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Dissertationes Oeconomicae

ILKKA KIEMA

ESSAYS ON THE ECONOMICS OF INTELLECTUAL PROPERTY RIGHTS

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Foreword

Ilkka Kiema's doctoral dissertation focuses on two important areas: the role of imperfect intellectual property in economic growth and commercial piracy of proprietary products. Kiema's first essay applies a macroeconomic perspective whereas the second and third essays discuss piracy in a microeconomic setting.

The first essay studies intellectual property (IP) policy with imperfect intellectual property rights in an endogenous growth model. Kiema applies a "pool of knowledge"-approach and his model includes a hazard rate of imitation and a patentability requirement. This model, like in many growth models, has the feature with multiple equilibria. Within the growth-theoretic framework Kiema characterizes growth-maximizing patents and the role of patentability requirement. In the second essay Kiema studies intellectual property policy and commercial piracy within the framework of a model where the higher risk of a punishment associated with a pirate copy is analogous to an advertising cost, the value of which is chosen by government. A major finding is that an increase in the price of the legal software increases the price dispersion in the market for illegal products and decreases their minimum price. In the third essay Kiema extends the second essay by including network externalities to provide a characterization of the optimal pricing policy of the copyright owners in the presence of commercial piracy. This essay provides a new perspective on the debate of commercial piracy under network externalities by showing how the profit-maximizing intellectual property protection strength increases with the quality of pirate copies.

This study is part of the research agenda carried out by the Research Unit of Economic Structure and Growth (RUESG). The aim of RUESG is to conduct theoretical and empirical research with respect to important issues in industrial economics, real option theory, game theory, organization theory, theory of financial systems as well as problems in labour markets, natural resources, taxation and time series econometrics.

RUESG was established at the beginning of 1995 and has been one of the National Centres of Excellence in research selected by the Academy of Finland. It has been financed jointly by the Academy of Finland, the University of Helsinki, the Yrjö Jahnsson Foundation, Bank of Finland and the Nokia Group. This support is gratefully acknowledged.

Helsinki, September 26, 2008

Erkki Koskela
Academy Professor
University of Helsinki
Director

Rune Stenbacka
Professor of Economics
Hanken School of Economics
Co-Director

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Another turning point in the development of the studies whose results are summarized in this monograph was reached in January, 2006. While I worked at the RUESG, a period of successfully completing the doctoral studies prescribed by the FDPE during 2002 and 2003 was followed by a long period of searching intensively and sometimes desperately for interesting and fruitful research topics within the economics of growth and of intellectual property rights. During this period I completed my licentiate thesis and studied the economics of information goods, but it was only on January 25, 2006, that I discovered the theoretical approach on which most of this dissertation is based.

At that time Juuso Välimäki presented devastating criticisms of my earlier attempts to model commercial piracy, and pointed out a more promising approach to modeling it. I am deeply grateful to Välimäki for suggesting to me the theoretical approach which has been developed in Chapters 3 and 4. While working on the microeconomic part of

the dissertation, I have also greatly benefited from the comments of Erkki Koskela and from the supervision and acute criticisms of Rune Stenbacka, which have essentially improved the quality of the Chapters 3 and 4 of this book.

In 2006, it also became clear to me how the model of my licenciate thesis could be reinterpreted and modified in order to turn it into a more interesting contribution to endogenous growth theory. The resulting model is discussed in the macroeconomic part (Chapter 2) of this book. I am grateful to Seppo Honkapohja and Erkki Koskela for supervising my licentiate thesis, and to Erkki Koskela and Matti Pohjola for the help and supervision that I received when I was working on Chapter 2.

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Chapter 1

INTRODUCTION

1.1. SOME BACKGROUND

According to a standard definition, an *information good* is a commodity whose main market value is derived from the information it contains (cf. Shapiro – Varian, 1999, p. 3). As the emergence of information technology has lowered the costs of storing and copying information, information goods have become infinitely expansible, and for all practical purposes *non-rival*. Non-rival goods have traditionally been called *public goods* in microeconomics, and according to microeconomic theory non-excludable public goods are associated with *market failures*, because they are subject to *increasing returns* and increasing returns imply *imperfect competition* (Jones, 1998, pp. 73-79).

In other words, the production of the first copy of an information good constitutes a large fixed cost, but after its emergence new copies can be produced at a small marginal cost, implying that the number of the produced copies grows faster than proportionally to the amount of resources which have to be put into producing them. This further implies that in a situation of perfect competition, in which the price of the information good is equal with the marginal costs of its production, the profits from producing it would have to be negative, so that there would be no incentives for producing it.

This familiar argument provides a justification for *intellectual property rights* which grant the producers of information goods a temporary monopoly during which they can obtain profit from their work. However, it is clear that intellectual property rights involve a *Nordhaus trade-off* between two negative effects: on the one hand, a weakening of intellectual property rights causes under-provision of information goods, whereas a strengthening of intellectual property rights increases the welfare loss which is caused by monopoly distortions (cf. Nordhaus, 1969, p. 76).

The emergence of modern information technology has had a two-fold effect on the Nordhaus trade-off. The increased possibilities of producing copies of information goods illegally – like e.g. the possibility downloading music files or software from peer-to-peer networks – has a direct negative effect on the revenue from selling them and, accordingly, on the incentives to produce them. On the other hand, the development of information technology has also led to the emergence of new business models – like e.g. the Open Source Software (OSS) business model – which allow firms to earn profits from freely distributable information goods.[1] It has also been argued that if an information good is subject to strong *network externalities* – like e.g. many software products are – and if the pirate copies are used only by consumers with a low valuation, piracy might be harmless or even useful for the copyright owner.[2]

Many information goods (like e.g. music files, or files containing movies) are valuable in a direct sense as consumption goods, whereas others have value because of their use in production (cf. Quah, 2003, p. 295). The information goods which are used in production can further be divided into two groups on the basis of the role that they have in the production function of an economy. In growth accounting, the output of an economy is modeled as being given by a function of labor, capital, and general productivity, and a distinction can be drawn between the information goods such as software, which appear as *capital* in this production function, and *innovations*. Also innovations can be viewed as information goods as soon as they have been given a linguistic expression which can be digitalized, and they show up in a production function only indirectly, through the increase in factor productivity that they cause.

In such traditional macroeconomic models as the Solow model the time development of factor productivity is exogenously given, but the more recently developed *endogenous growth models* have explained the growth of factor productivity as resulting from investments into research and development. Such investments are assumed to be motivated by the monopoly rent that results from them. However, in most endogenous growth models it is assumed that an inventor of a product of a new kind or an improved design for an existing one receives a permanent monopoly for producing it, ignoring the fact that such monopolies are can be lost because of expiry of patents or imitation.

---

[1] Cf. Lerner – Tirole (2002), pp. 223-227 and de Laat (2005), pp. 1522-1529, and Mustonen (2003).
[2] See Conner – Rumelt (1991), King – Lampe (2003), Slive – Bernhardt (1998), and Takeyama (1994). Cf. the discussion in Section 1.3. below.

This dissertation discusses two aspects of the economics of information goods and intellectual property rights. In each case, the aim will be to model the fact that intellectual property rights are imperfect. Chapter 2 is concerned with the macroeconomic problem of including imperfect intellectual property rights into an endogenous growth model. Chapters 3 and 4 discuss piracy in a microeconomic setting. Until now, most of the economic literature on piracy has been concerned with *end-user piracy*, i.e. illegal reproduction and distribution of information goods for free. In Chapter 3 I shall put forward a model of commercial (for-profit) piracy, which I shall in Chapter 4 generalize to the situation in which the considered information good is subject to network externalities.

The next two sections contains short a survey of endogenous growth theory and of the economic literature on piracy, and the subsequent sections outline the contents of the rest of the chapters of this book.

1.2. Endogenous Growth Theory

By definition, an endogenous growth model tries to explain the emergence of the production technology which is in use in an economy, whereas an exogenous growth model takes the production technology as given, without explaining it. E.g., the familiar Cass-Koopmans-Ramsey model is in this sense exogenous. In it a homogenous final good is produced in accordance with the production function[3]

$$Y = F\left(K, AL\right)$$

in which K is the amount of capital, L is the amount of labor, and the parameter A characterizes the general productivity of the economy. In this model the parameter A grows at an exogenously given, constant rate.

Historically, the first endogenous growth models postulated that the increase in the efficiency of production is determined by the accumulation of capital.[4] The obvious interpretation for a model of this type is that it represents a process of *learning-by-doing*, during which firms learn to utilize capital more efficiently as its amount increases. In more recent endogenous growth models the economy is divided into a

[3] Ramsey (1928), Cass (1965) and Koopmans (1965). Cf. Romer (2001), pp. 47-60.
[4] This is the case in e.g. the Frankel-Romer model, which was put forward in Frankel (1962) and Romer (1986). Cf. Aghion – Howitt (1998), pp. 25-29.

production sector and a research sector, and it is assumed that the improvements in technology depend on the amount of resources that the research sector is given. These normally include at least research labor, and sometimes also research capital. In a model of this type, the funding of the research sector is motivated by the monopoly rents from the new or improved products that the researchers invent.

It seems that the first model with these basic ideas was was put forward in Romer (1990; cf. also Romer, 1987). In this model a single final output is produced from an increasing variety of intermediate products in accordance with the production function (Romer, 1990, p. S83)

$$Y\left(H_Y, L, x\right) = H_Y^{\alpha} L^{\beta} \int_0^{\infty} x\left(i\right)^{1-\alpha-\beta} di$$

where L is a fixed stock of labor, H_Y is the human capital devoted to production, and $x(i)$ is the produced amount of the intermediate durable good i. Also the stock of human capital H is fixed in this model, but it is divided between two uses, production and research. Only the intermediate inputs i which for which $i \leq A$ are available at each moment of time, and A grows because of the human capital H_A employed in research. The employment in the research sector is funded by the monopoly rents which result from a permanent monopoly to each invented product, and the growth of their variety – i.e., the growth of A – makes the aggregate output Y of the economy grow.

Romer's model can be called *a model of growth through specialization,* since in it growth is based on the invention of new kinds of products. However, in actual economies growth is not based just on such specialization, but also on the *replacement* of old-fashioned products by new and better ones. As Figure 1.1 illustrates, the endogenous growth models can, broadly speaking, be divided into the models of growth through specialization and the Schumpeterian *models of growth through creative destruction*,[5] depending on whether they focus on the former or the latter aspect of economic growth.

[5] The models of this kind are called "Schumpeterian", because Joseph A. Schumpeter is famous for having claimed that "Creative Destruction", during which old consumers' goods, methods of production, and other features of the economy get replaced by new ones, is "the essential fact about capitalism" (Schumpeter, 1994, p. 83).

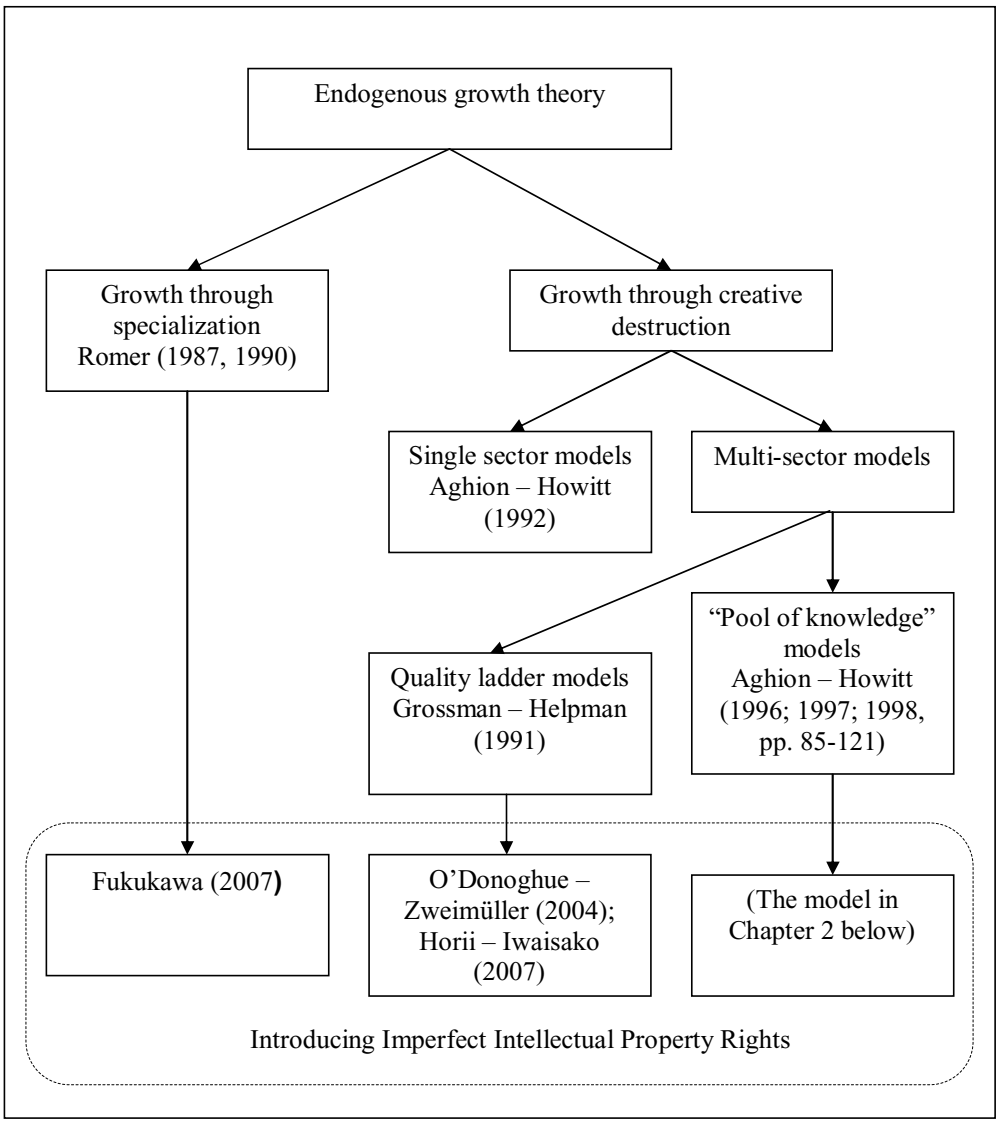

Figure 1.1. Endogenous Growth Models with Perfect and Imperfect Intellectual Property Rights.

A particularly simple Schumpeterian growth model is the one studied in Aghion – Howitt (1998, pp. 53-64; cf. also Aghion – Howitt, 1992). In this model a single final good is produced from an intermediate good in accordance with the production function

$$y = Ax^{\alpha}$$

where x is the amount of the intermediate good and A characterizes the level of the available technology. There is a fixed stock of labor, which can be used in either production or research. The workers of the production sector are involved in producing the intermediate good, whereas the workers of the research sector produce innovations. The arrival of innovations is a Poisson process, whose arrival rate is proportional to the size of the research labor force, and each innovation increases the parameter A by a constant factor. Again, the income of the workers of the research sector results from a monopoly rent to the innovations that they produce, and this monopoly always lasts until the emergence of the next innovation.

Unlike this simple model, the more realistic Schumpeterian growth models contain many sectors, but in all of them, the results of research efforts consist in the replacement of old-fashioned products or production technologies by new ones. The multi-sector Schumpeterian growth models can be divided into two groups on the basis of the way in which the size of the quality improvement that an innovation yields is determined.

Until now, the large majority of the Schumpeterian growth models have been *quality ladder models*, in which each innovation corresponds to a quality improvement of a fixed size to an earlier product of the same sector. For example, in the quality ladder model which was put forward in Grossman – Helpman (1991) there is a continuum $[0,1]$ of products, and the quality of a design of a product i is characterized by a number which is of the form $q_J(i) = \lambda^J$ for some J, where λ is a constant.[7] This equation can be interpreted as meaning that the considered design corresponds to the Jth step on the quality ladder. If in this model the highest-quality product in some sector has i corresponds to the Kth step on the quality ladder, an innovation in that sector will produce a design which corresponds to its $(K+1)^{\text{th}}$ step. In other words, the quality value which corresponds to the best available design gets always multiplied by the constant λ because of the innovation.

Since this assumption implies that the quality of a new design for a given product is determined solely by the quality of its currently used design, it rules out all forms of knowledge spill-over from the highly developed sectors of the economy to its less developed sectors. The "pool of knowledge" or *leapfrogging models* are based on another, equally extreme idealizing assumption. In these models the quality of a new

[7] Grossman – Helpman (1991), p. 45. Observe the change in notation.

design for a given product is determined by the total amount of available technological knowledge, and it is independent of the quality of the previous product of the same sector.

Aghion and Howitt have put forward several models with this basis idea, but in each of them there is a continuum $[0,1]$ of sectors, and the quality of the newest design for a product in a sector *i* at time *t* is denoted by A_{it}. In these models an innovation in a sector *i* yield a design which has the quality $A_{\max,t}$, i.e. the quality which corresponds to the highest value of A_{it} among all the sectors *i* of the economy. The value $A_{\max,t}$ increases at a pace which depends on the total amount of innovations in the economy. These assumptions are motivated by the idea that the value $A_{\max,t}$ represents "the research frontier", or "pool of technological knowledge", which all innovators utilize.[8]

However, all the endogenous growth models that were considered above are subject to an obvious criticism: they have *scale effects*. I.e., in these models the growth rate on a balanced growth path is an increasing function of the size of the economy. Further, if a constant proportion of the population is involved in research in two economies of a different size, according to these models the growth rates of the economies should be proportional to their sizes, and if this proportion stays constant in an economy in which the population grows exponentially, the growth *rate* of the economy should grow exponentially in time.

However, these implausible conclusions are not backed up by evidence. The number of the scientists and engineers who are involved in research and development has grown dramatically in most Western countries during the last decades, but this has not led to any comparable increase in the growth rates, which show no clear trend.[9] It also seems that the long run growth rates in developed Western countries are not significantly different, unlike one would expect if they were determined by the factors that appear in the endogenous growth models when they are applied to each country separately (Evans, 1996).

---

[8] Aghion – Howitt (1998), pp. 85-88. See also Aghion – Howitt (1996) and (1997).

[9] This is dramatically illustrated by Jones (1995a), pp. 517-518, which contrasts the growth of the number of scientists and engineers engaged in R&D in France, Germany, Japan, and U.S during the period 1960-1988 with the aggregate total factor productivity growth in these countries during the same period. In each case, the number of the scientists and engineers has more than doubled, whereas the aggregate productivity growth shows no clear trend. Cf. also Jones (1995b).

An obvious answer to the latter criticism is that the endogenous growth models should be applied to the world as a whole, rather than to its individual countries (cf. Jones, 1995a, p. 519). There are a variety of ways of answering former criticism and explaining why the increase in research labor force does not show up in a corresponding increase in the growth rate. These have been given a precise formulation in the *second-generation* endogenous growth models.

In some of these models, the number of the sectors of the economy is an increasing function of the population, and it turns out that a large economy does not grow faster than a small one because in the larger economy the research efforts have to be divided between a larger number of sectors (Young, 1998). In the quality ladder model which is due to Paul Segerström (1998) innovating becomes progressively more difficult in each industry, so that on the average the amount of resources spent on each innovation increase in the course of time.[10]

Below I shall not consider the problem of eliminating scale effects from endogenous growth models. Rather, I shall address another obvious weakness of the models that were described above: they assume that intellectual property rights are perfect. In each of these models, it is assumed that an innovator receives a monopoly rent from the invented good either permanently or at least until a better innovation emerges in the same sector of the economy. However, actually the monopoly of an innovator is not permanent because patents have a finite duration, and also because a monopoly may be lost already before the expiry of the patent because of imitation. In addition, there is empirical evidence which suggests that such appropriability mechanisms as *secrecy*, *lead time*, and *complementary sales and services* would in most industries be more important than patents.[11] Clearly, innovations which are protected by secrecy or lead time can be incorporated into the framework of endogenous growth theory just as well

[10] Cf. Jones (1995a), pp. 519-521, and Jones (2005), pp. 1090-1095.

[11] See Levin et al. (1987, p. 794), Mansfield (1986), and Cohen et al. (2000). Mansfield presents a survey according to which in most industries, a large majority (more than 80%) of the commercially introduced inventions would have been introduced even without the patent system. However, patents were according to this survey nevertheless essentially more important within the pharmaceutical and chemical industries (ibid., p. 175). Cohen et al. (2000) contains an analysis of a survey in which R&D unit or lab managers were asked to evaluate the effectiveness of various appropriability mechanisms in protecting the "firm's competitive advantage" from both product and process innovations. The considered mechanisms were secrecy, patents, lead time, complementary sales and services, and complementary manufacturing. It turned out that, on the average, patents were the least central of these mechanisms, whereas secrecy and lead time were the two most important ones (ibid., pp. 9-10; cf. also Figures 1-4). Levin et al. (1987) discusses a survey which has led to similar findings (see, in particular, ibid., pp. 793-798).

as patented innovations, but only if it is postulated that the monopoly of each innovator has a finite length.

In addition to the temporal length of a patent, the policy instruments that are considered in the literature on the economics of patents include the *required inventive step* – i.e. the minimum improvement that an innovation must make to the existing products if it counts as patentable – and the *lagging* and the *leading breadth* of a patent. With the lagging breadth, one means the minimum quality difference that the patented product must have with a lower-quality product if the latter may be produced without infringing on the patent. Similarly, the leading breadth of a patent is the quality improvement which a superior product must at least have if it does not infringe on the patent (cf. O'Donoghue et. al., 1998, p. 3).

None of these instruments of patent policy are explicitly considered in the early endogenous growth models that were described above, but growth models have subsequently been generalized by including them. Obviously, models of growth through specialization are unsuited for an analysis of the growth effects of the required inventive step of patents, since an innovation never replaces a lower-quality product in them, but specialization models have been used for analyzing the effects patent length in Chou – Shy (1991, 1993a) and Iwaisako – Futagami (2003), and of imitation in Helpman (1993), Kwan – Lai (2003), and Furukawa (2007).[12] In addition, Iwaisako – Futagami (2003) analyzes also the effects of *patent breadth* with a specialization model by postulating that all innovators are subject to compulsory licensing, and representing patent breadth with the royalty rate that such licensing yields (ibid., p. 248).

Unlike the models of growth through specialization, the Schumpeterian growth models could be used for analyzing the growth effects of all of the instruments of

[12] Each of these three papers is concerned with a model in which the strength of the intellectual property rights is represented by the rate of imitation. Helpman (1993) discusses the strength of intellectual property protection in the context of a model of trade between North and South, in which new products are invented in the North and get imitated in the South. Furukawa (2007) considers a model of growth through specialization in which the final sector productivity depends on past experience in using intermediate goods, and in which imitation may be growth-promoting, if it promotes the accumulation of experience. Kwan-Lai (2003) presents an account of the saddle paths of a specialization model, with which one can analyze the tradeoff between the short-run negative effects of an increase in IPR protection – i.e., that an increase in the rate of innovation causes a reduction in the resources which remain available for consumption – and its long-run positive effects, which are due to the increase in the variety of available goods in the future (p. 854). A specialization model of growth leaves out the fact that patents for the existing products decrease the incentives to produce improvements to them, and unsurprisingly, Kwan and Lai conclude that a strengthening of intellectual property rights (in the form of a decreased rate of imitation) will increase both growth and welfare when the model is calibrated by US data.

patent policy that were mentioned above. A *quality ladder model* has been used for analyzing the growth effects of imitation in Segerström (2007) and Horii – Iwaisako (2007), and O'Donoghue and Zweimüller (2004) use a quality ladder model for discussing the optimization of the patentability requirement and the leading breadth when these are allowed to differ. The growth effects of *patent length* have been investigated in the context of a single-sector quality ladder model in Horowitz – Lai (1996). A *pool of knowledge model* will be generalized to a situation of imperfect intellectual property rights in Chapter 2 of this book.

1.3. THE ECONOMICS OF PIRACY AND NETWORK EXTERNALITIES

The effects of commercial piracy on welfare and the profits of the copyright owner have been analyzed in an important, early paper Koboldt (1995), and in several papers by Dyuti S. Banerjee (see Banerjee 2003, 2006a, and 2006b). Koboldt (1995) discusses a model in which there are many information goods, and in which there is free entry to the market for pirate copies. The price of pirate copies stays positive, because their producers are faced with both costs of production and a danger of punishment, which is viewed as analogous with an increment in the production costs (ibid., pp. 136-137). In this setting the equilibrium price of pirate copies equals their "long-run marginal costs" (ibid., p. 139), which include the costs of punishments. Banerjee (2003), (2006a), and (2006b) discuss a setup with a single copyright owner and a single pirate, who is able to earn a positive profit for piracy.[13]

However, most of the rest of the fairly extensive theoretical literature on piracy is concerned with end-user piracy, rather than with commercial piracy. One strand of the literature studies the effects of piracy on *the quality and the variety* of the available information goods. E.g., Novos – Waldman (1984) show that piracy might decrease the quality of the information goods that maximize the revenue of a monopolist. This turns out to be the case in a model in which all consumers have the same valuation for the considered information good, but differing copying costs.[14]

[13] The basic features of Banerjee's model are presented in Banerjee (2003). Banerjee (2006a) generalizes the model to a situation in which the consumers and the software producers can affect government decisions by *lobbying*, whereas Banerjee (2006b) considers a case in which the monitoring costs are born by the copyright owner, rather than by the government.

[14] See Proposition 2 in Novos – Waldman (1984), p. 242.

Similarly, Johnson (1985) studies piracy in the context of a model of product differentiation, and concludes that it may decrease the variety of the available information goods in the long run.

Another part of the literature studies the effects of copyright protection on welfare and on the revenue of the copyright owner. It is clear that piracy reduces revenue in a model which contains no further elements besides the copyright owner who sells legitimate copies and end users who choose between buying one of them and using a pirate copy. However, the welfare effects of piracy turn out to be more complicated even in this simple setting.

The welfare effects of piracy have been analyzed in e.g. Yoon (2002) and Chen – Png (2003). Yoon (2002) considers a situation in which the intellectual property rights show up in a cost paid z by the consumers, i.e. in which copyright protection lowers the value that a pirate copy has for the consumers by the amount z, but does not cause extra costs for the copyright owner. In the context of this model it turns out that, also when the effects of lowering copyright protection on the incentives to produce new information goods are not taken into account, a decrease in copyright protection might increase welfare (ibid., Figure 2 on p. 337). Intuitively, a reduction in piracy has the effect of reducing the number of consumers who pay the extra cost z, and an increase in copyright protection increases welfare when this positive effect overrules the negative welfare effects of an increase in copyright protection.

Chen – Png (2003) study the welfare effects of software piracy in a more general setting, in which the policy instruments of the government include criminal sanctions against pirates, taxation of copying equipment and media, and a subsidy on the purchases of legitimate copies. They deduce the policy recommendations that a tax on the copying medium is preferable to a fine on copying, that it is optimal to subsidize the purchase of legitimate software, and also that when piracy occurs, it is welfare-increasing and yet harmless for the publisher's sales to reduce both the detection rate and the price of legitimate copies (ibid., pp. 116-117).

In each case, it turns out that piracy on the whole reduces the revenue of the copyright owner. However, it has also been suggested that piracy has also positive effects which might compensate for this negative effect (Peitz – Waelbroeck, 2006a, p. 450). Firstly, the demand for the *complementary products* (like e.g. concerts of an artist) might be increased by piracy (Gayer and Shy, 2006). In addition, the copyright owner can reduce the harmful of effects of piracy by bundling the information good

with complementary products and services (like e.g. product support for a software product) which are not available to the users of a pirate copy. Somewhat less obviously, when the considered information goods are *experience goods* and the consumers do not know in advance which products they prefer (e.g. to which recordings they would enjoy listening), the possibility to sample pirate copies might make them willing to pay more for their preferred product (Peitz – Waelbroeck, 2006b).

Further, the copyright owner might be able to *appropriate indirectly* a part of the revenue which is lost because of the pirates, and piracy might increase the value of the product because of *network externalities*. In this section, I shall still consider the last two of these possibilities in some more detail.

Indirect appropriability is possible when the copyright owner is able to charge a higher price for the shared copies of the information good. It was already pointed out in Liebowitz (1985) that the *journals which are available for photocopying in libraries* seem to be a case in point. When a publisher is able to charge a higher price for a journal from libraries than from the individuals who subscribe the journal for their private use, the publisher can appropriate indirectly a part of the value that the photocopies have for their users (ibid., pp. 949-950).

Indirect appropriation of this kind is possible when consumers are divided into "clubs", the same copy of an information good is used by all the members of each "club", and the "club members" share its costs. Situations of this kind have been studied more systematically in e.g. Bakos et al. (1999) and in Varian (2000). Varian considers a model in which the clubs are formed endogenously, i.e. in which potential club members join the club only if the information goods that they obtain by joining bring them a non-negative surplus. This is the case when, for example, a group of persons agrees to buy together a copy of a book and to share its costs equally.[15] Bakos et al. (1999) is concerned with a case in which club membership is determined exogenously, i.e. on grounds which are unrelated to the considered information goods, as it is the case when information goods are shared by a family or by a group of friends (ibid., p. 121).

[15] In Varian (2000), the clubs are endogenous in this sense, because Varian postulates – using the sharing of a book as an example – that the "willingness to pay for the book by all members of clubs that purchase the book exceeds the willingness to pay by members of clubs that do not purchase the book" (p. 475).

In Varian's setting it turns out that both the copyright owner and the consumers profit from sharing if the transaction costs of sharing are sufficiently low in comparison with the costs of producing new originals (Varian, 2000, p. 476). In the situation which is discussed in Bakos et al. (1999) there is a trade-off between an aggregation effect (ibid., p. 124) and a team diversity effect (p. 126). With the aggregation effect, Bakos et al. mean that the differences in valuations of clubs are smaller than the differences of the valuations of individuals, and that this reduction in "buyer diversity" increases profits. The team diversity effect refers to the fact that the differences in the sizes of the clubs tend to decrease the profits of the seller. Bakos et al. (1999) conclude that if all the clubs are of the same size in a model with exogenously formed clubs, sharing almost always increases profits of the seller (ibid., p. 127), but when their sizes differ, the sign of the effect that club formation has on profits depends on the relative magnitudes of the two effects.

Another obvious example of a case in which the copyright owner appropriates indirectly revenues from sharing is provided by the *video rental stores* and, more generally, by the business models which are based on renting information goods. Again, if the copyright owner can charge a higher price for the copies which are available for rent rather than used by a single individual, she will be able to appropriate a part of the extra value that sharing provides. In Varian's analysis, the possibility of sharing can even increase the profits of the copyright owner if each consumer either wishes to use the information good just for once (e.g. to watch a rentable movie just once) or if the consumers have heterogeneous tastes (Varian, 2000, 485-486).

By definition, a product exhibits *network effects* if its value to each user depends on the number of the other users that it has (Shapiro – Varian, 1999, p. 13). For example, many software products are subject to network effects, because when a consumer wishes to exchange files with other consumers, it is in her interest that the other consumers are using identical or at least compatible software products. In addition, the popularity of a software product improves the availability of complementary goods and services, such as plug-ins, product support, and training seminars, and this increases its value to its users in an indirect way (cf. Slive – Bernhardt, 1988, p. 888).

Several authors have pointed out that piracy might increase the profit of the copyright owner if the information good is subject to sufficiently strong network externalities. Clearly, piracy decreases the revenue of the copyright owner when a

consumer who would have bought a legitimate copy in the absence of piracy makes use of a pirate copy instead. However, when the consumers who would not have bought the product in any case make use of pirate copies, piracy may also have a positive effect on the revenue of the copyright owner, because in this case it increases the size of the network of its users and the valuation that the paying consumers give to it.

Piracy can be in the interest of the copyright owner when the latter effect is stronger than the former. Clearly, this cannot be the case if all consumers make use of pirate copies instead of legitimately bought ones. Slive – Bernhardt (1998) have put forward a model in which this does not happen because in it the consumers are divided into *business consumers* and *home consumers* who have a different ability to pirate. Takeyama (1994) considers a model in which the analogous result is valid because pirate copies are *of a lower quality* than legitimate ones, and it can turn out that low-valuation consumers make use of pirate copies while high-valuation consumers use legitimate ones. The model which is developed in Chapters 3 and 4 below will in this respect resemble the latter model, but its analysis turns out to be essentially more complicated, since pirate copies have a positive price in it.

1.4. Summary of the Essays

1.4.1. Intellectual Property Policy and Pool of Knowledge Growth Models

Chapter 2 discusses a pool of knowledge model in which intellectual property rights are imperfect. As it was explained above, a pool of knowledge model replaces an idealized assumption of the quality-ladder models, i.e. that all innovations constitute a quality improvement of the same size to the product on which they improve, with another, equally idealized assumption: in them it is assumed that all innovations result in a product with the topmost quality across all the sectors of the economy. Accordingly, the considered model is by itself unlikely to yield interesting quantitative predictions concerning the optimal IPR policy. However, the model may nevertheless produce qualitative insights concerning IPR policy, and insights into the prospects of developing more realistic models in the future.

The model differs from the earlier models of Aghion and Howitt in containing two instruments of patent policy, the *rate of imitation* ϕ and the *required inventive factor* δ_{SP}. It is easy to see that a pool of knowledge would not constitute an interesting framework for discussing the distinction between required inventive factor of patents and their breadth: if patents had in the model a leading breadth K which was larger than the patentability requirement δ_{SP}, in equilibrium this would have precisely the same consequences as the assumption that $\delta_{SP} = K$. However, the model could easily be generalized to include patents of a finite length. [16]

Aghion and Howitt assume in their pool of knowledge models that innovations are uniformly distributed across all sectors of the economy, which has the consequence that some innovations (i.e. the ones in the sectors in which the quality of the current product is quite close to the research frontier) are quite small. Nevertheless, in these models the innovator is always able to drive the previous incumbent out of the market and choose monopoly pricing without being faced with price competition with her (see e.g. Aghion and Howitt 1996, p. 16). However, it seems that it would be quite difficult to give a detailed account of the economic mechanism which makes the old products disappear in the context of these models.

The current model improves on Aghion and Howitt's pool of knowledge models in having two interpretations. One may follow Aghion and Howitt in assuming that the incumbent is always driven out of the market when a new innovation emerges, in which case the minimum inventive factor δ of the innovations that are actually made is always identical with the required inventive factor δ_{SP}. However, it is also possible to assume, more realistically, that each incumbent tries to compete with the entrant by lowering the price of its old-fashioned product. In this case no firms will choose to try to improve on products whose quality is too high, the inventive factor in the innovations that are actually made has a minimum δ_0 which is independent of δ_{SP}, and $\delta = \max\{\delta_0, \delta_{SP}\}$.

In the model each member of the labor force chooses whether to work in research or in production, and in equilibrium the wage w from working in production is identical with the wage w_R from working in research. The latter is determined by the value of

[16] This could be achieved by postulating that all products turn non-proprietary after the time *T*, or that the rate at which they get imitated changes after time *T*. Cf. discussion in Section 2.8 below.

patents, which depends both on the total amount of research in the future, and on the way in which research efforts are divided between different sectors. Recently Cozzi et al. (2007) have demonstrated that the decision to divide research efforts equally between sectors can be motivated by the ambiguity aversion of the investors, and I shall follow most of the previous endogenous growth literature in focusing on the equilibria in which research labor is divided equally between the sectors in which there is research. It will be seen below that after this restriction has been made, it becomes possible to determine the balanced growth paths of the model because, by definition, the size of the research sector stays constant on a balanced growth path. However, it is not obvious which assumptions should be made concerning the amount of research in the future when the system is not on a balanced growth path originally.

Segerstrom (1998, pp. 1298-1301) demonstrates that in his quality ladder model there is under reasonable restrictions to the parameter values a unique saddle path which approaches the situation of balanced growth. It is natural to ask whether Segerstrom's argument can be applied to pool of knowledge models. This question will be answered in Section 2.4. below: it turns out that this argument is applicable to Aghion and Howitt's earlier pool of knowledge models with perfect intellectual property rights, and also to a model with a positive imitation rate, but not to a model in which the required inventive factor is larger than 1.[17]

The balanced growth paths of the model can be characterized in a relatively simple way in terms of a function $F(g,\phi,\delta)$ called the *research incentive function*. This function expresses the ratio w_R/w of the wage in the research sector and in production for the constant growth rate g, imitation rate ϕ, and the required inventive step δ, so that on a balanced growth path $F(g,\phi,\delta)=1$. The graph $F(g,\phi,\delta)=\eta$ has a simple economic interpretation: each point on this graph represents a balanced

[17] This is because it is essential for Segerstrom's argument that the interest rate has an expression which does not explicitly contain $v(t)$, i.e. the value of an innovation. This turns out to be the case in Segerström's model, because in it $\dot{v}(t)/v(t)$ has an expression which does not explicitly depend on $v(t)$ or the interest rate (cf. Segerstrom, 1998, p. 1298 and p. 1300). However, no analogous expression exists in the currently considered model when $\delta > 1$.

growth path of a more general model in which the workers may have a preference for science or for production, shown in a wage differential between the two sectors.[18]

It also turns out that the model leads to simple definition of a growth trap which is due to slow growth in the past. Intuitively, this is a situation in which the equilibrium value of the growth rate g is zero if the state of the economy corresponds to very slow constant growth in the past, although a balanced growth path with a positive growth rate exists for the same parameter values. It turns out that growth traps which are due to slow growth in the past are possible when the imitation rate is small but positive.

As it seems plausible, the growth maximizing value of the imitation rate is zero, and this is also the welfare maximizing value of the imitation rate if the knowledge increase parameter γ, which characterizes the contribution that each innovation makes to the shared pool of knowledge, is sufficiently large. The growth-maximizing value of the required inventive step δ is always positive, but it is small in slowly growing economies. It also turns out that if the profitability of research is sufficiently large, the problem of choosing the growth-maximizing value of δ might fail to have an economically meaningful solution.

1.4.2. Commercial Piracy and Intellectual Property Policy

Chapter 3 presents a model of commercial piracy, which is utilized also in Chapter 4 in a modified form. The model aims at explaining a puzzling feature of the market for pirate copies: the pirate copies which are sold rather than distributed for free are homogenous goods with several manufacturers and – when they are distributed on the Internet – with almost zero reproduction costs, but they nevertheless often have a positive price.[19]

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[18] In other words, the situation in which $F(g,\phi,\delta)=\eta$ where $\eta \neq 1$ represents an equilibrium of a model in which the preference for science or for production shows itself in the fact that the wages in the two sectors satisfy the condition $w_R/w=\eta$. Such a more general model can be motivated by the fact that persons with a scientific education often seem to have a preference for research, which is shown in accepting employment in research also when it has a lower salary than employment of other kinds (cf. Stern, 2004).

[19] This is made apparent by e.g. the U.S. Department of Justice reports on court cases against large-scale commercial software pirates. See e.g. http://www.usdoj.gov/usao/vae/Pressreleases/12-DecemberPDFArchive/05/20051213petersonnr.pdf, http://www.usdoj.gov/usao/vae/Pressreleases/08-AugustPDFArchive/06/20060825ferrernr.pdf, and http://www.usdoj.gov/usao/vae/Pressreleases/04-AprilPDFArchive/07/20070420knott.pdf (accessed on October 9, 2008).

As it was mentioned above, the question what stops the price of pirate copies from sinking to zero because of a Bertrand competition between their manufacturers is not addressed by Banerjee's models of commercial piracy (Banerjee 2003, 2006a, 2006b), which contain only a single commercial pirate, whereas the model of Koboldt (1995) treats copyright protection as analogous with an increment in production costs. A model with the latter basic idea leads to the predictions that all pirate copies should have the same price and that they should be available to all the potential customers of commercial pirates. However, casual empiricism suggests that there is price dispersion in the market for pirate copies, and also that pirate copies are not available to all the consumers who would be willing to buy them (as one can easily verify by e.g. trying to find functioning, illegitimate copies of recently introduced software products on the Internet). However, both of these observations can be explained if the danger of getting caught and receiving a punishment with which the commercial pirates are faced is not modelled as a cost of production, but as an advertising cost. This is the approach which will be followed in Chapters 3 and 4 below.

The model which is developed in Chapter 3 postulates that commercial pirates – to whom I shall in what follows refer as *bootleggers* – have two kinds of costs. It will be assumed that the informing of consumers increases the risk of getting caught and, accordingly, the expectation value of the cost of a punishment, and that this increase *functions analogously with an advertising cost*. Secondly, the bootleggers may also have fixed costs either because a part of the risk of punishment may be independent of whether the bootleggers inform their potential customers, or because of digital rights management (DRM) systems.

The agents of the considered model are the copyright owner, K potential bootleggers, and a continuum of consumers whose valuations for a legitimate copy of the product are uniformly distributed in $[0,1]$, and whose valuations for a pirate copy are smaller by a factor q. These agents participate in a leader-follower game in which the copyright owner first sets the price p_M of a legitimate copy of the considered information good. In a next step, each potential bootlegger decides whether to enter the market and to pay a fixed cost F. Then the bootleggers who have entered (if any) choose the number of their advertisements and their price distribution, and send them to randomly chosen consumers, after which the consumers make their buying decisions.

Each advertisement causes the advertising cost *b*. The bootleggers are not constrained to specifying the same price in different advertisements and by assumption, they cannot keep track of the consumers to whom they have already sent an advertisement, so that the same bootlegger might send several advertisements to the same consumer. This implies that the form of advertising which is considered in the current model resembles the description of advertising in Butters's classical model of advertising (Butters, 1977), in which advertisers send advertisements of a homogenous product to randomly chosen consumers, and the sending of an advertisement causes a cost *b*.

It is easy to see that when only one bootlegger enters the market, in equilibrium she will specify the same price $p_{\max}$ in all her advertisements. However, when more than one bootleggers enter, there will be price dispersion: in this case the largest price in the advertisements will still have the value $p_{\max}$, but the prices in the advertisements are now distributed between $p_{\min}$ and $p_{\max}$, where $p_{\min}$ is a decreasing function of the number of the active bootleggers.

In the model the parameters *b* and *F* are viewed as *policy variables* which are affected by the government and by DRM systems. In Chapter 3 I analyze the effects of these variables on the markets for pirated and legitimate copies of information goods, and in particular, on the revenue of the copyright owner, both when the price of legitimate copies p_M is exogenously given and when the copyright owner has chosen it optimally.

It turns out that when both p_M and the number of the bootleggers *k* on the market are fixed, the demand of the copyright owner is an increasing function of the advertising cost *b* and a decreasing function of the quality *q* of the pirate copies. Further, the demand is a decreasing function of the number of bootleggers if p_M is not so large that all consumers prefer pirate copies to legitimate copies. Otherwise, the demand is independent of the number of the bootleggers.

These results make it natural to ask how the number *k* of the bootleggers who pay the fixed cost and enter the market depends on the policy variables *F* and *b*. It is easy to see that for each fixed value of p_M an increase in *F* has always a non-positive effect on *k*, but an increase in *b* may also increase *k*. Intuitively, this is because the advertising cost may *function as a collusive device* in the sense that some consumers

can buy a pirate copy from a single bootlegger only because of it and accordingly, the advertising cost allows the competing bootleggers to charge a price which is higher than their marginal costs.[20] This positive effect might be larger than the direct negative effect that an increase in the advertising costs causes.

The discussion of the optimization problem of the copyright owner, i.e. the problem of choosing p_M so that the revenue of the copyright owner is maximized in equilibrium, shows that the optimal value of p_M is never such that some pirate copies would be so cheap that all consumers would prefer buying one of them to buying a legitimate copy. Although the expression of the revenue of the copyright owner is, in general, complicated, it turns out that for each fixed number of the bootleggers k the revenue-maximizing price p_M is characterized by a surprisingly simple condition. This condition is

$$p_M = p_{\min,k} + \frac{1-q}{2}$$

where $p_{\min,k}$ is the minimum price of pirate copies when there are k active bootleggers, and q represents the quality of the pirate copies. When k is not fixed, but determined by the cost F of entering the pirate copy market, the revenue-maximizing price p_M is always *either* the price which is determined by the above condition for some k, *or* the largest price which suffices to block the entry of one more bootlegger.

In general, the revenue of the copyright owner is a non-decreasing function of the fixed cost F and a decreasing function of q. Hence, in the absence of network effects it is always in the interest of the copyright owner that the fixed costs of commercial pirates (caused by e.g. DRM systems) are increased. However, in equilibrium the revenue of the copyright owner is sometimes increased and sometimes decreased by an increase in the "advertising cost" b. This is because an increase in b might increase the profits of the bootleggers and the incentives to enter the pirate copy market.

The other testable predictions of the model include that an increase in the price of legitimate copies increases price dispersion in the market for pirate copies. Since low-price pirate copies can be viewed as a counterpart of non-commercial forms of piracy, a possible interpretation of this result is that high prices of legitimate copies

[20] This can be contrasted with the models of actual collusions, in which prices are kept above the marginal costs by the fact that selling the considered good at a price which is below the price set by the competitors would break the collusion in subsequent periods, and reduce future profits. See e.g. Stenbacka (1990).

correspond to the coexistence of commercial and non-commercial forms of piracy. Similarly, high-quality pirate copies can be expected to be subject to more price dispersion than low-quality pirate copies.

In what follows I shall not present a *welfare analysis* of this model or the closely related model of Chapter 4, which will be considered in the next subsection. This is, perhaps, in need of a separate justification. A welfare analysis would address the question how the choice of the policy variables F and b affects the value of a welfare function. However, it is questionable whether one should include the welfare which is obtained by illegal means (like the profits from selling pirate copies, or the utility from using them) in the welfare function of a social planner.[21] In addition, since the current model is concerned with an information good which already exists, rather than with the incentives for creating new information goods, a more general model would be needed for a discussion of the other side of the Nordhaus tradeoff, i.e., of the fact that a decrease in the revenue of the copyright owner decreases the incentives for creating new information goods.

1.4.3. Commercial Piracy, Network Externalities, and Intellectual Property Policy

Chapter 4 will be concerned with the extension of the above model to network industries. This extension is motivated by the fact that, although it has been proved in the earlier literature that piracy might in a variety settings increase the revenue of a copyright owner in the presence of sufficiently strong network externalities, until now this has been proved only in the context of models of non-commercial (end-user) piracy.[22] It is not *prima facie* obvious to which extent such results generalize to commercial piracy, since end-user piracy and commercial piracy have different effects on the profit of the copyright owner. The decrease of profit which is due to a loss of market share is smaller in the case of commercial piracy, because pirate copies

[21] Cf. Sandmo (1981), in which the analogous question is considered in the context of *tax evasion*. It is problematic whether a welfare function of a model of tax evasion should include the utility of tax-evaders since one might argue that the Pareto principle should not be extended to cases in which the utility of an individual is increased by illegal means (ibid., p. 275).

[22] See, however, Banerjee (2003), pp. 113-116. Banerjee presents an analysis of the effects of network externalities on the competition between a monopolist (i.e. the copyright owner) and a single commercial pirate. In the context of this model, it turns out that the pirate diminishes the profit of the copyright owner even in the presence of network externalities.

with a positive price are less attractive to consumers, but for the very same reason also the positive effect which is caused by network externalities is smaller.

The considered model differs from the one of Chapter 3 in so far that in it the bootleggers have no fixed costs and that their number is infinite, and also in so far that the valuations of the consumers are fixed only after the copyright owner has set the price p_M of legitimate copies. When p_M has been chosen, the consumers and the bootleggers form the expectation that the total market penetration of the product will have some value n_e. This determines the quantity ξ, which is called the *valuation parameter*, in accordance with $\xi = A + Bn_e$ (where *A*, *B* are non-negative constants), and the valuation parameter determines the valuations of the individual consumers.

Just like before, the consumers form a continuum $[0,1]$, but this time the consumer θ gives the valuation $\theta\xi$ to a legitimate copy and the valuation $q\theta\xi$ to a pirate copy of the product. A Nash equilibrium of the model is a situation in which the actual market penetration *n* of the product turns out to be n_e after the bootleggers have sent their advertisements and the consumers have made their buying choices, given these valuations.

It turns out that once p_M has been fixed, the values of ξ for which the bootleggers enter the market and send advertisements form a closed interval $[\xi_L, \xi_H]$. The values of ξ can be divided into intervals also on the basis of the pricing that the bootleggers choose in equilibrium if they enter the market for the given ξ. The function $\xi_2(p_M)$ will be defined in such a way that if $\xi < \xi_2(p_M)$, in equilibrium all the consumers who can buy a pirate copy will prefer it to a legitimate copy, but when $\xi > \xi_2(p_M)$ the highest-valuation consumers prefer a legitimate copy to some (at least the highest-price) pirate copies. The function $\xi_3(p_M)$ characterizes another borderline: when $\xi > \xi_3(p_M) > \xi_2(p_M)$, the highest-valuation consumers prefer legitimate copies to all (even the cheapest) pirate copies, but when $\xi_2(p_M) < \xi < \xi_3(p_M)$, all consumers prefer the cheapest pirate copies to legitimate copies.

Analogously with the results of the previous chapter, it turns out that it is never optimal for the copyright owner that $\xi < \xi_2(p_M)$. This result is quite intuitive: the

reason why piracy has in the earlier literature sometimes turned out to be profit-increasing for the copyright owner is that it acts as a surrogate for *price discrimination*. In the current model this would mean that low-valuation consumers make use of pirate copies while high-valuation consumers buy legitimate ones, but when $\xi < \xi_2(p_M)$, all consumers find pirate copies preferable.

Just like in the simpler models of non-commercial piracy, it turns out that for some parameter specifications the revenue-maximizing price p_M is sufficiently low to block the bootleggers from the market, whereas for others the optimal p_M is such that the bootleggers enter but $\xi > \xi_3(p_M)$. It also turns out that the third alternative, which is not present in the models of non-commercial piracy – i.e., the case in which $\xi_2(p_M) < \xi < \xi_3(p_M)$, and in which expensive pirate copies cause market segmentation but cheaper ones do not – is never revenue-maximizing for the copyright owner.

Since in this model the bootleggers have no fixed costs, the strength of intellectual property rights is represented in it solely by the "advertising cost" *b*. The current model approaches a model of non-commercial piracy in the limit in which $b \to 0$, whereas large values of *b* lead to a situation of no piracy. Accordingly, the problem of choosing the value of *b* so that the revenue of the copyright owner is maximized can be viewed a generalization of the question whether non-commercial piracy is beneficial to the copyright owner in the presence of network externalities. It turns out that when the pirate copies are of a sufficiently low quality, the solution of the problem of choosing the revenue-maximizing *b* approaches the situation of non-commercial piracy (i.e. the free availability of pirate copies to all consumers), and that when they are of a sufficiently high quality, a state of no piracy is revenue-maximizing. The revenue-maximizing value of *b* can also differ from both of these corner solutions, although this seems to be the case only when the effect of piracy on the revenue of the copyright owner is rather small.

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CHAPTER II

INTELLECTUAL PROPERTY POLICY AND POOL OF KNOWLEDGE GROWTH MODELS

ABSTRACT.[1]

I consider an endogenous "pool of knowledge" model of growth through creative destruction with imperfect intellectual property rights. The parameters of the model include the hazard rate of imitation and a patentability requirement. The model leads to a natural definition of growth traps which are due to slow growth in the past, and which are equilibria of the model when the rate of imitation is small but positive. The growth-maximizing value of the imitation rate is zero, but the growth-maximizing value of the patentability requirement can have any positive value below a theoretical maximum. A low patentability requirement is growth-maximizing in slowly growing economies.

2.1. INTRODUCTION

The policy instruments that are regularly considered in the literature on the economics of patents include the *required inventive step* and the *duration* of a patent, and its *lagging breadth* and *leading breadth*.[2] Until recently, economists have considered the problem of the optimal choice of these instruments mostly within a partial equilibrium framework. On the other hand, *endogenous growth theory* studies the incentives to conduct research and development within a general equilibrium framework. This makes it possible not just to explicitly discuss the effects of intellectual property rights on growth rate, but also to consider some aspects of IPR protection that do not have any obvious representation in a partial equilibrium model.

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[1] An earlier version of this paper has been published as Kiema, I., 2007, "Intellectual Property Protection Strength in a 'Pool of Knowledge' Endogenous Growth Model", HECER Discussion Paper No 184 (http://ethesis.helsinki.fi/julkaisut/eri/hecer/disc/184).

[2] Here lagging breadth refers to the minimum quality difference that the patented product must have with a lower-quality product if the latter may be produced without infringing on the patent, and similarly, leading breadth is the quality improvement which a superior product must at least have if it does not infringe on the patent (Cf. O'Donoghue et. al., 1998, p. 3).

However, in a large majority of the endogenous growth models that have been put forward until now intellectual property rights have been perfect in the sense that a research firm which makes an innovation always gets a permanent monopoly for producing the product which corresponds to it.

While surveying the earlier literature on endogenous growth theory in Section 1.2 above, I divided the endogenous growth models into the models of growth through *specialization*, in which each newly invented design of a product increases the number of the different products on the market, and Schumpeterian models of growth through *creative destruction* in which each newly invented design of a product replaces an existing design. As Figure 1.1 of Chapter 1 illustrates, the Schumpeterian models can further be divided into the the *single-sector* and *multi-sector* models on the basis of the number of the different designs that the products on the market have at each moment of time.

Until now, most of the multi-sector Schumpeterian models have been quality ladder models, in which each innovation causes a quality improvement of the same size to an earlier product. However, Philippe Aghion and Peter Howitt have put forward several versions of a "pool of knowledge" or leapfrogging model, in which the quality of a new design for a given product is determined by the total amount of available technological knowledge in the whole economy (Aghion and Howitt, 1996, 1997, and 1998, pp. 85-121),

At the end of Section 1.2. I also shortly surveyed the earlier literature in which restricted intellectual property rights have been incorporated into the framework of endogenous growth theory. As it was seen above, the Schumpeterian growth models have been used for analyzing the growth effects of illegal imitation and of the required inventive step, the leading breadth, and the length of patents. However, until now such analyses have been conducted only in the context of quality ladder models, rather than "pool of knowledge" models.

In a Schumpeterian growth model the innovators are motivated by the profit that results from a monopoly for producing a product with an improved design. In actual economies the improvements in the designs of goods do not just make consumption shift within each sector of the economy from old-fashioned products to new ones. Rather, they shift consumption also *across industries*. In other words, when there are dramatic innovations within some industries (like computer or communications industry) but not in others, the share of the innovative sectors within the economy

grows larger.[3] This seemingly trivial observation is somewhat problematic from the perspective of a Schumpeterian quality-ladder model, since it suggests that R&D investment into improving low-quality products would yield a smaller return than the attempts to improve on high-quality products, and if this were the case, in equilibrium the low-quality products would permanently retain their low quality.

In most quality-ladder models this difficulty is overcome either by choosing the relevant utility function or production function in such a way that the demand for the products of a given sector is independent of their quality,[4] or by postulating that the difficulty of producing an innovation is proportional to the profit that it yields, so that the expected returns from R&D turns out to be identical in all sectors.[5] A more natural explanation for the fact that R&D is no concentrated to improving on the highest-quality products is that the knowledge needed for improving on one product is useful also for improving on other products, so that improving on a low-quality product becomes easier when the products of other sectors reach higher quality. This idea has been implemented into the "pool of knowledge" models, since in them the innovators make use of a shared *pool of technological* knowledge, which is represented by the quality of the newest products on the market.

Clearly, it is natural to ask whether pool of knowledge models could form the basis of an interesting approach to intellectual property policy. In what follows I shall develop and evaluate such an approach. For the sake of simplicity, I shall follow Aghion and Howitt in making the extreme assumption that each improved product receives the quality which corresponds to the current leading-edge technology (see Aghion and Howitt, 1996; 1997; 1998, pp. 85-121). However, I will not assume that an innovation would always yield a permanent monopoly to producing the invented product. Rather, intellectual property rights are restricted by two parameters, the *required inventive step* and the *imitation rate*.

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[3] For a summary of the relevant macroeconomic data concerning U.S., see Jorgensen (2005). The rapid growth of output share of the computer and the software industries in the U.S. gross domestic product is dramatically illustrated by e.g. Figure 4 in ibid., p. 753.

[4] See e.g. Segerholm (1998), O'Donoghue and Zweimüller (2004) and Horii and Iwaisako (2007). Each of these papers is concerned with a model which resembles the classical model of Grossman and Helpman (1991) in so far that the demand for the goods of a sector is independent of its current quality. However, the two latter papers incorporate imperfect intellectual property rights: O'Donoghue and Zweimüller (2004) consider the optimization of the patentability requirement and the leading breadth when these are allowed to differ, and in the model of Horii and Iwaisako (2007) the holder of a patent loses its monopoly with an exogenously given probability in each period.

[5] For a simple implementation of this idea, see Aghion – Howitt (2005), p. 71-75. Cf. also Segerstrom (2007).

The required inventive step can be given two interpretations. In Aghion and Howitt's earlier pool of knowledge models the producer of the lower-quality product always exits the market when a higher-quality good emerges, even if the quality improvement is very small, and if one accepts this postulate, the required inventive step can be viewed simply as a *patentability requirement* imposed by the government. However, as it will be explained in more detail below, if each incumbent tries to compete with the entrant by lowering prices, it turns out that research is concentrated in the sectors in which research yields a quality improvement whose size suffices to make this infeasible. Accordingly, the required inventive step can also be viewed as representing the minimum quality improvement for which the inventor is not threatened by competition with the producer of the lower-quality product. Hence, even without considerations of restricted IPRs, the current model improves on the earlier pool of knowledge models by giving economic reasons for the assumption that lower-quality products disappear from the market.

Some of the issues that growth theory should address is, obviously, the fact that the current fast economic growth did not begin earlier than during the European industrial revolution of the 18th century, although other civilizations had reached a comparable technological level earlier,[6] and the closely related question under which circumstances a state of slow growth is *self-sustaining* in the sense that slow growth in the past is the cause of slow growth in the future. It turns out that a pool of knowledge model with imitation leads to a rigorous and elegant definition of a *growth trap which is due to slow growth in the past*. Below I shall analyze the conditions under which such growth traps emerge in the current model.

To keep this chapter at a manageable length, the breadth and the duration of patents have not been included among the parameters of the model. I shall nevertheless make below some comments on the results that their inclusion would yield. It will be seen below that a pool of knowledge model does not provide an interesting framework for analyzing patent breadth, and also that patents of a finite length could be incorporated to the current framework with relatively small modifications. I also do not consider the problem of removing *scale effects* from the model. It seems that in the current

[6] An often quoted example is formed by China during the Yuan dynasty. Cf. Quah (2001), pp. 79-82.

model the most natural way to address the problem of scale effects would be to postulate that the number of sectors of the economy increases with population. [7]

I shall present the main features of the model in Section 2.2 below. Section 2.3 analyzes the properties of the momentary equilibria of the production sector of the model, and Section 2.4 shortly discusses its dynamics. Sections 2.5 and 2.6 are concerned with its balanced growth paths and the circumstances under which growth traps and multiple equilibria are possible. Section 2.7 discusses the optimization of the policy variables, the imitation rate and the patentability requirement, and Section 2.8 concludes.

2.2. THE FRAMEWORK

The earlier pool of knowledge growth models by Philippe Aghion and Peter Howitt which were mentioned above are models in which a single final good is produced from a continuum $[0,1]$ of intermediate goods. In these models technological progress consists in the discovery of new, improved designs for the intermediate goods. In what follows, Aghion and Howitt's framework will be reinterpreted as describing a continuum $[0,1]$ of *final goods* which are used by consumers with a *utility function* which has a similar form with the production function of Aghion and Howitt's model.

In the current model the government has two policy instruments with which it can affect the long-run growth rate, the rate of imitation and the required inventive step, and it is easy to see that in the limit in which there is no imitation and no required inventive step, the mathematical structure of the current model becomes identical with the structure of one of the earlier models by Aghion and Howitt, despite of its different economic interpretation.[8] The structure of the current model has been depicted in Figure 2.1 (Cf. Figure 3.1 in Aghion and Howitt, 1998, p. 86).

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[7] Cf. Jones (2005), pp. 1093-1095.

[8] More specifically, the multisector "pool of knowledge" growth models that Aghion Howitt have put forward in Aghion and Howitt (1996), Aghion and Howitt (1997), and Aghion and Howitt (1998, pp. 85-121), can all be viewed as variants of a basic form of their model, which they present in ibid., pp. 86-91. It is easy to verify that if one assumes that there is no imitation, all the results in Section 2.3 except for the analysis between the competition of an entrant and an incumbent are valid also in this model, when the final goods are viewed as intermediate goods and the utility function (3) is interpreted as a production function of a single, homogenous final good, for which the consumers' utility function is given by (4), except for conventional definition (17), which fixed the unit of wealth. Hence, the

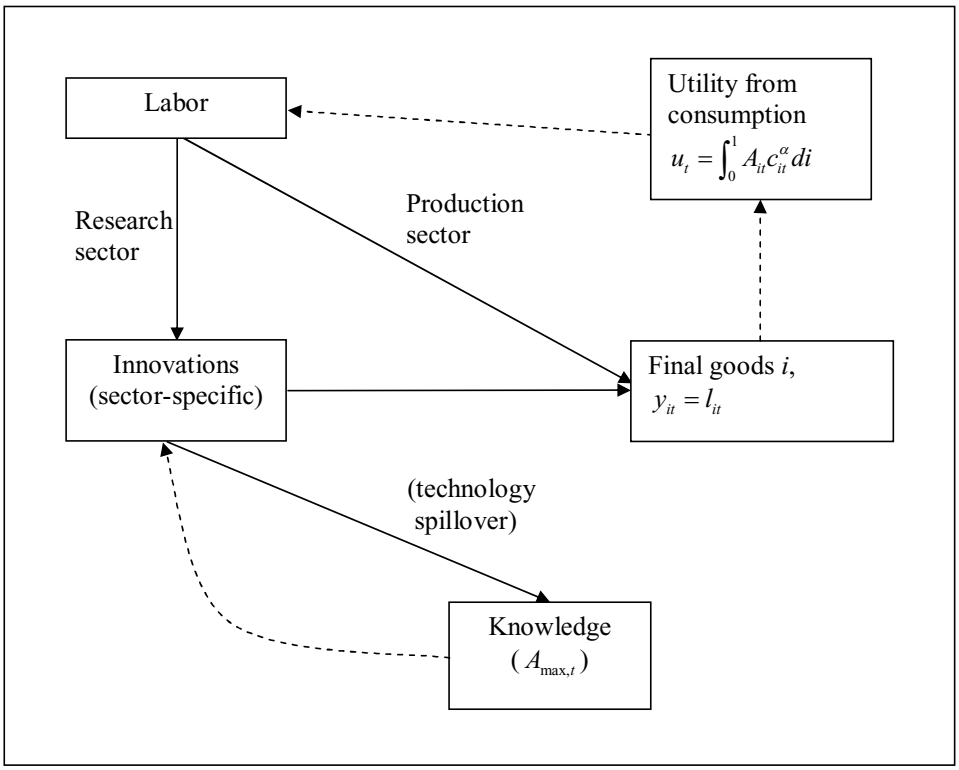

Figure 2.1. A multi-sector Schumpeterian growth model without capital.

In the current model there is a continuum of agents whose measure has been normalized to one and who can work either in production or in research. Time is continuous, and the amount of labor working in production is denoted by L. The size of the whole labor force has been normalized to 1 so that a labor force of size $1-L$ remains available for research. There is a continuum $[0,1]$ of final goods which the agents consume. Each final good i is produced using labor only in such a way that

(1) $y_i = l_i$

The instantaneous utility function of the agents is

(2) $u(t) = \int_0^1 A_{it} y_{it}^{\alpha} di$

current model becomes essentially identical with one of Aghion and Howitt's models, when one puts $\phi = 0$ (i.e. there is no imitation) and $\delta = 1$ (i.e. there is no required inventive step).

where A_{it} is the *quality parameter* which characterizes the level of technology in sector *i* at time *t* and y_{it} the consumed amount of the good *i* at time *t*. Here $0 < \alpha < 1$, so that the utility function of the final good is concave. It will be assumed that the consumption of two different variants of the good of the same sector brings no extra utility. (I.e., if the consumer consumed the amounts y_i' and y_i'' of two products of the same sector *i* with the respective quality parameters A_i' and A_i'', by assumption this makes only the contribution $\max\left\{A_i' y_i'^{\alpha}, A_i'' y_i''^{\alpha}\right\}$ to the integral in (2), so that consuming different products of the same sector is pointless). The total utility of each agent is given by the utility function

(3) $$U = \int_0^{\infty} e^{-\rho t} u(t)\, dt$$

where the rate of time preference $\rho > 0$ is an exogenously given constant.

The firms of the research sector produce sector-specific *innovations*. An innovation in a sector $i \in [0,1]$ is a new design for the good of sector *i* whose quality parameter A_i equals the current maximum value $A_{\max,t}$ of quality parameters, which represents the current stock of knowledge, or a "technology frontier". This assumption distinguishes pool of knowledge growth models from the quality ladder models, and it is motivated by the idea of a *technology spillover* from the other, more advanced sectors.

When an innovation happens in a given sector $i \in [0,1]$, the product which corresponds to the innovation becomes temporarily *proprietary*. This means that its inventor receives a temporary monopoly for producing it. This monopoly can end in one of two ways: either another innovation is made in the same sector, or the considered product turns into a *nonproprietary* product, which is produced under perfect competition. The latter event is governed by a Poisson process with the hazard rate ϕ.

In an actual economy a proprietary good which is produced by a monopolist can become a non-proprietary good, which is produced by several competing firms, in many ways. Most obviously, when the monopoly is based on a patent, this will happen if a competing firm brings an essentially identical product to the market, and the government fails to prosecute it for its violation of intellectual property rights. In what follows, I shall assume that the monopolies of innovators are based on patents,

but if one applied the current model also to the innovations which have not been patented, and which are protected only by e.g. a trade secret,[9] the parameter ϕ could be thought of as a measure of danger that the trade secret leaks out. Intuitively, the parameter ϕ is in each case a representation of the strength of intellectual property rights and other ways of protecting innovations, and its value can be affected by the government.

Innovations have also a *required inventive factor* δ_{SP} (where 'SP' refers to the social planner). The parameter δ_{SP} specifies the minimum that the ratio of the quality parameter values before and after the innovation must at least have if the innovation is patentable. In other words, since the quality parameter receives by assumption the value $A_{\max,t}$ after the innovation, an innovation in a sector *i* may be patented if and only if

(4) $$\frac{A_{\max,t}}{A_{it}} \geq \delta_{SP}$$

The flow of innovations in a sector *i* is governed by a Poisson process which has at time *t* the arrival rate

(5) $$\mu_{it} = \lambda d_{it}$$

where d_{it} is the amount of research labor in sector *i* at time *t* and the constant λ is the *efficiency parameter*, which represents the efficiency of the research sector in producing new innovations. It is also assumed that the Poisson processes that correspond to the innovations of the different researchers and the ones which turn proprietary innovations into nonproprietary ones are all independent of each other.

Finally, the innovations also increase the pool of knowledge which they utilize. The size of this pool of knowledge is represented by the "technology frontier", i.e. the maximum value $A_{\max,t}$ of the quality parameter, and the time development of $A_{\max,t}$ is determined by the equation

(6) $$\dot{A}_{\max,t} = A_{\max,t}\lambda\left(1-L\right)\ln\gamma$$

[9] A generalized interpretation of the current model in which the monopolies of inventors are not always based on patents can be motivated by the empirical evidence which suggests that such appropriability mechanisms as *secrecy*, *lead time*, and *complementary sales and services* would in most industries be more important than patents. See Levin et al. (1987, p. 794), Mansfield (1986), and Cohen et al. (2000).

Here $1-L$ is the size of the labor force in the research sector, so that $\lambda(1-L)$ is the rate at which innovations emerge in the economy as a whole, and the constant $\ln\gamma$ expresses the efficiency of the innovations in improving the level of technology.[10]

2.3. THE PRODUCTION SECTOR

In this section I shall deduce the properties of the momentary equilibrium of the production sector, taking the available labor force L and the quality parameters of both proprietary and non-proprietary goods as given. It will turn out to be convenient to specify the quality parameters in terms of the quantity

(7) $$a_i = \frac{A_i}{A_{\max}}$$

which will below be called the *relative quality* of the sector *i*. Each *non-proprietary product i* is produced under perfect condition, so that its price p_N equals its production costs. Given the production function (1), these are equal with the current wage *w*, so that

(8) $$p_N = w$$

The *proprietary products* are produced by a monopolist, i.e. the firm which has invented them. Since the number of the consumers has been normalized to 1, the consumption $y_i(t)$ of the good *i* by the representative consumer expresses also the demand of the good *i*. This easily implies that when the monopolist does not have to compete with the producers of lower-quality products, the profit of the monopolist is maximized when it chooses the price

(9) $$p_{Pr} = w/\alpha = p_N/\alpha$$

Together with the utility function (2) and the available work force *L*, the prices (8) and (9) and the labor market clearing condition determine the produced amount of each good. The labor market clearing condition can be formulated as

(10) $$\int_0^1 h_{Pr}(a)\, y_{Pr}(a)\, da + \int_0^1 h_N(a)\, y_N(a)\, da = L$$

[10] My motive for denoting this constant by $\ln\gamma$ is to keep my notation similar with the one used in Aghion – Howitt (1998; see p. 88).

where h_{Pr} and h_N are the density functions that the relative quality parameter a has among the proprietary and the nonproprietary products, and $y_{Pr}(a)$ and $y_N(a)$ denote the produced amounts of a proprietary and a non-proprietary good with relative quality parameter a.

The utility function (2) is such that in equilibrium the ratio between the consumed amounts of any two goods i and j must be identical for all consumers, and hence, it must also equal the ratio of the produced amounts of the two goods i and j. More specifically, (2), (8), and (9) imply that

$$(11)\qquad \begin{cases} y_N(a)/y_{Pr}(1) = \alpha^{-1/(1-\alpha)} a^{1/(1-\alpha)} \\ y_{Pr}(a)/y_{Pr}(1) = a^{1/(1-\alpha)} \end{cases}$$

Now one can conclude from (10) and (11) that

$$(12)\qquad y_{Pr}(1) = \frac{L}{\Psi_{Pr} + \alpha^{-1/(1-\alpha)}\Psi_N}$$

where the aggregators Ψ_{Pr} and Ψ_N are given by

$$(13)\qquad \begin{cases} \Psi_{Pr} = \int_0^1 h_{Pr}(a) a^{1/(1-\alpha)} da \\ \Psi_N = \int_0^1 h_N(a) a^{1/(1-\alpha)} da \end{cases}$$

Clearly, (11) and (12) suffice to determine the produced amounts of both proprietary and non-proprietary goods. The profit of the monopolist of a sector with the relative quality parameter value a is now seen to be

$$(14)\qquad \pi(a) = y_{Pr}(a)(p_{Pr} - w) = \frac{1-\alpha}{\alpha} a^{1/(1-\alpha)} \left(\frac{L}{\left[\Psi_{Pr} + \alpha^{-1/(1-\alpha)}\Psi_N\right]} \right) w$$

In what follows C will denote the total amount of wealth that is consumed at a time t. Since the size of population has been normalized to 1, C is numerically identical with the wealth consumed by each single agent at a moment of time t. Given the prices (8) and (9), and the optimal distribution of consumption between the different products which is given by (11), the budget constraint of a consumer whose consumption is C can be put into the form

$$(15)\qquad \begin{aligned} C &= \frac{w}{\alpha}\left(\int_0^1 h_{Pr}(a) y_{Pr}(a) da\right) + w\left(\int_0^1 h_N(a) y_N(a) da\right) \\ &= \frac{w}{\alpha}\left(\Psi_{Pr} + \alpha^{-\alpha/(1-\alpha)}\Psi_N\right) y_{Pr}(1) \end{aligned}$$

On the other hand, since in equilibrium each consumer distributes her consumption between the different goods in accordance with (11), one can conclude from (2), (11), and (12) that the instantaneous utility of the represtntative consumer equals

$$u(t)=\int_0^1 A_{it}y_{it}^{\alpha}di$$
$$\left(\int_0^1\left(aA_{\max,t}\right)h_N(a)\left(\alpha^{-1/(1-\alpha)}a^{1/(1-\alpha)}\right)^{\alpha}da+\int_0^1\left(aA_{\max,t}\right)h_{\Pr}(a)a^{\alpha/(1-\alpha)}da\right)\left(y_{\Pr}(1)\right)^{\alpha}$$
$$=A_{\max,t}\left(\Psi_{\Pr}+\alpha^{-\alpha/(1-\alpha)}\Psi_N\right)\left(y_{\Pr}(1)\right)^{\alpha}$$

When this result is combined with (15), it is seen that the consumption level C provides the utility

(16) $$u_t=\int_0^1 A_i y_i^{\alpha}di=A_{\max}\alpha^{\alpha}\left(\Psi_{Pr}+\alpha^{-\alpha/(1-\alpha)}\Psi_N\right)^{1-\alpha}\left(\frac{C}{w}\right)^{\alpha}$$

In a growth model with capital, the choice of the units of wealth at one instant of time would suffice to determine the units at all other moments of time, but since the current model does not contain capital, and since also the stock of the other goods varies in it constantly, in it the units of wealth must be chosen for each moment of time separately by convention, and also the interest rate is defined only relative to such a conventional choice. A customary choice would be to normalize the wage level to 1, but in what follows, I shall fix the units by choosing the wage level to be

(17) $$w=\alpha\left(\Psi_{Pr}+\alpha^{-\alpha/(1-\alpha)}\Psi_N\right)^{(1-\alpha)/\alpha}$$

Under this normalization of w, the utility of the representative consumer becomes simply

(18) $$u_t=A_{\max,t}C^{\alpha}$$

2.4. The Dynamics of the Model

When $V_{i,NEW}$ denotes the value of a patent to a new innovation in the sector i, the condition which characterizes the equilibrium of the research sector can be written as

(19) $$w=\lambda V_{i,NEW}$$

Since the arrival of innovations is a Poisson process, this immediately implies that if $V_{i,NEW}$ had a different value in different sectors *i*, there would be research only in the sectors in which $V_{i,NEW}$ is largest.

In general, $V_{i,NEW}$ depends on not just the total amount of research in the future, but also on the way in which research efforts are divided between the different sectors of the economy. Given that the expected profit from a proprietary innovation is identical in all the sectors in which there is research, it will be assumed below that there is the same amount of research labor in all the sectors in which there is research.[11] This implies that innovations emerge at the same rate in all the sectors in which there is research.

In what follows δ will denote the minimum value of the quality improvement $A_{\max}/A_i$ in the innovations which are actually made. If one follows Aghion and Howitt in assuming that the incumbent is always pushed out of the market when a product with a better quality emerges (even if the quality improvement should be very small), the above assumptions imply that there will be the same amount of research in all the sectors *i* in which the quality improvement $A_{\max}/A_i$ in the invented product exceeds the patentability requirement δ_{SP}. In this case δ and δ_{SP} are always identical.

However, as it was explained in Section 1, we wish to consider also an alternative interpretation for the model. Under this interpretation, the incumbent will attempt to stay on the market and sell its lower-quality product at a lower price p_{OLD}. It is easy to see that when the consumers maximize the utility function (2), they will prefer buying a new product with the quality parameter $A_{\max}$ for the price p_{NEW} to buying an old product with relative quality A_i for the price p_{OLD} only if

$$\frac{A_i}{A_{\max}} < \left(\frac{p_{OLD}}{p_{NEW}}\right)^{\alpha}$$

Hence, the monopolist will be able to charge the optimal price $p_{NEW} = w/\alpha$ even if the incumbent sinks it price to the level of the production costs $p_{OLD} = w$ if the relative quality of the old product satisfies the condition $A_{\max}/A_i > \delta_0$, where

[11] This assumption seems to be shared by all the Schumpeterian growth models which have been put forward in the earlier literature, and it seems very difficult to produce sensible estimates for the market value of patents without it. Cozzi et al. (2007) present an argument which motivates this assumption in the context of the model in Segerstrom (1998).

(20) $\delta_0 = \alpha^{-\alpha}$

Under the latter interpretation of the model, δ will be given by $\delta = \max\{\delta_0, \delta_{SP}\}$, since if in this case the social planner should choose a patentability requirement $\delta_{SP} < \delta_0$, $V_{i,NEW}$ will be maximal only in the sectors i in which the quality improvement exceeds δ_0, and the firms will act just like the patentability requirement was δ_0.

It will shortly be seen that on a balanced growth path C is a constant, so that the result (18) makes it natural to define the growth rate of the economy as

(21) $g = \dot{A}_{\max} / A_{\max}$

The main aim of this paper is to provide an analysis of the effect of the policy variables ϕ and δ on the balanced growth paths and growth traps of the current model and, in particular, their effects on g. Nevertheless, we shall also have a quick look at the out-of-equilibrium dynamics of the model.

In the discussion of its dynamics, the variable C can be viewed as a control variable, whereas the aggregators Ψ_{Pr} and Ψ_N are predetermined variables. It is easy to see that these variables suffice to determine the growth rate of the economy, since (12), (15), and (17) imply that

(22) $$L = \frac{\Psi_{Pr} + \alpha^{-1/(1-\alpha)}\Psi_N}{\left(\Psi_{Pr} + \alpha^{-\alpha/(1-\alpha)}\Psi_N\right)^{1/\alpha}} C$$

and according to (6) and (21) the growth rate is given by

(23) $g = \lambda(1-L)\ln\gamma$

A familiar argument shows that when the utility function U is given by (3) and (18), in equilibrium

(24) $$\frac{\dot{C}}{C} = \frac{g + r - \rho}{1-\alpha}$$

where r is the interest rate. Obviously, this result determines the time development of C for any given values of the predetermined variables and consumption at time $t = 0$, if the interest rate r can be expressed as a function of the current consumption and the predetermined variables. It turns out that r has such an expression when $\delta = 1$, but not necessarily otherwise.

To see why this is the case, assume that $\delta = 1$. Now patents become worthless for their owner because of the emergence of better innovations at rate $g/(\ln\gamma)$ and because of imitation at rate ϕ, so that the change in the value V_i of an innovation with relative quality *a* in a sector *i* satisfies the condition

$$\dot{V}_i = -\pi(a) + \phi V_i + \left(g/(\ln\gamma)\right)V_i + rV_i$$

When one puts $a = 1$, and applies the equilibrium condition $w = \lambda V_{i,NEW}$, (14), and (22), it turns out that when a new innovation has just emerged in sector *i* so that the value V_i of the current innovation in sector *i* equals $V_{i,NEW}$

$$(25) \qquad r = \frac{\dot{V}_i}{V_i} + \lambda\frac{1-\alpha}{\alpha}\left(\frac{C}{\left(\Psi_{Pr} + \alpha^{-\alpha/(1-\alpha)}\Psi_N\right)^{1/\alpha}}\right) - \phi - g/(\ln\gamma)$$

Here the derivative $\dot{V}_i$ should be interpreted as the change in the value of a patent *for some fixed, given new innovation*. It should be observed that when the value of a patent with the relative quality *a* is denoted by $V(a)$, $\dot{V}_i$ is different from $dV(1)/dt$ (i.e. the change in the value of an innovation with relative quality $a = 1$), since the relative quality of each innovation changes as a function of time.

Consider now two innovations which have at time *t* the relative qualities $a < 1$ and 1. Trivially, the ratio of their relative qualities remains $a/1 = a$ as long as they are on the market, so that according to (14) the ratio of the profits that they yield remains $a^{1/(1-\alpha)}$. On the other hand, when there is no patentability requirement, each innovation which is on the market at some time *t* has at that time the same probability of being on the market at any given time t' in the future. Putting these results together, it follows that

$$(26) \qquad V(a) = a^{1/(1-\alpha)}V(1)$$

and together with the equilibrium condition $w = \lambda V(1)$ and (17) this implies that

$$(27) \qquad \frac{\dot{V}}{V} = \frac{d}{dt}\left(a^{1/(1-\alpha)}\right) + \frac{(d/dt)V(1)}{V(1)} = \frac{1-\alpha}{\alpha}\frac{\dot{\Psi}_{Pr} + \alpha^{-\alpha/(1-\alpha)}\dot{\Psi}_N}{\Psi_{Pr} + \alpha^{-\alpha/(1-\alpha)}\Psi_N} - \frac{g}{1-\alpha}$$

The derivatives $\dot{\Psi}_{Pr}$ and $\dot{\Psi}_N$ which appear in (27) have simple expressions when $\delta = 1$. Clerarly, the aggregators Ψ_{Pr} and Ψ_N can be expressed in the form

$$(28)\quad \begin{cases} \Psi_{Pr} = \int_0^{A_{\max}} \left(A/A_{\max}\right)^{1/(1-\alpha)} H_{Pr}\left(A\right)dA \\ \Psi_N = \int_0^{A_{\max}} \left(A/A_{\max}\right)^{1/(1-\alpha)} H_N\left(A\right)dA \end{cases}$$

where H_{Pr} and H_N are the density functions of the parameter A among the proprietary and the non-proprietary products, respectively. Once more utilizing the fact that the rate at which old products get replaced by new ones is $g/\ln\gamma$ for all values of a when $\delta = 1$, it is now seen that

$$\dot{H}_{Pr}\left(A\right) = -\left(\phi + \frac{g}{\ln\gamma}\right)H_{Pr}\left(A\right) \text{ and } \dot{H}_N\left(A\right) = \phi H_{Pr}\left(A\right) - \frac{g}{\ln\gamma}H_N\left(A\right).$$

When these results are inserted into (28), a straightforward calculation shows that

$$(29)\quad \dot{\Psi}_{Pr} = \frac{g}{\ln\gamma} - \left(\frac{g}{1-\alpha} + \frac{g}{\ln\gamma} + \phi\right)\Psi_{Pr}$$

and

$$(30)\quad \dot{\Psi}_N = \phi\Psi_{Pr} - \left(\frac{g}{1-\alpha} + \frac{g}{\ln\gamma}\right)\Psi_N$$

Together (27), (28), and (29) imply that

$$(31)\quad \frac{\dot{V}}{V} = \frac{1-\alpha}{\alpha}\frac{g/\ln\gamma + \left(\alpha^{-\alpha/(1-\alpha)} - 1\right)\phi\Psi_{Pr}}{\Psi_{Pr} + \alpha^{-\alpha/(1-\alpha)}\Psi_N} - \frac{2g}{1-\alpha} - \frac{g}{\ln\gamma}$$

Obviously, when the initial values of the predetermined variables Ψ_{Pr} and Ψ_N and the control variable C have been given, the results (24), (25), (26), (29), (30), and (31) suffice to determine the development of the system in the future.

This above analysis constitutes the pool of knowledge analogy of the way in which the dynamics of a quality ladder model has earlier been discussed in Segerstrom (1998, pp. 1298-1302), and Segerstrom (2007). In each case, it is essential for the argument that $\dot{V}_i/V_i$, i.e. the rate at which the value of an innovation changes, has an expression which does not explicitly contain r or V_i. It is also easy to see that this construction does not generalize to the case in which $\delta > 1$, since (26) has no analogy in this case. Rather, it is now observed that unlike in e.g. the Segerstrom model, in this case $\dot{V}_i/V_i$ can only be evaluated after making some restrictive assumptions concerning the amount of research in the future.

2.5. THE BALANCED GROWTH PATHS

We now turn to the discussion of the balanced growth paths of the considered model, and for this reason we consider first in general its *steady states*, i.e. situations in which the growth rate $g = \dot{A}_{\max} / A_{\max}$ is a constant. When $g > 0$, the quality parameter of an innovation whose age is t is in this case $a = e^{-gt}$, and the innovations for which $a = e^{-gt} > 1/\delta$ cannot be replaced by a better innovation. The maximum age t_0 for which this condition is valid is

(32) $t_0 = (\ln \delta)/g = (\ln \delta)/(\lambda(1-L)\ln \gamma)$

If new innovations emerge at the constant rate $\lambda(1-L)$, the number of the innovations which are younger than this age is $(\ln \delta)/(\ln \gamma)$. However, this cannot be the case if $(\ln \delta)/(\ln \gamma) > 1$, i.e. if $\delta > \gamma$, since the measure of the sectors of the economy has been normalized to 1. The interpretation of this result is that *the current model has no balanced growth paths with a positive growth rate when* $\delta > \gamma$ since in this case the products of all sectors would end up being so close to the research frontier that improvements to them would not exceed the required inventive factor. In what follows, I shall assume that

(33) $\delta < \gamma$

When (33) is valid and the growth rate has the constant value g, within a finite time the number of the sectors in which there is research will obtain the constant value $1-(\ln \delta)/(\ln \gamma)$, so that the rate at which innovations happen in each of these sectors is

$$\frac{\lambda(1-L)}{1-(\ln \delta)/(\ln \gamma)} = \frac{g}{\ln \gamma - \ln \delta}$$

The other sectors are protected from innovation, but in both kinds of sectors, a monopolist might loose the monopoly because her product might become non-proprietary through imitation.

In what follows, $\Psi_{Pr}(g)$ and $\Psi_N(g)$ shall denote the values of Ψ_{Pr} and Ψ_N in a steady state with growth rate g. These can be determined by calculating as the limits

that Ψ_{Pr} and Ψ_N approach when g stays constant. [12] It is easy to see that if the growth rate has a constant value $g > 0$, the functions h_{Pr} and h_N approach the limits

$$(34) \qquad \lim_{t\to\infty} h_{Pr}(a) = \begin{cases} \left(1/(\ln\gamma)\right) a^{\phi/g-1} (a\delta)^{1/(\ln\gamma/\delta)}, & a < 1/\delta \\ \left(1/(\ln\gamma)\right) a^{\phi/g-1}, & a \geq 1/\delta \end{cases}$$

and

$$(35) \qquad \lim_{t\to\infty} h_N(a) = \left(a^{-\phi/g} - 1\right) h_{Pr}(a)$$

When (34) is inserted into the definition of Ψ_{Pr}, it turns out that

$$(36) \qquad \begin{aligned} \Psi_{Pr}(g) &= \frac{1}{\ln\gamma}\int_0^{1/\delta} a^{1/(1-\alpha)+\phi/g-1} (a\delta)^{1/(\ln(\gamma/\delta))}\, da + \frac{1}{\ln\gamma}\int_{1/\delta}^{1} a^{1/(1-\alpha)+\phi/g-1} da \\ &= \frac{(1-\alpha)/\ln\gamma}{1+(1-\alpha)(\phi/g)}\left(1-\left(\frac{1}{\delta}\right)^{1/(1-\alpha)+\phi/g}\left(\frac{(1-\alpha)/\ln(\gamma/\delta)}{1+(1-\alpha)(\phi/g)+(1-\alpha)/\ln(\gamma/\delta)}\right)\right) \end{aligned}$$

The results (34) and (35) also imply that

$$h_{Pr}(a) + h_N(a) = \begin{cases} \left(1/(a\ln\gamma)\right)(a\delta)^{1/(\ln\gamma-\ln\delta)}, & a < 1/\delta \\ 1/(a\ln\gamma), & a \geq 1/\delta \end{cases}$$

Hence, the sum $h_{Pr}(a) + h_N(a)$, i.e. the total number of product with a given relative quality *a*, is independent of ϕ as it, of course, should be the case. This allows us to conclude that also the sum $\Psi = \Psi_{Pr}(g) + \Psi_N(g)$ is independent of ϕ. When $\phi = 0$, there are no non-proprietary goods so that in this case $\Psi_N(g) = 0$ and $\Psi = \Psi_{Pr}(g)$. Inserting the value $\phi = 0$ into (36), it now follows that

$$(37) \qquad \Psi = \frac{1-\alpha}{\ln\gamma}\left(1-\left(\frac{1}{\delta}\right)^{1/(1-\alpha)}\left(\frac{1-\alpha}{\ln(\gamma/\delta)+(1-\alpha)}\right)\right)$$

The value of Ψ_N is now determined by the results (36) and (37) and the fact that

$$(38) \qquad \Psi_N(g) = \Psi - \Psi_{Pr}(g)$$

In general, a steady state is a balanced growth path if it satisfies the equilibrium condition $w = \lambda V(1)$, i.e. if the wage from working in production is identical with the

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[12] Obviously, in the special case in which $\delta = 1$ the steady state values of Ψ_{Pr} and Ψ_N could also be determined by inserting the values $\dot{\Psi}_{Pr} = \dot{\Psi}_N = 0$ into (29) and (30) and solving for Ψ_{Pr} and Ψ_N. It is easy to verify that the values which are obtained in this manner are identical with (36) and (38) when $\delta = 1$.

expected profit per worker from research. Here the profit flow is given by (14), and the wage is fixed by the convention (17), so that the wage is a constant on a balanced growth path. Since also the labor force involved in production must be a constant if g is constant, (14) implies that on a balanced growth path $\dot{C}=0$, and one can conclude from (24) that

(39) $\quad r=\rho-g$

As we saw above, the hazard rate with which a product is replaced by a better one is 0 as long as its age is smaller than the value t_0 defined by (32), and $g/\ln(\gamma/\delta)$ after that. Given that the relative quality of a product which is invented at t will be $a^{-g(t'-t)}$ at a subsequent point of time t', in a steady state

$$\left(V(1)\right)_t=\int_t^{t+t_0} e^{-(r+\phi)(t'-t)}\pi_{t'}\left(e^{-g(t'-t)}\right)dt' + \int_{t+t_0}^{\infty} e^{-(r+\phi)(t'-t)-(g/\ln(\gamma/\delta))(t'-(t+t_0))}\pi_{t'}\left(e^{-g(t'-t)}\right)dt'$$

Since the labor force L, the wage w, and the aggregators Ψ_{Pr} and Ψ_N all stay constant in a steady state, (14) implies that

$$\pi_{t'}\left(e^{-g(t'-t)}\right)=e^{-(g/(1-\alpha))(t'-t)}\pi_t(1)$$

and remembering that $r=\rho-g$ it is seen that

(40) $\quad \left(V(1)\right)_t=\dfrac{\pi_t(1)}{\tilde{r}(g)}$

where

(41) $\tilde{r}(g)=\left(\int_0^{t_0} e^{-(\rho+\phi+\alpha g/(1-\alpha))t}dt+\int_{t_0}^{\infty} e^{-(\rho+\phi+\alpha g/(1-\alpha))t-(g/\ln(\gamma/P))(t-t_0)}dt\right)^{-1}$

Clearly, the quantity $\tilde{r}$ is time independent, and if it were the case that $g=\phi=0$ i.e. if there was no growth and no imitation, $\tilde{r}(g)$ would be simply $\rho=r$. Accordingly, a natural way to think of $\tilde{r}(g)$ is to view it as a generalized discount factor for future profits, which takes into account not just the interest rate, but also the other effects which lower the expected value of the future profits from a patent.

When (40), (14) and the fact that $L=1-g/(\lambda\ln\gamma)$ are inserted into the equilibrium condition $w=\lambda\left(V(1)\right)$, it receives the form

(E1) $\quad F(g,\phi,\delta)=1$

where F is given by

$$(42)\quad F(g,\phi,\delta)=\lambda\frac{1-\alpha}{\alpha}\frac{\left(1-g/(\lambda\ln\gamma)\right)}{\left(\Psi_{Pr}(g)+\alpha^{-1/(1-\alpha)}\Psi_{N}(g)\right)\tilde{r}(g)}$$

In this definition it is explicitly mentioned that F depends also on the parameters ϕ and δ, which describe the strength of IPR protection and which implicitly affect $\tilde{r}(g)$, Ψ_{Pr}, and Ψ_{N}, since below we shall investigate the growth effects of the choice of ϕ and δ.

In what follows I shall refer to F as the *research incentive function*. This is because of a simple economic interpretation that one can give to the steady states in which the growth rate is positive but $F(g,\phi,\delta)\neq 1$. The equilibrium condition $w=\lambda V(1)$ is an expression of the idea that the wage is identical in production and in the research sector. However, there is empirical evidence which suggests that persons with a scientific education have a preference for research which is shown in accepting employment in research even when it has a lower salary than employment of other kinds.[13] Within growth theory, this phenomenon could be modeled by postulating a wage difference between the research sector and production.[14]

In the current framework, this could be achieved by postulating that the utility of an individual depends on the sector that she works in (because of e.g. a social status associated with science) in such a way that in equilibrium the quantity $\eta=\left(\lambda V(1)\right)/w$ is not 1. The preference for science could be modeled by giving ζ a value which is smaller than 1 and similarly, a preference for employment in production can be modeled by setting $\zeta>1$. Under this interpretation of the model, each value ζ of the function $\zeta=F(g,\phi,\delta)$ can be viewed as the strength of preference which would make g the growth rate on a balanced growth path, when the intellectual property policy is represented by ϕ and δ.

[13] Cf. Stern (2004).
[14] A growth model with this feature has been put forward in e.g. Fershtman et al. (1996; cf. p. 114).

2.6. THE GROWTH TRAPS

With a *growth trap* I shall in what follows mean an equilibrium in which the growth rate is zero, although also a balanced growth path with a positive growth rate would be possible for the same parameter values. There is a fairly trivial sense in which the current model has growth traps. In a pool of knowledge model researchers cannot freely choose the size of the quality improvement that their research yields in a given product. Rather, its size is determined by the closeness of its current design to the "research frontier" and given the idealizing assumption that the shift of the "research frontier" that each single innovation causes is infinitesimal, this implies if all products are sufficiently close to the research frontier, there are no incentives for improving on any of them.[15]

However, the growth traps of this kind can be viewed as an artifact of the modeling technique that we have chosen, i.e. of the fact that in the model it is impossible to make large improvements to the highest-quality products. As it was pointed out in the introduction, it is more interesting to ask whether a state of no growth could be caused by slow growth in the past. We are now in the position to pose this question in a precise manner.

A situation of very slow growth in the past can be represented by postulating that the values of the aggregators Ψ_{Pr} and Ψ_N are the limits of their steady-state values $\Psi_{Pr}(g)$ and $\Psi_N(g)$ when $g \to 0$, i.e. that they have the values

$$\Psi_{Pr}(0) = \lim_{g\to 0} \Psi_{Pr}(g) = \begin{cases} \Psi, & \phi = 0 \\ 0, & \phi > 0 \end{cases} \tag{43}$$

and

$$\Psi_N(0) = \lim_{g\to 0} \Psi_N(g) = \begin{cases} 0, & \phi = 0 \\ \Psi, & \phi > 0 \end{cases} \tag{44}$$

Now *a growth trap which is due to slow growth in the past* can be defined to be a situation in which $g = 0$, $\Psi_{Pr} = \Psi_{Pr}(0)$, and $\Psi_N = \Psi_N(0)$ is an equilibrium, although there is a balanced growth path with a positive growth rate for identical values of all the parameters of the model (i.e. for identical ϕ, δ, λ, α, ρ, and γ).

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[15] More specifically, when the patentability requirement δ_{SP} is larger than 1, there are no incentives for improving on proprietary products whose relative quality parameter *a* is larger than $1/\delta_{SP}$. Hence, if $a > 1/\delta_{SP}$ for all products, there are no incentives to improve on any of them, so that *g* will remain permanently zero.

Geometrically, a situation in which $g=0$ and the aggregators Ψ_{Pr} and Ψ_N have their low-growth values $\Psi_{Pr}=\Psi_{Pr}(0)$ and $\Psi_N=\Psi_N(0)$ is an equilibrium if the research incentive function satisfies the condition $F(0,\phi,\delta)<1$, so that the kind of growth traps that we are considering do not exist when *F* is a decreasing function of *g*. However, it is easy to see that *F* is not in all cases decreasing in *g*.

More specifically, definition (42) shows that a change in the steady-state growth rate *g* has three kinds of effects on the research incentive function. Firstly, larger values of *g* correspond to smaller values of multiplier $\left(1-g/(\lambda\ln\gamma)\right)$. Intuitively, this means that the supply of productive labor is smaller when the growth rate is larger, because a larger part of the population works in the research sector, and this diminishes the profits from new products. Secondly, (41) easily implies that

$$\frac{\partial\tilde{r}(g)}{\partial g}>0$$

i.e. that an increase of *g* increases the effective discount factor of future profits. Intuitively, this means that when the growth rate is higher, the expected profits are lower because the danger of loosing the monopoly for a product is larger, and because the quality of the products of the other sectors increases faster.

Finally, (36), (37), and (38) imply that

$$\frac{\partial}{\partial g}\left(\Psi_{Pr}(g)+\alpha^{-1/(1-\alpha)}\Psi_N(g)\right)=-\left(\alpha^{-1/(1-\alpha)}-1\right)\frac{\partial\Psi_{Pr}(g)}{\partial g}<0$$

so that an increase of *g* decreases the denominator in (42). Intuitively, this means that a higher growth rate *has also a positive effect* for a monopolist, because when growth is faster, a larger part of the products of competitors are proprietary, and have higher prices.

Figure 2.2 depicts a situation in which the last, positive effect is for small values of *g* larger than the two negative effects, causing the model to have multiple equilibria. Given that the main aim of this paper is to evaluate the changes of an approach based on "pool of knowledge" models in general terms, and to deduce qualitative results concerning the growth effects of intellectual property rights protection from them, in this and the subsequent figures the parameter values have not been calibrated to any particular economy. Rather, they have been chosen to illustrate the different possibilities that the current model allows for.

In Figure 2.2 there are two positive values g, $g = g_1$ and $g = g_2$, which satisfy (E1) and which accordingly correspond to balanced growth paths of the model. In addition, since $F(0,\phi,\delta) < 1$, also the situation in which $g = 0$ and the aggregators Ψ_{Pr} and Ψ_N have their low-growth values is an equilibrium of the model, so that it constitutes a growth trap which is due to slow growth in the past.

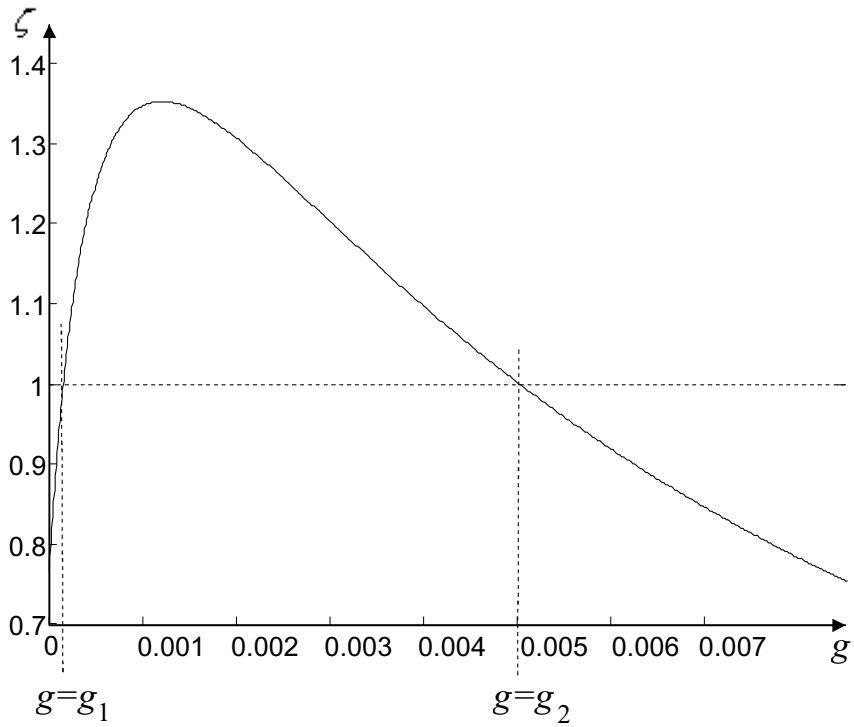

Figure 2.2. The research incentive function $\zeta = F(g,\phi,\delta)$ when $\gamma = 1.5$, $\lambda = 0.5$, $\phi = 0.005$, $\alpha = 0.95$, $\rho = 0.1$ and $\delta = \delta_0 = \alpha^{-\alpha} \approx 1.0499$.

Obviously, the positive effect of growth on the research incentive function does not exist if there is no imitation (because in this case all products are proprietary, independently of the growth rate), and it can be expected to be small also when the imitation rate is too high (because in this case most products will be non-proprietary even for high values of the growth rate). The following proposition confirms this

intuition by showing that multiple equilibria exist when ϕ is small but positive, but not necessarily otherwise.

PROPOSITION 1. *(a) If $\phi = 0$, the research incentive function is decreasing in g, so that there cannot be multiple equlibria with $g > 0$, or growth traps due to slow growth in the past.*
(b) For all values $\phi > 0$ which are sufficiently close to 0 (a sufficient condition being $\phi < \left(\alpha^{-\alpha/(1-\alpha)} - \alpha\right)\rho$), the model has for some values of the efficiency parameter λ multiple equilibria, one of which is a growth trap caused by slow growth in the past.

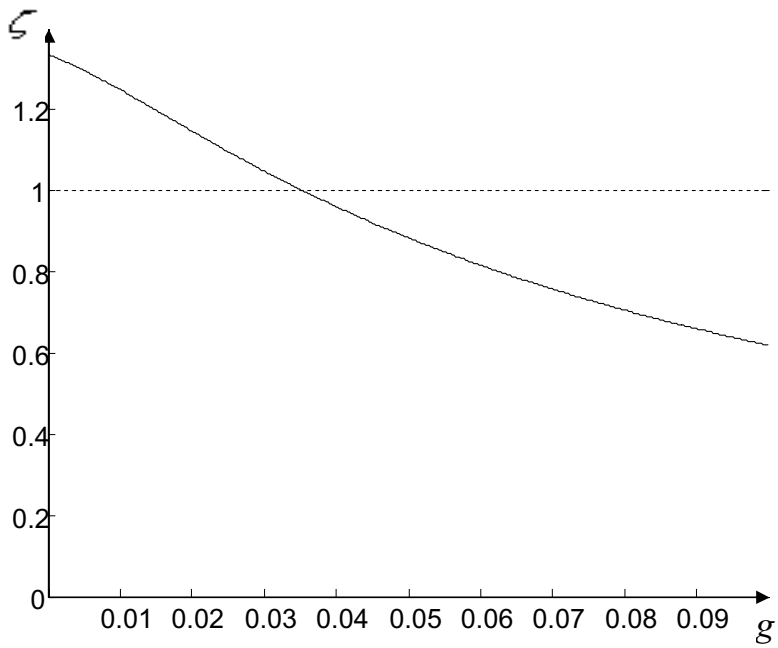

Figure 2.3. The research incentive function $\zeta = F(g, \phi, \delta)$ when $\gamma = 1.5$, $\lambda = 5$, $\phi = 0.5$, $\alpha = 0.95$, $\rho = 0.1$, and $\delta = \delta_0 = \alpha^{-\alpha} \approx 1.0499$.

Proposition 1 is illustrated with Figure 2.3, in which both the imitation rate ϕ and the efficiency parameter λ are larger than in Figure 2.2, but which corresponds to the

same parameter values otherwise. A larger imitation rate decreases the incentives for research, implying that a larger value of λ is needed for obtaining a positive growth rate. It also decreases the positive effect of growth on research incentives via the larger market share of proprietary products, and this is shown in the fact that now F is everywhere decreasing, and multiple equilibria are impossible.

2.7. The Growth and Welfare Effects of Intellectual Property Policy

In the current model intellectual property policy affects growth and welfare via the required inventive factor δ and the imitation rate ϕ. Given that the model allows for multiple equilibria, the function $g(\phi,\delta)$ is now defined to be the largest value of g which for the given ϕ and δ satisfies (E1) if such values exist, and 0 otherwise. In this section I study the growth-maximizing and welfare-maximizing choice of ϕ and δ, assuming that $g = g(\phi,\delta)$.

The practice of restricting attention to the equilibrium with the largest growth rate can be motivated by observing that in e.g. Figure 2.2 the balanced growth path which corresponds to the smaller of the two possible positive long-run growth rates – i.e. the one for which $g = g_1$ – has some quite implausible features. In this equilibrium an arbitrarily small increase in the amount of research and the corresponding increase in g would cause a situation in which the profits of research firms per worker would be larger than the wage in production so that the firms would have an incentive to increase their research efforts even more. The situation in which $g = g_1$ is nevertheless an equilibrium, because just like in most other endogenous growth models, in the current model the contribution of each firm to the growth rate of the economy is infinitesimal. In all actual economies the decisions of each firm have an effect on the growth rate of the economy (however small this effect might be), and this suggests making the following restriction on the considered equilibrium value of the growth rate g:

(E2) $\partial F(g,\phi,P)/\partial g < 0$

Excluding the implausible case in which $\partial F(g,\phi,P)/\partial g$ is precisely 0 when $F(g,\phi,P)=1$, it is clear that the largest of the equilibrium values, $g=g(\phi,P)$, will always satisfy the additional condition (E2). Accordingly, for the rest of this section I shall assume that (E1) and (E2) are both valid for the considered balanced growth paths.

Since (3) and (18) imply that the utility of each agent is infinite and a reasonable welfare analysis is impossible when $g>\rho$, I shall also assume that

(45) $g<\rho$

In the current model there is no disutility of labor and the agents have identical income, and it is natural to define the welfare function to be given simply by the utility (3) of the agents. Restricting attention to balanced growth paths, I shall below consider the normalized utility function

(46) $\tilde{U}=U/A_{\max,0}$

The definition (3) and the results (18), (22), and (38) easily imply that this is given by

(47) $$\tilde{U}=\frac{C^{\alpha}}{\rho-g}=G(\Psi_{Pr})\frac{L^{\alpha}}{\rho-g}$$

where

(48) $$G(\Psi_{Pr})=\frac{\left(\Psi-\left(1-\alpha^{\alpha/(1-\alpha)}\right)\Psi_{Pr}\right)}{\left[\Psi-\left(1-\alpha^{1/(1-\alpha)}\right)\Psi_{Pr}\right]^{\alpha}}$$

In what follows, I shall take $\tilde{U}$ to be the welfare function that the social planner wishes to maximize. The formula (48) shows that if the social planner adjusts δ and ϕ in such a way that the growth rate on the balanced growth path is increased, this has three kinds of effects on the function $\tilde{U}$. The increased growth increases welfare in the future, which is shown by the increase of the term $1/(\rho-g)$, but it demands a larger work force in research, so that the term $L^{\alpha}=\left(1-g/(\lambda\ln\gamma)\right)^{\alpha}$ is decreased in (47). Thirdly, a larger growth rate corresponds to different quality parameter distributions $\Psi_{Pr}(g)$ and $\Psi_{N}(g)$, which is shown in the change in $G(\Psi_{Pr})$. This effect is characterized by the following lemma.

LEMMA 1. *The function* $G(\Psi_{Pr})$ *receives its minimum for some value* $\underline{\Psi}_{Pr}$ *which belongs to the interval* $[0,\Psi]$. *The function* $G(\Psi_{Pr})$ *receives its largest value in this interval both when* $\Psi_{Pr}=0$ *(i.e. when all products are non-proprietary) and when* $\Psi_{Pr}=\Psi$ *(i.e. when all products are proprietary).*

In other words, the resources for producing the different products are allocated most efficiently when the products are either all proprietary or all non-proprietary, This result should be contrasted with the familiar results concerning the deadweight loss that microeconomic analyses of a monopoly yield. As Lemma 1 shows, in the current model there is nothing that would correspond to the deadweight loss when all the goods in the economy are produced by monopolists. In order to understand intuitively why this is the case, it should be observed that when the labor force L is fixed, the consumed amount of each good is the same when all the products are proprietary and when they are all non-proprietary, although the prices of the goods differ in the two cases. The two cases are otherwise different only in so far that when the goods are proprietary, the owners of the firms receive a part of the wealth that is spent on buying the goods, whereas when they are non-proprietary, all the wealth that is spent on the goods is given to their producers.

However, as we are not considering the welfare effects of the distribution of wealth between labor and the owners of firms, but implicitly assume that each consumer owns an equal share of the firms,[16] this difference does not show up in a welfare calculation. Rather, the only way in which the monopolies affect welfare is that, when monopolies cause price differences, the consumers fail to distribute their wealth between the products in an optimal fashion. (E.g., if there are two goods with the same quality, it would be socially optimal to consume the same amount of each of them, but this will not happen if one of them is competitively priced and the other one has a monopoly price.)

The growth and welfare effects of the rate of imitation ϕ are characterized by the following proposition. In the current model the effects of imitation on the research incentive function are purely negative, since larger values of ϕ correspond to a larger

[16] This assumption is made implicitly when it is assumed that welfare can be evaluated by considering a single representative consumer, whose consumption is equal with the average consumption of all consumers.

danger of the loss of monopoly, and also to lower prices of competing products, and accordingly, the growth maximizing value of ϕ is 0.

PROPOSITION 2. *Keeping the other parameters fixed, the growth rate* $g(\phi,\delta)$ *is a decreasing function of the rate of imitation* ϕ *whenever* $g(\phi,\delta)$ *is positive, so that the growth maximizing value of the rate of imitation is* $\phi = 0$. *If the knowledge increase parameter* γ *is sufficiently large (if* $\ln\gamma > \rho/\lambda$*) this is also the welfare maximizing value of* ϕ.

The assumption which appears in this proposition – i.e., $\ln\gamma > \rho/\lambda$ – means, intuitively, that the negative welfare effect of growth which is due to the decrease of the labor in production is smaller than the positive effect which is due to increased future welfare.

There is no similar general and simple answer to the question which value of the required inventive factor δ is growth-maximizing. The following proposition characterizes the optimal value of δ in general terms.

PROPOSITION 3. *Assume that the imitation rate* ϕ *and the other parameters of the model except for the required inventive factor* δ *are fixed.*
(a) If $g(\phi,\delta)$ *is positive when* $\delta = 1$, *the value* $g(\phi,\delta)$ *will be increased by a sufficiently small increase of* δ.
(b) If $\delta \geq \gamma$, *the model does not have balanced growth paths. If balanced growth paths with a positive growth rate exist for some value of* δ, *the value of* δ *which maximizes the growth rate is smaller than* $e^{2(1-\alpha)}$.

Part (a) of this proposition shows that the growth-maximizing choice of δ is never 1, and part (b) shows that the optimal patentability requirement is small when the profitability (which is measured by $1/\alpha$) is low. Clearly, Proposition 3 is compatible with both a situation in which the growth-maximizing value of δ is smaller than γ, and with a situation in which growth is an increasing function of δ in the whole interval $[1,\gamma)$. Figure 2.4 illustrates the former possibility.

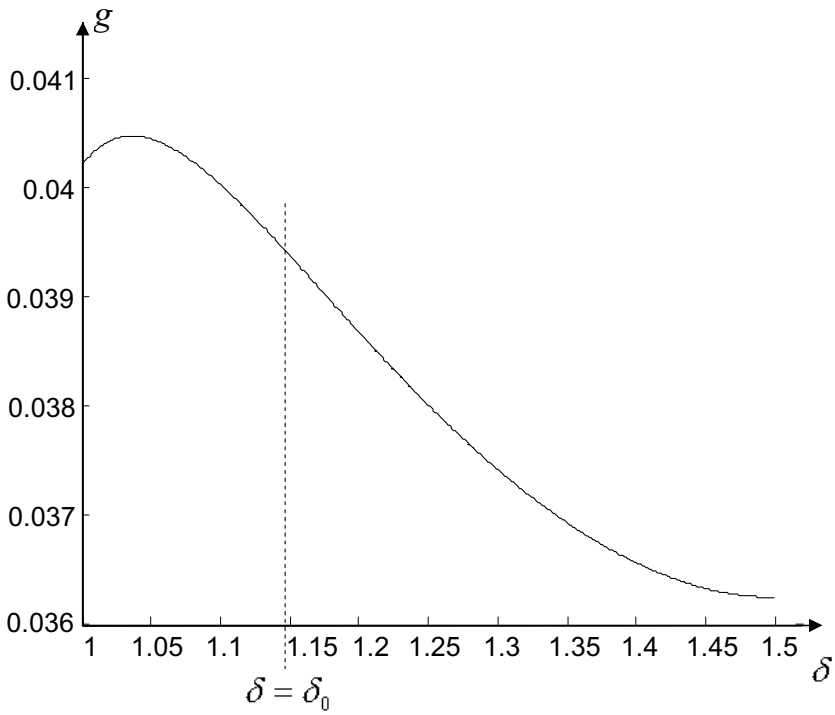

Figure 2.4. The growth rate as a function of the required inventive factor, when $\gamma = 1.5$, $\phi = 0.05$, $\lambda = 1$, $\alpha = 0.85$, and $\rho = 0.1$.

In this case $e^{2(1-\alpha)} < \gamma$, and one can conclude from Proposition 3(b) that the problem choosing δ so that growth is maximized must have a well-defined solution. When this is the case, (38) and Lemma 2 imply that the welfare-maximizing value of δ will be larger or smaller than the growth-maximizing δ, depending on the sign of $G(\Psi_{Pr})$ which represents the welfare effects of the price distribution of the products. In the situation of Figure 2.4 the growth-maximizing choice of δ is smaller than δ_0, and as it was explained in Section 4, δ_0 is the minimum inventive step for which makes it unprofitable for the incumbent to sell its product at a lower price after the new product has been invented. Hence, Figure 2.4 can be interpreted as meaning that the growth-maximizing policy would be to protect the holder of the newest patent not just from imitation but also from competition with inferior products.

Figure 2.5 represents a case in which $e^{2(1-\alpha)} > \gamma$. In this case Proposition 3 does not imply that there was a growth-maximizing value of δ. No such value exists in the situation of Figure 2.5, since in it growth is increased by an increase in δ in the whole interval $[1,\gamma)$, but the model fails to have an equilibrium if $\delta \geq \gamma$.

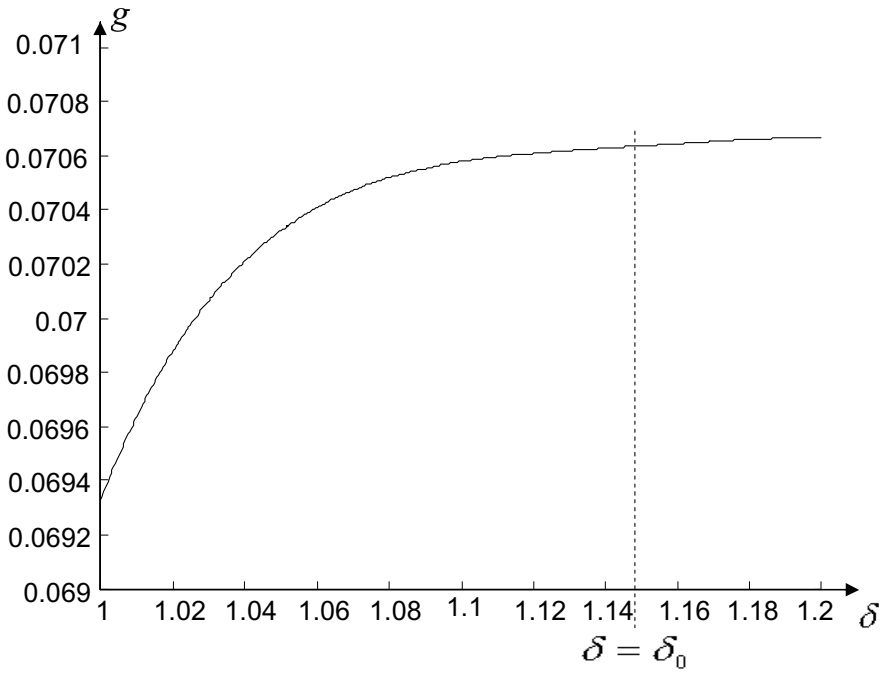

Figure 2.5. The growth rate as a function of the required inventive factor, when $\gamma = 1.2$, $\phi = 0.05$, $\lambda = 3$, $\alpha = 0.85$, and $\rho = 0.1$.

Intuitively, an increase of δ has a positive effect on the research incentive function because it increases *future* profits by lengthening the time during which the innovation is protected from being replaced by a superior product, and a negative effect because it increases the average *current* quality of the products of competitors, by shifting research efforts to the worst products on the market. Since the former effect is small in an economy with few innovations, it is to be expected that small values of δ would be optimal in an economy with a small growth rate.

I shall conclude this section with a proposition which shows that this is, indeed, the case. The proposition is concerned with the effects of the lowering of the efficiency parameter λ. A decrease in λ shifts the research incentive function *F* downwards and decreases $g(\phi,\delta)$, and as Figures 2.2 and 2.3 illustrate, such a shift will make $g(\phi,\delta)$ decrease to zero *continuously* when *F* is decreasing in *g* (as in Figure 2.3) but not necessarily otherwise (e.g. not in the situation of Figure 2.2). We wish to consider the limit in which the growth rate is small but positive, and for this reason the following proposition contains the restrictive assumption that *F* is decreasing in *g*.

PROPOSITION 4. *Assume that the imitation rate* ϕ *is fixed and suppose F is a decreasing function of the growth rate g. Define* λ_0 *by* $\lambda_0 = \inf\left\{\lambda' \middle| \left(g(\phi,\delta)\right)_{\lambda=\lambda'} > 0 \text{ for some } \delta\right\}$ *(i.e., let* λ_0 *be the threshold value of the efficiency parameter* λ *below which the growth rate will be zero, independently of how the social planner chooses the inventive factor* δ *). Now the growth-maximizing value of the required inventive factor* δ *approaches 1 when* $\lambda \to \lambda_0 +$. *If the welfare maximizing value of* δ *corresponds to a positive growth rate when* $\lambda \to \lambda_0 +$, *also this value approaches 1 when* $\lambda \to \lambda_0 +$.

2.8. CONCLUDING REMARKS

Above I have studied the generalization of a pool of knowledge growth model to a situation of imperfect intellectual property rights. In the model the very idealized assumptions of the quality ladder models – i.e. that all innovations are of the same size – has been replaced with an equally idealized assumption – i.e., that the available pool of knowledge is equally useful for developing all products – and, as it stands, the model is unlikely to produce interesting quantitative predictions concerning e.g. the optimal required inventive step or the extent to which R&D should be subsidized or taxed. Nevertheless, the above analysis has produced both several interesting qualitative results and insights into the prospects of developing more realistic pool of knowledge models in the future.

While considering the dynamics of the model, I presented the pool of knowledge version of a standard analysis of dynamics of a quality ladder model when it is not initially on its balanced growth path. It turned out that when there is no required inventive step, this analysis has an analogy for arbitrarily values of the imitation rate, but not otherwise. In the presence of a patentability requirement the analysis of the dynamics of the model is essentially more difficult, and it seems to be possible only after introducing restrictive assumptions concerning the amount of R&D in the future.

It was seen that the model provides conceptual tools for understanding growth traps, and it led to a simple definition of a growth trap which is caused by slow growth in the past. It turned out such growth traps are possible when the rate of imitation is small but positive. It also turned out that on a balanced growth path growth is always increased by a decrease of imitation, so that the growth-maximizing imitation rate is zero.

However, these results were deduced assuming that the efficiency with which the available pool of knowledge can be used for making new innovations is independent of whether the products are proprietary. It should be observed that in an actual economy the "proprietary" products are often not protected by patents but by e.g. trade secrets, and in this case it might be easier to utilize the current pool of knowledge when there are more non-proprietary products on the market. It is clear that the growth-maximizing imitation rate might be positive in a generalized model in which this effect is taken into account.

The analysis of comparative statics revealed that the growth-maximizing value of the required inventive step is always positive. The model led to the plausible prediction that the growth-maximizing required inventive step is small in a slowly-growing economy, and when the profit margins of the monopolists are small. It was also seen that for large values of profitability the problem of choosing the growth-maximizing required inventive step might fail to have an economically meaningful solution in a pool of knowledge growth model.

These results can be compared with the results of such microeconomic analyses of the required inventive step as e.g. O'Donoghue (1998), Hunt (2004), and Denicolò – Zanchettin (2002). O'Donoghue (1998) and Hunt (2004) conclude from their models

that the growth-maximizing required inventive step is positive,[17] but in the context of the model of Denicolò – Zanchettin (2002) it turns out that a positive novelty requirement is not needed in the optimal patent policy, if the leading breadth is chosen optimally.[18]

There are several ways in which one might wish to generalize the current model. To make the model more realistic, one might replace the assumption of a single pool of knowledge with the idea that the products are distributed into "bundles" which utilize separate but interconnected pools of knowledge. A less dramatic and easier modification would be to include the instruments of patent policy which were not considered above. As it was pointed out in the introduction, patent literature distinguishes between patent length, the patentability requirement, and the lagging and the leading breadth of a patent. As a natural next step, one might wish to generalize the current framework in such a way that it allowed for a discussion of patent breadth and patent length.

However, it seems that a pool of knowledge growth model does not allow for an interesting discussion of the distinction between the leading breadth of a patent and the patentability requirement. If one assumed that patents have a leading breadth K which is larger than the patentability requirement δ, this would in the current model have precisely the same consequences as the assumption that the patentability requirement was K: since in the model all new products are of the quality which

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[17] O'Donoghue, T. (1998) analyzes the effects of a patentability requirement on research spending and on welfare in a model in which the firms choose their level of R&D spending, and also the size of the innovations that they are targeting (ibid,. p. 659). Proposition 1 in ibid., p. 667, shows that when there is no required inventive step and patents have no leading breadth,the amount of investment in research may be smaller than the socially optimal investment. O'Donoghue also concludes that a positive patentability requirement may increase R&D spending and welfare (Propositions 2 and 3 in ibid., pp. 669-670), and also that patentability requirement can have a positive effect on R&D spending even when patents have a leading breadth (ibid., p. 673).

The main difference between the models of O'Donoghue et al (1998) and Hunt (2004) is that whereas in the former model each firm chooses a targeted innovation size, in the latter paper each firm chooses only a R&D intensity, and both the size of the resulting innovation and its time of arrival are random phenomena (Hunt, 2004, p. 403). In Hunt's model, the leading breadth and patentability requirement are identical (ibid., p. 404). It turns out that also in the context of this model the growth-maximizing patentability standard is positive (Proposition 4 in p. 413). One of the interesting points that Hunt (2004) makes is that the optimal patentability standard depends on the rate of innovation in the industry, so that the optimal patentability requirement is different in different industries (ibid., p. 414).

[18] See Proposition 1 on p. 812 in Denicolò – Zanchettin (2002). This paper analyzes a simple two-period model of sequential innovation. In this model the innovation of the second period builds upon the innovation of the first period, and the first innovation may be protected by both a patentability requirement and leading breadth.

corresponds to the research frontier, there would in both cases be research in only those sectors whose distance from the research frontier was larger than *K*.

On the other hand, it would be fairly easy to include patents of a finite duration into the current framework, since in it the size of an innovation exceeds the patentability requirement δ if and only if it replaces a product whose age is larger than a constant t_0. Hence, many features of the current model would remain unchanged if one replaced the patentability requirement δ with a minimum age t_0 which a product must exceed before it can be replaced by a new one. However, in a model with this interpretation one should either assume that all products become non-proprietary when they reach the age t_0, or that the rate of imitation gets higher when the products reach the age t_0, since imitating becomes legal at that time, and these changes would affect the results of the above analysis.

APPENDIX. PROOFS OF THE PROPOSITIONS IN CHAPTER 2.

PROOF OF PROPOSITION 1. The definition (41) easily implies that $\partial\tilde{r}(g)/\partial g > 0$. Further, when $\phi = 0$, $\Psi_{Pr}(g) = \Psi$, which is according to (37) independent of *g*. Plugging these results into (42), it turns out that when $\phi = 0$,

$$\frac{1}{F}\frac{\partial F}{\partial g} = \frac{-1}{\lambda \ln\gamma - g} - \frac{1}{\tilde{r}(g)}\frac{\partial \tilde{r}(g)}{\partial g} < 0$$

This proves the validity of (a).

Turning to (b), assume now that $\phi > 0$. First, it is observed that if it for some value of λ it is the case that $F(0,\phi,\delta) = 1$ and $\left(\partial F(g,\phi,\delta)/\partial g\right)_{g=0} > 0$, then the model must have an equilibrium for some positive value g_2 of *g*, since $\lim_{g\to(\lambda\ln\gamma)-} F(g,\phi,P) = 0$. Since $F(g,\phi,\delta)$ is a decreasing function of λ, it follows that when λ is given a slightly smaller value, there will still be a positive value $g \approx g_2$ which corresponds to an equilibrium, but $F(g,\phi,\delta) < 1$ so that $g = 0$ is a growth trap due to slow growth in the past.

Accordingly, we now consider the condition which must be valid if λ can be chosen so that $F(0,\phi,\delta)=1$ and $\left(\partial F(g,\phi,\delta)/\partial g\right)_{g=0}>0$ .Clearly, (41), (42), (43), and (44) easily imply that the former of these conditions is equivalent with.

$$\text{(A1)}\qquad \lambda\frac{(1-\alpha)\alpha^{\alpha/(1-\alpha)}}{(\rho+\phi)\Psi}=1$$

Define now the function *I* by

$$\text{(A2)}\qquad I(Q,R,t_0)=\int_0^{t_0}e^{-Qt}dt+\int_{t_0}^{\infty}e^{-Qt-R(t-t_0)}dt=\frac{1}{Q}-\frac{Re^{-Qt_0}}{Q(Q+R)}$$

Clearly, when one puts $R=g/(\ln\gamma-\ln\delta)$ and $t_0=(\ln\delta)/g$, it turns out that $\Psi=\left(g/(\ln\gamma)\right)I(Q_1,R,t_0)$, $\Psi_{Pr}=\left(g/(\ln\gamma)\right)I(Q_2,R,t_0)$, and $\tilde{r}(g)=1/I(Q_3,R,t_0)$, where $Q_1=g/(1-\alpha)$, $Q_2=\phi+g/(1-\alpha)$, and $Q_3=\phi+\rho+(\alpha g)/(1-\alpha)$.

Plugging these expressions into (42), and remembering that $\Psi_N=\Psi-\Psi_{Pr}$ where Ψ is independent of *g*, it now follows that

$$\text{(A3)}\qquad \begin{aligned}\frac{\partial F(g,\phi,P)/\partial g}{F(g,\phi,P)}&=\frac{-1}{\lambda\ln\gamma-g}+\frac{1}{I(Q_3,R,t_0)}\frac{\partial I(Q_3,R,t_0)}{\partial g}\\&+\frac{\left(1-\alpha^{1/(1-\alpha)}\right)}{(\ln\gamma)\left(\Psi-\left(1-\alpha^{1/(1-\alpha)}\right)\Psi_{Pr}\right)}\left[I(Q_2,R,t_0)+g\frac{\partial I(Q_2,R,t_0)}{\partial g}\right]\end{aligned}$$

When $g\to 0$, the computation of the value of $\partial I(Q,R,t_0)/\partial g$ is complicated by the fact that, both when $Q=Q_2$ and when $Q=Q_3$

$$\lim_{g\to 0}\left(e^{-Qt_0}\right)=\lim_{g\to 0}\left(\delta^{-Q/g}\right)=\begin{cases}1, & \delta=1\\0, & \delta>1\end{cases}$$

so that is necessary to consider the cases in which $\delta=1$ and $\delta>1$ separately. A tedious but straightforward calculation shows that

$$\text{(A4)}\qquad \begin{aligned}&\lim_{g\to 0}\left(\frac{1}{I(Q_3,R,t_0)}\frac{\partial I(Q_3,R,t_0)}{\partial g}\right)\\&=\begin{cases}\left(-1/(\phi+\rho)\right)\left(\alpha/(1-\alpha)+1/\ln(\gamma/\delta)\right), & \delta=1\\\left(-1/(\phi+\rho)\right)\left(\alpha/(1-\alpha)\right), & \delta>1\end{cases}\end{aligned}$$

Now (A3) and (A4) imply that

$$\text{(A5)}\qquad \lim_{g\to 0}\left(\frac{\partial F(g,\phi,P)/\partial g}{F(g,\phi,P)}\right)=\frac{-1}{\lambda\ln\gamma}-\frac{\xi}{\phi+\rho}+\frac{\left(1-\alpha^{1/(1-\alpha)}\right)}{(\ln\gamma)\Psi\phi}$$

where

(A6) $$\xi = \begin{cases} \alpha/(1-\alpha) + 1/\ln\gamma, & \delta = 1 \\ \alpha/(1-\alpha), & \delta > 1 \end{cases}$$

As explained above, we wish to find out under which circumstances the derivative (A5) is positive when $F(0,\phi,P)=1$, i.e. when (A1) is valid. When λ is chosen so that it satisfies (A1), the expression (A5) will be positive if and only if

(A7) $$\frac{-(1-\alpha)\alpha^{\alpha/(1-\alpha)}}{(\ln\gamma)(\rho+\phi)} - \frac{\xi\Psi}{\phi+\rho} + \frac{1-\alpha^{1/(1-\alpha)}}{(\ln\gamma)\phi} > 0$$

Considering separately the case in which $\delta = 1$ and the case in which $\delta > 1$, it easily follows from (A6) and (37) that in both cases $\xi\Psi < 1/\ln\gamma$. This implies that (A7) is valid whenever

$$\frac{-(1-\alpha)\alpha^{\alpha/(1-\alpha)}}{\rho+\phi} - \frac{1}{\phi+\rho} + \frac{1-\alpha^{1/(1-\alpha)}}{\phi} > 0$$

This is equivalent with $\phi < \left(\alpha^{-\alpha/(1-\alpha)} - \alpha\right)\rho$, and it can be concluded that in this case the model has an equilibrium which is a growth trap due to slow growth in the past, as it is stated in part (b) of this proposition. □

PROOF OF LEMMA 1. The definition (48) implies that

(A8) $$G'(\Psi_{Pr}) = \frac{(1-\alpha)\left(1-\alpha^{1/(1-\alpha)}\right)\left(1-\alpha^{\alpha/(1-\alpha)}\right)\Psi_{Pr} - \left(1-\alpha-\alpha^{\alpha/(1-\alpha)}+\alpha^{1+1/(1-\alpha)}\right)\Psi}{\left(\Psi - \left(1-\alpha^{1/(1-\alpha)}\right)\Psi_{Pr}\right)^{1+\alpha}}$$

This further implies that there is just one value $\underline{\Psi}_{Pr}$ of Ψ_{Pr} for which $G'(\Psi_{Pr})=0$, and that G is a decreasing function when $\Psi_{Pr} < \underline{\Psi}_{Pr}$ and an increasing function when $\Psi_{Pr} > \underline{\Psi}_{Pr}$, so that the value $G(\underline{\Psi}_{Pr})$ is the minimum of G. The definition (48) also implies that $G(0) = G(\Psi)$, so that it must be the case that $0 < \underline{\Psi}_{Pr} < \Psi_{Pr}$, and the value $G(0) = G(\Psi)$ is the maximum of G within the interval $\left[0, \Psi_{\phi=0}\right]$. □

PROOF OF PROPOSITION 2. The definitions (36) and (41) immediately imply that $\partial\Psi_{Pr}/\partial\phi < 0$ and that $\partial\tilde{r}(g)/\partial\phi > 0$, but according to (38) Ψ is independent of ϕ. Hence, one can conclude from (42) that

$$\text{(A9)}\quad \frac{\partial F(g,\phi,P)}{\partial\phi}=\left(\frac{-1}{\tilde{r}(g)}\frac{\partial\tilde{r}(g)}{\partial\phi}+\frac{1-\alpha^{1/(1-\alpha)}}{\Psi-\left(1-\alpha^{1/(1-\alpha)}\right)\Psi_{Pr}}\frac{\partial\Psi_{Pr}}{\partial\phi}\right)F(g,\phi,P)<0$$

By definition, for any given values of ϕ and P the value $g(\phi,P)$ satisfies the condition $F(g(\phi,P),\phi,P)=1$ so that

$$\frac{dg}{d\phi}=-\frac{(\partial F)/(\partial\phi)}{(\partial F)/(\partial g)}<0$$

since we are assuming (E2). Hence, the growth maximizing value of the imitation rate is $\phi=0$.

Turning to the claim concerning welfare, it is observed that

$$\frac{d}{dg}\left(\frac{L^{\alpha}}{\rho-g}\right)=\frac{d}{dg}\left(\frac{\left(1-g/(\lambda\ln\gamma)\right)^{\alpha}}{\rho-g}\right)=\frac{1-\left(1/(\lambda\ln\gamma)\right)\left((1-\alpha)g+\alpha\rho\right)}{\left(1-g/(\lambda\ln\gamma)\right)^{1-\alpha}(\rho-g)^{2}}$$

Since we are assuming the validity of (45), i.e. that $g<\rho$, and that $\ln\gamma>\rho/\lambda$, this implies that

$$\frac{d}{dg}\left(\frac{L^{\alpha}}{\rho-g}\right)>\frac{1-\left(1/(\lambda\ln\gamma)\right)\rho}{\left(1-g/(\lambda\ln\gamma)\right)^{1-\alpha}(\rho-g)^{2}}>0.$$

This implies that the term $L^{\alpha}/(\rho-g)$ receives its maximal value when $\phi=0$, since the growth rate is largest in this case. On the other hand, Lemma 1 states that when $\phi=0$, also the function $G(\Psi_{Pr})$ receives its maximal value $\Psi^{1-\alpha}$, which is independent of g. Putting these results together, it follows from the definition (47) that the choice of ϕ which maximizes the welfare function $\tilde{U}$ is $\phi=0$. □

PROOF OF PROPOSITION 3. Let the function I be still given by (A2), define Q_1, Q_2, Q_3, R, and t_0 just like in the proof of Proposition 1, so that $\Psi=\left(g/(\ln\gamma)\right)I(Q_1,R,t_0)$, $\Psi_{Pr}=\left(g/(\ln\gamma)\right)I(Q_2,R,t_0)$, and $\tilde{r}(g)=1/I(Q_3,R,t_0)$. Now (42) implies that

$$\text{(A10)}\quad \begin{aligned}&\frac{1}{F(g,\phi,\delta)}\frac{\partial F(g,\phi,\delta)}{\partial\delta}=\\&=\frac{1}{I(Q_3,R,t_0)}\frac{\partial I(Q_3,R,t_0)}{\partial\delta}-\frac{\partial I(Q_1,R,t_0)/\partial\delta-\left(1-\alpha^{1/(1-\alpha)}\right)\partial I(Q_2,R,t_0)/\partial\delta}{I(Q_1,R,t_0)-\left(1-\alpha^{1/(1-\alpha)}\right)I(Q_2,R,t_0)}\end{aligned}$$

Since each of the values $Q = Q_1$, $Q = Q_2$, and $Q = Q_3$ is independent of δ it easily follows from (A2) the definitions $R = g/\left(\ln\left(\gamma/\delta\right)\right)$ and $t_0 = \left(\ln\delta\right)/g$ that, for each of these values of Q,

$$\text{(A11)} \quad \frac{\partial I\left(Q,\delta,t_0\right)}{\partial\delta} = \frac{RQe^{-Qt_0}}{g\delta\left(Q+R\right)^2}$$

Together with (A2), this implies that

$$\text{(A12)} \quad \frac{1}{I\left(Q,R,t_0\right)}\frac{\partial I\left(Q,R,t_0\right)}{\partial\delta} = \frac{RQ^2e^{-Qt_0}}{g\delta\left(Q+R\right)\left(Q+R-Re^{-Qt_0}\right)}$$

We now put $I_i = I\left(Q_i,R,t_0\right)$ and

$$\text{(A13)} \quad J_i = \frac{1}{I\left(Q_i,R,t_0\right)}\frac{\partial I\left(Q_i,R,t_0\right)}{\partial\delta}$$

when $i = 1,2,3.$ Using these notations, (A10) receives the form

$$\text{(A14)} \quad \frac{1}{F\left(g,\phi,\delta\right)}\frac{\partial F\left(g,\phi,\delta\right)}{\partial\delta} = J_3 - \frac{J_1I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)J_2I_2}{I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)I_2}$$

In order to prove (a), assume that $\delta = 1$. In this case $R = g/\ln\gamma$ and $t_0 = 0$, so that

$$J_i = \frac{Q_i}{\delta\left(Q_i\ln\gamma + g\right)}$$

Since $g/\left(1-\alpha\right) = Q_1 \le Q_2 = g/\left(1-\alpha\right)+\phi$, it now follows that $J_1 \le J_2$, and one can conclude from (A14) that

$$\frac{1}{F\left(g,\phi,\delta\right)}\frac{\partial F\left(g,\phi,\delta\right)}{\partial\delta} = J_3 - \frac{J_1I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)J_2I_2}{I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)I_2} \ge J_3 - J_1$$

Further, since $g/\left(1-\alpha\right) = Q_1 < g/\left(1-\alpha\right)+\left(\rho-g\right)+\phi = Q_3$, it must also be the case that $J_1 < J_3$, so that it can further be concluded that

$$\frac{1}{F\left(g,\phi,\delta\right)}\frac{\partial F\left(g,\phi,\delta\right)}{\partial\delta} > 0$$

Hence, also $\partial F\left(g,\phi,\delta\right)/\partial\delta$ is positive, and since we are assuming that (E2) is valid,

$$\frac{dg}{d\delta} = -\frac{\partial F/\partial\delta}{\partial F/\partial g} > 0$$

This completes the proof of (a).

Turning to part (b) of this proposition, it was demonstrated in Section 5 above that $\delta < \gamma$ whenever the model has balanced growth paths. Turning to the other claim which was made in part (b), assume that $e^{2(1-\alpha)} \le \delta$. It can be concluded from (A12) that

$$\text{(A15)} \quad \begin{aligned} &\frac{\partial}{\partial Q}\left(\frac{1}{I(Q,R,t_0)}\frac{\partial I(Q,R,t_0)}{\partial \delta}\right) = \frac{\partial}{\partial Q}\left(\frac{RQ^2 e^{-Qt_0}}{g\delta(Q+R)\left(Q+R-Re^{-Qt_0}\right)}\right) \\ &< \left(\frac{2}{Q}-t_0\right)\frac{RQ^2 e^{-Qt_0}}{g\delta(Q+R)\left(Q+R-Re^{-Qt_0}\right)} \end{aligned}$$

Hence, this derivative is negative whenever

$$\text{(A16)} \quad 2/Q \le t_0$$

Since $t_0 = (\ln\delta)/g$ and $Q_1 = g/(1-\alpha)$, the condition $e^{2(1-\alpha)} \le \delta$ implies that (A16) is valid for all $Q \ge Q_1$. Hence, (A15) implies that

$$\frac{1}{I(Q,R,t_0)}\frac{\partial I(Q,R,t_0)}{\partial \delta}$$

is a decreasing function of Q when $Q \ge Q_1$, and hence, $J_1 \ge J_2$ and $J_1 > J_3$. Together with (A14), these results imply that

$$\frac{1}{F(g,\phi,\delta)}\frac{\partial F(g,\phi,\delta)}{\partial \delta} = J_3 - \frac{J_1 I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)J_2 I_2}{I_1 - \left(1-\alpha^{1/(1-\alpha)}\right)I_2} \le J_3 - J_1 < 0$$

Together with (E2), this implies that

$$\frac{dg}{d\delta} = -\frac{\partial F/\partial \delta}{\partial F/\partial g} < 0$$

This completes the proof of (b). □

PROOF OF PROPOSITION 4. Assume that $\delta > 1$. The definition (41) implies that $\tilde{r}(0) = \phi + \rho$. When this result, (43), and (44) are inserted into (42), it follows that

$$\text{(A17)} \quad F(0,\phi,\delta) = \begin{cases} \lambda \dfrac{1-\alpha}{\alpha(\phi+\rho)\Psi}, & \phi = 0 \\ \dfrac{\lambda(1-\alpha)\alpha^{\alpha/(1-\alpha)}}{(\phi+\rho)\Psi}, & \phi > 0 \end{cases}$$

However, it easily follows from (37) that $\partial\Psi/\partial\delta > 0$, and one can conclude that

(A18) $\dfrac{\partial F(0,\phi,\delta)}{\partial \delta} < 0$

Hence, since F is by assumption a continuous and decreasing function of g,

$$\begin{aligned}\lambda_0 &= \inf\left\{\lambda' \middle| \left(g(\phi,\delta)\right)_{\lambda=\lambda'} > 0 \text{ for some } \delta\right\}\\ &= \inf\left\{\lambda' \middle| F(0,\phi,\delta) > 1 \text{ for some } \delta, \text{ when } \lambda = \lambda'\right\}\\ &= \inf\left\{\lambda' \middle| F(0,\phi,1) > 1, \text{ when } \lambda = \lambda'\right\}\end{aligned}$$

Obviously, since F is continuous in λ, $\left(F(0,\phi,1)\right)_{\lambda=\lambda_0} = 1$. Let $\delta' > 1$ be arbitrary. Now it must be the case for all values $\lambda' > \lambda_0$ which are sufficiently close to λ_0, for all $\delta \geq \delta'$ and for all growth rates g that

$$\left(F(g,\phi,\delta)\right)_{\lambda=\lambda'} \leq \left(F(0,\phi,\delta')\right)_{\lambda=\lambda'} < \left(F(0,\phi,1)\right)_{\lambda=\lambda_0} = 1$$

In other words, when λ' is sufficiently close to λ_0 there will be no growth if $\delta \geq \delta'$, so that the growth-maximizing value of δ must in this case be smaller than δ'. Since $\delta' > 1$ was arbitrary, it follows that the growth-maximizing value of δ approaches 1 when $\lambda \to \lambda_0 +$. Similarly, also the welfare-maximizing value of δ must approach 1 if it is a value for which the growth rate is positive. □

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CHAPTER III

COMMERCIAL PIRACY AND INTELLECTUAL PROPERTY POLICY

ABSTRACT[1]

I discuss the competition between a copyright owner and several commercial pirates who sell copies of the same information good to consumers. I view the increased risk of a punishment that offering a pirate copy to a consumer causes as an *advertising cost*, whose value is chosen by the government. The structure of the market for pirate copies is affected also by fixed costs which are caused by punishments or DRM systems. I present a systematic analysis of the effects of these policy variables and the quality of pirate copies on the market for the considered information good.

3.1. INTRODUCTION

The distribution of illegal *pirate copies* of information goods might have a variety of motives. Such copies are distributed on the one hand by the members of *peer-to-peer networks*, who deliver digital goods on the Internet without monetary compensation and who are motivated by e.g. a feeling of identification with the other network members, and on the other hand by commercial pirates who are, more conventionally from the perspective of the economist, motivated by the revenue that results from their activities. Somewhat less obviously, the consumers of an information good might also form *clubs* each of which buys a single copy of the information good, produces further copies of it, and distributes one of them to all club members.[2]

Given that both commercial and non-commercial forms of piracy are illegal, there is no obvious way of estimating the extent to which pirate copies of information copies

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[1] An essentially identical version of this paper has earlier been published as Kiema, I., 2008, "Commercial piracy and intellectual property policy", *Journal of Economic Behavior & Organization* **68**, 304-318.

[2] For the producer of the first copy of the information good, this practice resembles a situation in which the club members buy a single copy of an information good and use it successively. The latter practice is legal when the considered product is e.g. a book, a journal, or a video tape, and in both cases, the producer of the first copy might be able to appropriate indirectly a part of the value that the good has for its consumers. Cf. Varian (2000).

get sold rather than distributed for free, or the effects of commercial piracy on the profits of copyright owners. Nevertheless, e.g. the International federation of the Phonographic Industry (IFPI) has estimated that approximately 37% of all the [music] CDs that were purchased in 2005 globally were pirate copies.[3] However, in the case of the *software industry* it is more difficult to find estimates of the prevalence of commercial piracy.

The Business Software Alliance (BSA) publishes yearly a piracy study which contains estimates for the piracy rate (i.e. the ratio of the number of pirated software units to the total number of installed software units) for different countries of the world, and also for different regions of the world as a whole. For example, according to the BSA the worldwide piracy rate was 35% in 2005.[4] However, such estimates do not make a distinction between commercial and non-commercial forms of piracy.[5] Nevertheless, e.g. the other surveys of the BSA suggest that both the commercial and non-commercial forms of software piracy are of a considerable economic significance.[6]

There is a relatively large economic literature on *end-user copying*.[7] *Dyuti S. Banerjee* has recently put forward several closely related models of the competition between a monopolist (i.e., the copyright owner) and a single commercial pirate in (Banerjee, 2003, 2006a, 2006b), but it nevertheless seems that until now economists

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[3] International federation of the Phonographic Industry (IFPI), *The Recording Industry 2006 Piracy report*, p. 4, available at http://www.ifpi.org/site-content/library/piracy-report2006.pdf, accessed on October 29, 2006.

[4] *Third Annual BSA and IDC Global Software Piracy Study*, http://www.bsa.org/globalstudy/upload/2005%20Piracy%20Study%20-%20Official%20Version.pdf, accessed on April 13, 2007, p. 13.

[5] This is because the estimates of the BSA have been calculated from an estimate of the total number of installed software units, which is based on the number of the sold hardware units and surveys concerning their average software load, and an estimate of the number of the sold software units, which is based on information concerning the market revenues of software vendors and software pricing. Cf. ibid., p. 14.

[6] In one of such surveys, the BSA has investigated the attitudes of the online consumers from six different countries towards *spam*, i.e. commercial emails that they have received without requesting or signing up for them (BSA, *Consumer Attitudes Toward Spam in Six Countries*, 2004, available at http://www.bsa.org/usa/events/loader.cfm?url=/commonspot/security/getfile.cfm&pageid=20654, accessed on April 13, 2007). In each country, more than 80% of the respondents stated that they had received spam which was concerned with computer software (ibid., p. 6), and 27% reported that, in the product category "computer software", they had "purchased an item or taken advantage of an offer" which was suggested to them in spam (ibid., p. 12). Only 31% of the respondents stated that they agreed with the statement that they would "never buy commercial software using this method because it is most likely unlicensed and illegal" (ibid., p. 16).

[7] For a survey, see Peitz and Waelbroeck (2006a).

have given much less attention to *commercial piracy* than to end-user copying.[8] Below I shall put forward a model of the competition between the copyright owner and several commercial pirates, to whom I shall refer as *bootleggers*.

The production costs of pirate copies are low, and in the case of the pirate copies which are distributed in an electric form via the Internet they are almost zero. Accordingly, if intellectual property rights are not enforced, the prices of pirate copies can be expected to fall to zero via Bertrand competition. However, when bootleggers are in danger of being punished for their activities, it may be costly for them to inform potential consumers of their products, since this may increase the risk of getting caught and receiving a punishment. For example, if an illegally operating Internet site which offers pirate copies of software products for sale informs its potential customers by sending e-mail messages to randomly chosen addresses, the risk of a punishment is increased by each message. In this case the expected cost from a punishment is analogous with an *advertising cost*, which explains the positive price of pirated information goods.

Information goods can be protected not only by copyright and other intellectual property rights but also by *digital rights management* (DRM) systems. Digital rights management tools can, broadly speaking, be divided into *cryptography* (i.e. the distribution of information goods in an enciphered format) and *watermarking* (i.e. embedding information into a digital product in such a way that each copy of the good becomes different).[9] Watermarks can be used for tracking down the person who has originally bought the legitimate copy of an information good from which the pirate copies on the market have been produced, which makes it easier penalize commercial pirates.[10] Clearly, a cryptographic device causes a *fixed cost* for a commercial pirate, but the costs of watermarking can be either fixed or variable: if a bootlegger removes the watermark, its removal causes a fixed cost, but if she sells watermarked information goods, the risk caused by the watermark increases with the number of the copies that she sells.

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[8] See, however, Yao (2005), which discusses counterfeiting on a more general level, without restricting attention to counterfeited information goods.

[9] For a survey, see Eskicioglu and Delp (2001).

[10] Since a large-scale commercial pirate can be expected to be able to break down a cryptographic system, watermarking seems to be the more relevant DRM tool in the context of large-scale commercial piracy (cf. ibid., pp. 683-684). See also Park and Scotchmer (2005), contains an analysis of DRM and of the different effects of the use of *shared* and *independent* DRM systems on end-user piracy.

Below I shall analyze the effects of DRM systems and the policy instruments of the government on the profits of the bootleggers and the copyright owner. In the model the "*advertising costs*" which are caused by an increased risk of legal sanctions keep the price of pirate copies positive also when there are several bootleggers on the market. Accordingly, the model differs from previous work in the same field in so far that it provides tools for analyzing also *the market structure* of the market for pirate copies and its effects on the market for legitimate copies.[11]

3.2. The Main Features of the Model

The agents of the model that is considered below are 1) a *copyright owner* who sells copies of an information product legally, 2) *K potential bootleggers* who wish to sell illegitimate pirate copies of the same good, and 3) a unit mass of consumers, which is indexed by $\theta \in [0,1]$. The bootleggers can inform the consumers of the availability of their products by sending them advertisements at random. As a paradigmatic example of a situation of this type, one might think of an illegally operating Internet site which sends advertisements of pirated software products to randomly chosen e-mail addresses. The sending of an advertisement is associated with a cost b which should be interpreted as the increase in expected cost of punishment that sending a single advertisement causes.

More precisely, I shall assume that sending an advertisement causes an increase α in the risk of getting caught, that the bootlegger receives a punishment G if she gets caught, and that the bootleggers are risk neutral. In this case the "*advertising cost*" b is given by[12]

(1) $b = \alpha G$

Any combination of α and G which corresponds to the same value of b has the same effects on the markets for both legitimate and pirate copies, and accordingly I shall

[11] Cf. (Banerjee, 2003, 2006a, 2006b), which discuss several closely related models of the competition between the copyright owner and a single pirate. In these models the pirate copies are of a lower quality than legitimately bought copies, and this stops the price of the good from sinking to zero through Bertrand competition. However, in these models the number of the pirates is by assumption 1, and they do not address the question why the price of the pirate copies does not sink to zero through a Bertrand competition *between the pirates* (rather than between them and the copyright owner).

[12] For a discussion of the econometric problem of actually constructing an index which measures the strength of legal software protection in a given country, see Andrés (2006, pp. 34-37).

below analyze the effects of the choice of b (rather than of α and G) for these markets.[13]

I shall assume that the bootleggers cannot keep track of the consumers to whom they have already sent an advertisement. Rather, each of the bootleggers sends each of the advertisements with the same probability to each consumer.[14] This implies that a bootlegger might send to the same consumer several advertisements in which the product is offered for sale at different prices. This assumption is particularly plausible in the context of trade on the Internet, since the potential customers of bootleggers might have several e-mail addresses. In addition, if the consumers are divided into groups whose members inform each other of the advertisements that they have received, a single advertisement might reach individuals with different e-mail addresses, and in this case a bootlegger cannot eliminate the possibility that the same group of consumers receives many advertisements from her. In this case one should interpret b as the average cost of reaching a single consumer with a single advertisement.

This implies that the description of the competition between the bootleggers resembles the classical *model of advertising by Gerard R. Butters* which was put forward in Butters (1977),[15] and similarly with Butters's model, in the current model there will be *price dispersion* in the market for the advertised product.[16] Clearly, this result is quite plausible in the context of the illegal business model of the bootlegger: since the bootleggers do not necessarily know the prices set by their competitors or the demand that they face, they have an incentive for finding out the optimal price

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[13] It is easy to see that the problem of choosing α and G so that the monitoring costs of the government are minimized, given the constraint (1), does not have a well-defined solution: if an increase in monitoring causes costs for the monitoring authorities, but an increase in punishments (like fines) does not, any increase of G and decrease in α which keeps (1) valid seems always beneficial from the perspective of the government. Hence, it seems that a meaningful discussion of the optimal choice of α and G would require a more general model.

[14] In a more general model could be assumed that the bootleggers would be able to target their advertisements at the consumers who are likely to buy a pirate copy. This would change the demand function (8), but it is easy to see that the analysis of the market for pirate copies for a fixed value of p_M – i.e. the results which are summarized in Proposition 1 below – would still remain valid. However, the analysis of the optimization problem of the copyright owner would become essentially more complicated.

[15] See also Tirole (1988, pp. 290-294). The current model and Butters's model of advertising can both be contrasted with *Varian´s model of sales* (Varian, 1980), in which the potential customers are divided into *informed* and *uninformed consumers*, and the informed consumers know the prices chosen by *all* the sellers, whereas the uninformed consumers know only the price set by the seller from whom they make their purchase.

[16] More precisely, there is price dispersion whenever there are more than one active bootleggers on the market. See Proposition 1 below.

level experimentally, by specifying different prices in different advertisements. Further, when *peer-to-peer networks* are viewed as a limiting case of commercial piracy, also the fact that identical information goods are sold by pirates and distributed for free on peer-to-peer networks is seen to exemplify price dispersion in the market for pirate copies.

However, the current model differs from Butters's model in several essential respects. In the currently considered model there is an oligopoly in the market for pirate copies,[17] and since the "advertising costs" have been meant to represent the expected costs from a punishment for copyright violation, it will be for simplicity assumed that the copyright owner may advertise for free, and that all consumers have the option of buying the product from her. In addition, since I wish to model a situation in which only a part of the consumers prefers buying a pirate copy to buying a legitimate copy, unlike Butters I shall assume that the reservation prices of the consumers differ.

I shall assume that reservation price of the consumer θ (where $0 \leq \theta \leq 1$) for a legitimate copy of the good is θ and that for a pirate copy her reservation price is $q\theta$, where $q \in (0,1)$ is a constant.[18] In Banerjee (2003), from which I have borrowed this notation, it was assumed that q corresponds to the probability with which a pirate copy is operational.[19] However, here the parameter q has been meant to represent not only the fact that a pirated product might be technically of a worse quality than a legal one or not operational at all, but also features like that the consumers might prefer legally bought copies also for ethical reasons, because there might be legal sanctions against using (and not just against selling) pirate copies, or because buying a pirate copy requires giving credit card information to criminals.

Different values of q seem plausible in the different applications of the model. One might expect that e.g. music files downloaded from a peer-to-peer network are

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[17] Butters (1977) is for the most part concerned with a model with a large number of sellers with no market power. See, however, Appendix A (ibid., pp. 483- 488) for a short discussion of an oligopoly of N sellers.

[18] Clearly, one could also assume that consumers differ also with respect to q and not just with respect to θ. E.g. one could follow Chen and Png (2003, p. 110) by assuming that some consumers reject pirate copies on ethical grounds while others do not. The remarks made in footnote 14 above apply also to models of this type: our discussion of the market for pirate copies for each fixed value of p_M can be generalized in an obvious way to such models, but choosing the optimal p_M would be essentially more complicated in their context.

[19] Banerjee (2003, p. 100).

experienced by their users to be of an almost identical quality with legally bought ones, and in this case it seems plausible to assume that $q \approx 1$. However, the other considerations besides the technical quality seem more relevant in the case of e.g. illegally bought software products, and this motivates the assumption that they correspond to essentially lower values of q. Finally, if one applied the current model to pirate copies of other branded articles instead of information goods, they could be associated with even lower values of q.

In the model a copyright owner and K potential bootleggers play the following four-stage *leader-follower game.*[20]

1) The copyright owner sets the price p_M of legitimate copies.
2) Each potential bootlegger decides whether to enter the market and to pay a fixed cost F.
3) The bootleggers (if any) who have entered decide the number of the advertisements that they send and send them to randomly chosen consumers. The bootleggers are not constrained to offering the product at the same price in different advertisements.
4) The consumers choose whether to buy the product. If a consumer θ has not received any advertisements, she will buy the product from the copyright owner if $\theta - p_M \geq 0$, and she will buy nothing otherwise. If a consumer θ has received at least one advertisement, and if the lowest price suggested in the advertisements that she has received is p, she will buy the product from the copyright owner if $\theta - p_M \geq \max\{0, q\theta - p\}$. If this is not the case, she will buy the product from a bootlegger at price p if $q\theta - p \geq 0$. If neither of these conditions is valid, she will not buy anything. If the consumer buys the product from a bootlegger at price p and if there are several bootleggers who have offered the product at price p to her, she will choose one of them at random.

In this game the aim of the copyright owner is to maximize her profit, which is simply equal to her revenue

[20] Cf. ibid., which contains a discussion of both a leader-follower game and a Bertrand game in the context of a model with a single pirate.

(2) $R_M(p_M) = p_M D_M(p_M)$

where $D_M(p_M)$ is the demand function of the copyright owner, and each bootlegger $i \in \{1, 2, ..., k\}$ aims at maximizing the profit $P_i = R_i - bA_i$, where R_i is the revenue of the *i*th bootlegger and A_i is the number of the advertisements that she sends. Obviously, in equilibrium the number of the bootleggers who actually enter the market will be

(3) $k(p_M) = \max\{k' | R_i - bA_i \geq F \text{ when the number of the bootleggers is } k'\}$

Part 4) of the definition of the considered game implies that if the cheapest price at which a consumer θ can buy the considered good from a bootlegger is *p*, she will buy it from the copyright owner if and only if

(4) $\theta \geq \dfrac{p_M - p}{1-q}$ and $\theta \geq p_M$

and from the bootlegger if and only if

(5) $\theta < \dfrac{p_M - p}{1-q}$ and $q\theta \geq p$

Clearly, (4) implies that there are consumers who are willing to buy the product from a bootlegger for the price p only if $(p_M - p)/(1-q) > p/q$. This condition, which must be valid for all advertisements in equilibrium, is equivalent with

(6) $qp_M > p$

If the condition (6) is valid for the cheapest price p which is suggested in the advertisements that the consumer θ has received, then it will be the case that:

If $\theta < p/q$, the consumer does not buy anything.

(7) If $p/q \leq \theta < (p_M - p)/(1-q)$, the consumer buys a pirate copy for price *p*.

If $\theta \geq (p_M - p)/(1-q)$, the consumer buys a legitimate copy.

I shall denote the demand function of the pirate copies by $x(p)$. More precisely, I shall let $x(p)$ denote the proportion of the consumers who would buy a pirate copy at the price p if p was the cheapest price which is suggested to them in the advertisements that they have received. Clearly, (7) implies that

$$(8) \qquad x(p)=\begin{cases} x_A(p), & p<q+p_M-1 \\ x_B(p), & p\geq q+p_M-1 \end{cases}=\begin{cases} (q-p)/q, & p<q+p_M-1 \\ (qp_M-p)/\left[q(1-q)\right], & p\geq q+p_M-1 \end{cases}$$

Here the case in which $x(p)=x_A(p)$ corresponds to the situation in which there are no consumers who would buy a legitimate copy if they are offered a pirate copy at price *p*. In Section 4 below it will be seen that in equilibrium the choice of p_M by the copyright owner and the choices of the values of *p* by the bootleggers will be such that $x(p)=x_B(p)$ for all the advertisements that the bootleggers send.

In the next section I deduce the equilibrium distribution of the prices of pirate copies for a given value of p_M and a corresponding number of bootleggers $k=k(p_M)$. This will be utilized in the subsequent discussion of the optimization problem of the copyright owner who chooses p_M. However, as it was explained in the introduction, reliable estimates of the size of the market for pirate copies and of the risk of getting caught which the bootleggers face are not available, and accordingly it is also interesting to study the situation in which p_M is exogenously given. This situation can be thought of as a model of a case in which the copyright owner has no information concerning the bootleggers, or bases her decisions on incorrect estimates of the parameters that characterize the market for pirate copies.

3.3. The Market for Pirate Copies

Below I shall assume that there are *k* bootleggers, labeled $1,...,k$, who have entered the market. Analogously with notation used in Butters (1977),[21] I define $A_i(p)$ to be the measure of the advertisements sent by bootlegger *i* at a price smaller than or equal with *p*, $a_i(p)$ to be the derivative of $A_i(p)$ whenever it exists, and *A* to be the total number of the advertisements.

As it was explained in Section 2 above, I assume that the bootleggers cannot keep track of the consumers to whom they have already sent an advertisement – which is quite plausible in the context of trade on the Internet, since most potential consumers have several e-mail addresses – and that a bootlegger might send several

[21] Cf. Butters (1977, p. 469).

advertisements to the same consumer. If there are N consumers to whom the bootleggers send altogether M advertisements at random, the probability with which a consumer does *not* receive any advertisements is

$$\left(1-\frac{1}{N}\right)^{M}=\left(\left(1-\frac{1}{N}\right)^{N}\right)^{\eta}$$

where $\eta = M/N$ is the number of the advertisements per consumer. Clearly, in the limit in which the number of the consumers approaches infinity this becomes

$$\lim_{N\to\infty}\left(\left(1-\frac{1}{N}\right)^{N}\right)^{\eta}=e^{-\eta}$$

In the currently considered case, the mass of the consumers has been normalized to one, and the measure of the advertisements which contains a price not larger than p is denoted by $\sum_{i=1}^{k} A_i(p)$. Hence, the probability with which a consumer receives at least one of these advertisements is

$$1-\exp\left(-\sum_{i=1}^{k} A_i(p)\right)$$

It is easy to see that if $k=1$, i.e. if there is just one active bootlegger on the market, she cannot have an incentive to specify different prices in different advertisements. If the single bootlegger sends A advertisements which all contain the price p, her profit will be

(9) $$P_{(k=1)} = px(p)\left(1-e^{-A}\right)-bA$$

Hence, in this case it is optimal for the bootlegger to advertise at the price p which maximizes $px(p)$. For reasons which will become soon obvious, I shall denote this price by $p_{\max}$, and I shall put

(10) $$r_{\max} = p_{\max}x\left(p_{\max}\right)$$

Further, one can also conclude from (9) that when $k=1$, the optimal number of the advertisements is

(11) $$A=\ln\left(p_{\max}x\left(p_{\max}\right)/b\right)=\ln\left(r_{\max}/b\right)$$

However, it is clear that when there are more than one active bootleggers on the market, there must be price dispersion in the market for pirate copies: if two bootleggers specified the same price in all their advertisements, each of them could

increase her profits by making a sufficiently small reduction in the price.[22] By the same argument, it can be seen that there cannot be any set of advertisements with a positive measure which would all contain the same price, when there are at least two bootleggers. It is also clear that it can never be optimal for a bootlegger to specify a price which is larger than $p_{\max}$, i.e. the price which is optimal in the absence of competition. Hence, when $k \geq 2$, the prices specified by the bootleggers correspond to a continuous distribution function $A_i(p)$, so that the revenue of each bootlegger is

$$(12) \qquad R_i = \int_{p_{\min}}^{p_{\max}} px(p) \exp\left(-\sum_{i=1}^{k} A_j(p) \right) a_i(p)\, dp$$

for some $p_{\min}$. The problem of maximizing this quantity can be solved by the standard tools of the analysis of variations, and its solution is given by the following proposition.

PROPOSITION 1. Suppose that the price of legitimate copies is p_M, that there are k active bootleggers on the market, and that their strategies for a symmetric Nash equilibrium relative to p_M. The largest price suggested in the advertisements has the value $p = p_{\max}$ for which $px(p)$ is maximal, and the smallest price $p_{\min}$ suggested in them is determined by the condition

$$(13) \qquad r_{\min} = b^{1-1/k} r_{\max}^{1/k}$$

where $r_{\min} = p_{\min} x(p_{\min})$ and $r_{\max} = p_{\max} x(p_{\max})$. When $k \geq 2$, the number of the advertisements with a price not larger than $p \in [p_{\min}, p_{\max}]$ sent by a bootlegger i is given by

$$(14) \qquad A_i(p) = \frac{1}{k-1} \ln \frac{px(p)}{p_{\min} x(p_{\min})} = \frac{1}{k-1} \ln \frac{px(p)}{r_{\min}}$$

Proposition 1 implies that when $k \geq 2$ the total number of advertisements sent by each bootlegger is given by

[22] As it was explained in Section 2, the conclusion that the prices of pirate copies vary is quite plausible: since the bootleggers operate on an illegal market concerning which it is difficult to find detailed information, they can be expected to try to receive such information by specifying different prices in different advertisements.

$$(15) \qquad A_i = \frac{1}{k-1}\ln\left(\frac{r_{\max}}{r_{\min}}\right) = \frac{1}{k-1}\ln\left(\left(\frac{r_{\max}}{b}\right)^{(k-1)/k}\right) = \frac{1}{k}\ln\left(r_{\max}/b\right)$$

A comparison of this result with (11) shows that the total number of the advertisements that the bootleggers send is independent of their number *k*. However, according to Proposition 1 an increase in *k* shifts their price distribution downwards. The condition (13) which determines the value of smallest price $p_{\min}$ which occurs in the advertisements can be made more intuitive by observing that (12) and (14) together imply that the revenue of each bootlegger is

$$(16) \qquad \begin{aligned} R_i &= \int_{p_{\min}}^{p_{\max}} px(p)\exp\left(-\sum_{i=1}^{k} A_j(p)\right) a_i(p)\,dp \\ &= \int_{p_{\min}}^{p_{\max}} r_{\min}\left(\frac{r_{\min}}{px(p)}\right)^{1/(k-1)} \frac{dA_i(p)}{dp}dp = \int_{p_{\min}}^{p_{\max}} r_{\min} e^{-A_i(p)} \frac{dA_i(p)}{dp}dp = r_{\min}\left(1-e^{-A_i}\right) \end{aligned}$$

This result states that the revenue of each bootlegger is identical with the revenue that she would receive if she was the only bootlegger in the market and she specified the price $p_{\min}$ in all her advertisements. As *k* increases, the increased competition in the market for pirate copies in shown in the fact that, in accordance with (13), this price sinks from the price which would be optimal in the absence of the other bootleggers, $p_{\max}$, towards the cost of sending a single advertisement, *b*.

Combining (13), (15), and (16), the revenue of the bootlegger can be also written

$$(17) \qquad R_i = r_{\min}\left(1-e^{-A_i}\right) = r_{\min} - r_{\min}\left(\frac{b}{r_{\max}}\right)^{1/k} = r_{\min} - b$$

so that profit of each bootlegger is seen to be given by

$$(18) \qquad P_i = R_i - bA_i = r_{\min} - b - b\ln\frac{r_{\min}}{b}$$

These results have been proved without assuming that the "demand function" $x(p)$ for pirate copies – i.e., the probability with a consumer buys a pirate copy if the lowest price suggested to her is *p* – is of the particular form (8). However, a more detailed analysis of the comparative statics of the market for pirate copies will depend on the form of $x(p)$.

The optimal price of a pirate copy in the absence of other bootleggers, $p_{\max}$, can be deduced from (8) by elementary means. Clearly, some of the consumers who receive an advertisement with price *p* will prefer a legitimate copy to a pirate copy if

$p > q + p_M - 1$ and $x(p) = x_B(p)$, but there will no such consumers when $p < q + p_M - 1$ and $x(p) = x_A(p)$. It is also clear that $px_A(p)$ obtains its largest value when $p = p_A = q/2$, and that $px_B(p)$ obtain its largest value when $p = p_B = qp_M/2$. Since $p_A > p_B$, it is possible to distinguish between three cases: if $p_A > p_B \geq p_M + q - 1$, the optimal price is p_B, if $p_M + q - 1 \geq p_A > p_B$, the optimal price is p_A, and if $p_A > p_M + q - 1 > p_B$, and the optimal price must be the corner solution $p = p_M + q - 1$. Solving for p_M, these results receive the form

$$(19) \qquad p_{\max} = \begin{cases} (qp_M)/2, & p_M \leq 2((1-q)/(2-q)) \\ p_M + q - 1, & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ q/2, & p_M \geq 1 - q/2 \end{cases}$$

Hence,

$$(20) \qquad x(p_{\max}) = \begin{cases} p_M/(2(1-q)), & p_M \leq 2((1-q)/(2-q)) \\ (1-p_M)/q, & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ 1/2, & p_M \geq 1 - q/2 \end{cases}$$

Now it can be concluded from (15) that the total number of the advertisements is

$$(21) \qquad A = \begin{cases} \ln\left[qp_M^2/(4b(1-q))\right], & p_M \leq 2((1-q)/(2-q)) \\ \ln\left[((p_M + q - 1)(1-p_M))/(qb)\right], & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ \ln(q/(4b)), & p_M \geq 1 - q/2 \end{cases}$$

The following proposition, which follows easily from these results together with Proposition 1, summarizes the comparative statics of the market for pirate copies for a fixed value of p_M and a fixed number of active bootleggers *k*. The additional assumption $p_M < 1 - q + p_{\min}$ which is made in this proposition means that some consumers prefer buying a legitimate copy to buying a pirate copy at $p_{\min}$, i.e. the cheapest price that pirate copies have on the market. In the next section it will be seen that this additional assumption is always valid when p_M is chosen optimally.

PROPOSITION 2. Suppose that there are *k* active bootleggers, that their strategies form a Nash equilibrium relative to the price of legitimate copies p_M, and that $p_M < 1 - q + p_{\min}$.

(a) An increase in the price p_M of legitimate copies increases the revenue of each bootlegger, the number of the advertisements, and the largest price $p_{\max}$ that pirate copies have on the market. If $k \geq 2$, an increase in p_M *decreases* the smallest price $p_{\min}$ of pirate copies on the market.

(b) An increase in the number of the bootleggers k does not affect the total number of the advertisements or the largest price of pirate copies $p_{\max}$, but it decreases $p_{\min}$, the revenue of each single bootlegger, and their total revenue.

(c) An increase of the advertising cost b does not affect $p_{\max}$, but it decreases the number of the advertisements and when $k \geq 2$, it shifts $p_{\min}$ upwards. In this case the profit P_i of each bootlegger receives its maximum value for a single value b_E of b within the interval $b \in (0, r_{\max})$. The profit P_i is an increasing function of b when $b < b_E$ and a decreasing function of b when $b > b_E$, and

$$\lim_{b \to 0} P_i = \lim_{b \to r_{\max}} P_i = 0 .$$

(d) An increase in the quality parameter q increases the maximum price $p_{\max}$ of pirate copies, the number of the advertisements, and the profits of the bootleggers. The effect of an increase of q on $p_{\min}$ can be either positive or negative, but it is always the case that $\partial p_{\min} / \partial q < 1/2$.

The intuitive explanation of the last of these results is that an improvement in the quality of pirate copies on the one hand allows the bootleggers to charge higher prices, but on the other hand it also intensifies competition by increasing the number of advertisements, and this tends to lower the prices of pirate copies. This leads to the prediction that when the pirates have a considerable risk of receiving a punishment, an improvement in the quality of pirate copies can be expected to increase their price dispersion. The fact that $\partial p_{\min} / \partial p_M < 0$ when $k \geq 2$ has a similar intuitive interpretation.

Part (c) of this result, which is illustrated by Figure 1 below, is concerned with the question how the profits of each bootlegger depend on the parameter b. This question is important for obvious reasons: when the number of the bootleggers on the market depends on their profits in accordance with (3), it is in the interest of the copyright owner that their profits are low. Part (c) of this result implies that when $k \geq 2$, the

profits earned by the bootleggers are for sufficiently low values b increased by an increase of b, which might increase the incentives to enter the market for pirate copies. I formulate this observation as a separate corollary.

COROLLARY 1.

(a) For each fixed value of p_M an increase in F has always a non-positive effect on the number of the bootleggers.

(b) For each fixed value of p_M an increase in b has sometimes a positive and sometimes a negative effect on the number of the bootleggers.

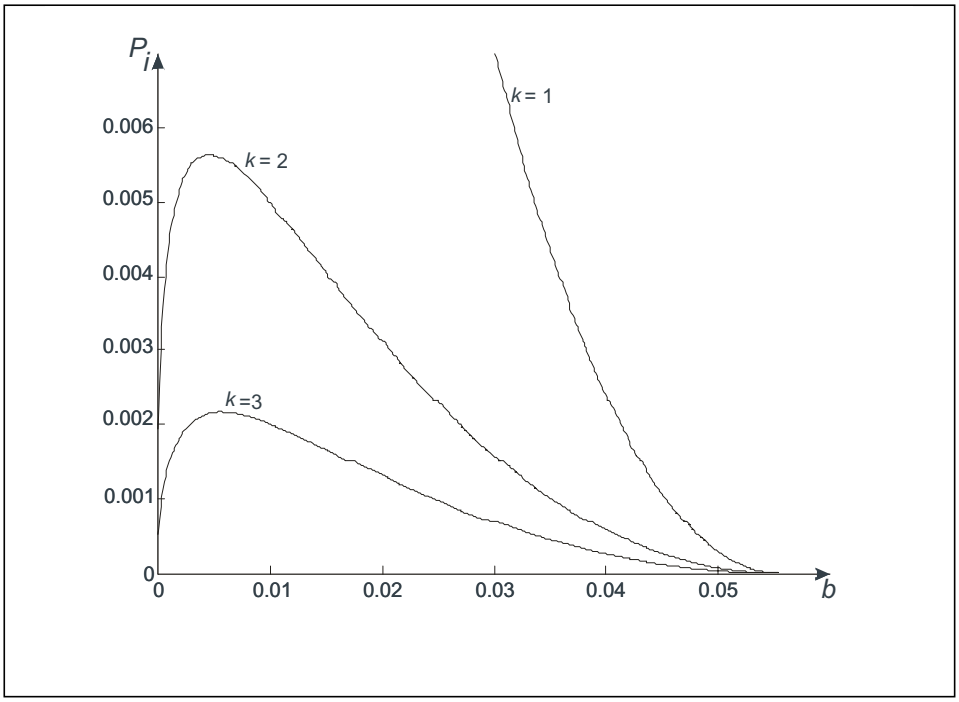

Figure 1. The profit P_i of each bootlegger as a function of the "advertising cost" b for some values of k when $p_M = 1/3$ and $q = 2/3$.

3.4. THE MARKET FOR LEGITIMATE COPIES

In the current model the profit of the copyright owner is equal with her revenue $R_M(p_M) = p_M D_M(p_M)$. The characterization of the demand $D_M(p_M)$ of the copyright owner is easy when $p_M > 2((1-q)/(2-q))$. In this case $p_{\max}$ is according to (19) such that none of the consumers who have the option of buying a pirate copy will choose a legitimate copy, and the demand of the copyright owner is simply

(22) $$D_M(p_M) = e^{-A}(1-p_M)$$

Here the total number of the advertisements A is given by (21), so that

(23) $$R_M(p_M) = \begin{cases} (qbp_M)/(p_M+q-1), & 2((1-q)/(2-q)) < p_M < 1-q/2 \\ (4bp_M(1-p_M))/q, & p_M \geq 1-q/2 \end{cases}$$

It is now immediately observed that when $R_M(p_M)$ is given by either of these formulas, the copyright owner can increase her profits by lowering p_M, so that these choices of p_M cannot be optimal for her.

It is essentially more complicated to calculate $D_M(p_M)$ in the more interesting case in which $p_M \leq 2((1-q)/(2-q))$. Clearly, in this case the consumers θ for whom $p_M \leq \theta < (p_M - p_{\max})/(1-q)$ will buy a legitimate copy if and only if they do not receive any advertisements. The demand from such consumers is

(24) $$D_L(p_M) = \left(\frac{p_M - p_{\max}}{1-q} - p_M\right)e^{-kA_1} = \frac{2b}{p_M}$$

On the other hand, a consumer would buy a legitimate copy even if she could buy a pirate copy for $p_{\min}$ if $\theta \geq (p_M - p_{\min})/(1-q)$. Putting

(25) $$p'_{\min} = \max\{p_{\min}, p_M + q - 1\}$$

the demand from such consumers (if any) is seen to be

(26) $$D_H(p_M) = \max\{0, 1-(p_M - p_{\min})/(1-q)\} = 1-(p_M - p'_{\min})/(1-q)$$

Finally, the consumers θ for whom $(p_M - p_{\max})/(1-q) \leq \theta < (p_M - p_{\min})/(1-q)$ will buy a legitimate copy if they do not receive an advertisement which contains a price which is lower than $p = p_M - (1-q)\theta$. The demand from such consumers is

(27) $$D_{Med}\left(p_M\right)=\int_{(p_M-p_{\max})/(1-q)}^{(p_M-p'_{\min})/(1-q)} e^{-kA_1\left(p_M-(1-q)\theta\right)}d\theta$$

Here the indexes 'L', 'Med', and 'H' refer to low, medium, and high valuation, respectively. The total demand of the copyright owner is the sum of the three demand components, i.e.

(28) $$D_M\left(p_M\right)=D_H\left(p_M\right)+D_{Med}\left(p_M\right)+D_L\left(p_M\right)$$

Unfortunately, it seems that one cannot express the value of the integral which occurs in (27) in terms of elementary functions. The results in the rest of this section are based on a change of variables in this integral. If one puts

(29) $$z=\left(p_M-(1-q)\theta\right)/\left(qp_M\right)$$

the demand component $D_{Med}\left(p_M\right)$ is seen to equal

(30) $$D_{Med}\left(p_M\right)=\frac{qp_M}{1-q}\int_{p'_{\min}/(qp_M)}^{1/2} e^{-kA_1(qp_M z)}dz$$

Using (8), (13), and (14) , the integrated function can be expressed in the form

(31) $$\begin{aligned} e^{-kA_1(qp_M z)} &= \left(\frac{p_{\min}x\left(p_{\min}\right)}{\left(qp_M\right)zx\left(\left(qp_M\right)z\right)}\right)^{k/(k-1)} \\ &= \frac{b(1-q)}{4^{1/(k-1)}qp_M^2}\frac{1}{\left(z(1-z)\right)^{k/(k-1)}}=\frac{1-q}{qp_M^2}f(z) \end{aligned}$$

where

(32) $$f(z)=\frac{b}{4^{1/(k-1)}\left(z(1-z)\right)^{k/(k-1)}}$$

Putting these results together, the demand of the copyright owner can now be expressed in the form

(33) $$D_M\left(p_M\right)=\frac{2b}{p_M}+\frac{1}{p_M}\int_{p'_{\min}/qp_M}^{1/2} f(z)dz+\left(1-\frac{p_M-p'_{\min}}{1-q}\right)$$

These results make it easy to analyze the comparative statics of the profit of the copyright owner relative to the variables b, q, and k when the price set by the copyright owner is fixed.

PROPOSITION 3. Assume that the price p_M of legitimate copies is fixed, that there are k active bootleggers, and that their strategies form a symmetric Nash equilibrium relative to p_M.

(a) The demand $D_M(p_M)$ of copyright owner is a decreasing function of the quality parameter q.

(b) When $p_M \geq 2((1-q)/(2-q))$, the demand $D_M(p_M)$ is independent of the market structure in the market for pirate copies, but when $p_M < 2((1-q)/(2-q))$, $D_M(p_M)$ is a decreasing function of k.

(c) An increase in the fixed cost F has a non-negative effect on $D_M(p_M)$. An increase in the advertising cost b increases $D_M(p_M)$ if it does not affect the number of the bootleggers. If this is not the case, an increase in b can also decrease $D_M(p_M)$.

According to part (c) of this proposition, an increase in the punishments for piracy can decrease the profits of the copyright owner also in the absence of *network effects*, which would make the market penetration of the product affect the value that it has for the consumers, because an increase in the "advertising cost" might increase the profits of the bootleggers and the number of the bootleggers who enter the market.

I now turn to the optimization problem of the copyright owner, which is the problem of maximizing the revenue $R_M(p_M) = p_M D_M(p_M)$, where $D_M(p_M)$ is given by (33). It turns out that although the integral which occurs in (33) cannot be evaluated explicitly, the derivative of the revenue $R_M(p_M)$ has a surprisingly simple expression.

PROPOSITION 4. The optimal price p_M of legitimate copies is never such that some pirate copies would be so cheap that there would be no consumers who prefer buying a legitimate copy to buying one of them. If p_M is such that this is not the case, and if there are pirate copies on the market,

$$\frac{dR_M(p_M)}{dp_M} = 1 - \frac{2p_M}{1-q} + \frac{2p_{\min}}{1-q} \tag{34}$$

This proposition implies that for each fixed value of k the derivative $dR_M(p_M)/dp_M$ has the value zero when

(35) $$p_M = p_{\min,k} + \frac{1-q}{2}$$

where $p_{\min,k}$ denotes the minimum price of pirate copies for the given p_M and *k*. Further, together with the fact that $\partial p_{\min,k} / \partial p_M < 0$ (see Proposition 2(a) above) this implies that for each *k* there can be at most one value of p_M for which (35) is valid.

However, the actual number of the pirates $k(p_M)$ is determined by p_M in accordance with (3), and it is a non-decreasing function of p_M. According to Proposition 2(b), $p_{\min,k}$ is a decreasing function of *k* and hence, also the equilibrium value of p_M determined by (35) is a decreasing function of *k*. These observations yield the following characterization of the equilibrium price p_M of legitimate copies.

COROLLARY 2. (a) Assume that there are $K = k$ potential bootleggers and that they do not have fixed costs. If there are pirate copies on the market in equilibrium, the equilibrium value of p_M is determined by the condition (35).
(b) More generally, when there are *K* potential bootleggers with fixed costs $F \geq 0$ there can be at most one value of p_M for which (35) is valid with $k = k(p_M)$. The optimal choice of p_M is either this value (if it exists) or the value which maximizes $R_M(p_M)$ in $\{\overline{p}_M(k) | k = 0,1,...,K-1\}$, where $\overline{p}_{M,k}$ denotes the largest price for which the number of the active bootleggers stays below *k*, i.e.

$$\overline{p}_{M,k} = \max\{p_M | P_i(p_M;k) \leq F\}$$

Figure 2 illustrates the first part of this result. It shows the revenue curve $R_M(p_M)$ of the copyright owner when the number *k* of the bootleggers is 0, 1, and 2, and the limit of $R_M(p_M)$ when $k \to \infty$. In the situation of the figure the copyright owner would have to lower p_M to the value p'_M in order to prevent the emergence of a market for pirate copies, and in equilibrium she will let all potential bootleggers enter. In the limit in which the number of the active bootleggers approaches infinity the optimal price p_M of legitimate copies approaches the value $p_{M,opt}$ shown in Figure 2.

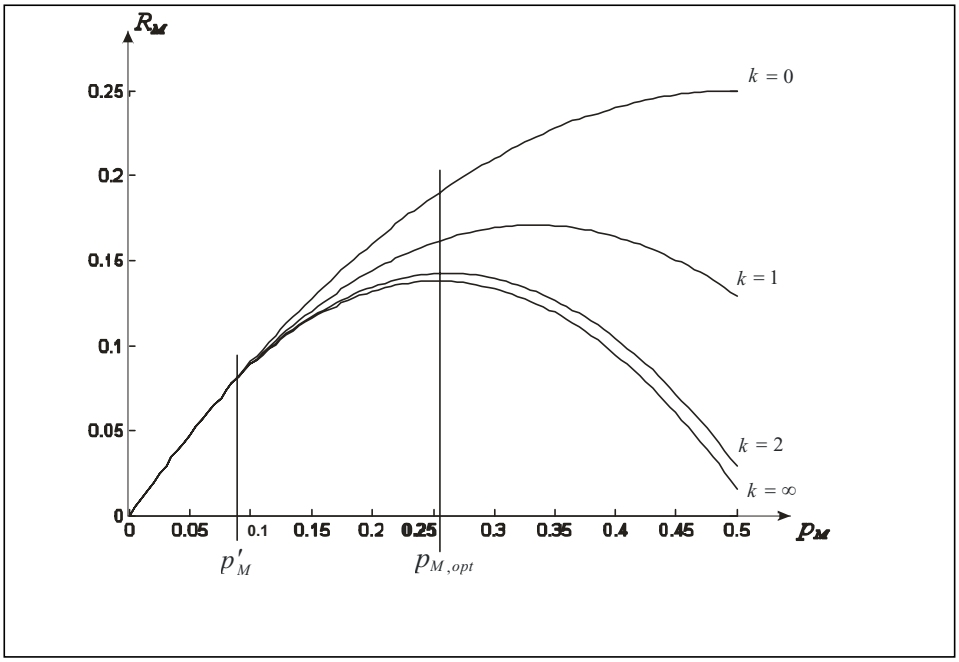

Figure 2. The revenue curve of the copyright owner for some values of k when b=0.002, q=0.5, and F=0.

Figure 3 represents a situation in which the potential bootleggers have fixed costs, but which is similar in other respects. The dotted lines show the revenue of the copyright owner in the presence of 0, 1, and 2 bootleggers. The revenue curve of the copyright owner – which is shown as a solid line – has a discontinuity at the points $\overline{p}_{M,1}$ and $\overline{p}_{M,2}$ at which the number of the bootleggers changes. According to the latter part of the above corollary, the equilibrium value of p_M is either one of the points of discontinuity or the maximum of the curve which corresponds to one of the values $k = 0$, $k = 1, \ldots$. Figure 3 illustrates the former possibility, since in it the optimal price p_M is the largest price which suffices to block the entry of the second bootlegger, i.e. $\overline{p}_{M,2}$.

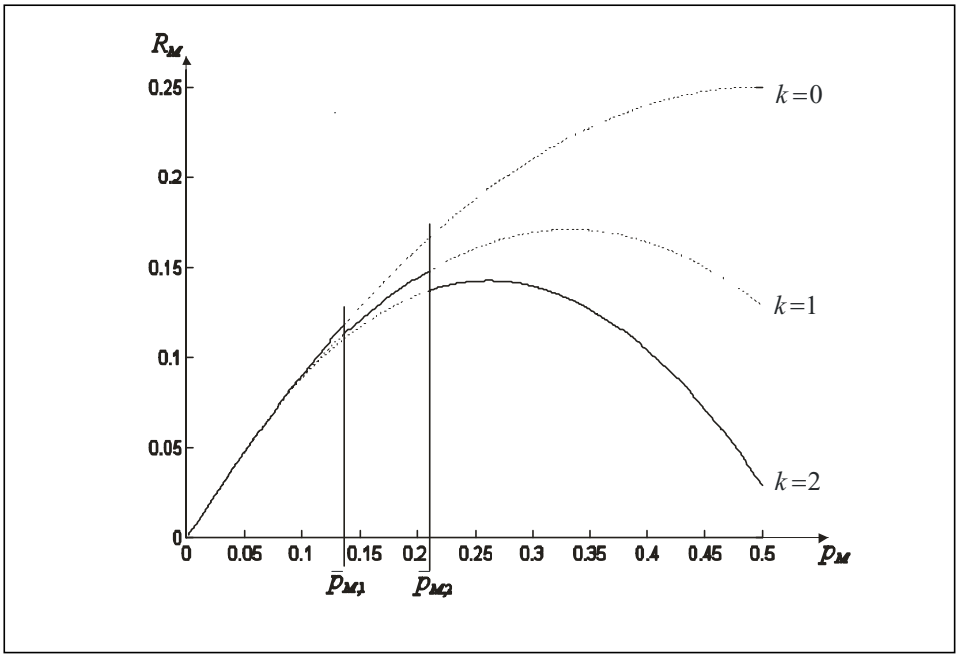

Figure 3. The revenue curve of the copyright owner when b=0.002, q=0.5, and F=0.001.

When the number of the bootleggers k is fixed, according to Proposition 3(c) an increase of b shifts the revenue curve $R_M(p_M)$ upwards and hence, it must correspond to an increase of the revenue of the copyright owner. However, if b is sufficiently small and $k \geq 2$, an increase of b will increase the profits of the bootleggers. This decreases the p_M values $\bar{p}_{M,2}$, $\bar{p}_{M,3}, \ldots$ at which the bootleggers enter, and if one of these values is the equilibrium value of p_M, this decreases of the revenue of the copyright owner in equilibrium.

The following corollary follows immediately from these observations and Proposition 3.

COROLLARY 3. Suppose that the strategies of the copyright owner and the bootleggers form a Nash equilibrium of the game 1) - 4).

a) The revenue of the copyright owner is a non-decreasing function of the fixed cost F and a decreasing function of q.
b) The revenue of the copyright owner is sometimes increased and sometimes decreased by an increase in the advertising cost b.

Hence, our earlier conclusion that an increase in the punishments for piracy might decrease the revenue of the copyright owner generalizes to the equilibriums of the model as a whole. I shall conclude this section by presenting some other comparative static results which are concerned with these equilibriums. Clearly, the comparative static properties of the model will be different in the equilibriums in which p_M is optimal relative to some fixed number of bootleggers, and the "corner solutions" in which p_M has the largest value which suffices to block the entry of one more bootlegger. In the following proposition, attention has been restricted to the equilibriums of the former kind. These equilibriums are the ones in which the revenue of the copyright owner is maximal also when the number of the active bootleggers is exogenously given, which corresponds to setting $F = 0$.

PROPOSITION 5. Suppose that the strategies of the copyright owner and the bootleggers form a Nash equilibrium of a game of the form 1) - 4) in which $F = 0$. Suppose further that there are $K = k$ active bootleggers who send advertisements.

(a) The price p_M of legitimate copies and the minimum and maximum prices of pirate copies, $p_{\min}$ and $p_{\max}$, are decreased by an increase in k.

(b) When $k = 1$, the prices p_M and $p_{\max}$ are independent of b. When $k \geq 2$, the prices p_M, $p_{\max}$, and $p_{\min}$ are increasing functions of the advertising cost b, and

$$\frac{dp_M}{db} = \frac{dp_{\min}}{db}$$

(c) A raise in the quality parameter q shifts p_M downwards, but the derivatives $dp_{\max}/dq$ and $dp_{\min}/dq$ can be either positive or negative.

3.5. Concluding Remarks

Above the role of government policy and DRM systems in preventing commercial piracy was analyzed in a setting in which the profitability of piracy was restricted not just by these factors and the competition with the copyright owner, but also by the competition between the commercial pirates. The at first glance puzzling fact that the prices of pirate copies do not always sink to zero via Bertrand competition between the pirates was explained by drawing a distinction between the different effects that government monitoring and DRM systems have on the illegal business model of the commercial pirate. The costs of breaking DRM systems and the expected costs of a punishment were viewed as consisting of a *fixed cost of production* and an "*advertising cost*" which depends on the number of the consumers to whom a bootlegger – i.e., a commercial pirate – offers her products and which keeps the prices of pirate copies above their production costs.

Above the markets for legitimate and illegitimate copies of information goods were studied both when the copyright owner chooses the price of legitimate copies optimally and when this is not the case, e.g. because of lack of information concerning the pirate copy market. The comparative statics of these markets was studied with respect to the quality of the pirate copies and the policy variables, i.e. the fixed and the variable costs of the bootleggers. Some of the results of this analysis were to be expected, but others were more surprising.

For example, it turned out that the revenue of the copyright owner is decreased by an increase in the quality of pirate copies both when the price of legitimate copies is optimal and when it is exogenously given, and that – as long as this does not affect the number of the bootleggers on the market – an increase in their "advertising costs" increases the revenue of the copyright owner and the prices of pirate copies. More interestingly, it also turned out that when there are several bootleggers on the market, an increase in the price of legitimate copies increases price dispersion in the market for pirate copies and decreases their minimum price. Since the pirate copies which have a very low price might be viewed as the counterpart of non-commercial forms of piracy in the current model, this result can be taken to mean that if the copyright owner chooses a sufficiently high price for an information good, it will simultaneously be subject to both commercial and non-commercial forms of piracy.

Our analysis also revealed that the effects of the two policy variables on the revenue of the copyright owner were different: whereas it was always in the interest of the copyright owner that the fixed costs of the bootleggers were increased, this was not true of the "advertising costs", because an increase in the "advertising costs" might increase the profitability of commercial piracy and the incentive to enter the market for pirate copies. As the current model contains just a single information good without any network effects, this result is distinct from the familiar results that the producer of an information good might profit from piracy if this makes it easier to sell complementary goods or services to consumers,[23] or if this leads network effects which increase the popularity of the product.[24]

Since the fixed costs of the bootleggers can be caused by technical protection devices whereas – at least in the case of commercial piracy on the Internet – the "advertising costs" result almost completely from the increased risk of a punishment, this result can be interpreted to mean that an improvement in the DRM systems which make an information good technically difficult to copy is always in the interest of the copyright owner, but this is not necessarily true of an increase in the legal protection of information goods.

Above I did not present a detailed *welfare analysis* from which one could have deduced the optimal values of the policy variables of the model. Such an analysis is made problematic by the fact that it is not clear whether a social planner should aim at maximizing welfare which is obtained by illegal means, like through commercial piracy. I also did not analyze the effects of piracy on the *market penetration* of the product, which would be quite essential if one wanted to include network effects in the current model. Inclusion of network effects would lead to several interesting questions that we have not considered. E.g., above it was seen that an improvement in the quality of pirate copies tends to lower the revenue of the copyright owner, and it would be interesting to find out whether this effect could in the presence of network effects be balanced by the positive effects of the increase in market penetration that a quality improvement would cause.

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[23] For example, the demand for concerts by an artist might be increased by the pirate copies of her recordings (Gayer and Shy, 2006). Somewhat less obviously, when the consumers do not know in advance which information products they prefer (e.g. which musical recordings they would enjoy listening), the possibility to sample pirate copies might make them willing to pay more for their preferred product. Cf. Peitz and Waelbroeck, 2006b.

[24] See e.g. Conner and Rumelt (1991, Proposition 4 on p. 133).

APPENDIX. PROOFS OF THE PROPOSITIONS IN CHAPTER 3.

PROOF OF PROPOSITION 1. When one puts

$$f\left(A_i, a_i, p\right) = px\left(p\right)\exp\left(-\sum_{j\neq i} A_j\left(p\right)\right)e^{-A_i}a_i\left(p\right)$$

the Euler equation which must be valid in the interval $\left(p_{\min}, p_{\max}\right)$ for a function $A_i\left(p\right)$ for which the function R_i given by (12) obtains its maximum value turns out to be

(A1) $$\frac{\partial f}{\partial A_i} - \frac{d}{dp}\frac{\partial f}{\partial a_i} = 0$$

This Euler equation is equivalent with

$$-px\left(p\right)\exp\left(-\sum_{j\neq i} A_j\left(p\right)\right)e^{-A_i}a_i\left(p\right) - \frac{d}{dp}\left[px\left(p\right)\exp\left(-\sum_{j\neq i} A_j\left(p\right)\right)e^{-A_i}\right] = 0$$

and further with

(A2) $$x\left(p\right) + px'\left(p\right) = px\left(p\right)\sum_{j\neq i} a_j\left(p\right)$$

Since we are considering a symmetric equilibrium, it can be concluded that

(A3) $$a_i\left(p\right) = \frac{1}{k-1}\left(\frac{1}{p} + \frac{x'\left(p\right)}{x\left(p\right)}\right)$$

and further that

$$A_i\left(p\right) = \frac{1}{k-1}\int_{p_{\min}}^{p}\left(\frac{1}{u} + \frac{x'\left(u\right)}{x\left(u\right)}\right)du = \frac{1}{k-1}\ln\frac{px\left(p\right)}{p_{\min}x\left(p_{\min}\right)}$$

This proves the validity of (14).

The result (13) is can be deduced from (14) by observing that (14) and (12) imply that

(A4) $$\begin{aligned} R_i &= \int_{p_{\min}}^{p_{\max}} px\left(p\right)\left(\frac{p_{\min}x\left(p_{\min}\right)}{px\left(p\right)}\right)e^{-A_i(p)}a_i\left(p\right)dp \\ &= p_{\min}x\left(p_{\min}\right)\left(1 - e^{-A_i}\right) \end{aligned}$$

and that the profit of each bootlegger is given by

(A5) $P_i = R_i - bA_i = p_{\min} x\left(p_{\min}\right)\left(1-e^{-A_i}\right) - bA_i$

Since each bootlegger could choose to send arbitrarily many advertisements with the price $p_{\min}$ without having to compete with the other bootleggers, A_i must have the value which maximizes profits for the given value of $p_{\min}$. In other words, in equilibrium it must be the case that

$$\frac{\partial P_i}{\partial A_i} = p_{\min} x\left(p_{\min}\right) e^{-A_i} - b = 0$$

so that

(A6) $A_i = \ln \frac{p_{\min} x\left(p_{\min}\right)}{b} = \ln \frac{r_{\min}}{b}$

On the other hand, the values of A_i and $p_{\min}$ are connected also by the fact that the total number of the advertisements that the advertiser *i* sends is $A_i\left(p_{\max}\right)$. The result (14) implies that

(A7) $A_i = A_i\left(p_{\max}\right) = \frac{1}{k-1} \ln \frac{p_{\max} x\left(p_{\max}\right)}{p_{\min} x\left(p_{\min}\right)} = \frac{1}{k-1} \ln \frac{r_{\max}}{r_{\min}}$

Together the results (A6) and (A7) imply (13). □

PROOF OF PROPOSITION 2. First it is observed that since by assumption $p_{\max} \geq p_{\min} > p_M + q - 1$, (19) implies that $p_M < 2\left(\left(1-q\right)/\left(2-q\right)\right)$. Keeping this in mind, it is observed that part (a) of this proposition follows trivially from (13), (16), (19), (20), and (21), except for the statement that $\partial p_{\min} / \partial p_M < 0$. It can be proved by observing that the when $k \geq 2$ and $p_{\min} > q + p_M - 1$, the condition (13), i.e. $r_{\min} = b^{1-1/k} r_{\max}^{1/k}$, can be expressed in the form

$$\frac{\left(qp_M - p_{\min}\right) p_{\min}}{q\left(1-q\right)} = b^{1-1/k} \left(\frac{qp_M^2}{4\left(1-q\right)} \right)^{1/k}$$

Hence,

$$\frac{\left(qp_M - 2p_{\min}\right)}{q\left(1-q\right)} \frac{\partial p_{\min}}{\partial p_M} + \frac{p_{\min}}{1-q} = \frac{2}{kp_M} r_{\min}$$

and

$$\frac{\left(qp_M - 2p_{\min}\right)}{q\left(1-q\right)}\frac{\partial p_{\min}}{\partial p_M} = p_{\min}\left(\frac{2x\left(p_{\min}\right)}{kp_M} - \frac{1}{1-q}\right)$$
$$= \frac{p_{\min}}{1-q}\left(\frac{2\left(qp_M - p_{\min}\right)}{qkp_M} - 1\right) < 0$$

Since $qp_M - 2p_{\min} > qp_M - 2p_{\max} = 0$, it now follows that $\partial p_{\min}/\partial p_M < 0$.

The parts (b) and (c) of this proposition follows trivially from (13), (15), (16), (19), and (21), except for except for the claim concerning the profit P_i of the bootleggers. In order to demonstrate it, we assume that $k \geq 2$ and observe that (18) and (13) imply that

(A8) $$\frac{\partial P_i}{\partial b} = \left(1 - 1/k\right)\frac{r_{\max}^{1/k}}{b^{1/k}} - 1 - \frac{1}{k}\ln\frac{r_{\max}}{b} + \frac{1}{k}$$

and that

(A9) $$\frac{\partial^2 P_i}{\partial b^2} = -\frac{k-1}{k^2}\frac{r_{\max}^{1/k}}{b^{1+1/k}} + \frac{1}{kb}$$

This immediately implies that there is precisely one value of $b \in \left(0, r_{\max}\right]$ for which $\partial^2 P_i/\partial b^2 = 0$, and this further implies that there are at most two values of $b \in \left(0, r_{\max}\right]$ for which $\partial P_i/\partial b = 0$. It is now observed that

(A10) $$\left[\frac{dP_i}{db}\right]_{b=r_{\max}} = \left(1 - 1/k\right) - 1 + \frac{1}{k} = 0$$

so that within the interval $\left(0, r_{\max}\right)$ there can be at most one value of b for which $\partial P_i/\partial b = 0$. On the other hand, since the function P_i is by construction positive in the interval $\left(0, r_{\max}\right)$, and since it has the limit 0 when $b \to 0$ and when $b \to r_{\max}$, it must be the case that $\partial P_i/\partial b = 0$ for at least one value $b_E \in \left(0, r_{\max}\right)$. Hence, P_i is an increasing function of b when $b < b_E$ and that P_i is a decreasing function of b when $b > b_E$.

Finally, turning to the part (d) of this proposition, also all the statements in it follow trivially from (13), (18), (19), (20) and (21), except for the statement concerning $\partial p_{\min}/\partial q$. It can be demonstrated by using (8) for expressing (13) in the form

$$\frac{qp_M - p_{\min}}{q^{1+1/k}\left(1-q\right)^{1-1/k}}p_{\min} = \frac{b^{1-1/k}p_M^{2/k}}{4^{1/k}}$$

Since the right-hand side of this equation is constant for each constant value of q, it must be the case that

(A11) $$\left(\frac{qp_M - 2p_{\min}}{(qp_M - p_{\min})p_{\min}}\right)\frac{\partial p_{\min}}{\partial q} = -\left(\frac{p_M}{qp_M - p_{\min}} - \frac{1+1/k}{q} + \frac{1-1/k}{1-q}\right)$$

Clearly, the multiplier of $\partial p_{\min}/\partial q$ in (A11) is always positive, but the term on the right-hand side is sometimes positive and sometimes negative: the right-hand side is negative e.g. when $k = 2$ and $q > 3/4$, but it has a positive value when $q \approx 0$ and also $p_{\min}/(qp_M)$ is very small, since in this case

$$-\left(\frac{p_M}{qp_M - p_{\min}} - \frac{1+1/k}{q} + \frac{1-1/k}{1-q}\right) \approx -\frac{1}{q} + \frac{1+1/k}{q} = \frac{1}{kq}$$

This proves the statement that $\partial p_{\min}/\partial q$ is sometimes positive and sometimes negative. In order to prove the claim concerning its upper limit we now introduce the notation $\gamma = p_{\min}/(qp_M)$ and observe that the right-hand side of (A11) can be positive only if

$$\frac{1}{q(1-\gamma)} - \frac{1+1/k}{q} + \frac{1-1/k}{1-q} < 0$$

This is equivalent

(A12) $$\gamma < \frac{1/k - q}{1 - 2q + 1/k}$$

The partial derivative $\partial p_{\min}/\partial p_M$ can be positive only when this condition is valid, and if this is the case,

(A13) $$\frac{qp_M - 2p_{\min}}{qp_M - p_{\min}} = \frac{1-2\gamma}{1-\gamma} > \frac{(1-2q+1/k) - 2(1/k - q)}{(1-2q+1/k) - (1/k - q)} = \frac{1-1/k}{1-q} > 1 - 1/k \geq \frac{1}{2}$$

Hence, now one can conclude from (A11) that if $\partial p_{\min}/\partial p_M$ is positive,

$$\frac{\partial p_{\min}}{\partial p_M} < 2p_{\min}\left(\frac{1+1/k}{q} - \frac{p_M}{qp_M - p_{\min}}\right) < \frac{2p_{\min}}{kq}$$

Given that $p_{\min} < p_{\max} = qp_M/2$ and that $k \geq 2$, it now follows that

$$\frac{\partial p_{\min}}{\partial p_M} < \frac{p_M}{k} < \frac{1}{2}. \square$$

PROOF OF PROPOSITION 3. When $p_M \geq 2\left((1-q)/(2-q)\right)$ the statements of this proposition follow trivially from (23) and the continuity of $D_M\left(p_M\right)$. Suppose then that $p_M < 2\left((1-q)/(2-q)\right)$. According to (33) in this case

$$\text{(A14)} \quad D_M\left(p_M\right) = 1 - \frac{p_M - p'_{\min}}{1-q} + \frac{1}{p_M}\int_{p'_{\min}/qp_M}^{1/2} f\left(z\right)dz + \frac{2b}{p_M}$$

where $f\left(z\right) = b\Big/\left(4^{1/(k-1)}\left(z\left(1-z\right)\right)^{k/(k-1)}\right)$. Below $f\left(z\right)$ will be viewed as given by this formula for arbitrary real values of k although, of course, (A14) has a meaningful economic interpretation only when k is an integer. Clearly, (A14) is formally valid also when $k=1$, since in this case $p'_{\min}/\left(qp_M\right) = p_{\max}/\left(qp_M\right) = 1/2$, so that the integral on the right-hand side of (A14) vanishes.

When $p'_{\min} = p_M + q - 1$, (A14) implies that

$$D_M\left(p_M\right) = \frac{1}{p_M}\int_{1/p_M-(1-p_M)/(qp_M)}^{1/2} f\left(z\right)dz + \frac{2b}{p_M}$$

and it immediately follows that $\partial D_M\left(p_M\right)/\partial b > 0$ and $\partial D_M\left(p_M\right)/\partial q < 0$. When it is observed that $4z\left(1-z\right) < 1$ when $z < 1/2$, it follows that

$$\frac{\partial f\left(z\right)}{\partial k} = \frac{1}{z\left(1-z\right)}\frac{\partial}{\partial k}\left(\frac{1}{4z\left(1-z\right)}\right)^{1/(k-1)} < 0$$

so that also $\partial D_M\left(p_M\right)/\partial k < 0$. This proves (a) and (b) when $p'_{\min} = p_M + q - 1$.

When $p'_{\min} = p_{\min}$, the analysis of the considered partial derivatives is made more complicated by the implicit dependence of $p_{\min}$ on b, q, and k. However, (31) implies that

$$\text{(A15)} \quad f\left(\frac{p_{\min}}{qp_M}\right) = \frac{qp_M^2}{1-q}$$

so that one can conclude from (A14) that

$$\frac{\partial D_M\left(p_M\right)}{\partial p'_{\min}} = \frac{1}{1-q} + \frac{-1}{p_M\left(qp_M\right)} f\left(\frac{p_{\min}}{qp_M}\right) = 0$$

and this makes it legitimate to leave this dependence out of consideration below.

Now it follows that when $p'_{\min} = p_{\min}$,

(A16) $$\frac{\partial D_M(p_M)}{\partial b} = \frac{1}{p_M}\int_{p'_{\min}/qp_M}^{1/2} \frac{\partial f(z)}{\partial b} dz + \frac{2}{p_M} > 0$$

so that the demand of the copyright owner is an increasing function of *b*. Further, since $\partial f(z)/\partial k < 0$, must be the case that

(A17) $$\frac{\partial D_M(p_M)}{\partial k} = \frac{1}{p_M}\int_{p'_{\min}/qp_M}^{1/2} \frac{\partial f(z)}{\partial k} dz < 0$$

Finally, (A14) and (A15) together imply that

(A18) $$\begin{aligned}\frac{\partial D_M(p_M)}{\partial q} &= -\frac{p_M - p_{\min}}{(1-q)^2} - \frac{1}{p_M}\left(\frac{\partial}{\partial q}\left(\frac{p_{\min}}{qp_M}\right)\right) f\left(\frac{p_{\min}}{qp_M}\right) \\ &= -\frac{p_M - p_{\min}}{(1-q)^2} + \frac{1}{p_M}\frac{p_{\min}}{q^2 p_M}\left(\frac{qp_M^2}{1-q}\right) = -\frac{p_M}{(1-q)^2} + \frac{1}{q(1-q)^2} p_{\min} < 0\end{aligned}$$

This completes the proof of the parts (a) and (b) of the proposition. Turning to part (c), it is first observed that an increase in *F* can affect the demand of the copyright owner only by decreasing *k*, and as it just has been proved, a decrease of *k* has a non-negative effect on $D_M(p_M)$. Finally, above it was seen that $\partial D_M(p_M)/\partial b > 0$, so that an increase in *b* increases $D_M(p_M)$ if it does not affect the number of the active bootleggers. However, since according to Corollary 1(b) an increase in *b* can increase *k*, and since an arbitrarily small change in *b* can change *k* by 1, it must sometimes be the case that an increase in *b* decreases $D_M(p_M)$. □

PROOF OF PROPOSITION 4. First it is observed that if $p_M > 2((1-q)/(2-q))$, (23) immediately implies that $R_M(p_M) = p_M D_M(p_M)$ is a decreasing function of p_M and that the copyright owner can increase her profits by lowering the price p_M. Suppose now that $p_M \le 2((1-q)/(2-q))$. In this case (33) implies that

(A19) $$R_M(p_M) = p_M\left(1 - \frac{p_M - p'_{\min}}{1-q}\right) + \int_{p'_{\min}/qp_M}^{1/2} f(z)\,dz + 2b$$

and the derivative of $R_M(p_M)$ is seen to be

(A20) $$\frac{dR_M(p_M)}{dp_M} = \frac{d}{dp_M}\left[p_M\left(1 - \frac{p_M - p'_{\min}}{1-q}\right)\right] - \frac{d(p'_{\min}/qp_M)}{dp_M} f\left(\frac{p'_{\min}}{qp_M}\right)$$

If $p'_{\min} = p_M + q - 1$, this implies that

$$\frac{dR_M(p_M)}{dp_M} = -\frac{d}{dp_M}\left(\frac{1-(1-q)/p_M}{q}\right) f\left(\frac{p'_{\min}}{qp_M}\right) < 0$$

and the copyright owner has an incentive to lower the price. Hence, in equilibrium $p'_{\min} = p_{\min} > p_M + q - 1$, and some consumers would buy a legitimate copy even if they could get a pirate copy for $p_{\min}$.

When $p'_{\min} = p_{\min}$, the price $p_{\min}$ satisfies the condition (A15) and one can conclude from (A20) that

$$\frac{dR_M(p_M)}{dp_M} = \left(1 - \frac{2p_M - p_{\min}}{1-q}\right) + \frac{p_M}{1-q}\frac{dp_{\min}}{dp_M} - \left[\frac{1}{qp_M}\frac{dp_{\min}}{dp_M} - \frac{p_{\min}}{qp_M^2}\right]\frac{qp_M^2}{1-q}$$
$$= 1 - \frac{2p_M - p_{\min}}{1-q} + \frac{p_{\min}}{1-q} = 1 + \frac{2p_{\min}}{1-q} - \frac{2p_M}{1-q}$$

This completes the proof. □

PROOF OF PROPOSITION 5. According to Proposition 2(b) , for each value of p_M the minimum price $p_{\min,k}$ is decreased by an increase in *k*. Hence, Proposition 4 implies that an increase in *k* decreases the derivative $dR_M(p_M)/dp_M$ for each value of p_M, so that the value of p_M for which $dR_M(p_M)/dp_M = 0$ grows smaller if *k* increases. Now (19) and (35) imply that also $p_{\max}$ and $p_{\min}$ will decrease. This proves part (a) of the proposition.

Turning to part (b), (35) is written in the form

(A21) $$p_M(q,b,k) = p_{\min}(p_M,q,b,k) + \frac{1-q}{2}$$

When $k = 1$, $p_{\min} = p_{\max}$ is for each fixed value of p_M independent of *b*, so that also the solution of this equation is independent of *b*. Suppose then that $k \geq 2$. Differentiating (A21) with respect to *b* one gets

(A22) $$\frac{dp_M}{db} = \frac{\partial p_{\min}}{\partial b} + \frac{\partial p_{\min}}{\partial p_M}\frac{dp_M}{db}$$

According to Proposition 2, $\partial p_{\min}/\partial p_M < 0$ and $\partial p_{\min}/\partial b > 0$, and one can conclude from (A22) that

$$\frac{dp_{\min}}{db} = \frac{dp_M}{db} = \frac{\partial p_{\min}/\partial b}{1 - \partial p_{\min}/\partial p_M} > 0$$

Now (19) implies that also $dp_{\max}/db > 0$. Finally, (A21) also implies that

(A23) $$\frac{dp_M}{dq} = \frac{\partial p_{\min}}{\partial q} + \frac{\partial p_{\min}}{\partial p_M}\frac{dp_M}{dq} - \frac{1}{2}$$

Since $\partial p_{\min}/\partial q < 1/2$ according to Proposition 2(d), this implies that

$$\frac{dp_M}{dq} = \frac{\partial p_{\min}/\partial q - 1/2}{1 - \partial p_{\min}/\partial p_M} < 0$$

It must still be demonstrated that no similar general results are valid for $dp_{\max}/dq$ and $dp_{\min}/dq$. In the case of $dp_{\max}/dq$, perhaps the easiest way to see this is to observe that – as one can demonstrate by elementary means – when $k = 1$, $p_M = (1-q)/(2-q)$ and $p_{\max} = q(1-q)/(2(2-q))$, and that the sign of $dp_{\max}/dq = (d/dq)\left[q(1-q)/(2(2-q))\right]$ depends on the value of q. Assume then that $k \geq 2$. Clearly, (A23) implies also that

(A24) $$\frac{dp_{\min}}{dq} = \frac{dp_M}{dq} + \frac{1}{2} = \frac{\partial p_{\min}/\partial q - 1/2}{1 - \partial p_{\min}/\partial p_M} + \frac{1}{2}$$

so that since $1 - \partial p_{\min}/\partial p_M > 1$, $dp_{\min}/dq$ must be positive whenever $\partial p_{\min}/\partial q$ is positive. Perhaps the easiest way to see that $dp_{\min}/dq$ can be negative is to assume that k is very large and to write the condition (A21) in the form

(A25) $$\frac{qp_M - p_{\min}}{(1-q)q} = \frac{1}{2} - \frac{p_{\min}}{q}$$

Clearly, the left-hand side of this equation equals $x(p_{\min})$, and in the limit in which $k \to \infty$ it will be the case that $x(p_{\min})\,p_{\min} = b$. Hence, in the limit in which $k \to \infty$

$$\left(\frac{1}{2} - \frac{p_{\min}}{q}\right)p_{\min} = p_{\min}x(p_{\min}) = b$$

Clearly, the left-hand side of this equation is an increasing function of both q and $p_{\min}$ but the right-hand side is a constant, and this immediately implies that when k is sufficiently large $dp_{\min}/dq$ must be negative. □

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CHAPTER IV

COMMERCIAL PIRACY, NETWORK EXTERNALITIES, AND INTELLECTUAL PROPERTY POLICY

ABSTRACT

It has often been pointed out that *end-user piracy* may increase the profit of the copyright owner in the presence network externalities. I consider a model of *commercial piracy* with network externalities in which the positive price of pirate copies is explained by the increase in the risk of punishment that the informing of potential customers causes. The model yields a characterization of the optimal pricing policy of the copyright owner in the presence of commercial piracy, and shows how the profit-maximizing intellectual property protection strength increases with the quality of pirate copies.

4.1. INTRODUCTION

There is a relatively large economic literature on both the legal and the illegal ways in which consumers can share information goods.[1] The legal business models that are based on the sharing of information goods include e.g. resale markets of books, video rental stores, and the for-profit circulating libraries that existed in England in the 18th and 19th centuries. It is easy to see that if the copyright owner is able to charge a higher price for those copies of an information good which are shared by several consumers, such legal forms of sharing do not necessarily lower her profits (cf. Varian, 2000, pp. 475-7).

It has also been pointed out by several authors that if an information good is subject to sufficiently large network externalities, even illegal end-user copying might

[1] For a survey on the literature on illegal sharing (i.e., end-user piracy), see Peitz and Waelbroeck (2006).

increase the profit of the copyright owner.[2] This may be the case if piracy increases the valuation that the consumers give to the information good, and if this positive effect overrules the negative effect of losing consumers to the pirates, so that the net effect on the profit of the copyright owner is positive. Takeyama (1994) shows that this is sometimes the case in a setting in which pirate copies are of a lower quality than legitimate copies,[3] whereas Slive and Bernhardt (1998) prove a similar result in a model in which pirate copies are of the same quality with legitimate ones, and in which consumers are divided into two groups (business consumers and home consumers) who have a different ability to pirate.[4]

In each case, piracy is increases the revenue of the copyright owner if it is a surrogate for *price discrimination*: when the copyright owner cannot sell the information good at a different positive price to the low-valuation and high-valuation consumers, it might be in her interest to let the low-valuation consumers pirate it. The same point is illustrated also by the analysis of King and Lampe (2003). King and Lampe restrict attention to a case in which pirate copies are of the same quality with legitimate copies, and conclude that in this case piracy can be in the interest of the copyright owner only if there is a substantial body of high-valuation consumers who are not able to make use of a pirate copy. [5]

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[2] See also Shy and Thisse (1999), who prove a closely related result according to which allowing for piracy might be in the interest of a copyright owner for strategic reasons, in order to make the information good more attractive than a similar product of a competing manufacturer.

[3] Takeyama (1994, pp. 158-162) considers a model in which the consumers are divided into two homogenous groups, the high-valuation and the low-valuation consumers, and pirate copies are of a lower value than originals for each of these groups. If the copyright owner is unable to price discriminate, in Takayama's model she has she in the absence of piracy the options of choosing a price which is so low that also the low-valuation consumers will buy the product, or selling the product to the high-valuation consumers only at a higher price. It turns out that when the network externalities are sufficiently strong, letting the low-valuation consumers pirate the product yields higher profit than either of these alternatives (see formula (8) in ibid., p. 160).

[4] See Proposition 1 in Slive and Bernhard (1998), p. 893. Cf. also Proposition 4 in Conner –Rumelt (1991), p. 133.

[5] According to King and Lampe (2003), the results of Slive and Bernhard (1998), Conner and Rumelt (1991), and Takeyama (1994) are based on four assumptions. These are that "customers have a differential ability to pirate", "the seller does not have an ability to directly price discriminate between different customers", "the number of potential pirates is relatively small", and "the ability to pirate is inversely correlated with customer willingness-to-pay" (King and Lampe, 2003, p. 274). It should be observed that this is not quite rigorously true of the analysis of Takeyama, in which all consumers have an identical ability to pirate. In Takeyama's model market segmentation is caused by the fact that pirate copies are of a lower quality than legitimate copies, and when this is the case, the high-valuation consumers sometimes buy legitimate copies even when low-valuation consumers make use of pirate copies (Takeyama, 1994, p. 159).

Analogously, it is essential for the results that are deduced below that the difference in the surplus from buying a legitimate copy at price p_M, i.e. $\theta\xi - p_M$, and the surplus from buying a pirate copy for the price *p*, i.e. $q\theta\xi - p$, is an increasing function of the parameter θ, so that the consumers with

Software products are obvious examples of information goods which are subject to network externalities, since their popularity affects their value in many ways. Most obviously, when a consumer wishes to *exchange files* (like e.g. documents produced by a word processor) with other consumers, it is in her interest that the other consumers are using identical or at least compatible software products. Also the availability of complementary goods and services (such as plug-ins, product support, and training seminars) can be expected to improve with the popularity of a software product. Finally, the popularity of a software product that a company is using also improves the chances that the company will be able to hire employees who are familiar with the product in advance, and this reduces the expected costs of training employees.[6]

The economic analyses of software piracy in the presence of network externalities have until now been almost exclusively concerned with end-user piracy rather than with commercial piracy.[7] However, pirate copies of software products are not just distributed for free, but they are also sold at a positive price. Since both commercial and non-commercial forms of piracy are illegal, it is difficult to estimate to which extent the pirate copies that are currently in use result from commercial piracy rather than end-user piracy,[8] but e.g. the profits that major commercial pirates have earned suggest that also the commercial forms of software piracy are of a considerable economic significance.[9]

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a large θ value prefer a legitimate copy also when the consumers with a small θ value prefer a pirate copy. However, it will be not necessary to assume that the consumers with a higher θ value would have a smaller ability to make use of pirate copies (i.e. that they would receive fewer advertisements).

[6] Cf. Slive – Bernhardt, 1988, p. 888. Also the fact that *evaluation versions* of many commercial software products are freely available on the Internet suggests that the free distribution of software must, for one reason or another, be in the interest of software manufacturers (ibid, p. 887).

[7] An exception is formed by Banerjee (2003), pp. 113-116, which contains a short analysis of the effects of network externalities on the competition between a monopolist (i.e. the copyright owner) and a single commercial pirate. Banerjee assumes that the utility that a consumer draws from an information good consists of two components, a stand-alone value which is uniformly distributed in an interval $[\theta_L, \theta_H]$ and a component which results from network externalities, and which is identical for all consumers. It turns out that for a utility function of this kind, the profit of the monopolist is always smaller in the presence of the pirate than in her absence, independently of the strength of the network externalities. (The former is $\pi_m^{lf'*}$, given by formula (B.8) in ibid., Appendix, B on p. 124, and the latter is $\pi_m'^{*}$, given by formula (B.2) on p. 123, and it is easy to see that $\pi_m^{lf'*} < \pi_m'^{*}$).

[8] The Business Software Alliance publishes yearly estimates for the software piracy rates in the different countries of the world, and in the world as a whole (see http://w3.bsa.org/globalstudy/, accessed on March 18, 2008). However, these estimates do not draw a distinction between the commercial and the non-commercial forms of piracy.

[9] For reports by the U.S. Department of Justice on court cases against large-scale commercial software pirates, see e.g.

Intuitively, it seems that when the pirate copies are sold at a positive price, the effects of piracy on the revenue of the copyright owner might well differ from its effects when pirate copies are distributed for free. Piracy has a direct negative on the revenue of the copyright owner, when some of the consumers who would have bought a legitimate copy in the absence of piracy make use of a pirate copy instead, and it has an indirect positive effect, when it increases the valuation that the paying consumers give to the product. *Both* of these effects decrease as the price of pirate copies increases, since high-price pirate copies are less attractive to the customers of the copyright owner than low-price pirate copies, and since their availability causes smaller network effects. Hence, it is interesting to ask to which extent the earlier results concerning the effect of piracy on the profit of the copyright owner carry over to commercial piracy. Below I shall develop a model which addresses this question.

Pirate copies are by definition information goods that are not protected by copyright, and since they have several producers, one might expect that their prices would always sink to the level of their production costs via Bertrand competition. Accordingly, a satisfactory model of commercial piracy should explain why all pirated information goods do not resemble the copyrighted files that are illegally distributed via peer-to-peer networks in having zero price. In the current model the positive price of the pirate copies that are sold rather than distributed for free is explained by postulating that *commercial piracy is made possible by the punishments for piracy*.

If the pirate's risk of getting caught and receiving a punishment for piracy is increased whenever she informs her potential customers of the availability of pirate copies, the risk of a punishment functions analogously with an *advertising cost*. In this case each pirate might be able to offer her product only to a part of the potential consumers, and some consumers might be able to buy a pirate copy from a single commercial pirate only. Clearly, in this case the prices of pirate copies cannot be expected to sink to zero. A model with these features but without network

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http://www.usdoj.gov/usao/vae/Pressreleases/12-DecemberPDFArchive/05/20051213petersonnr.pdf , http://www.usdoj.gov/usao/vae/Pressreleases/08-AugustPDFArchive/06/20060825ferrernr.pdf, and http://www.usdoj.gov/usao/vae/Pressreleases/04-AprilPDFArchive/07/20070420knott.pdf (accessed on March 18, 2007).

externalities was put forward in Kiema (2008).[10] Below I shall consider a model which contains network effects, but which has an otherwise similar logical structure.[11]

In Section 4.2, I shall present the basic features of the current model. Section 4.3 characterizes the equilibria of the market for pirate copies, and Section 4.4 studies the optimization problem of the copyright owner. In the model the strength of copyright protection is represented by an "advertising cost" (i.e. expected disutility from punishment) which affects both the price and the availability of pirate copies, and in Section 4.5 I shall consider the problem of choosing its strength so that the revenue of the copyright owner is maximized. Section 4.6 concludes.

4.2. The Basic Features of the Model

The agents of the current model are the monopolist, a continuum $[0,1]$ of consumers which is indexed by θ, and an infinite number of commercial pirates, to whom I shall refer as *bootleggers*. The consumer θ gives the valuation $\theta\xi$ to a legitimately bought copy of the good, where the valuation parameter ξ is defined by $\xi = A + Bn_e$. Here $A, B \geq 0$ are constants, and n_e is the expected market penetration of all (both legally bought and pirated) copies of the product. Further, the consumer θ gives the valuation $q\theta\xi$ to a pirate copy of the product, where $0 < q < 1$ is a constant called the *quality parameter*.[12]

This specification of the network externality is similar with the one postulated in e.g. de Palma – Leruth (1996, p. 239) in so far that in the current model the utility from the network externality is higher for the high-valuation consumers. The specification differs from the one in the classical paper Katz – Shapiro (1985), although just like in the current model, in ibid. (p. 426) the value of a network good has two components, a stand-alone value which is different for different consumers, and a component which is due to the network externalities. However, in Katz and Shapiro's model the latter

[10] Essentially the same paper has been presented also as Chapter 3 above.

[11] More precisely, the two models differ also in so far that the model in Kiema (2008) was a model of an oligopoly of k commercial pirates, whereas below I shall for the sake of simplicity restrict attention to a situation in which the number of the pirates is infinite.

[12] Here the parameter q has been meant to represent not only the fact that a pirated product might be technically of a worse quality than a legally bought one or not operational at all. Rather, it can be also be viewed as a representation of the fact that some consumers prefer legally bought copies also for ethical reasons or because there might be legal sanctions against using (and not just against selling) pirate copies, or because buying a pirate copy requires giving credit card information to criminals.

component is identical for all consumers. It is clear that the specification that is used below is more natural in the context of information goods. If the considered good is e.g. a word processor and the considered network externality consists of the possibility to exchange files, it is clearly plausible to assume that the utility which is due to the network externality is small for the consumers for whom the stand alone value of the product is small.[13]

The bootleggers inform consumers of the availability of their products by sending them advertisements at random, and – as it will shortly be seen – the market share of each bootlegger must be infinitesimal in equilibrium. As it was explained above, in the current model pirate copies have a positive price because the informing of consumers increases the risk of getting caught and receiving a punishment. More specifically, I postulate that the cost of the punishment is G and that the sending of an advertisement increases its probability of a punishment by η, so that the "*advertising cost*" b for informing a single consumer is given by[14]

(1) $b = \eta G$

This "advertising cost" represents the strength of intellectual property rights in the current model. The monopolist, the bootleggers, and the consumers participate in the following game.

1) The copyright owner sets the price p_M of legitimate copies.

2) Observing p_M, the copyright owner, the consumers and the bootleggers form the expectation that the market penetration of (the legitimate and pirated copies of) the product will be n_e, where n_e has some value in $[0,1]$. This expectation determines the valuation parameter ξ in accordance with $\xi = A + Bn_e$, where $A, B \geq 0$.

3) The bootleggers choose whether to advertise pirate copies. If they do, they send advertisements to randomly chosen consumers. Sending a single advertisement causes an increase b in the expected disutility of punishment. The bootleggers are not constrained to offering the product at the same price in different advertisements.

4) The consumers choose whether to buy the product. Each consumer has the options of buying the product from the copyright owner for p_M or from one of the

[13] Accordingly, in the context of the current model this specification of the network externality seems more natural than the one use d in Banerjee (2003). Cf. also footnote 7 above .

[14] For a discussion of the econometric problem of actually constructing an index which measures the strength of legal software protection in a given country, see Andrés (2006, pp. 34-37).

bootleggers (if any) from whom they have received an advertisement. In equilibrium each consumer makes the choice which maximizes her surplus.

It will shortly be seen that the price p_M of the legitimate copies of the product and the valuation parameter ξ of the consumers will suffice to determine both the advertising and pricing decisions of the bootleggers and the buying decisions of the consumers, so that the realized market penetration *n* of the product can be expressed as a function $n(\xi, p_M)$ of ξ and p_M. On the other hand, according to 2) each value of ξ corresponds to the expected market penetration $n_e(\xi)$ which is given by

(2) $$n_e(\xi) = \frac{\xi - A}{B}$$

Below we shall be interested in fulfilled expectations equilibria, i.e. situations which satisfy the equilibrium condition

(E) $$n(\xi, p_M) = n_e(\xi)$$

which states that the actual market penetration $n(\xi, p_M)$ of the (legally bought and pirated) copies of the product is identical with its expected market penetration $n_e(\xi)$. Obviously, here the range of the possible values of ξ is $[A, A+B]$.

In what follows $x(p)$ will denote the probability with which a randomly chosen consumer is willing to buy a pirate copy for the price *p* when she does not receive any advertisements with a lower price. Obviously, $x(p)$ depends implicitly on both p_M and ξ. The value of the price of *p* that maximizes $px(p)$ for given values of p_M and ξ is below denoted by $p_{\max}$. It is clear that the bootleggers will send some advertisements if and only if

$$p_{\max} x(p_{\max}) \geq b$$

When this condition is valid, the equilibrium that emerges in the pirate copy market resembles the equilibria of the classical model of advertising by Butters (1977). I shall conclude this section by discussing the situation in which this condition is *not* valid, and the bootleggers do not enter the market. Clearly, in this case a consumer θ will buy the consider good from the monopolist if and only if $\theta\xi - p_M \geq 0$, so that the realized market penetration turns out to be

(3) $$n(\xi, p_M) = \max\left\{0, 1 - \frac{p_M}{\xi}\right\}$$

It is clear that when $p_M > A$, the model has a corner solution in which $n = n_e = 0$ and $\xi\theta = (A + Bn_e)\theta = A\theta < p_M$ for all consumers θ. This represents a case in which there are no consumers who would be willing to buy the product, because each consumer assumes that no one else will buy it. The formulas (2) and (3) also immediately imply that the equilibrium condition (E) can be valid for at most two other values of ξ.

Figure 4.1 depicts a situation in which there are altogether three equilibria of the model. In this figure the region in which the curve $n(\xi)$ is above the curve $n_e(\xi)$ corresponds to cases in which the consumers underestimate the popularity (and, accordingly, the value) of the considered good, and the two regions in which $n(\xi)$ is below $n_e(\xi)$ correspond to cases in which consumers overestimate it. The points at which the curves $n = n(\xi)$ and $n = n_e(\xi)$ cross represent equilibria of the model, and the corresponding values of ξ have been denoted by ξ' and ξ'' in the figure. The corner solution $\xi = A$, $n_e = 0$, i.e. the case in which no one buys the product because each consumer believes that no one else will buy it, corresponds to the origin in the figure, because the figure represents a case in which $A = 0$.

However, in the situation of Figure 4.1 the equilibrium $\xi = \xi'$ is *unstable* in a sense in which the two other equilibria (i.e., $\xi = A = 0$ and $\xi = \xi''$) are not. This idea can be made rigorous by considering a game in which the considered good is repeatedly offered for sale for the price p_M, and in which the valuation parameter ξ is determined by the actual market penetration n' of the product in the previous round, i.e. in which $\xi = A + Bn'$. In Figure 4.1, the arrows which have been drawn between the curves $n(\xi)$ and $n_e(\xi)$ represent the time development of the valuation parameter ξ and the market penetration of the product in this repeated game. As the figure illustrates, if the value of ξ is originally below ξ', in the repeated game it will approach A, and the market penetration of the product will approach 0. Similarly, if the valuation parameter is originally above ξ', it will approach ξ''.

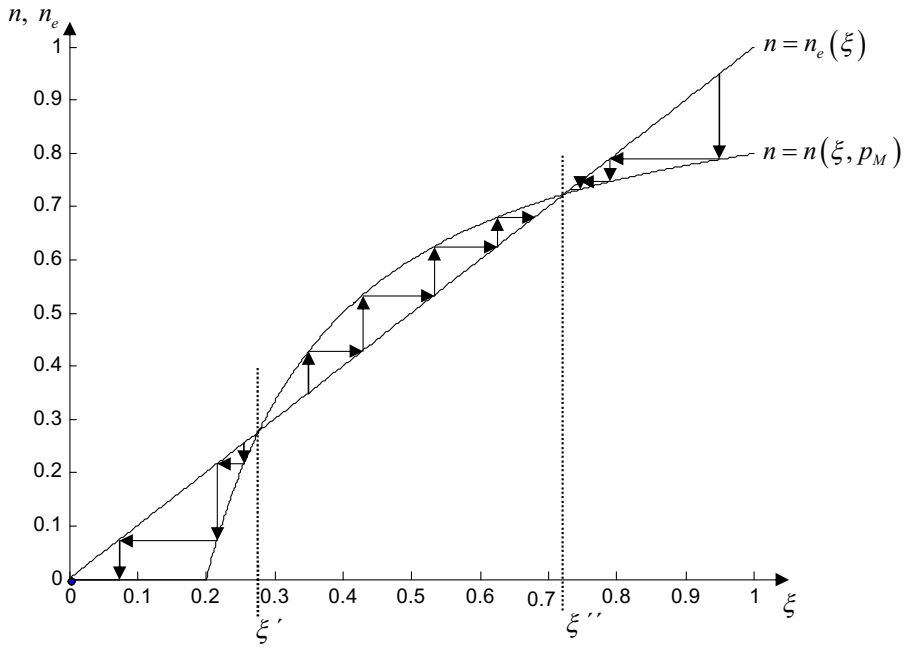

Figure 4.1. The expected market penetration n_e and the actual market penetration n as functions of ξ in the absence of piracy when $A = 0$, $B = 1$, and $p_M = 0.2$.

However, both the equilibrium in which no consumers buy the product and $\xi = A$ and the equilibrium in which $\xi = \xi''$ are stable in the sense that if ξ is originally sufficiently close to the equilibrium value, ξ will start to approach the equilibrium value in the repeated game. As Figure 4.1 illustrates, an equilibrium with positive market penetration corresponds to an equilibrium which is stable in this sense if and only if it meets the *stability condition*[15]

(S) $$n_e'(\xi) > \frac{\partial n(\xi, p_M)}{\partial \xi}.$$

[15] More precisely, the geometry of the situation makes it obvious that the equilibria which satisfy the condition (S) are stable if both $n_e(\xi)$ and $n(\xi, p_M)$ are increasing functions of ξ, which is, of course, trivially the case when they are given by (2) and (3).

4.3. THE EQUILIBRIUM OF THE PIRATE COPY MARKET

We now turn to the case in which the price set by the monopolist and the valuation parameter $\xi = A + Bn_e$ are such that the bootleggers send at least some advertisements. As it was explained above, this will be the case whenever

$$p_{\max} x(p_{\max}) \geq b$$

where $p_{\max}$ is the price which maximizes $px(p)$, i.e. the expected revenue from a single advertisement in the absence of all other advertisements. Since the consumer θ gives the valuation $\theta\xi$ to a legitimate copy and the valuation $q\theta\xi$ to a pirate copy, the probability with which a randomly chosen consumer buys a pirate copy for the price p when this is the lowest price that is suggested in the advertisements that she receives is given by

$$(4) \qquad x(p) = \begin{cases} x_A(p), & p < p' \\ x_B(p), & p \geq p' \end{cases} = \begin{cases} 1 - p/(q\xi), & p < p' \\ (qp_M - p)/[q\xi(1-q)], & p \geq p' \end{cases}$$

where $p' = p_M - (1-q)\xi$. Intuitively, when $x(p) = x_A(p)$, there are no consumers who would buy a legitimate copy if they can get a pirate copy for the price p, but when $x(p) = x_B(p)$, the price p is so high that such consumers exist. The result (4) implies that, somewhat surprisingly, in the latter case an increase in the expected market penetration of the product *decreases* the popularity of pirate copies.

Putting $\xi_1(p_M) = (2/(2-q))p_M$ and $\xi_2(p_M) = ((2-q)/(2(1-q)))p_M$, a straightforward calculation shows that $p_{\max}$ is given by

$$(5) \qquad p_{\max} = \begin{cases} q\xi/2, & \xi < \xi_1(p_M) \\ p_M - (1-q)\xi, & \xi_1(p_M) \leq \xi \leq \xi_2(p_M) \\ (qp_M)/2, & \xi > \xi_2(p_M) \end{cases}$$

Intuitively, when $\xi > \xi_2(p_M)$, the price $p_{\max}$ is so high that some consumers who can get a pirate copy for $p_{\max}$ will nevertheless choose a legitimate copy, but when $\xi_1(p_M) \leq \xi_2 \leq \xi_2(p_M)$, $p_{\max}$ is the largest price for which this is not the case. When $\xi \leq \xi_1(p_M)$, $p_{\max}$ is even lower than this limit price. Table 4.1 summarizes these

facts, together with intuitive interpretations for some of the other functions that will defined below.

Function	Interpretation
$\xi_L(p_M)$	The smallest value of ξ for which the bootleggers advertise.
$\xi_H(p_M)$	The largest value of ξ for which the bootleggers advertise.
$\xi_1(p_M)$	The smallest value of ξ for which $p_{\max}$ is the limit price which makes all consumers prefer a pirate copy to a legitimate copy; if $\xi < \xi_1(p_M)$, p_M is so high (relative to ξ) that $p_{\max}$ is even lower than this limit price.
$\xi_2(p_M)$	The largest value of ξ for which $p_{\max}$ is the limit price which makes all consumers prefer a pirate copy to a legitimate copy; if $\xi > \xi_2(p_M)$, some consumers prefer legitimate copies to pirate copies with the price $p_{\max}$.
$\xi_3(p_M)$	If $\xi_H(p_M) > \xi_2(p_M)$ (i.e., if the bootleggers advertise also for some values ξ for which $\xi > \xi_2(p_M)$), $\xi_3(p_M)$ is the largest value of ξ for which all consumers prefer *some* (at least the cheapest) pirate copies to legitimate copies. If $\xi_H(p_M) < \xi_2(p_M)$, $\xi_3(p_M)$ is defined by convention as $\xi_3(p_M) = \xi_2(p_M)$.

Table 4.1.

Now it easily follows from (4) and (5) that in the absence of other advertisements the expected revenue from an advertisement with the price $p_{\max}$ is

$$(6) \qquad r_{\max} = p_{\max} x(p_{\max}) = \begin{cases} q\xi/4, & \xi < \xi_1(p_M) \\ (\xi - p_M)(p_M - (1-q)\xi)/(q\xi), & \xi_1(p_M) \le \xi \le \xi_2(p_M) \\ qp_M^2/(4(1-q)\xi), & \xi > \xi_2(p_M) \end{cases}$$

Forming the derivative $\partial r_{\max}/\partial\xi$, it is easily observed that when the revenue $r_{\max}$ is viewed as a function of ξ, it has a single maximum, and that this corresponds to the value $\xi_M = p_M/\sqrt{1-q}$ which is located between $\xi_1(p_M)$ and $\xi_2(p_M)$. The revenue $r_{\max}$ is an increasing function when $\xi < \xi_M$ and a decreasing function when $\xi > \xi_M$ and hence, when p_M is fixed, the values of ξ for which $r_{\max} \geq b$ – i.e. the values for which the pirates enter the market – form a closed interval.

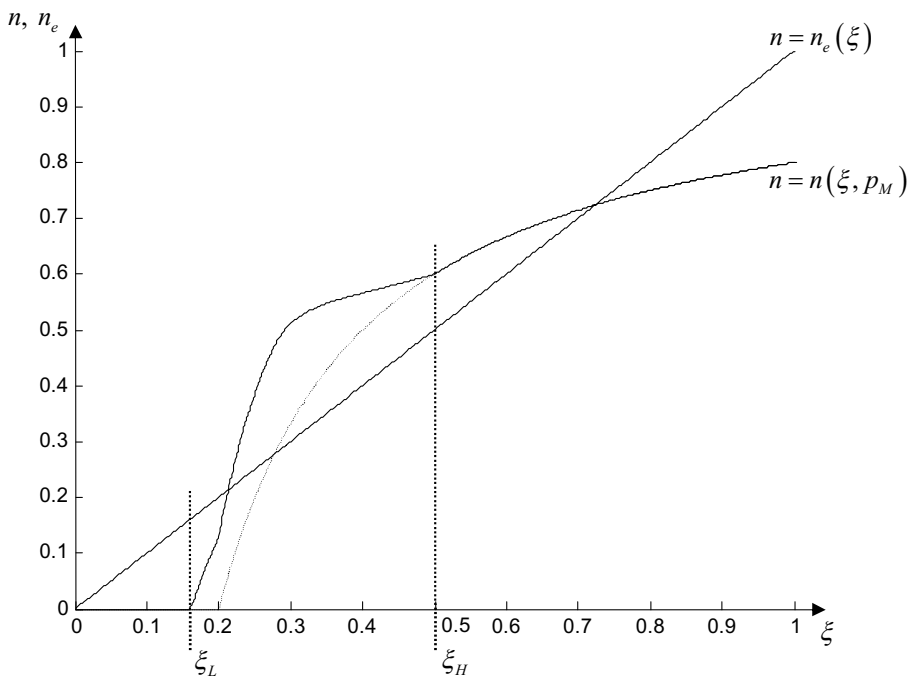

Figure 4.2. The expected market penetration n_e and the actual market penetration n as functions of ξ when $A = 0$, $B = 1$, $p_M = 0.2$, $q = 0.5$, and $b = 0.02$.

This makes it clear how Figure 4.1 gets modified when commercial pirates are included in it. Figure 4.2 corresponds to a fixed value of p_M, and it shows the realized market penetration $n(\xi, p_M)$ as a function of the valuation parameter ξ, when the

"advertising cost" b is so high that the bootleggers do not enter the market, and when b is reduced so much that they enter for some values of ξ. In the latter case the values of ξ for which the bootleggers enter form an interval $[\xi_L, \xi_H]$. Within this interval the curve $n(\xi, p_M)$ shifts upwards, but outside of it the curve remains unchanged.

Clearly, if there was just a single bootlegger, she would maximize her revenue by specifying the price $p = p_{\max}$ in all her advertisements. However, in the current model the number of the bootleggers is infinite, and this easily implies that there has to be price dispersion in the market for pirate copies. To see this, let $A(p)$ denote the number of the advertisements which have a price which is smaller than or equal with p. Since the number of the consumers has been normalized to 1 and since the advertisements are sent at random, it easily follows that the probability with which a consumer does *not* receive any advertisements with a price smaller than or equal with p is $e^{-A(p)}$. Hence, the expected revenue $r(p)$ from sending one more advertisement with a price p is

(7) $$r(p) = e^{-A(p)} p x(p)$$

This must be equal with b for each p at which some bootleggers advertise (since if $r(p) < b$, some bootleggers would be advertising with a negative profit, and if $r(p) > b$, each bootlegger with an infinitesimal market share would have an incentive to send one more advertisement at the price p).

It is also clear that in equilibrium the largest price in the advertisements must always have the value $p_{\max}$. Trivially, no bootleggers can have an incentive to advertise with a price $p > p_{\max}$, since in this case $A(p) \geq A(p_{\max})$ and $px(p) < p_{\max} x(p_{\max})$ so that $r(p) < r(p_{\max})$. On the other hand, if all advertisements contained prices lower than $p_{\max} - \varepsilon$ for some $\varepsilon > 0$, profits would be increased by changing the highest prices in the advertisements from $p_{\max} - \varepsilon$ to $p_{\max}$ (because this would not change the risk that the consumer gets an offer with a lower price, but it would increase $px(p)$). Analogous arguments show that the smallest price $p_{\min}$ that appears in the advertisements is determined by the condition

(8) $p_{\min} x(p_{\min}) = b$

and also that some bootleggers must be advertising at each price p in the interval $[p_{\min}, p_{\max}]$.

Now it also follows that (7) is valid for each p for which $p_{\min} \le p \le p_{\max}$, so that A is a continuous function and the number of the advertisements that are sent at each price p is infinitesimal. This further implies that the market share of each single bootlegger is infinitesimal (since if a bootlegger with a finite market share decided not to send her higher-price advertisements when (7) is valid, this would not affect her profit from those advertisements, but it would allow her to earn a positive profit from her lower-price advertisements).

In particular, (7) and the fact that $r(p) = b$ imply that $A(p)$ is given by

(9) $$A(p) = \ln\left(\frac{px(p)}{b}\right)$$

for all prices p in $[p_{\min}, p_{\max}]$. The result (9) also implies that the total number of the advertisements is given by

(10) $$A = A(p_{\max}) = \ln\left(\frac{r_{\max}}{b}\right)$$

As the next step, we shall deduce from (8) an explicit formula for $p_{\min}$. When $\xi \le \xi_2(p_M)$, (4) and (5) imply that $p_{\min} < p_{\max} \le p'$ so that $x(p_{\min}) = x_A(p_{\min})$. Intuitively, this condition means that all consumers prefer buying a pirate copy to buying a legitimate copy.

Assume now that the bootleggers advertise for some values of ξ for which $\xi > \xi_2(p_M)$. Now (4) and (5) imply that $p_{\max} > p'$ and $x(p_{\max}) = x_B(p_{\max})$, which means that *some* (but not necessarily all) pirate copies cause market segmentation in the sense that the highest-valuation consumers prefer legitimate copies to them, but lower-valuation consumers do not. *All* pirate copies cause market segmentation in this sense if also $p_{\min} > p'$. This will be case if

(11) $p'x(p') < b$,

i.e. if the limit price p' is so small that it is unprofitable to advertise at it. Keeping in mind that p_M is fixed and that we are considering the case in which $\xi > \xi_2(p_M)$, it

can be verified by elementary means that the condition (11) valid if and only if $\xi > \xi_3(p_M)$, where

$$(12) \qquad \xi_3(p_M) = \frac{(2-q)p_M - bq + \sqrt{q^2(p_M+b)^2 - 4qp_M b}}{2(1-q)}$$

It is clear that the expression under the square root sign is positive in the case in which we are considering, i.e. when the bootleggers send advertisements for some values of ξ for which $\xi > \xi_2(p_M)$.[16] When there are no such values of ξ, we define, by convention, $\xi_3(p_M) = \xi_2(p_M)$.

Now it can be concluded from (4) and (8) that whenever $\xi_L(p_M) \le \xi \le \xi_H(p_M)$ (i.e., whenever the valuation parameter is such that the bootleggers send at least some advertisements),

$$(13) \qquad p_{\min} = \begin{cases} \dfrac{q\xi}{2} - \sqrt{\left(\dfrac{q\xi}{2}\right)^2 - q\xi b}, & \xi \le \xi_3(p_M) \\ \dfrac{qp_M}{2} - \sqrt{\left(\dfrac{qp_M}{2}\right)^2 - bq\xi(1-q)}, & \xi > \xi_3(p_M) \end{cases}$$

After these preparations it has become possible to deduce an explicit formula for the realized market penetration $n(\xi, p_M)$ of the product when the bootleggers are on the market. For this purpose, the potential consumers of the monopolist and the bootleggers are divided into three groups. Firstly, the consumers θ for whom $\xi\theta \ge p_M$ will all buy *either* a legitimate copy *or* a pirate copy. Such consumers will exist if and only if $\xi \ge p_M$, and the demand component which corresponds to them is

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[16] We are considering a case in which the bootleggers advertise when the valuation parameter is ξ, where $\xi > \xi_2(p_M)$. According to (6), this implies that $qp_M^2 / 4(1-q)\xi \ge b$ or, equivalently, $qp_M^2 \ge 4b(1-q)\xi$. Given that $\xi > \xi_2(p_M) = \left((2-q)/(2(1-q))\right)p_M$, this condition can only be valid if $qp_M^2 \ge 2b(2-q)p_M$. The latter condition is equivalent with $qp_M^2 + 2bqp_M \ge 4bp_M$, and it further implies that

$$q(p_M+b)^2 = qp_M^2 + 2bqp_M + qb^2 > 4bp_M$$

However, this is equivalent with $q^2(p_M+b)^2 - 4qp_M b > 0$. Hence, the expression under the square root sign is positive, if $\xi > \xi_2(p_M)$ and the bootleggers advertise when the valuation parameter has the value ξ.

(14) $$n_1(\xi, p_M) = \max\{0, 1 - p_M/\xi\}$$

The second group of potential customers consists of the consumers who will not buy a legitimate copy but who are willing to buy a pirate copy even for the price $p_{\max}$. Obviously, a consumer of this group buys a pirate copy if she gets at least one advertisement. A consumer θ belongs to this group if and only if $p_{\max} \le q\theta\xi$ and $\theta\xi < p_M$, i.e. if and only if $p_{\max}/(q\xi) \le \theta < p_M/\xi$, and consumers for whom this condition is valid will exist whenever $p_{\max} < qp_M$. However, the latter condition must always be valid in equilibrium, since in general a consumer θ can prefer buying a pirate copy for the price p to buying a legitimate copy for p_M only if $q\xi\theta - p > \xi\theta - p_M$, and this condition cannot be valid for any value of θ if $p \ge qp_M$. Hence, in equilibrium there are always consumers which belong to the second group. Clearly, since the probability with which a consumer gets no advertisements is $e^{-A} = b/r_{\max}$, the market penetration component which corresponds to these consumers is

(15) $$n_2(\xi, p_M) = \left(1 - e^{-A}\right)\left(\min\left\{\frac{p_M}{\xi}, 1\right\} - \frac{p_{\max}}{q\xi}\right)$$

Finally, the third group of potential customers consists of the consumers whose reservation price for pirate copies lies between $p_{\min}$ and $p_{\max}$. Since the maximal price that the consumer θ is willing to pay for a pirate copy is $q\theta\xi$, the market penetration component which corresponds to these consumers is

(16) $$n_3(\xi, p_M) = \int_{p_{\min}/(q\xi)}^{p_{\max}/(q\xi)} \left(1 - e^{-A(q\xi\theta)}\right) d\theta = \frac{1}{q\xi}\int_{p_{\min}}^{p_{\max}} \left(1 - e^{-A(p)}\right) dp$$

Putting these results together, it can be concluded that when there are pirate copies on the market, the total market penetration of the product is

(17) $$\begin{aligned} n(\xi, p_M) &= n_1(\xi, p_M) + n_2(\xi, p_M) + n_3(\xi, p_M) \\ &= 1 - \frac{p_{\min}}{q\xi} - \frac{b}{r_{\max}}\left(\min\left\{\frac{p_M}{\xi}, 1\right\} - \frac{p_{\max}}{q\xi}\right) - \frac{1}{q\xi}\int_{p_{\min}}^{p_{\max}} e^{-A(p)} dp \end{aligned}$$

This can be evaluated using (4), (5), (6), and (9). As these results show, the expressions of $p_{\max}$ and $r_{\max}$ depend on whether $\xi < \xi_1(p_M)$, $\xi_1(p_M) \le \xi \le \xi_2(p_M)$, or $\xi > \xi_2(p_M)$, and also the function $A(p)$ which appears in

the integral in (17) receives a different form depending on whether $x(p)=x_A(p)$ or $x(p)=x_B(p)$ for the prices between $p_{\min}$ and $p_{\max}$. Keeping this in mind, a tedious but straightforward calculation leads to the following expressions for the market penetration (17) of the product when the pirates are on the market:

$$n(\xi,p_M)=1-\frac{p_{\min}}{q\xi}-\frac{2b}{q\xi}-\frac{b}{q\xi}\ln\left(\frac{q\xi-p_{\min}}{p_{\min}}\right) \qquad (\xi\le p_M) \tag{18}$$

$$n(\xi,p_M)=1-\frac{p_{\min}}{q\xi}-\frac{2b(2p_M-\xi)}{q\xi^2}-\frac{b}{q\xi}\ln\left(\frac{q\xi-p_{\min}}{p_{\min}}\right) \qquad (p_M\le\xi\le\xi_1(p_M)) \tag{19}$$

$$\begin{aligned} n(\xi,p_M)&=1-\frac{p_{\min}}{q\xi}-\frac{b(1-q)}{p_M-(1-q)\xi}\\ &-\frac{b}{q\xi}\ln\left(\frac{(p_M-\xi+q\xi)(q\xi-p_{\min})}{p_{\min}(\xi-p_M)}\right) \end{aligned} \qquad (\xi_1(p_M)<\xi\le\xi_2(p_M)) \tag{20}$$

$$\begin{aligned} n(\xi,p_M)&=1-\frac{p_{\min}}{q\xi}-\frac{2b(1-q)}{qp_M}\\ &-\frac{b}{q\xi}\ln\left(\frac{(q\xi-p_{\min})(p_M-(1-q)\xi)}{p_{\min}(\xi-p_M)}\right)\\ &-\frac{b(1-q)}{qp_M}\ln\left(\frac{(1-q)(\xi-p_M)}{p_M-(1-q)\xi}\right) \end{aligned} \qquad (\xi_2(p_M)<\xi<\xi_3(p_M)) \tag{21}$$

$$\begin{aligned} n(\xi,p_M)&=1-\frac{p_{\min}}{q\xi}-\frac{2b(1-q)}{qp_M}\\ &-\frac{b(1-q)}{qp_M}\ln\left(\frac{qp_M-p_{\min}}{p_{\min}}\right) \end{aligned} \qquad (\xi\ge\xi_3(p_M)) \tag{22}$$

In what follows I shall be interested in the fulfilled expectations equilibria which satisfy the stability condition, i.e. situations in which both (E) and (S) are valid for $n_e(\xi)$ and $n(\xi,p_M)$.[17] In the absence of the bootleggers the market penetration $n(\xi,p_M)=\max\{0,1-p_M/\xi\}$ is a concave function, and as Figure 4.1 illustrates, in

[17] As it was pointed out that in Section 2, the geometry of Figure 1 makes it obvious that the equilibria which satisfy the condition (S) are stable as long as both $n_e(\xi)$ and $n(\xi,p_M)$ are increasing functions of ξ. It should be observed that although this condition is trivially valid in the absence of the bootleggers, there is no obvious economic reason why $n(\xi,p_M)$ would always have to increase as a function of ξ when the bootleggers are on the market. However, a straightforward calculation shows that $\partial n(\xi,p_M)/\partial\xi>0$ also when $n(\xi,p_M)$ has one of the expressions (18)-(22).

this case each price p_M of legitimate copies corresponds to at most one equilibrium with $\xi > 0$ which satisfies the stability condition (S). However, when the bootleggers are taken into account, $n(\xi, p_M)$ is no longer in general a concave function of ξ. Accordingly, it is conceivable that there are more than one stable fulfilled expectations equilibria with a positive market penetration which correspond to the same price p_M.

In what follows I shall restrict attention to the fulfilled expectations equilibrium which has the largest market penetration and, accordingly, the largest ξ value, and for each p_M, I shall let $\xi(p_M)$ denote the valuation parameter in this fulfilled expectations equilibrium. However, it is straightforward to verify that when there are many equilibria which satisfy (S), the results concerning the function $\xi(p_M)$ below remain valid if $\xi(p_M)$ is an arbitrary continuous function which gives for each p_M a value of ξ which corresponds to a stable equilibrium.

It is also easy to see that the equilibrium with the largest market penetration satisfies the stability condition (S), except for an implausible special case. To see this, one should first observe that $A \le \xi(p_M) < A + B$, since when $\xi = A + Bn_e$, the value $\xi = A$ corresponds to the market penetration $n_e = 0$ and the value $\xi = A + B$ corresponds to the market penetration $n_e = 1$. When $\xi(p_M) > A$ so that the equilibrium market penetration is positive, the equilibrium condition (E) receives the form

(23) $n_e(\xi(p_M)) - n(\xi(p_M), p_M) = 0$

Since for each positive price p_M

$$n_e(A+B) - n(A+B, p_M) = 1 - n(A+B, p_M) > 0$$

and since the value $\xi = \xi(p_M) < A + B$ is the largest value of ξ which satisfies (23), it must be the case that

(24) $\frac{\partial}{\partial \xi}\left(n_e(\xi) - n(\xi, p_M)\right)_{\xi=\xi(p_M)} \ge 0$

In other words, the equilibrium $\xi = \xi(p_M)$ satisfies the stability condition (S), except for the case in which $\partial n(\xi, p_M)/\partial \xi$ and $n_e'(\xi)$ are identical in it. (The latter

possibility will be realized, in particular, if the curves $n(\xi, p_M)$ and $n_e(\xi)$ meet at just one point, and $n(\xi, p_M) < n_e(p_M)$ elsewhere.)

4.4. The Optimization Problem of the Monopolist

We now turn to the optimization problem of the monopolist, i.e. the problem of choosing p_M so that the revenue of the monopolist is maximized when $\xi = \xi(p_M)$. Clearly, solution of this problem might either be a value of p_M for which the bootleggers enter the market, in which case $\xi(p_M)$ is given by one of the formulas (18)-(22), or it might be a value of p_M for which the bootleggers do not enter. According to (3), in the latter case the demand of the monopolist is given by $n(\xi, p_M) = 1 - p_M/\xi$, so that the revenue of the monopolist is

$$(25) \qquad R_M = p_M\left(1 - \frac{p_M}{\xi}\right) \qquad (r_{\max} < b)$$

In order to find an expression for R_M when the bootleggers are on the market, we now assume that $r_{\max} \geq b$, which implies that the bootleggers advertise in equilibrium, and divide potential customers of the monopolist into the high-valuation, medium-valuation, and the low-valuation customers.

The *high-valuation customers*, are, by definition, consumers who will buy a legitimate copy even if they can get a pirate copy for $p_{\min}$. Such consumers exist only if ξ is larger than $\xi_3(p_M)$, and a consumer θ belongs to this group only if $\theta\xi - p_M \geq q\theta\xi - p_{\min}$. This is equivalent with $\theta \geq (p_M - p_{\min})/((1-q)\xi)$, so that the demand from the high-valuation consumers is

$$(26) \qquad D_H = \begin{cases} 0, & \xi < \xi_3(p_M) \\ 1 - \dfrac{p_M - p_{\min}}{(1-q)\xi}, & \xi \geq \xi_3(p_M) \end{cases}$$

By definition, the *low-valuation customers* are the ones who will buy a legitimate copy if and only if they do not receive any advertisements. Clearly, a consumer θ is a low-valuation consumer if and only if $\theta \geq p_M/\xi$ and $\theta\xi - p_M < q\theta\xi - p_{\max}$. When $\xi < \xi_2(p_M)$, the latter condition is valid for all the consumers $\theta < 1$ so that

$$D_L = e^{-A}\left(1-\frac{p_M}{\xi}\right) = \left(\frac{b}{r_{\max}}\right)\left(1-\frac{p_M}{\xi}\right)$$

When $\xi \geq \xi_2(p_M)$, (5) implies that the condition $\theta\xi - p_M < q\theta\xi - p_{\max}$ is equivalent with $\theta < \left((1-q/2)/((1-q)\xi)\right)p_M$, and the demand from the low-valuation consumers is

$$D_L = e^{-A}\left(\left(\frac{1-q/2}{1-q}\right)\frac{p_M}{\xi} - \frac{p_M}{\xi}\right) = \frac{b}{r_{\max}}\left(\frac{qp_M}{2(1-q)\xi}\right)$$

Together with (6), these results imply that

$$(27) \qquad D_L = \begin{cases} \dfrac{4b(\xi - p_M)}{q\xi^2}, & p_M \leq \xi < \xi_1(p_M) \\ \dfrac{bq}{p_M - (1-q)\xi}, & \xi_1(p_M) \leq \xi \leq \max\{\xi_2(p_M), \xi_3(p_M)\} \\ \dfrac{2b}{p_M}, & \xi > \max\{\xi_2(p_M), \xi_3(p_M)\} \end{cases}$$

Finally, the *medium-valuation customers* are by definition consumers who are willing to buy a legitimate copy even if they can get a pirate copy for $p_{\max}$, but not if they can get a pirate copy for $p_{\min}$. Clearly, when $\xi \leq \xi_2(p_M)$, no consumers with $\theta < 1$ are medium valuation customers. Assume then that $\xi > \xi_2(p_M)$. Now a consumer θ is a medium-valuation customer if $q\xi\theta - p_{\min} > \xi\theta - p_M \geq q\xi\theta - p_{\max}$. This is equivalent with

$$(28) \qquad \frac{p_M - p_{\max}}{(1-q)\xi} \leq \theta < \frac{p_M - p_{\min}}{(1-q)\xi}$$

Remembering that $p_{\max} = (qp_M)/2$ when $\xi > \xi_2(p_M)$, and putting

$$\theta' = \min\left\{\frac{p_M - p_{\min}}{(1-q)\xi}, 1\right\}$$

(28) is easily seen to be equivalent with

$$\xi_2(p_M)/\xi < \theta \leq \theta'$$

Hence, when $\xi > \xi_2(p_M)$, (9) implies that the demand from the medium-valuation consumers is

$$D_{med} = \int_{\xi_2(p_M)/\xi}^{\theta'} e^{-A(p_M-(1-q)\xi\theta)} d\theta = \int_{\xi_2(p_M)/\xi}^{\theta'} \frac{bd\theta}{(p_M-(1-q)\xi\theta)x(p_M-(1-q)\xi\theta)}$$

Remembering that $\xi_2(p_M) = \left((2-q)/(2(1-q))\right)p_M$, it is readily calculated that

$$D_{med} = \frac{b}{p_M}\ln\left(\frac{(1-q)(\xi\theta' - p_M)}{p_M - (1-q)\xi\theta'}\right)$$

The definitions of $\xi_3(p_M)$ and θ' imply that $\theta' = 1$ if $\xi_2(p_M) < \xi \le \xi_3(p_M)$, but $\theta' = (p_M - p_{\min})/((1-q)\xi)$ if $\xi > \xi_3(p_M)$. Hence, D_{med} has, in general, the expression

$$(29) \qquad D_{med} = \begin{cases} 0, & \xi \le \xi_2(p_M) \\ \dfrac{b}{p_M}\ln\dfrac{(1-q)(\xi - p_M)}{p_M - (1-q)\xi}, & \xi_2(p_M) < \xi \le \xi_3(p_M) \\ \dfrac{b}{p_M}\ln\left(\dfrac{qp_M}{p_{\min}} - 1\right), & \xi > \xi_3(p_M) \end{cases}$$

In each case, the revenue R_M of the monopolist is given by $R_M = p_M(D_L + D_{med} + D_H)$. Combining (26), (27), and (29), this it is seen to equal

$$(30) \qquad R_M = \begin{cases} 0, & \xi \le p_M \\ \dfrac{4b(\xi - p_M)p_M}{q\xi^2}, & p_M < \xi < \xi_1(p_M) \\ \dfrac{bqp_M}{p_M - (1-q)\xi}, & \xi_1(p_M) \le \xi \le \xi_2(p_M) \\ 2b + b\left(\ln\dfrac{(1-q)(\xi - p_M)}{p_M - (1-q)\xi}\right), & \xi_2(p_M) < \xi \le \xi_3(p_M) \\ 2b + b\ln\left(\dfrac{qp_M}{p_{\min}} - 1\right) + \left(1 - \dfrac{p_M - p_{\min}}{(1-q)\xi}\right)p_M, & \xi > \xi_3(p_M) \end{cases}$$

Having deduced an expression for R_M – which given by (25) or (30), depending on whether the bootleggers enter the market – we now turn to the problem of choosing p_M so that R_M is maximized, when $\xi = \xi(p_M)$. Clearly, in either case

$$(31) \qquad \frac{dR_M}{dp_M} = \frac{\partial R_M}{\partial p_M} + \frac{d\xi}{dp_M}\frac{\partial R_M}{\partial \xi}$$

so that the effect of a change of p_M on revenue consists of the direct effect, which is given by $\partial R/\partial p_M$, and the indirect effect, which is represented by the latter term and which is caused by the change in the valuations of the consumers.

We begin our analysis of this optimization problem by considering the latter effect. The derivative $d\xi/dp_M$, which appears in (31), is determined by the equilibrium condition (E), i.e. $n\left(\xi\left(p_M\right), p_M\right) = n_e\left(\xi\left(p_M\right)\right)$. Clearly, this condition implies that

$$(32) \quad \frac{d\xi}{dp_M} = \frac{\partial n\left(\xi, p_M\right)/\partial p_M}{n_e'\left(\xi\right) - \partial n\left(\xi, p_M\right)/\partial \xi}$$

This further implies that $d\xi/dp_M$ and $\partial n/\partial p_M$ have the same sign if the stability condition (S) is valid. This observation will be formulated as a separate proposition.

PROPOSITION 1. In a stable equilibrium $\partial n/\partial p_M > 0$ implies that $d\xi/dp_M > 0$ and $dn/dp_M > 0$, and $\partial n/\partial p_M < 0$ implies that $d\xi/dp_M < 0$ and $dn/dp_M < 0$. (I.e. the sign of the change of the market penetration caused by a small change in p_M is identical in a stable fulfilled expectations equilibrium, and in a case in which the valuations of the consumers are fixed.)

This makes it possible to determine the sign of $d\xi/dp_M$ directly on the basis of the expressions (18)-(22) of the function $n\left(\xi, p_M\right)$, by studying the signs of the partial derivatives $\partial n\left(\xi, p_M\right)/\partial p_M$. These are given by the following lemma.

LEMMA 1. Suppose that ξ and p_M are such that the pirates are on the market. If $p_M < \xi < \xi_2\left(p_M\right)$, $\partial n/\partial p_M < 0$, but if $\xi = \xi_2\left(p_M\right)$, $\partial n/\partial p_M = 0$. If $\xi > \xi_2\left(p_M\right)$, $\partial n/\partial p_M > 0$, so that if the valuation parameter ξ is fixed, in this case an *increase* in p_M *increases* market penetration.

The result (30) easily implies that when $\xi < \xi_2\left(p_M\right)$ – i.e., when none of the consumers who can get a pirate copy will buy a legitimate one – the direct effect of a decrease of price p_M on revenue R_M is positive (since $\partial R_M/\partial p_M < 0$) and also the direct effect of an increase in ξ is positive (since $\partial R_M/\partial \xi > 0$). However,

Proposition 1 and Lemma 1 imply that in this case a decrease in p_M increases ξ, and combining the results, it follows that the monopolist has always an incentive to lower the price if $\xi < \xi_2(p_M)$.[18] This statement will be formulated as the following corollary.

COROLLARY 1. Suppose that the model has a fulfilled expectations equilibrium with bootleggers in which $\xi < \xi_2(p_M)$ (which means that none of the consumers who can buy a pirate copy will prefer to buy a legitimate copy). The revenue of the monopolist is increased when the monopolist lowers the price p_M until the resulting equilibrium is either such that $\xi \geq \xi_2(p_M)$, or such that the bootleggers do not enter the market.

This corollary corresponds to a clear intuition. As it was explained in the introduction, the reason why the pirates are sometimes useful for the monopolist is that they constitute a *surrogate for price discrimination*: if the copyright owner cannot sell the information good at a lower price to the low-valuation consumers, it might be in her interest to distribute it to them for free and charge a larger price from the high-valuation consumers. Obviously, there is no market segmentation of this kind when $\xi \leq \xi_2(p_M)$ and all the consumers who can get hold of a pirate copy will prefer it to a legitimate copy.

The above corollary leaves three possibilities, two of which are intuitive. Firstly, it may be the case in equilibrium that the monopolist blocks the entry of the bootleggers by choosing a price which is so low that piracy is unprofitable, and secondly, it may also be the case that the bootleggers enter but $\xi > \xi_3(p_M)$. The latter possibility leads to the kind of market segmentation which exists also in the models of non-commercial piracy, since in this case all pirate copies are such that the consumers with a sufficiently high valuation prefer legitimate copies to them.

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[18] As it was stated at the end of Section 3, the fulfilled expectations equilibrium in which $\xi = \xi(p_M)$ either satisfies the stability condition (S) or is such that $n_e'(\xi) = \partial n(\xi, p_M)/\partial \xi$. To be precise, Corollary 1 follows from Proposition 1 and Lemma 1 only in the former case, but – as the proof of Corollary 1 in the Appendix shows – it is more or less trivial to generalize this proof so that Corollary 1 gets demonstrated in the latter case as well.

A third possibility which has not been eliminated yet is that $\xi_2(p_M) < \xi \le \xi_3(p_M)$. In this case, some (i.e. the cheapest) pirate copies are such that all consumers prefer them to legitimate copies, whereas other (i.e. the more expensive) pirate copies lead to the kind market segmentation which might increase the revenue of the monopolist. There seems to be no obvious economic reason why a situation of this kind could not be revenue-maximizing for the monopolist. Nevertheless, the following proposition demonstrates that this cannot be the case in the current model. According to it, the profit-maximizing equilibrium is always either of the two more plausible alternatives.

PROPOSITION 2. The revenue-maximizing choice of p_M is always either such that the bootleggers do not enter the market, or such that they enter but $\xi \ge \xi_3(p_M)$ (i.e., all pirate copies have prices which are such that the highest-valuation consumers prefer buying a legitimate copy to buying one of them).

When (30) and Proposition 2 are combined, they lead to a relatively simple characterization of the optimal price in an equilibrium in which the bootleggers are on the market. Given that $\xi > \xi_3(p_M)$, one can conclude from (4) and (8) that $b = p_{\min}\left((qp_M - p_{\min})/(q(1-q)\xi)\right)$, and from (30) that

$$\frac{\partial R_M}{\partial p_{\min}} = -b\frac{qp_M}{p_{\min}(qp_M - p_{\min})} + \frac{p_M}{(1-q)\xi} = 0$$

Hence, although a change p_M changes also $p_{\min}$, this effect can be left out of consideration when one forms the derivative dR_M/dp_M. Accordingly, the equilibrium condition

$$\frac{dR}{dp_M} = \frac{\partial R}{\partial p_M} + \frac{\partial R}{\partial \xi}\frac{d\xi}{dp_M} = 0$$

is equivalent with

$$\text{(33)} \qquad \left(1 - \frac{2(p_M - p_{\min})}{(1-q)\xi}\right) + \frac{(p_M - p_{\min})p_M}{(1-q)\xi^2}\frac{d\xi}{dp_M} = 0$$

Intuitively, this condition states that the direct effect on profit that a small change in p_M would cause, and which is represented by the former term, and its indirect effect which is due to network externalities, and which is represented by the latter term, must cancel each other out in equilibrium.

4.5. Optimal Copyright Protection in The Presence of Commercial Piracy

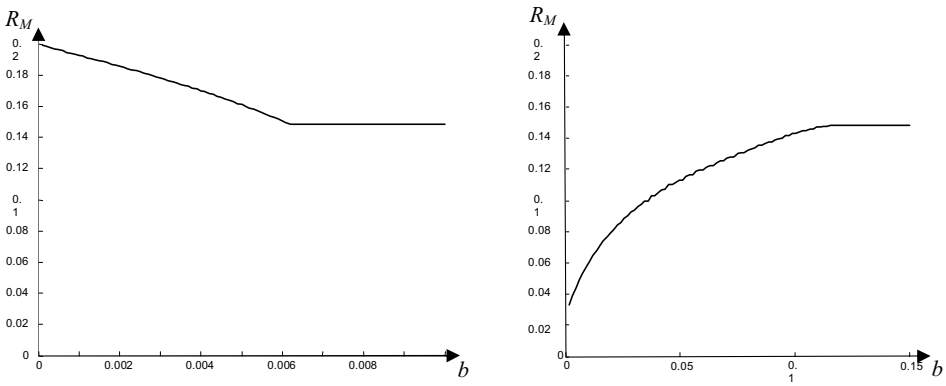
I shall conclude the analysis of the model by studying the dependence of the incentives to create information goods on the amount of copyright protection. In the current model the incentives to create information goods are represented by the revenue of the monopolist R_M, and – as it was explained in Section 4.2 above – copyright protection is represented by the "advertising cost" *b*, which corresponds to the disutility from a punishment for copyright violation. Hence, the problem of choosing copyright protection so that the incentives to create information goods are maximized corresponds in the current model to the problem of maximizing the equilibrium value of R_M as a function of *b*.

Intuitively, since the negative effect of piracy which is caused by the loss of paying consumers is small when the pirate copies are of a low quality, but the positive effect which is due to network effects is in the current model independent of their quality, the revenue-maximizing level of IPR protection can be expected to be small when the quality of pirate copies is low. This intuition is confirmed by the following proposition which is concerned with low-quality pirate copies.[19]

Proposition 3. In the limit in which the quality *q* of pirate copies approaches zero, the revenue-maximizing equilibrium approaches the situation of free availability of pirate copies. I.e., the revenue-maximizing advertising cost *b* and the minimum price of pirate copies $p_{\min}$ approach zero, and the market penetration approaches 1.

[19] To get a concrete idea of the situation in which the parameter *q* is small, one can once more consider the sales of pirate copies of software products via the Internet. As it was stated in footnote 12, the parameter *q* has not been intended to represent only a physically low quality of pirate copies, but also other reasons why consumers might prefer legitimate copies to pirate copies (like e.g. ethical considerations, or unwillingness to give credit card information to criminals). Clearly, even if all the consumers would have a low willingness to pay for pirate copies because of such considerations, such considerations would not affect the utility that the users of the legitimate copies obtain from the pirate copies via network effects.

Proposition 3 is illustrated by Figure 4.3(a). It represents an extreme case in which $q = 0.2$, so that the pirate copies are of a low quality, and $A = 0$ and $B = 1$, so that that the considered good is a pure network good which has no intrinsic value for a consumer if it is not used by others. In the situation of the figure the revenue-maximizing value of b is $b = 0$, meaning that it would be in the interest of the copyright owner that pirate copies would be available for everyone for free.



(a) **(b)**

Figure 4.3. The revenue R_M of the monopolist for the optimal price p_M as a function of the strength of IPR protection *b*, when $A = 0$ and $B = 1$. In Figure 4.3(a) the quality parameter $q = 0.2$, but in Figure 4.3(b) $q = 0.45$.

On the other hand, if pirate copies are of a high quality, the negative effect which is caused by the loss of consumers to the bootleggers can be expected to overrule the positive effect which is due to network externalities. The following proposition shows that this is, indeed, the case.

PROPOSITION 4. When the quality of the pirate copies q is sufficiently large, the equilibrium revenue of the monopolist receives its maximum in the absence of piracy, i.e. when the "advertising cost" b is so high that the pirates do not advertise.

Proposition 4 is illustrated with Figure 4.3(b), which shows that when pirate copies are of a high quality, the revenue of the monopolist R_M is an increasing function of the "advertising cost" b, and R_M obtains its largest value when b is so large that the pirate copies are no longer available.

The revenue of the monopolist is a monotonous function of IPR protection strength b in both Figures 4.3(a) and 4.3(b). These figures and Propositions 3 and 4 make it natural to ask whether this is always the case, or whether the optimal amount of copyright protection is sometimes located between complete protection (represented by the values of b which eliminate piracy) and no protection (represented by $b = 0$).

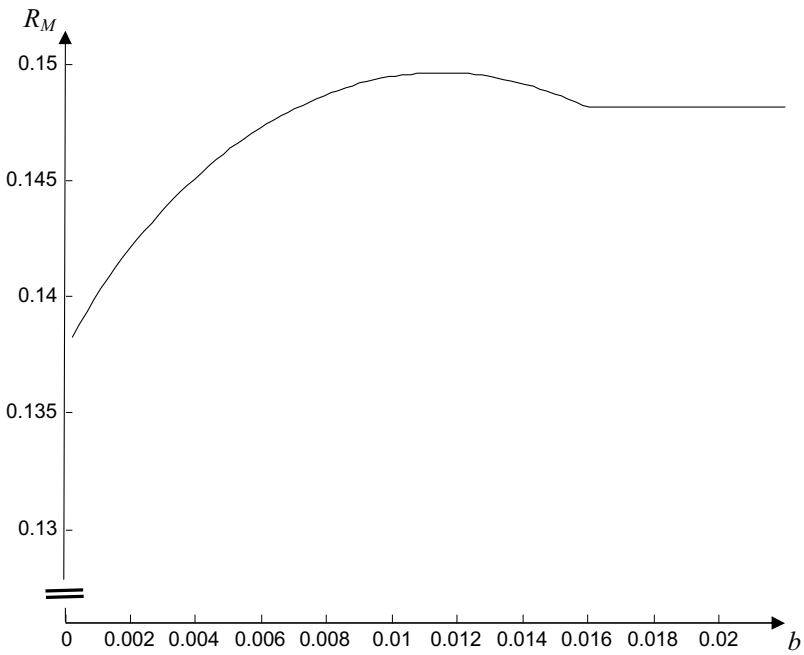


Figure 4.4. The revenue R_M of the monopolist for the optimal price p_M as a function of the strength of IPR protection b, when $A = 0$, and $B = 1$, and the quality parameter $q = 0.45$.

Figure 4.4 shows that also this is possible, if the quality of the pirate copies is neither too large nor too small. For the parameter specification of this figure a situation of commercial piracy is for the monopolist preferable to both a situation of the free availability of pirate copies and a situation of no piracy. However, the numerical

examples worked out by the author seem to indicate that the effects of piracy on the revenue of the monopolist are relatively small when this happens: e.g. in the situation of Figure 4.4, the revenue-maximizing equilibrium of commercial piracy yields only approximately 9% larger revenue than the situation in which pirate copies are freely available, and only approximately 1% larger revenue than complete IPR protection.

4.6. Concluding Remarks

It has repeatedly been pointed out that non-commercial piracy can increase the profits of a copyright owner when an information good is subject to sufficiently strong network externalities. Above I studied a model of commercial piracy, which constitutes, in a sense, a generalization of the models which lead to this result. A situation of free availability of pirate copies corresponds in the current model to the limiting case in which the "advertising cost" b, which represents the strength of copyright protection, approaches zero, whereas a situation of no piracy correspond to the values of copyright protection strength b which are sufficiently large to make piracy unprofitable. The earlier models allow for a comparison of these two cases, but in the current model it is possible to discuss, more generally, the effects of piracy when the strength of copyright protection has been arbitrarily chosen.

Most of the results that were deduced above in this more general setting were quite intuitive. For example, according to Corollary 1 the revenue-maximizing choice of the price of legitimate copies is never such that all the consumers who get the opportunity to buy a pirate copy would buy one. This result was to be expected, since the reason why piracy has in the earlier literature sometimes turned out to be beneficial for the copyright owner is that it acts as a surrogate for *price discrimination*: the free distribution of pirate copies can be thought of as selling low-quality versions of an information good to low-valuation consumers at price zero, and this might be in the interest in the copyright owner if no other methods of price discrimination are available. However, the optimal choice of the copyright owner might well be such there are pirate copies on the market, if the higher-valuation consumers prefer a legitimate copy to all (even the lowest-price) pirate copies.

Since the prices of the pirate copies vary in the current model, the model allows also for a third possibility: it might be the case that all (even the highest-valuation

consumers) prefer the lowest-price pirate copies to legitimate copies, although the higher-price pirate copies cause the kind of market segmentation which is in the interest of the copyright owner. There seems to be no obvious economic intuition on whether a situation of this kind can be in the interest of the copyright owner. However, Proposition 2 showed that a situation of this kind cannot emerge in the current model if the copyright owner chooses the price of legitimate copies optimally.

Above the strength of copyright protection was represented with an "advertising cost" b which was an adjustable parameter and accordingly, the considered model allowed for a discussion of its optimal choice. Confirming to the earlier results by Takeyama (1994), it was seen that when the pirate copies are of a sufficiently low quality, the revenue-maximizing value of copyright protection strength b is zero, and that when they are of a sufficiently high quality, the revenue of the copyright owner is maximized when b is sufficiently high to prevent piracy altogether. [20] This made it natural to ask whether the revenue of the copyright owner always receives its maximum in either of these corner solutions, or whether it is possible that from the perspective of the copyright owner a situation of commercial piracy is sometimes preferable to both the free availability of pirate copies and a situation of no piracy. Above it turned out that also this case is possible, although is seems that this will happen only for the parameter specifications for which the (positive or negative) effects of piracy on the revenue of the copyright owner are rather small.

APPENDIX. PROOFS OF THE PROPOSITIONS IN CHAPTER 4.

PROOF OF PROPOSITION 1. The fact that $\partial n/\partial p_M$ and $d\xi/dp_M$ have the same sign when the stability condition (S) is valid follows immediately from (32). Trivially, since in any fulfilled expectations equilibrium the actual market penetration $n = n_e(\xi) = (\xi - A)/B$, it must also be the case that $dn/dp_M = (1/B)(d\xi/dp_M)$ has the same sign with $\partial\xi/\partial p_M$.

[20] Cf. results (9) and (10) in Takeyama, pp. 160-161.

PROOF OF LEMMA 1. It might seem that since $p_{\min}$ depends of ξ in accordance with (13), one should take this dependence into account when forming the partial derivative $\partial n/\partial p_M$. However, when e.g. $p_M < \xi < \xi_1$ so that

$$b = p_{\min} x_A\left(p_{\min}\right) = \frac{p_{\min}\left(q\xi - p_{\min}\right)}{q\xi}$$

(19) implies that

$$\frac{\partial n}{\partial p_{\min}} = -\frac{1}{q\xi} - \frac{b}{q\xi}\frac{\partial}{\partial p_{\min}}\ln\left(\frac{q\xi - p_{\min}}{p_{\min}}\right) = -\frac{1}{q\xi} + \frac{b}{\left(q\xi - p_{\min}\right)p_{\min}} = 0$$

A similar computation shows that

$$\text{(A1)} \qquad \left(\frac{\partial n\left(p_M,\xi\right)}{\partial p_{\min}}\right)_{p_{\min}=p_{\min}\left(p_M,\xi\right)} = 0$$

in all of the cases (18)-(22). (The intuitive explanation for the result (A1) is that in each case, the explicit dependence of n on $p_{\min}$ results from the integral in the expression (16) of $n_3\left(\xi, p_M\right)$, and trivially, $\partial n_3\left(\xi, p_M\right)/\partial p_{\min} = 0$, since in (16) the integrated function receives the value 0 at the lower limit of the integral.)

The result (A1) implies that all terms containing $\partial n\left(\xi, p_M\right)/\partial p_{\min}$ cancel out in the derivatives $\partial n\left(\xi, p_M\right)/\partial p_M$. Keeping this in mind, (19) trivially implies that when $p_M \le \xi \le \xi_1\left(p_M\right)$, $\partial n/\partial p_M < 0$. Moving to the case in which $\xi_1\left(p_M\right) < \xi \le \xi_2\left(p_M\right)$, it is readily calculated that in this case (20) implies that

$$\frac{\partial \xi}{\partial p_M} = \frac{2b\left(1-q\right)\left(\xi - \left(\left(2-q\right)/2\left(1-q\right)\right)p_M\right)}{\left(p_M - \left(1-q\right)\xi\right)^2\left(\xi - p_M\right)}$$

and since by assumption $\xi \le \xi_2\left(p_M\right) = \left(\left(2-q\right)/\left(2\left(1-q\right)\right)\right)p_M$, it turns out that $\partial n/\partial p_M \le 0$, and that $\partial n/\partial p_M = 0$ if and only if $\xi = \xi_2\left(p_M\right)$.

When $\xi_2\left(p_M\right) < \xi < \xi_3\left(p_M\right)$, it follows from (21) with a straightforward calculation that

$$\frac{\partial n\left(\xi, p_M\right)}{\partial p_M} = \frac{2b\left(1-q\right)}{qp_M^2} + \frac{b\left(1-q\right)}{qp_M^2}\ln\left(\frac{\left(1-q\right)\left(\xi - p_M\right)}{p_M - \left(1-q\right)\xi}\right) - \frac{b}{p_M}\frac{1}{\left(\xi - p_M\right)}$$

Now the assumption that $\xi > \xi_2\left(p_M\right) = \left(\left(2-q\right)/\left(2\left(1-q\right)\right)\right)p_M$ implies that

$$\frac{\partial n(\xi, p_M)}{\partial p_M} > \frac{2b(1-q)}{qp_M^2} - \frac{b}{p_M}\frac{1}{(\xi_2(p_M) - p_M)} = \frac{2b(1-q)}{qp_M^2} - \frac{b}{p_M}\left(\frac{2(1-q)}{qp_M}\right) = 0$$

Finally, when $\xi \geq \xi_3(p_M)$, (22) implies that

$$\frac{\partial n(\xi, p_M)}{\partial p_M} = \frac{2b(1-q)}{qp_M^2} + \frac{b(1-q)}{qp_M^2}\ln\left(\frac{qp_M - p_{\min}}{p_{\min}}\right) - \frac{b(1-q)}{p_M(qp_M - p_{\min})}$$

Since in this case $p_{\min} < p_{\max} = qp_M/2$, it follows that

$$\frac{\partial n(\xi, p_M)}{\partial p_M} > \frac{2b(1-q)}{qp_M^2} - \frac{b(1-q)}{p_M(qp_M - (qp_M)/2)} = 0 . \square$$

PROOF OF COROLLARY 1. If p_M has been chosen so that $\xi < \xi_2(p_M)$ in a stable equilibrium with bootleggers, according to Lemma 1 a sufficiently small decrease Δp in price p_M will cause an increase Δn in the market penetration n. In this case $n(\xi, p_M - \Delta p) \geq n_e(\xi)$, and since $n_e(\xi, p_M)$ and $n_e(\xi)$ are continuous in ξ, there must be some $\xi' > \xi$ for which $n(\xi', p_M - \Delta p) = n_e(\xi')$. Hence, a sufficiently small decrease in p_M causes an increase in $\xi(p_M)$. According to (30), both a decrease in p_M and an increase in ξ increase R_M when $\xi < \xi_2(p_M)$, so that the considered choice of p_M cannot be optimal. □

PROOF OF PROPOSITION 2. The previous corollary already demonstrated that the revenue-maximizing choice of p_M cannot be such that $\xi < \xi_2(p_M)$, and Lemma 1, (30), and (32) easily imply that that the price $\xi = \xi_2(p_M)$ cannot be optimal either, since for this price

$$\frac{dR_M}{dp_M} = \frac{\partial R_M}{\partial p_M} + \frac{\partial R_M}{\partial \xi}\frac{d\xi}{dp_M} = \frac{\partial R_M}{\partial p_M} < 0 .$$

In order to demonstrate this proposition, it must still be shown that if the price set by the monopolist has a value $p_M = \tilde{p}_M$ for which $\xi_2(\tilde{p}_M) < \xi(\tilde{p}_M) < \xi_3(\tilde{p}_M)$, then $p_M = \tilde{p}_M$ cannot be the revenue-maximizing price. In order to show this, we put $\tilde{\xi} = \xi(\tilde{p}_M)$ and $\tilde{n} = n(\tilde{\xi}, \tilde{p}_M) = n_e(\tilde{\xi})$, and assume that $\xi_2(\tilde{p}_M) < \tilde{\xi} < \xi_3(\tilde{p}_M)$, and

demonstrate that the revenue R_M is not maximal for $p_M = \tilde{p}_M$ by considering separately the cases in which $\tilde{n} \le 1/2$ and $\tilde{n} > 1/2$.

Assume first that $\tilde{n} \le 1/2$, and let p_{M0} be the price which satisfies

(A2) $1 - p_{M0}/\tilde{\xi} = \tilde{n}$

Since for the given values of $\tilde{p}_M$ and $\tilde{\xi}$ the market penetration in the absence of the bootleggers would be smaller than the actual market penetration, i.e. since

(A3) $1 - \tilde{p}_M/\tilde{\xi} < \tilde{n}$,

it must be the case that, $p_{M0} < \tilde{p}_M$. Next we shall demonstrate that the bootleggers do not enter the market when the monopolist chooses the price $p_M = p_{M0}$ by deducing a contradiction from the assumption that they do.

Assume that $p_M = p_{M0}$ and the bootleggers send advertisements. In this case there must be some $\xi' \ge \tilde{\xi}$ for which $(r_{\max})_{\xi=\xi', p_M = p_{M0}} \ge b$. Obviously, in this case $\xi' \ge \tilde{\xi} > \xi_2(\tilde{p}_M) > \xi_2(p_{M0})$, and now (6) implies that $(r_{\max})_{\xi, p_M} \ge b$ also when $\xi = \tilde{\xi}$ and p_M has an arbitrary value in the interval $[p_{M0}, \tilde{p}_M]$. Obviously, for all of these values of p_M $\tilde{\xi} > \xi_2(\tilde{p}_M) > \xi_2(p_M)$, and one can now conclude from Lemma 1 that $\partial n(\xi, p_M)/\partial p_M > 0$ whenever $p_{M0} < p_M < \tilde{p}_M$. Hence,

$$1 - \frac{p_{M0}}{\tilde{\xi}} \le n(\tilde{\xi}, p_{M0}) < n(\tilde{\xi}, \tilde{p}_M)$$

This contradicts (A2), however. Hence, if the monopolist chooses the price p_{M0}, the bootleggers will not enter the market. In this case the resulting valuation parameter $\xi(p_{M0})$ must nevertheless be at least $\tilde{\xi}$, because according to (A2) in the absence of the bootleggers one of the equilibria of the model is such that $p_M = p_{M0}$ and $\xi = \tilde{\xi}$. Hence, in this case the revenue of the monopolist R_{M0} satisfies the condition

(A4) $R_{M0} \ge p_{M0}(1 - p_{M0}/\tilde{\xi})$.

On the other hand, for the price $\tilde{p}_M$ the demand of the monopolist is smaller than $1 - \tilde{p}_M/\tilde{\xi}$, so that in this case the revenue of the monopolist must satisfy the condition

(A5) $R_M(\tilde{p}_M) < (1 - \tilde{p}_M/\tilde{\xi})\tilde{p}_M$

The assumption that $\tilde{n} \le 1/2$ and (A3) together imply that $p_{M0} \ge \tilde{\xi}/2$. However, the expression $\left(1 - p_M/\tilde{\xi}\right) p_M$ is a decreasing function of p_M when $p_M \ge \tilde{\xi}/2$. Hence,

$$R_M\left(p_{M0}\right) \ge \left(1 - p_{M0}/\tilde{\xi}\right) p_{M0} > \left(1 - \tilde{p}_M/\tilde{\xi}\right) \tilde{p}_M > R_M\left(\tilde{p}_M\right).$$

In other words, if $\tilde{n} \le 1/2$ the revenue of the monopolist is increased if she lowers the price of legitimate copies to such an extent that the bootleggers do not enter the market.

Assume now that $\tilde{n} > 1/2$. In this case we prove that $p_M = \tilde{p}_M$ cannot be the optimal choice by deducing a contradiction from the assumption that R_M receives a local maximum when $p_M = \tilde{p}_M$. Clearly, this assumption can only be valid if *either*

(A6) $$\left(\frac{dR_M}{dp_M}\right)_{p_M = \tilde{p}_M} = 0$$

or the function $\left(R_M\right)_{p_M, \xi\left(p_M\right)}$ has a discontinuity when $p_M = \tilde{p}_M$. Also the latter possibility must be taken into account, because according to Lemma 1 a decrease of p_M shifts the curve $n = n\left(\xi, p_M\right)$ downwards when $\xi_2\left(p_M\right) < \tilde{\xi} < \xi_3\left(p_M\right)$, and it is conceivable that a decrease of p_M might make the equilibrium market penetration $\xi\left(p_M\right)$ jump discontinuously to a smaller value, in which case also R_M would be discontinuous.

In what follows, we shall eliminate each of these possibilities in turn. Assume first that (A6) was valid. Putting $\zeta = \xi/p_M$, (30) implies that

$$R_M = 2b + b\left(\ln\frac{(1-q)(\zeta - 1)}{1-(1-q)\zeta}\right)$$

Obviously, this is an increasing function of ζ, so that $dR_M/dp_M = 0$ is equivalent with

(A7) $$\frac{d\zeta}{dp_M} = \frac{d\left(\xi/p_M\right)}{dp_M} = \frac{1}{p_M}\left(\frac{d\xi}{dp_M} - \frac{\xi}{p_M}\right) = 0$$

and further with

(A8) $$\frac{d\xi}{dp_M} = \frac{\xi}{p_M}$$

Next it is observed that (A8) implies that

(A9) $$\left(\frac{dn_e(\xi)}{dp_M}\right)_{p_M=\tilde{p}_M} = \left(\frac{d\xi}{dp_M}\right)_{p_M=\tilde{p}_M} n_e'(\tilde{\xi}) = \frac{\tilde{\xi}}{\tilde{p}_M}\frac{1}{B} > \frac{\tilde{n}}{\tilde{p}_M}$$

and also that

$$\left(\frac{d}{dp_M}\left(\frac{1}{\xi}\right)\right)_{p_M=\tilde{p}_M} = \frac{-1}{\tilde{\xi}\tilde{p}_M}$$

On the other hand, (A7) implies that

$$\frac{d}{dp_M}\left(\frac{p_M-(1-q)\xi}{\xi-p_M}\right)_{p_M=\tilde{p}_M} = \frac{d}{dp_M}\left(\frac{1-(1-q)\zeta}{\zeta-1}\right)_{p_M=\tilde{p}_M} = 0$$

When the two last results are inserted into (21), it turns out that

(A10) $$\begin{aligned}\left(\frac{dn(\xi,p_M)}{dp_M}\right)_{p_M=\tilde{p}_M} &= -\left(\frac{d}{dp_M}\left(1-n(\xi,p_M)\right)\right)_{p_M=\tilde{p}_M} \\ &= \frac{1-n(\tilde{\xi},\tilde{p}_M)}{\tilde{p}_M} - \frac{b}{q\tilde{\xi}}\frac{d}{dp_M}\left(\ln\left(q\xi-p_{\min}\right)\right)_{p_M=\tilde{p}_M} < \frac{1-\tilde{n}}{\tilde{p}_M}\end{aligned}$$

Combining (A9) and (A10), it now follows that

$$\frac{\tilde{n}}{\tilde{p}_M} < \left(\frac{dn_e(\xi)}{dp_M}\right)_{p_M=\tilde{p}_M} = \left(\frac{dn(\xi,p_M)}{dp_M}\right)_{p_M=\tilde{p}_M} < \frac{1-\tilde{n}}{\tilde{p}_M}$$

This contradicts the assumption that $\tilde{n} > 1/2$, however.

Secondly, consider the possibility that $\xi(p_M)$ is discontinuous when $p_M = \tilde{p}_M$. In this case $n(\tilde{p}_M,\xi)$ and $n_e(\xi)$ must have the same slope when $\xi = \tilde{\xi}$, so that

(A11) $$\left(\partial n(\xi,\tilde{p}_M)/\partial\xi\right)_{\xi=\tilde{\xi}} = dn_e'(\tilde{\xi}) = 1/B \geq \tilde{n}/\tilde{\xi}$$

It is readily calculated that

$$\frac{\partial}{\partial\xi}\ln\left(\frac{p_M-(1-q)\xi}{\xi-p_M}\right) = -\frac{qp_M}{\left(p_M-(1-q)\xi\right)(\xi-p_M)}$$

Using this result, it is easy to see that (21) implies that

(A12) $$\begin{aligned}\left(\frac{\partial n(\xi,\tilde{p}_M)}{\partial\xi}\right)_{\xi=\tilde{\xi}} &= -\left(\frac{\partial\left(1-n(\xi,\tilde{p}_M)\right)}{\partial\xi}\right)_{\xi=\tilde{\xi}} \\ &< \frac{1-n(\tilde{\xi},\tilde{p}_M)}{\tilde{\xi}} - \frac{2b(1-q)}{q\tilde{p}_M\tilde{\xi}} - \frac{b\left(\tilde{p}_M-(1-q)\tilde{\xi}\right)}{q\tilde{\xi}\tilde{p}_M}\frac{\partial}{\partial\xi}\left(\ln\left(\frac{\tilde{p}_M-(1-q)\xi}{\xi-\tilde{p}_M}\right)\right)_{\xi=\tilde{\xi}} \\ &= \frac{1-\tilde{n}}{\tilde{\xi}} - \frac{2b(1-q)}{q\tilde{p}_M\tilde{\xi}} + \frac{b}{\tilde{\xi}\left(\tilde{\xi}-\tilde{p}_M\right)}\end{aligned}$$

The assumption $\xi(\tilde{p}_M) > \xi_2(\tilde{p}_M)$ implies that $\tilde{p}_M < \left(2(1-q)/(2-q)\right)\tilde{\xi}$, and also that

$$\tilde{\xi} - \tilde{p}_M > \tilde{\xi} - \left(2(1-q)/(2-q)\right)\tilde{\xi} = \left(q/(2-q)\right)\tilde{\xi}.$$

Plugging these results into (A12), it turns out that

$$\text{(A13)} \quad \left(\frac{\partial n(\xi, \tilde{p}_M)}{\partial \xi}\right)_{\xi=\tilde{\xi}} < \frac{1-\tilde{n}}{\tilde{\xi}} - \frac{b(2-q)}{q\tilde{\xi}^2} + \frac{(2-q)b}{q\tilde{\xi}^2} = \frac{1-\tilde{n}}{\tilde{\xi}}$$

Together the conditions (A11) and (A13) imply that

$$\frac{\tilde{n}}{\tilde{\xi}} \le \left(\frac{\partial n(\xi, \tilde{p}_M)}{\partial \xi}\right)_{\xi=\tilde{\xi}} < \frac{1-\tilde{n}}{\tilde{\xi}}$$

which contradicts the assumption that $\tilde{n} > 1/2$. This eliminates the possibility that $\xi(p_M)$ would be discontinuous when $p_M = \tilde{p}_M$, and completes the proof. □

PROOF OF PROPOSITION 3. If q is fixed and the advertising cost b approaches zero, for each p_M the market penetration of the product approaches 1, and $\xi \to \xi_{\max}$, where $\xi_{\max} = A + B$. In this case the revenue of the monopolist approaches the revenue from the consumers who prefer buying a legitimate copy to getting a pirate copy for free. I.e., it approaches

$$\left(1 - \frac{p_M}{(1-q)\xi_{\max}}\right) p_M$$

Hence, the revenue of the monopolist when p_M is chosen optimally approaches

$$\frac{(1-q)\xi_{\max}}{4}$$

On the other hand, for a fixed value of ξ which is smaller than $\xi_{\max}$ the revenue of the monopolist cannot be larger than $(1-\xi/p_M)p_M$, which further implies that it cannot be larger than $\xi/4$. Hence, for each q the revenue-maximizing choice of b must be such that it leads to a market penetration which satisfies the condition

$$\frac{\xi}{4} \ge \frac{(1-q)\xi_{\max}}{4}$$

Hence, the valuation ξ in the revenue-maximizing equilibrium approaches $\xi_{\max}$ when $q \to 0$, which means that the market penetration of the product approaches 1.

Trivially, this can only be the case if the minimum price $p_{\min}$ and the advertising cost b approach zero. □

PROOF OF PROPOSITION 4. Denote the revenue-maximizing price and the corresponding valuation parameter value be $p_{M,opt}$ and ξ_{opt} in the absence of piracy, so that the maximum revenue in the absence of piracy is given by

(A14) $$R_{M,opt} = p_{M,opt}\left(1-\frac{p_{M,opt}}{\xi_{opt}}\right)$$

Corollary 1 states if the corresponding values are p_M and ξ in an equilibrium with pirates, the monopolist can increase her revenue by lowering the price p_M if $\xi < \xi_2(p_M) = \left((2-q)/(2(1-q))\right)p_M$. Hence, if for some q the revenue-maximizing equilibrium is an equilibrium with pirates, in it the price p_M satisfies the condition $p_M < \left((2(1-q))/(2-q)\right)\xi$, implying that the revenue of the monopolist can be at most

(A15) $$R_M < p_M\left(1-\frac{p_M}{\xi}\right) < p_M < \left((2(1-q))/(2-q)\right)\xi < \left((2(1-q))/(2-q)\right)\xi_{\max}$$

where $\xi_{\max} = A + B$. Since (A14) does not depend on q, it must for sufficiently large values of q be larger than the revenue R_M which satisfies (A15). □

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