R&D INVESTMENTS OF A DUOPOLY UNDER A PERMISSIVE PATENT SYSTEM

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Abstract

This thesis considers an investment game, in which two firms invest in R&D in order to obtain an innovation. It is assumed that a permissive patent system is in use, which allows the fragmentation of immaterial property rights among several innovators in cases of simultaneous discovery. The objective is to shed light into the firms' investment behavior using a game-theoretical model.

It is assumed that initially there is an inter-firm difference in product quality, observable by the utility-maximizing consumers. The game consists of two stages: In the first stage, each firm decides on the sum that it is going to invest in R&D. Investment may result in a product quality-improving discovery. In the second stage, the firms engage in either Bertrand or Cournot competition, which are examined as separate cases.

By examining the Nash equilibrium of the firms' investments it is determined, under which parameter configurations each firm invests a positive sum in R&D. There are three parameters in the model: (i) the initial inter-firm difference in product quality, (ii) the quality-improvement resulting from an innovation and (iii) a parameter reflecting the relationship between the sum invested and the probability of making a discovery, thereby reflecting the cost of innovation. Because the mathematical expressions arrived at are complex, numerical computations done on a computer are applied in order to obtain useful results. Based on the numerical results, conclusions are drawn regarding the three parameters' influence on the firms' investment decisions.

The following results are arrived at: (i) Both firms are more likely to invest in cases of inexpensive and/or highly significant discoveries; (ii) The firm producing the higher-quality product is more likely to invest; (iii) In the case of the firm producing the lower-quality product, a large initial inter-firm difference in product quality reduces the firm's willingness to invest under Cournot competition, but increases it under Bertrand competition; (iv) Both firms are more likely to invest under Cournot competition than under Bertrand competition; (v) Under Bertrand competition, the firm selling the lower-quality product is less likely to invest if obtaining an innovation might make the products similar in quality.

Keywords
patent, investment, research and development, immaterial property rights

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Additional information

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Tiivistelmä

Työssä tutkitaan kahden yrityksen välistä investointipeliä, jossa yritykset investoivat tutkimukseen ja kehittämistoimintaan tarkoituksenaan tehdä innovaatio. Tutkimuksessa oletetaan, että käytössä on niin sanottu salliva patenttijärjestelmä (permissive patent system), joka mahdollistaa keksintöjä koskevan immateriaalioikeuden jakamisen usean innovoijan kesken, mikäli keksinnöt on tehty usealla taholla samanaikaisesti. Työn tarkoituksena on selventää yritysten investointikäyttäytymistä peliteoreettisen mallin avulla.

Tutkimuksessa oletetaan, että tarkasteltavien yritysten myymien tuotteiden välillä vallitsee alkutilanteessa laatuero, jonka hyötyä maksimoivat kuluttajat pystyvät havaitsemaan. Tutkimuksessa muodostettavassa mallissa peli jaetaan kahteen vaiheeseen: Ensimmäisessä vaiheessa kukin yritys tekee päätöksen summasta, jonka tämä aikoo investoida tutkimukseen ja kehittämistoimintaan. Investoinnin seurauksena yritys voi saada käyttöönsä tuotteen laatua parantavan keksinnön. Toisessa vaiheessa yritysten oletetaan käyvän joko Bertrand- tai Cournot-kilpailua. Näitä kahta kilpailun muotoa tarkastellaan erillisinä tapauksina.

Tutkimuksessa selvitetään tarkastelemalla investointien Nash-tasapainoa, millaisissa asetelmissa yritykset investoivat positiivisia summia tutkimukseen ja kehittämistoimintaan. Asetelma määrittyy kolmen parametrin avulla, joita ovat (i) yritysten välinen laatuero alkutilanteessa, (ii) mahdollisesti tehtävän keksinnön tuotteen laatua lisäävän vaikutuksen suuruus ja (iii) parametri, joka kuvaa investoidun rahasumman vaikutusta keksinnön tekemisen todennäköisyyteen eli kuvaa siten keksinnön tekemisen kustannuksia. Matemaattiset lausekkeet, joihin tutkimuksessa päädytään, ovat monimutkaiset, minkä seurauksena käyttökelpoisten tulosten saamiseksi sovelletaan tietokoneella tehtävää numeerista laskentaa. Numeeristen tulosten perusteella tehdään päätelmiä siitä, kuinka tarkasteltavat kolme parametria vaikuttavat yritysten investointipäätöksiin.

Tutkimuksessa päädytään seuraaviin tuloksiin: (i) Kummankin yrityksen kannustimet investoida ovat suuremmat, jos keksinnön tekemisen kustannukset ovat matalat tai jos keksintö on merkittävä; (ii) Halukkuus ryhtyä investoimaan on suurempi parempilaatuista tuotetta myyvän yrityksen tapauksessa; (iii) Suuri tuotteiden välinen laatuero vähentää heikompilaatuista tuotetta myyvän yrityksen investointihalukkuutta Cournot-kilpailun vallitessa mutta lisää sitä Bertrand-kilpailun vallitessa; (iv) Cournot-kilpailun vallitessa kummankin yrityksen halukkuus ryhtyä investoimaan on suurempi kuin Bertrand-kilpailun vallitessa; (v) Bertrand-kilpailun vallitessa heikompilaatuista tuotetta myyvä yritys on vähemmän halukas investoimaan, jos keksinnön tekeminen voi johtaa tilanteeseen, jossa yritykset myyvät laadultaan samankaltaisia tuotteita.

Avainsanat
patentit, investoinnit, tutkimus- ja kehittämistoiminta, immateriaalioikeus

Säilytyspaikka

Muita tietoja

Contents

1 Introduction

1.1 Background

Technological change is often considered the ultimate source of economic growth. In Solow's (1956) model of technological growth, technological change is considered as an exogenous input. Newer models (e.g. Romer 1990), however, often consider endogenous technological change, where market incentives lead agents to devote resources to R&D. Technology differs from many other goods in that it is not excludable by its nature, i.e. its use by others than the innovator is hard to restrict. For example, an agent may be able to imitate technology developed by someone else. The lack of excludability reduces the incentives to engage in innovative activities. This may, in turn, have a negative influence on economic growth. The patent system is intended to increase the incentive to innovate by granting the innovator a monopoly over the innovation for a certain period of time. Another motivation for the existence of the patent system is that it promotes the disclosure of technological knowledge: since patents are public, patented innovations become available for unrestricted, public usage when the patent expires.

Empirical evidence suggests that it is relatively common for different innovators to make the same discovery at nearly the same time (see e.g. Kingston 2004). Patent systems, however, grant the monopoly to one innovator only, even if several independent innovators were to file patent applications for the same discovery within a short while. In such cases (called situations of "interference") patents are granted either according to a first-to-invent or a first-to-file rule, depending on the country. The first-to-invent rule means that the patent is granted to the first inventor, provided that the date of first invention can be documented. The first-to-file rule, which is applied in all countries except the United States, means that the patent is granted to the first one to file the patent application. (Scotchmer and Green 1990, p.133)

It has been proposed by La Manna et al. (1989), that the traditional patent system is not socially optimal: a better system would be a *permissive patent system*, i.e. one which would allow the (horizontal) fragmentation of immaterial property rights (IPRs) among several inventors who have made the discovery within a short while. Also Kultti and Takalo (2003) investigate the implications of different IPR systems, and conclude that, in general, it is not optimal to grant the IPR to one innovator only.

The discussion regarding the fragmentation of IPRs brings forth the question of how firms would invest under an IPR system that allowed fragmentation. There is a sizeable amount of microeconomic research concerning IPRs, but most of the research in the field assumes that a patent is granted to one innovator only – most models do not allow for the fragmentation of IPRs. Also, in most of the earlier research, the revenues received by the firms are not derived explicitly, e.g. by considering a Cournot or Bertrand equilibrium (see e.g. Varian 1999, pp. 478–483).

A patent race is a competition between firms, in which "the first firm to acquire sufficient knowledge to make an innovation is granted a valuable patent" (Harris and Vickers 1987, p.1). Loury (1979) was one of the first to study patent races. Since then, different forms of patent races have been studied extensively. A very common feature in the numerous papers concerning patent races is that the models used in them allow an IPR to be granted to one innovator only. Many of the models do, however, allow for the imitation or licensing of a discovery, which may increase the number of agents using the discovery.

Singh and Vives (1984) derive the Bertrand and Cournot equilibria in a situation similar to the one considered in this thesis: they too consider a duopolistic market structure involving inter-firm differences in product quality. In their article the equilibria are, however, derived somewhat differently, as they take as their starting point the utility function of a representative consumer, whereas in this thesis the derivation of the equilibria starts by defining an (aggregate) demand curve associated with each firm's product. Singh and Vives do not present all the algebra behind their results as explicitly as it is presented in this thesis, and unlike this thesis, investments are not considered in their article.

Denicolò (2000) considers a two-stage patent race, where the IPR concerning the first-stage innovation does not necessarily extend to subsequent, second-stage innovations that are improvements to the first-stage innovation. Thus, the first-stage innovator does not necessarily have an advantage in the second stage over the other participants, since others may be equally capable of making patentable improvements on the first-stage innovation. Denicolò examines the welfare implications of different regimes of "forward protection", in which the IPR concerning the first-stage innovation may or may not extend to protect the first-stage innovator's interests in the second stage of the game. Denicolò's model does not explicitly consider the formation of the revenues of the firms. A similar setting to that of Denicolò has been studied by Green and Scotchmer (1995). In another article by Denicolò (2001), he considers a patent race where innovations are cost-reducing; that is, they do not affect product quality, but they lower production costs. In the model, a patent can be granted to one innovator only. The firms' are assumed to engage in Bertrand competition, but the Bertrand equilibrium is not considered explicitly in the research. Kortum (1997) develops a general-equilibrium model of technological change, where patents advance the technological frontier representing the "best techniques for producing each good in the economy". As a general-equilibrium model, Kortum's model considers the IPR issues on a far more general level than the model developed in this thesis. With his model, Kortum explains some trends that can be observed in industrial research, patenting and productivity growth. Aghion et al. (2001) consider a dynamic process of step-by-step innovation under duopoly, where both firms engage in R&D activities. They develop a model for determining how competition and imitation affect economic growth. They find that some imitation is not necessarily bad for growth, but lots of it is always bad.

This thesis differs from most of the related research in several aspects. Most of the earlier research considers the welfare implications of different parameter settings of the respective models, and the models usually involve cumulative innovation with more than one decision period. However, most of the models use simplifying assumptions regarding the formation of the firms' revenues. In contrast to most of the earlier research, this thesis considers a more limited game, involving only one period of investment decisions. Welfare implications are not considered either. However, the post-investment situation, where the revenue is generated, is modeled on a more detailed level in this thesis than in most of the earlier research, as Bertrand and Cournot competition are considered explicitly here.

1.2 Objectives and methods

The objective of this Master's thesis is to *determine the Nash equilibrium of R&D investments in a model that (i) allows the horizontal fragmentation of immaterial property rights, (ii) involves quality-improving discoveries, and (iii) explicitly considers the post-discovery situation as a Bertrand or Cournot game*.

The investment behavior of two competing firms is examined at different combinations of the values of the three parameters: (i) the initial inter-firm difference in product quality, (ii) the cost of making a discovery by investing in R&D, and (iii) the size of the product quality improvement gained by making a discovery. Studying the Nash equilibrium of R&D investments clarifies how these three parameters influence equilibrium R&D investments. This, in turn, should be indicative of how firms' incentives to invest are formed under an IPR system, which allows the horizontal fragmentation of immaterial property rights.

It turns out, that algebraic expressions are very complex when expressed as functions of the three parameters. Therefore, it is extremely difficult to analyze these expressions analytically, and algebra alone is an insufficient means of determining how the parameters influence the firms' incentives to invest in R&D. One solution to this problem would be to build a different model that would produce results that are easier to analyze. This would, however, mean that some key element of the problem would have to be simplified, which in turn might lead to conclusions different from the original model. This is not desirable. Therefore, in order to be able to use the chosen model, *numerical computation done by computer* is applied in this Master's thesis: for each combination of the three parameters, calculations are performed to determine how the firms invest in the respective Nash equilibrium. The results of the numerical computations illustrate in which regions of the three-dimensional parameter space R&D investments are made by both firms, by only one of the firms, or by neither of the firms. Based on the numerical results, conclusions are drawn about how the parameters determine the firms' incentives to invest in R&D.

1.3 The investment game considered in this Master's thesis

The patent race considered here is not of a cumulative type; there is only one decision period for making investment decisions. After the firms know whether each of them has been granted a patent to the new discovery, they engage in Bertrand or Cournot competition in which they maximize their profits. In Bertrand competition the firms set their prices and the quantities sold are then determined on the market, whereas in Cournot competition the firms set the quantities that they sell and the prices are then determined on the market. Both forms of competition are considered in this Master's thesis.

The main part of this Master's thesis consists of deriving the sub-game perfect Nash equilibrium (see e.g. Dutta 2001, pp. 196–197) of a two-stage game in which the first decision is an investment decision and the second is a price or quantity decision (depending on whether Bertrand or Cournot competition is considered). The equilibrium is calculated using backward-induction: First the Bertrand or Cournot equilibrium and the associated profits are derived in the situation after the investment game has taken place, describing the post-investment situation with parameters that reflect the post-investment quality difference between the firms' products.

Then the expressions of the profits from the Bertrand/Cournot competition stage (which are expressed as functions of the product quality associated with each firm) are used in finding the Nash equilibrium of R&D investments. The Bertrand and Cournot equilibria are derived in Chapter 2 and the equilibrium of investment decisions is derived in Chapter 3. The results of the research, and their interpretations, are considered in Chapter 4.

The game is illustrated by Figure 1. The notation used in the picture is that of decision tree analysis (see e.g. Brealey and Myers 2000, pp. 275–276), in this case applied to a game-theoretical situation.[1] Decision points are depicted as squares, and stochastic events are depicted as circles. The decisions at both stages of this game are continuous values, and the dark triangles after the decision points represent the continuum of possible decisions. In this game, making a discovery is considered a stochastic event, the probability of which depends on the size of the investment.

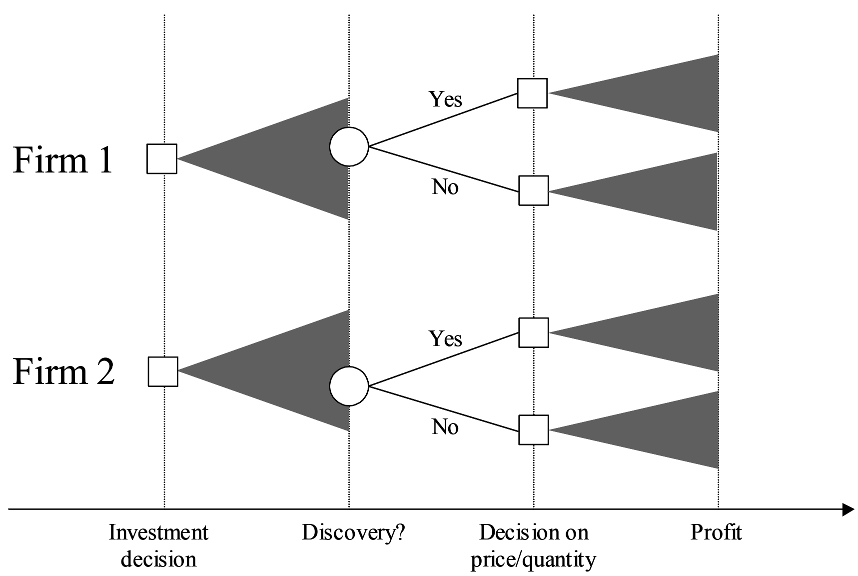

Figure 1. An illustration of the game considered in this Master's thesis.

In the Bertrand/Cournot competition of the second stage of the game, the demand curves and the inter-firm difference in product quality are modeled in a way similar to that of Koboldt (1995,

[1] It should be noted that even though the firms' decision trees are drawn separately in the figure, each firm's profit is also influenced by the other firm's decisions. The figure should also *not be confused* with the so-called "extensive form" illustration often used to depict games.

pp. 137–139). In his model, Koboldt assumes that the consumers' willingness to pay for each product is uniformly distributed on the value range reaching from zero to the willingness to pay of the consumer who has the highest valuation for the products. This results in linear demand curves for the two firms' products, which intersect with the horizontal axis at the same point. Such demand curves are also assumed in the game considered in this thesis. The increase in product quality resulting from innovation is modeled as an upward shift of the intercept of the respective demand curve, which represents an increase in every consumer's willingness to pay. One critical assumption of the model considered in this thesis is that of zero marginal costs. While this assumption is unrealistic, it is a good approximation at least if the products considered are information goods (Shapiro and Varian 1999, p. 3).

2 Bertrand and Cournot equilibrium

2.1 The model

The model considered here involves a market where two firms (firm 1 and firm 2) operate. Both firms produce the same product but there is a difference in the quality of the products, as the product produced by firm 1 is superior in quality to the product produced by firm 2. The assumptions of the model are as follows:

1. Firm 1 and firm 2 sell their products at prices p_1 and p_2, respectively, where $p_1, p_2 > 0$.

2. The firms have zero costs.

3. Both firms engage in either Bertrand or Cournot competition (these two forms of competition are considered as two separate cases).

4. There is a continuum of consumers, indexed by $q \in [0,1]$. The demand curves[2] associated with each of the two products are assumed to be linear. The inverse demand curves, which can be interpreted as exhibiting each individual consumer's willingness to pay, are:

   $P_1(Q) = a - aQ$, for the product of firm 1, and

   $P_2(Q) = b - bQ$, for the product of firm 2,

   where $a > b > 0$. An example of such demand curves is presented in Figure 2.

5. The utility that consumer q gets from buying one of the products is defined to equal his individual willingness to pay (read from the inverse demand curve) for the product minus the price of the product:

[2] The word "demand curve" here refers to the demand that the respective product would meet if the other firm of the duopoly did not sell any of its own products. Thus, it reflects the consumers' willingness to pay for the product.

$u_1(q) = P_1(q) - p_1$, for the product of firm 1, and

$u_2(q) = P_2(q) - p_2$, for the product of firm 2.

6. Each consumer buys one product at most. A consumer always buys the product that will give him the greatest utility, assuming that this utility is positive.

The demand curves of firm 1 and firm 2 intersect with the vertical axis at $p = a$ and $p = b$, respectively, since $P_1(0) = a - a \cdot 0 = a$, and $P_2(0) = b - b \cdot 0 = b$. The demand curves defined in assumption 4 intersect with the horizontal axis at $q = 1$, since $P_1(1) = a - a \cdot 1 = 0$, and $P_2(1) = b - b \cdot 1 = 0$.

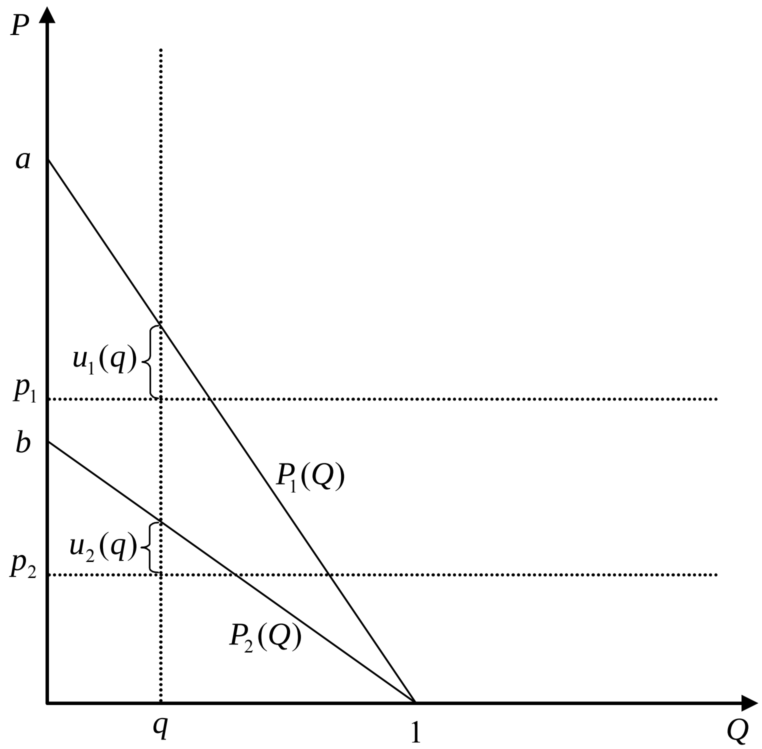

Figure 2. The demand curves associated with each firm's product.

2.2 The relationship between prices and quantities

The goal here is to deduce how prices determine the quantities sold. Substituting the inverse demand curves of assumption 4 into the utility functions of assumption 5 gives

(1) $$u_1(q) = P_1(q) - p_1 = a - aq - p_1$$

(2) $$u_2(q) = P_2(q) - p_2 = b - bq - p_2 \,.$$

These functions are linear and decreasing in q, since $a > 0$ and $b > 0$. The function u_1 is decreasing more rapidly than u_2, since according to assumption 4, $a > b$. Figure 3 shows one possible realization of these functions.

Two conditions need to be satisfied in order for a consumer to buy a product: (i) buying the product in question brings the consumer a greater utility than buying the other product, and (ii) the utility obtained by buying the product is positive. The fulfillment of these conditions is considered next.

2.3 Which purchase brings the greatest utility?

Figure 3 depicts the functions that determine the utility received by each consumer from buying one of the products.

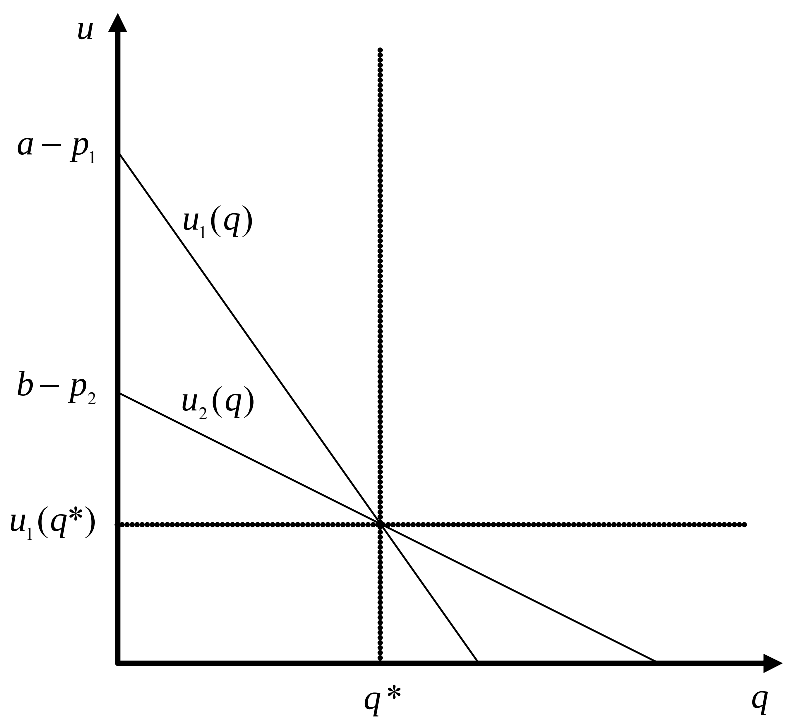

Figure 3. The functions that determine the utility received from buying one of the products.

According to assumption 6, consumer q buys the product of firm 1 only if this brings him at least as large a utility as buying the product of firm 2. Therefore consumer q buys the product of firm 1 only if

(3) $$u_1(q) \geq u_2(q).$$

Substituting the functions u_1 and u_2 from expressions (1) and (2) gives

(4) $$a - aq - p_1 \geq b - bq - p_2.$$

This can be solved for q:

(5) $$q \leq \frac{p_1 - p_2 + b - a}{b - a}.$$

The right-hand side of expression (5) is denoted with q^*, which represents the point on the q-axis, at which $u_1(q) = u_2(q)$, that is, the point, where the respective consumer is indifferent between the firms. Thus, the consumer q chooses the product of firm 1 only if

(6) $$q \leq q^*, \qquad \text{where } q^* = \frac{p_1 - p_2 + b - a}{b - a}.$$

2.4 Is the obtained utility positive?

The consumer q gets positive utility from buying the product of firm 1 if and only if $u_1(q) > 0$, that is, when $a - aq - p_1 > 0$. Solving this for q yields

(7) $$q < \frac{a - p_1}{a} = q_1^0.$$

Similarly, the consumer q gets positive utility from buying the product of firm 2 if and only if $u_2(q) > 0$, that is when $b - bq - p_2 > 0$. Solving this for q yields

(8) $$q < \frac{b - p_2}{b} = q_2^0.$$

2.5 The quantities sold by each firm

As noted earlier, consumer q buys the product from one firm if the utility thus received is (i) positive and (ii) greater than the utility that would have been obtained by buying the product of the other firm. Figure 4 depicts the alternatives as to how the "utility functions" u_1 and u_2 can lie in the (q,u) coordinate plane, and how the quantities sold by each company are determined in these different situations. Depending on the prices, the outcome may be one of six alternatives, each of which results in different expressions for the quantities sold. The curves may intersect in any of the four quadrants of the coordinate plane. If the intersection point lies in the second or the fourth quadrant of the coordinate plane, the outcome also depends on the sign of $u_2(0)$ or $u_1(0)$, respectively.

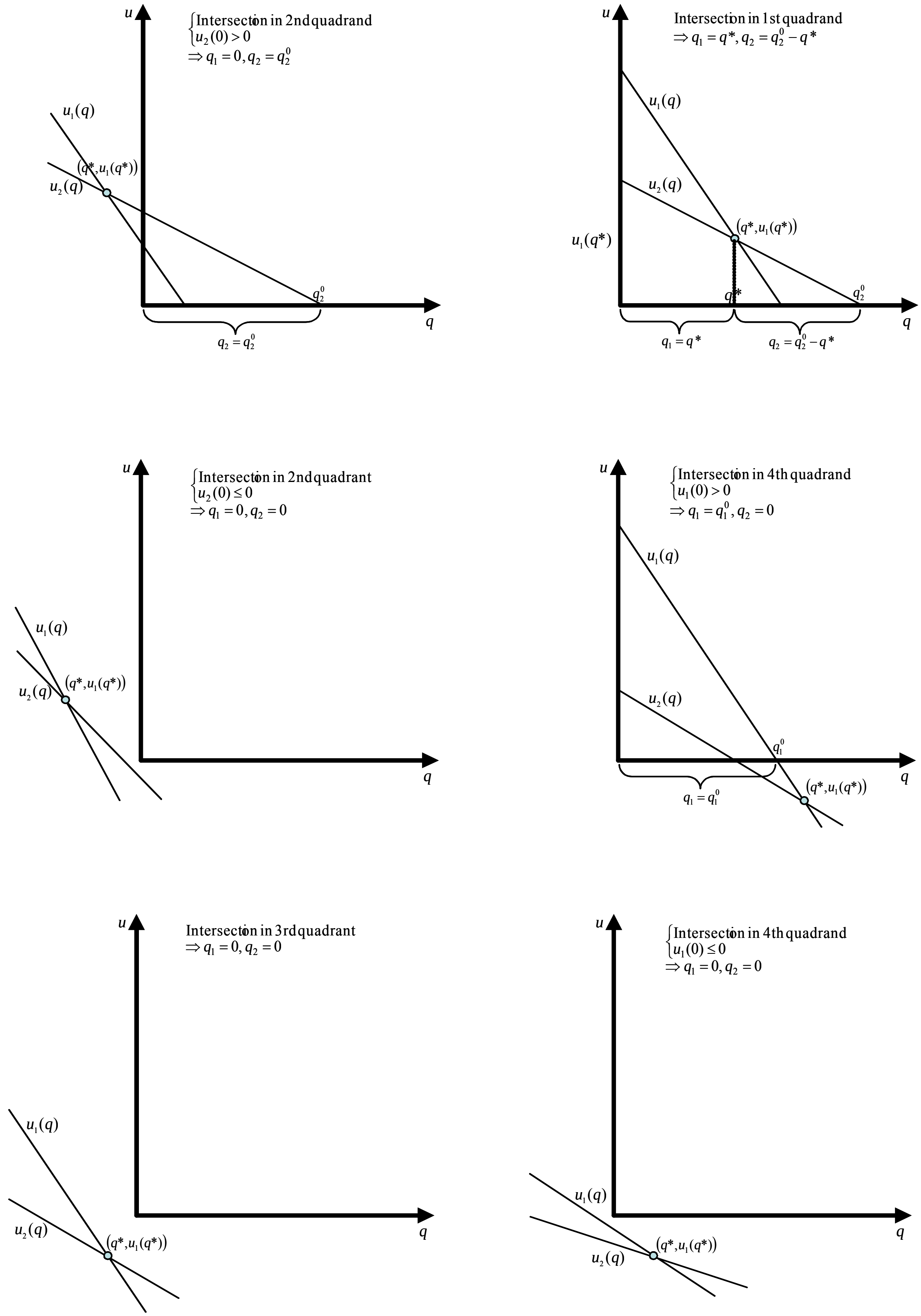

Figure 4. The alternatives as to how the "utility functions" may lie in the coordinate plane.

Table 1. The relationship between prices and quantities.

	$q^* \le 0$	$q^* > 0$
$u_1(q^*) > 0$	$q_1 = 0, q_2 = q_2^0$, if $u_2(0) > 0$ $q_1 = 0, q_2 = 0$, if $u_2(0) \le 0$	$q_1 = q^*, q_2 = q_2^0 - q^*$
$u_1(q^*) \le 0$	$q_1 = 0, q_2 = 0$	$q_1 = q_1^0, q_2 = 0$, if $u_1(0) > 0$ $q_1 = 0, q_2 = 0$, if $u_1(0) \le 0$

The quantities sold by the firms associated with the six alternatives depicted in Figure 4 are summarized in Table 1. The upper-right cell of the table represents a duopolistic situation where both firms are able to sell a positive quantity. In the other three cells, either one firm has monopoly or neither firm is able to sell any products at the prevailing prices.

Substituting q^*, q_1^0 and q_2^0 from expressions (6), (7) and (8) into the quantities q_1 ja q_2 expressed in Table 1 gives, after simplification[3]:

$$(9)\qquad \begin{cases} q_1 = \dfrac{p_1 - p_2 + b - a}{b - a},\ q_2 = \dfrac{p_2 a - p_1 b}{b(b-a)} & \text{for } q^* > 0 \text{ and } u_1(q^*) > 0 \\ q_1 = 0,\ q_2 = \dfrac{b - p_2}{b} & \text{for } q^* \le 0 \text{ and } u_1(q^*) > 0 \text{ and } u_2(0) > 0 \\ q_1 = 0,\ q_2 = 0 & \text{for } q^* \le 0 \text{ and } u_1(q^*) > 0 \text{ and } u_2(0) \le 0 \\ q_1 = 0,\ q_2 = 0 & \text{for } q^* \le 0 \text{ and } u_1(q^*) \le 0 \\ q_1 = \dfrac{a - p_1}{a},\ q_2 = 0 & \text{for } q^* > 0 \text{ and } u_1(q^*) \le 0 \text{ and } u_1(0) > 0 \\ q_1 = 0,\ q_2 = 0 & \text{for } q^* > 0 \text{ and } u_1(q^*) \le 0 \text{ and } u_1(0) \le 0 \end{cases}$$

[3] $q_2^0 - q^* = {}^{b-a)}\dfrac{b - p_2}{b} - {}^{b)}\dfrac{p_1 - p_2 + b - a}{b - a} = \dfrac{b^2 - ba - p_2 b + p_2 a - p_1 b + p_2 b - b^2 + ab}{b(b-a)} = \dfrac{p_2 a - p_1 b}{b(b-a)}$

After combining those alternatives of expression (9), in which both firms sell zero quantity, substituting the simplified[4] inequality conditions and writing the expressions of q_1 and q_2 separately, the equations of expression (9) take the following form:

$$(10)\qquad q_1 = \begin{cases} \dfrac{p_1 - p_2 + b - a}{b-a} & \text{for } p_1 < p_2 + a - b \text{ ja } p_1 > \dfrac{a}{b} p_2 \\ \dfrac{a - p_1}{a} & \text{for } p_1 < p_2 + a - b \text{ ja } p_1 \le \dfrac{a}{b} p_2 \text{ ja } p_1 < a \\ 0 & \text{otherwise} \end{cases}$$

$$(11)\qquad q_2 = \begin{cases} \dfrac{p_2 a - p_1 b}{b(b-a)} & \text{for } p_1 < p_2 + a - b \text{ ja } p_1 > \dfrac{a}{b} p_2 \\ \dfrac{b - p_2}{b} & \text{for } p_1 \ge p_2 + a - b \text{ ja } p_1 > \dfrac{a}{b} p_2 \text{ ja } p_2 < b \\ 0 & \text{otherwise} \end{cases}$$

These expressions give to each combination of prices p_1 and p_2 the respective quantities sold by the firms. Figure 5 depicts how the p_1p_2 coordinate plane is divided by expressions (10) and (11) into areas in which there is either monopoly or duopoly, or a situation in which no products are sold by either company.

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[4] Substitute $u_1(q)$, $u_2(q)$ and q^* with expressions (1), (2) and (6).

$$q^* > 0 \Leftrightarrow \frac{p_1 - p_2 + b - a}{b - a} > 0 \Leftrightarrow p_1 < p_2 + a - b$$

$$u_1(q^*) > 0 \Leftrightarrow a - aq^* - p_1 > 0 \Leftrightarrow a - a\frac{p_1 - p_2 + 1 - a}{1 - a} - p_1 > 0 \Leftrightarrow p_1 > \frac{a}{b} p_2$$

$$u_1(0) > 0 \Leftrightarrow a - a \cdot 0 - p_1 > 0 \Leftrightarrow p_1 < a$$

$$u_2(0) > 0 \Leftrightarrow b - b \cdot 0 - p_2 > 0 \Leftrightarrow p_2 < b$$

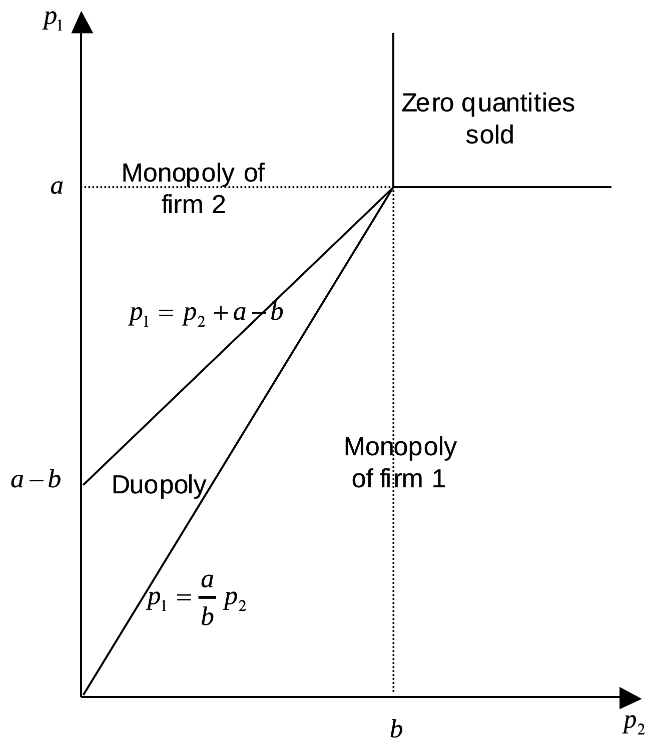

Figure 5. The areas of the p_1p_2 coordinate plane.

2.6 Bertrand equilibrium

Since costs are zero and prices are assumed positive, a positive quantity sold brings a firm a greater profit than selling zero, regardless of what the other firm does. Therefore, in the kind of Bertrand competition considered here, the best response price can never be such that it leads to zero quantity being sold. Thus, it is evident that the Bertrand equilibrium can only lie in the area of the p_1p_2 plane, where the price setting results in positive quantities sold by both firms. It was observed earlier, that this area is constrained by the conditions $p_1 < p_2 + a - b$, $p_1 > ap_2$, $p_1 > 0$ and $p_2 > 0$. In this area, the quantities sold, according to expressions (10) and (11), are

(12) $$q_1 = \frac{p_1 - p_2 + b - a}{b-a},$$ for firm 1, and

(13) $$q_2 = \frac{p_2 a - p_1 b}{b(b-a)},$$ for firm 2.

Since costs are zero, the profits of the companies are

(14) $\pi_1 = p_1 q_1$, for firm 1, and

(15) $\pi_2 = p_2 q_2$, for firm 2,

which when substituting q_1 and q_2 from expressions (12) and (13) take the form

(16) $$\pi_1 = p_1 \frac{p_1 - p_2 + b - a}{b - a},$$ for firm 1, and

(17) $$\pi_2 = p_2 \frac{p_2 a - p_1 b}{b(b - a)},$$ for firm 2.

The first order conditions for maximum profits to the firms, with respect to the prices of their own products, are

(18) $$\frac{\partial \pi_1}{\partial p_1} = \frac{2 p_1 - p_2 + b - a}{b - a} = 0,$$ for firm 1, and

(19) $$\frac{\partial \pi_2}{\partial p_2} = \frac{2 p_2 a - p_1 b}{b(b - a)} = 0,$$ for firm 2.

Solving for prices transforms the first order conditions into

(20) $$p_1 = \frac{p_2 + a - b}{2},$$ for firm 1, and

(21) $$p_2 = \frac{p_1 b}{2a},$$ for firm 2.

The respective second order conditions for maximum are

(22) $$\frac{\partial^2 \pi_1}{\partial p_1^{\,2}} = \frac{2}{b - a} < 0,$$ for firm 1, and

(23) $$\frac{\partial^2 \pi_2}{\partial p_2^{\,2}} = \frac{2a}{b(b - a)} < 0,$$ for firm 2.

Since $a > b$ and $a, b > 0$, it is evident that the second order conditions are satisfied everywhere in the respective domains and thus the first order conditions give the maximums. Therefore, the best response curves for Bertrand competition are

(24) $$p_1 = P_1(p_2) = \frac{p_2 + a - b}{2}, \qquad \text{for firm 1, and}$$

(25) $$p_2 = P_2(p_1) = \frac{p_1 b}{2a}, \qquad \text{for firm 2.}$$

The Bertrand equilibrium is found at the intersection point of these best response curves. Therefore, the following pair of equations has to hold true at the equilibrium:

(26) $$\begin{cases} p_1 = \dfrac{p_2 + a - b}{2} \\ p_2 = \dfrac{p_1 b}{2a} \end{cases}$$

Solving the pair of equations gives the Bertrand equilibrium:

(27) $$\begin{cases} p_1{*} = p_1^B = \dfrac{2a(a-b)}{4a-b} \\ p_2{*} = p_2^B = \dfrac{b(a-b)}{4a-b} \end{cases}$$

The solution is feasible since $p_1{*} = \dfrac{\overbrace{2a}^{>0}\overbrace{(a-b)}^{>0}}{\underbrace{4a-b}_{>0}} > 0$ and $p_2{*} = \dfrac{\overbrace{b}^{>0}\overbrace{(a-b)}^{>0}}{\underbrace{4a-b}_{>0}} > 0$.

The profits in Bertrand equilibrium can be obtained by substituting the equilibrium prices from expression (27) into expressions (16) and (17):

(28) $$\pi_1^B = \frac{2a(a-b)}{4a-b} \frac{\dfrac{2a(a-b)}{4a-b} - \dfrac{b(a-b)}{4a-b} + b - a}{b-a} = (a-b)\left(\frac{2a}{4a-b}\right)^2$$

(29) $$\pi_2^B = \frac{b(a-b)}{4a-b} \frac{\dfrac{b(a-b)}{4a-b}a - \dfrac{2a(a-b)}{4a-b}b}{b(b-a)} = (a-b)\frac{ab}{(4a-b)^2}$$

2.7 Cournot equilibrium

It was observed in last section that since costs are zero and prices are positive, a positive quantity sold brings a firm a greater profit than selling zero, regardless of what the other firm does. For Cournot competition the implication is the same as in the case of Bertrand competition: the equilibrium quantities must be positive, and therefore the prices resulting from the firms setting their quantities must lie in the area of the p_1p_2 plane, which is constrained by the conditions $p_1 < p_2 + a - 1$, $p_1 > ap_2$, $p_1 > 0$ and $p_2 > 0$. In this area, according to expressions (10) and (11), the prices of the firms' products and the quantities sold have the following relationship:

(30) $$\begin{cases} q_1 = \dfrac{p_1 - p_2 + b - a}{b - a} \\ q_2 = \dfrac{p_2 a - p_1 b}{b(b - a)} \end{cases}$$

Solving this pair of equations for p_1 and p_2 gives

(31) $$\begin{cases} p_1 = a - aq_1 - bq_2 \\ p_2 = b - bq_1 - bq_2 \end{cases}$$

Since costs are zero, the profits of the companies are

(32) $\pi_1 = p_1 q_1,$ for firm 1, and

(33) $\pi_2 = p_2 q_2,$ for firm 2,

which when substituting p_1 and p_2 take the form

(34) $\pi_1 = (a - aq_1 - bq_2)q_1,$ for firm 1, and

(35) $\pi_2 = (b - bq_1 - bq_2)q_2,$ for firm 2.

The first order conditions for maximum profits to the firms with respect to the quantities sold by the firms are

(36) $\dfrac{\partial \pi_1}{\partial q_1} = a - 2aq_1 - bq_2 = 0,$ for firm 1, and

(37) $$\frac{\partial \pi_2}{\partial q_2} = b - bq_1 - 2bq_2 = 0, \qquad \text{for firm 2.}$$

Solving for the quantities transforms the first order conditions into

(38) $$q_1 = \frac{a - bq_2}{2a}, \qquad \text{for firm 1, and}$$

(39) $$q_2 = \frac{1 - q_1}{2}, \qquad \text{for firm 2.}$$

The respective second order conditions for maximum are

(40) $$\frac{\partial^2 \pi_1}{\partial {q_1}^2} = -2a < 0, \qquad \text{for firm 1, and}$$

(41) $$\frac{\partial^2 \pi_2}{\partial {p_2}^2} = -2b < 0, \qquad \text{for firm 2.}$$

Since a,b > 0, it is evident that the second order conditions are satisfied everywhere in the respective domains and thus the first order conditions give the maximums. Therefore, the best response curves for Cournot competition are

(42) $$q_1 = Q_1(q_2) = \frac{a - bq_2}{2a}, \qquad \text{for firm 1, and}$$

(43) $$q_2 = Q_2(q_1) = \frac{1 - q_1}{2}, \qquad \text{for firm 2.}$$

The Cournot equilibrium is found at the intersection point of the best response curves. Therefore, the following pair of equations has to hold true at the equilibrium:

(44) $$\begin{cases} q_1 = \dfrac{a - bq_2}{2a} \\ q_2 = \dfrac{1 - q_1}{2} \end{cases}$$

Solving the pair of equations gives the Cournot equilibrium:

(45) $$\begin{cases} q_1{*} = q_1^C = \dfrac{2a-b}{4a-b} \\ q_2{*} = q_2^C = \dfrac{a}{4a-b} \end{cases}$$

The solution is feasible since $q_1{*} = \dfrac{\overbrace{2a-b}^{>0}}{\underbrace{4a-b}_{>0}} > 0$ and $q_2{*} = \dfrac{\overbrace{a}^{>0}}{\underbrace{4a-b}_{>0}} > 0$.

The profits in Cournot equilibrium can be obtained by substituting the equilibrium quantities from expression (45) into expressions (34) and (35):

(46) $$\pi_1^C = \left(a - a\frac{2a-b}{4a-b} - b\frac{a}{4a-b}\right)\frac{2a-b}{4a-b} = a\left(\frac{2a-b}{4a-b}\right)^2$$

(47) $$\pi_2^C = \left(b - b\frac{2a-b}{4a-b} - b\frac{a}{4a-b}\right)\frac{a}{4a-b} = b\left(\frac{a}{4a-b}\right)^2$$

2.8 Summary

The expressions for Bertrand and Cournot equilibria and the respective profits derived in this chapter are summarized in Table 2.

Table 2. Summary of the Bertrand and Cournot equilibria and the respective profits.

	Bertrand competition		**Cournot competition**	
	Firm 1	***Firm 2***	***Firm 1***	***Firm 2***
Equilibrium	$\frac{2a(a-b)}{4a-b}$	$\frac{b(a-b)}{4a-b}$	$\frac{2a-b}{4a-b}$	$\frac{a}{4a-b}$
Profit	$(a-b)\left(\frac{2a}{4a-b}\right)^2$	$(a-b)\frac{ab}{(4a-b)^2}$	$a\left(\frac{2a-b}{4a-b}\right)^2$	$b\left(\frac{a}{4a-b}\right)^2$

3 Nash equilibrium of investment decisions

3.1 The model

In this section the model considered in Chapter 2 is extended to involve product quality-improving R&D investments. A quality improvement is modeled as an upward shift of the respective demand curve by a predefined amount, as is depicted in Figure 6. The probability that a firm gets a quality improvement, i.e., an innovation, depends on the amount it has invested in R&D.

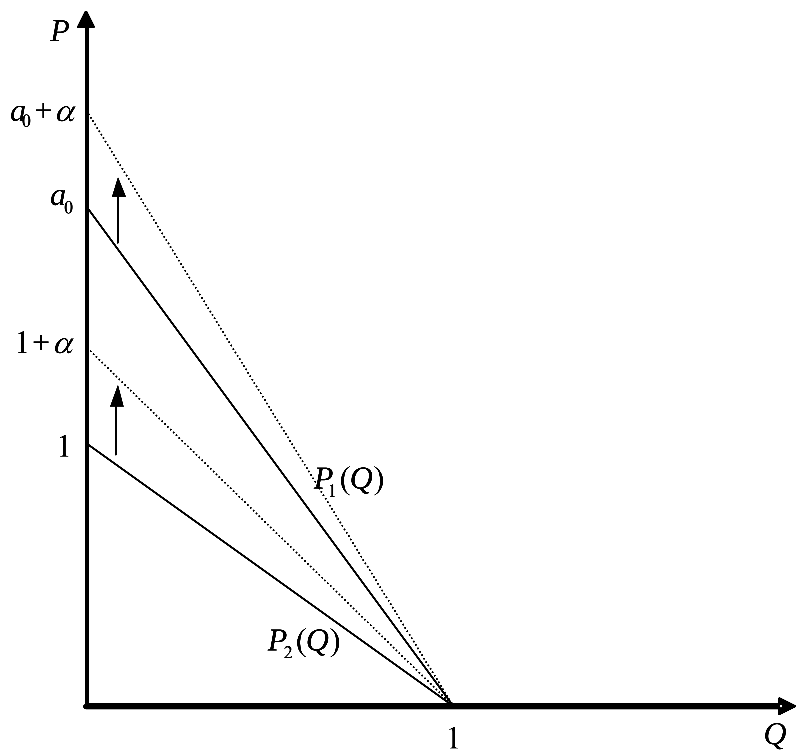

Figure 6. The influence of innovations on product quality.

The model considered here is an investment game involving two firms, firm 1 and firm 2, which act as Cournot or Bertrand (both cases are considered) competitors as described in Chapter 2. For the investment game, four additional assumptions are added to assumptions 1–6 listed in Chapter 2:

7. Firm 1 and firm 2 make the investment decisions (about the sums of money to invest) i_1 and i_2, respectively. Each firm has a certain probability $p(i)$ of obtaining an innovation, and the probability depends on the amount of money invested according to the formula[5]

   $$p(i) = 1 - e^{-\mu i},$$

   where $\mu > 0$. An innovation can be obtained by both firms, or by just one of the firms, or by neither of the firms.

8. It is assumed that initially the values of parameters a and b (defined in Chapter 2) are $a = a_0 > 1$ and $b = 1$. If a firm obtains an innovation, the quality of its product increases by a certain amount, which is considered constant. The increase in quality is modeled as increasing each consumer's willingness to pay for the firm's product, that is, increasing the intercept parameters a and b by $\alpha > 0$:

   - If firm 1 obtains an innovation, the parameter a increases from a_0 to $a_0 + \alpha$.
   - If firm 2 obtains an innovation, the parameter b increases from 1 to $1 + \alpha$.

   The quality improvement considered here shifts up the respective firm's demand curve in such a way that the intercept of the demand curve rises by the amount α, and the intersection point with the horizontal q-axis remains at $q = 1$, as before. The demand curves continue being linear in q. The influence of a quality improvement is depicted in Figure 6.

9. The firms act either as Cournot or Bertrand competitors after the investment game has taken place. These two forms of competition are considered as two separate cases of the investment game.

10. The firms are assumed to be risk-neutral. Therefore, each firm strives to maximize the expected value of its profit, which consists of the revenue gained after the innovation has taken place and the cost of the investment made by the firm.

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[5] $\frac{\partial p(i)}{\partial i} > 0$, $\frac{\partial^2 p(i)}{\partial i^2} < 0$ and $\lim_{i \to \infty} p(i) = 1$

3.2 Expected profits

The investment game has for alternative outcomes. These outcomes, labeled A, B, C and D, are the following:

A. Both firms obtain the innovation

B. Only firm 1 obtains the innovation

C. Only firm 2 obtains the innovation

D. Neither firm obtains the innovation

3.2.1 Determining the probabilities

The probability $p(i)$, that a firm obtains the innovation depends on the size of the investment it has made, as defined in Assumption 7. Therefore, the probabilities associated with each of these four cases depend on the investments made by both firms. The probabilities of the four cases, denoted with P^A, P^B, P^C, and P^D, are easy to calculate:[6]

(48) $$P^A = p(i_1)p(i_2) = \left(1-e^{-\mu i_1}\right)\left(1-e^{-\mu i_2}\right)$$

(49) $$P^B = p(i_1)\left(1-p(i_2)\right) = \left(1-e^{-\mu i_1}\right)\left(1-\left(1-e^{-\mu i_2}\right)\right) = \left(1-e^{-\mu i_1}\right)e^{-\mu i_2}$$

(50) $$P^C = \left(1-p(i_1)\right)p(i_2) = \left(1-\left(1-e^{-\mu i_1}\right)\right)\left(1-e^{-\mu i_2}\right) = e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)$$

(51) $$P^D = \left(1-p(i_1)\right)\left(1-p(i_2)\right) = \left(1-\left(1-e^{-\mu i_1}\right)\right)\left(1-\left(1-e^{-\mu i_2}\right)\right) = e^{-\mu i_1}e^{-\mu i_2}$$

3.2.2 Determining the profits

The profits associated with the cases depend on whether Bertrand or Cournot competition is considered. The profits of firm 1 and firm 2 in both forms of competition were expressed as

[6] Since the events are independent, the probability of two events happening is the product of their individual probabilities. The probability of a certain event not happening is the probability of the event, subtracted from 1.

functions of a and b in Table 2 of last chapter. The form of competition is irrelevant to the algebra in the following sections. Therefore, the same expressions are used in the following sections to represent the firms' profits under both forms of competition:

(52) $\pi_1(a,b)$, for Firm 1, and

(53) $\pi_2(a,b)$, for Firm 2.

Case A: Both firms obtain the innovation

As is declared in Assumption 8, if both firms obtain the innovation, parameters a and b obtain the values

$a = a_0 + \alpha$, and $b = 1 + \alpha$.

The condition $a > b$ is satisfied, since $a_0 > 1$. Therefore, the profits of the firms are

(54) $\pi_1^A = \pi_1(a,b) = \pi_1(a_0 + \alpha, 1 + \alpha)$, and

(55) $\pi_2^A = \pi_2(a,b) = \pi_2(a_0 + \alpha, 1 + \alpha)$.

Case B: Only firm 1 obtains the innovation

If only firm 1 obtains the innovation, parameters a and b obtain the values

$a = a_0 + \alpha$, and $b = 1$.

Since only a increases, and $a_0 > 1$, the condition $a > b$ is satisfied. Therefore, the profits of the firms are

(56) $\pi_1^B = \pi_1(a,b) = \pi_1(a_0 + \alpha, 1)$, and

(57) $\pi_2^B = \pi_2(a,b) = \pi_2(a_0 + \alpha, 1)$.

Case C: Only firm 2 obtains the innovation

If only firm 2 obtains the innovation, the parameters a and b obtain the values

$a = a_0$,   and $b = 1 + \alpha$.

However, if α is large enough, it is possible for b to obtain a value so large that the condition $a > b$ is not satisfied, and the expressions for equilibria calculated in Chapter 2 cannot be used. It turns out, however, that it is possible to get around this problem. **First, consider the case where the condition $a > b$ is satisfied**. Here the profits can be determined in the same way as before:

(58) $\pi_1^C = \pi_1(a,b) = \pi_1(a_0, 1+\alpha)$,   and

(59) $\pi_2^C = \pi_2(a,b) = \pi_2(a_0, 1+\alpha)$.

Then consider the case where $a < b$,[7] that is, $a_0 < 1 + \alpha$. In this case, firm 2's products are superior in quality to those of firm 1. Therefore, in order to be able to use the results of Chapter 2, the firms have to be temporarily swapped. This is done with the following substitution:

(60) $a' = b$,   and

(61) $b' = a$.

The condition $a' > b'$ is satisfied and the profits of the firms can be calculated[8]:

(62) $\pi_1^C = \pi_2(a', b') = \pi_2(b, a)$,   and

(63) $\pi_2^C = \pi_1(a', b') = \pi_1(b, a)$.

Summarizing the results from both cases gives:

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[7] The expressions of profit derived in last chapter can also handle the case a = b correctly. Therefore, the condition $a_0 \geq 1 + \alpha$ is used instead of $a_0 > 1 + \alpha$ in order for the expressions to be complete.

[8] Remember, the profit function π_1 describes the profit of the firm producing the higher-quality product, while π_2 describes the profit of the other firm. Therefore, since in the case considered here, firm 1 is the producer of the lower-quality product, the profit of firm 1 is calculated using the profit function π_2 and the profit of firm 2 is calculated using the profit function π_1.

(64) $$\pi_1^C = \begin{cases} \pi_1(a,b), & \text{for } a \geq b \\ \pi_2(b,a), & \text{for } a < b \end{cases}$$ and

(65) $$\pi_2^C = \begin{cases} \pi_2(a,b), & \text{for } a \geq b \\ \pi_1(b,a), & \text{for } a < b \end{cases}$$

Substituting *a* and *b* gives:

(66) $$\pi_1^C = \begin{cases} \pi_1(a_0,1+\alpha), & \text{for } a_0 \geq 1+\alpha \\ \pi_2(1+\alpha,a_0), & \text{for } a_0 < 1+\alpha \end{cases}$$ and

(67) $$\pi_2^C = \begin{cases} \pi_2(a_0,1+\alpha), & \text{for } a_0 \geq 1+\alpha \\ \pi_1(1+\alpha,a_0), & \text{for } a_0 < 1+\alpha \end{cases}$$

Case D: Neither firm obtains the innovation

If neither firm obtains the innovation, the parameters *a* and *b* retain their initial values

$a = a_0$, and $b = 1$.

Since $a_0 > 1$, the condition $a > b$ is satisfied. Therefore, the profits of the firms are

(68) $$\pi_1^D = \pi_1(a_0,1),$$ and

(69) $$\pi_2^D = \pi_2(a_0,1).$$

3.2.3 Expected profits

The profit functions defined above consider the profits that the firms get after the investments have been made, but do not take into account the costs of the investments. When taking into account the probabilities and profits associated with the four cases considered above, as well as the costs of the investments, the expected profit functions take the form

(70) $$\Pi_1^e = P^A\pi_1^A + P^B\pi_1^B + P^C\pi_1^C + P^D\pi_1^D - i_1,$$ for firm 1, and

(71) $$\Pi_2^e = P^A\pi_2^A + P^B\pi_2^B + P^C\pi_2^C + P^D\pi_2^D - i_2,$$ for firm 2.

Substituting the probabilities from expressions (48)–(51) into expressions (70) and (71) gives the following expected profits:

(72) $$\Pi_1^e = \left(1-e^{-\mu i_1}\right)\left(1-e^{-\mu i_2}\right)\pi_1^A + \left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_1^B + e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_1^C + e^{-\mu i_1}e^{-\mu i_2}\pi_1^D - i_1, \text{ and}$$

(73) $$\Pi_2^e = \left(1-e^{-\mu i_1}\right)\left(1-e^{-\mu i_2}\right)\pi_2^A + \left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_2^B + e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_2^C + e^{-\mu i_1}e^{-\mu i_2}\pi_2^D - i_2.$$

3.3 The best response functions for investment

The firms' best response functions for investment decisions are denoted with

$I_1(i_2)$, for firm 1, and

$I_2(i_1)$, for firm 2,

where $I_1, I_2 \geq 0$, since investments cannot be negative. The firms are risk-neutral according to assumption 10. Thus, a firm's best response function can be found by maximizing the expected value of the firm's profits, for each investment decision of the other firm. The maximization problem is constrained by the fact that only non-negative investment decisions are possible. Thus, the problem takes the following form:[9]

(74) $$I_1(i_2) = Arg\max_{i_1} \Pi_1^e, \quad \text{where } i_1 \geq 0, i_2 \geq 0, \text{ and}$$

(75) $$I_2(i_1) = Arg\max_{i_2} \Pi_2^e, \quad \text{where } i_1 \geq 0, i_2 \geq 0.$$

[9] Since the variables i_1 and i_2 are restricted to non-negative values, the optimal values of these variables cannot be negative. Therefore, the best response functions I_1 and I_2 can only take non-negative arguments and obtain non-negative values.

3.3.1 The best response function of firm 1

The first order condition for maximum, in the maximization problem faced by firm 1, is

$$(76) \qquad \frac{\partial \Pi_1^e}{\partial i_1} = 0 .$$

After substituting Π_1^e from expression (70) and differentiating it with respect to i_1, the condition takes the form

$$(77) \qquad \mu e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_1^A + \mu e^{-\mu i_1} e^{-\mu i_2}\pi_1^B - \mu e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_1^C - \mu e^{-\mu i_1} e^{-\mu i_2}\pi_1^D - 1 = 0 .$$

Solving this for i_1, and writing the solution as a function of i_2 denoted with $J_1(i_2)$,[10] gives

$$(78) \qquad J_1(i_2) = i_1{*} = \frac{\ln \mu\left(e^{-\mu i_2}(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D) + (\pi_1^A - \pi_1^C)\right)}{\mu},$$

assuming that $e^{-\mu i_2}(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D) + (\pi_1^A - \pi_1^C) > 0$.[11]

The second order condition for global maximum is

$$(79) \qquad \frac{\partial^2 \Pi_1^e}{\partial {i_1}^2} < 0 \text{ for all } i_1.$$

After substituting Π_1^e from expression (70) and differentiating it twice with respect to i_1, the second order condition takes the form

$$(80) \qquad -\mu^2 e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_1^A - \mu^2 e^{-\mu i_1} e^{-\mu i_2}\pi_1^B + \mu^2 e^{-\mu i_1}\left(1-e^{-\mu i_2}\right)\pi_1^C + \mu^2 e^{-\mu i_1} e^{-\mu i_2}\pi_1^D < 0,$$

which simplifies to

$$(81) \qquad e^{-\mu i_2}\left(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D\right) + \left(\pi_1^A - \pi_1^C\right) > 0 .$$

---

[10] Unlike the best response functions, $J_1(i_2)$ is allowed to take negative arguments and obtain negative values.

[11] It will be shown that this condition is satisfied if the second order condition for maximum is satisfied.

If the value of i_2 satisfies this condition, the condition holds for all values of i_1. Therefore, the first order condition gives the global maximum, as long as (i) the second order condition is satisfied, and (ii) the maximum is feasible, that is, $i_1^* = J_1(i_2) \geq 0$. If the second order condition is satisfied, but the maximum is unfeasible, $i_1^* = 0$ is the solution to the maximization problem[12]. For the values of i_2, which do not satisfy the second order condition, the optimal value of i_1 lies either at zero or infinity.

Since for $i_1 = 0$, $\Pi_1^e = \underbrace{\left(1 - e^{-\mu i_2}\right)}_{\geq 0} \underbrace{\pi_1^C}_{\geq 0} + \underbrace{e^{-\mu i_2}}_{>0} \underbrace{\pi_1^D}_{\geq 0} \geq 0$, and

$$\lim_{i_1 \to \infty} \Pi_1^e = \lim_{i_1 \to \infty} \underbrace{\left(1 - e^{-\mu i_1}\right)}_{\to 1} \underbrace{\left(1 - e^{-\mu i_2}\right)}_{\geq 0} \underbrace{\pi_1^A}_{\geq 0} + \underbrace{\left(1 - e^{-\mu i_1}\right)}_{\to 1} \underbrace{e^{-\mu i_2}}_{>0} \underbrace{\pi_1^B}_{\geq 0} + \underbrace{e^{-\mu i_1}}_{\to 0} \underbrace{\left(1 - e^{-\mu i_2}\right)}_{\geq 0} \underbrace{\pi_1^C}_{\geq 0} + \underbrace{e^{-\mu i_1}}_{\to 0} \underbrace{e^{-\mu i_2}}_{>0} \underbrace{\pi_1^D}_{\geq 0} - \underbrace{i_1}_{\to \infty} = -\infty,$$

the objective function obtains its maximum value at $i_1 = 0$, if the second order condition for maximum is not satisfied. Based on these results, the best response function of firm 1 can be written as follows:

$$(82) \qquad I_1(i_2) = \begin{cases} J_1(i_2) & \text{for } e^{-\mu i_2}\left(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D\right) + \left(\pi_1^A - \pi_1^C\right) > 0 \text{ and } J_1(i_2) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The conditions of expression (82) can be simplified further. Substituting $J_1(i_2)$ from expression (78) into the condition $J_1(i_2) \geq 0$ transforms the condition into

$$(83) \qquad \frac{\ln \mu\left(e^{-\mu i_2}(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D) + (\pi_1^A - \pi_1^C)\right)}{\mu} \geq 0,$$

which is equivalent to

$$(84) \qquad e^{-\mu i_2}(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D) + (\pi_1^A - \pi_1^C) \geq \frac{1}{\mu}.$$

---

[12] The objective function is concave if its second order condition is satisfied. If, in addition, the maximum is unfeasible, i.e., if the maximum lies on the negative i_1 axis, the first derivative is negative everywhere on the non-negative i_1 axis. Thus, the objective function is decreasing in i_1, if $i_1 \geq 0$. Therefore, the solution to the optimization problem is $i_1 = 0$.

Now it can be seen that since $\mu > 0$, the first condition on the first line of expression (82) is rendered redundant by the second condition on the same line. Therefore expression (82) simplifies to

$$(85) \qquad I_1(i_2) = \begin{cases} J_1(i_2) & \text{for } e^{-\mu i_2}\left(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D\right) + \left(\pi_1^A - \pi_1^C\right) > \dfrac{1}{\mu} \\ 0 & \text{otherwise} \end{cases}$$

3.3.2 The best response function of firm 2

The deriving of the best response function of firm 2 is almost identical to that of firm 1. Therefore it is left out from the main text and presented in Appendix A. The best response function of firm 2 is

$$(86) \qquad I_2(i_1) = \begin{cases} J_2(i_1) & \text{for } e^{-\mu i_1}\left(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D\right) + \left(\pi_2^A - \pi_2^B\right) > \dfrac{1}{\mu} \\ 0 & \text{otherwise} \end{cases},$$

where

$$(87) \qquad J_2(i_1) = \frac{\ln \mu\left(e^{-\mu i_1}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B)\right)}{\mu}.$$

3.4 *Nash equilibrium of investments*

A Nash equilibrium of investments is defined as any point where the best response curves (i.e. the best response functions graphed in the $i_1 i_2$ plane) intersect, that is, any pair of values (i_1, i_2) which satisfies the condition

$$(88) \qquad \begin{cases} i_1 = I_1(i_2) \\ i_2 = I_2(i_1) \end{cases}.$$

It turns out that there may be one or more intersection points, and thus, equilibria. They may lie either in the positive quadrant of the $i_1 i_2$ plane, or at one of its axes. An example of the best response curves and the equilibrium is depicted in Figure 7.

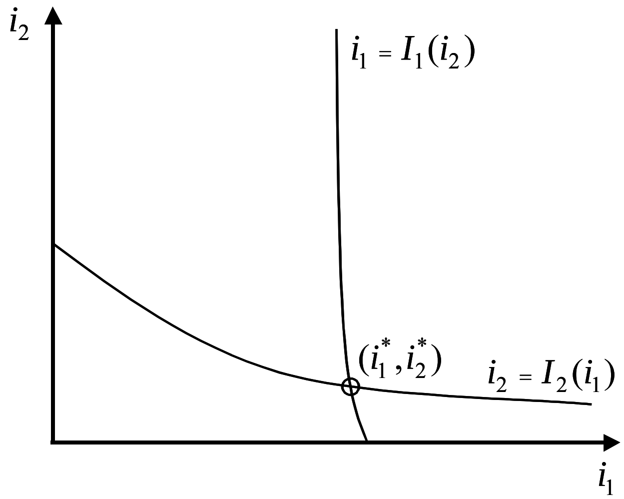

Figure 7. An example of the best response curves and the equilibrium of the investment game.

The intersection points can be calculated by solving the pair of equations (88). Since investments must be non-negative, all feasible equilibria must have non-negative values of i_1 and i_2. Thus, at least one of the following four conditions is satisfied in every feasible equilibrium:

- $i_1 > 0$ and $i_2 > 0$ in the equilibrium
- $i_1 = 0$, $i_2 \geq 0$ in the equilibrium
- $i_2 = 0$, $i_1 \geq 0$ in the equilibrium
- $i_1 = 0$ and $i_2 = 0$ in the equilibrium

Next, these four conditions are written in terms of the parameters of the investment game. It should be noted that some equilibria satisfy more than one of these conditions.

Case: $i_1 > 0$ and $i_2 > 0$ in the equilibrium

Expressions (85) and (86) imply, that both best response functions can be positive only when $I_1(i_2) = J_1(i_2)$ and $I_2(i_1) = J_2(i_1)$. Thus, if equilibria exist, for which both coordinates are positive, they are solutions to the following pair of equations:

$$(89) \qquad \begin{cases} i_1 = J_1(i_2) \\ i_2 = J_2(i_1) \end{cases}$$

After substituting $J_1(i_2)$ and $J_2(i_1)$ form expressions (78) and (87), the pair of equations takes the following form:

$$(90) \qquad \begin{cases} i_1 = \dfrac{\ln \mu\left(e^{-\mu i_2}(-\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D) + (\pi_1^A - \pi_1^C)\right)}{\mu} \\ i_2 = \dfrac{\ln \mu\left(e^{-\mu i_1}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B)\right)}{\mu} \end{cases}$$

The expression is easier to handle after the following substitutions:

$E_1 = -\pi_1^A + \pi_1^B + \pi_1^C - \pi_1^D \qquad F_1 = \pi_1^A - \pi_1^C$

$E_2 = -\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D \qquad F_2 = \pi_2^A - \pi_2^B$

Rewriting the pair of equations (90) gives

$$(91) \qquad \begin{cases} i_1 = \dfrac{\ln \mu\left(e^{-\mu i_2} E_1 + F_1\right)}{\mu} \\ i_2 = \dfrac{\ln \mu\left(e^{-\mu i_1} E_2 + F_2\right)}{\mu} \end{cases}$$

After substituting the expression of i_2 into the expression of i_1, and simplifying, the pair of equations (91) transforms into

$$(92) \qquad F_2\left(e^{\mu i_1}\right)^2 + \left(E_2 - E_1 - \mu F_1 F_2\right)e^{\mu i_1} - \mu F_1 E_2 = 0$$

The expression is easier to handle after a few substitutions:

$A = F_2$

$B = E_2 - E_1 - \mu F_1 F_2$

$C = -\mu F_1 E_2$.

After these substitutions, equation (92) takes the following form:

$$(93) \qquad A\left(e^{\mu i_1}\right)^2 + Be^{\mu i_1} + C = 0$$

Using the quadratic formula to solve equation (93) gives[13]

$$(94) \qquad e^{\mu i_1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

In order for the algebra encountered later to be simpler, the two solutions are substituted with R^A and R^B, representing the solutions with positive and negative signs before the square root, respectively. Thus, expression (94) takes the following form:

$$(95) \qquad e^{\mu i_1} = \begin{cases} \dfrac{-B + \sqrt{B^2 - 4AC}}{2A} = R^A & \text{or} \\ \dfrac{-B - \sqrt{B^2 - 4AC}}{2A} = R^B & \end{cases}$$

Solving expression (95) for i_1 gives

$$(96) \qquad i_1 = \begin{cases} \dfrac{\ln\left(R^A\right)}{\mu} = i_1^A & \text{or} \\ \dfrac{\ln\left(R^B\right)}{\mu} = i_1^B & \end{cases}$$

Substituting i_1 from expression (96) into the expression of i_2 in the original pair of equations (90) results in two solution values of i_2. These solutions are denoted with i_2^A and i_2^B, corresponding to the solution values i_1^A and i_1^B, respectively. After simplification, the solution to the original pair of equations (90) takes the following form:

$$(97) \qquad (i_1, i_2) = \begin{cases} \left(i_1^A, i_2^A\right) = \left(\dfrac{\ln\left(R^A\right)}{\mu}, \dfrac{\ln \mu\left(\left(R^A\right)^{-1} E_2 + F_2\right)}{\mu} \right) & \text{or} \\ \left(i_1^B, i_2^B\right) = \left(\dfrac{\ln\left(R^B\right)}{\mu}, \dfrac{\ln \mu\left(\left(R^B\right)^{-1} E_2 + F_2\right)}{\mu} \right) & \end{cases}$$

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[13] Assuming that $B^2 - 4AC \geq 0$ and $A \neq 0$. The first condition will be returned to below. The second condition will not cause problems in this treatment of the investment game.

Solutions exist, if and only if the discriminant of the quadratic formula (94) is non-negative, that is $B^2 - 4AC \geq 0$. In addition, a solution is feasible if and only if it satisfies the condition associated with the case considered here, i.e. the case, in which $i_1 > 0$ and $i_2 > 0$. Therefore, the following conditions apply:

- (i_1^A, i_2^A) is an existing, feasible solution, if and only if $B^2 - 4AC \geq 0$, $i_1^A > 0$, and $i_2^A > 0$
- (i_1^B, i_2^B) is an existing, feasible solution, if and only if $B^2 - 4AC \geq 0$, $i_1^B > 0$, and $i_2^B > 0$

After substituting the expressions of i_1^A, i_2^A, i_1^B and i_2^B from expression (97), and simplifying, these conditions take the following form:

- The necessary and sufficient conditions for the existence of the equilibrium $(i_1^A, i_2^A) > 0$:[14]

(98) $$B^2 - 4AC \geq 0,\ R^A > 1,\ \text{and}\ \left(R^A\right)^{-1} E_2 + F_2 > \frac{1}{\mu}$$

- The necessary and sufficient conditions for the existence of the equilibrium $(i_1^B, i_2^B) > 0$:

(99) $$B^2 - 4AC \geq 0,\ R^B > 1,\ \text{and}\ \left(R^B\right)^{-1} E_2 + F_2 > \frac{1}{\mu}$$

If both of the conditions (98) and (99) are satisfied, there are two equilibria, in which both coordinates are positive. If only one of the two is satisfied, there is only one such equilibrium. If neither condition is satisfied, then no such equilibrium exists.

Case: $i_1 = 0$ and $i_2 \geq 0$ in the equilibrium

Substituting the lower equation of the pair of equations (88) into the upper one gives the following equation:

(100) $$i_1 = I_1(I_2(i_1)).$$

Equation (100) holds if and only if i_1 is an equilibrium investment of firm 1. Thus, there exists an equilibrium, in which $i_1 = 0$, if and only if

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[14] The > sign means, in the case of a coordinate pair, that both coordinates are positive.

(101) $I_1(I_2(0)) = 0$.

In this equilibrium, i_2 = $I_2(0)$, which is non-negative, since I_2 is non-negative by definition. Therefore, expression (101) is a sufficient and necessary condition for the existence of an equilibrium, in which i_1 = 0 and $i_2 \geq 0$.

Case: i_2 = 0 and $i_1 \geq 0$ in the equilibrium

Substituting the upper equation of the pair of equations (88) into the lower one gives the following equation:

(102) $i_2 = I_2(I_1(i_2))$

Equation (102) holds if and only if i_2 is an equilibrium investment of firm 2. Thus, there exists an equilibrium, in which i_2 = 0, if and only if

(103) $I_2(I_1(0)) = 0$.

In this equilibrium, i_1 = $I_1(0)$, which is non-negative, since I_1 is non-negative by definition. Therefore, expression (103) is a sufficient and necessary condition for the existence of an equilibrium, in which i_2 = 0 and $i_1 \geq 0$.

Case: i_1 = 0 and i_2 = 0 in the equilibrium

The coordinate pair $(0,0)$ is an equilibrium if and only if it satisfies the definition of an equilibrium, i.e. the pair of equations (88). Therefore, the sufficient and necessary condition for the existence of an equilibrium, in which i_1 = 0 and i_2 = 0, is

(104) $I_1(0) = 0$ and $I_2(0) = 0$.

3.5 Conditions characterizing the equilibrium

There are three parameters that affect the outcome of the investment game: a_0, α and μ. The sufficient and necessary conditions for the existence of different kinds of equilibria were derived in Section 3.4. These conditions are summarized in Table 3, and they have been given the labels C1–C5. It should be noted, that these conditions could, in theory, be written in terms of the parameters a_0, α and μ, but the expressions would then be very complex.

Table 3. Summary of the conditions derived in Section 3.4.

Condition C1: There exists an equilibrium $\left(i_1^A, i_2^A\right) > 0$	$B^2 - 4AC \geq 0$, $R^A > 1$, and $\left(R^A\right)^{-1} E_2 + F_2 > \frac{1}{\mu}$
Condition C2: There exists an equilibrium $\left(i_1^B, i_2^B\right) > 0$	$B^2 - 4AC \geq 0$, $R^B > 1$, and $\left(R^B\right)^{-1} E_2 + F_2 > \frac{1}{\mu}$
Condition C3: There exists an equilibrium $\left(0, i_2\right)$	$I_1(I_2(0)) = 0$
Condition C4: There exists an equilibrium $\left(i_1, 0\right)$	$I_2(I_1(0)) = 0$
Condition C5: There exists an equilibrium $\left(0, 0\right)$	$I_1(0) = 0$ and $I_2(0) = 0$.

Logical combinations of the conditions presented in Table 3 are used to determine the type of the equilibrium of the investment game. This characterization of the equilibrium is presented in Table 4 as a set of logical expressions. It is explained in the footnotes how the logical expressions in the right column of Table 4 have been arrived at. The logical operators used in the logical expressions of the table are *and* ($\wedge$), *or* ($\vee$), and *negation* ($\neg$).

Table 4. Conditions that characterize the equilibrium.

A verbal description of the type of equilibrium	**A logical expression that determines the type of equilibrium**
There exists exactly one equilibrium and both firms invest positive amounts in it.[15]	$((C1 \wedge \neg C2) \vee (C2 \wedge \neg C1)) \wedge \neg(C3 \vee C4)$
There exists exactly one equilibrium. In this equilibrium, firm 1 invests a positive amount, and firm 2 invests zero.[16]	$C4 \wedge \neg(C1 \vee C2 \vee C3)$
There exists exactly one equilibrium. In this equilibrium firm 2 invests a positive amount, and firm 1 invests zero.[17]	$C3 \wedge \neg(C1 \vee C2 \vee C4)$
There exists exactly one equilibrium, and both firms invest zero in it.[18]	$C5 \wedge \neg(C1 \vee C2)$
There are several equilibria.[19]	If none of the conditions above hold true.

[15] Of the two possible equilibria, in which both coordinates are positive, exactly one exists, and there are no equilibria on the axes.

[16] There is an equilibrium, in which $i_2 = 0$, and there are no equilibria having two positive coordinates, nor any equilibria, in which $i_1 = 0$.

[17] There is an equilibrium, in which $i_1 = 0$, and there are no equilibria having two positive coordinates, nor any equilibria, in which $i_2 = 0$.

[18] There is an equilibrium at the origin, and there are no equilibria with two positive coordinates. Since there can be one equilibrium at most on each axis, and the origin belongs to both axes, the equilibrium at the origin is the only equilibrium, if the equilibria with two positive coordinates are excluded.

[19] All alternatives for unique equilibria have been examined in the first four cases. If none of the conditions associated with these four cases holds true, then there cannot be a unique equilibrium, and thus there must be several of them (there is always at least one equilibrium, since the best response curves are continuous and defined in the value range $[0,\infty[$).

4 Results

The algebra involved with the expressions discussed in last chapter is far too complex for algebraic analysis of the equilibrium, i.e., it is not reasonable to write the conditions of Table 4 as expressions of the parameters a_0, α and μ. However, information about the outcomes of the model can still be extracted by numerical analysis. Numerical computations done by computer are used here in order to obtain information about how the values of the three parameters affect the equilibrium of investments. The computations are performed separately for the cases of Bertrand and Cournot competition. The analyses of these two cases differ only in that the functions π_1 and π_2, which determine each firm's profit from the Cournot/Bertrand game, are different in the two forms of competition.

4.1 The numerical computations

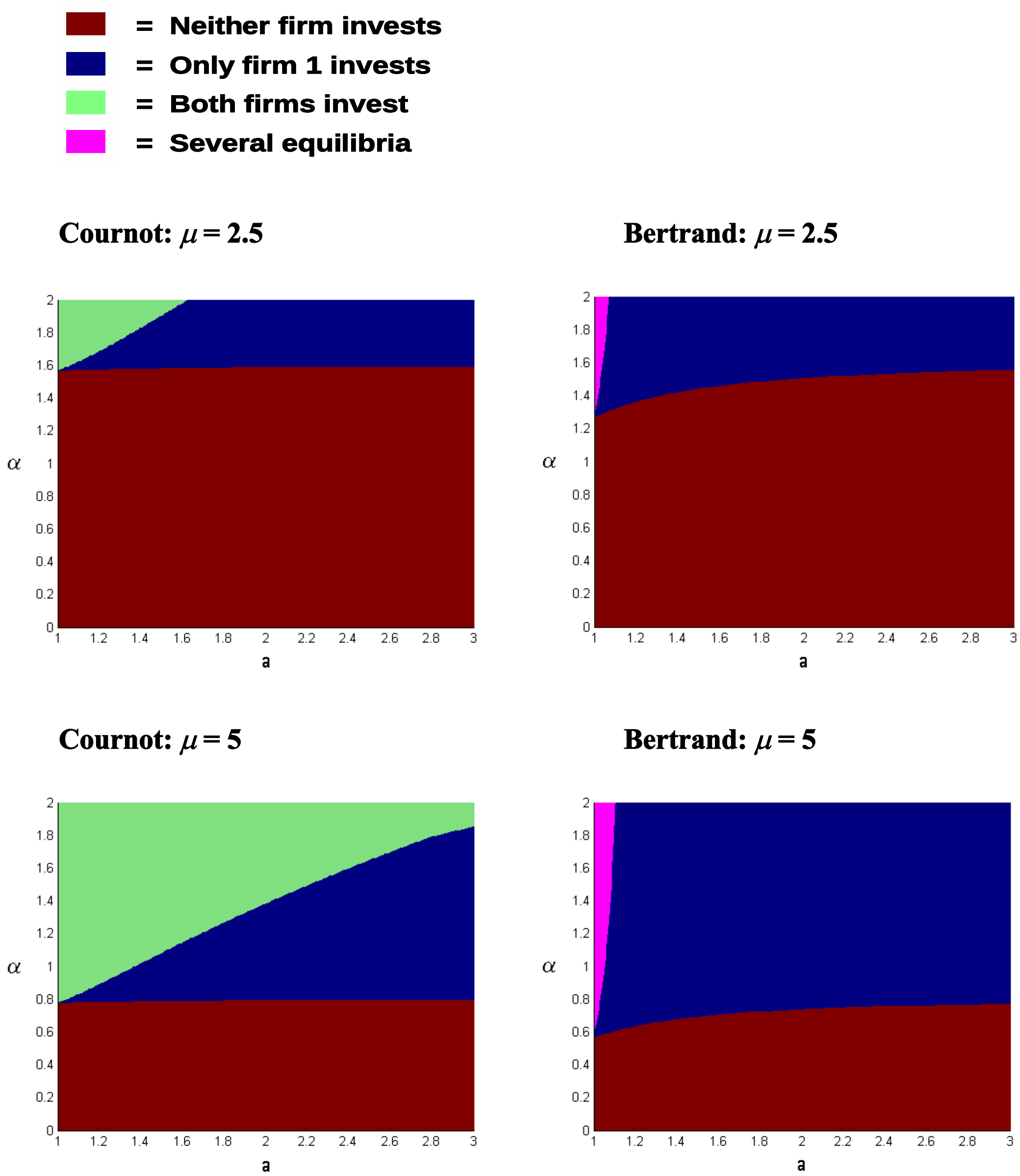
The first part of the numerical analysis involved the numerical computation of the logical expressions of Table 4 for certain combinations of the three parameters. These computations had to be performed for a large number of combinations of the parameters, in order to get an accurate view on the parameters' influence on the investment behavior of the two firms.

A computer program in the *C programming language* was created to perform the actual numerical computations. This program took the value of the parameter μ as an input and calculated, for different values of the parameters a_0 and α,[20] whether the combination of parameter values (a_0,α,μ) resulted in investments by (i) neither firm, (ii) only firm 1, (iii) only firm 2, (iv) both firms, or in (v) several equilibria. The output of the program consisted of (a_0,α) coordinate pairs and for each of these coordinate pairs the respective investment behavior of the firms (coded as an integer value). The output of the program was then loaded into the mathematical modeling software *Matlab*, which was used to plot diagrams based on the output. These diagrams are presented and discussed in the following section.

[20] The values of the parameters a and α were selected at even intervals from the value ranges $a_0 \in [1,3]$ and $\alpha \in [0,2]$.

4.2 Analysis of the output of the numerical computations

Figure 8 shows the output of the numerical computations, plotted as diagrams. The figure illustrates the firms' investment behavior in equilibrium, at different values of the parameters a_0, α and μ. The value of the parameter μ is indicated above each diagram in the figure, as well as whether the diagram in question illustrates the case of Bertrand or Cournot competition. Different colors are used to indicate the kind of investment behavior associated with each combination of the parameters.



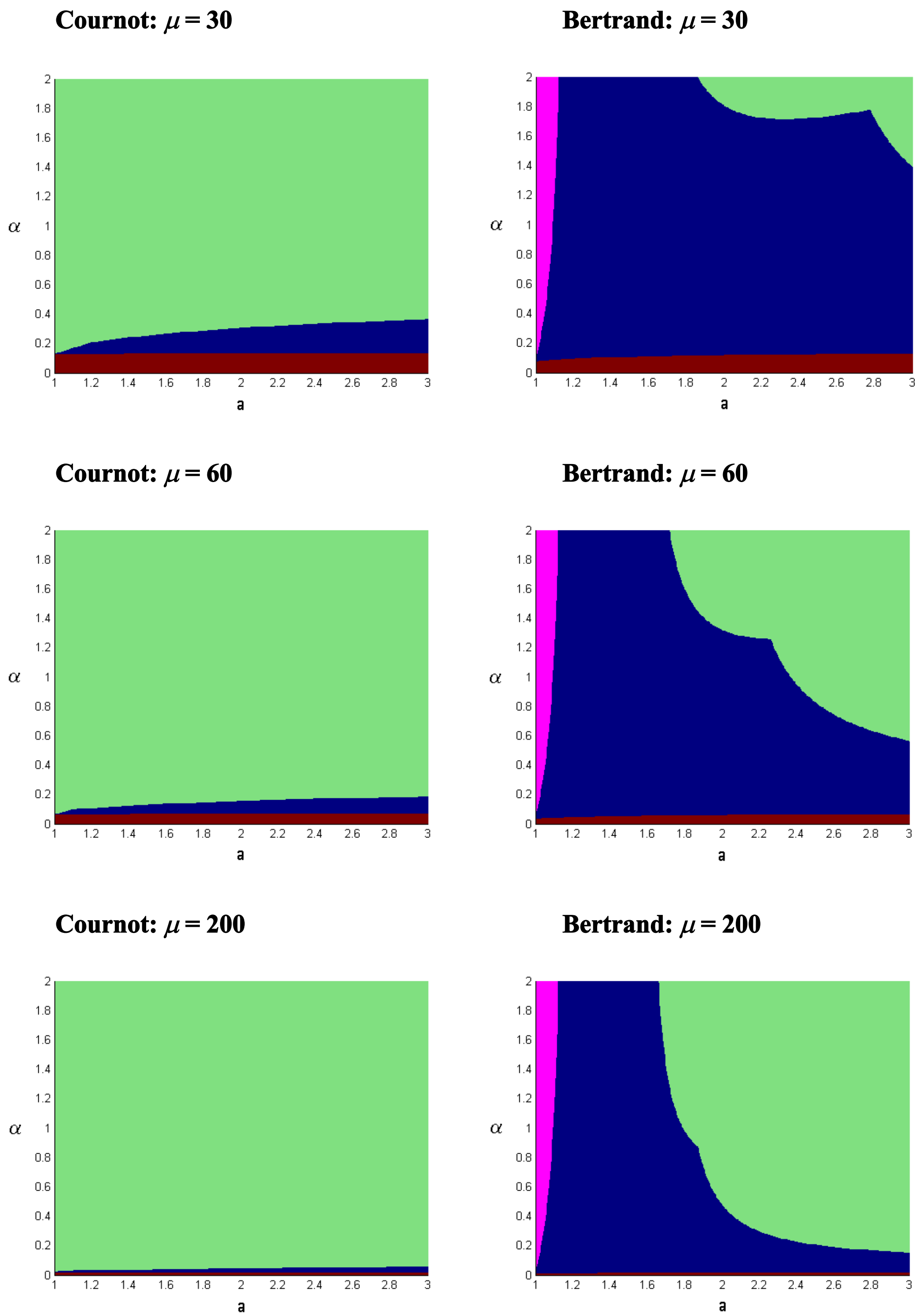


Figure 8. The outcome of the investment game at different values of the parameters a_0, α and μ. The axis labeled "a" in the diagrams of the figure is the a_0 axis.

By examining the shapes of the differently colored areas in the diagrams of Figure 8, and observing how the areas change as the parameter μ changes, the following observations can be made[21]: (1) In order for the firms to invest, α and μ have to be large enough; (2) As α or μ increases, firm 1 is the first to start investing, followed by firm 2 only when α or μ reaches a sufficiently large value (parameter combinations where only firm 2 invests do not exist); (3) Under Cournot competition, a_0 has to be sufficiently small for firm 2 to invest, while under Bertrand competition, investment of firm 2 requires a_0 to be sufficiently large[22]; (4) Both firms are more likely to invest under Cournot competition than under Bertrand competition; (5) Under Bertrand competition, firm 2 is less likely to invest if the coordinate pair (a_0,α) lies close to the line $\alpha = a_0 - 1$ [23].

In the case of Bertrand competition, there is a region in the parameter space, in which several equilibria occur. Therefore the results presented here may not apply under Bertrand competition in cases where the value of the parameter α is particularly large in comparison to the value of the parameter a_0. The case of several equilibria is not discussed further.

4.3 The results expressed qualitatively

In last section, observations were made concerning the parameters' influence on the firms' investment behavior. In this section, the observations are expressed qualitatively. The parameters can be interpreted as follows: The parameter a_0 reflects the initial inter-firm difference in product quality, and the parameter α reflects the size of the quality improvement potentially

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[21] The observations apply under both Cournot and Bertrand competition, if not stated otherwise.

[22] This is accurate only in general. There is a small region in the parameter space, where an increase in a_0 actually lowers firm 2's incentives to invest. It can be seen in Figure 8, in the case of Bertrand competition, where $\mu = 30$. In the upper right corner of this graph, there is a small region, where both firms invest, and an increase of a certain size in parameter a would result in investment only by firm 1. Since this is a relatively rare exception to the observation in question, attention is not paid to it in the examination of the outcomes of the model.

[23] This observation is based on the "corner" in the area where both firms invest. This "corner" is located on the diagonal of the diagrams that illustrate the case of Bertrand competition. In Figure 8□, τηε ∀χορνερ∀ χαν βε οβσερϖεδ ιν τηε διαγραμσ, ωηερε μ is at least 30.

brought by the discovery, which can be interpreted as the significance of the discovery. The parameter μ reflects how rapidly the probability of making the discovery grows as the sum invested grows, i.e., μ reflects the inexpensiveness of making the discovery. Based on these interpretations, observations (1)–(5) made in last section can be expressed as the following five results, respectively:

Result (1): The firms invest only if the potential discovery is sufficiently significant and inexpensive. This applies under both forms of competition.

Result (2): The potential discovery needs to be more significant and inexpensive in order to attract investment from firm 2 than in order to attract investment from firm 1, and situations where only firm 2 invests do not occur. This applies under both forms of competition.

Result (3): Under Cournot competition, investment by firm 2 is less likely if the initial difference in product quality is large, while under Bertrand competition, a large initial difference makes investment by firm 2 more likely.

Result (4): Both firms are more likely to invest under Cournot competition than under Bertrand competition.

Result (5): Under Bertrand competition, firm 2 is less likely to invest if being the only one to obtain the discovery would bring firm 2 close to firm 1 in terms of product quality.

4.4 Interpretation of the results

Some of the results can be interpreted using economic intuition, while others are hard to interpret because of the complexity of the game-theoretical situation. Also, the fact that the results do not reveal information about the exact size of each firm's investment increases the difficulty of making accurate interpretations of how the parameters affect the firms' incentives. Interpretation is, however, attempted below, in order to give some understanding of the results, even though there is a risk that the interpretations are not all entirely accurate.

The part of Result (1) concerning the inexpensiveness of investments appears pretty obvious: the firms can indeed be expected to avoid very expensive investments. However, the part of the result concerning the significance of the potential discovery is not as obvious. One might think that highly significant discoveries would be associated with increased incentives for each firm to invest in R&D, since obtaining such a discovery would increase the consumers' willingness to pay. There are, however situations, where increased willingness to pay is not beneficial to the firms: If, after the innovation, firm 2 got close to firm 1 in terms of product quality, the firms' products would be less differentiated from each other, which would reduce both firms' ability to use market power. As the firms would be selling nearly identical products, competition would drive down the prices, thereby reducing both firms' profits. Thus, it should not be automatically assumed, that the consumers' increased willingness to pay for a firm's product is always beneficial to the firm in question. However, since the results show that both firms are more likely to invest if the significance of the potential discovery is high, the positive forces, such as the consumers' increased willingness to pay, are apparently stronger than the negative forces, such as firm 2's risk of ending up selling products similar to those of firm 1.

Result (2) can be explained as follows: Firm 1 is not willing to give up its position of quality leadership, since this would decrease its profits. Therefore, it will invest in order to keep ahead of firm 2, and also in order to be able to sell a more valuable product. Since firm 1 is ahead of firm 2 in the beginning, it is easier for firm 1 to keep its leading position than it is for firm 2 to get ahead of firm 1. Therefore, firm 1 can expect a larger increase in revenue as a result of making an investment than firm 2 could expect from an investment of equal size. Thus, it is more profitable for firm 1 to invest than it is for firm 2 to invest, which implies that in some cases investing is profitable only for firm 1.

Result (3) is very interesting, but difficult to explain comprehensively. Part of the result can be explained by firm 2's attempt to keep the firms' products differentiated from each other under Bertrand competition. If the initial difference in product quality is sufficiently large, firm 2 does not run the risk of ending up selling products too similar to those of firm 1. However, this only explains firm 2's investment behavior in the case that the potential discovery is not very significant. If the significance of the potential discovery is large, firm 2 may run the risk of catching up with firm 1 even under a large initial difference in product quality. Therefore, the explanation given above is incomplete. In contrast to Bertrand competition, selling similar products does not result in low prices under Cournot competition. This explains why firm 2's incentives to invest do not increase, like they did under Bertrand competition, as the difference in initial product quality grows. However, the reason why a large initial difference actually *reduces* firm 2's incentives to invest under Cournot competition remains unexplained.

One explanation of Result (4) is that both firms' profits are lower in Bertrand competition than in Cournot competition, since Bertrand competition forces the firms to push down the prices of their products, thereby lowering their revenues. Thus, investments that can be expected to be profitable under Cournot competition may not be profitable under Bertrand competition. Therefore, profit-maximizing firms are less willing to invest under Bertrand competition than under Cournot competition.

Result (5) can be explained as follows: In Bertrand competition, firms producing identical products would be totally unable to use market power. Competition in prices would force the firms to push prices down to the level of competitive equilibrium (zero in the case of this model). The larger the difference between the products of the firms, the better they are able to use their market power. In a scenario in which firm 2 succeeds in making a discovery but firm 1 does not succeed (despite its investments), the quality of firm 2's product may rise to a level close to that of firm 1's product, and the prices of both products would then be low. In an attempt to avoid this situation, firm 2 is less willing to invest if making a discovery would bring the quality of firm 2's product close to the quality of firm 1's product.

5 Discussion and conclusions

The research considers a situation in which two firms invest under a permissive patent system. Results are obtained on the firms' investment behavior at different parameter settings. Interpretations based on economic intuition could be given for most of the results. The complexity of the game-theoretical situation makes interpretation of some of the results difficult, but most results are in accordance with intuition.

Much of the research related to the topic of this thesis involves patent races. The model applied in this thesis can also be interpreted as a patent race. However, there are significant differences between this thesis and earlier research, which make it difficult to compare the results. Most of the earlier research in the field considers the welfare implications of different IPR regimes and parameter settings. However, this thesis does not consider the general-level welfare implications, but rather focuses on modeling the duopolistic situation on a detailed level by considering the post-discovery situation explicitly as a case of Bertrand or Cournot competition.

Unlike most patent races, the model considered in this thesis does not either consider cumulative investments involving several periods of investment decisions. Welfare implications, general equilibrium analysis and a larger number of decision periods could, of course, be included in this model as well, but this would result in even more complex algebraic expressions than those of the present model.

Besides research concerning patent races, this thesis also relates to research concerning the Bertrand and Cournot equilibrium. Singh and Vives (1984) examine cases of Bertrand and Cournot competition, that are very similar to those considered in Chapter 2 of this thesis. Comparing their expressions regarding the Bertrand and Cournot equilibria to those derived here is, however, difficult because they have defined the problem somewhat differently and with different parameters.

The research of this Master's thesis could be extended in many ways, in order to make the model more realistic and useful. First, the model could be extended to several periods of investment decisions. This would, however, make the already complex algebra even more complex. Therefore, extending the model to cover a larger number of decision periods might require even more comprehensive application of computerized numerical computations: a multi-decision-period game could e.g. be modeled using computer simulations. Second, the game could be extended to involve a larger number of firms. Third, the welfare implications of different scenarios could be considered. Fourth, the assumption of zero costs could be relaxed. Since the present model assumes zero marginal costs, the results of this research may be best applicable in industries producing information goods, where marginal costs are very low. Fifth, the role of the patent system or IPR system in the model could be made more explicit, and additional features could be added, such as imitation or licensing. Sixth, the outcomes of the model could be studied on a more exact level: while this thesis only determined whether the equilibrium outcome of the game involves positive investments of each firm or not, further research could focus more on *how much* the firms invest under different parameter settings.

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Appendix A: The best response function of firm 2

The first order condition for maximum, in the maximization problem faced by firm 2, is

(A.1) $$\frac{\partial \Pi_2^e}{\partial i_2} = 0 .$$

After substituting Π_2^e from expression (71) and differentiating it with respect to i_2, the condition takes the form

(A.2) $$\mu\left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_2^A - \mu\left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_2^B + \mu e^{-\mu i_1}e^{-\mu i_2}\pi_2^C - \mu e^{-\mu i_1}e^{-\mu i_2}\pi_2^D - 1 = 0 .$$

Solving this for i_2, and writing the solution as a function of i_1 denoted with $J_2(i_1)$, gives

(A.3) $$J_2(i_1) = i_2{}^* = \frac{\ln \mu\left(e^{-\mu i_1}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B)\right)}{\mu},$$

assuming that $e^{-\mu i_1}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B) > 0$.

The second order condition for global maximum is

(A.4) $$\frac{\partial^2 \Pi_2^e}{\partial i_2{}^2} < 0$$ for all i_2.

After substituting Π_2^e from expression (71) and differentiating it twice with respect to i_2, the second order condition takes the form

(A.5) $$-\mu^2\left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_2^A + \mu^2\left(1-e^{-\mu i_1}\right)e^{-\mu i_2}\pi_2^B - \mu^2 e^{-\mu i_1}e^{-\mu i_2}\pi_2^C + \mu^2 e^{-\mu i_1}e^{-\mu i_2}\pi_2^D < 0 ,$$

which simplifies to

(A.6) $$e^{-\mu i_1}\left(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D\right) + \left(\pi_2^A - \pi_2^B\right) > 0 .$$

If the value of i_1 satisfies this condition, the condition holds for all values of i_2. Therefore, the first order condition gives the global maximum, as long as (i) the second order condition is satisfied, and (ii) the maximum is feasible, that is, $i_2{}^* = J_2(i_1) \geq 0$. If the second order condition is satisfied, but the maximum is unfeasible, $i_2{}^* = 0$ is the solution to the maximization problem.

For the values of i_1, which do not satisfy the second order condition, the optimal value of i_2 lies either at zero or infinity.

Since for $i_2 = 0$, $\Pi_2^e = \underbrace{\left(1-e^{-\mu i_1}\right)}_{\geq 0}\underbrace{\pi_2^B}_{\geq 0} + \underbrace{e^{-\mu i_1}}_{\geq 0}\underbrace{\pi_2^D}_{\geq 0} \geq 0$, and

$$\lim_{i_2\to\infty} E(\pi_2) = \lim_{i_2\to\infty} \underbrace{\left(1-e^{-\mu i_1}\right)}_{\geq 0}\underbrace{\left(1-e^{-\mu i_2}\right)}_{\to 1}\underbrace{\pi_2^A}_{\geq 0} + \underbrace{\left(1-e^{-\mu i_1}\right)}_{\geq 0}\underbrace{e^{-\mu i_2}}_{\to 0}\underbrace{\pi_2^B}_{>0} + \underbrace{e^{-\mu i_1}}_{\geq 0}\underbrace{\left(1-e^{-\mu i_2}\right)}_{\to 1}\underbrace{\pi_2^C}_{\geq 0} + \underbrace{e^{-\mu i_1}}_{\geq 0}\underbrace{e^{-\mu i_2}}_{\to 0}\underbrace{\pi_2^D}_{\geq 0} - \underbrace{i_2}_{\to\infty} = -\infty$$

the objective function obtains its maximum value at $i_2 = 0$, if the second order condition for maximum is not satisfied. Based on these results, the best response function of firm 2 can be written as follows:

$$\text{(A.7)} \qquad I_2(i_1) = \begin{cases} J_2(i_1) & \text{for } e^{-\mu i_1}\left(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D\right) + \left(\pi_2^A - \pi_2^B\right) > 0 \text{ and } J_2(i_1) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The conditions of expression (A.7) can be simplified further. Substituting the $J_2(i_1)$ from expression (A.3) into the condition $J_2(i_1) \geq 0$ transforms the condition into

$$\text{(A.8)} \qquad \frac{\ln \mu\left(e^{-\mu i_2}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B)\right)}{\mu} \geq 0,$$

which is equivalent with

$$\text{(A.9)} \qquad e^{-\mu i_2}(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D) + (\pi_2^A - \pi_2^B) \geq \frac{1}{\mu}.$$

Now it can be seen that since $\mu > 0$, the first condition on the first line of expression (A.7) is rendered redundant by the second condition on the line. Therefore the expression simplifies to

$$\text{(A.10)} \qquad I_2(i_1) = \begin{cases} J_2(i_1) & \text{for } e^{-\mu i_1}\left(-\pi_2^A + \pi_2^B + \pi_2^C - \pi_2^D\right) + \left(\pi_2^A - \pi_2^B\right) > \dfrac{1}{\mu} \\ 0 & \text{otherwise} \end{cases}$$