable superpotential terms above both give rise to a contribution to the scalar potential of the form

$$V(\phi) \supset \frac{|\lambda|^2}{M^{2n-6}}|\phi|^{2n-2}. \tag{3.29}$$

In addition to the superpotential, supersymmetry breaking will lift the degeneracy of the scalar field potential. In general, the contribution is given by soft supersymmetry breaking terms, which in the simplest case take the form [79, 80]

$$V(\phi) \supset m^2|\phi|^2 + \frac{\lambda A}{nM^{n-3}}\phi^n, \tag{3.30}$$

where n is larger than or equal to the number of fields in the flat direction. The explicit form of the contribution once again depends on the particular supersymmetry breaking mechanism and the details of the model.

Let us, for example, consider a flat direction lifted by a superpotential term of the form of Eq. (3.27). As above, we assume hidden sector F-term supergravity breaking. The leading terms that are induced in the potential can then be expressed as [79, 80]

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + \cos(n\theta + \theta_A)\frac{|A\lambda|}{nM^{n-3}}|\phi|^n + \frac{|\lambda|^2}{M^{2n-6}}|\phi|^{2n-2}, \tag{3.31}$$

where θ and θ_A are the complex phases of the flat direction and $A\lambda$, respectively. The parameters m, λ, A and θ_A are in principle functions of the hidden sector fields. Their explicit expressions can be derived from Eqs. (3.16)–(3.19) when the Kähler potential and the hidden sector superpotential are given. In Chapter 5 we will discuss a particular model of this kind in more detail.

3.3.3 Flat directions in the early universe

In the early universe supersymmetry is broken by the presence of a non-zero energy density, which can give a much larger contribution to the flat direction potential than the ones listed above. During inflation, for example, the energy density by definition takes on a large positive value. Also after inflation, when the universe is dominated by the oscillations of the inflaton, the vacuum energy is non-zero, hence breaking supersymmetry. Furthermore, supersymmetry is broken even during the radiation dominated era, by the distinct thermal occupation numbers of fermions and bosons. Such finite energy supersymmetry breaking is then transmitted to the flat direction by non-renormalizable interactions.

Including all possible contributions, the generic potential along a supersymmetric flat direction ϕ is given by [79, 80]

$$V(\phi) = \left(m^2 - cH^2\right)|\phi|^2 + \left(\lambda\frac{A + aH}{nM^{n-3}}\phi^n + \text{h.c.}\right) + |\lambda|^2\frac{|\phi|^{2n-2}}{M^{2n-6}}, \tag{3.32}$$

where c, a and λ are dimensionless model-dependent couplings, and m and A are the soft breaking terms. The contribution from the finite energy supersymmetry breaking is encoded in the Hubble parameter dependent terms, which generically dominate the scalar potential as long as $H > m$. The location of the minimum of the potential depends on the sign of the parameter c. For negative c, the minimum resides at the origin, while for positive c and $H \gg m$, the minimum is at finite values of ϕ. In this case, the flat direction condensate acquires a non-zero vev

$$|\phi_0| = \left(\frac{\beta H M^{n-3}}{\lambda}\right)^{\frac{1}{n-2}}, \tag{3.33}$$

where β is a numerical constant which depends on a, c, and n. Depending on the order n at which the flat direction is lifted, the value of the non-zero vev $|\phi_0|$ can be as large as the Planck scale. During inflation the Hubble parameter is nearly constant, and the flat direction condensate consequently sits at its minimum at ϕ_0. After the end of inflation, however, when the inflaton oscillates around the bottom of its potential, the Hubble rate and hence also the flat direction vev decrease with time. When $H \sim m/3$, the effective mass term changes sign, and the condensate rolls down towards its new minimum at $\phi = 0$.

Non-adiabatic particle production

If, at this point, the flat direction condensate passes close enough to the origin, production of particles coupled to the flat direction can take place through parametric resonance, analogously to preheating. Such non-adiabatic particle production from oscillating flat directions can have important repercussions in the early universe, see for example Ref. [81].

In general, however, the terms proportional to A and to aH in the potential Eq. (3.32) are complex with a non-vanishing relative phase $\theta_a - \theta_A$. In this case the condensate will spiral around the origin at $\phi = 0$ with a non-zero $\dot{\theta}$ and particle production can take place only if several flat directions are excited simultaneously [82, 81]. If the phase-dependent terms are absent, on the other hand, particle production may occur from only one excited flat direction. This is the case for example if the flat direction is lifted by a non-renormalizable superpotential term of the form Eq. (3.28), so that F_ψ is non-zero along the flat direction, but $W = 0$ along it. Furthermore, the superpotential may vanish along the flat direction, possibly due to a discrete R-symmetry. In such a case, when W identically vanishes, the potential during inflation is no longer given by Eq. (3.32), but takes the form [79, 80]

$$V(\phi) = H^2 M_p^2 f(\frac{|\phi|^2}{M_p^2}) + H^2 M_p^2 g(\frac{\phi^n}{M_p^n}). \tag{3.34}$$

In this case the typical value of the flat direction during inflation, rather than being given by Eq. (3.33), is of the order of the Planck scale.

It is also worth noting that the coefficients A and a of Eq. (3.32) depend on the specific form of the Kähler potential couplings and there are cases in which they are suppressed by inverse powers of M_p. For instance, if the inflaton is a composite field, it will appear in the Kähler potential only through bilinear combinations and $a \sim H/M_p$. Moreover, in the case of D-term inflation [83] (see Section 5.1) a vanishes identically and no phase-dependent terms are generated when $W = 0$ along the flat direction. We have placed particular emphasis on identifying situations in which non-adiabatic particle production from flat directions is feasible here, since we will explore one possible consequence of this effect in the following chapter.

Papers

Chapter 4

Thermal leptogenesis and supersymmetry

In Chapter 2 we have presented the observational evidence for a baryon asymmetry, and argued that it must have been generated through some dynamical process during the early radiation dominated era. Since Standard Model physics alone is not sufficient to fulfill the criteria necessary for a dynamical generation of the asymmetry, the process of baryogenesis must take place within some theory beyond the SM. In the current chapter we consider the possibility that the baryon asymmetry is generated through leptogenesis, which can be realized when the Standard Model is extended with a set of heavy right-handed neutrinos, as is suggested by the see-saw mechanism for neutrino masses [84].

We begin by presenting a few of the most commonly proposed baryogenesis models below. In Section 4.2 we then review the see-saw mechanism and explain how it can fulfill the Sakharov criteria for baryogenesis. This is followed by a crude quantitative presentation of the thermal leptogenesis scenario in Section 4.3. Subsequently, we review the consequences for the model if supersymmetry is realized in the early universe, and discuss, in particular, the inherent conflict with the gravitino bound. In Section 4.5, we finally outline the possible solution to this problem which was presented in the enclosed research paper [1], making use of the non-adiabatic production of particles from flat directions discussed above. For more thorough reviews of thermal leptogensis, we refer the reader to Refs. [85, 86].

4.1 Models of baryogenesis

The Standard Model fails to produce the observed baryon asymmetry during the electro-weak phase transition for two distinct reasons. Firstly, the *CP* violation is too small to induce an asymmetry of the observed magnitude. Secondly, the experimental lower bound on the Higgs mass implies that the phase transition cannot be of first order, as required. The new physics responsible for baryogenesis must consequently provide additional sources of *CP* violation, and either modify the Higgs sector to enhance the phase transition, or provide a completely different process for departure from equilibrium. In addition, if the asymmetry is produced before the electro-weak phase transition by a process other than sphalerons, this processes must violate $B-L$ in order to avoid sphalerons subsequently erasing the asymmetry.

The simplest option in the search for a viable baryogenesis model is then perhaps to look for modifications to the Standard Model with new ingredients in the Higgs sector, which could alter the strength of the phase transition. One such example is the two Higgs doublet model [87], where the Higgs potential has more parameters and is *CP*-violating, unlike the Higgs potential of the Standard Model. The most well-known example, however, is probably the MSSM electroweak baryogenesis [45, 88], where the presence of a light stop would modify the Higgs potential in the required way. Light Higgsinos and gauginos, on the other hand, would provide the required enhancement of the *CP*-violation. Whereas the scenario has a drawback in the small window of the supersymmetric parameters for which baryogenesis is viable, it has the virtue of soon being subject to experimental tests at the LHC.

Giving up on electro-weak baryogenesis [89], there is no shortage of models proposed in the literature. The first scenarios were formulated in terms of grand unified theories (GUT), where baryon number is violated by the interactions of heavy gauge bosons and leptoquarks, while the departure from equilibrium could be provided in the decay of these heavy fields, see Ref. [10]. The GUT baryogenesis scenario, however, generally has difficulties with proton decay and unwanted relics, enforcing a lower bound on the mass of the decaying boson, which is usually much higher than the reheating temperature in generic models inflation. Other early proposals include the Affleck-Dine baryogenesis [90], where the asymmetry arises in the potential of a classical scalar field, for example a supersymmetric flat direction, which later decays into particles.

Thermal leptogenesis [91] is another scenario where the out-of-equilibrium decay of heavy particles is the source of the baryon asymmetry. However, in this case the asymmetry is produced in the lepton sector. Electro-weak sphaleron transitions can then convert the lepton asymmetry into a baryon asymmetry [92], allowing for baryogenesis through leptogenesis. The minimal successful leptogenesis scenario requires only the addition of heavy right-handed neutrinos, as suggested by the see-saw mechanism for neutrino masses [84]. The Yukawa couplings of the heavy neutrinos provide the necessary new source of *CP* violation. Our focus in this chapter will be on the simplest and, perhaps, theoretically best motivated realization of leptogenesis: thermal leptogenesis with hierarchical singlet neutrinos.

4.2 Neutrino mass and leptogenesis

Before getting into the details of the thermal leptogenesis scenario, we here briefly present the observational evidence for finite neutrino masses, and explain how the see-saw mechanism with heavy right-handed neutrinos can account for them. Furthermore, we show that the same ingredients required to generate the light neutrino masses are sufficient to satisfy the Sakharov conditions necessary for baryogenesis.

Evidence for neutrino masses

Although, in the Standard Model neutrinos are massless, there is by now ample evidence from neutrino oscillation experiments that at least two neutrinos have non-zero masses. Measurements of solar and atmospheric neutrino fluxes, confirmed also by reactor and accelerator experiments, determine two mass-squared differences [32]

$$\Delta m^2_{\rm sol} = (7.59 \pm 0.20) \times 10^{-5}\ {\rm eV}^2, \tag{4.1}$$
$$\Delta m^2_{\rm atm} = (2.43 \pm 0.13) \times 10^{-3}\ {\rm eV}^2. \tag{4.2}$$

While the oscillation experiments measure only the mass differences, an obvious implication is that the mass of the heaviest neutrino must be at least $m_{\rm atm} \equiv \sqrt{m^2_{\rm atm}} \sim 0.05$ eV. The absolute mass scale can furthermore be probed through β- and 2β-decays as well as cosmology. The consensus of all these measurements, assuming three neutrino flavours, is for masses in the sub-eV range

$$m_i < 1\ {\rm eV}, \tag{4.3}$$

where $i = (1, 2, 3)$. For a more detailed discussion of the observational constraints on the neutrino mass, the reader is referred to e.g. Ref. [93].

The see-saw mechanism

The Standard Model neutrinos can acquire Dirac mass terms if a set of right handed neutrinos is added to the theory, but the smallness of the masses is difficult to explain without fine-tuning the Yukawa couplings. In high-energy see-saw models [84], on the other hand, sufficiently small Majorana neutrino masses are naturally induced, when heavy particles are integrated out.

In the simplest realization of the see-saw mechanism, the so-called Type I see-saw,[1] the Standard Model is extended by two or three singlet fermions, or right-handed (RH) neutrinos, N_i with large Majorana masses M_i. Expressed in the basis in which the Yukawa couplings for the charged leptons as well as the Majorana mass matrix are diagonal, the extended Lagrangian reads schematically

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \left(i\bar{N}_i \partial\!\!\!/ N_i - \frac{1}{2} M_i N_i^2 + h_{i\alpha} \bar{N}_i \ell_\alpha\, H + {\rm h.c.} \right). \tag{4.4}$$

Here ℓ_α indicates the lepton doublet with flavour $\alpha = (e, \mu, \tau)$ respectively, and H is the Higgs field. The effective light neutrino mass matrix m_ν is related to the RH neutrino mass matrix M through the see-saw formula,

$$m_\nu = -m_D \frac{1}{M} m_D^T, \tag{4.5}$$

where $m_D = hv$ is the Dirac neutrino mass matrix generated after spontaneous symmetry breaking by the Yukawa coupling matrix h and the Higgs vev $v = 174$ GeV.

With a Dirac mass at the electroweak scale, light neutrino masses $m_i \lesssim 1$ eV are generated, as long as the RH neutrino masses are large, typically close to the grand unified scale. The see-saw mechanism thus elegantly solves the problem of generating small neutrino masses in a natural way. The attractiveness of the model is further enhanced by the fact that heavy RH neutrinos have a natural connection with grand unification; e.g. the grand unified group SO(10) [96] predicts the existence of such fermionic singlets.

[1] Also type II (with $SU(2)$-triplet scalars) [94] and type III (with $SU(2)$-triplet fermions) [95] see-saw models exist.

The Sakharov conditions

Generating small neutrino masses is not the only virtue of the see-saw mechanism: it also contains all the necessary ingredients for baryogenesis through thermal leptogenesis, as first proposed by Fukugita and Yanagida [91]. Let us now see how the Sakharov conditions are satisfied in thermal leptogenesis.

1. The RH neutrinos violate lepton number through their decay into leptons and antileptons $N \to \ell H, \bar{\ell}\bar{H}$. In addition, sphaleron transitions, which break B and L but conserve $B-L$, can transfer lepton number into baryon number. Within the temperature range [97]
$$T_{\rm EW} \lesssim T \lesssim 10^{12} \text{ GeV}, \tag{4.6}$$
when sphalerons are in thermal equilibrium and unsuppressed, lepton number produced in neutrino decays can thus be transferred into baryon number.

2. The Yukawa matrix $h_{i\alpha}$ in the high-energy Lagrangian Eq. (4.4) is in general complex and thus contains *CP*-violating phases. Out of nine complex parameters, three can be absorbed into the lepton wave function, leaving six physical *CP*-violating phases. Consequently, the neutrinos decay with different rates into leptons and antileptons, producing a net *CP* asymmetry
$$\varepsilon_i = -\frac{\sum_\alpha \Gamma(N_i \to \ell_\alpha H) - \sum_\alpha \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})}{\sum_\alpha \Gamma(N_i \to \ell_\alpha H) + \sum_\alpha \Gamma(N_i \to \bar{\ell}_\alpha \bar{H})}. \tag{4.7}$$

3. The out-of-equilibrium condition is provided by the expansion of the universe: interaction rates slower than the Hubble rate are incapable of keeping particle distributions in equilibrium. It is convenient to define the decay parameters
$$K_i \equiv \frac{\Gamma_D(N_i \to \ell H, \bar{\ell}\bar{H})}{H(T = M_i)}, \tag{4.8}$$
so that the heavy neutrino N_i is out of thermal equilibrium for $K_i < 1$. If this is the case during the decay of the heavy neutrinos, all the Sakharov conditions are indeed fulfilled.

Consequently, if the light neutrino masses are indeed generated through the see-saw mechanism, all the necessary ingredients for baryogenesis are also provided.

4.3 Thermal leptogenesis

Assuming that the light neutrino masses are indeed generated through heavy RH neutrinos, a population of these neutrinos will be produced by scattering processes during or after reheating, as long as the reheating temperature is above the mass of the neutrinos. As the temperature drops below the neutrino mass, their equilibrium number density becomes exponentially suppressed, and the neutrinos begin to decay. Since the RH neutrino decay is *CP*-violating, an asymmetry in the number of leptons and anti-leptons is produced. In general, however, the neutrinos do not decay fast enough to maintain their equilibrium density. If this is indeed the case, some of the produced asymmetry remains, and is partly transferred into a baryon asymmetry by sphaleron transitions.

In this section, we estimate the amount of baryon asymmetry that can be generated through this mechanism of thermal leptogenesis and compare it to the observations. We consider the minimal scenario, where the mass spectrum for the heavy neutrinos is hierarchical $M_1 \ll M_2, M_3$, and the lightest neutrino N_1 is assumed to give the main contribution to the asymmetry. This approximation is well justified since neutrino mixing data seems to favor a picture where the asymmetry generated by the two heavier neutrinos is depleted before decay and can be safely ignored, even for a mild hierarchical RH neutrino masses. To further simplify the treatment, we also ignore the effects of lepton flavor. For a more detailed account of the estimate performed here, see Refs. [98, 99]. Exhaustive reviews on the implications of going beyond the simplifying assumptions made here can be found in Refs. [85, 86]

4.3.1 The produced baryon asymmetry

Given an initial N_1 abundance, there are three main factors that affect the final amount of baryon asymmetry that is produced, each corresponding roughly to one of the Sakharov conditions. These are the *CP* asymmetry, the efficiency of various interactions, as well as the effect of the sphaleron processes. Below we briefly estimate the magnitude of the effect of each of these factors on the final asymmetry.

The *CP* asymmetry

The *CP* asymmetry ε parametrizes the amount of lepton asymmetry that is generated in the RH neutrino decay. In particular, for every $1/\varepsilon$ neutrinos that decay, the asymmetry in lepton number is increased by one. The *CP* asymmetry Eq. (4.7) arises from the interference between the tree-level amplitude and the one-loop self-energy and vertex correction diagrams contributing to the neutrino decay. In a basis where the RH neutrino mass matrix is diagonal, a perturbative calculation for hierarchical neutrinos yields [100, 101]

$$\varepsilon_1 \simeq \frac{3}{16\pi} \sum_{i=2,3} \frac{\mathrm{Im}\left[(h^\dagger h)^2_{i1}\right]}{(h^\dagger h)_{11}} \frac{M_1}{M_i}. \tag{4.9}$$

Since we are ignorant of the neutrino Yukawa couplings, it is useful to write the *CP* asymmetry in the form

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_1(m_3 - m_1)}{v^2} \sin\delta_L, \tag{4.10}$$

where $\sin\delta_L$ is known as the leptogenesis phase. The remaining part of the expression reaches its maximum value when the neutrinos are fully hierarchical, implying $m_1 = 0$ and $m_3 = m_{\mathrm{atm}}$, giving the maximal *CP* asymmetry [102]

$$\varepsilon_1^{\max} = \frac{3}{16\pi} \frac{M_1 m_{\mathrm{atm}}}{v^2}, \tag{4.11}$$

where $v = 174$ GeV is the electroweak breaking scale.

At this point, let us note that the *CP* asymmetry in the processes that produce the neutrino population is very closely related to the *CP* asymmetry of the neutrino decays presented here. In particular, in the case of hierarchical neutrinos, the *CP* asymmetry in the scattering interactions by which the N_1 population is produced is equal in magnitude but opposite in sign to the *CP* asymmetry in N_1 decay. A priori, this may seem to imply that the two asymmetries cancel, making the final lepton asymmetry vanish; however, a non-zero final lepton asymmetry survives if the initial anti-asymmetry produced with the neutrino population is depleted by scattering, decays, and inverse decays. This depletion is called washout, and is crucial for the success of thermal leptogenesis.

The efficiency factor

The final lepton number depends on what amount of the initial anti-asymmetry generated in N_1 production is washed out, as well as on the final amount of N_1 neutrinos that remains present at the time of decay. The evolution of the number densities of the heavy neutrinos and the lepton number both depend on the relation between the neutrino decay rate and the Hubble expansion rate. Since these are inherently non-equilibrium processes, they are studied in terms of the Boltzmann equations.

To keep the treatment at a sufficient level of accuracy, there are four classes of processes that need to be taken into account: decays and inverse decays with$\Delta L = \pm 1$, $\Delta L = 1$ scattering processes, and $\Delta L = 2$ scattering processes mediated by the heavier neutrinos. The relevant Boltzmann equations, describing the dynamics of these non-equilibrium processes, can be written in the form [103, 104]

$$\frac{\mathrm{d}N_{N_1}}{\mathrm{d}z} = -(D+S)\,(N_{N_1} - N_{N_1}^{\mathrm{eq}}), \tag{4.12}$$

$$\frac{\mathrm{d}N_{B-L}}{\mathrm{d}z} = \varepsilon_1\,(D+S)\,(N_{N_1} - N_{N_1}^{\mathrm{eq}}) - W_1\,N_{B-L}, \tag{4.13}$$

where N_X denotes the abundance of X per RH neutrino in ultra-relativistic thermal equilibrium. Furthermore, we have introduced the notation $D \equiv \Gamma_D/(Hz)$, similarly also for S and W, where $z = M_1/T$ and H is the Hubble parameter. The D term accounts for the effect of decays and inverse decays on the neutrino abundance, while S represents the effect of$\Delta L = 1$ scatterings. Finally, W accounts for the washout due to inverse decays,$\Delta L = 1$ scatterings and the non-resonant $\Delta L = 2$ scattering processes. Note that the second equation governing the asymmetry is written in terms of $B - L$, which is unaffected by sphaleron processes.

Assuming that the asymmetry is dominantly generated by the lightest RH neutrino, which is typically the case for a hierarchical heavy neutrino spectrum [105], the solution for the final $B - L$ asymmetry reads [106]

$$N_{B-L}^{\mathrm{out}} = \varepsilon_1\,\kappa = -\varepsilon_1 \int_{z_in}^{\infty} \mathrm{d}z'\,(D+S)\,(N_{N_1} - N_{N_1}^{\mathrm{eq}})\, e^{-\int_{z'}^{\infty}\mathrm{d}z''\,W(z'')}, \tag{4.14}$$

where we have defined the efficiency factor κ. For given neutrino masses, the relevance of the various terms in the Boltzmann equations and hence the size of κ depends crucially on the value of the decay parameter K_1 defined in Eq. (4.8).

In the weak washout regime, $K_1 \ll 1$, the produced asymmetry is not considerably reduced by washout effects. However, since in this case any initially existing asymmetry is not entirely washed either, the scenario is very sensitive to initial conditions as well as thermal corrections [107]. In the strong washout regime, $K_1 \gtrsim 3$, on the other hand, a thermal neutrino abundance is produced regardless of the initial conditions, rendering the scenario far more predictive. In particular for $5 \lesssim K_1 \lesssim 100$, the efficiency factor can be approximated as [108]

$$\kappa \simeq \frac{1}{2K_1^{1.2}} \sim 0.1 - 0.01. \tag{4.15}$$

In the weak washout regime, this result can be decreased by up to two orders of magnitude [98, 99].

Which regime is most relevant depends on the values of the neutrino masses through the decay parameter. It is useful to rewrite in terms of two dimensionful parameters of the order of the light neutrino masses, so that $K_1 = \tilde{m}_1/m_*$, where $\tilde{m}_1$ is the effective neutrino mass [103]

$$\tilde{m}_1 \equiv \frac{(m_{\rm D}^\dagger m_{\rm D})_{ii}}{M_i} = 8\pi \frac{v^2}{M_1^2}\Gamma_D, \tag{4.16}$$

and $m_\star$ is the equilibrium neutrino mass

$$m_\star \equiv 8\pi \frac{v^2}{M_1^2} H(T = M_1) \simeq 10^{-3}\ \text{eV}. \tag{4.17}$$

It can be shown [109] that for most types of neutrino spectra $\tilde{m}_1 \gtrsim m_{\rm sol}$, where $m_{\rm sol}$ is given by Eq. (4.1). In this case leptogenesis occurs in the strong washout regime, and the efficiency factor is well approximated by Eq. (4.15).

The sphaleron contribution

Finally, only a part of the final lepton asymmetry is converted into a baryon asymmetry by the sphalerons, expressed through the sphaleron constant $C_{\rm sph}$. The constant can be calculated by an analysis of the chemical potentials of all particle species in the high temperature phase where sphalerons are active. The asymmetry in lepton flavor α contributes through sphaleron processes to $B/N_f - L_\alpha$, N_f being the number of flavors, which is conserved in the Standard Model. The baryon number can be related to the conserved $B - L$ through the sphaleron constant [92]

$$C_{\rm sph} = \frac{8N_f + 13N_H}{22N_f + 13N_H}, \tag{4.18}$$

where N_H the number of Higgs doublets, yielding $C_{\rm sph} = 28/79$ for the Standard Model.[2]

For simplicity we have considered in our estimate mainly the effects of processes occurring on a time scale comparable to the expansion rate of the universe, which are

[2]Here we have assumed that the sphalerons go out of equilibrium before the electroweak phase transition. If, instead, the sphalerons remain in equilibrium until after the phase transition, we would have $C_{\rm sph} = 12/37$ [110].

accounted for by the Boltzmann equations. Thus we have neglected the so-called spectator processes [111], which are not directly related to the generation of the asymmetry, but are fast and could affect it indirectly by changing the densities of the particles involved. These processes impose certain relations among the chemical potentials of various particle species, as in the case of the sphaleron mentioned above. Indeed, supposing that the sphalerons are in equilibrium during the leptogenesis process, they are responsible for transmitting the L asymmetry produced in the neutrino decays into a $B-L$ asymmetry in a non-trivial way, whereas here we have simply assumed that the asymmetry is directly produced in $B-L$ instead of L. Other such processes are the gauge interactions, Yukawa interactions involving the heavier fermions, as well as the QCD sphalerons. However, the effect of spectator processes are not expected to induce corrections larger than at most 30% [112].

4.3.2 Bounds from observations

The baryon asymmetry predicted by leptogenesis should be compared to the observed baryon-to-photon ratio η, which is measured at recombination. Putting the results of the previous section together, the value at recombination of the baryon-to-photon ratio produced in thermal leptogenesis can be expressed as

$$\eta = C_{\rm sph} \frac{N_{B-L}^{\rm out}}{N_\gamma^{\rm rec}} = C_{\rm sph} \frac{\varepsilon_1 \, \kappa}{N_\gamma^{\rm rec}}. \tag{4.19}$$

Assuming a standard thermal history, the number of photons per RH neutrino in ultra-relativistic thermal equilibrium at recombination reads

$$N_\gamma^{\rm rec} = \frac{4}{3} \frac{g_*^{\rm SM} + \frac{7}{4}}{g_*^{\rm rec}} \simeq 37. \tag{4.20}$$

For a given efficiency factor, the maximal *CP* asymmetry Eq. (4.11), corresponds to a maximal baryon asymmetry $\eta^{\rm max}$. Demanding that this maximal asymmetry is large enough to explain the observed asymmetry, $\eta^{\rm CMB} = (6.225 \pm 0.170) \times 10^{-10}$, imposes a lower bound on the RH neutrino mass

$$M_1 \gtrsim \frac{16\pi}{3} \frac{N_\gamma^{\rm rec}}{C_{\rm sph}} \frac{v^2}{\kappa} \frac{\eta^{\rm CMB}}{m_{\rm atm}} \simeq 5 \times 10^8 \ {\rm GeV} \, \kappa^{-1}. \tag{4.21}$$

The lowest possible M_1 is obtained for $\kappa = 1$, corresponding to the limit of thermal initial abundance of heavy neutrinos N_1 and zero washout. For κ evaluated at $K_1 \simeq 3.5$, on the limit to the strong washout regime, the lower bound becomes [113]

$$M_1 \gtrsim 4 \times 10^9 \, {\rm GeV}. \tag{4.22}$$

The lower bound on M_1 also translates into a lower bound on the initial temperature of leptogenesis. Since we are assuming thermal production of the N_1 neutrinos, the minimal initial temperature that allows for successful leptogenesis can be identified with a lower bound on the reheating temperature $T_{\rm RH}$ after inflation. In the weak washout regime, the bound on the neutrino mass roughly coincides with the minimal reheating temperature, whereas in the strong washout regime the minimal temperature can be up to one order of magnitude smaller [99, 107, 86]. In either case, there is a lower bound on the reheating temperature of the order

$$T_{\rm RH} \gtrsim 10^8 - 10^9 \text{ GeV}. \tag{4.23}$$

4.4 Leptogenesis and supersymmetry

Unlike many of the baryogenesis models proposed in the literature, which in one way or another rely on supersymmetry to produce a sufficient baryon asymmetry, thermal leptogenesis is a perfectly viable scenario in the absence of supersymmetry. In fact, it even appears to be more viable when supersymmetry is not present. This is not because supersymmetry would drastically change the predictions of leptogenesis, but due to the strict upper bound on the reheating temperature in supersymmetric theories. Nevertheless, it is useful to explore the option of supersymmetric leptogenesis. Not only because supersymmetry is a very plausible extension to the Standard Model, but also to alleviate the hierarchy problem that arises through the introduction of the see-saw mass scale. Let us take a brief look at how supersymmetry affects the scenario.

4.4.1 Modifications to the standard scenario

The minimal supersymmetric leptogenesis model consists of the MSSM with three added heavy neutrinos, which are complemented by their superpartners, the sneutrinos $\tilde{N}_i$. The leptonic part of the MSSM superpotential Eq. (3.4) is then extended to read

$$W = \frac{1}{2} M_i N_i N_i + h_{i\alpha} N_i L_\alpha H_u - h_e \bar{e} L H_d, \tag{4.24}$$

where N_i is now a singlet neutrino superfield, containing the RH neutrino and sneutrino. Although the addition of the sneutrinos brings new structure in the supersymmetric version of leptogenesis, quantitatively the scenario turns out to be very similar to the standard case.

For the *CP* asymmetry, the addition of the superpartners brings new contributing diagrams. Firstly, there is the addition from the decay of the heavy neutrinos into sleptons, practically doubling the number of loop diagrams, which in the hierarchical limit amounts to an increase of the *CP* asymmetry approximately by a factor of two. Furthermore, there are additional sources coming from the decay of sneutrinos into leptons and sleptons. In general also the soft supersymmetry breaking terms give relevant contributions, but for thermal leptogenesis they amount only to small corrections and can be neglected. In this case, the heavy neutrinos and sneutrinos have equal masses and decay rates, leading to equal *CP*-asymmetries. In the hierarchical limit, the total *CP* asymmetry including both the neutrino and sneutrino contribution can thus be written as [101]

$$\varepsilon_1 \simeq -\frac{3}{4\pi} \sum_{i=2,3} \frac{\text{Im}\left[(h^\dagger h)^2_{i1}\right]}{(h^\dagger h)_{11}} \frac{M_1}{M_i}. \tag{4.25}$$

Comparing this result to Eq. (4.9), we see that the new decay channels in the MSSM yield an enhancement of the *CP* asymmetry by a factor of four.

However, there are also changes in the other factors affecting the final asymmetry [85]. Due to the additional slepton final states, the decay rate of N_1 is twice faster than in the standard scenario. In the strong washout regime, the associated inverse decay reactions enhance the washout by a factor of two. In the weak washout regime, on the other hand, the neutrino production becomes more efficient, increasing the asymmetry by the same factor. Furthermore, since the expansion rate of the universe is proportional to $\sqrt{g_*}$, it is roughly faster by a factor of $\sqrt{2}$ in the supersymmetric version. This reduces the time during which the strong washout processes can erase the asymmetry, enhancing the final asymmetry by this same factor. In the weak washout regime, on the other hand, it reduces the time for neutrino production, yielding a final suppression of $\sqrt{2}$.

Finally, since in the MSSM there are two Higgses, the coefficient $C_{\rm sph}$ is altered from its value in the SM to 8/23, but remains of similar size. Altogether, the modifications presented here amount only to an overall change in the final asymmetry by approximately a factor of two. This implies that the constraints on M_1 and $T_{\rm RH}$, Eqs. (4.21) and (4.23) respectively, derived in the previous section remain essentially unchanged [107].

4.4.2 On the gravitino bound

In the supersymmetric scenario, the lower bound Eq. (4.23) on the reheating temperature is in conflict with the upper bound Eq. (3.22), which is necessary to avoid the overproduction of gravitinos during reheating. Practically, the severe upper bound makes the thermal generation of heavy neutrinos impossible, rendering the supersymmetric scenario with a hierarchical RH neutrino spectrum unviable. This is a serious drawback for the thermal leptogenesis scenario, even if several modifications leading out of this dilemma have been proposed in the literature.

To avoid this conflict, one can firstly modify the assumptions on the nature of the gravitino. If the gravitino is stable, the bound from nucleosynthesis depends on the nature of the next-to-lightest supersymmetric particle, see Eq. (3.23), and the bound can be relaxed even up to 10^9 GeV [74]. If, in addition, one assumes that there is a small violation of R-parity, the NLSP can decay before the onset of nucleosynthesis, thus evading the bound on the reheating temperature entirely. The stringent limit on the reheating temperature can be also be avoided by relaxing the assumptions on the gravitino mass. This is the case, for example, for gravitinos lighter than 1 keV, which may occur in gauge mediated supersymmetry breaking, as well as for gravitinos heavier than about 50 TeV, which can arise through anomaly mediation [114].

Alternatively, some properties of the neutrino spectrum can be modified. Soft leptogenesis [115] is a supersymmetric scenario, where the violation of lepton number as well as the required *CP* violation are sourced by the soft supersymmetry breaking terms A and b, see Eq. (3.8), respectively associated with the Yukawa coupling and mass of the lightest RH neutrino. The lower bound on the reheating temperature in this scenario, which requires only one heavy neutrino, can be as low as 10^6 GeV [107]. In resonant leptogenesis [100, 116], on the other hand, the RH neutrinos are nearly degenerate in mass and the self-energy contributions to the CP asymmetries are enhanced; consequently, the correct baryon asymmetry can be produced even at temperatures as low as a TeV. Another interesting variation is the scenario where the right-handed sneutrino develops a large amplitude, dominating the total energy density [117]. In this case the sneutrino decay reheats the universe, producing a lepton asymmetry, and values of the reheating temperature even as low as 10^6 GeV avoid the gravitino problem.

Finally, one can give up the assumption of thermal production of RH neutrinos. The lightest RH neutrino can be produced non-thermally for example during preheating [118], or from the decay of the inflaton [119], relaxing the lower bound on the reheating temperature by up to two orders of magnitude compared to Eq. (4.23).

4.5 Leptogenesis with flat directions

The solutions to the tension between the minimal supersymmetric leptogenesis scenario and the gravitino bound presented above, all rely on modifying the theory by changing the properties or interactions of some of the fields involved. In the enclosed research paper [1] we present another possible solution to the problem, which does not require the modification of any component. To the contrary, this solution is already incorporated in the model, due to the presence of flat directions in the supersymmetric scalar potential. In the previous chapter, we mentioned how, after inflation when the flat direction oscillates around the new minimum of its potential, particle production can occur. Following Ref. [1], we here investigate the possibility of producing the heavy neutrino population through this mechanism, and estimate under which conditions such a scenario can lead to successful baryogenesis.

4.5.1 Choosing the setup

In order for particle production to take place, the oscillating flat direction condensate must pass very close to the origin. In general this is not the case, but the flat direction spirals around the origin, possibly leading to a large baryon asymmetry through the Affleck-Dine mechanism [90, 80]; however, we will not consider that possibility further here. Instead, following the discussion at the very end of the previous chapter, we are lead to consider flat directions along which the induced A-terms of Eq. (3.32) are suppressed. Examples of such directions are the $n = 4$ direction ue as well as the $n = 9$ direction Que [77], both of which are lifted by non-renormalizable superpotential terms of the form of Eq. (3.28).

The most efficient non-adiabatic production of heavy states occurs through the process analogous to instant preheating [39], which is thus the mechanism we focus on here. When the flat direction passes by the origin, non-adiabatic production of particles coupled to the flat direction occurs. As discussed in Section 2.3, when the condensate then continues its oscillation, the produced particles become massive and may efficiently decay into other massive states.

Our aim is to estimate the amount of right-handed N_1 neutrinos that can be produced in the final state, from the non-perturbative decay of a flat direction ϕ into some intermediate state: $\phi \to X \to N_1$. We choose to focus on the scalar Higgs H_u, which couples to N_1 via the superpotential term $h_{1\alpha} N_1 L_\alpha H_u$, although other states will be produced as well. In order to guarantee a sufficient H_u production, we will furthermore focus on flat directions involving the third generation quark u_3.

4.5.2 N_1 production through instant preheating

With our choice of flat directions, the up-Higgs is coupled to the condensate through the Lagrangian term $h_t^2 |\phi|^2 |H_u|^2$. Consequently, the Higgs has an effective mass

$$m_{H_u}^2 = \widetilde{m}_{H_u}^2 + h_t^2 |\phi|^2 , \tag{4.26}$$

where $\widetilde{m}_{H_u}^2$ is the corresponding soft-breaking mass parameter. When the flat direction condensate first passes through the origin, particle production takes place in the region where adiabaticity is violated [39]. Following Eq. (2.33), production of up-Higgses with the mass Eq. (4.26), is thus efficient within the region

$$|\phi| \lesssim |\phi_*| = \left(\frac{m|\phi_0|}{h_t}\right)^{1/2} , \tag{4.27}$$

where m is the soft mass of the flat direction, and $|\phi_0|$ its initial amplitude. The number density of up-Higgses that is produced in this interval can be estimated to

$$n_{H_u} \simeq \frac{(h_t m|\phi_0|)^{3/2}}{8\pi^3}, \tag{4.28}$$

where we have assumed that the bare mass of the up-Higgs $\widetilde{m}_{H_u}^2$ is negligible compared to the average momentum of the produced particles

Once the flat direction condensate has passed through the origin on its first oscillation, the produced up-Higgses become heavier and heavier, as the condensate continues its oscillation up the other side of its potential. When the effective up-Higgs mass Eq. (4.26) becomes larger than the mass M_1 of the lightest RH neutrino, the up-Higgses can promptly decay into N_1 neutrinos through the superpotential coupling $h_{i\alpha} N_i L_\alpha H_u$. In particular, if at least one of the Yukawa couplings $h_{i\alpha}$ is not very small and the field Q_3 is not included in the flat direction, most of the up-Higgs's decay channels are blocked by the large effective masses induced by the flat direction at large ϕ, and H_u will decay dominantly into N_1 neutrinos and leptons.

Let us estimate the maximal N_1 mass $M_1^{\max} \simeq h_t \phi^{\max}$ that can be generated through this production process. To this end we need to determine the maximum value $\phi^{\max}$ that the condensate can reach during its first oscillation, after its passage through the origin. The equation of motion for ϕ takes the form

$$\ddot{\phi} + m^2 \phi = -h_t \frac{|\phi|}{\phi} n_{H_u}, \tag{4.29}$$

where the term on the right-hand side corresponds to the ϕ-dependent energy density generated by the produced H_u particles when ϕ crosses the origin. It acts as a friction term damping the ϕ oscillations. Solving the equation of motion Eq. (4.29), we obtain an estimate for the maximal mass

$$M_1^{\max} \simeq \frac{4\pi^3 \sqrt{m\phi_0}}{h_t^{3/2}} = 4 \times 10^{12}\ \text{GeV} \left(\frac{\phi_0}{M_P}\right)^{1/2} \left(\frac{m}{100\ \text{GeV}}\right)^{1/2} , \tag{4.30}$$

where the numerical estimate assumes a top-Yukawa coupling $h_t \simeq 0.6$ at high energy scales. As expected, the production mechanism of instant preheating can indeed lead to very heavy RH neutrinos.

4.5.3 The baryon asymmetry

After reaching its maximum value at the first oscillation, the flat direction vev starts decreasing again, with the consequence that also the mass of the up-Higgs Eq. (4.26) decreases. Eventually the up-Higgs mass becomes small enough that the RH neutrinos produced at the top of the flat direction oscillation may efficiently decay back into up-Higgses and leptons. As in the standard thermal leptogenesis scenario, the heavy neutrino decay produces a lepton asymmetry, dictated by the *CP* asymmetry ε, which can later be partly transformed into a baryon asymmetry by sphaleron processes.

Here we evaluate the maximum amount of baryon asymmetry that can be produced in this way. To correspond with the notation of the original article [1], we use the alternative expression for the baryon asymmetry $Y_B = (8.75 \pm 0.23) \times 10^{-11}$, which is related to η as follows

$$Y_B = \frac{n_B}{s} = \frac{n_\gamma}{2\pi^2 g_* T^3/45}\eta \simeq \frac{\eta}{7.04}. \tag{4.31}$$

This definition, which is normalized with respect to the entropy density s, is useful in that it remains constant as long as the number of degrees of freedom in the plasma are unchanged.

As a first approximation, we assume that all the up-Higgses decay into N_1 neutrinos at the top of the first oscillation. We then estimate that the decay of the RH neutrinos into up-Higgses and leptons produces a lepton asymmetry

$$n_L \simeq \varepsilon\, n_{N_1} \simeq \varepsilon\, \frac{(h_t m|\phi_0|)^{3/2}}{8\pi^3}. \tag{4.32}$$

As soon as the RH neutrinos decay, their energy density $\rho_{N_1} = M_1 n_{N_1}$ gets converted into a "thermal" bath with an effective temperature $T^4 \sim (30\rho_{N_1})/(g_*\pi^2)$, where g_* is the corresponding number of relativistic degrees of freedom. We estimate that T is smaller than M_1 for

$$M_1 > 10^9 \text{ GeV} \left(\frac{|\phi_0|}{M_p}\right)^{1/2} \left(\frac{m}{100\,\text{GeV}}\right)^{1/2}, \tag{4.33}$$

implying that no washout factor should appear in the estimate of the final lepton asymmetry Eq. (4.32): since the mass of the RH neutrinos generated during the preheating stage is expected to be much larger than this lower bound,$\Delta\ L = 1$ inverse decays are not taking place. Similarly, it can be argued that the$\Delta\ L = 2$ processes are out of equilibrium. Furthermore, since only processes with leptons in the final state can generate an asymmetry, there is no cancellation between the asymmetry produced in the N_1 decays and the possible asymmetry produced in the N_1 production through inverse decays. Finally, flavor effects are not expected to be important in determining the final baryon asymmetry, since the$\Delta\ L = 1$ inverse decays are out of equilibrium.

During the non-perturbative decay of the flat direction, the inflaton field continues to oscillate around the minimum of its potential, until eventually reheating takes over and the inflaton decays into relativistic degrees of freedom. To be able to compare the baryon density to the relativistic entropy density, we need to determine the baryon asymmetry at the time of reheating, which after sphaleron conversion reads

$$Y_B = C_{\rm sph}\, \frac{n_L}{s}\, \frac{H^2_{\rm RH}}{H_{\rm osc}} \simeq \frac{8}{23}\varepsilon\, \frac{n_{N_1}}{s}\, \frac{H^2_{\rm RH}}{H^2_{\rm osc}}. \tag{4.34}$$

Here the sphaleron constant is given by Eq. (4.18) with $N_H = 2$, and the last factor accounts for the dilution of the asymmetry due to the expansion of the universe between the time of neutrino production until the beginning of reheating. At the time of the first flat direction oscillation, when the lepton asymmetry is produced, the Hubble constant reads $H_{\rm osc} \simeq m/3$, see Eq. (3.32), whereas the value at reheating $H_{\rm RH}$ can be inferred from Eq. (2.29). Expressing the neutrino number density as in Eq. (4.32), we finally reach an estimate for the baryon asymmetry

$$Y_B \simeq \frac{9\;\varepsilon\; h_t^{3/2}\, T_{\rm RH}\, |\phi_0|^{3/2}}{92\pi^3 m^{1/2} M_P^2} = 10^{-6}\,\varepsilon \left(\frac{T_{\rm RH}}{10^7 \text{ GeV}}\right) \left(\frac{|\phi_0|}{M_P}\right)^{3/2} \left(\frac{100 \text{ GeV}}{m}\right)^{1/2}. \tag{4.35}$$

In the supersymmetric case, the maximal *CP* asymmetry for normal hierarchical light neutrinos is given by Eqs. (4.25) and (4.11), where $m_{\rm atm} \simeq 0.05$ eV. From Eq. (4.35) we therefore estimate that a sufficient baryon asymmetry is generated if the mass M_1 of the lightest RH neutrino satisfies the bound

$$M_1 \gtrsim 2 \times 10^{11} \text{ GeV} \left(\frac{10^7 \text{ GeV}}{T_{\rm RH}}\right) \left(\frac{M_P}{|\phi_0|}\right)^{3/2} \left(\frac{m}{100 \text{ GeV}}\right)^{1/2}. \tag{4.36}$$

This lower bound on the neutrino mass together with the estimated maximal mass in Eq. (4.30), implies that successful baryogenesis can occur only for

$$\phi_0 \gtrsim 0.2\, M_P \left(\frac{10^7 \text{ GeV}}{T_{\rm RH}}\right)^{1/2}. \tag{4.37}$$

In other words, the flat direction condensate has to start its oscillation from field values close to the Planck scale. Although this limit on ϕ_0 is independent of h_t, the presence of the top Yukawa coupling is crucial to guarantee that the flat direction decays abundantly into up-Higgses H_u.

4.5.4 Discussion and conclusions

To conclude, we have shown that the observed baryon asymmetry can be explained within the supersymmetric leptogenesis scenario for low reheating temperatures and a RH hierarchical mass spectrum, thus avoiding the gravitino bound. Although the proposed solution may be considered as a non-thermal production of RH neutrinos, the main result is that it does not involve any extra assumption, such as a large coupling between the RH neutrinos and the inflaton field.

However, we found that the solution requires two conditions to be met: the initial value of the flat direction must be close to the Planck scale, and the phase-dependent terms in the flat direction potential must either vanish, or be sufficiently small for particle production to be efficient. The mechanism might, nevertheless, work also in models with smaller values of M_1, since the baryon asymmetry could be generated by the decays of the heavier RH neutrinos. The up-Higgs could decay into the RH neutrino N_2 (or N_3) instead of into the lightest RH neutrino N_1, if the condensate reaches the value $\phi = M_2/h_t$ before the up-Higgs decays into N_1 neutrinos. This condition can be satisfied if the Yukawa couplings $h_{i\alpha}$ are hierarchical and $|h_{1\alpha}| \ll 1$. If this is the case, one should replace M_1 with M_2 (or M_3) in Eqs. (4.33) and (4.36).

Moreover, in hybrid inflation, the flat direction might couple differently to the slow-rolling inflaton and to the field responsible for the end of inflation. In this case, the curvature of the flat direction potential might change its sign promptly at the end of the slow-roll period. The flat direction would then oscillate in its potential with a frequency of the order of the inflationary Hubble scale [120], with the consequence that a much larger number of up-Higgs quanta could be produced, relaxing the bound on ϕ_0.

Chapter 5

Inflation with supersymmetric flat directions

In Chapter 2 we have presented the virtues of the inflationary paradigm, and explained how it can be realized in terms of a scalar field. Furthermore, we discussed observational constraints on the form of the inflaton potential. From a theoretical point of view, however, a satisfactory model of inflation cannot be limited to specifying the inflationary potential, but should address the relation of the inflaton to the Standard Model, or at least to other physics expected to lie there beyond. In this chapter we will address this question further.

Our main focus will be on a particular inflationary model, in which the inflaton is a flat direction of the MSSM [121]. However, we begin the chapter by a general overview of the problems related to identifying an inflaton in high-energy physics. In Section 5.2 we then present the MSSM inflation model, and review its construction and inflationary predictions. We also discuss some of its major drawbacks, mainly the large fine-tuning required to ensure the flatness of the inflationary potential. In the final section, we present a supergravity embedding of the MSSM inflationary model, which was proposed in the included research paper [2], in order to alleviate the fine-tuning problem.

5.1 Embedding inflation into particle physics

The most apparent problem in connecting *scalar* field inflation to known physics is perhaps just that — it requires a scalar field, yet to date none has ever been observed. Moreover, the Standard Model Higgs has not proved its worth as a suitable inflaton candidate, despite courageous attempts [122]. Inflationary model builders are thus directed to consider higher energies, beyond the Standard Model in their quest for a scalar field inflaton. In this regard supersymmetry with the large number of scalar fields it brings about appears a particularly fruitful framework.

Another potential difficulty is the very flat inflaton potential that is necessary for slow-roll inflation to produce a sufficient number of e-folds. Even from a phenomenological perspective, where the main challenge is in constructing internally consistent slow-roll potentials, fine-tuning of parameters is sometimes required. When inflation is considered in a particle physics framework, however, one furthermore has to ensure that the potential remains flat also under radiative corrections. Supersymmetry provides a useful framework also in this respect, since one of the main motivations for supersymmetry beyond the Standard Model is the cancellation of quadratic divergencies that it provides for scalar masses. In addition, the multitude of flat directions present in the supersymmetric scalar field space provide ample inflaton candidates with nearly flat potentials, when lifted by non-renormalizable terms or quantum effects. In light of all this, it seems there should be no obstacles to identifying inflationary models within supersymmetry. Indeed, viable inflationary theories, mainly of the polynomial or hybrid type, can be constructed with fairly simple choices of super- and Kähler potentials [53]. There is, nevertheless, a caveat to all of this: at the fundamental level supersymmetry is expected to be local, and hence supergravity effects must be taken into account.

Inflation in supersymmetric theories

We have seen in Chapter 3 that the supersymmetric scalar potential can be written as the sum of two contributions, the F-term and the D-term. In most supersymmetric inflation models, the potential is dominated by the F-term. These models, however, are plagued by what is called the η-problem. As discussed in Section 3.3, supergravity generically induces a mass for scalar fields during inflation of the order of H, see Eq. (3.32). For the inflaton itself, this mass renders $\eta \sim 1$, in conflict with the slow-roll requirement. In order to avoid the problem, one can invoke non-generic forms of the super- and Kähler potentials, which guarantee that the mass becomes suppressed, or simply suppose that the term is accidentally suppressed. Alternatively, one can rely on quantum corrections to drive the mass to a small value. A different strategy is to suppose that the D-term dominates [83], but also these models suffer from difficulties. While the supergravity correction to the inflaton mass is absent in D-term inflation, the inflaton field value is required to be at least of the order of M_P, making the model susceptible to large contributions from non-renormalizable terms. More details on the subjects mentioned here, and an extensive list of references can be found in Refs. [48, 53].

At the end of the day, we seem to be faced with the limitations of the supersymmetric framework. While it may provide the scalar field inflaton with a raison d'être, it fails to yield a complete field theory picture. Most of the supersymmetric models of inflation proposed to date require high enough energies that the connection to low-energy supersymmetry, motivated by the Standard Model remains at most elusive. Nevertheless, without providing any guidance, the models require us to make educated guesses about their specific properties in the regime where non-renormalizable quantum effects become important. For a more rigid top-down approach to inflationary model building, one may thus want to turn to string theory.

In string theory, there is no lack of potential inflaton candidates, as the various parameters describing the background geometries and particular compactifications, the moduli, are all scalar fields. With the added amount of freedom that comes from fluxes, torsion and possible non-perturbative effects, one might expect viable inflationary potentials in string theory to be abundant. However, identifying an inflaton within the moduli sector has proved surprisingly difficult. Due to some groundbreaking work, such as the KKLT scenario [123], where the joint contribution of non-perturbative effects and an explicit supersymmetry breaking term induced by anti-D3 branes allows for de-Sitter vacua with all but one degree of freedom stabilized, some viable examples of modular inflation have recently been constructed [124]. Moreover, a more general understanding of the conditions necessary for realizing slow-roll inflation in the moduli sector of string models has been achieved [125]. Nevertheless, any inflationary model in string theory as of yet comes with the price of high intricacy, and the lack of a well-formulated Standard Model sector of particle physics.

In this respect, the inflationary model recently proposed by Allahverdi et al., where inflation is driven by a flat direction of the MSSM [121], provides a welcome contrast. Both the inflationary period and the generation of the Standard Model fields during reheating are explained in a self-consistent manner by the physics of the flat direction. Furthermore, the inflaton couplings to Standard Model particles are known and, at least in principle, measurable in laboratory experiments such as the LHC or a future Linear Collider; even the inflaton mass is directly related to the slepton or squark masses and can thus be probed in the laboratory [126].

5.2 MSSM inflation

Here we review the construction as well as inflationary parameters of the MSSM inflation model, following closely the original references [121, 127]. While the setting of this model into the MSSM provides its many virtues, it also gives rise to some major drawbacks. The exceptionally low scale of inflation requires the inflaton potential to be extremely flat, with the usual consequence of fine-tuning problems. We also briefly discuss this issue here, before attempting to address it in Section 5.3.

5.2.1 The potential

Assuming that only one flat directions is excited, and that any other fields are relaxed at the origin, phenomenologically viable slow roll inflation can arise along the flat directions $LL\bar{e}$ and $\bar{u}\bar{d}\bar{d}$. Both directions are of dimension six, and are lifted by a superpotential term of the form $W = \lambda\phi^6/6M^3$. Assuming hidden sector F-term supersymmetry breaking, the leading order potential along these directions is then given by Eq. (3.31) and reads

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + \cos(6\theta + \theta_A)\frac{|A\lambda|}{6M^3}|\phi|^6 + \frac{|\lambda|^2}{M^6}|\phi|^{10}. \tag{5.1}$$

While both the first and the last term in the potential are positive definite, the A-term will give a negative contribution whenever $\cos(n\theta + \theta_A) < 0$. Choosing the phase such that the A-term is minimized along the angular direction: $\cos(n\theta + \theta_A) = -1$, one obtains, for the radial direction, the inflationary potential of Ref. [121]

$$V(|\phi|) = \frac{1}{2}m^2|\phi|^2 - \frac{|A\lambda|}{6M^3}|\phi|^6 + \frac{|\lambda|^2}{M^6}|\phi|^{10}. \tag{5.2}$$

As such, this potential is generically not flat enough to yield a successful period of inflation. It is noticed, however, that the potential has a secondary minimum at

$$|\phi_0| = \left(\frac{mM_P^3}{\sqrt{10}|\lambda|}\right)^{1/4}, \tag{5.3}$$

provided that $A^2 \geq 40m^2$. Furthermore, this minimum becomes a saddle point if the soft parameters satisfy the condition [121, 127]

$$A^2 = 40m^2. \tag{5.4}$$

In this case $V'(\phi_0) = V''(\phi_0) = 0$ and the potential in the vicinity of the saddle point is flat enough to support inflation. Provided that the flat direction is initially close to the saddle point ϕ_0, with $\dot{\phi} \simeq 0$, and that the energy density of the flat direction dominates the universe, a sufficient period of slow roll inflation ensues.

5.2.2 Inflationary parameters and predicitions

The energy scale of MSSM inflation effectively depends on the soft mass of the flat direction and on the coupling λ, through the value of the inflaton at the saddle point. For the phenomenologically motivated value of the soft mass $m \simeq 1$ TeV, and taking $|\lambda| \simeq 1$, the inflaton field value at the saddle point becomes

$$|\phi_0| \simeq 3 \times 10^{14} \text{ GeV}, \tag{5.5}$$

which is considerably lower than typical field values in single-field inflation. The corresponding energy scale, which can be approximated by the potential Eq. (5.2) evaluated at the saddle point, reads

$$V_{\text{inf}} \simeq V(|\phi_0|) = \frac{4}{15}m^2|\phi_0|^2 \sim 10^{34} \text{ (GeV)}^4. \tag{5.6}$$

This corresponds to the inflationary Hubble scale [121, 127]

$$H_{\text{inf}} \simeq \left(\frac{V_{\text{inf}}}{3M_P^2}\right)^{1/2} \simeq 0.1 \text{ GeV}. \tag{5.7}$$

Although the Hubble scale is very low compared to typical slow roll models, the inflationary predictions turn out to be very similar. Moreover, thanks to the correspondingly low value of the inflaton vev, the problem of Planckian field values, which causes a challenge for many conventional models, is entirely avoided here.

Slow-roll parameters and number of e-folds

The number of e-folds during the inflationary period, as well as the expressions for the slow-roll parameters, can be determined from the expansion of the inflationary potential Eq. (5.2) around the saddle point

$$V(|\phi|) \simeq V(|\phi_0|) + \frac{1}{6}V'''(|\phi_0|)(|\phi| - |\phi_0|)^3 = V(|\phi_0|) + \frac{16}{3}\frac{m^2}{|\phi_0|}(|\phi| - |\phi_0|)^3. \tag{5.8}$$

The slow roll parameters, Eqs. (2.24) and (2.25), thus take the form

$$\epsilon \simeq 1800\frac{M_P^2}{|\phi_0|^2}\left(\frac{|\phi|-|\phi_0|}{|\phi_0|}\right)^4, \qquad (5.9)$$

$$\eta \simeq 120\frac{M_P^2}{|\phi_0|^2}\left(\frac{|\phi|-|\phi_0|}{|\phi_0|}\right). \qquad (5.10)$$

The slow-roll condition Eq. (2.26) becomes violated when $|\eta| \simeq 1$, at the field value

$$|\phi| \simeq |\phi_0| - \frac{|\phi_0|^3}{120M_P^2} \equiv |\phi_{\rm end}|, \qquad (5.11)$$

which defines the end of inflation.

The number of e-folds of inflation Eq. (2.48) calculated from some field value $|\phi|$ during inflation then reads

$$N(|\phi|) = \int_{|\phi|}^{|\phi_{\rm end}|}\frac{H}{|\dot{\phi}|}{\rm d}|\phi| \simeq \frac{|\phi_0|^3}{60M_P^2(|\phi_0|-|\phi|)}. \qquad (5.12)$$

When $|\phi|$ is very close to the saddle point $|\phi_0|$, quantum fluctuations are stronger than the classical motion, and the field is in the regime of eternal inflation [128]. The classical force begins to dominate when $H^2 \simeq |\dot{\phi}|$, which corresponds to the field value

$$|\phi| \simeq |\phi_0| - \left(\frac{m}{M_P}\right)^{1/2}\frac{|\phi_0|^2}{M_P}. \qquad (5.13)$$

Taking this as the initial inflaton field value, we obtain an estimate for the total amount of e-folds during inflation

$$N_{\rm tot} \simeq \frac{|\phi_0|}{60(mM_P)^{1/2}} \sim 10^3. \qquad (5.14)$$

The numerical estimate, which corresponds to the same values for the soft parameters as above, is sufficient to explain the observations of flatness, homogeneity and absence of topological relics. Due to the low scale of inflation, the number e-folds N_* since the largest currently visible scales exited the horizon will be much lower than the typical 60 or so [129]. If the inflaton decays immediately after the end of inflation, we obtain [121, 127]

$$N_* \sim 50, \qquad (5.15)$$

or slightly under, depending on the energy scale of inflation.

Amplitude and spectral index

For the observed amplitude of perturbations to be produced with such a low scale of inflation, the potential must be extremely flat, which is indeed the case here. With the given potential, the amplitude of the curvature perturbations Eq. (2.50), evaluated at the horizon crossing of the cosmic scales reads

$$\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \frac{1}{2\pi}\frac{H_{\rm inf}^2}{|\dot{\phi}_*|} \simeq \frac{4\sqrt{5}}{\pi}N_*^2\frac{mM_P}{|\phi_0|^2} \sim 10^{-5}, \qquad (5.16)$$

and the numerical estimate is again performed for $m \sim 1$ TeV and $|\phi_0| \sim 3 \times 10^{14}$ GeV. With a slight tuning of the parameters m and λ, the exact observational value can easily be obtained. That this is indeed possible for phenomenologically viable values of the soft parameters is a non-trivial result. In fact this is what singles out the $n = 6$ flat directions from higher order ones as the only feasible inflaton candidates [127]. For lower order flat directions ($n \leq 5$), on the other hand, no A-term arises in the potential, which obviously makes them unsuitable. Furthermore, the spectral index of the curvature perturbation reads

$$n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{4}{N_*} \sim 0.92, \tag{5.17}$$

with a running of -0.002. Due to the low inflationary scale, no observable tensor perturbations are produced. In the absence of tensor modes, the scalar spectral index deviates with about 3σ from the observed value $n_s = 0.96$ [4]. The tilt can, nevertheless, be enhanced to fit the observational value by tuning the effective coupling λ to be smaller than 1 . Moreover, slight deviations from the saddle point condition Eq. (5.4), to the extent that the success of the model allows for, can modify the spectral index up to $n_s \simeq 1$ [130, 127].

5.2.3 The problem of fine-tuning

The MSSM inflationary model is unique in providing phenomenologically viable inflation within the MSSM, without the inclusion of any extra gauge singlets. It is important to point out, however, that the success of the model depends crucially on the saddle point condition Eq. (5.4) being fulfilled. Because of the low scale of inflation, the condition must be satisfied to an accuracy of about 10^{-18}, in order to produce the observed amplitude for the curvature perturbations [130, 127, 2, 131]. In other words, the MSSM inflationary scenario seems to require an unsatisfactory fine-tuning of the soft supersymmetry breaking parameters in the inflaton potential to the accuracy of 10^{-9}. Since the soft parameters are determined by the supersymmetry breaking mechanism, however, it is reasonable to ask whether the saddle point condition could simply be a consequence of the realisation of a particular supersymmetry breaking scheme. This issue has been investigated in the enclosed research paper [2], and will be discussed in more detail in the following section.

Another problem within this inflationary model, equally due to the exceptionally low inflationary scale, is the required fine-tuning of the inflaton initial conditions. The slow roll conditions are satisfied only in the immediate vicinity of the saddle point, and the initial value of the inflaton must thus be set very close to the saddle point in order to obtain a sufficiently long period of inflation. Also in this question, a rigorous solution must involve physics at higher energies, which govern the stage before the onset of the inflationary period. For example, the initial conditions might be connected to the details of the supersymmetry breaking mechanism. The problem has also been addressed within the string theory landscape [132], where the MSSM inflation could occur as a last stage of a chain of inflationary periods driven by energy densities of several false vacua, but the issue remains unresolved.

5.3 A supergravity origin for MSSM inflation

Since in the MSSM the soft supersymmetry breaking parameters are put in by hand, there can in that context be no explanation for the saddle point condition Eq. (5.4), other than simple fine-tuning. It is nevertheless clear that the relation reflects physics that is beyond the MSSM, in particular the mechanism of supersymmetry breaking. In gravity mediated supersymmetry breaking, where the soft parameters are usually expected to be of similar order of magnitude, such a relation might arise naturally. In the enclosed research paper [2], we study what constraints the saddle point condition places on the supergravity model, and identify a class of hidden sector supergravity models, in which the saddle point condition is indeed identically satisfied.

5.3.1 Identifying the model

We work in the hidden sector picture and consider F-term supersymmetry breaking, since D-flat directions remain so also in supergravity. For the dimension six flat directions that we are considering as the inflaton, the superpotential to lowest order reads

$$W = \hat{W} + I = \hat{W}(h_a) + \frac{\hat{\lambda}(h_a)}{6}|\phi|^6, \tag{5.18}$$

where h_a refers to the hidden sector fields and the hat denotes quantities that are functions of the hidden sector fields only, cf. Eqs (3.15) and (3.27) in Chapter 3. The superpotential may also contain any higher order terms allowed by symmetries, but these will not affect the analysis and are therefore suppressed. In order to proceed we assume a generic perturbative form for the Kähler potential

$$K = \hat{K}(h_a, h_a^*) + \hat{Z}_2(h_a, h_a^*)\frac{|\phi|^2}{M_P^2} + \hat{Z}_4(h_a, h_a^*)|\frac{|\phi|^4}{M_P^4} + \ldots \tag{5.19}$$

In Eqs. (3.16) - (3.19), the scalar potential in hidden sector supergravity is expressed in terms of the superpotential W and the Kähler potential K. For the form of the functions given here, the lowest order terms correspond to the inflaton potential Eq. (5.1) with the soft parameters

$$\begin{aligned}
m^2 &= 2M_P^2 e^{\hat{G}}\left(\hat{Z}_2(\hat{G}^a\hat{G}_a - 2) + \hat{G}_a\hat{G}_{\bar{a}}(\hat{Z}_2^a\hat{Z}_2^{\bar{b}}\hat{Z}_2^{-1} - \hat{Z}_2^{a\bar{b}})\right), && (5.20)\\
|A| &= 2M_P e^{\hat{G}/2}\hat{Z}_2^{1/2}\left|\hat{G}^a(\hat{K}_a + \hat{\lambda}_a\hat{\lambda}^{-1} - 6\hat{Z}_{\bar{a}}\hat{Z}_2^{-1}) + 3\right|, && (5.21)\\
|\lambda| &= e^{\hat{K}/2}|\hat{\lambda}|\hat{Z}_2^{-1/2}, && (5.22)\\
\theta_A &= \arg\left(\hat{G}^a(\hat{K}_a + \hat{\lambda}_a\hat{\lambda}^{-1} - 6\hat{Z}_{\bar{a}}\hat{Z}_2^{-1}) + 3\right) + \arg(\hat{W}*) + \arg(\hat{\lambda}). && (5.23)
\end{aligned}$$

Recall that lower indices indicate derivatives, in particular indices a, b refer to derivates with respect to the hidden sector fields. The lower case indices are raised with the inverse of the hidden sector Kähler metric $\hat{K}^{a\bar{b}}$.

The saddle point condition

The saddle point condition $A^2 = 40m^2$ can now be written as a partial differential equation for the hidden sector dependent functions in the potential

$$\begin{aligned}\left|\hat{G}^a\left(\hat{K}_a + \hat{\lambda}_a\hat{\lambda}^{-1} - 6\hat{Z}_{\bar{a}}\hat{Z}_2^{-1}\right) + 3\right|^2 &= 20\Big(\hat{G}^a\hat{G}_a - 2 + \hat{G}_a\hat{G}_{\bar{b}} \times \\ &\quad \left(\hat{Z}_2^a\hat{Z}_2^{\bar{b}}\hat{Z}_2^{-2} - \hat{Z}_2^{a\bar{b}}\hat{Z}_2^{-1}\right)\Big). \qquad (5.24)\end{aligned}$$

Since the the functions $\hat{K}$, $\hat{Z}_2$ in the Kähler potential are non-analytic by definition, whereas the superpotential and the functions $\hat{W}$ and $\hat{\lambda}$ are analytic, the differential equation only has trivial solutions for arbitrary values of the hidden sector fields. However, by neglecting the hidden sector dependence of the superpotential, in other worlds by treating the quantities $\hat{W}$ and $\hat{\lambda}$ as constants, non-trivial solutions can be found. Inspired by the no-scale supergravity model, we make an Ansatz for an unspecified number of hidden sector fields

$$K = \sum_a \beta_a \log(h_a + h_a^*) + \kappa \prod_a (h_a + h_a^*)^{\alpha_a}\phi^2 + \mathcal{O}(\phi^4) \qquad (5.25)$$

where α_a, β_a and κ are constants. The Ansatz solves Eq. (5.24) provided that the parameters $\alpha = \sum_a \alpha_a$ and $\beta = \sum_a \beta_a$ satisfy

$$\alpha(36\alpha + 16 - 12\beta) + (\beta + 7)^2 = 0\ , \qquad (5.26)$$

while κ can take on arbitrary values. For Kähler potentials of this form, the saddle point condition in MSSM inflation thus holds irrespectively of the values of the hidden sector fields.

5.3.2 Higher order corrections

At this point, it is important to recall that the success of the MSSM inflationary model requires the potential to be extremely flat in the vicinity of the saddle point. In solving the saddle point condition, however, we have considered only the leading order part of the supergravity scalar potential. Thus, one may wonder whether corrections arising from the expansion of the potential Eq. (3.16) to higher orders will destroy this flatness. In Ref. [2], it is shown that supergravity corrections to the potential may be orders of magnitude larger than the allowed deviation from the exact saddle point condition. Therefore, it is not enough to determine the Kähler potential up to second order, but also higher order terms must be taken into account.

In order to guarantee the flatness of the inflaton potential, constraints must be put also on the next to leading as well as the next to next to leading order supergravity corrections. Analogously to the leading order results, we find a form of the Kähler potential for which all these conditions are satisfied identically, i.e. irrespective of the values of hidden sector fields. This class of Kähler potentials can be written in the form

$$\begin{aligned}K &= \sum_a \beta_a \log(h_a + h_a^*) + \kappa \prod_a (h_a + h_a^*)^{\alpha_a}\phi^2 + \mu\Big(\kappa \prod_a (h_a + h_a^*)^{\alpha_a}\Big)^2\phi^4 + \\ &\quad \nu\Big(\kappa \prod_a (h_a + h_a^*)^{\alpha_a}\Big)^3\phi^6 + \mathcal{O}(\phi^8)\ , \qquad (5.27)\end{aligned}$$

where we have introduced the new parameters μ and ν. The introduction of these additional terms amounts to further constraints on the parameters α_a and β_a, while μ and ν remain adjustable. Some examples of these constraints are given in Table (5.1) below.

Table 5.1: Constraints on the parameters of the Kähler potential Eq. (5.27) imposed by the flatness of the MSSM inflaton potential.

$\beta = \sum_a \beta_a$	$\alpha = \sum_a \alpha_a$	$\gamma = \sum_a \alpha_m^2/\beta_a$	$\delta = \sum_a \alpha_a^3/\beta_a^2$
-3	$-\frac{4}{9}$	$\frac{1}{9}$	$\frac{91}{324}$
-7	0	0	δ
-7	$-\frac{25}{9}$	$-\frac{10}{9}$	$-\frac{1654}{1863} + \frac{162}{23}\nu$
-11	$-\frac{1}{9}$	$\frac{1}{21}$	$-\frac{8465}{75411} + \frac{162}{19}\nu$
-11	-4	$-\frac{29}{21}$	$-\frac{2491}{2940} + \frac{36}{5}\nu$

5.3.3 Discussion and conclusions

To summarize, we have found a class of supergravity hidden sector Kähler potentials for which the saddle point condition Eq. (5.4) is identically satisfied to the required degree. This strongly suggest that the extremely flat MSSM inflaton potential could indeed arise naturally as a consequence of the structure of the underlying supergravity model. However, it is worth noting that our analysis is far from complete, since the dependence of the hidden sector on the superpotential has been entirely neglected [133]. Thus we are not allowing for a proper dynamical treatment of the stabilization of the hidden sector fields into their vacuum expectation values. Nevertheless, since the model requires that the flat direction is the only dynamical variable during inflation, it is implicitly assumed that the hidden sector fields are stabilized before the beginning of inflation, either by the neglected superpotential terms or through some other mechanism. A detailed analysis of these effects requires precise knowledge of the nature and dynamics of the hidden sector fields, which can only be provided by some fundamental theory at large scales.

In this context, it is interesting to note that Kähler potentials of the form found here appear fairly commonly in various compactifications of string theory. For example, Abelian orbifold compactifications of heterotic string theory [134], as well as intersecting D-brane models [135] yield Kähler potentials of this form, although Eq. (5.26) represents a non-trivial constraint for the parameters. In the string context $-\beta$ generically measures the number of hidden sector fields, in which case one would expect an integer value for β. In Table (5.1) we list some solutions to Eq. (5.26), which admit nonzero soft supersymmetry breaking terms as well as integer values for β.

Furthermore, the flatness of the inflation should be guaranteed also in the presence of radiative corrections in supergravity. An analysis of one-loop corrections to the leading order potential shows that the location of the saddle point is shifted by the running of the soft parameters [127]; however, the running of the supergravity corrections to the leading order potential, discussed in the previous section, should also be taken into account. The supergravity embedding that we have proposed here does not as such address this problem, since the requirement of the flatness of the potential is simply translated into the constraints on the Kähler potential; however, we point out that there is a continuous trajectory in the Kähler potential parameters for which the saddle point condition is identically satisfied, which could have some underlying interpretation, for example as a consequence of some symmetry [2, 131].

Chapter 6

Beyond inflation with string theory

In this final substantial chapter of the thesis, we leave the standard model of cosmology, which has provided the frame for the discussion so far, to one side and venture into a slightly more speculative domain. In Section 2.5 we discussed some of the conceptual shortcomings of the inflationary hot big bang cosmology, and suggested that they might have a resolution in the context of an ultraviolet complete theory at the Planck scale. Here we focus on a particular string theoretical scenario of the very early universe: string gas cosmology, which suggests a resolution to the initial singularity plaguing the standard cosmological scenario.

Following the general introduction, here below, to the string gas cosmology scenario and the very early universe in string theory, we present the quantitative side of the scenario in Section 6.2. Subsequently, we shift gears and review some basic results concerning the decay of unstable D-branes in string theory. This is to prepare for the discussion in Section 6.4, of the resolution to the initial condition problem of string gas cosmology, which is proposed in the enclosed research paper [3].

6.1 String theory and the initial singularity

String theory is expected to describe physics on scales up to the Planck scale or beyond, possibly resolving some of the shortcomings of the standard cosmological model. In a truly fundamental theory, it would be possible to track the cosmological perturbations for all times, implying that the trans-Planckian physics and its signatures would be known. Furthermore, such a theory should be able to resolve the initial singularity, which cannot be addressed within the big bang model. Several proposals using the ingredients of string theory to address some of these issues have been made, most notably perhaps the pre-Big Bang cosmology [136] and various bouncing cosmologies [137], often involving the dynamics of branes [138]. Another proposed scenario is string gas cosmology [139, 140], which originates in an early attempt of Brandenberger and Vafa to naturally explain the dimensionality of spacetime.

String gas cosmology

String gas cosmology focuses, in particular, on the effect of the winding degrees of freedom in string theory as well as the implications of the T-duality symmetry. The basic postulate of the model is that the universe is a compact space, usually taken to be a torus, which is filled with a thermal gas of closed strings [140]. Furthermore, it is usually assumed that the universe begins string sized and close to the Hagedorn temperature, which is believed to be the maximal temperature for a perturbative string ensemble.

Already a simple qualitative consideration of the string gas scenario has quite fundamental implications on the picture of the early universe. The degrees of freedom in the string gas consist of the three types of closed string modes: momentum modes, describing the center of mass motion of strings; winding modes, expressing the number of times a string is wound around a given torus one-cycle; and finally oscillatory modes. For comparison, the only degrees of freedom which point particles have are momentum modes. The energy of the momentum modes is quantized in units of the inverse torus radii, while the energy of winding modes is directly proportional to the radii. The oscillatory modes have radius-independent energies.

The presence of the string winding modes leads to a symmetry of the string mass spectrum under the inversion of the torus radii, along with the interchange of the corresponding momentum and winding quantum numbers, which is a special case of the more general T-duality symmetry. This symmetry implies the existence of an effective minimal length scale, which in turn suggests a resolution of both the spatial and temperature singularities of the early universe. While the string gas scenario provides a resolution to the singularity problem in this way, the origin of the scenario itself is not explained, and the string gas is implicitly assumed to have an infinite history. The fact that this assumption is indeed consistent, is perhaps a part of the models main asset, but within a fundamental theory there should be a more rigorous explanation for the initial conditions.

Initial conditions

Let us break the question of initial conditions into two parts: the origin of spacetime itself, and the origin of the matter or energy within the spacetime. The first question amounts to how to resolve the initial spacelike singularity of big bang cosmology. To this end, let us mention a model describing the disappearance of space by closed string tachyon condensation, which has been proposed some time ago [141]. The time-reversal of this process could be interpreted as the emergence of space from nothing, and has indeed recently been connected to string gas cosmology [142].

However this still leaves open the second question of how the string gas came to exist. In another recently proposed scenario, energy in a spacetime can originate from a decaying brane, for which an initial condition can be prepared at the origin of time [143]. Basically, the spacetime comes into existence with a large amount of stored energy which gets released immediately after. In a similar manner, we address the second part of the question of initial conditions within string gas cosmology in the third enclosed research paper [3], by studying specifically if a thermal gas of closed strings could arise as a consequence of the decay of a setup of unstable D-branes, through open string tachyon condensation.

6.2 String gas cosmology

String gas cosmology is usually formulated within the framework of dilaton gravity, which is the simplest modification of Einstein's gravity that respects the T-duality symmetry. Although it cannot account for all of the degrees of freedom in the Hagedorn phase, replacing the full string theory by this low-energy effective theory is cosmologically motivated by the assumption of slowly varying fields, i.e. the adiabatic approximation, which allows one to stay at tree level in α' and at weak string coupling $g_s \ll 1$. Here we briefly recapitulate the main features of the model; several more extensive reviews, following a similar treatment, can be found in the literature [144].

6.2.1 The cosmological evolution

Working in the critical string dimension and assuming there are no fluxes present, the D-dimensional dilaton gravity action in the string frame takes the form

$$S_{\text{string}} = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} e^{-2\phi} \left(R + 4(\nabla\phi)^2 \right). \tag{6.1}$$

Here κ_D is the D-dimensional reduced gravitational constant, G is the determinant of the background spacetime metric, R the usual Ricci scalar, and ϕ the dilaton. Assuming furthermore, as in FRW cosmology, a homogeneous spacetime, the background fields G and ϕ are at most functions of time, and the background metric can be written in the familiar RW form $ds^2 = -dt^2 + \sum_i a_i^2(t) dx_i^2$. Rewritten in terms of the number of e-folds $N_i = \log a_i(t)$ and the shifted dilaton $\varphi = 2\phi - \sum_i N_i$, the action Eq. (6.1) takes a form that is manifestly invariant under the T-duality transformation

$$N_i \rightarrow -N_i, \tag{6.2}$$

$$\varphi \rightarrow \varphi. \tag{6.3}$$

When the dilaton gravity action Eq. (6.1) is coupled to the matter action of a gas of free strings

$$S_m = \int dt \sqrt{-G_{00}}\; F(N_i, \beta\sqrt{-G_{00}}), \tag{6.4}$$

where F denotes the string gas free energy, and is varied with respect to the fields, one obtains the evolution equations of string gas cosmology [145]

$$\dot{\varphi}^2 - \sum_{i=1}^{D-1} \dot{N}_i^2 = e^{\varphi} E, \tag{6.5}$$

$$\ddot{N}_i - \dot{\varphi}\dot{N}_i = \frac{1}{2} e^{\varphi} P_i, \tag{6.6}$$

$$\ddot{\varphi} - \sum_{i=1}^{D-1} \dot{N}_i^2 = \frac{1}{2} e^{\varphi} E. \tag{6.7}$$

Here $E = F + TS$ is the total energy of the string gas and $P_i = -\partial F/\partial N_i$ the pressure in the i-th direction multiplied by the total volume.

The cosmology that emerges from these equations is determined by the behavior of the string gas energy and pressure as functions of the scale factor. Due to the different form of the scale factor dependence of the energy levels of the winding and momentum modes, the winding modes give a negative contribution to the total pressure, while the momentum modes give a positive one. Hence Eq. (6.6) implies that winding modes tend to prevent expansion, whereas the momentum modes induce it. In the assumed initial state, where the universe is string scale sized and filled with a dense string gas with a temperature close to the Hagedorn temperature, the energy is nearly constant, $E \sim T_H S$, and the numbers of winding and momentum modes are equal, so that the total pressure vanishes. As a consequence the scale factor remains constant on average, making this initial phase a semi-stable period.

In the absence of winding modes, space is free to expand, in which case the temperature drops, so that the massive string modes eventually go out of equilibrium and the universe enters a standard radiation dominated era. During this period, the radii evolve as $a_i \sim t^{2/D}$ as is usual during radiation domination while the original dilaton $\phi = (\varphi + \sum_i N_i)/2$ approaches a constant.

6.2.2 A three-dimensional universe

String gas cosmology has been advocated as a mechanism that dynamically generates a universe with precisely three large spatial dimensions [139, 145]. This argument is based on the observation that the winding modes give a negative contribution to the string gas pressure, and thus oppose expansion. In order for a spatial dimension to be able to grow large, the winding modes wrapped around this dimension must therefore be annihilated by intersecting with winding modes of opposite orientation. Since string worldsheets are two-dimensional, the argument goes, a pair of strings have non-zero probability of intersecting only in four or less spacetime dimensions, so that at most three spatial dimensions can grow large. The conclusion of this qualitative consideration has been confirmed in various studies [146, 147], but when cosmological dynamics, in particular the effect of the dilaton, are taken into consideration, the simple argument no longer holds.

In a numerical study of the Boltzmann equations governing the string annihilation in the framework of dilaton gravity [148], it was found that the coupling to the dilaton, which is rolling towards weak coupling, in general causes the strings to freeze out too fast for the anisotropic annihilation to take place. In particular, for initial conditions that admit a large number of winding modes, i.e. for small initial values of the dilaton, the strings tend to freeze out so that all dimensions remain small. For a small initial number of winding modes, on the other hand, all strings typically annihilate and the whole compact space grows large. Only for a very narrow range of intermediate initial conditions is it likely that three dimensions grow large.

6.3 String production from unstable D-branes

While string gas cosmology may provide a finite history for three large dimensions, the string gas itself is implicitly assumed to have an infinite history. In the enclosed research paper [3] we study whether such a gas of strings could arise as a consequence of the decay of unstable D-branes, in order to address this conceptual shortcoming of string gas cosmology. Before going in on the details of the proposed scenario, let us begin by reviewing some facts relating to the decay of D-branes. For an extensive review, the reader is referred to Ref. [149].

6.3.1 D-brane decay

The D-branes of bosonic string theory are unstable. As a sign of this, the spectrum of open strings on a brane contains a tachyonic mode. Furthermore, supersymmetric Type II string theories contain unstable non-BPS D-branes, but also pairs of stable BPS D-branes of opposite charge become unstable at subcritical separation. Here we nevertheless restrict ourselves to bosonic string theory for simplicity.

In bosonic string theory, unstable branes will decay into closed strings. We analyze the decay at tree level, where interactions between emitted closed strings can be neglected. Since the mass of a D-brane is proportional to the inverse of the string coupling g_s, in the limit of weak coupling D-branes admit a simple description in closed string theory as extra objects inserted in the theory. Therefore an unstable D-brane acts as a classical time-dependent source for closed string fields. The final state for a p-dimensional D-brane, or Dp-brane, is a coherent state of closed strings [150],

$$|\psi\rangle \sim: \exp\left\{-i\sum_s \int d^{p+1}x\ J_s(x)\cdot\phi_s(x)\right\} : |0\rangle, \tag{6.8}$$

where $J_s(x)$ is the source terms for the closed string fields $\phi_s(x)$, and the sum is over all possible fields. In the case of full brane decay, the source J_s reads, after a Fourier transformation,

$$\tilde{J}_s = \pi T_p \frac{\sin(E_s \ln(\lambda))}{\sinh(\pi E_s)}. \tag{6.9}$$

Here T_p is the tension of the p-dimensional brane, which is inversely proportional to the closed string coupling constant, so that at weak coupling the brane stores a large energy density. The parameter λ controls the brane lifetime

$$\tau \sim -\ln[\sin(\pi\lambda)], \tag{6.10}$$

and can take values in the range $0 \leq \lambda \leq 1/2$.

6.3.2 The decay products

To begin with, we consider decaying branes in a non-compact space, so that there are no winding modes. In this case, the string energy is given by

$$E_s = E_s(N, k_\perp) = \sqrt{4l_s^{-2}(N-1) + \vec{k}_\perp^2}, \tag{6.11}$$

where l_s is the string length. The total energy and number density of emitted closed strings from the decay of a p-dimensional D-brane then read [150]

$$\frac{\bar{E}}{V_p} = \pi^{11}(2\pi)^{2(6-p)} \sum_{N=0}^{\infty} d(N) \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \frac{|\tilde{J}_s|^2}{2} \tag{6.12}$$

$$\frac{\bar{N}}{V_p} = \pi^{11}(2\pi)^{2(6-p)} \sum_{N=0}^{\infty} d(N) \int \frac{d^{25-p}k_\perp}{(2\pi)^{25-p}} \frac{|\tilde{J}_s|^2}{2E_s(N,k_\perp)}. \tag{6.13}$$

The sum in the equations is over all final closed string states of symmetric oscillator excitations between left- and right-moving sectors, where $d(N)$ denotes the density of states at level N.

Due to the exponential growth of the density of left-right symmetric closed string states

$$d(N) \sim N^{-27/4} e^{4\pi\sqrt{N}} , \tag{6.14}$$

the main contribution to the total number and energy of strings produced in the decay comes from the highest energies. Evaluating the large energy behavior of Eqs. (6.12) and (6.13), one finds that the total amount of energy per unit p-volume carried by all the closed string modes emitted during the decay [151] is infinite for $p \leq 2$. Since the intial D-brane has a finite energy, the total energy carried by the closed strings cannot really be infinite, but the divergence is due to the breakdown of perturbation theory. The finite result for $p \geq 3$, on the other hand, implies that the single closed string channel alone does not carry away all of the initial energy of the brane. Since multi-string emission channels are suppressed by powers of the string coupling, the result means that higher dimensional branes do not decay completely – the final state contains a lower dimensional brane. This implies that the decay must be inhomogeneous, although the process is not fully understood at the moment [152].

Let us study as an example the $p = 0$ case. By Eq. (6.12) the total energy produced is infinite, but we expect that, once the backreaction of the closed string emission process is taken into account, there will be a natural cut-off on the sum over N, rendering the answer finite. In particular, since the original brane energy is of the order of g_s^{-1}, the cut-off should be of the same order. Let us reinterpret the results adopting the cut-off M, such that

$$\sum_{N \leq M^2/4} N^{-1/2} \sim M. \tag{6.15}$$

Since we are looking at a D0-brane, there is no other decay channel, and all the energy is converted into closed string radiation. The total energy carried by the strings is then of the order of M. For $M \ll g_s^{-1}$ this energy is small compared to the energy of the brane, but if we cut the sum off at $M \sim g_s^{-1}$, all the brane energy will be emitted into closed strings.

6.4 An origin for the string gas

Above we have described the decay of unstable branes into an ensemble of closed strings. The analysis suggests that decaying D-branes may indeed produce a state that is similar to the initial state of string gas cosmology. In the research paper [3] we propose a particular D-brane setup as a possible initial state, and study its evolution during and after the brane decay.

6.4.1 A proposal for the initial state

For simplicity, we continue to work in bosonic string theory, and hence in 26 dimensions. In order to make contact with the setup of string gas cosmology, we take all the spatial directions to be compactified on the torus T^{25}, with equal string scale radii in all directions. The most natural initial state on this torus would be a space-filling unstable D25-brane, or a stack of them. However, we have seen above that branes with $p \geq 3$ presumably decay into lower dimensional branes, and the process is currently poorly understood. For a pure string gas without any branes, we are thus directed to consider lower dimensional branes which decay completely. Hence we choose an initial configuration consisting of D1-branes wrapped in each direction X^i. For homogeneity we assume an equal number of D1-branes in every direction, all having the same lifetime.

A Dp-brane with all its tangential and perpendicular directions compactified on a torus is related to a D0-brane via T-duality. Thus we expect similar results to hold for this system as those obtained in the previous section. In particular, since under a T-duality transformation momentum along a circle gets mapped to the winding charge along the dual circle, we expect that all the energy of a Dp-brane wrapped on a torus will be converted into closed string radiation, with most of the energy is stored in the highly wound closed string modes of mass around g_s^{-1} [153].

6.4.2 Thermalization of the string gas

To arrive at the initial state of string gas cosmology, the decay should produce a string ensemble with a thermal distribution. The number distribution of the strings produced through the brane decay can be expressed as follows

$$\bar{n}(N, k_\perp) = \frac{|\tilde{J}_s|^2}{2E_s(N, k_\perp)} \ . \tag{6.16}$$

Generically this does not correspond to a thermal distribution of strings, although the deviation becomes small for highly energetic strings near the Hagedorn temperature. This is due to the fact that $|\tilde{J}_s|^2 \sim e^{-2\pi E_s}$ for large energies, implying that the number distribution $\bar{n}$ approaches a thermal distribution at the Hagedorn temperature $T_H = 1/2\pi$.

In the analysis of the brane decay, however, one assumes zero coupling for the closed strings. This is of course an idealization, as the emitted strings will interact and backreact on the decay. Under suitable circumstances, these interactions will thermalize the string distribution. We estimate the thermalization time scale t_{th} of the string ensemble to

$$t_{\text{th}} \simeq \frac{\sqrt{E_s}}{g_s^4} l_s^{-1} \sim g_s^{-9/2} l_s^{-1}, \tag{6.17}$$

where we have used the fact that most of the energy is stored in slowly moving heavy strings of mass $E_s \sim g_s^{-1}$.

Let us make a comparison between this result and the lifetime of the initial unstable brane Eq. (6.10), controlled by the parameter λ. For a generic choice of λ, the lifetime is very short: of the order of the string scale, and the brane decay is a very rapid process. Since the thermalization timescale Eq. (6.17) in this case is much longer than the timescale of the brane decay, it is justified to treat the brane decay and the subsequent thermalization of the strings separately. Thus we have good reason to believe that the decay of unstable branes may indeed give rise to a thermal string gas. Furthermore, the spacetime stress tensor for the outgoing closed strings [154] has been computed and is known to quickly settle to zero total pressure [149], which is indeed one of the assumptions on the initial phase of string gas cosmology. Consequently, it is also natural to assume that the volume of the torus remains essentially unchanged during the brane decay.

6.4.3 Time evolution of the dilaton

So far we have argued that our proposed setup of decaying branes produces a universe that looks qualitatively like the initial state of string gas cosmology. In addition, we need to make sure that the process is internally consistent and that it creates a final state that fulfills the adiabatic and weak coupling assumptions of dilaton gravity. During the decay, the brane sources the low-energy effective fields; in particular the dilaton time evolves. Hence, one might worry that the dilaton ends up being too large, contradicting the initial assumption of weak string coupling and insignificant back-reaction.

In the brane decay process, the evolution of the dilaton is governed by an equation of motion given by the details of the brane setup. During the period of thermalization following the decay, the exact evolution of the dilaton is unknown. However, one can argue that the explicit form of the distribution of the background strings has only a small effect on the propagation of the dilaton. What counts are the ensemble averaged quantities such as the mean comoving energy of the gas, which does not change during thermalization. Hence, we are led to assume that the dilaton should reach values that are consistent with the dilaton gravity era already at end of the brane decay process.

Neglecting any back-reaction from emitted strings, the evolution of the dilaton during the full brane decay is governed by the equation of motion [150]

$$-\partial_t^2 \phi = a\Big[\frac{1}{1+\hat{\lambda}e^{t}} + \frac{1}{1+\hat{\lambda}e^{-t}} - 1\Big] \ . \tag{6.18}$$

Here $\hat{\lambda} = \sin \pi\lambda$ and a is a positive constant, which is related to the initial tension of the unstable brane. From now on we set $a = 1$. Solving Eq. (6.18) we find the time dependence of the dilaton

$$\phi(t) = Li_2(-\hat{\lambda}^{-1}e^{t}) - Li_2(-\hat{\lambda}e^{t}) + C_1 t + C_2 \ , \tag{6.19}$$

where $Li_2(z)$ is the dilogarithm, and C_1 and C_2 are constants of integration. In our notation, the branes begin to decay at $t = 0$ and have lifetimes of $\tau = -\log \hat{\lambda}$, which are input parameters. The constants C_1 and C_2 can be determined by assigning $\phi(t)$ and $\dot{\phi}(t)$ initial or final values, corresponding to initial values for the string gas cosmology era.

Consistency with the dilaton gravity approximation calls for final values of the time derivative in the range $-1 \lesssim \dot{\varphi}(\tau) < 0$. Note that the dilaton solution above is for the string frame dilaton ϕ, whereas these values are for the shifted dilaton φ. Assuming that the torus radii stay fixed during the brane decay, however, these are simply related as $\dot{\varphi}(\tau) = 2\dot{\phi}(\tau)$. Furthermore, the value of the dilaton itself is required to be negative, both during the brane decay process and the string gas cosmology scenario, in order to preserve weak coupling.

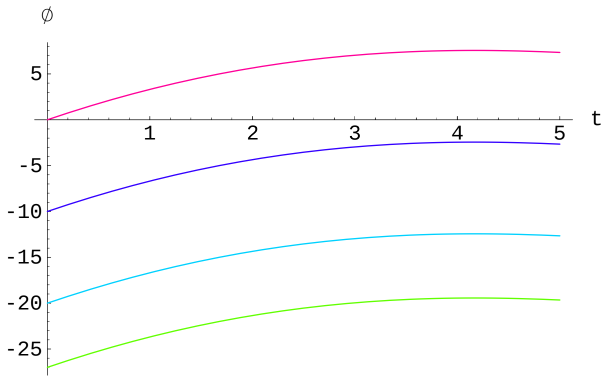

Figure 6.1: $\phi(t)$ for varying $\phi(0)$. From top to bottom we have $\phi(0) \simeq \ln 1$, $\ln 10^{-4}$, $\ln 10^{-8}$ and $\ln 10^{-12}$, with $\tau = 5t_s$ and $\dot{\varphi}(\tau) = -1$.

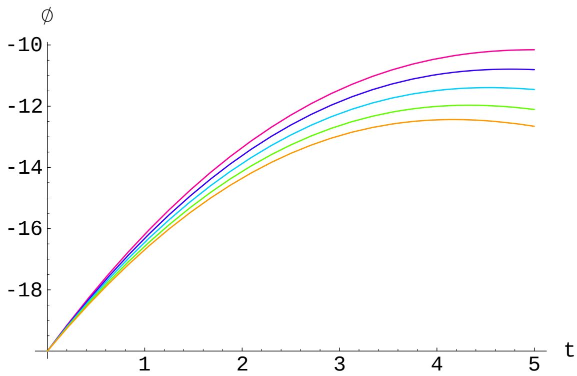

Figure 6.2: $\phi(t)$ for varying $\dot{\varphi}(\tau)$. From top to bottom we have $\dot{\varphi}(\tau) = 0$, -0.25, -0.5, -0.75 and -1, with initial value $\phi(0) = -20$, corresponding to $\phi(0) = -20 \simeq \ln 10^{-8}$ and $\tau = 5t_s$.

The Figures (6.1) and (6.2) show the dilaton evolution during brane decay for different initial values and different final values of its derivative, respectively. The figures address the consistency of the weak string coupling. Fig. (6.1) illustrates how, starting from large negative values of the dilaton, the typical decay time is too short for the dilaton to grow significantly. Fig. (6.2) shows that different choices for time derivatives of the dilaton in the end of the decay also cause no significant change in the dilaton evolution. Fig. (6.3), on the other hand, illustrates the dependence of the dilaton evolution on the lifetime of the decaying brane.

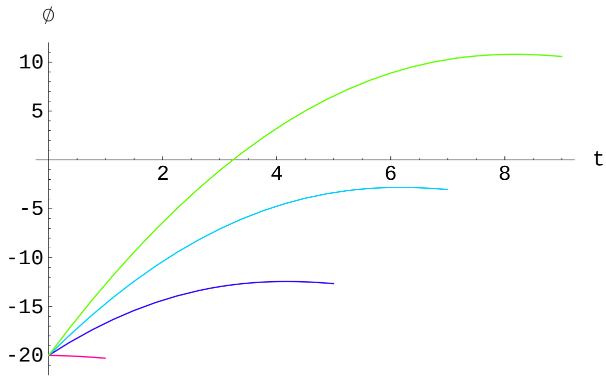

Figure 6.3: $\phi(t)$ for varying τ. From bottom to top we have $\tau = 1,\ 5,\ 7$ and 9, with initial value $\phi(0) = -20$ and $\dot{\varphi}(\tau) = -1$.

6.4.4 Three large dimensions

In the previous section, we studied the evolution of the dilaton during brane decay, and found that it can indeed lead to values that are consistent with the dilaton gravity picture. Let us now take one step further and investigate under what circumstances it might lead to the values favored for three dimensions to grow large. As argued above, it is reasonable to assume that the dilaton gravity era and the study of the Boltzmann equations sets in just after the branes have decayed. In the analysis of Ref. [148], the direction of time is chosen so that $\dot{\varphi} < 0$, as is done here. Furthermore, the initial value of the derivative is chosen to be $\dot{\varphi(\tau)} = -1$, which is just the borderline value allowed by the dilaton gravity approximation. In this case, there is a gap of initial values$\Delta\ \varphi(\tau) \simeq 0.5$, around the value $\varphi(\tau) \sim -2.5$, for which three dimensions are likely to become unwrapped [148].

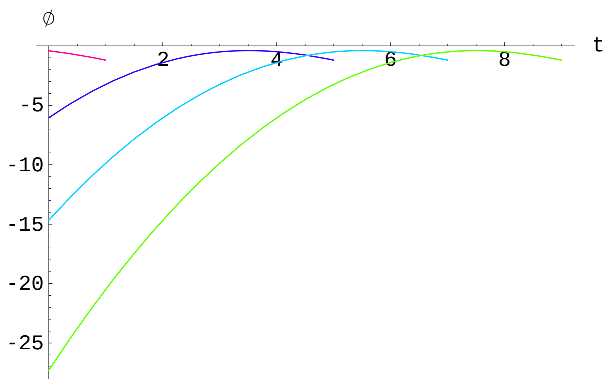

Figure 6.4: $\phi(t)$ plotted for different combinations of $\phi(0)$ and τ. From top to bottom, we have $\phi(0) = -0.5, -6, -15, -27$ and $\tau = 1, 5, 7, 9$, with $\dot{\varphi}(\tau) = -1$.

In Fig. 4, we have plotted a number of combinations of $\phi(t)$ and τ, all of which lead the dilaton to values within the favored range. The figure illustrates that it is possible to reach the favored range of initial values for a variety of decay configurations. In particular, one can start from very weak coupling and still reach the preferred range for a decay time $\tau \simeq 10t_s$ that is short in comparison to the time of thermalization. This once again verifies that the decay may be discussed separately from the other dynamics. Thus we may conclude that the somewhat arbitrary fine-tuning of the dilaton initial value that is required in the study of Ref. [148] here has a physical interpretation in terms of the lifetimes of the unstable branes.

6.4.5 Discussion and conclusions

In this paper we discussed the issue of the origin of the thermal string gas, whose existence is usually assumed in string gas cosmology. We studied low-dimensional unstable D-branes wrapped on a torus in bosonic string theory, and found that the string gas could indeed naturally be produced by the decay process of the branes. One virtue of the scenario is that there is freedom in choosing the details of the initial condition, and this is reflected in the subsequent development. For example, a space filling brane appears to produce string-brane gas [147], while a configuration of lower dimensional branes can produce pure string gas.

Besides adding to the control of various properties of the string gas through the physical properties of the branes, the introduction of the brane setup provides a resolution to the problem of initial conditions for the string gas. In contrast to simply assuming an infinite history for the string gas, the setup allows for the possibility of initial conditions at some finite point in time. These initial conditions fall into three categories: a scenario, where the unstable brane is first created as a condensate of incoming closed strings; a scenario, where the unstable brane pops out from imaginary time; and a scenario, where the brane initial state is prepared by a complex time contour at the initial spacelike singularity. As a consequence of having different possibilities for controllable initial conditions for the string gas, in turn, there is a better chance of understanding any possible imprints or signatures of observable interest that the gas might leave behind.

While such features certainly seem promising, let us recall that we have in our treatment made a number of simplifying assumptions. For example we initially assume a weak string coupling so that any back-reaction can be neglected, although we then find that during the decay the dilaton grows, implying that interactions with the emitted closed strings should be taken into account. Furthermore we focus only on low-dimensional branes that are wrapped on a torus with all its radii equal, assuming an equal number of branes at every direction of the torus, each decaying equally fast. Finally, we argue that the thermalization timescale of the produced strings is much longer than the decay time, and hence treating the brane decay and string dynamics separately is justified. In a more rigorous treatment, any of the assumptions mentioned here can be challenged, but we leave this and other modifications of the scenario for future work. Especially the details of the thermalization leave room for more detailed analysis, with possibly interesting effects.

Chapter 7

Summary

The main purpose of the thesis at hand has been to study various problems at the interface between high energy particle physics, in particular supersymmetry, and cosmology. The interplay between these two historically quite distinct fields has proven a fruitful enterprise from either point of view. While particle theory points cosmological models into the right direction, the early universe provides a unique laboratory for studying particle physics at very high energies. However, there is a more subtle level of interplay as well, where problems of naturalness, or seemingly arbitrary assumptions can be seen in a more favorable light, by looking at the problem from another angle or placing it into a different context. The topics treated in this thesis can, in one way or another, all be placed into this category.

The thermal leptogenesis scenario is very well motivated from a low-energy point of view, through the see-saw mechanism for neutrino masses; however, the conflict between the scenario and the gravitino bound in the presence of supersymmetry, has always been a severe issue from a theoretical perspective. Although several modifications rendering the scenario compatible with the bound have been proposed, they have all involved adding some new structure to the MSSM with heavy neutrinos, so that the simplest and theoretically most favored model has remained incompatible. Nevertheless, we have shown that just by taking into consideration the flat directions, which might in any case be present in the scenario, a sufficient lepton asymmetry can indeed be produced even when the reheating temperature is low enough to avoid overproduction of gravitinos.

Furthermore, by embedding the MSSM inflation model into specifically chosen models of hidden sector supergravity, we have been able to provide an origin for the cancellation of parameters that causes the extremely flat potential required for sufficient inflation at such low energy scales. From a theoretical point of view, this considerably lessens the gravity of the strong fine-tuning that the model is plagued with, although many questions regarding the high energy continuation of these supergravity models still remain unaddressed. Similarly, in the case of string gas cosmology, taking into account the presence of unstable branes in string theory, we are able to provide the scenario with well-defined initial conditions at some finite point in time, as opposed to assuming it has an infinite history. Furthermore, the details of this initial condition are reflected in the subsequent development as distinct properties of the resulting cosmology, which can thus be controlled more accurately.

Naturally, also the solutions that we have proposed are set within a particular formalism, which imposes some limits on the applicability of their conclusions. While this is important to keep in mind, it should not be allowed to undermine the progress that can be made through such progressive steps. At some point down the line, any of the scenarios presented here might be set into a broader context that might affirm or contradict the assumptions and conclusions made here. In the end, whether any of the topics treated in this work actually provide descriptions true to nature remains to be determined by future observations.

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