A model for money as a store of value[1]

Klaus Kultti
Discipline of Economics
University of Helsinki
P.O. Box 17
00014 Helsinki
Finland
E-mail:Klaus.kultti@helsinki.fi

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Abstract. We present a model in which the use of perfectly divisible money is endogenous, and the agents can hold any amount of it. Its rationale stems from the need for a store of value. Introduction of money induces more efficient production. The model combines features from random matching and market models. This makes it easy to keep track of the distribution of money holdings. The model also exhibits non-trivial prices.

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1. Introduction

Historical evidence shows that money is very adaptable, performs many functions, and is able to perform functions not originally intended (Davies, 1994, p. 27). The two most commonly discussed functions of money are medium of exchange and store of value. Unit of account and common measure of value are also frequently mentioned. The flexibility of money makes its formalisation and modelling problematic. Any model that illustrates one function of money seems to be deficient in depicting some of its other functions.

There are two main classes of models that have been used to analyse money and monetary economies, and in which the use of money is endogenously determined. Search-theoretic models (e.g. Kiyotaki and Wright, 1989, 1991, 1993) formalise the double coincidence of wants problem in which trade takes place only if both parties desire what the other possesses. Introduction of money speeds up exchange as a single coincidence of wants between parties is sufficient for trade. The medium of exchange function of money emanates from the difficulties of barter. In the standard models of this kind it is assumed that both money and goods are indivisible, and that the agents are able to carry only one unit of money or a commodity at a time. Questions of price formation are then left unanswered as the price of any good is one unit of another good or one unit of money. There have been efforts to address this question, too. Trejos and Wright (1995) as well as Shi (1995) have constructed monetary models along search-theoretic lines in which the relative values of goods are determined non-trivially by bargaining over the terms of trade. In their models money is indivisible and goods, or services, are produced on the spot after the agreement on the terms of trade has been reached. The agreement specifies the amount of goods produced but the agents can hold only one unit of money at a time. It is notable that indivisibility of money is required to keep track of the distribution of money holdings. It is not clear that the models can be easily altered in such a way that money is divisible and goods indivisible. The bargaining story in these models becomes even more difficult if one assumes that the production costs of goods are sunk at the time of trading.

Price distribution with divisible money and costly production is constructed in a model by Zhou (1999). She shows the existence of a continuum of single-price equilibria indexed by the aggregate real-money balances if one such equilibrium exists. The model is not particularly easy or operational.

Overlapping generation models are the other major example of monetary models. It is thought that they depict the store of value function of money. In the simplest models there is no trade without money, with money the economy can achieve a Pareto-optimum, and if storage technology is introduced trade ceases again. The only thing money accomplishes, is consumption smoothing.

The aim of the present model is twofold. First, we show how the medium of exchange function of money emanates from the need for a store of value in an economy where exchanges are decentralised. Decentralisation means that credit arrangements cannot replace money. Secondly, we demonstrate how prices emerge when agents are allowed to hold any amount of money. Introduction of storage capacity in the form of money provides an incentive to invest in the production of high quality goods. Their price is, of course, higher than that of low quality goods, and money is the means of charging higher prices.

The problem with arbitrary monetary holdings in search models is to keep track of their distribution. However, if we want to have non-trivial prices the agents must be allowed to hold various quantities of money. Our model solves this problem by combining features from both random matching and market models. The result is a neat way to keep track of money holdings. The model only depicts equilibrium, and neither the origin of money nor initial monetary distribution is dealt with. We assume that money is divisible unlike goods, and that production costs are sunk at the time of exchange. The last assumption seems to correspond to reality more often than production on the spot, which Trejos and Wright (1995), and Shi (1995) postulate to attain non-trivial prices with indivisible money.

It should be emphasised that in the model money is valued since there is no better way to store value. The meetings of the agents are anonymous and the agents are able to carry only one unit of a good at a time. If the former assumption were relaxed credit arrangements would be possible. If the latter assumption were relaxed non-trivial prices in terms of low quality commodities could emerge. However, as we want to emphasise the store of value function of money we want to limit the storage possibilities. Also, if there were divisible interest bearing assets in the economy the model would be subject to the criticism on the absence of return dominance.

It has long been understood that the positive consequences of monetary exchange are not limited to gains from trade. Adam Smith (1776) noted that specialisation is limited by the extent of the market. Use of money extends the market, encourages specialisation, and results in gains in productivity. In our model, a particular consequence of the use of money is an increase in productivity. This takes place as the agents become more willing to invest, not because they specialise. Before going into details it is perhaps worthwhile to present the basic idea of the model.

There are two types of goods and agents. Both types consume only what the other type produces. They can produce and carry one unit of a commodity at a time. Thus, in pure barter, the most they can do is to swap goods when they meet each other. One of the goods can be produced in different qualities. The higher the quality the more costly the good is to produce. The agents would naturally require more than just one unit of their consumption good in compensation for producing a high quality good. This is, however, impossible because of the lack of storage facilities. Introduction of money provides an incentive for the production of high quality goods, since producers can be compensated with money that they can later use to buy consumption goods.

We assume that there exists a location for the trade of each type of commodity. These locations can be regarded as markets or market places which coordinate the agents' actions. The idea of different trading zones for the trade of different goods has been used in a search model by Iwai (1988) and Kultti (1994). The markets coordinate the agents' expectations and actions, and enable us to keep track of the distribution of money holdings without difficulty. Markets also ensure that the need for money does not arise from the double coincidence of wants problem.

In markets trades are executed in pairwise meetings. The seller quotes a price and the buyer either accepts or rejects it. In either case the match dissolves, and the agents are matched with new partners next period. As such this is too simple a process to determine prices. We impose an equilibrium condition which guarantees that no agent finds it profitable to change into the production of the other type of good. This is sufficient to guarantee that the number of monetary equilibria is finite.

In markets the agents meet randomly which guarantees a degree of anonymity that is desirable to highlight some features of monetary economies. As money is a generally accepted means of payment, random matching ensures that the agents cannot condition their behaviour on the identity of other agents, and that the acceptance of money is not based on personal relationships between individuals. It also precludes credit arrangements. The existence of markets simplifies the analysis but is not essential to the results.

The article is organised as follows. In section 2 we describe the model, in section 3 we analyse pure barter, and in section 4 we study a monetary economy. Section 5 contains discussion on welfare, and the conclusions are in section 6.

2. The Model

There are two types of goods, *I*s and *II*s. The agents specialise in the production of one type of good, and consume the other type of good. It can be thought that the model depicts trade and production in excess of production for own consumption. We call those who specialise in the production of type *I* good type *I* agents, and those who specialise in the production of type *II* good type *II* agents. Production of type *I* good costs $c > 0$ in utility, and if nothing is produced there are no costs. Type *II* good can be produced in various qualities. The higher the quality the more expensive (in terms of utility) it is to produce the good. The production cost of type *II* good is denoted by

$\gamma \in \{0\} \cup [c, \infty)$. Type *II* agents get utility *U* from the consumption of type *I* agents' production good while type *I* agents derive utility $u(\gamma)$ from the consumption of a good whose production cost is γ. Thus, the quality of the good is indexed by its production cost. For simplicity we assume that the utility derived from goods with production cost *c* is the same for both types $u(c) = U$, and that consuming nothing gives zero utility, $u(0) = 0$. As the argument of the utility function is the disutility of producing a good of certain quality, $u(.)$ can also be regarded as a production function. The production technology exhibits fixed costs; every time an agent produces he has to incur a cost of at least size *c*.

Both types of agents derive positive utility only from goods produced by the other type. Otherwise they are identical in all respects. Type *I* agents either incur production costs zero or *c*. Their production possibility set $\{(0,0),(c,U)\}$ is a subset of type *II* agents' production function.

We normalise the utility from the consumption of type *I* good as well as the lowest positive production cost to unity, $U = c = 1$. From point $\gamma = 1$ on the utility or production function $u(.)$ is increasing, but eventually its second derivative becomes negative.We also assume that $u'(1) > 1$ so that for some values of the argument $\gamma > 1$ the production function is above the 45 degree line. At these values the utility from the consumption of the good is greater than the disutility from its production. We also assume that the agents can produce several goods in a row without consuming. This assumption differs from the typical search models, but there is nothing inherently dubious about it. Especially, if the model is thought to depict production and trade in excess of self-sufficiency.

The agents can hold at most one unit of any commodity at a time, and it can be carried from one period to the next, i.e. goods are not perishable. The economy proceeds in discrete time, and there is an infinite number of periods. The agents' common discount factor is δ, $0 < \delta < 1$. The number of both types of agents is of Lebesgue-measure one. Trade takes place in markets, and every period the agents go to the appropriate market either to sell their production good or to buy a consumption good. Every agent meets a randomly determined agent of the opposite type. When a pair of agents meet they agree on the terms of trade. The timing of the agents' actions within a period is the following: Production, departure to the markets, agreement on price, trading, and consumption.

Money is assumed to be perfectly durable, non-producible, recognisable, easy to carry, perfectly divisible, and intrinsically worthless. Being easy to carry, means that the agents can possess any amount of money in contrast to goods which they can carry only one unit at a time. The agents cannot observe how much money the other agents possess. We assume perfect divisibility of money in order to make price formation uncomplicated, and to contrast money with goods that are indivisible. As there is no smallest unit of money it can be assumed, without loss of generality, that the total amount of money in the economy is of Lebesgue-measure one. Prices adjust to make any amount of money compatible with monetary equilibrium. We do not discuss how money came into existence, even though it would be interesting. The initial monetary holdings may determine which equilibrium results, but these questions are outside the scope of this article.

There exist markets for each good. Different qualities of type *II* goods are considered different goods, and there is a different market for each good produced in equilibrium. The existence of markets is significant only in a monetary economy.

The bargaining procedure is critical in price determination. For our purposes it is sufficient to consider a simple procedure. We assume that in each market the seller of a good quotes a price and the buyer either accepts or rejects it. If the buyer accepts the agents execute the trade. If the buyer rejects the offer the match is dissolved, and the agents are matched with new partners in the next period. If the buyer faces a seller who offers a good of some other quality than expected in the market, it takes one period to ascertain its quality. Then, in the beginning of the next period the buyer can either accept the stated price or the match is dissolved, and he meets a new partner in the same period. This is a slightly akward assumption but it simplifies the analysis. Its main purpose is to make deviation by producing goods of lower than expected quality unprofitable in the monetary equilibrium. The same effect can be achieved with a more complicated bargaining procedures, or by making quality a discontinuous variable.

We assume that the agents can decide which good to produce. Thus, in equilibrium the expected utility from the production of either good has to be equal. This condition restricts the number of equilibria. In equilibrium the choice of quality hinges on what quality other agents produce. The seller has to take into account that buyers can always resort to other sellers who have goods of certain quality.

There are numerous equilibria in the model. For technical tractability and because it is sufficient to convey the ideas we study only symmetric steady state equilibria in both pure barter and monetary economy.

3. Pure Barter

In pure barter type *I* agents produce what they can, and type *II* agents produce the lowest quality good that still guarantees trade. There is no trade if production costs exceed the expected utility from consumption. From type *I* agents' point of view, production and trade take place if $u(\gamma)-c=u(\gamma)-1\geq 0$ holds, and from type *II* agents' point of view if $U-\gamma=1-\gamma\geq 0$ holds. The only value of γ that satisfies both inequalities is $\gamma=1$, the lowest positive production cost. Thus, both types produce at the same point on the production function.

The agents never give up their good for nothing. Thus, we simply assume that they always produce rather than enter the markets without anything for trade. The prices are trivial as type *I* agents' strategy just tells whether to swap goods with a type *II* agent who possesses a good of quality γ, and type *II* agents' strategy states the quality of the good to be produced, and whether to exchange this good for the good offered by his partner.

The expected lifetime utilities are given by the following expressions for type *I* agents and for type *II* agents, respectively

$$E\sum_{t=0}^{\infty}\delta^t\left[\chi(t)u(\gamma)-\phi(t)c\right] \quad (1)$$

$$E\sum_{t=0}^{\infty}\delta^t\left[\chi(t)U-\phi(t)\gamma\right]. \quad (2)$$

In (1) and (2) E is the expectation operator, and χ and ϕ are indicator functions; the former equals one if the agent consumes a good, and the latter equals one if the agent produces. Otherwise they equal zero.

DEFINITION 1. A symmetric steady state equilibrium in pure barter is a strategy σ_i for each type of agents, $i = I, II$, such that σ_i maximises the expected life time utility of type i agent given the other agents' strategies.

In equilibrium both types of agents produce a good of quality $\gamma = c = 1$. Clearly, it is in the interest of type *I* agents to accept any type *II* good as its quality γ is at least unity. Type *II* agents cannot do better than accept the type *I* good offered. The only way to deviate from the equilibrium strategies is for type *II* agents to produce a good of quality $\gamma > 1$, but that is not profitable.

We use standard techniques and evaluate the expected life time utilities using value functions. Denote the expected discounted utilities of type *I* and type *II* agents, who follow maximising strategies, by *V* and *W*, respectively. The value functions are evaluated at the end of each period after consumption, and they are determined by the following equations:

$$V = \delta\left\{u(\gamma) - c + V\right\} \quad (3)$$

$$W = \delta\left\{U - \gamma + W\right\} \quad (4)$$

In (3), the left hand side is the discounted life time utility of type *I* agent before entering the next period, and making a production decision . The right hand side is the utility next period; the agent produces incurring cost *c*, trades, consumes receiving utility $u(\gamma)$, and is ready to enter next period whose value is *V*. The interpretation of (4) is analogous. From (3) and (4) we get

$$V = \frac{\delta}{1-\delta}(u(\gamma) - c) \tag{5}$$

$$W = \frac{\delta}{1-\delta}(U - \gamma) \tag{6}$$

In order to have production and trade, *V* and *W* have to be non-negative, but this is guaranteed as $U = c = \gamma = 1$. Type *I* agents' life time utility, as well as that of type *II* agents' is zero, which can also be attained by ceasing production altogether. For simplicity, we assume that the agents always choose to produce when they are indifferent between producing and not producing. That barter does not improve on autarky may seem strange. This is the result of the normalisation assumptions, and simplifies analysis. We could assume that the consumption of the lowest quality good yields more utility than its production takes, which would make barter preferable to autarky.

4. Monetary Economy

The agents are able to carry only one unit of any good with them, and they meet each other again with probability zero. In the absence of money, the best they can do is to swap goods one for one. In these circumstances, it does not pay for type *II* agents to produce anything more costly than the minimum quality necessary. Introduction of money makes it possible for the economy to move to a more efficient equilibrium. This does not necessarily happen as there is always an equilibrium in which the agents do not accept money. However, in a monetary equilibrium type *II* agents produce goods of higher quality than in pure barter while type *I* agents produce the same product as before. The price of type *II* good is higher than that of type *I* good, and type *I* agents have to accumulate enough money (i.e. produce and sell sufficiently many products) to buy consumption goods from type *II* agents. After selling their goods type *II* agents are able to consume without having to produce anything for several periods.

In a symmetric equilibrium the monetary economy works as follows. Type *II* agents produce a good of certain quality, and every period the agents go to an appropriate market where they are matched with an agent of the opposite type. Type *I* agents sell their production good at a certain price, say x, to type *II* agents, i.e. type *II* agents give them x units of money in exchange for the good. Type *II* agents, who produce a good of quality $\gamma > 1$, require $(n-1)x$ units of money plus one unit of type *I* good for it. Heuristically, the price of type *II* good is n units of the type *I* good; since the agents can carry only one unit of a good, type *I* agents pay only part of the price in goods, and the remainder in money. In order to accumulate enough money, type *I* agents have to produce and sell $n-1$ units of their production. As long as type *II* agents have money they buy from type *I* agents. Only after they have spent all of it they produce again. Note that type *II* agents can buy only one unit per period. When they have $(n-1)x$ units of money they buy a unit of their consumption good during the next n-1 periods.

In the monetary economy, the agents are in various states depending on how much money they have, and whether they have a commodity in their possession or not. To formally establish the existence of a monetary equilibrium, we determine the proportions of agents in various states, the prices of both goods, and the agents' equilibrium strategies. The monetary economy has more structure than pure barter, and the strategy spaces are correspondingly more complex.

First type *II* agents decide the quality γ of their production good. Then, the agents choose which market they go to. In the market for type *I* good type *I* agents choose the price of their good, and similarly in the market for type *II* good type *II* agents choose the price of their good. In equilibrium no agent should find it profitable to move from the production of one good to the production of another good. We do not content ourselves with just Nash equilibrium strategies where the agents' strategies are best responses against each other. We want the strategies also to satisfy a perfectness requirement, i.e. the threats not to accept deviating offers should be credible. This is a reasonable requirement since it is easy enough to sustain just about any monetary equilibrium without requiring perfectness; if type *II* agents, for instance, adopt a strategy that specifies some quality level that they always produce and some price for it, the best type *I* agents can do, is to pay the price as long as this results in positive utility for them.

We study a rational expectations equilibrium where the agents' expectations about **d**, the distribution of agents in various states, coincides with the true distribution that results from the agents' strategies.

DEFINITION 2. A symmetric steady state monetary equilibrium consists of (i) two markets, one for type *I* good and one for type *II* good, (ii) a strategy σ_i for each type of agents, *i* = *I*, *II*, such that given the other agents' strategies and **d**, σ_i maximises the expected utility of an agent of type *i*, (iii) equality of supply and demand in both markets.

At the time the value functions are evaluated both types of agents can be in *n* different states. The agents can, in equilibrium, possess zero goods and zero units of money, zero goods and *x* units of money, and so on up until zero goods and $(n-1)x$ units of money, where *x* is the price of type *I* good, and $(n-1)x$ units of money plus a type *I* good is the price of type *II* good. Since the agents can hold only one commodity at a time, equality of supply and demand in both markets is guaranteed when one *n*th of the agents of both types are in any of the possible states. This is shown in the appendix.

Next we shall specify the agents' strategies in a monetary equilibrium. Assume that type *II* agents produce a good of quality $\gamma^* > c$ whose price is $(n-1)x$ units of money plus a unit of type *I* good, where *x* is the price of type *I* good.

Type *I* agents produce a good every period, go to the market for type *I* good as long as they possess less than $(n-1)x$ units of money, and ask *x* units of money for their good. Having accumulated $(n-1)x$ units of money they again produce and enter the market for type *II* good. They accept any price that does not exceed $(n-1)x$ units of money plus a unit of type *I* good for a good of quality γ^*. If a good of some other quality is offered at some price they ascertain its quality and accept it only if it is more profitable than acquiring a good of quality γ^* next period.

Type *II* agents produce a good of quality γ^* if they possess less than *x* units of money, and go to the market for type *II* good. They ask $(n-1)x$ units of money plus a unit of type *I* good for it. If they possess *x* units of money or more, they do not produce but go to the market for type *I* good. They pay no more than *x* units of money for a type *I* good.

Note that the price of type *I* good *x* depends on the amount of money in the economy. In equilibrium *x* adjusts so that all the money is used. If the amount of money is halved the new price of type *I* good will be *x*/2 in the corresponding equilibrium.

The agents maximise their expected lifetime utilities as before. We evaluate their value functions with the suggested equilibrium strategies, and then we check that one time deviations from the strategies are not profitable.

Denote by *V*(.,.) the value function of a type *I* agent. $V(j,k)$ is the life time utility of an agent who holds *j* units of a commodity and *kx* units of money, *j*=0,1, and *k*=0,1,... The value function can be described by the following equation

$$V(j,k) = \delta\left\{\max_{\sigma} E(\chi \mathrm{u}(\gamma) - \phi c + V(g,h))\right\} \qquad (7),$$

where the maximisation is over strategies, *E* is the expectation operator, χ is an indicator function that equals one if the agent consumes a good, and ϕ is an indicator function that equals one if the agent produces a good. In all other cases the indicator functions equal zero. $V(g,h)$ is the agent's life time utility in the next period's state (g,h). When the agents follow the suggested equilibrium strategies the life time utilities of type *I* agents in various states are determined by the following equations.

$$
\begin{aligned}
&V(0,0) = \delta\{V(0,1) - c\} \\
&V(0,1) = \delta\{V(0,2) - c\} \\
&. \\
&. \qquad\qquad\qquad\qquad\qquad\qquad (8) \\
&. \\
&V(0, n-1) = \delta\{u(\gamma^*) - c + V(0,0)\}
\end{aligned}
$$

$V(0,0)$, for example, is the expected life time utility of type *I* agent who has no good and no money. Next period he produces a good and sells it for *x* units of money; production costs *c* and the agent is ready to enter the next period possessing one unit of size *x* (or *x* units) of money and no good. In the last equation the agent possesses $n-1$ units of money of size *x* and no good. In the following period he produces a good, enters the market for type *II* agents' production good, and buys a unit of good paying everything he has for it; he consumes the good and enters the following period possessing no good and no money. From these *n* equations we solve $V(0,0)$ which is needed to check the optimality of the strategies, and to derive conditions for the existence of monetary equilibrium.

$$V(0,0) = \frac{\delta^n}{1-\delta^n} u(\gamma^*) - \frac{\delta}{1-\delta} c \qquad (9)$$

Type *I* agents are in the worst possible state when they have no money, since it takes the longest before they get to consume. To ensure production condition $V(0,0) \geq 0$ has to hold. Otherwise type *I* agents would find it best not to produce at all when they have no money, and monetary equilibrium could not be sustained.

We denote by $W(.,.)$ the value function of type *II* agent. $W(j,k)$ is his life time utility when he has j units of a commodity and kx units of money, j=0,1, and k=0,1,2,... The value function is described by the following equation

$$W(j,k) = \delta\left\{\max_{\sigma} E(\chi U - \phi\gamma + W(g,h))\right\} \tag{10}$$

Maximisation is again over strategies, E is an expectation operator, χ is an indicator function that equals one if the agent consumes and zero otherwise, ϕ is an indicator function that equals one if the agent produces, and $W(g,h)$ is the agent's life time utility in the random state (g,h) next period. In equilibrium type *II* agents' life time utilities in various states are determined by the following equations.

$$\begin{aligned} W(0,0) &= \delta\{U - \gamma^* + W(0,n-1)\} \\ W(0,n-1) &= \delta\{U + W(0,n-2)\} \\ &. \\ &. \\ &. \\ W(0,1) &= \delta\{U + W(0,0)\}. \end{aligned} \tag{11}$$

Their interpretation is analogous to (8). The life time utility of type *II* agents in the crucial state is

$$W(0,0) = \frac{\delta}{1-\delta}U - \frac{\delta}{1-\delta^n}\gamma^* \tag{12}$$

Type *II* agents are in the worst possible state when they have no good and no money. Condition $W(0,0) \geq 0$ ensures that type *II* agents produce. The non-negativity of (9) and (12) bound the discount factor from below; it must be close enough to unity. Otherwise, type *II* agents do not find it profitable to invest in the production of high quality goods, and type *I* agents do not desire to spend a long time accumulating money.

We also assume that the following condition holds

$$\frac{\delta^{m-1}(1+\delta)}{1+\delta+\ldots+\delta^{n-1}}\gamma^* > U,\ 1<m\leq n-1 \tag{13}$$

It guarantees that type *II* agents will not pay 2*x* (or anything more) for a type *I* good. Consider a type *II* agent who has *mx* units of money, $1 < m \leq n-1$. If he pays 2*x* his utility is $U + W(0, m-2)$, and if he refuses to pay more than *x* and waits for one period to buy a good for *x*, his utility is $W(0,m) = \delta[U + W(0, m-1)]$. This is greater than the former if condition (13) holds.

For a given γ the possible equilibrium prices are bounded from below and above by the non-negativity of (9) and (12). We assume that when the agents have no money nor goods they may switch from the production of one type of good to the production of the other type of good. In equilibrium both types of agents have to be indifferent between producing a type *I* and a type *II* good. This is guaranteed when $V(0,0) = W(0,0)$ which implies that the price-quality pair has to satisfy the following condition

$$\delta^{n-1}u(\gamma^*) + \gamma^* = 2(1+\delta+\ldots+\delta^{n-1}). \tag{14}$$

We get the following result whose proof is relegated to the appendix.

PROPOSITION 1. i) Let *u*(.) be such that $u'(1) > 1$, there exists $\tilde{\gamma} = \arg\max \frac{u(\gamma)}{\gamma}$, and $\left\lfloor \frac{u(\tilde{\gamma}) + \tilde{\gamma}}{2} \right\rfloor = k > 2$ where $[.]$ is the greatest integer function, and ii) that for all γ_1 and $\gamma_2, \gamma_1 < \gamma_2 < \tilde{\gamma}$ implies $\frac{u(\gamma_1)}{\gamma_1} < \frac{u(\gamma_2)}{\gamma_2}$. Then for large enough δ there exist at least one and at most $k-1$ monetary equilibria.

Assumption i) says that the economy could, in principle, move to a more efficient production than pure barter. In particular, it would be possible to produce a high quality good such that type *I* agents would be willing to pay at least two units of their good in compensation for it, and type *II* agents' would find the arrangement profitable, too. Assumption ii) guarantees that type *II* agents cannot deviate by producing lower quality goods, since the average utility, or average product, is lower than in equilibrium for any good with lower production cost. Of course, when the average product declines it cannot be divided between two parties in such a way that both get more. A reduction in quality (disutility of production) results in such a large reduction in utility that type *I* agents are not willing to accept the low quality good at any price that is acceptable to the deviating type *II* agent. It is noteworthy that the condition does not restrict the utility function to be either convex or concave, from point $\gamma = 1$ onward, because of the fixed cost in production. It is just required that average productivity $\frac{u(\gamma)}{\gamma}$ grows upto $\tilde{\gamma}$. We also note that in the proof of *Proposition 1* the use of money is a choice variable. Its acceptance decision is, however, not made explicit. It comes as a byproduct of the decision to accept the quoted prices. The multiplicity of equilibria reflects a co-ordination problem. A monetary equilibrium is viable on the portion of the production function where average productivity is increasing. If this portion is large there are many points which can sustain a monetary equilibrium. The assumptions of *Proposition 1* are by no means necessary, but they are reasonably simple and their meaning is clear.

5. Welfare

An obvious welfare measure for the economy is the average utility of the agents. As both the monetary and non-monetary equilibria, we are concerned with, are stationary the average utility in any period is the same. We utilise the value functions already calculated, and evaluate the utility in the end of period. The agents are evenly distributed between different states one *n*th of them being in each state. Thus, the average utilities of type *I* and type *II* agents are expressed by the following formulae

$$\frac{1}{n}[V(0,0)+V(0,1)+\ldots+V(0,n-1)]=\frac{\delta}{n(1-\delta)}u(\gamma^*)-\frac{\delta}{1-\delta}c \tag{15}$$

$$\frac{1}{n}[W(0,0)+W(0,1)+\ldots+W(0,n-1)]=\frac{\delta}{1-\delta}U-\frac{\delta}{n(1-\delta)}\gamma^* \tag{16}$$

Consider two equilibria $(\gamma_1,n_1)<(\gamma_2,n_2)$ ordered so that both coordinates of the former equilibrium are strictly less than corresponding coordinates of the latter equilibrium. It can be easily shown that for δ close enough to unity the average utility of both types of agents is higher in the larger equilibrium, and consequently the same applies to the total average utility that serves as a social welfare measure. In a non-monetary equilibrium the agents' value functions are zero in the end of each period. In a monetary economy the agents' value functions are at least zero in the worst possible state. These observations give us the following proposition.

PROPOSITION 2. For sufficiently high discount factors welfare is higher in any monetary equilibrium than in the pure barter equilibrium. The monetary equilibria (γ,n) can be Pareto-ranked such that larger equilibria dominate lower equilibria.

In the best equilibrium, the total average utility is maximised. Condition (13) limits the possibilities of reaching this equilibrium. In the limit as δ approaches unity the price of type *II* good in terms of type *I* good cannot exceed twice the production costs. If the price difference is too large type *I* agents can credibly ask more than the suggested equilibrium price for their good. Thus, in this model money does not completely overcome the imperfections of the economy. However, introduction of money improves everybody's welfare compared to pure barter. This is in contrast to the models in which the use of money alleviates the double coincidence of wants problem. In these models the maximum holdings of indivisible money is one unit, and too much money slows the trading process which may lead to outcomes worse than barter. In this model the trading process functions well, and introduction of money induces the agents to move to more efficient production.

As there are many equilibria in this economy there is not much point in conducting comparative statics. However, it can be readily seen from equation (12) that decreasing the discount factor δ, reduces the highest possible quality of type *II* agents' production good that can be sustained in a monetary equilibrium. This is quite intuitive, as investment in quality is compensated over several periods, and low values of δ result in low present utility from consumption that takes place long after the production cost has been incurred.

We have assumed that money is perfectly divisible; this means that its quantity is not a constraint. Should there be a smallest money unit, it would limit the possible type *II* good prices which, of course, would limit the quality of the good type *II* agents would be willing to produce. In some cases then, the lack of money would prevent the economy from achieving the most efficient production patterns.

EXAMPLE 1. The model can be used to gain some insight into how differences in time preference affect the production and consumption patterns. Assume that type *I* and type *II* agents are divided into two groups of equal size. In one group the agents are more impatient than in the other group. Denote the discount factor of the more patient agents by δ_1, and that of the less patient agents by δ_2, $\delta_1 > \delta_2$. Let us examine a situation in which the the more patient type *II* agents produce a good of quality γ_1 and the less patient agents a good of quality γ_2 with prices (n-1)x units of money plus a unit of type *I* good, and (m-1)x units of money plus a unit of type *I* good, respectively. Assume that the more patient type *I* agents consume good of quality γ_1, and the less patient a good of quality γ_2. Thus, there are three markets in this economy. Assume further that the agents make a decision about which good to produce and to consume when they have neither money nor goods. In equilibrium the following "incentive" constraints must hold

$$\frac{\delta_1^m}{1-\delta_1^m}u(\gamma_2)-\frac{\delta_1}{1-\delta_1}<\frac{\delta_1^n}{1-\delta_1^n}u(\gamma_1)-\frac{\delta_1}{1-\delta_1} \tag{17}$$

$$\frac{\delta_2^n}{1-\delta_2^n}u(\gamma_1)-\frac{\delta_2}{1-\delta_2}<\frac{\delta_2^m}{1-\delta_2^m}u(\gamma_2)-\frac{\delta_2}{1-\delta_2} \tag{18}$$

$$\frac{\delta_1}{1-\delta_1}+\frac{\delta_1}{1-\delta_1^n}\gamma_1<\frac{\delta_1}{1-\delta_1}+\frac{\delta_1}{1-\delta_1^m}\gamma_2 \tag{19}$$

$$\frac{\delta_2}{1-\delta_2}+\frac{\delta_2}{1-\delta_2^m}\gamma_2<\frac{\delta_2}{1-\delta_2}+\frac{\delta_2}{1-\delta_2^n}\gamma_1 \tag{20}$$

Constraint (17) says that the more patient type *I* agents do not find it advantageous to consume what the less patient agents consume, and constraint (19) says that the more patient type *II* agents do not desire to produce what the less patient agents produce. Constraints (18) and (20) say that the less patient should concentrate their consumption and production on good of quality γ_2. The first two constraints result from (9), and the last two from (12). Simplifying (17)-(20) and dividing the left hand side of (17) with the right hand side of (19), and dividing the right hand side of (17) with the left hand side of (19) we get

$$\frac{u(\gamma_2)}{\gamma_2}<\frac{u(\gamma_1)}{\gamma_1}\delta_1^{n-m} \tag{21}$$

Similarly, dividing the corresponding sides of (18) by the the corresponding sides of (20) we get

$$\frac{u(\gamma_1)}{\gamma_1}\delta_2^{n-m}<\frac{u(\gamma_2)}{\gamma_2}. \tag{22}$$

Using the second assumption of *Proposition 1* we see that $\gamma_1>\gamma_2$, and $n>m$. Thus, any equilibrium in which patient and impatient agents produce and consume different commodities is such that the former produce and consume higher quality goods. The more patient agents attain a higher welfare than the less patient agents, since they are more willing to invest and deter consumption.

6. Conclusion

We have constructed a model in which money emerges from a need for a store of value. The existence of storage facilities would make it possible to induce some agents to produce high quality, and high cost, goods as they could be compensated with several units of their consumption good, but there are no storage facilities in the economy. Money functions as a substitute for these and its use results in more effective production which is both individually and socially desirable. We believe that our model, by combining decentralised exchanges with random matching, illustrates the store of value function in a novel way.

The other novel feature of the model is to exhibit prices when goods are indivisible, money is divisible, production costs are sunk at the time of trading, and agreement on price is made pairwise. Price determination is associated with the problem of keeping track of the distribution of money holdings. The existence of markets and the ensuing requirement of the equality of supply and demand in the markets makes this problem manageable. It is a well established view in economics that prices are determined in the markets but their characterisation remains far from complete. The present model offers a connection between markets and money. While in frictionless markets there is no need for money, here markets coordinate the agents' actions and restrict their behaviour by making them less dependent on their current partner.

It can be argued that there is too much ambiguity in the model as there are many monetary equilibria. The other way to look at this feature is to say that this is typical of monetary economies. They are characterised by more ambiguity and instability than barter economies (Davies, 1994).

We have to introduce more structure into the model if we strive at uniqueness of the price and quality of goods. For example, demand and supply, which are normally thought to determine prices, are restricted in the model by several assumptions. First, the agents hold only one unit of a commodity at a time. Secondly, there are equal numbers of the agents, and thirdly, their preferences are particularly simple reflecting the first assumption. Relaxing these assumptions may lead to more definitive results. A more sophisticated bargaining procedure than the rather simplistic one that is studied here may reduce the number of equilibria. However, the simple procedure in which the seller of a good quotes the price and the buyer either accepts or rejects has the flavour of competitive market models, and makes the analysis of the model relatively simple.

Appendix

Calculation of **d**. Denote by $v(0),...,v(n-1)$ the proportions of type *I* agents in various states in the end of a period. $v(k)$ denotes those who possess *kx* units of money. Denote by $w(0),...,w(n-1)$ the proportions of type *II* agents in various states in the end of a period. $w(k)$ denotes those who possess *kx* units of money. In equilibrium supply and demand are equal in each market. In the market for type *II* good there are $v(n-1)$ type *I* agents and $w(0)$ type *II* agents. Thus $v(n-1) = w(0)$. Next period those type *I* agents who, in this period, possess $(n-2)x$ units of money, and those type *II* agents, who possess *x* units of money, go to the market for type *II* good. In a steady state equilibrium $v(n-2) = v(n-1) = w(0) = w(1)$. By the same logic, the proportions of type *I* agents in all possible states are equal, as well as those of type *II* agents.

Proof of proposition 1. i) We first show that there exist price-quality pairs that satisfy condition (14). Consider $n = k = \left[\frac{u(\tilde{\gamma}) + \tilde{\gamma}}{2}\right]$. From Proposition 1 we know that $\frac{u(\tilde{\gamma}) + \tilde{\gamma}}{2} > k$ which implies that for sufficiently large δ and $\delta^{n-1}u(\tilde{\gamma}) + \tilde{\gamma} > 2(1 + \delta + ... + \delta^{n-1})$. Thus, for δ sufficiently close to unity the LHS of

(14) is larger than the RHS at point $\tilde{\gamma}$. As the LHS is decreasing in γ there exists a $\gamma^* < \tilde{\gamma}$ such that (14) holds as an equality. Note that regarding *n* as a continuous variable (14) implicitly defines *n* as a function of γ unambiguously. We can continue the above process for quality γ* and price $n = \left[\frac{u(\tilde{\gamma}) + \tilde{\gamma}}{2}\right] - 1$ to find another price-quality pair that satisfies (14). Going all the way down to $n = 2$ we see that there are $\left[\frac{u(\tilde{\gamma}) + \tilde{\gamma}}{2}\right] - 1$ potential equilibria. However, the price-quality pair has to satisfy also (13) to qualify as an equilibrium. (13) restricts the price from being too high compared to γ*. As the average productivity decreases as γ decreases there exists a largest price-quality pair among the potential pairs that satisfies (13). Every smaller pair also satisfies (13), and there always exists a $\hat{\gamma} > 1$ such that $\delta u(\hat{\gamma}) + \hat{\gamma} = 2(1 + \delta)$, and for $n = 2$ this price-quality pair constitutes the smallest equilibrium.

ii) It is verified that unilateral one time deviations are not profitable. Then money matters only in units of size *x* for the agent who responds to the deviating offer. Consequently, when deviations from the equilibrium are studied it is thought that the agents deviate by offering or accepting a price that differs from the equilibrium price by a multiple of *x*.

It is not profitable to a type *I* agent to go to the market for type *II* agents' production good with less than $(n-1)x$ units of money and a unit of good, since this is what type *II* agents always ask for their good. Neither is it profitable to a type *I* agent to demand more than *x* units of money for his production good, since type *II* agents will not pay more. This is guaranteed by condition (13).

iii) It is not profitable to a type *II* agent to demand more than $(n-1)x$ units of money plus a unit of good for his production good since type *I* agents never have more. It not profitable to produce a good while holding at least *x* units of money since this results in incurring production costs sooner rather than later without affecting the consumption stream. The only relevant way to deviate is to produce a lower quality good $\gamma < \gamma^*$.

When a good of lower quality is offered it takes one period to ascertain its quality. Thus, the deal will be made only one period later, if ever. One period later, the type *I* agent has the option to leave his current partner and buy a good of quality γ^* at price $(n-1)x$ units of money plus a unit of good. The type *II* agent has to offer the type *I* agent a better deal. If the agents strike a deal in which the type *I* agent pays $(m-1)x$ units of money plus a unit of good $(m<n)$ the following condition has to hold.

$$u(\gamma)+V(0,n-m)>u(\gamma^*)+V(0,0) \quad \text{(A1)}$$

Utility from accepting the lower quality good at the lower price has to be greater than trading in the market. Condition (A1) reduces to

$$u(\gamma)>\frac{1-\delta^m}{1-\delta^n}u(\gamma^*) \quad \text{(A2)}$$

The deviation should be profitable to the type *II* agent, too. This is ensured by the following condition.

$$-\gamma+\delta[U+W(0,m-1)]>-\gamma^*+U+W(0,n-1), \quad \text{(A3)}$$

where the left hand side is the utility from producing a lower quality good this period and waiting for one period to sell it at price $(m-1)x$ units of money plus a unit of good. The right hand side is the utility from not deviating. Condition (A3) reduces to

$$-\gamma+\frac{1-\delta^{m+1}}{1-\delta^n}\gamma^*>U=1 \quad \text{(A4)}$$

As the coefficient of γ^* is at most unity it can be seen that γ has to be at least unity less than γ^* regardless of the value of δ. Clearly, there exist n and $\gamma^*<\tilde{\gamma}$ such that $\gamma^* < n < u(\gamma^*)$ and $\frac{\delta^{n-1}(1+\delta)}{1+\delta+\ldots+\delta^{n-1}}\gamma^* - U > 0$ for large enough δ, i.e. (13) holds. It can be seen immediately that (9) and (12) are non-negative for sufficiently large δ, too. Suppose that the type *II* agent produces a good of quality $\gamma < \frac{1-\delta^{m+1}}{1-\delta^n}\gamma^* - 1$ and intends to charge (*m*-1)*x* units of money plus a good for it. Now $u(\gamma) < u\left(\frac{1-\delta^{m+1}}{1-\delta^n}\gamma^* - 1\right) < \left[\frac{1-\delta^{m+1}}{1-\delta^n} - \frac{1}{\gamma^*}\right]u(\gamma^*)$ where the second inequality results from the assumption ii) of proposition 1. $1/\gamma^*$ is larger than $1/n$ and $\frac{1-\delta^{m+1}}{1-\delta^n} - \frac{1}{n}$ is as close to $\frac{1-\delta^m}{1-\delta^n}$ as desired when δ is sufficiently close to unity. This shows that condition (A2) cannot hold for δ large enough. Similarly, it can be shown that there is not a deal of the following kind that is advantageous to both agents; type *I* agent pays a lower than market price and keeps his production good, and type *II* agent produces a lower quality good γ, $\gamma < \gamma^*$. Thus, there exists a monetary equilibrium in which type *II* agents produce a good of quality γ*, and whose price is $(n-1)x$ units of money plus a unit of good.n

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