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Nonlinear Dynamics of Optically Injected Semiconductor Lasers

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ACADEMIC DISSERTATION

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ABSTRACT

Semiconductor lasers subjected to external optical injection from a single mode tunable laser have been investigated in a master-slave configuration. The resulting dynamics has been studied by measuring the output of the slave laser as the frequency detuning between the lasers, the injection strength, and the slave laser current are varied. These parameters constitute the experimentally accessible control parameters of the system.

A new method of condensing the information obtained during a sweep of one parameter has been presented. Output spectra are plotted in a two-dimensional image where the qualitative changes in the dynamics of the slave laser can easily be observed. The method has been utilized to explore the dynamical characteristics of the optically injected semiconductor laser. Images sampled for a range of fixed frequency detunings as the injection strength is increased have been analysed to create a map of the nonlinear regions in the parameter plane spanned by the injection strength and the frequency detuning. The resulting map shows how the dynamics is organized in the parameter plane, with main features including islands of chaotic oscillation. Within the chaotic islands pockets of periodic oscillation are encountered. Similar maps have been measured for different values of the slave laser's operating current. As a whole, the extent of the nonlinear regions has been found to scale with respect to the relaxation oscillation frequency and the injection strength relative to the free-running slave laser power. Particular regions have been found to deviate from this general scaling by growing in size as the current is increased.

The results have been interpreted in terms of the rate-equation model for the injected laser. Values of the dynamical parameters of the rate-equations have been obtained by probing the laser with weak optical injection. The maps of the dynamical regions are in a good qualitative agreement with theoretical results for lasers with similar values of the linewidth-enhancement factor.

A home-made extended cavity diode laser and a polarization stabilized He-Ne laser have been used as master lasers in the experiment. The properties of the extended cavity laser have been determined.

CONTENTS

1 INTRODUCTION

Self-sustained periodical oscillators synchronize if there is a sufficiently strong coupling between them. This common effect is found throughout nature in a vast amount of systems, ranging from very simple mechanical constructions to highly complicated biological organisms [1]. A classic example of the former is the synchronization of two pendulum clocks on a wall, first observed by Christiaan Huygens (1629-1695) in 1665. As an everyday example of the latter, consider the circadian clock. It regulates body temperature, schedules sleep and other physiological patterns in humans, and it is found to synchronize to the length of the day. Besides being used in countless electronic circuits containing phase-locked loops, the phenomenon is further encountered in complex systems with large numbers of oscillators such as pacemaker cells in the heart, neurons in the cortex, or in entire populations of tropical fireflies flashing mating calls in synchrony. The dynamical properties of coupled or forced oscillators are equally diverse. An experienced traveller may for instance recognize the effects, commonly referred to as jet-lag, associated with a rapid disturbance in the circadian rhythm that occurs when too many time-zones are crossed at once. The body protests in various ways, but eventually its daily pattern is recovered with the phase reset to the new environment. A perhaps more serious example is the interaction between healthy pacemaker cells, and collections of abnormal, so called ectopic pacemaker cells, causing cardiac arrhythmias that may become detrimental to the individual's health.

Although the locking phenomena encountered in natural systems are fascinating, and certainly important in science in general, the study of the mechanisms behind the phenomena in these systems is limited by the very little external control that can be exerted into them. These systems are therefore impractical for the physicist, whose task is to construct simple models and render them to systematic study in controlled environments. While Huygens' early discovery initiated the entire research field of coupled oscillators, studies on the locking behaviour of two tuning forks as a function of frequency and coupling strength in the beginning of the last century [2] could be viewed upon as the beginning of the systematic investigation of synchronization or locking phenomena in modern times. The advent of radio communication brought on a vigorous research into electrical circuits that also lead to advances in the understanding of locking phenomena. Any realistic self-sustained oscillator must contain a nonlinear element in order to stabilize its oscillation amplitude. The nonlinear aspects of oscillator locking were recognized early on in work that still stands as a foundation for much research today [3, 4].

The laser can without a doubt be regarded as a very complicated device, from both a theoretical and a technological perspective. The physics behind a laser's operation can satisfactorily be described only by quantum mechanics, and the manufacturing of lasers requires utmost skills in many fields. In contrast to many other *dynamical* systems however, lasers are relatively simple. A laser's degrees of freedom are limited to only a few experimentally controllable ones, constituting a highly confined system. Moreover, the timescale is considerably faster, allowing a clear distinction of true dynamical states rather than transients. In this respect, the laser is an ideal candidate for dynamical investigations. In the simplest picture, the laser is just a good example of a self-sustained oscillator. It is then hardly surprising that the locking of two helium-neon laser oscillators was demonstrated shortly after the invention of the laser [5].

The dynamical properties of lasers can in certain cases be modelled in terms of the well known Lorenz equations that give a truncated description of convection flows in fluids [6]. Thus, interesting nonlinear phenomena, such as the strange attractors that the Lorenz model is famous for, can be anticipated in lasers. Many practical lasers, however, have less degrees of freedom than the full set of Lorenz equations, which limits the range of nonlinear phenomena that can be seen [7, 8]. Additional degrees of freedom can be introduced externally by a perturbation such as an injected optical signal. Under favourable conditions the laser output locks to the injected signal. The perturbation can also cause unstable oscillations, and a complete destabilization of the steady state is possible. Lasers are used in numerous applications where these situations occur either deliberately, such as in injection locking applications, or unwantedly, such as in back-reflections or cross-coupling in optical fibres. It is in part for this reason that the physical mechanisms governing the dynamics of the injected laser need to be understood. More importantly, the results of recent efforts have shown that the optically injected laser is of profound interest from a fundamental nonlinear physics point of view, showing bistability and chaos among others [9].

The semiconductor laser master-slave configuration, which is studied in this work, refers to a situation where the injection is taken from a separate source that usually closely approximates a monochromatic wave. Further, the injection source (master) is undisturbed by the target (slave), i.e. the coupling is unidirectional. This highly simplified system exhibits a wealth of nonlinear phenomena principally due to the intrinsic properties specific to semiconductor lasers. It is of great technological importance, and may offer further insight to more complicated systems such as mutually coupled lasers or lasers with optical feedback.

2 STRUCTURE AND PURPOSE OF THIS STUDY

This study consists of an introductory section, together with four articles published or to be published in refereed international scientific journals. The following papers will be referred to by Roman numerals in the main text.

Summaries of the original papers

Paper I: A simple extended cavity diode laser for spectroscopy around 640 nm, S. Eriksson, Å. M. Lindberg and B. Ståhlberg, *Opt. Las. Technol.* **31**, 473 (1999).

> We report on a laser source with a high degree of coherence, based upon feedback from a reflection grating. The basic properties of the laser, such as the fundamental linewidth, frequency stability, and tunability are determined. The performance of the laser is demonstrated by studying the closed $1s_5$-$2p_9$ transition in a neon discharge in a saturation-spectroscopy experiment.

Paper II: Periodic oscillation within the chaotic region in a semiconductor laser subjected to external optical injection, S. Eriksson and Å. M. Lindberg, *Opt. Lett.* **26**, 142 (2001).

> A new method of gathering and presenting spectra of optically injected lasers is presented. The method condenses the information obtained in a sweep of one control parameter in a two-dimensional image from which the dynamical characteristics can easily be deduced. The experimental procedure and time required to study injection dynamics is greatly reduced by this method. The technique is used to show period-three oscillation in a pocket within one of the chaotic regions of the semiconductor slave laser. A polarization stabilized He-Ne laser was used as master laser in the experiment.

Paper III: Observations on the dynamics of semiconductor lasers subjected to external optical injection, S. Eriksson and Å. M. Lindberg, *J. Opt. B: Quantum Semiclass. Opt.* **4**, 149 (2002).

> The nonlinear regions of the slave laser are mapped in the parameter plane spanned by the injection strength and frequency detuning between master and slave lasers. The dynamical characteristics are determined from images constructed from optical and radio-frequency spectra by the method described in Paper II. The map shows clearly where the chaotic islands with the periodic pockets are located on the large scale.

Paper IV: Dynamics of optically injected semiconductor lasers explored for wide parameter ranges, S. Eriksson, Submitted (2002).

A rigorous study of the dynamical characteristics of an optically injected semiconductor laser is presented. Weak injection levels are used to determine the parameters that influence the dynamics. The nonlinear dynamics is studied experimentally for different parameter values by varying the operating point of the slave laser. It was found that the nonlinear regions scale with respect to the relaxation-oscillation frequency and relative injection strength on the large scale. Specific regions differ from this general trend by growing in size as the current is increased.

In recent years, there has been tremendous advances in the field concerning the theory of nonlinear dynamics of semiconductor lasers. Theoretical considerations have clearly outnumbered experimental investigations, which still remain few. The purpose of this study is to bring further insight to the dynamics of semiconductor lasers subjected to external optical injection through experiments. Emphasis is laid on finding and utilizing new methods of exploring the rich dynamics of semiconductor lasers with injected signals. Throughout this thesis, a basic laser physics view-point is adopted, with only main results from nonlinear bifurcation analysis cited where necessary.

The remainder of this thesis is organized as follows. In section 3, a general overview of optical injection is given. Part of section 3 is reserved to aspects of optical self-injection, i.e. optical feedback, with an emphasis on how feedback can be used to improve the output properties of semiconductor lasers. Feedback techniques were used in the construction of an extended cavity diode laser, reported on in Paper I. This laser was subsequently used as the master laser in the injection experiments described in Papers III and IV. In section 4, the basic dynamical properties of semiconductor lasers are discussed. The central concepts of relaxation-oscillations and the linewidth-enhancement factor are introduced, together with methods of obtaining information on the laser characteristics in experiments. Section 5 is devoted to the dynamics of semiconductor lasers subjected to external optical injection. The section is begun with a summary of previous research in the field which enables the reader to place the work in this thesis in its proper context. It is further described how the nonlinear bifurcations can be deduced from images constructed of the slave laser's output spectra. The main results of Papers II, III and IV are reviewed based on the mappings of the nonlinear regions. Conclusions are given in section 6.

Author's contribution

All experiments were devised, constructed and performed by the author on his initiative. The experimental technique of gathering and presenting data in easily interpreted images was developed by the author. All papers were written by the author.

3 OPTICAL INJECTION

3.1 General overview

Optical injection refers to the situation when an electromagnetic field is applied to the laser through one of its partially reflecting facets. The injection field is in most cases nearly resonant with the oscillating laser field. The characteristics of the injection system depends on the properties and origin of the injection field. Three main categories can be defined:

- **External optical injection.** The injected field originates from another light source, usually a single mode laser which operates undisturbed by the injected laser.
- **Conventional feedback.** The output of the laser is back-reflected from a mirror or from a frequency selective reflector such as a grating.
- **Phase conjugate feedback.** The output of the laser is back-reflected from a phase conjugating mirror.

Semiconductor lasers are particularly sensitive to all three types of injection. Consider a laser oscillating in a single mode with an external signal applied. The situation is schematically depicted in Fig. 1, where the injection field denoted by E_i impinges from the right. The cavity length is given by

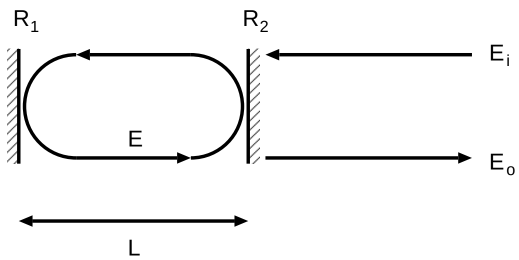


Figure 1: Principles of the optically injected laser.

L and the facet power reflectivities by R_1 and R_2. After each round-trip time given by $\tau_0 = 2nL/c$, the external field adds a fraction $t_2 E_i$ to the circulating field E. The refractive index of the active laser medium is given by n and the front facet amplitude transmission by $t_2 = \sqrt{1 - R_2}$. Thus, the injection rate is given by $\eta = t_2/\tau_0$. Semiconductor lasers have in general short cavity lengths and low facet reflectivities with typical values such as L=500 μm and $R_2 = 0.32$ respectively. This leads to injection rates of the order of 10^{11} s^{-1}. In contrast, the relaxation rates of the gain and population inversion of a semiconductor laser is normally of the order of 10^9 s^{-1}.

In the case of external optical injection, the injected field is independent of the circulating field, which simplifies the theoretical modelling. The differential equation for the field envelope of the solitary laser is complemented with a term $\eta E_i \exp(i\Omega t)$, where Ω is the frequency difference between the injection field and the solitary laser [10, 11]. In mathematical terms, the phase-space dimension is increased by one, since amplitude and phase become coupled by the added term. The injection field depends both in the cases of conventional and phase conjugate feedback on its prior history. The injection term for conventional feedback takes the form $\kappa E(t - \tau)$ [12], where τ is the round trip time of the cavity formed by the external reflector and the back facet of the laser. The coupling rate κ will now also depend on the properties of the reflector. A new timescale enters the system in form of the delay time τ. In this case the system becomes infinite dimensional, since a uniquely defined solution would in principle require the initial values to be specified on a continuous function on the time interval τ. The case of phase conjugate feedback is similar, with the exception that the injection field is now given by the conjugate. If the finite response time of the conjugating reflector is taken into account, the problem becomes complicated further. Out of the three different categories, external optical injection is by far the simplest to model. Consequently, interpreting the experimental results in terms of existing theory is rewarding, and may even prove helpful in understanding other types of driven systems.

The majority of this work deals with the dynamics of semiconductor lasers subjected to external injection. Conventional feedback is a crucial element in the master laser used in the injection experiments in Papers III and IV. Therefore, some aspects of the spectral properties of semiconductor lasers with feedback from a reflection grating are presented in the next subsection. The topic of phase conjugate feedback or the dynamics of delayed feedback systems in general is beyond the scope of this study.

3.2 The extended cavity diode laser

Solitary semiconductor lasers have a rather poor spectral purity. The output typically consists of a main oscillating mode, with a linewidth of the order of 10 - 100 MHz. The excessively broad linewidth is due to the coupling between amplitude and phase of the amplifying medium in the semiconductor laser (cf. section 4). Besides the main mode, several other longitudinal modes oscillate near threshold. Because of these properties, semiconductor lasers do not in general qualify as good light sources in precision experiments. Optical feedback can be used to improve the spectral characteristics of semiconductor lasers. The principle is to improve the quality factor of the laser's cavity, thereby reducing the linewidth. The non-vanishing reflectivity of the laser's front facet (R_2 in Fig. 1) complicates the matter. With feedback applied, essentially single mode operation can be obtained and the linewidth is reduced dramatically [13, 14], depending on fraction and phase of the back reflected light [15–17]. The feedback should, however, be applied with great care since it can induce instabilities and lead to severe broadening of the emission, the so called coherence collapse [18]. The behaviour can coarsely be divided into five different regions according to the fraction of back reflected light f_e and the distance of the reflector L_e [19, 20]. For weak feedback ($f_e < 10^{-5}$) and short cavity lengths ($L_e < 10$ cm) the emission is stable with linewidth reduction depending on the feedback phase (regime I). If the distance is made longer, multiple modes of the external cavity become allowed. Several modes may oscillate at once or the output may hop from mode to mode depending on feedback phase (regime II). Less feedback is needed at longer cavities for a transition between regimes I and II. If the feedback is increased above $f_e = 10^{-5}$, the laser will stabilize its output on the mode with the highest linewidth reduction (regime III). Further increasing the feedback above $f_e = 10^{-4}$, leads to coherence collapse with a very noisy output (regime IV). For very strong feedback levels $f_e > 10^{-1}$, the facet reflectivity R_2 acts as a small perturbation in a long external cavity and the output is again stable and the linewidth can be reduced by several orders of magnitude (region V). The division into regions can be further refined to take into account the so called low-frequency fluctuations (LFF) [21–23], a topic that has recently been under much debate [24–27]. The LFF phenomenon occurs within regime IV close to the stable emission regime V, when the laser is pumped close to the solitary laser's threshold current [27].

Although the effects of optical feedback can be complicated, they are well studied. Great value can be obtained from inexpensive commercial semiconductor diode lasers by following a few common guidelines, yielding single mode tunable light sources with narrow linewidths [28]. These devices are commonly referred to as external or extended cavity diode lasers (ECDLs). The availability of high quality light-sources at low cost greatly benefitted the field of atomic physics when the apparatus in cooling and trapping experiments was simplified by the use of diode lasers [29]. Advances in the field ultimately led to the recent experimental observation of Bose-Einstein condensation in dilute gases [30–32].

Typical solutions employing direct feedback operate in region V. Achieving feedback ratios this high normally requires an anti-reflection coating on the front facet of the semiconductor laser. Many commercially available high power semiconductor lasers (~ 15 mW) already have reduced front facet

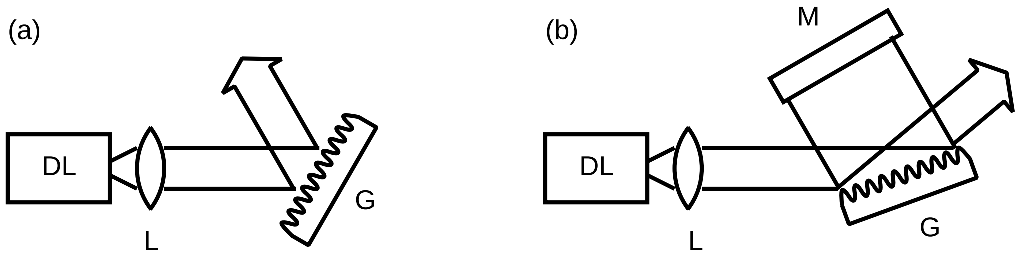


Figure 2: Extended cavity designs for diode lasers. The grating (G) is in mounted in the Littrow configuration in (a) and in grazing incidence with feedback from a mirror (M) (Littman-Metcalf design) in (b). The diode laser (DL) output is collimated by a lens (L).

reflectivities and can readily be utilized without opening the packaging. The most widely used design is based on feedback from a dispersive element such as a grating, providing strong frequency selective feedback. Since the first reported external cavity diode laser in 1972 [33], numerous variations have been realized. Many laboratories have created their own versions based on one of two common cavity configurations previously used for tunable dye-lasers [34–36]. In the Littrow configuration, Fig. 2 (a), the first order diffraction of the grating is counter-reflected, and the undiffracted beam serves as the output. The cavity is tuned by rotating and translating the grating. In the Littman-Metcalf configuration, Fig. 2 (b), the first order diffraction is back-reflected from a mirror that can be rotated or translated for tuning. The advantage of the Littman-Metcalf design is that the cavity can be tuned without laterally shifting the output beam, without the inclusion of an additional beamsplitter. Moreover, because of the grazing incidence, a larger number of grating lines is illuminated, giving a better frequency selection.

The construction and operation of an ECDL based on the Littman-Metcalf design is described in Paper I. Approximately 20 % of the light from a commercial semiconductor laser is back-reflected from a nearly 5 cm long extended cavity. While a longer cavity would yield a greater linewidth reduction, the longitudinal mode spacing of the extended cavity becomes smaller. If the mode spacing falls within the modulation bandwidth of the semiconductor laser, the extended cavity modes can excite resonances at the mode spacing frequency. The modulation bandwidth is approximately given by the relaxation oscillation frequency, which is typically a few GHz in semiconductor lasers (cf. section 4). The corresponding mode spacing frequency of the constructed extended cavity is 3 GHz, which is just outside the modulation bandwidth of the solitary semiconductor lasers that were used. The feedback mirror is fastened on a piezo-electric translator on a three-point mirror mount. By rotating the mirror, coarse wavelength tuning is achieved while finetuning is provided by the piezo.

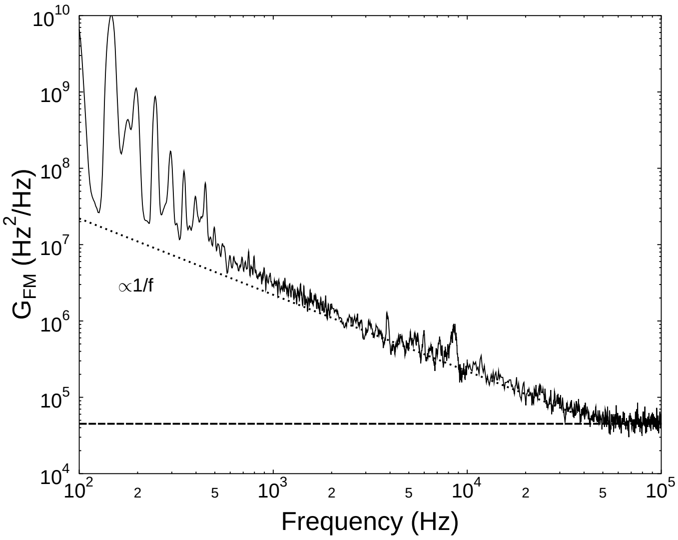


Figure 3: Frequency noise spectrum of the ECDL. The dotted line is proportional to $1/f$. The white noise level is indicated by the dashed line. The background noise floor of the measurement is below 10^2 Hz^2/Hz (not shown in the figure).

The laser's performance was evaluated by measuring its linewidth, and by using it in a saturation spectroscopy experiment. The linewidth of a laser can be deduced from the frequency noise spectrum, which can be measured by the use of a frequency discriminator [37]. Here, a Fabry-Pérot (FP) cavity was used for this purpose. The frequency noise spectrum G_{FM} of the laser is shown in Fig. 3. The spectrum has a $1/f$ behaviour persisting up to 50 kHz after which the noise flattens out to become governed by white noise. The flat portion of the spectrum has its origin in fluctuations due to spontaneous emission, and results in a Lorentzian shape of the oscillation line. The FWHM-linewidth is given by $\pi G_{FM} = 140$ kHz. The origin of the $1/f$ noise may be due to the small fraction of fluctuating power in neighboring longitudinal modes of the external cavity [38]. However, it may also be due to thermal fluctuations, which is typically the case for free running lasers. Various other sources such as current driver noise may further add to the total noise figure. For a recent study of external effects see e.g. reference [39]. All noise sources add to the total emission line. In order to perform the injection experiments, the original semiconductor laser inside the ECDL was replaced with a different model, better suited for operation at the slave laser wavelength near 633 nm. This permitted a direct observation of the laser line in a heterodyne measurement, where a polarization stabilized He-Ne laser was used as a local oscillator. The polarization stabilized He-Ne laser has a good long term stability [40] and sufficiently low linewidth for this purpose. The short term (0.1 s) lineshape of the ECDL was recorded with a radio-frequency spectrum analyser. A Lorentzian fit to the data gives a linewidth of 260 kHz, confirming a linewidth reduction of two to three orders of magnitude depending on diode chips.

Since no additional anti-reflection coating is applied to the front facet of the semiconductor laser, it is necessary to synchronize the tuning of the extended cavity with the semiconductor laser, i.e. the phase difference needs to be constant. This is achieved by controlling the injection current of the semiconductor laser with a small fraction of the piezo tuning voltage. With this arrangement, the ECDL can be continuously tuned in 10 GHz ranges without mode hops or change in the linewidth. The tuning characteristics were demonstrated in a saturation spectroscopy measurement of the closed $1s_5$-$2p_9$ transition in a neon discharge (Paper I).

The results of the characterization show that the constructed ECDL has a narrow linewidth, and that the laser frequency can be tuned in sufficiently wide ranges. Moreover, it was found that the frequency drift was less than 20 MHz during typical measurement times that were of the order of 10 s in this work. Thus, the constructed laser readily qualifies as a light source in precision experiments such as the optical injection experiments reported on in Papers III and IV.

4 DYNAMICS OF SEMICONDUCTOR LASERS

4.1 The class B laser

The temporal evolution of a single mode laser's output, in the absence of noise, can be described by a set of three coupled autonomous first order differential equations [8, 41, 42]. These are the equations for the electric field envelope, the electric polarization of the amplifying medium, and the population inversion responsible for the gain. The equations are governed by the relaxation rates of the respective dynamical variables, and are therefore called rate equations. The loss rate for the field envelope consists of main contributions from outcoupling and scattering, which in the case of semiconductor lasers can both be large. The polarization decays through dephasing mechanisms, typically due to collisions within the amplifying medium. The population inversion decay rate originates in the relaxation rates inherent to the laser material. The decay rates for a given laser can differ in magnitude from one another. If one or more dynamic variables decay considerably faster than the others, their long term temporal evolution will follow the slower variable(s). The number of equations, or equivalently the dimension of the phase space, can then be reduced accordingly by adiabatic elimination. Lasers can be classified according to which variables have been eliminated [7]. Class A lasers have rapidly relaxing polarization and population inversion compared to the field, leaving only one equation for the field amplitude. In class B lasers, the polarization relaxes more rapidly than the field and the population inversion, resulting in a two-dimensional phase space. Finally, in class C lasers all variables relax on the same timescale, leaving the full set of equations.

In semiconductor gain media, the scattering rates of the carriers are typically of the order of 10^{13} s^{-1} [43], which is two orders of magnitude faster than the electric field and four orders of magnitude faster than the carrier relaxation rate. The scattering results in a rapid dephasing of the polarization, which is adiabatically eliminated. The remaining dynamical variables are the electric field amplitude and the carrier density. Hence, the semiconductor laser falls under the category of class B lasers.

It is well known that the trajectory of a well behaved solution in a two-dimensional phase space cannot intersect with itself, except when the solution reaches a fixed point [44]. The trajectory can only tend towards either a fixed point or closed orbits which can be periodic limit-cycles, as time approaches infinity. Consequently, no quasi-periodic or strange attractors can be found in solitary class B lasers. The attractors for a class B laser operated above threshold are the unstable off point (zero output intensity) and the stable on point (nonzero output intensity). In semiconductor lasers the electric field relaxes considerably faster than the carriers, resulting in the characteristic relaxation-oscillation resonance, also found in many other class B lasers such as the Nd:YAG laser, and the CO_2 gas laser. In general terms, the relaxation-oscillation corresponds to the periodic exchange of energy between the oscillating electric field and the population. The laser responds to a small perturbation by damped oscillations eventually reaching the steady state; the stable fixed point becomes a stable focus in the phase plane. A large perturbation tuned near the relaxation-oscillation frequency can, however, result in instabilities and a destabilization of the steady state is possible, as will be seen in section 5. The relaxation-oscillation dynamics is a central element in understanding the behaviour of optically injected semiconductor lasers.

4.2 Key parameters of the semiconductor laser rate-equations

Besides the relaxation rates for the field amplitude and the carrier density, a number of intrinsic parameters enter the rate equation for semiconductor lasers. Here, the term intrinsic is used as opposed to extrinsic, or control parameters. The semiconductor gain is a nonlinear function of the carrier density and the field intensity. In theoretical considerations relating to the work in this thesis, the gain is normally expanded around the steady state value, and the first order terms are retained. The laser gain function is then characterized by the first order partial derivatives with respect to the carrier density and field intensity, denoted differential and nonlinear gain respectively. The nonlinear gain can be seen as a measure of how the gain saturates, and has been shown to be of crucial importance in describing the relaxation-oscillation dynamics in semiconductor lasers [45–47].

The active material in semiconductor lasers has a highly asymmetric gain profile with transparency at wavelengths above the amplifying band and strong absorption at shorter wavelengths. This bears consequences to the refractive index, which can be related to the gain by the Kramers-Kronig relations. Contrary e.g. to gas lasers, an increase of the gain in semiconductor lasers by increasing the carrier density, leads to a decrease of the refractive index. The strength of the coupling between gain and refractive index can be described as [48]

$$\alpha = -\frac{\partial\chi'/\partial N}{\partial\chi''/\partial N}, \tag{1}$$

where $\chi = \chi' + i\chi''$ is the susceptibility and N is the carrier density. Typical values for semiconductor lasers are α = 2...10. The coupling between amplitude and phase will also affect the noise characteristics of semiconductor lasers. Eq. (1) was originally introduced to explain the excessively broad linewidth of single mode semiconductor lasers [49, 50]. For semiconductor lasers, the famous Schawlow-Townes formula [51] for the linewidth needs to be corrected by a factor $(1 + \alpha^2)$. As a consequence, α is commonly known as the linewidth-enhancement factor.

Because an injected field modifies the gain conditions of the solitary laser, the α-factor has a profound significance in understanding the dynamics of optically injected semiconductor lasers. The origin of the phenomena that can be seen in an injected semiconductor laser for higher injection levels can intuitively be understood as follows. The injected field lowers the threshold gain required for lasing, and the laser adjusts its gain accordingly. A lower gain leads through Eq. (1) to an increased refractive index that causes a red-shift in the laser's instantaneous frequency. A new frequency enters the system in form of a beat frequency between the instantaneous frequency and the injected signal (c.f. subsection 4.3). The interplay of the new frequency and the relaxation-oscillations then leads to complicated dynamics.

4.3 Experimental techniques of determining parameter values

The relaxation-oscillation resonance governs the modulation response of a class B laser. In practice, the relaxation frequency sets an upper limit to the modulation bandwidth, which for semiconductor lasers typically ranges from one to several tens of GHz. Because of the importance of semiconductor lasers in communications applications, this aspect has been studied extensively. Due to the combined effects of intrinsic semiconductor laser properties, the noise characteristics of the output falls into a range that is measurable with existing technology. Hence, the semiconductor laser is of interest also from a basic laser physics and quantum optics point of view. In this work, the parameter values are needed as a reference in order to compare results with theoretical work by other groups. Furthermore, the influence of the intrinsic parameters on the dynamics is studied in Paper IV.

Spontaneous emission noise in solitary semiconductor lasers excites the relaxation-oscillation resonance, and results in a peak near the relaxation oscillation frequency in the AM-spectrum of a direct measurement of the output with a photodetector [52–54]. An analytic expression for the AM-noise spectrum can be obtained from the linearized rate-equations for a single-mode laser, containing the parameters for the gain relaxation rates. The parameter values can then be determined by numerical fitting of the spectrum to data measured at radio or microwave frequencies. The direct measurement of noise yields, however, rather low signals and phase sensitive detection needs to be used. Moreover, the very weak side-modes that most single-mode diode lasers exhibit, will influence the total noise. The so called mode-partition noise [55] can considerably alter the appearance of the noise-spectrum. If the oscillations of the individual side-modes are anti-correlated, the total noise will be reduced at low frequencies, and simple analytic expressions cannot in general be obtained [20].

The AM-response of semiconductor lasers can also be studied by direct modulation of the carrier density through the injection current. In this case, the intrinsic response is obscured by the parasitic response due to the laser package and chip impedances. This can be overcome by direct modulation of the carriers through photo-mixing the signal of two tunable lasers at the active layer of the subject laser [56, 57], or through optical modulation with non-resonant injection [58].

Excited by spontaneous emission noise, the relaxation oscillation resonance induces sidebands around the main oscillating mode [59, 60], which can be measured in a heterodyne setup or by the use of an FP interferometer. The sidebands have an asymmetry in amplitude due to the α-factor. The linearized rate-equations can again be used to obtain an analytical expression that besides the gain parameters also includes the α-factor [47]. The excitation can be enhanced by four-wave mixing of a weak

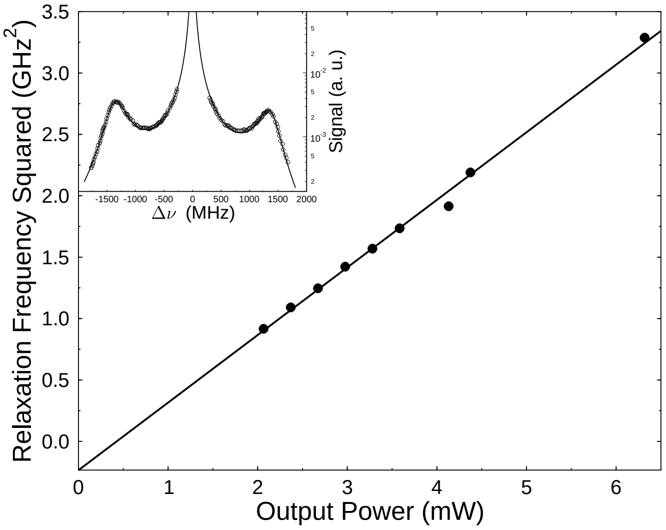


Figure 4: The relaxation oscillation frequency squared as a function of laser output power. The full circles were obtained from a fit of the regeneratively amplified signal to data, of which a typical example is shown in the inset.

nearly resonant injection signal [61, 62]. In the process, an injection signal at the optical frequency $\nu_{ML} = \nu_{SL} + \Delta\nu$ is regeneratively amplified, and a four-wave mixing signal appears at $\nu_{SL} - \Delta\nu$. The indices ML and SL stand for master and slave lasers respectively. One can then either measure the four-wave mixing signal, or the regeneratively amplified signal, as a function of detuning, $\Delta\nu$, between the lasers. Both signals contain in principle the same information, but the four wave-mixing signal was found to be symmetric and insensitive to α [62]. As discussed in subsection 4.2, the process can be viewed upon as entering a new frequency into the system at frequency $\Delta\nu$, with the amplitude corresponding to the injection signal. For weak enough amplitudes, the response can be calculated from the linearized rate-equations.

In this work, the regeneratively amplified signal was measured by heterodyning the output of the subject laser with the frequency shifted injection beam (Paper IV). The experimental setup was the same as in the injection dynamics experiments, and is described thoroughly in Paper III. A typical example of the measured signal, together with a fit of the theoretical signal [62], is shown in the inset of Fig. 4. The signal was not sampled inside the injection locking range, where the laser locks to the injection signal. An excellent agreement with theory is obtained, and the asymmetry of the sidebands is clearly observable. The squared relaxation oscillation frequency has a linear dependence

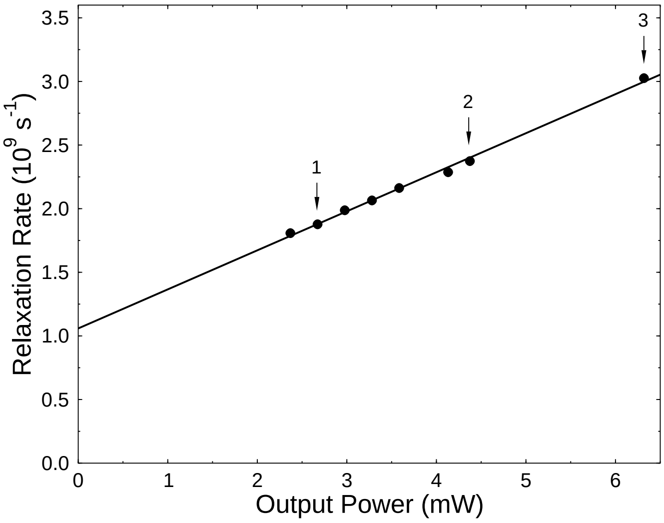


Figure 5: Total damping rate of the relaxation oscillations as a function of laser output power. The arrows denote points where the dynamical regions are mapped, see section 5.

on the output power, which can be seen in the main plot of Fig. 4. The total damping rate of the relaxation oscillations can also be extracted from the regeneratively amplified signal. The damping rate has a linear dependence on output power as can be seen in Fig. 5. The squared relaxation-oscillation frequency and the total damping rate are functions of the output power with the gain relaxation parameters as coefficients. As discussed in detail in Paper IV, a measurement of the cavity decay rate is necessary to completely determine the gain parameters. It was also shown in Ref. [62], that the α-factor cannot be determined with good accuracy from the regeneratively amplified signal.

The coupling between amplitude and phase leads to an asymmetry in the injection-locking range with respect to $\Delta\nu = 0$ [11]. The α-factor acts as a direct measure of the amount of asymmetry, and can therefore be determined from a measurement of the locking-range. Injection-locking leads to a change in the carrier density that can be observed as a change in the bias voltage across the current-driven semiconductor laser. The locking-range can be determined by measuring the voltage change as a function of $\Delta\nu$ [63]. The method was used in this work to obtain $\alpha = 3.9 \pm 0.5$, for the lasers under study. This value lies within the error limits of values found from the fitting of regenerative spectra.

With the knowledge of α, the cavity decay rate can be obtained from the modified Schawlow-Townes formula. The formula needs to be further corrected to take into account the excess noise due to outcoupling, and the spontaneous emission factor in semiconductor lasers. The excess noise is a consequence of the non-orthogonality of the resonator eigenmodes of a laser with outcoupling [64, 65]. The spontaneous emission factor describes the deviation from an ideal four level laser [20].

The parameters that affect the nonlinear dynamics and their values are summarized in Table 1. The unabridged list containing parameters used in the characterization process can be found in Paper IV. The results confirm the expected two order of magnitude difference in the decay rates of the cavity and

Parameter	Symbol	Value
Linewidth enhancement factor	α	3.9 ± 0.5
Cavity decay rate	γ_c	$4.7 \cdot 10^{11}$ s^{-1}
Carrier decay rate	γ_s	$1.06 \cdot 10^{9}$ s^{-1}
Differential gain relaxation rate	γ_n	$0.046P \cdot 10^{9}$ s^{-1}
Nonlinear gain relaxation rate	γ_p	$0.26P \cdot 10^{9}$ s^{-1}

Table 1: Parameter summary. The laser power P is given in units of mW, and ranges between 2 to 6 mW in the experiment.

carrier density, typical for lasers exhibiting relaxation oscillations. The differential and nonlinear gain relaxation rates are power dependent, and different values can therefore be accessed in an experiment by choosing the operating point of the semiconductor laser. The influence of these two parameters on the dynamics were studied in Paper IV. Together with the operating wavelength (634 nm) the knowledge of the parameters in Table 1 are sufficient for a numerical study of the rate-equations if the pump current, frequency detuning and injection strength are taken as variable parameters.

5 SEMICONDUCTOR LASERS WITH EXTERNAL OPTICAL INJECTION

5.1 Background

Frequency locking aspects of lasers with external optical injection were previously an extensively studied subject, motivated by the spectral purity transfer from the master laser to the slave in the locked condition [66]. Since high output power and spectral purity are qualities that rarely meet in a single laser, the purity transfer can be utilized to create high power laser systems with narrow linewidths from a stable narrow-band master oscillator and a separate high power slave laser. The technique has also been incorporated to semiconductor lasers for various applications [67–70]. It can be shown both theoretically and experimentally, that the linewidth transfer is complete for sufficiently high injection levels [71–73]. A complete linewidth transfer was also verified in this work, to the extent of experimental limitations (see Paper III), although this well known phenomenon was not investigated further. It is known from early theoretical considerations that injection which is too weak to achieve locking at nonzero frequency detunings can lead to pulsing in the intensity [10]. Consequently, effort was previously laid on finding the appropriate values for these parameters to achieve locking, i.e. finding the locking range.

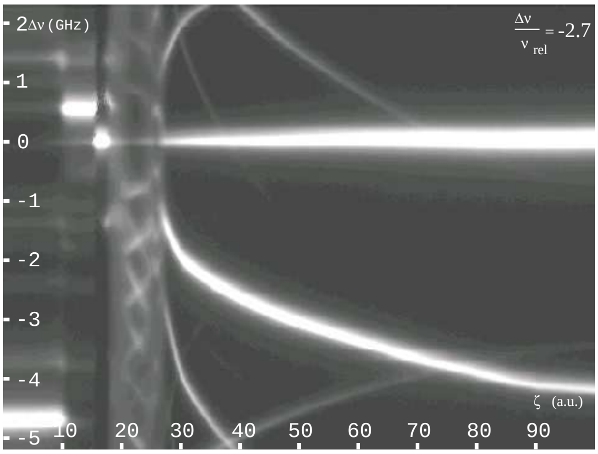
The effect of optical injection is, as already mentioned in section 3, to provide a reference point for the phase, thereby coupling an additional dynamical variable to the system. The phase space dimension is increased by one, which means that interesting dynamical behaviour can be anticipated not only in lasers with optical injection in general [74], but also in the class B laser [75, 76]. For higher injection intensities the relaxation oscillations may become undamped and lead to a destabilization of the steady-state [7, 77]. Whereas earlier experimental studies of the dynamical effects of optical injection in other lasers were few [78, 79], the semiconductor laser has received increasing interest recently. After it became clear that a nonzero linewidth-enhancement factor leads to new dynamics [11, 80, 81], several studies, both experimental and theoretical, were undertaken [82–92]. The complicated nature of the three-dimensional equations governing the system prevents a full analytical treatment of the problem. Therefore, most of the effort was concentrated on studying isolated phenomena in certain regions of the parameter space, allowing for an approximation and simplification of the rate-equations. The problem was also approached by simulating the full equations for specific parameter values. Among important results were predicting [82] and later observing [86] a period-doubling route to chaos in the semiconductor laser with external optical injection. From a nonlinear bifurcation theory point-of-view, it was established that the stable locked state is created in a saddle-node bifurcation. Further, the relaxation oscillations become undamped via a Hopf-bifurcation [75, 76, 93], creating a limit cycle which may further bifurcate by period-doublings as a parameter is changed [83, 88]. From simplified rate-equations, analytical expressions can be obtained showing that the saddle-node and Hopf-bifurcations take place on curves formed in the parameter plane consisting of the detuning between lasers and the injection strength [90, 91]. The injection strength is usually defined as the ratio of the injected intensity and the free running laser intensity.

The analysis of the simplified system already indicated a need for viewing the dynamics in a broader perspective [84, 90]. The effect of a nonzero α-factor was also recognized as a symmetry breaking feature [84]. A detailed map of the dynamics of injected semiconductor lasers in the parameter space was first obtained experimentally [94], showing that the locations where chaotic oscillation is found, form islands in the parameter space. Recently, fundamental bifurcation theory was utilized in a numerical study of the full set of rate-equations [95]. In this landmark paper, it is clearly shown how the bifurcations organize in the parameter plane for a wide range of parameter values. A gradual lifting of the symmetry with respect to detuning between lasers is seen as the α-factor is increased from zero. The rate-equations were also simulated for parameter values in the same range as those used in the experiment, with results again showing a sensitive dependence on the value of α [96]. The theoretical results [95, 96] agree well with the experiment [94] on a qualitative level. This suggests that the rate-equations reproduce the dynamics of an optically injected laser correctly, at least to the accuracy in the experiment. This fact has very recently encouraged further theoretical exploration of the rate-equations, showing an enormous wealth of phenomena, including multistability [97] and excitability [98]. It has also been predicted that the transition to chaos depends on the direction taken in the parameter plane [99], which illustrates the necessity in viewing the dynamics from a broad perspective. Another interesting development is the synchronization of chaotic lasers [100], which has now also been applied to semiconductor lasers [101–103]. Chaos synchronization is currently under consideration as a form of secure communication [104, 105].

5.2 Dynamical characteristics from continuously sampled spectra

The combination of intrinsic properties of semiconductor lasers leads to high relaxation-oscillation frequencies. Here, *high* should be interpreted in the context of frequencies that can be measured with present technology. As can be seen in Fig. 4, the relaxation-oscillation frequency of the particular type of laser used in this work ranges roughly between 1 and 2 GHz. Moreover, optical injection will increase the resonance frequency significantly, a quality which holds technological potential for high bandwidth communication [106]. The high frequencies, or equivalently the fast time-scales, lead to a difficulty in obtaining useful time-series by measuring the oscillations of the slave laser's output. A reliable time-series analysis requires a long time-series. On the other hand, one needs sufficiently high-frequency sampling in order to capture the full waveform, which very often is non-sinusoidal and contains several harmonics. Due to technological issues such as limitations of detection and memory speed, it is at present time difficult to meet both requirements, resulting in either short time-segments with good resolution or long time-segments with poor resolution, both of little use. As a consequence much of what is known experimentally about the dynamics in semiconductor lasers, is based on measuring either optical spectra or radio-frequency spectra. In the present study both optical and radio-frequency spectra were recorded in order to distinguish amplitude and phase oscillation.

(a)



(b)

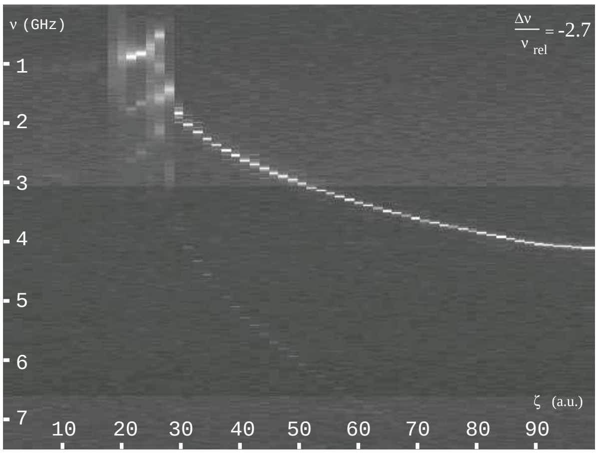


Figure 6: Example of the constructed images from one sweep. (a) optical and (b) radio-frequency spectra for the dimensionless detuning $\Delta\nu/\nu_{rel} = -2.7$, where ν_{rel} is the relaxation oscillation frequency. The origin of the vertical axis coincides with the master laser frequency. Note that the free running laser mode appears at the bottom left in (a) because it has been folded into the next fringe of the FP.

Many investigators have captured interesting dynamics by recording the spectrum of some specific isolated behaviour for certain parameter values. The map of Ref. [94] was constructed in this fashion, by varying the parameters by hand. While this method is certainly useful, it quickly becomes tedious for an extensive study. Keeping in mind that the laser's behaviour for a specific set of parameters may depend on which route was taken [99], certain aspects may be overlooked, or may not be observed by this method. In this work both the optical and radio-frequency spectrum of the slave laser is measured and recorded on a PC continuously as one parameter is increased, while the others are kept fixed. From the obtained spectra, a two-dimensional image is formed by plotting the spectra next to one-another with a shade on the grey-scale representing intensity. An example of the resulting images is seen in Fig. 6. In the figure, the injection strength relative to the power of the free running slave laser, denoted ζ, is swept upward from zero. The following features can be identified; free running laser below $\zeta = 10$, multimode behaviour between $\zeta = 10$ and $\zeta = 17$, frequency locking at $\zeta = 17$, undamped relaxation oscillations at $\zeta = 18$, structure-less spectra typical to chaos between $\zeta = 18$ and $\zeta = 28$, and limit cycle oscillation above $\zeta = 28$. Within the chaotic region a clear structure for $\zeta = 21$ to $\zeta = 25$, and at $\zeta = 27$, is seen. These features constitute the return to periodic orbits, more specifically period-two and period-three orbits for the respective regions.

The method of gathering and presenting spectra was first reported in Paper II, where it was used to demonstrate that the periodic windows form pockets within the chaotic regions of the injected laser. The periodic pockets are analogous to the periodic windows found in many nonlinear systems, e.g. the logistic map [8, 44]. Period-three oscillation was previously predicted from the rate-equations [85, 107], but was not reportedly observed experimentally. Very recently, period-three oscillation has received further interest [108].

5.3 Nonlinear regions in the parameter plane

Despite the recent advances in the field, the only extensive experimental map of the nonlinear dynamics in an injected semiconductor laser has up until now been the one found in Ref. [94][1]. The method of recording spectra and presenting them in a concise image can be used as a tool in constructing such a map. The procedure described in subsection 5.2 is repeated for a whole range of frequency detunings, and the images are analyzed. Each observation where there is a change in the spectrum is recorded as a point in the parameter plane consisting of detuning and injection strength. Since the images are stored it is easy to recognize similarities in the dynamics from detuning to detuning and as the procedure advances, clear regions with distinct dynamics form. The complete picture summarizing the basic dynamical behaviour in the parameter plane is seen in Fig 7. In the figure, a region labelled 'S' with stable injection locking opens up towards negative detunings. This is the region most often utilized in applications. Below the locking region frequency pulling phenomena 'P' can be observed. At large negative detunings the power is shifted to other longitudinal modes 'OM'. This occurs because the injection modifies the gain conditions which become more favourable for other modes. Above the locking region single-mode operation prevails, which was confirmed by monitoring the spectrum with a separate Fabry-Pérot interferometer with a large enough free-spectral range to show the nearest longitudinal side modes. Note that on a frequency scale of the mode separation ($\sim$ 60 GHz), the emission of the slave laser is still narrow band even in the chaotic regions marked

[1]A map of the dynamics of a DFB laser has been published during the writing of this thesis [109].

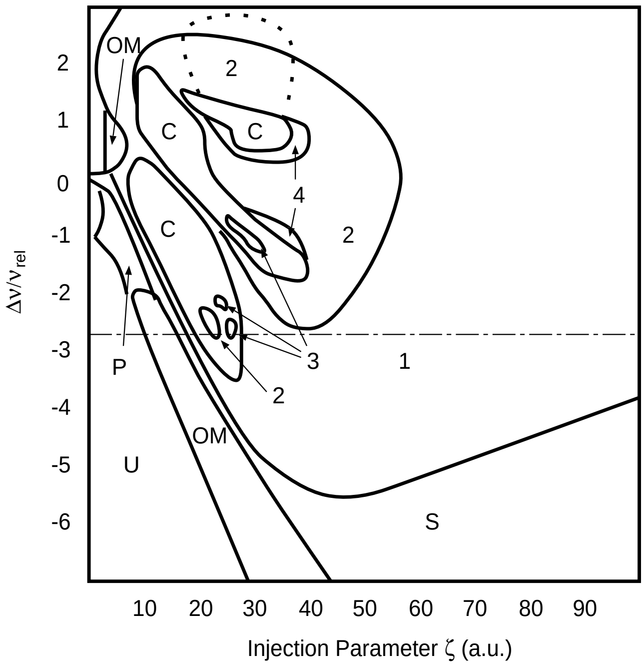


Figure 7: Nonlinear regions for the injected laser biased at 1.16 times threshold current. The dot-dashed line indicates the detuning in Fig. 6.

by 'C'. It is the intensity and phase of the single mode that oscillates violently. The integers in Fig. 7 denote oscillation with periods times the integer of the basic period, thus '1' denotes a limit-cycle, '2' a doubled oscillation etc. The length of the period is found by counting the number of peaks. A period-doubled spectrum has a peak at one half of the main frequency etc. Because of the shifting resonance frequency, which can also be seen e.g. for high ζ in Fig. 6, recognizing the main frequency can be difficult. Having access to images for different detunings eases once again the task considerably, as the behaviour can be followed throughout the parameter plane. The constructed map shows clearly how the dynamics is organized in the parameter plane. The chaotic oscillations are found in three distinct islands with periodic pockets inside them. The map shows remarkable similarities with those obtained from simulations [96], and compares well with the maps obtained with bifurcation analysis [95].

5.4 The laser current as an additional experimental parameter

In section 4 it was shown how the gain parameters depend on the output power of the laser. The power depends on the injection current which can be included in the study. The subject was approached by numerical simulation of the rate-equations in Ref. [96]. It was shown that specific regions may grow or move in the parameter plane as the pump current is increased. The results were presented on an absolute scale of the detuning which complicates the interpretation. In this work, maps of the dynamical regions were recorded for three values of the pump current at the corresponding output power and relaxation rates indicated by arrows in Fig. 5 (section 4). Now the method of presenting spectra in images described in the previous sections is fully utilized. Because the origin of the dynamics lies in the relaxation-oscillations, it is a natural choice to present the results relative to the relaxation-oscillation frequency, which is consistently done in the maps of this thesis. The constructed maps are shown to detail in three separate panels in Paper IV. It was found that the overall location of the dynamical regions is fairly constant on this chosen scale, but that specific regions grow with increasing pump current. As an example, the stable locked region is not altered significantly. The rather surprising result is that for the laser type studied here, it is the chaotic regions that grow, although the system becomes more damped (see Fig. 5). An explanation for this phenomenon can be found by considering that the total damping rate γ_r is a sum of the gain parameters, $\gamma_r = \gamma_s + \gamma_n + \gamma_p$, and that γ_n and γ_p depend linearly on the current (c.f. Table 1), but with different slopes. The effect of the differential gain is to drive an excitation, while the effect of the nonlinear gain is to dampen it through saturation. These two opposing effects may balance each other on the whole, leading to similar dynamical characteristics for different laser currents. On the other hand, the deviation from this balance may result in certain regions to change shape, as observed in this work.

Since the relaxation-oscillation frequency increases with the square root of the current, the regions grow correspondingly on the absolute frequency scale. The results of this work indicates that it is advantageous for applications, or for further research of a particular behaviour, to operate the slave laser at a high bias, since a less stable master laser is required to stay within one dynamical region.

6 CONCLUSIONS

This work deals with the experimental observation on the dynamics of optically injected semiconductor lasers. Detailed investigations have been performed in a master-slave configuration.

A simple, yet versatile laser source has been constructed utilizing semiconductor lasers with optical feedback techniques. The laser output has been characterized and found to have a high degree of coherence, and broad tunability, which both are prerequisites in high precision experiments. The constructed laser has been used to probe the modulation response of a separate semiconductor laser, yielding important information on basic dynamical characteristics. The constructed laser has also served as master laser in experiments with external optical injection.

A method of recording and presenting output spectra of semiconductor lasers subjected to external optical injection has been developed. The strength of this method has first been demonstrated in the observation of periodic pockets within the chaotic regions in a semiconductor laser. The method has further been utilized in an extensive study of the dynamical properties of the injected laser. Distinct dynamical regions regions have been mapped in the parameter plane consisting of detuning between master and slave lasers, and the injection strength. The obtained map compares well with theoretical predictions for lasers with similar values of the linewidth-enhancement factor. The study of nonlinear regions has been extended to include different operating points of the slave laser. The results show how the extent of the dynamical regions scales as a whole.

The laser with external optical injection is the subject of intense research at present time. The results of this work may serve as an excellent reference point for further exploration of the very rich nonlinear dynamics that this system exhibits.

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Stefan Eriksson

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