



## Photon recollision probability in heterogeneous forest canopies: Compatibility with a hybrid GO model

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[1] Photon recollision probability, or the probability by which a photon scattered from a phytoelement in the canopy will interact within the canopy again, has previously been shown to approximate well the fractions of radiation scattered and absorbed by homogeneous plant covers. To test the applicability of the recollision probability theory to more complicated canopy structures, a set of modeled stands was generated using allometric relations for Scots pine trees growing in central Finland. A hybrid geometric-optical model (FRT, or the Kuusk-Nilson model) was used to simulate the reflectance and transmittance of the modeled forests consisting of ellipsoidal tree crowns and, on the basis of the simulations, the recollision probability ( $p$ ) was calculated for the canopies. As the recollision probability theory assumes energy conservation, a method to check and ensure energy conservation in the model was first developed. The method enabled matching the geometric-optical and two-stream submodels of the hybrid FRT model, and more importantly, allowed calculation of the recollision probability from model output. Next, to assess the effect of canopy structure on the recollision probability, the obtained  $p$ -values were compared to those calculated for structureless (homogeneous) canopies with similar effective LAI using a simple two-stream radiation transfer model. Canopy structure was shown to increase the recollision probability, implying that structured canopies absorb more efficiently the radiation interacting with the canopy, and it also changed the escape probabilities for different scattering orders. Most importantly, the study demonstrated that the concept of recollision probability is coherent with physically based canopy reflectance models which use the classical radiative transfer theory. Furthermore, it was shown that as a first approximation, the recollision probability can be considered to be independent of wavelength. Finally, different algorithms for calculation of the recollision probability from measured or modeled radiation fluxes are presented and discussed in the article.

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### 1. Introduction

[2] Recently, parameterization of the radiation budget of vegetation canopies in terms of so-called spectral invariants, that is, wavelength-independent canopy structural parameters, has received increased attention. Theoretical basis for the spectral invariants has been presented by *Knyazikhin et al.* [1998] and *Panferov et al.* [2001], who were the first to apply these parameters in their algorithm for retrieval of leaf area index (LAI) and fraction of photosynthetically active radiation absorbed by vegetation (FPAR) from MODIS surface reflectance data. The spectral invariant  $p$ , proposed to govern canopy absorption, also has an intuitively appealing interpretation. Namely, it corresponds approximately to

the probability that a photon scattered from a phytoelement in the canopy will interact within the canopy again, the “photon recollision probability.”

[3] In a Monte Carlo simulation study, *Smolander and Stenberg* [2005] calculated the recollision probability ( $p$ ) for horizontally homogeneous canopies of varying leaf area index (LAI) and showed that, to a very good approximation, the portions of radiation absorbed and scattered by the canopy depended only on  $p$  and the leaf albedo at a given wavelength. The theory of spectral invariants (“ $p$ -theory”) has also been applied in estimating canopy reflectance [*Zhang et al.*, 2002; *Shabanov et al.*, 2003; *Wang et al.*, 2003; *Disney et al.*, 2005; *Rautiainen and Stenberg*, 2005]. All of the above studies give support to the  $p$ -theory proposed by *Knyazikhin* and coworkers. However, to extend the interpretation of  $p$  as photon recollision probability to realistic cases, this interpretation should be tested in heterogeneous canopies.

[4] The aim of this paper is to test the compatibility of the recollision probability with commonly used physically based canopy reflectance models that take into account

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the grouped character of natural canopies. Our hypothesis is that because of their physically based nature, both methods should be applicable to heterogeneous canopies. No attempts of comparing these two approaches have been reported in scientific literature. Besides testing this hypothesis, we present a method of calculating the recollision probability from common model output.

[5] The portions of absorbed and scattered radiation for different values of leaf albedo were simulated for a set of modeled forest stands with varying canopy structure using the forest reflectance and transmittance (FRT) model developed by *Kuusik and Nilson* [2000]. Next, the relationship of canopy absorption to leaf albedo was derived. Then, the mean photon recollision probability and its dependence on leaf single-scattering albedo were calculated from the standard output parameters of the model. The physically based FRT model was chosen to be used in this study since it builds upon a fairly detailed and realistic description of forest canopy structure, involving stand variables such as tree size, shape and spatial pattern. At the same time, it retains a generalized picture of the canopy without a need to describe the exact distribution of foliage elements. Such an abstraction of the canopy structure allows a direct analysis of the dependency of the recollision probability on basic stand and tree parameters (crown length and width, leaf area density inside a crown, number of crowns per unit ground area). FRT is used in many research projects worldwide for forest reflectance modeling because of its speed, invertibility and the availability of its source code. It has also performed well in the third phase of the Radiation Model Intercomparison Exercise [*Widlowski et al.*, 2006]. However, as energy conservation is a key prerequisite for calculating  $p$  from model-predicted reflectance and transmittance, a method was first developed and applied to normalize the FRT model to conserve energy on the basis of a wavelength-independent correction factor. The new, central feature of this method, which can be applied to other FRT-like models as well, is that the correction factor can be calculated from model output without modifying its internal algorithms.

## 2. Definitions

### 2.1. Canopy Radiation Budget

[6] The following notations are used for the parameterization of the shortwave radiation budget of a vegetation canopy above a black soil. Of the radiation incident on the canopy, a fraction is transmitted to underlying surface without any interactions with phytoelements. This directly transmitted fraction is termed “uncollided radiation.” The rest of incoming photons, termed “collided radiation,” will interact at least once with phytoelements and will be absorbed or scattered by the canopy. These wavelength-independent fractions are called the zero-order canopy transmittance ( $t_0$ ) and canopy interceptance ( $i_0$ ), respectively ( $i_0 + t_0 = 1$ ). The collided photons ( $i_0$ ) will eventually be absorbed ( $a$ ) by the canopy or scattered ( $s$ ) out from the canopy, but may interact several times with phytoelements before this happens. The number of interactions depends on the wavelength or specifically, because the phytoelements are mainly leaves, on the leaf single-scattering albedo ( $\omega$ ) at the considered wavelength. Using these notations, the law of

conservation of shortwave radiative energy inside a plant canopy can be written as

$$a(\omega) + s(\omega) + t_0 = 1. \quad (1)$$

By subtracting the fraction  $t_0$  of incident radiation that is transmitted through the canopy without interactions from both sides of this equation, we obtain the law of conservation of energy for intercepted radiation,

$$a(\omega) + s(\omega) = 1 - t_0 = i_0. \quad (2)$$

The scattered part ( $s(\omega) = i_0 - a(\omega)$ ) can further be divided into upward and downward scattered components, that is, canopy spectral reflectance ( $r(\omega)$ ) and transmittance ( $t_s(\omega)$ ) which together with the uncollided zero-order transmittance ( $t_0$ ) comprise total canopy transmittance. Inserting  $s(\omega) = r(\omega) + t_s(\omega)$  into equation (1) gives us the canopy radiation budget,

$$a(\omega) + s(\omega) + t_0 = a(\omega) + r(\omega) + t_s(\omega) + t_0 = 1. \quad (3)$$

### 2.2. Photon Recollision Probability

[7] The recollision probability  $p$  is defined as the (mean) probability by which a photon scattered from a phytoelement in the canopy will interact within the canopy again [*Smolander and Stenberg*, 2003, 2005]. This allows calculating normalized canopy absorption  $a(\omega)/i_0$  as

$$a(\omega)/i_0 = (1 - \omega) + \omega p_1(1 - \omega) + \omega^2 p_2^2(1 - \omega) + \dots, \quad (4)$$

where  $p_i$  is the recollision probability after  $i$ th interaction of a photon with a canopy phytoelement. On the assumption that the recollision probability remains constant in successive interactions,  $p_1 = p_2 = \dots = p$ , equation (4) becomes

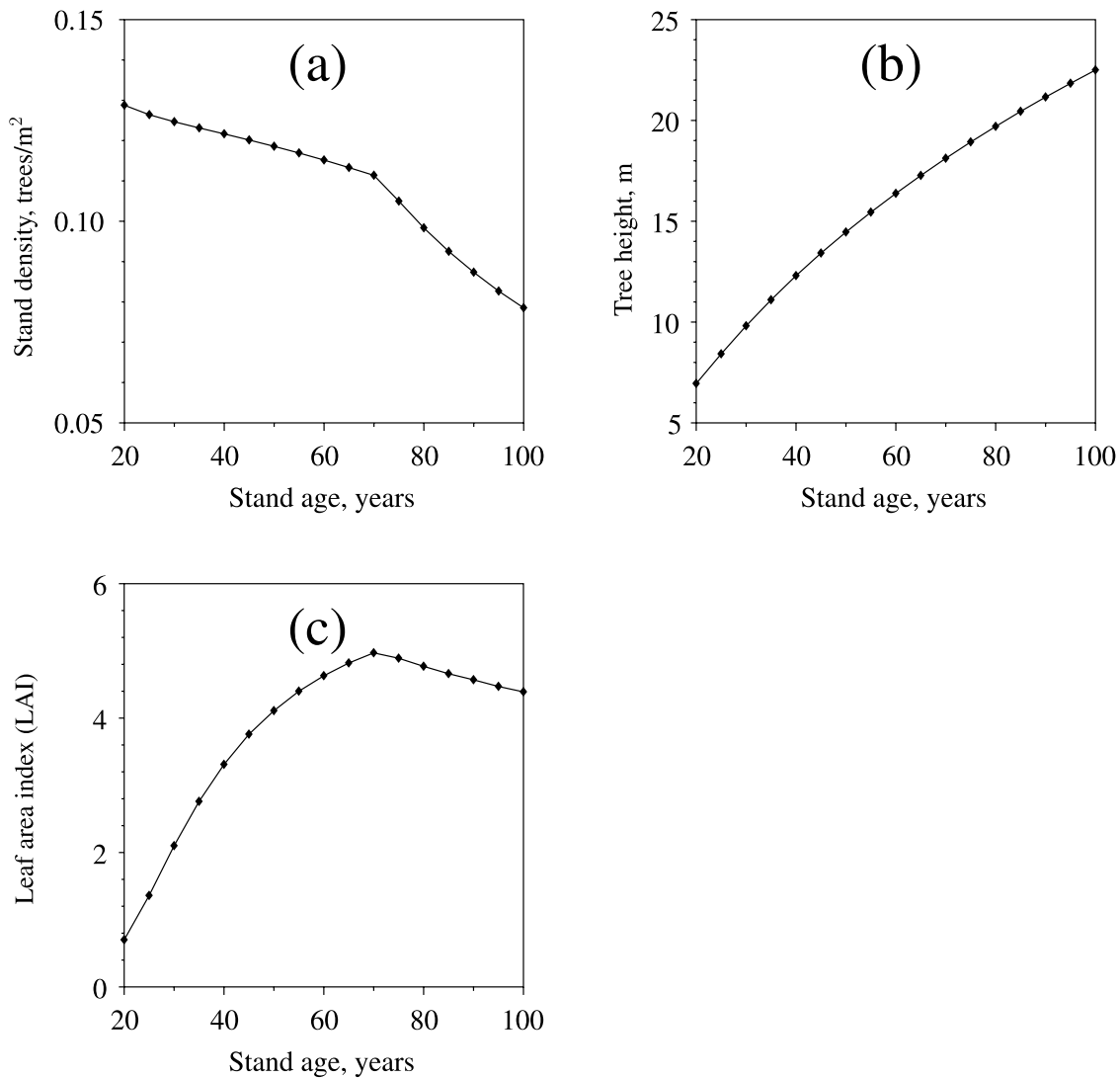
$$a(\omega)/i_0 = (1 - \omega) + \omega p(1 - \omega) + \omega^2 p^2(1 - \omega) + \dots = \frac{1 - \omega}{1 - p\omega}. \quad (5)$$

Equation (5) is used to relate the canopy internal parameter (a parameter that cannot be measured directly)  $p$  to three measurable quantities ( $a$ ,  $i_0$ ,  $\omega$ ).

## 3. Materials and Methods

### 3.1. Input Data

[8] The simulations with the FRT model were carried out for a set of 17 modeled forest stands with an age range of 20 to 100 years. The stand data (Figure 1) were simulated according to the allometric relationships of Scots pine trees growing in central Finland with models of the MELA system [*Hynynen et al.*, 2002], a system developed in Finland for generating stand growth and yield on the basis of simple stand and tree variables. The growth models for Scots pine were used only to generate a realistic set of input stands with natural ranges of variation in crown dimensions and LAI. Otherwise, no species specific parameters were applied.



**Figure 1.** Basic stand characteristics calculated using a growth model for Scots pine trees in central Finland: (a) stand density, (b) tree height, and (c) leaf area index (LAI).

[9] To give the reader an idea of the structure of the stands, a brief description will now follow (see Figure 1 and Table 1). The stands started with an initial density of 1300 trees per hectare at the age of 20 years, decreasing to 800 trees per hectare by the age of 100 years. A Poisson distribution of tree locations was assumed all throughout the developmental course of the stands; that is, the tendency of older stands to have a more regular tree pattern was ignored. LAI for the stands ranged from 0.7 to 5, reaching its maximum value at 70 years and decreasing marginally after that. Tree height in the stands ranged from 7 to 23 m, and canopy depth (crown length) from 5 to 10 m. The largest stand canopy volume was reached at the end of the studied age course.

[10] Even though the structure is generated here as a function of stand age, most model results will be presented as functions of LAI which is more a general and comprehensive canopy variable than the model-specific stand age. This may cause ambiguity as LAI is not a unique function of the main input parameter, stand age. However, when

necessary, this is cleared in the figure captions and article text.

### 3.2. Model Description

[11] The Kuusk-Nilson forest reflectance and transmittance model, FRT [Kuusk and Nilson, 2000], is a hybrid geometric-optical radiative transfer model that calculates reflectance and transmittance factors for canopies consisting of geometrical crown envelopes. A uniform spatial distribution of leaf area (constant leaf area density) inside the crown envelopes is assumed. First-order scattering is calculated by numerical integration over the envelopes using canopy bidirectional gap probabilities. Diffuse fluxes (i.e., fluxes of multiple-scattered radiation) are calculated using the analytical solution of a two-stream radiative transfer model, which assumes homogeneous (structureless) canopies. Thus, to be coherent with the geometric-optical part of the FRT model, the two-stream model is parameterized using an effective LAI instead of the actual LAI of the canopy. The effective LAI is defined as the LAI of a homogeneous canopy that has the same canopy intercept-

**Table 1.** Age-Dependent Characteristics of the Modeled Scots Pine Stands Used as Input for the FRT Model

Stand Age, years	LAI	Tree Density, trees/m <sup>2</sup>	Crown Height, m	Crown Radius, m
20	0.70	0.129	5.18	1.00
25	1.36	0.126	5.84	1.18
30	2.10	0.125	6.29	1.35
35	2.76	0.123	6.60	1.49
40	3.31	0.122	6.86	1.61
45	3.76	0.120	7.09	1.72
50	4.11	0.119	7.30	1.81
55	4.40	0.117	7.54	1.89
60	4.63	0.115	7.78	1.97
65	4.82	0.113	8.05	2.03
70	4.97	0.111	8.32	2.10
75	4.89	0.105	8.63	2.16
80	4.77	0.098	8.92	2.21
85	4.66	0.093	9.22	2.27
90	4.57	0.087	9.48	2.32
95	4.47	0.083	9.74	2.37
100	4.39	0.079	9.97	2.42

tance and direct transmittance as the real (structured) canopy, and it is calculated by the geometric-optical model using a solar zenith angle of 40°. Since only a subset of the FRT model's features is used in the current study, only the relevant characteristics of the model are briefly described here; for a more detailed description see [Kuusk and Nilson, 2000].

[12] Crowns were modeled as ellipsoids with a uniform (Poisson) distribution of trees in the horizontal coordinates. Leaves were treated as bi-Lambertian scatterers with no specular reflection component, and the reflectance and transmittance of leaves were taken equal. A spherical leaf angle distribution was assumed.

[13] The two-stream (2S) model embedded in FRT was also used separately to compare the effect of canopy structure on photon recollision probability. The model was run with the same values of effective LAI as when used as a submodel of the hybrid FRT model. The only difference in the scattered radiation field as predicted by the FRT and 2S models is thus the way in which first-order scattering is calculated.

[14] Because the FRT model uses different approaches for single and higher-order scattering, it is not bound to be energy-conservative. The recollision probability theory on the other hand assumes energy conservation, i.e., that the absorption of a conservative medium is zero. Therefore, to calculate the recollision probability from the output of the FRT model it had to first be renormalized to meet the criterion of energy conservation. The procedure by which this was done is described in the following section.

### 3.3. Achieving Energy Conservation

[15] As described previously, FRT uses a geometric-optical model to compute first-order scattering and a two-stream submodel for the remaining scattering orders. These two parts of the FRT model, although both based on the theory of radiative transfer, use different descriptions and parameterizations of the canopy structure and, as a result, the law of energy conservation is not strictly followed. To compensate for this, we developed a technique to ensure energy conservation and to enable the matching of the geometric-optical and two-stream submodels of the FRT

model. The energy conservation procedure was carried out on the various canopy reflectance and transmittance components calculated by FRT while keeping the internal algorithms of the model intact. In other words, the procedure only recalibrated the model output.

[16] The law of conservation of energy (equation (2)) for an ideal FRT-like model (i.e., a combined model that uses geometric-optical submodel to compute first-order scattering and absorption, and a submodel based on the solution of the two-stream approximation of the radiative transfer equation to approximate the higher-order components) can be written as

$$i_0 = s_1^{GO} + a_1^{GO} + s_h^{2S} + a_h^{2S}, \quad (6)$$

where  $i_0$  is the canopy interceptance,  $s_1$  is the fraction of photons that escape the canopy after the first interaction with a canopy element (first-order scattering),  $a_1$  is the fraction of photons absorbed at the first interaction with a canopy element (first-order absorption),  $s_h$  is the fraction of photons that escape the canopy after two or more interactions with canopy elements (higher-order scattering), and  $a_h$  is the fraction of photons absorbed after more than one interaction with a canopy element (higher-order absorption). The superscripts GO and 2S refer to the geometric-optical and two-stream submodels, respectively. For the geometric-optical submodel,  $s_1^{GO}$  is integrated over all view directions.

[17] We assumed that the geometric-optical model that uses a more realistic description of canopy structure produces the “true” values for direct transmittance, first-order scattering and first-order absorption while the two-stream submodel produces only approximations for higher-order scattering and absorption, i.e.,  $S_h^{2S}$  and  $A_h^{2S}$  instead of  $s_h^{2S}$  and  $a_h^{2S}$ . To fulfill the law of conservation of energy (equation (2)), the FRT model must be recalibrated to satisfy the condition:

$$i_0 = s_1^{GO} + a_1^{GO} + f(S_h^{2S} + A_h^{2S}) = i_0^{GO}, \quad (7)$$

where  $f$  is a correction factor and  $i_0 = i_0^{GO}$  denotes canopy interceptance calculated using the geometric-optical model. Using the law of conservation of energy (equation (2)) for a two-stream model,  $S_h^{2S} + A_h^{2S} = I_0^{2S} - S_1^{2S} - A_1^{2S}$ , gives

$$i_0^{GO} = s_1^{GO} + a_1^{GO} + f(I_0^{2S} - S_1^{2S} - A_1^{2S}), \quad (8)$$

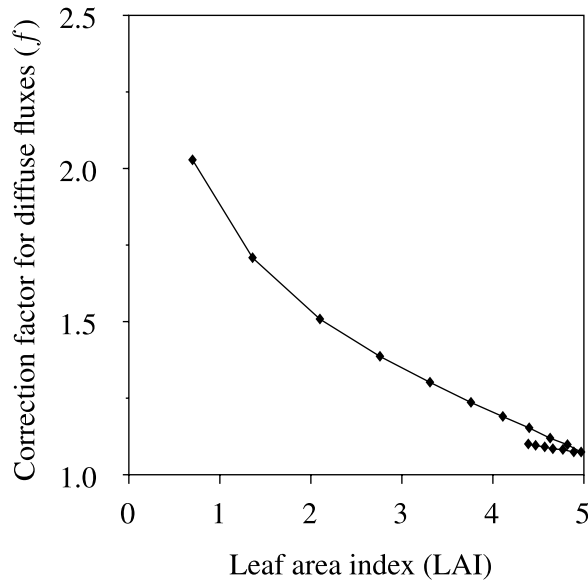
or, after solving for  $f$ ,

$$f = \frac{i_0^{GO} - s_1^{GO} - a_1^{GO}}{I_0^{2S} - S_1^{2S} - A_1^{2S}}. \quad (9)$$

Capital letters  $I_0^{2S}$ ,  $S_1^{2S}$ , and  $A_1^{2S}$  denote the approximations of  $i_0^{2S}$ ,  $s_1^{2S}$ , and  $a_1^{2S}$ , respectively, calculated by the two-stream submodel.

[18] The first-order scattering coefficient for a physically based canopy reflectance model is calculated by integrating the bidirectional view factor and leaf scattering phase function ( $\Gamma$ ) first over the whole canopy and then over all possible exit directions (or view angles),

$$s_1 = \int_{4\pi} \left[ \int_C \Gamma(\omega, \Omega_1, \Omega_2) p(\mathbf{r}, \Omega_1, \Omega_2) d\mathbf{r} \right] d\Omega_1, \quad (10)$$



**Figure 2.** Energy conservation correction factor ( $f$ , equation (9)) for the two-stream submodel of FRT as a function of leaf area index. The correction factor compensates for the differences in first-order scattering in the geometric-optical and two-stream algorithms.

where  $C$  is canopy volume (i.e., the volume filled with scattering elements),  $\mathbf{r}$  is a point in  $C$ ,  $\Omega_1$  is the view angle,  $\Omega_2$  is the illumination angle, and  $p(\mathbf{r}, \Omega_1, \Omega_2)$  is the bidirectional gap probability at the point  $\mathbf{r}$  in the directions  $\Omega_1$  and  $\Omega_2$  [Kuusk and Nilson, 2000]. If  $\Gamma$  does not depend on the spatial coordinates  $\mathbf{r}$ ,

$$s_1 = \int_{4\pi} \Gamma(\omega, \Omega_1, \Omega_2) \left[ \int_C p(\mathbf{r}, \Omega_1, \Omega_2) d\mathbf{r} \right] d\Omega_1. \quad (11)$$

For a more detailed treatment of the leaf scattering phase function and its dependence on the directions  $\Omega_1$  and  $\Omega_2$ , and the leaf single-scattering albedo  $\omega$  [see, e.g., Knyazikhin and Marshak, 1991]. The important property of  $\Gamma$  used in the following derivations is that, if the ratio of leaf reflectance to transmittance does not vary with wavelength, the bi-Lambertian leaf scattering model allows the scattering phase function to be factorized to separate the geometric and optical parts,  $\Gamma(\omega, \Omega_1, \Omega_2) = \omega \Gamma'(\Omega_1, \Omega_2)$ . On this assumption, the fraction  $s_1$  can be written as

$$s_1 = q_1 \omega, \quad (12)$$

where  $q_1$  is a factor depending on the illumination angle and canopy structure calculated from equation (11). Similarly, first-order absorption can be written as

$$a_1 = \int_C (1 - \omega) p(\mathbf{r}, \Omega_2) d\mathbf{r}, \quad (13)$$

where  $p(\mathbf{r}, \Omega_2)$  is gap probability in the illumination direction  $\Omega_2$  at  $\mathbf{r}$ . If  $\omega$  does not depend on  $\mathbf{r}$ ,

$$a_1 = (1 - \omega) \int_C p(\mathbf{r}, \Omega_2) d\mathbf{r}. \quad (14)$$

The integral  $\int_C p(\mathbf{r}, \Omega_2) d\mathbf{r}$  is equal to canopy interception (and also to the total canopy absorption  $a$  if  $\omega = 0$ ),  $\int_C p(\mathbf{r}, \Omega_2) d\mathbf{r} = i_0$ . This equation holds for all  $\omega$  as  $i_0$  is wavelength-independent, and

$$a_1 = (1 - \omega) i_0. \quad (15)$$

These results (equations (12) and (15)), derived from the basic principles of the radiative transfer theory, are valid for both geometric-optical and two-stream models. Now, equation (9) can be rewritten as

$$f = \frac{i_0^{GO} - s_1^{GO} - a_1^{GO}}{I_0^{2S} - S_1^{2S} - A_1^{2S}} = \frac{i_0^{GO} - q_1^{GO}}{I_0^{2S} - q_1^{2S}}. \quad (16)$$

Thus, assuming a constant ratio of leaf transmittance to leaf reflectance, the correction factor  $f$  is independent of wavelength. It can be calculated using equation (9) and eliminating the terms containing canopy absorption (which is not an output of FRT) by setting  $\omega$  to 1,

$$f = \frac{i_0^{GO} - s_1^{GO}}{I_0^{2S} - S_1^{2S}} \Big|_{\omega=1} = \frac{i_0^{GO} - s_1^{GO}}{S_h^{2S}} \Big|_{\omega=1}. \quad (17)$$

Now, this correction factor can be applied to canopy scattering calculated with FRT for any wavelength (or leaf single-scattering albedo  $\omega$ ):

$$s(\omega) = s_1^{GO}(\omega) + f S_h^{2S}(\omega). \quad (18)$$

The normalized canopy absorption  $a(\omega)/i_0$  (equation (4)) can be calculated as

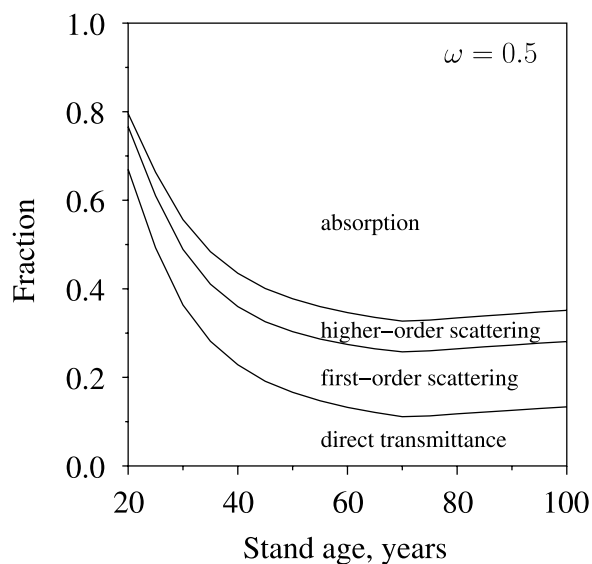
$$a(\omega)/i_0 = \frac{i_0^{GO} - s(\omega)}{i_0^{GO}} = 1 - \frac{s_1^{GO}(\omega) + f S_h^{2S}(\omega)}{i_0^{GO}}. \quad (19)$$

[19] The energy conservation correction factors (equation (9)) for all modeled canopies are plotted as a function of LAI in Figure 2. Since the correction factor depends on the effective LAI, which (at the same true leaf area) varies with canopy structure, it is not a unique function of LAI. To illustrate the contribution of the geometric-optical and two-stream submodels of FRT, the fractions of absorbed radiation and first- and higher-order scattering for a canopy with  $\omega = 0.5$  are plotted as a function of stand age in Figure 3. The stands that have the highest correction factors have also the smallest contribution of diffuse radiation; this diminishes the effect of the correction on total canopy reflectance and transmittance.

[20] For the two-stream model (2S), canopy absorption is calculated assuming a homogeneous layer of vegetation and, as expected, energy is conserved and no correction of fluxes is required.

### 3.4. Calculation of Recollision Probability

[21] As both the FRT and 2S models are based on the classical radiative transfer theory, they cannot calculate the recollision probability ( $p$ ) directly. Instead, the average recollision probability is calculated from the sum of reflectance and transmittance ( $a(\omega)/i_0 = (1 - t_s - t_0 - r)/i_0$ ) by finding the best fit for equation (5).



**Figure 3.** An illustration of the division of radiation into the parts that are directly transmitted, escaped after first interaction, escaped after several interactions, and absorbed, as a function of stand age. Calculations are at the leaf single-scattering albedo ( $\omega$ ) value of 0.5.

[22] To assess the possible variation of recollision probability with single-scattering albedo, equation (5) was solved for  $p(\omega)$ :

$$p(\omega) = \frac{1}{\omega} - \frac{1 - \omega}{\omega a(\omega)/i_0}. \quad (20)$$

Using equation (20), another expression relating the mean recollision probability and leaf single-scattering albedo to normalized canopy absorption, slightly different from equation (5), can be obtained by averaging  $p(\omega)$  over wavelengths,

$$\langle p \rangle = \frac{1}{N} \sum_{i=1}^N p_i = \frac{1}{N} \sum_{i=1}^N \left( \frac{1}{\omega_i} - \frac{1 - \omega_i}{\omega_i a_i / i_0} \right), \quad (21)$$

where  $N$  is the number of wavelengths (or leaf albedos) for which canopy absorption is either measured or modeled.

### 3.5. Simulations With FRT and 2S

[23] FRT was used to simulate the reflectance and transmittance of the 17 modeled forest stands consisting of ellipsoidal tree crowns, and the recollision probability for the different canopies was calculated on the basis of the simulations. Next, to compare this recollision probability with that of structureless (homogeneous) canopies with the same effective LAI, the 2S model was utilized.

[24] For a fixed canopy structure, the simulation results depended only on the boundary conditions and the spectral properties of canopy elements. As the purpose of this study was to investigate the compatibility of geometric-optical models with the theory based on spectrally invariant recollision probability, instead of simulating the forest reflectance for a selected set of wavelengths, the leaf albedo ( $\omega$ ) was varied from 0 to 1 in steps of 0.02. The contribution

from branches and stems was ignored. The canopy was assumed to be bounded by black soil from below and incident radiation consisted of only the direct solar beam with the Sun at  $45^\circ$  from the zenith. FRT output was integrated over all directions to obtain total canopy absorption and scattering, 2S simulated these quantities directly.

[25] In the simulations, energy conservation was achieved with the method described in section 3.3. Note that the energy conservation correction factors that were obtained are site-specific and the values obtained in the simulations are thus not universally applicable. However, we believe that these simulations indicate the general behavior of the FRT model and illustrate the problems encountered when it is applied to highly clumped canopies. The large values of the correction factors indicate that matching the two-stream submodel to the geometric-optical part is a weak spot of FRT. As the multiply scattered field is more relevant in the near-infrared region, the results obtained in that part of the spectrum are most vulnerable to energy loss. As can be seen from Figure 2, and in agreement with expectations, the loss was larger for the younger unclosed canopies. However, these difficulties do not arise from the physical foundations of the model but from a mismatch between the two submodels used by FRT and we hope they can be solved in the near future.

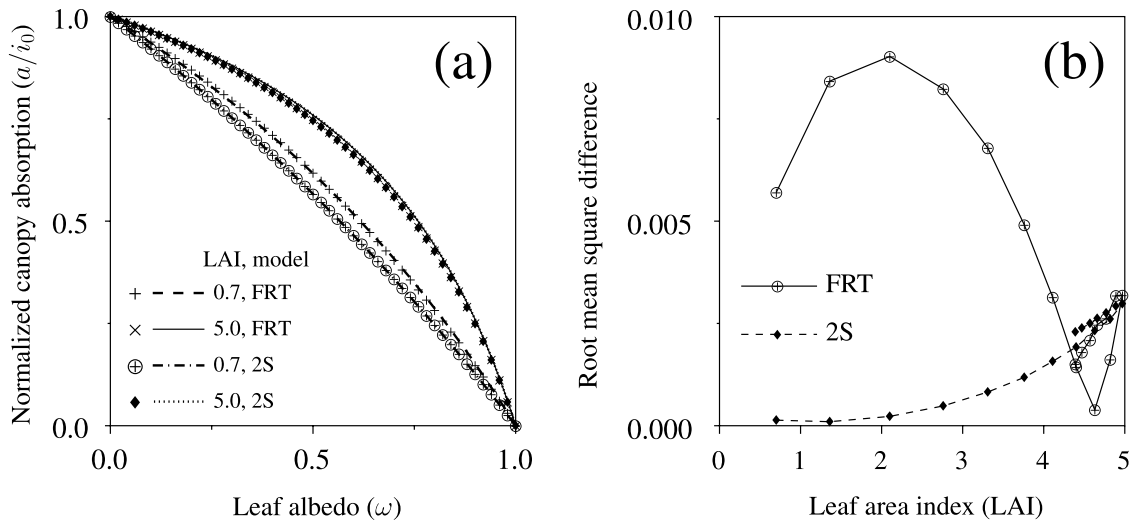
## 4. Results and Discussion

### 4.1. Simulated Canopy Absorption

[26] Normalized canopy absorption ( $a/i_0$ ) was simulated by FRT for the set of modeled forest stands and also for their homogeneous counterparts by 2S. As an example, we will now examine canopy absorption as a function of leaf albedo for the two canopy types at two extreme values of LAI: LAI = 0.7 and LAI = 5.0 (Figure 4). For a sparse canopy (LAI = 0.7), the two models, FRT and 2S, produce different relationships between canopy absorption and leaf albedo. In a dense canopy (LAI = 5), on the other hand, the curves practically coincide. Nevertheless, for all cases, equation (5) provides a very good approximation to the dependence of  $p$  on  $\omega$ . In Figure 4b, the root mean square (RMS) differences between the normalized canopy absorption predicted by the  $p$ -theory (equation (5)) and the two radiative transfer based models are shown. As the differences are small (compared to the errors caused by various modeling assumptions), the  $p$ -theory can be considered coherent with both types of canopy reflectance models, regardless of whether these models contain canopy structure or not. Generally, FRT results were best approximated using the spectral invariant at large LAI values and closed canopies, whereas the 2S model results were fitted best at small LAI values (Figure 4b).

[27] The dependence of  $p$  on LAI (calculated by line fitting) is plotted in Figure 5a. The overall shape of the curve is similar for the two models, but the 2S model produces consistently smaller values of  $p$ . For small LAI values, where the effect of canopy clumping should be more evident, the difference between the two models is the largest. For a dense canopy (LAI = 5), on the other hand, the two models produce almost identical  $p$ -values.

[28] The larger  $p$ -values predicted by FRT indicate that structured forest canopies absorb more of the once collided



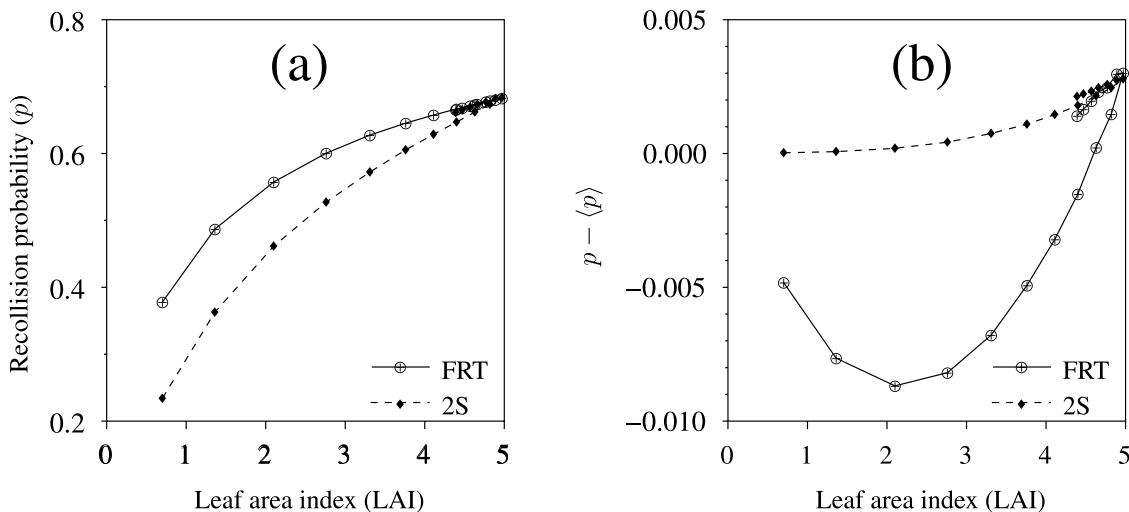
**Figure 4.** Comparison of the  $p$ -theory with radiative transfer models. (a) Normalized canopy absorption  $(a(\omega)/i_0)$  for the two extreme LAI values (stand age = 20 years, LAI = 0.7; stand age = 40 years, LAI = 5.0) as a function of leaf albedo. Shown are results of the FRT and 2S models (plotted using symbols) together with the curves calculated using the  $p$ -theory (equation (5)), plotted using lines). For LAI = 5, the FRT and 2S models produce nearly identical results and the fitted lines overlap. (b) Root mean square (RMS) differences of normalized canopy absorption  $(a(\omega)/i_0)$  between the two radiative-theory based models (FRT and 2S) and the  $p$ -theory as functions of LAI.

radiation irrespective of wavelength. It is somewhat unexpected that all the points are on the same curve, i.e., that  $p$  predicted by FRT for a stand is a single-value function of LAI and not directly influenced by other canopy structural parameters. As the 2S model uses effective LAI that is corrected for stand structure and is a multiple value function of the true LAI, the  $p$ -values for older forests with decreasing LAI follow a different curve. The difference in  $p$ -values for a homogeneous and heterogeneous model is also an indicator of the importance of taking into account the 3-D structure of a vegetation canopy, an issue that has recently gained wide attention in the remote sensing modeling community [e.g., Widłowski *et al.*, 2005].

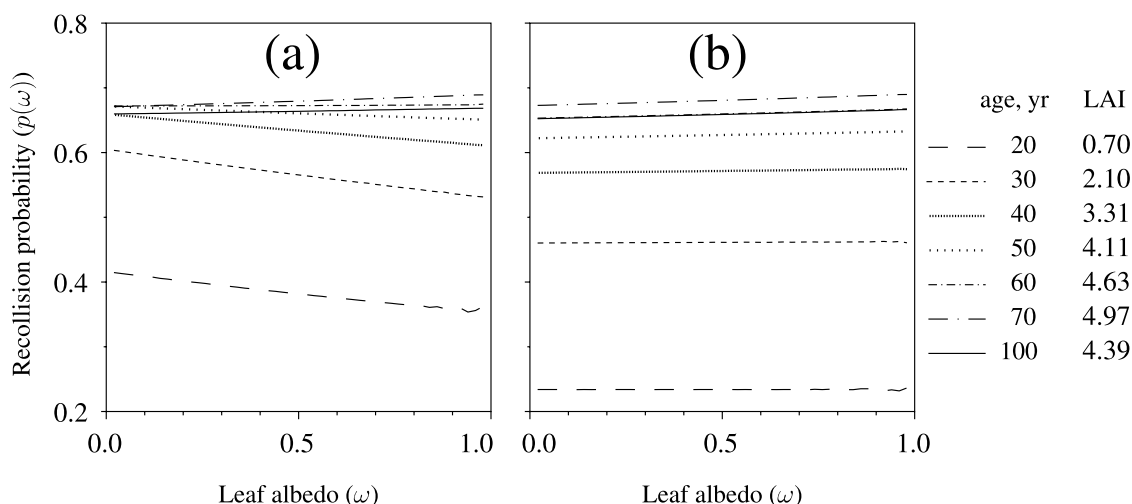
**4.2. Comparison of Two Algorithms for Calculating  $p$**

[29] Two different algorithms for calculating the recollision probability from modeled radiation fluxes were examined next. The algorithms are applicable also to measured radiation fluxes, and would be interesting to test with an empirical data set.

[30] Besides the mean recollision probability estimated by line fitting (equation (5)),  $p$  can be calculated using equation (20) for all leaf albedos and averaged using equation (21) to yield the average recollision probability  $\langle p \rangle$ . The differences between the recollision probabilities  $p$  (calculated by line fitting), and  $\langle p \rangle$  (the average recollision



**Figure 5.** Recollision probability calculated using the physically based FRT and 2S models: (a) recollision probability calculated by line fitting ( $p$ , equation (5)) and (b) difference  $p - \langle p \rangle$  between the line fitting result and the averaged  $\langle p \rangle$  (equation (21)) as functions of leaf area index (LAI).



**Figure 6.** Dependence of the recollision probability  $p(\omega)$  calculated from equation (20) on leaf albedo ( $\omega$ ) as predicted by the (a) FRT and (b) 2S model.

probability) are plotted against LAI in Figure 5b. As expected, the differences are small compared to the values of  $p$  and depend only marginally on LAI. Although in the current study the two estimates of recollision probability can be considered equivalent, under some circumstances, one approach for calculating it may have clear advantages over the other.

[31] If recollision probability is calculated from canopy absorption by line fitting using the least squares method, the sum of the squares of the differences between the measured and modeled absorptions is minimized, i.e.,

$$\frac{d}{dp} \sum_i (A - \bar{A}_i)^2 = 0, \quad (22)$$

where  $A_i = a_i/i_0$  is a measured value of canopy absorption and  $\bar{A}_i$  is the approximation calculated using the  $p$ -theory (equation (5)). After expanding  $A_i$  into a Taylor series around the point  $p_i = p$ , where  $p_i$  is the recollision probability calculated from a single measurement point using equation (20), and keeping only the first term, this condition can be shown to yield

$$p \approx \frac{\sum_i \left( \left. \frac{dA}{dp} \right|_{\omega=\omega_i} \right)^2 p_i}{\sum_i \left( \left. \frac{dA}{dp} \right|_{\omega=\omega_i} \right)^2}, \quad (23)$$

where  $\omega_i$  is the leaf single-scattering albedo for the wavelength at which the  $i$ th measurement was taken. Equation (23) is similar to equation (21) for calculating  $\langle p \rangle$ , but instead of equal weights  $1/N$ , the square of the derivative of canopy absorption with respect to  $p$  is used. Thus both methods yield a “mean” value for  $p$ , and the difference is only in the weights used in the averaging procedure.

[32] If recollision probability does not depend on  $\omega$ , i.e., it is truly spectrally invariant, the values of  $p$  and  $\langle p \rangle$  should coincide. Furthermore, even if the assumption of the constancy of  $p$  does hold, because of measurement errors, the

two algorithms could still give different results. If  $p_i$  is calculated from measured  $a_i$  with a measurement uncertainty  $\Delta a_i$ , the uncertainty of  $p_i$  can be calculated as  $\Delta p_i = \frac{dp}{da} \Delta a_i$ . If recollision probability is calculated by line fitting (i.e., equation (23)), the weights for calculating the mean value are inversely proportional to the square of the uncertainties of  $p_i$ , and the mean recollision probability  $p$  calculated by line fitting can be considered more insensitive to errors than  $\langle p \rangle$ . Also, if the purpose of calculating recollision probability is its implementation for modeling canopy absorption, the value obtained by line fitting minimizes, because of the way it is calculated, the modeling errors of canopy absorption ( $a$ ). If the recollision probability is not truly spectrally invariant but varies with  $\omega$ , both  $p$  and  $\langle p \rangle$  depend on the set of wavelengths (or leaf albedos) over which they are averaged. Averaging weights should be chosen on the basis of practical considerations, either the ones given above (equations (21) and (23)) or some others, depending on the application.

#### 4.3. Spectral Invariance of the Recollision Probability

[33] Equation (5) underlying the theory of photon recollision probability for vegetation canopies assumes that  $p$  does not depend on wavelength. However, recollision probability may be a function of scattering order and therefore can also have an interpretation in which it is nonconstant.

[34] In the current modeling experiment, if the recollision probability does not depend on  $\omega$ , the difference  $p - \langle p \rangle$  is 0 (Figure 5b). According to the results, the spectral invariance of  $p$  is thus best satisfied at small LAI values for the 2S model and at large LAI values for FRT. This is consistent with the observed minima of the RMS difference curves at the same LAI values (Figure 4b). In other cases,  $p$  and  $\langle p \rangle$  are functions of  $\omega$ . The dependence of  $\langle p \rangle$  on  $\omega$  (calculated from equation (20)) is quasilinear and stronger for FRT than for 2S (Figure 6). While the slope is always positive for 2S, it is mainly negative for FRT with positive values only at the highest LAI values (LAI  $\approx$  5).

[35] Both radiative transfer models used in the current study are physically based and, after a correction procedure,



guaranteed to be energy-conservative. Although the hybrid geometrical-optical FRT model can be said to represent a forest canopy in a more “natural” way, the two-stream equations are the exact solutions of the radiative transfer equation for a canopy of infinitesimally small horizontal Lambertian scatterers and thus provide a valuable insight into the reflecting properties of vegetation. As the concept of recollision probability also has clear physical foundations, it is not unexpected that it is coherent with physically based radiative transfer models and equation (5) does, indeed, approximate equation (4) quite well.

[36] It is natural to assume that the vertical distribution of scattering events depends on the order of scattering. If the spatial distribution of photons that have undergone at least one interaction is different from the distribution of incident unscattered photons inside the canopy, the probability of interacting again can be expected to be different for these photons. Furthermore, since the average number of interactions of a photon with leaves depends on the leaf albedo ( $\omega$ ), the mean recollision probability can also be assumed to vary with  $\omega$ .

[37] For the 2S model which represents the simplest case of a horizontally homogeneous canopy, the positive slope of  $p$  as a function of  $\omega$  indicates that for a photon that has undergone at least one interaction, it is harder to escape from the canopy after interacting again than for an unscattered photon. The extinction of direct radiation is exponential and first-order scattering occurs more likely near the top of the canopy where it is relatively easy for the photons to escape. By surviving interactions with canopy elements, photons get diffused deeper into the canopy. This leads to higher-order photons being more evenly distributed in the homogeneous canopy layer and forcing them to penetrate a thicker leaf layer before managing to escape. A similar qualitative explanation can be given for the variation in the slope of  $p$  as a function of  $\omega$  (Figure 6b, the slope is larger for larger LAI values): for a thin canopy, higher-order photons can relatively easily exit through the bottom of the canopy, thus equalizing the escape probabilities for different scattering orders.

[38] The negative slope of  $p$  as a function of  $\omega$  produced by FRT, on the other hand, indicates that in structured canopies it was easier for higher-order photons to escape than for the single-scattered ones. Although this might sound controversial, it can be a result of canopy structure. Inside a single crown, because of the exponential character of radiation attenuation, a single-scattered photon should always have a higher chance of escaping the crown through the side it entered the canopy through when compared to a higher-order photon. When viewing the whole canopy instead of a single tree, photons can use different escape paths between the crowns compared to the single entrance path associated with a monodirectional radiation beam. However, the energy conservation correction factor applied to the diffuse radiation submodel of FRT may have influenced the slope, and therefore these results are not conclusive.

[39] Considering the complex nature of radiation scattering in plant canopies, the simplifications used in the present study exclude the possibility of applying these results directly to natural canopies. Yet we think that the results

justify the application of photon recollision probability at canopy scale.

#### 4.4. Energy Conservation of Hybrid Canopy Reflectance Models

[40] This specific energy conservation technique described in section 3.3 is not the only possible approach to achieve energy conservation. Its physical meaning can be explained on the basis of the general characteristics of physically based canopy reflectance models. The output vector of  $x$  such models, concisely described using a function  $R$ ,

$$x = R(\psi_{struc}, \psi_{opt}, \psi_{bound}), \quad (24)$$

depends on a variety of input parameters:  $\psi_{struc}$  contains canopy structural properties (LAI, leaf angle distribution, etc.),  $\psi_{opt}$  the optical properties of canopy elements, and  $\psi_{bound}$  boundary conditions (illumination conditions, ground reflectance, etc). In equation (8), the correction factor  $f$  is applied to all three radiation field components ( $I_0^{2S}$ ,  $S_1^{2S}$ ,  $A_1^{2S}$ ). Because of the linearity of the function  $R$  (equation (24)) with the intensity of incident radiation field  $I$  in  $\psi_{bound}$ , this correction is equivalent to adjusting the input parameter  $I$ :

$$\begin{aligned} f \cdot s^{2S} &= f \cdot R^{2S}(\psi_{struc}^{2S}, \psi_{opt}^{2S}, \psi_{bound,rest}^{2S}, I) \\ &= R^{2S}(\psi_{struc}^{2S}, \psi_{opt}^{2S}, \psi_{bound,rest}^{2S}, f \cdot I), \end{aligned} \quad (25)$$

where  $\psi_{bound,rest}^{2S}$  contains all other boundary conditions except  $I$  for the two-stream model. In the current study, the two-stream submodel was run to produce a single value, canopy scattering  $s^{2S}$ , in the output vector  $x^{2S}$ .

[41] Other possible energy conservation techniques could be designed by adjusting different input variables. Probably the most evident is a modification of the effective LAI used by the two-stream submodel. However, this modification would not be spectrally invariant and would also require a modification of the internal algorithms of FRT. In addition, the working principle of a normalization which modifies the energy input of one submodel is easy to interpret: energy that is lost (if  $f > 1$ ) in FRT at the interface of the two models (i.e., after first-order scattering) is reinserted into the submodel that calculates higher-order scattering. On the other hand, if  $f < 1$ , excess energy is removed from FRT by normalization. The fact that, under the assumptions used in the current study, first-order scattering depends linearly on leaf single-scattering albedo explains the independence of  $f$  from  $\omega$ .

[42] As  $q_1$  in equation (12) is calculated by integration over the whole canopy, normalization forces the model to conserve energy at the interface of the two submodels but it sets no constraints on how this energy is distributed. In other words, the energy sinks (or sources, if  $f < 1$ ) at model interfaces are distributed differently from the compensatory sources (or sinks, if  $f < 1$ ) that the normalization procedure creates to conserve energy. However, this problem is inevitable as the two submodels use intrinsically different methods to describe a canopy. Even without normalization, the interface cannot guarantee a consistent description of the field inside the canopy. The two-stream model is just a more

crude approximation to facilitate a quick calculation of higher-order scattering and it cannot be completely compatible with a more complex geometric-optical model used to enhance the calculation of the directional and spectral distribution of first-order scattering.

[43] The use of the energy conservation correction factors is the only solution available today for hybrid models such as FRT. It requires an additional step of running the model using totally reflecting canopy elements. The issues of energy conservation should be considered (and solved) for all physically based canopy radiative transfer models, since, besides allowing calculation of the recollision probability, it makes the models more universally applicable. A model that conserves energy can be used to predict consistently and simultaneously reflectances at different view angles, e.g., to simulate multiangular measurements or calculate energy balance, for which exiting radiation intensity has to be integrated over all view angles.

## 5. Conclusions

[44] The current study demonstrates that the physically based concept of recollision probability ( $p$ -theory) is coherent with physically based canopy reflectance models. However, the application of  $p$ -theory in a radiative transfer model requires that the model conserves energy. If this criterion is met, recollision probability can be used to relate canopy absorption at different wavelengths, even in the spectral regions where leaves have very different optical properties (e.g., red and infrared).

[45] As a first approximation, the recollision probability can be considered independent of wavelength. Usually, it is most reasonable to calculate the mean recollision probability by fitting a line to measured canopy absorption rather than to take the average value of  $p_i$  calculated for single data points.

[46] The effect of structure is evident both in the value of the recollision probability and also in the slope of  $p$  plotted as a function of leaf albedo. Structure makes canopies more efficient absorbers of the collided radiation and changes the escape probabilities for different scattering orders.

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