Optimal Information Management: Organizations versus Markets*

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Abstract

This paper examines the optimal structure of delegation when a decision-maker must have some mass of information processed to make a decision. She can either delegate to agents working in her own organization, in which case she retains full authority, or she hands over this authority to an outside supplier, and outsources these tasks. By incorporating authority in a stylized model of information processing, we endogenize the comparative advantage of each form of delegation, and provide novel microfoundations for the make-or-buy decision. We outline precise conditions under which giving up authority is optimal. We also show which tasks must be outsourced to align the preferences of the outside supplier on their own preferences, and thereby maximize the benefits accruing from outsourcing.

Keywords: information processing, boundaries of the firm, authority, delegation.
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1 Introduction

This paper examines how a decision-maker optimally acquires and processes information before making a decision. When the amount of information becomes too large, she will find it profitable to delegate an increasing number of tasks to agents in her organization, or even contract them out. This gives rise to a nexus of hierarchical "processing units," where units lower in the hierarchy process information for their superiors, who eventually channel it to the principal. The information processing literature analyzes this type of problems, and helps us understand why tasks are delegated, why hierarchical structures efficiently centralize authority while decentralizing processing, among other issues.[1] However, a traditional limitation of such approach is that "*the optimal hierarchies derived could apply just as well to the organization of production in the U.S.A. as to the organization of production in Microsoft*" (Hart and Moore, 1999).[2,3] A second limitation is that the hierarchical position of a unit does not relate to any concept of authority: some agent A is "hierarchically above" another agent B when B provides information to A, not because A has authority over B.

To address these limitations, we propose a model of information processing in which we incorporate a concept of *authority* in the spirit of Aghion and Tirole (1997) and Hart and Moore (1999). In our setup, the amount of authority the principal has over agents depends on her decision whether to retain activities in-house or to outsource them to independent organizations. In the former case the principal has, by definition, full *authority* over the agents: she controls both their hierarchical position and their task. In the latter case instead she hands over authority to an independent decision-maker (the outside supplier). To the best of our knowledge, our paper is the first one bringing in the issues of the boundaries of the firm into the information processing literature.[4]

Using this approach allows us to go beyond the internal structure of each organization and

[1] See, e.g. Radner (1992, 1993), Bolton and Dewatripont (1994), Garicano (2000) or Van Zandt (1997, 1998, 1999).

[2] Note that this limitation is also stressed by the original authors. For instance: "[...O]*ur model* [...] *could concern subsets of firms, as well as some interfirm relations*" (Bolton and Dewatripont 1994, p810), or "[...T]*he boundaries of firms are determined, in part, by the relative efficiency of markets and bureaucracies for processing information. However,* [the authors] *did not make a direct comparison with how markets perform similar tasks.*" (Van Zandt 1997, p4)

[3] Holmström and Roberts (1998) detail that "*by leveraging its control over software standards, using an extensive networks of contracts and agreements [...], Microsoft has gained enormous influence in the computer industry and beyond*" (p85), which shows that such hierarchies go beyond a single organization. Moreover, thanks to this delegation process, the stock market value of Microsoft was at the time as high as $10 million per employee, which, they argue, cannot be attributed to asset ownership.

[4] For instance, Hart and Moore (1999) choose to ignore the informational approach and focus on the authority provided by asset ownership. They however point out that "in future work, it would be desirable to combine the informational approach in the literature and the authority approach in this [their] paper".

provide novel microfoundations for the relative advantage of in-house processing versus outsourcing. Quite clearly, this is closely connected to the vast literature on vertical contracting and holdup problems (Coase 1937, Williamson 1975 and 1985, Grossman and Hart 1986, Hart and Moore 1990, and many others) that addresses the issues of transaction costs, ownership, relationship-specific investments, and incomplete contracts. In the main analysis, however, our model will abstract from these issues, to isolate the effects of the loss of authority on the outsourcing decision and show how it affects the boundaries of the firm as well as the equilibrium structure of market.

We show that the principal generally benefits from relinquishing authority, and thus from outsourcing. Yet, outsourcing all activities is never optimal. The reason is that, if too many tasks were outsourced, there would be a conflict of interests among the principals whose preferences are not perfectly aligned. Put differently, principals would then suffer from losing authority. By contrast, if only some tasks are outsourced –namely those for which each of the principals have aligned interests– principals will induce the outside supplier to become more efficient than what they could achieve in-house.

To gain further understanding of these results, it is useful to provide details of the model. A commonly accepted benefit of task delegation is to free up time for the principal and reduce the total delay of information processing. Our benchmark model –inspired by Radner (1993) and Bolton and Dewatripont (1994)– rationalizes delegation with the same argument. It only departs from their approach in two respects. First, since we want to incorporate incentive issues into the model, we let the performance of an agent depend on the amount of effort he exerts: the more effort, the better information is processed (i.e. the smaller, the more summarized is the information transmitted to the superior). In this way, we endogenize the productivity of each agent, given the incentives of the agents and of the principal. Second, to focus on the issues we choose to emphasize, we simplify the model of the hierarchy: we abstract from the possibility of skip-level reporting (see, *e.g.* Radner, 1993), and adopt a simplified version of communication costs. Moreover, we assume identical agents (unlike Prat, 1997) and information elements (Garicano, 2000), abstract from agents making errors (Sah and Stiglitz, 1986), from environmental uncertainty (Meagher et al. 2001), and other considerations we do not mention here for the sake of brevity.

As a benchmark, we first consider the case in which the principal retains all tasks in-house. We find that in-house processing generates minimal delays when agents are willing to work for free. If meeting their participation constraint requires they are paid a strictly positive wage, the principal adjusts the structure of her organization to trade off the benefit of reduced delay against the cost of a higher wage bill. The next question we address is how outsourcing may deliver more efficient outcomes. If the principal is alone on the market, replacing the organization by an outside supplier is shown to be a dominated action: at best, the outside supplier reproduces the organization of the principal. Yet, industries are seldom composed of one firm. Several principals (possibly with different needs) coexist on the market. If the market is sufficiently large, that is if there are sufficiently many principals who consider outsourcing tasks, the incentives of the outside supplier become substantially different.

If different principals choose to outsource tasks to a same supplier, the latter becomes their *common agent* (Bernheim and Whinston, 1986). The question is then whether this loss of authority is costly or beneficial. We first show that, if the preferences of all principals are perfectly congruent, that is if they want exactly the same information set processed, their common supplier always sets up an organization that reduces delay below the level obtained under in-house processing. In other words, there is room for efficiency-improving outsourcing. In addition, outsourcing generates scale economies at the industry-wide level. Hence, losing authority is generally beneficial in the perfect congruence case.

Yet, assuming perfect congruence lacks realism. Introducing heterogeneity among the principals shows that outsourcing may become less efficient than in-house processing, even if market size is large: losing authority becomes prohibitively costly. Under in-house processing at least, each principal dictates her agents to exert a sufficient amount of effort on tasks that are of direct interest to her. Under outsourcing instead, the supplier focuses less on the idiosyncratic needs of a given principal when the number of outsourcing principals increases.[5] For this reason, principals may prefer to retain all tasks in house.

The value of outsourcing thus appears to be increasing in the level of congruence between the principals. As we further show, however, even principals with differing objectives can make their preferences *appear* perfectly congruent in the eyes of the common supplier, and thereby reduce the costs of losing authority. This simply requires that only *common business activities* –activities for which all principals have common needs– be outsourced. Instead, remaining tasks (which become the "core business" of the principals' organizations) must always be maintained in house, so that the principal retains entire control on them. The eventual outcome is thus that the "hierarchy" is actually a set of different organizations, which themselves are a collection of an endogenous number of agents. Comparative statics on exogenous outsourcing costs and on market size then allow us to make predictions about the evolution of organization size and of total employment in an industry when external conditions vary, as well as on when principals switch from in-house processing to outsourcing or conversely.

The remainder of the paper is structured as follows. Section 2 lays out our model of information processing in organizations, and Section 3 shows how organizations should be structured, given the processing technology and the other parameters of the model. In Section 4, we turn to outsourcing and show that, under certain conditions, principals prefer dealing

[5] This parallels the analysis of Hart and Holmström (2002), where the scope of the firm affects the preferences and the behavior of "bosses".

with independent suppliers even though this means giving up a part of their authority. In Section 5, we discuss issues that are more closely related to the contracting literature, and incorporate the possibility of hold up or ex post bargaining. Finally, Section 6 concludes.

2 The Model

Our benchmark model considers a decision-maker (or "principal" – we use the two terms interchangeably), who has to process an exogenous amount of information in order to reach a decision. She is free to establish an organization to speed up this process. In that case, she hires agents who will pre-process information. In this section, we derive the objective function of such an organization when the amount of effort exerted by these agents is endogenous. The basic trade-off that arises is one in which processing delays can only be reduced at the expense of a higher wage bill. In the next section, we analyze how this trade-off is solved, before showing in Section 4 that outsourcing can help circumventing the tension between delay and wages.

2.1 The Setup

Information processing. To make a decision, our principal must have some exogenous amount of information processed. Information items are atomless and the *initial set of information* has mass M. Agents can reduce the size of this set by pre-processing information, following a technology we describe below. Then, like in Bolton and Dewatripont (1994), if the information set reaching the principal has a mass M_0 ($\leq M$), she must spend $e_0\, M_0$ units of time at processing this set before making her decision. In the literature, e_0 is commonly called the *overload* of the principal.

The information set can be freely subdivided into subsets with positive mass m_i among the different agents, who are indexed by i. Individual agents can exert a variable amount of *effort* at screening each piece of information. Assuming that the amount of effort per unit of time is constant, the more time (total effort) they spend on these items, the smaller the size of their output, and thus the simpler the task of their superior. To interpret this, think for instance of the news: the mass of information to be processed is the set of events taking place in the world. An organization (TV channel or newspaper) produces summary information. The task of each agent/journalist is to gather the information to be broadcasted, and the more time spent by a subordinate journalist at preparing the set of articles, the easier the work of the presenter or editor.

One difference between our setup and that of Bolton and Dewatripont (1994) or Radner (1993) is thus that the agents' productivity will be endogenized. Technically, we assume that an agent i who processes some mass of information can reduce this mass to a fraction $f(e_i) \leq 1$ of its initial mass by exerting a variable amount of effort e_i. Effort is productive, in the sense that $f(0) = 1$ and $f'(e_i) \leq 0$.[6] On another hand, effort is costly: the amount of time needed to process a set of information ("gather news") with mass m_i while exerting an effort e_i is equal to $e_i\, m_i$ units of time. There are decreasing returns to effort ($f''(e_i) > 0$), and we also assume that $\lim_{e_i \to \infty} f(e_i) > 0$.

Costs. For the sake of tractability, the marginal opportunity cost of an agent's time is set equal to some exogenous value $w (\geq 0)$. Thus, the principal must pay a wage at least as large as $w\ e_i\, m_i$ to satisfy the participation constraint of an agent who works $e_i\, m_i$ units of time. Similarly, we assume that the marginal cost associated with delay is an exogenous constant $r\,(> 0)$.

Organizations. The principal is free to hire as many agents as she wants in her organization. The organization is made of information processing agents who are positioned in *layers*, denoted by $l = 1, 2, ..., L$, which work sequentially.[7] This means that information is first processed by the lowest layer L. Once agents in layer L completed their task, their output is sent to layer $L-1$, and then to layer $L-2$, and so on, until layer 1 transmits the remaining amount of information to the principal (layer 0).[8] Figure 1 below illustrates such a hierarchy.

Figure 1: The general shape of a hierarchy

	Principal
	↑
Layer 1:	$a_{1,1}.....a_{1,n_1}$
	↑
Layer 2:	$a_{2,1}.....a_{2,n_2}$
	⋮
Layer L:	$a_{L,1}.....a_{L,n_L}$
	↑
Initial information batch:	M

Importantly, we assume that the principal has full *authority* over the agents inside the organization. More precisely, we define an organization as an entity in which the principal

[6] Note that this functional form implicitly assumes that productivity is independent of the number of times information has been processed before, like in Radner (1993).

[7] That is, we abstract from skip-level reporting (see Radner (1993) or Bolton and Dewatripont (1994) for a comparison).

[8] In Castanheira and Leppämäki (2003), we analyze a simpler version of the model, and show that the essence of the results remains identical with a just-in-time version of the model.

can directly contract on the amount of effort exerted by each agent.[9] The agent's only choice will then be to accept or reject the contract offered by the principal (see below).

A priori, increasing the number of agents in a layer can only speed up processing, thanks to an increased division of labor. For instance, consider the simplest organization, with one layer only. If the initial set of information is equally divided among the agents, and if n_1 agents are hired in layer 1 to exert an effort level e_1 (> 0), the delay needed by that layer is: $d_1 = e_1\, M/n_1$. Thus, the larger is n_1, the shorter is the delay d_1. Yet, we know that big organizations eventually become more difficult to manage and face higher total (production) costs. One of the reasons is that increased coordination and communication problems eventually reduce their performance (Radner 1993, Bolton and Dewatripont 1994). For the sake of tractability, we introduce these costs in a reduced form, and assume that each agent slows down the organization by some fixed delay λ.[10] To avoid confusion, we call this cost λ "coordination costs".

2.2 The objective function

We are now in a position to derive the objective function of the principal. Delegating tasks to agents may speed up information processing, although at the expense of an increased wage bill. Her problem is thus one of task assignment; each agent must be requested to work on some information subset and exert some amount of effort in order to minimize the joint cost of delay and of wages. For any given organizational structure with N agents, the principal thus faces an optimization problem with $2N$ control variables. Lemma 1 however shows that any agent belonging to a given layer l should be assigned a same task $\{e_l, m_l\}$:

Lemma 1 *To minimize processing costs, all agents i in a given layer l must process the same mass of information and exert the same level of effort: $e_i = e_l$ and $m_i = m_l$ $\forall i \in l$.*

Proof. *See Appendix 1.* ■

This result implies that all agents in a given layer become "homogenous," in the sense that they perform a similar task. It follows that:

$$M_l = M \cdot \prod_{j=l+1}^{L} f(e_j), \ \forall l < L \text{ and } d_l = e_l \frac{M_l}{n_l} + \lambda\, n_l.$$

[9] Note that, if effort was exogenous, the principal would implicitly be also able to contract on the effort exerted by agents *outside* the organization. Such an assumption is the one generally made in the literature. By endogenizing effort, instead, we shall be able to distinguish between the costs and benefits of outsourcing.

[10] Formally, communication costs à la Radner-Dewatripont-Bolton could be considered, if we were assuming that the cost of adding one agent in layer l were some function $\lambda(n_l, n_{l-1}, n_{l+1})$, which they derive from the fundamentals of their model.

That is, the mass of information reaching layer l is only determined by the effort levels in each layer below l and, since tasks are identical in a layer, the delay needed to process information is inversely proportional to the number of agents in that layer ($m_l = M_l/n_l$). This stems from increased division of labor, or parallel processing. However, the more agents in the layer, the higher are coordination costs, $\lambda\, n_l$.

Adding up these delays, the *total delay* needed by the organization, D_{Org}, becomes:

$$D_{\text{Org}}\left(\{e_l\}, \{n_l\}, L\right) = e_0\, M_0 + d_{\text{Org}}, \text{ where } d_{\text{Org}} = \sum_{l=1}^{L} d_l,$$

in which $\{e_l\}$ denotes the vector of effort levels, $\{n_l\}$ the vector of the number of agents per layer, and L stands for the total number of layers in the organization.

Finally, since effort is contractible inside the organization, wages must only satisfy the participation constraint of the agents: $w_i \geq e_i\, m_i$. Thus, total wage costs are equal to:

$$W_{\text{Org}}\left(\{e_l\}, L\right) = w \cdot \sum_{l=1}^{L} n_l\left(e_l\, m_l\right) = w \cdot \sum_{l=1}^{L} e_l\, M_l,$$

and the objective function of the principal is:

$$\min_{\{e_l\},\{n_l\},L} TC_{\text{Org}}\left(\{e_l\}, \{n_l\}, L\right) = r\, D_{\text{Org}}\left(\{e_l\}, \{n_l\}, L\right) + W_{\text{Org}}\left(\{e_l\}, L\right). \tag{1}$$

Solving this optimization problem will allow us to endogenize the cost of decision-making, as well as the associated structure of the organization. This is the purpose of the next section.

3 In-house processing

In this section, we derive the optimal structure of the organization as well as the optimal effort levels under in-house processing. In this setup, the principal has full authority over the agents, and controls all choice variables. We show that, despite full authority, inefficiencies arise –both in terms of increased delay and wage costs– when wages are positive. By contrast, Section 4 will analyze when and how outsourcing can overcome such inefficiencies.

Most of the analysis in the literature focuses on delay minimization, which will prove to be a useful benchmark in our analysis as well. Given the control variables at hand, delay minimization imposes:

$$\frac{\partial D_{\text{Org}}}{\partial e_l} = 0; \quad \frac{\partial D_{\text{Org}}}{\partial n_l} = 0. \tag{2}$$

From (1), however, it is straightforward to see that *total cost* minimization imposes instead:

$$r\frac{\partial D_{\text{Org}}}{\partial e_l} = -\frac{\partial W_{\text{Org}}}{\partial e_l}; \quad \frac{\partial D_{\text{Org}}}{\partial n_l} = 0. \tag{3}$$

That is, the only case in which cost minimization coincides with delay minimization is when wages are zero. Substituting for D_{Org} and W_{Org} into (3), and solving for optimal effort levels, layers size, and hierarchical structure, we can state our first proposition, which lays foundations for later analysis:

Proposition 1 *Under in-house processing, optimal effort levels are determined recursively:*

$$f'\left(e_1^*\right) = -\frac{1/n_1 + w/r}{e_0}, \tag{4}$$

$$f'\left(e_l^*\right) = -\frac{1/n_l + w/r}{1/n_{l-1} + w/r}\left(e_{l-1}^* - \frac{f\left(e_{l-1}^*\right)}{f'\left(e_{l-1}^*\right)}\right)^{-1}, \forall l \geq 2, \tag{5}$$

and each effort e_l^* is increasing in e_0 and in layer size, n_l. In turn, optimal layer size, n_l^*, as well as the optimal number of layers L^*, are strictly increasing in M and e_0. Finally, the equilibrium delay needed by the organization is monotonically increasing in w/r.

Proof. See Appendix 2. ■

Thus, rather unsurprizingly, total delay is minimized either when $w = 0$ or $r \to \infty$. For w/r strictly positive, cost minimization calls for a different structure of the organization. Higher wages not only increase the total costs of decision-making but also total delay: to limit the impact of increased wage costs, the principal adapts the organization, delegates less, and hence processes a larger fraction of the initial information set by herself.[11] Conversely, given the wage level, a higher r induces the principal to select an organizational structure that generates shorter delays.

Even though we are treating r as exogenous in our analysis, it is easy to relate it to some economic fundamentals and examine how changes in those affect the level of this marginal cost of delay. Assume for instance that processing any information set M generates a return equal to R, which is itself a function of aggregate demand, market size, and the like. The quicker one such set is processed, the earlier the organization can start processing another set, thereby increasing total profits over some period of time. Hence, the higher is aggregate demand or the more "global" is the market, the closer to delay minimization is the objective function of the principal. Yet, the next section will show that higher aggregate demand is not necessary for an outside supplier to value delay minimization higher.

Proposition 1 also shows that, when decision-making becomes more time-consuming (e_0 or M increases), the principal increases the size of her organization along two dimensions. Both the size of each layer and the number of layers are increased. This shows that time-consuming

[11] In Castanheira and Leppämäki (2003), we show that, in the case of *exogenous* effort levels, only the optimal number of layers is affected by wages.

tasks call for increased "task specialization" (the number of different layers/tasks increases). Prat (1997) showed that, if different types of agents coexist, each different layer should be staffed with agents of a different type. In contrast with his approach, we consider agents that are ex ante completely homogenous. Yet, depending on their hierarchical position, the agents' task becomes different ex post.

4 Outsourcing information acquisition

As shown in the previous section, the best the principal can do is to adapt the structure of her organization to the current economic environment. She can generally not maintain speed and profitability at the same time. Can she resolve this tension by contracting tasks outside the organization? At first glance, since inefficiencies cannot be avoided under full authority, outsourcing should only make matters even worse. In our setup, losing authority should be akin to an increase in "transaction costs," in the terminology of Coase and Williamson.

Yet, as Abraham and Taylor (1996), Domberger (1998) or Hummels et al. (1998) illustrate, outsourcing is a widespread phenomenon. In the US, for instance, "*the market for outsourced services was estimated to be $100 billion in 1996*", and "*outsourcing is particularly widespread in the IT and business services area*" (Domberger 1998, p21). Outsourcing typically involves more than production: the whole decision-making process (R&D, choice of production methods) tends to be contracted out. Furthermore, it is not uncommon to observe that competing companies (say, Nokia and Ericsson) outsource some activities to a common supplier (here: Elcoteq, a Finnish producer of communication equipment).[12] Other examples abound. Think for instance about the car industry, where electrical component producers (like Bosch) supply competing car producers; or about international news agencies that sell the same pre-processed information to various local newspapers and other forms of media.

These observations raise several questions. First, incentive problems should become more severe under outsourcing, since outsourcing necessarily increases the number of intermediaries: unless the mass of information is trivially small, the outside supplier will also have to hire agents within his own organization. Hence, the principal will be dealing with these agents only *indirectly*, through the authority of the outside supplier. A first question to address is thus why principals prefer to trap themselves into such arm's length relationships, even though they could hire agents directly. Second, different principals are shown to outsource to common suppliers. That is, they choose to link themselves to a *common agent.* Hence, a second question to address is why common agency is efficient.

[12] Elcoteq's advertisement (www.elcoteq.com) states that it "*provides engineering and manufacturing services, supply chain management and after-sales services to international high-tech companies,*" which highlights its stress on making decisions ("engineering services") for its customers.

To address these questions, we extend the model of Section 3 and allow the principal to contract out processing tasks to an outside supplier, in which case the supplier dictates the hierarchical position and effort level of the agents in his organization. Outsourcing thus creates an incentive problem, in the sense that the supplier maximizes his own profits, and not that of the principal. As we show below, the magnitude of this incentive problem will depend on how different the principals' preferences are.

The contract between the principal and the outside supplier specifies the delivery of an information set of mass M^S within a given delay d^S, against the payment of some price p. That is, contracting is made on the output of the supplier, not on the task of the agents. Hiring an outside supplier entails some non-negative fixed costs C (see Domberger 1998, p60-67), which we assume are borne by the principal and expressed in delay units. Clearly, trade between a principal and a supplier can only happen when it generates a surplus. We assume that this surplus goes to the supplier.[13] In the spirit of Bernheim and Whinston (1986), this makes the supplier (common agent) residual claimant in equilibrium, which gives him high-powered incentives – see below.

The industry is composed of several principals, who are indexed by $j = 1, 2, ..., \Omega$, where Ω represents the *number of principals* in the industry or the *size of the market.* Each principal j is characterized by the information set M_j she needs to have processed. For symmetry, all information sets M_j are assumed to have a same mass M. Yet, these sets may only partially overlap: $\alpha \in [0, 1]$ is a *congruence parameter*, that represents the fraction of items that is common to *all* principals in the industry. The remaining fraction $(1 - \alpha)$ is specific to *one* principal only.[14] Accordingly, we call the former fraction "*industry-wide*" or "*common*" information, and the latter "*specific*" information. Importantly, note that this notion of specificity does not relate to any exogenous comparative advantage at processing different parts of the information set, since any organization has access to the same technology.

4.1 Full Outsourcing

We first consider the case in which principals are constrained to outsource either *all* or *none* of their activities to a common supplier.[15] Moreover, to facilitate comparisons with the in-house processing case analyzed earlier, it is assumed here that the principals do not hire any agent "in-house" if they opt for full outsourcing. The total processing costs faced by a given

[13] Results immediately extend to the case in which ex post bargaining implies that some of the surplus goes to the principal – see section 5.

[14] A more general case is presented in section 5.

[15] Clearly, to make matters interesting, we assume that M and e_0 are sufficiently large, so that (for $w = 0$) the optimal level of delegation is strictly positive under in-house processing.

principal j thus become:

$$TC_{\text{Full}}(j) = r\left(d^S_{\text{Full}}(j) + M^S_{\text{Full}}(j)\ e_0\right) + rC + p(j),$$

where d^S_{Full} and M^S_{Full} are respectively the delay needed by the supplier, and the mass of his output. The functional shape of these is introduced below, and will be seen to depend on the degree of congruence among principals. Next, C is the cost of hiring an outside supplier, and $p(j)$ is the price paid for the service supplied. After the supplier completed his task, the principal still has to process the remaining items, which entails an additional delay $M^S_{\text{Full}}(j)\ e_0$.

Clearly, the principal has an incentive to fully outsource her activities if and only if $TC_{\text{Full}}(j) \leq TC_{\text{Org}}(j)$, that is if outsourcing reduces processing costs. Using the results of Section 3, this condition amounts to:

$$p(j) \leq r\,D_{\text{Org}} + W_{\text{Org}} - r\left(d^S_{\text{Full}}(j) + M^S_{\text{Full}}(j)\ e_0\right) - rC,$$

and the supplier clearly selects the price that makes this condition binding.

The equilibrium values of d^S_{Full} and M^S_{Full} depend on the outside supplier's behaviour. Following the same methodology as in the previous section, we derive his objective function and, from there, his equilibrium behaviour. His profits are:

$$\begin{aligned}\pi_{\text{Supplier}} &= \sum_j p(j) - W^S_{\text{Full}} \\ &= \sum_j \left[K(j) - r\left(d^S_{\text{Full}}(j) + M^S_{\text{Full}}(j)\ e_0\right)\right] - W^S_{\text{Full}}, \qquad (6)\end{aligned}$$

with $K(j) = r\,D_{\text{Org}}(j) + W_{\text{Org}}(j) - rC$, and W^S_{Full} being the total wage cost entailed by the supplier under full outsourcing. The latter has high-powered incentives since he reaps all the benefits from improving information processing *i.e.* from reducing processing delays, wage costs and/or size of the remaining information set (M^S_{Full}). For instance, if there were only one principal ($\Omega = 1$), the supplier would find it optimal to exactly reproduce the organization of the principal: wage costs and the marginal return to effort are identical. Hence, the optimal effort levels, number of agents per layer, and total number of layers would be identical, by Proposition 1. However, the outsourcing cost C implies that contracting out cannot profitable in that situation.

When more than one principals outsource to the supplier, the latter is alone to process the union of his customers' information sets. Therefore, outsourcing may end up increasing delay, since the latter, as well as the size of the supplier's organization, are also increasing in the mass of information processed, for the same reasons as in Proposition 1. We shall say that outsourcing *increases efficiency* if it reduces processing delay below the one obtained under in-house processing. On the other hand, outsourcing concentrates wage costs into a single organization (that of the supplier) instead of duplicating them in each principal's organization. This is an endogenous *scale economy effect*, already highlighted in the literature (see e.g. Domberger 1998).

Case 1 below presents a case in which the needs of all principals are exactly aligned (the congruence parameter α is 1). In that case, outsourcing is shown to both increase efficiency and generate scale economies. Case 2 instead considers a situation in which principals have differing needs ($\alpha < 1$). In that case, the benefit of scale economies is still present but, for a large industry size, losing authority generates costs larger than this benefit.

Case 1: No specific information. There is no specific information when $\alpha = 1$, that is when all principals use the same information set. In this case, congruence is perfect. All information is industry-wide, and there is no conflict of interest between principals; all share a common goal.

If several principals outsource to a common supplier, the latter becomes their common agent. From the results of Bernheim and Whinston (1986), we know that the common agent (supplier) maximizes the joint profit of such an industry. Assuming that all principals outsource to the same supplier, and using a superscript 'S' to denote a variable under the control of the supplier, his objective becomes (we drop the index j from now on):

$$\max_{L^S,\{e_l^S\},\{n_l^S\}} \pi_{\text{Supplier}} = \Omega\left[K - r\left(d_{\text{Full}}^S + M_{\text{Full}}^S\, e_0\right)\right] - w\sum_{l=1}^{L^S} e_l^S\, M_l^S, \text{ with:}$$
$$\begin{cases} d_{\text{Full}}^S = \sum_{l=1}^{L^S}\left(\frac{e_l^S}{n_l^S} M \prod_{l'=l}^{L^S} f\left(e_{l'}^S\right) + \lambda\, n_l^S\right), \\ M_{\text{Full}}^S = M \prod_{l=1}^{L^S} f\left(e_l^S\right). \end{cases} \tag{7}$$

Since all principals are perfectly congruent, as one reads from (7), the supplier processes an information set of mass M, which is why both delay and M_{Full}^S are only proportional to M. It does not depend on the number of principals who outsource to this supplier, which in turn maximizes the scale economy effect of outsourcing. Under in-house processing, Ω principals would separately process a set of size M. Under full outsourcing, this set is processed by one organization only.

The objective function of the supplier (7) directly compares with (1). The only relevant difference between the two objective functions is that the marginal cost of time taken into account by the supplier is Ω times larger than under in-house processing. Applying Lemma 2 (see Appendix 1), this yields:

$$f'\left(e_1^{S*}\right) = -\frac{1/n_1^{S*} + w/\left(r\Omega\right)}{e_0}; \tag{8}$$

$$f'\left(e_l^{S*}\right) = -\frac{1/n_l^{S*} + w/\left(r\Omega\right)}{1/n_{l-1}^{S*} + w/\left(r\Omega\right)}\left(e_{l-1}^{S*} - \frac{f\left(e_{l-1}^{S*}\right)}{f'\left(e_{l-1}^{S*}\right)}\right)^{-1}, \forall l \geq 2. \tag{9}$$

From these conditions, it follows that the common supplier behaves exactly as a principal who would be facing a marginal cost of delay equal to Ωr instead of r. Given the results of

Proposition 1, this implies that full outsourcing must reduce processing delays when wages are strictly positive:

Proposition 2 *For $\alpha = 1$ and $w = 0$, full outsourcing is always dominated by in-house processing. For $w > 0$, $\Omega > 1$, and $rC \leq (1 - 1/\Omega) W_{Org}$, all principals outsource to a common supplier, and outsourcing reduces delays.*

Proof. If $w = 0$, a principal creates the organization that minimizes processing delays (see Proposition 1). Hence, outsourcing can only reduce profits. For $w > 0$ and $\Omega > 1$, by (8) and (9), outsourcing reduces processing delays compared to in-house processing. Hence, $\forall rC \leq W_{\mathrm{Org}}$, principals are willing to pay a strictly positive price to the supplier and $rC \leq (1 - 1/\Omega) W_{\mathrm{Org}}$ ensures that this price is high enough for the supplier to make positive profits. ■

Proposition 2 shows that full outsourcing may be preferred to in-house processing when wages are high. When wages are zero, outsourcing cannot bring any additional benefit.[16] For positive wages instead, in-house processing may simply be too costly. The rationale for this result is that high wages have less of a distortionary effect on a common supplier. As we stressed in the previous section, distortionary effects are inversely proportional to the marginal cost of delay, and this cost proves to be higher for a common supplier, since any reduction in delay allows him to increase his price on *each* principal. It thus appears that market provides better incentives than authority per se. In this case, losing authority can only be beneficial.

The second benefit of outsourcing is that wage costs are concentrated into one single organization (that of the supplier). This is the scale economy effect stressed above. Yet, scale economies only play a secondary role in motivating outsourcing: even if we eliminate these, *e.g.* by increasing C up to $(1 - 1/\Omega) W_{\mathrm{Org}}/r$, outsourcing remains a dominant strategy: in the absence of specific information, outsourcing always increases efficiency.

Conversely, normalizing r to 1 and letting Ω be sufficiently large, one can check that there is a threshold $\bar{w}(C)$ such that, for any value of w below this threshold, information is processed in-house. Within the range $[0, \bar{w}(C)]$, the closer to zero is w, the more efficient is information processing (see Proposition 1). However, if w increases above the threshold $\bar{w}(C)$, information processing will be outsourced. In the limit (for $\Omega \to \infty$), the supplier minimizes total processing delays, for any finite w. High wage levels hence imply that information is still processed quickly, although at the expense of higher outsourcing costs (each of the Ω principals bears a cost C). Thus, for moderate levels of C, larger market size increases the value of outsourcing.

[16] Results would of course be different at the margin had the principals and the supplier access to different technologies. This latter aspect is often at the heart of the firm's specificities in a Williamsonian type of analysis of the boundaries of the firm.

The predictions of this proposition may sound somewhat counterintuitive, since only two types of outcomes can arise: either all principals process all information in-house, or all outsource to one single supplier. Such a prediction is unlikely to be met in actuality. As the second case shows below, however, this all-or-nothing outcome only results from the assumption of perfect congruence.

Case 2: Introducing specific information. In the above setting, processing delays under outsourcing can never exceed those obtained under in-house processing. Profit maximization induces the supplier to make decisions that maximize efficiency, because all principals share a common goal. Things are quite different when the objectives of the different principals are not perfectly aligned: dedicating extra resources to one principal then means that delays must increase for all other principals. If some principal j had authority over the agents who work for the supplier, she would require them to focus on her own information set, and discard the information needed by the others. Under outsourcing, however, principals do not have this authority. Only the supplier controls the allocation of tasks within his organization.

In this extended case, a supplier with Ω customers must process the union of all the information sets of the principals, that is an information set with mass:

$$M_{\text{Total}} = \left[\Omega\left(1-\alpha\right)+\alpha\right]\, M,$$

since there are Ω disjoint subsets with mass $\left(1-\alpha\right)M$, and one common, industry-wide, subset with mass αM. Therefore, the delay needed by the supplier will now be proportional to M_{Total} (and hence depend on Ω). By contrast, under in-house processing each principal dictates her agents to focus exclusively on her information set, and delays remain proportional to M. On the other hand, outsourcing still generates scale economies at the industry-wide level, since $M_{\text{Total}} < \Omega M$, and wages are concentrated in one organization. A priori, full outsourcing could still be profitable for both parts. Yet, we find that lack of congruence between the principals magnifies incentive problems; it makes it costly to lose authority. Denoting again by $M^S_{\text{Full}} \equiv M \prod_{l=1}^{L^S} f\left(e^S_l\right)$ the mass of information reaching each principal under full outsourcing, the objective function of the common supplier indeed becomes:

$$\min_{L^S,\{e^S_l\},\{n^S_l\}} \Omega r \left(\sum_{l=1}^{L^S}\left(M_{\text{Total}}\times e^S_l\left(\frac{1}{n^S_l}+\frac{w}{\Omega r}\right)\prod_{l'=l+1}^{L^S} f\left(e^S_{l'}\right)+\lambda\, n^S_l\right)+M^S_{\text{Full}}\, e_0\right) \tag{10}$$

with the only difference between (10) and (7) above being that the delay is proportional to M_{Total}, whereas each principal only cares about her own information set, M^S_{Full}. Rewriting (10) by factoring in M_{Total} and substituting for M^S_{Full} yields:

$$\Omega r\, M_{\text{Total}}\left(\sum_{l=1}^{L^S}\left(e^S_l\left(\frac{1}{n^S_l}+\frac{w}{\Omega r}\right)\prod_{l'=l+1}^{L^S} f\left(e^S_{l'}\right)+\lambda\, n^S_l\right)+\frac{M}{M_{\text{Total}}}e_0\prod_{l'=1}^{L^S} f\left(e^S_{l'}\right)\right).$$

Applying again the results of Lemma 2, the first order conditions for the supplier follow immediately:

$$f'\left(e_1^{S^*}\right) = -\frac{1/n_1^{S^*} + w/[r\Omega]}{e_0/[\Omega(1-\alpha)+\alpha]}; \quad (11)$$

$$f'\left(e_l^{S^*}\right) = -\frac{1/n_l^{S^*} + w/(r\Omega)}{1/n_{l-1}^{S^*} + w/(r\Omega)} \left(e_{l-1}^{S^*} - \frac{f\left(e_{l-1}^{S^*}\right)}{f'\left(e_{l-1}^{S^*}\right)}\right)^{-1}, \forall l \geq 2, \quad (12)$$

that is, like in Case 1, the supplier behaves "as if" the marginal cost of delay were Ω times larger than its actual value, which should make process information more intensely. However, he also behaves "as if" the overload of each principal were a fraction $1/[\Omega(1-\alpha)+\alpha]$ of its actual value, e_0. Altogether, (11) and (12) imply that effort levels are strictly increasing in the level of congruence, α. The rationale for this result is as follows. The profits of the common supplier increase when he can, at a relatively low wage cost, reduce processing delays for all principals in his portfolio of clients. However, when his clients do not have perfectly congruent interests, processing information for one given principal implies that delay is increasing for all other principals, which reduces his profits. Thus, the larger the clientele, the less important it is to pay attention to the specific needs of each single customer. In other words, the more clients the supplier has, the costlier it is to lose authority. Hence:

Proposition 3 *For $\alpha < 1$ and $\Omega \to \infty$, there is no equilibrium in which all principals fully outsource their activities to a common supplier, even for arbitrarily low outsourcing costs ($C \to 0$). In addition:*
i) *For w/r and α sufficiently small, in-house processing dominates full outsourcing, $\forall\Omega$.*
ii) *For intermediate values of w/r, $C \to 0$, and α sufficiently close to 1, full outsourcing dominates in-house processing. However, several suppliers (with a finite number of customers) must coexist on the market if Ω is large.*
iii) *For large values of w/r and $\alpha < 1$, each principal prefers to process all information by herself rather than fully outsourcing or hiring agents in-house.*

Proof. See Appendix 3. ■

Reading from (11), it is immediate to see why all principals never outsource to a common supplier if Ω is too large: $\Omega \to \infty$ implies that effort levels are equal to zero, and hence that the supplier does not process information at all. Thus, bearing the outsourcing cost C is a dominated action. Put differently, the number of customers served by a single supplier must remain sufficiently small, such that the costs borne by the loss of authority remain below the benefits of scale economies; different suppliers may co-exist on the market.

Proposition 2 already showed that full outsourcing is a dominated action for $w \to 0$. Since outsourcing is even less profitable when preferences are not fully congruent, part *i* of

Proposition 3 follows easily. Conversely, part *ii* simply states that, if outsourcing costs are sufficiently low, and if the preferences of the principals are sufficiently congruent, intermediate wages ensure that full outsourcing is still efficiency enhancing. Finally, since the number of customers served by a given supplier is bounded above, high wages also reduce the efficiency of the supplier, and hence the value of delegation altogether. Importantly, the comparison between Propositions 2 and 3 for $\Omega \to \infty$ demonstrates that this latter result only arises when losing authority over the agents is costly. The next question we address is thus how principals can reduce these costs.

4.2 Partial Outsourcing

The above results show that, except for perfectly aligned preferences (that is, if $\alpha = 1$), full outsourcing can be quite costly. It might thus be in their interest not to outsource *all* of their activities, but only the industry-wide, common, activities. In this case, principals retain "in-house" the fraction $(1-\alpha)$ of the information which is specific to them. The processing of this information thus becomes, in a sense, the "core business" of their organization, whereas they outsource the remaining fraction α, for which preferences are perfectly aligned throughout the industry. We call this case *partial outsourcing.*

Under partial outsourcing, both the principals and the supplier will hire agents, but process a different part of the information set. Hence, agents in different organizations will work in parallel, and we cannot determine in advance which organization will finish its task first. To maintain coherence with the assumptions of Section 2, we keep assuming that the principal can start processing information only when all agents have finished processing their respective batch. Hence, the delay needed to process all information becomes:

$$D_{\text{Partial}} = \max\left[d^P_{\text{Partial}}, d^S_{\text{Partial}}\right] + e_0 \left[M^P_{\text{Partial}} + M^S_{\text{Partial}}\right], \tag{13}$$

where a superscript P denotes a variable under the control of the principal and a superscript S a variable under the control of the supplier. M^P_{Partial} and M^S_{Partial} are the respective *output* of the two organizations.

In words, the total delay under partial outsourcing, D_{Partial}, is the maximum between 1) the delay needed by the principal's organization when it only processes a part of the whole batch (d_{Partial}) and 2) the delay needed by the supplier (d_{Supplier}) *plus* the delay needed by the principal to process the information transmitted to her by the two organizations together –the last term in (13).

Hence, under partial outsourcing, each principal bears a cost equal to:

$$TC_{\text{Partial}} = rD_{\text{Partial}} + W^P_{\text{Partial}} + rC + p, \tag{14}$$

in which W^P_{Partial} denotes the wages paid by the principal in the case of partial outsourcing. Like under full outsourcing, we compute the price that binds the incentive condition of the principal:

$$p_{\text{Partial}} = \tilde{K} - \left(r \max\left[d^P_{\text{Partial}}, d^S_{\text{Partial}}\right] + e_0\, M^S_{\text{Partial}}\right),$$

in which $\tilde{K} = r\left[d_{\text{Org}} + e_0\left(M_{\text{Org}} - M^P_{\text{Partial}}\right)\right] + \left(W_{\text{Org}} - W^P_{\text{Partial}}\right) - rC$, where a subscript 'Org' denotes the equilibrium value of a variable under in-house processing, as before. Given p_{Partial}, the profits of the supplier are:

$$\pi_{\text{Supplier}} = \Omega\left[\tilde{K} - r\left(\max\left[d^P_{\text{Partial}}, d^S_{\text{Partial}}\right] + M^S_{\text{Partial}}\, e_0\right)\right] - w\sum_{l=1}^{L^S} e^S_l\, M^S_l, \text{ with:}$$

$$\begin{cases} d^S_{\text{Partial}} = \sum_{l=1}^{L^S}\left(\frac{e^S_l}{n^S_l}\,\alpha M\,\prod_{l'=l}^{L^S} f\left(e^S_{l'}\right) + \lambda\, n^S_l\right) \\ M^S_{\text{Partial}} = \alpha M\,\prod_{l=1}^{L^S} f\left(e^S_l\right) \end{cases} \tag{15}$$

The difference between (15) and (10) is thus essentially that the delay needed to process information under partial outsourcing is now only proportional to αM, instead of $[\Omega(1-\alpha)+\alpha]M$ above. Importantly, partial outsourcing achieves more than a reduction in the mass of information to be processed by the supplier. By maintaining "core business" activities in house, principals only outsource the fraction of the information set for which their preferences are perfectly aligned. In this way, they manage to recover from the loss of authority they were experiencing under full outsourcing:

Proposition 4 *For any $w \geq 0$, $\alpha \in (0,1)$, there exists a non-empty set $(0, C^{\max}_{Partial}]$ such that if C belongs in this set, there exists an equilibrium in which all principals outsource to a common supplier. Moreover, $C^{\max}_{Partial}$ is monotonically increasing in Ω and M.*

Proof. See Appendix 4. ■

The results of Proposition 4 contrast with those of Propositions 2 and 3: whereas full outsourcing is only profitable when wages and congruence are sufficiently large, partial outsourcing proves profitable even if congruence and wages are small. The reason for this result is that different organizations can now work in parallel. This increases specialization at the industry level, and reduces processing delays. Moreover, the amount of information processed by the supplier is independent of the size of the market, Ω. Put differently, partial outsourcing allows the creation of smaller and more efficient organizations, but maintains high scale economy benefits. Last, since principals only outsource the fraction of the information set for which their preferences are perfectly congruent,[17] the outside supplier provides further efficiency improvements, by Proposition 2.

[17] For that reason, partial outsourcing and in-house processing become formally equivalent if $\alpha \to 0$, whereas full and partial outsourcing become formally equivalent for $\alpha \to 1$.

It is useful to stress that the benefits of specialization are purely endogenous here. Be the agent working on a "specialized" or a "general" information set, his productivity is identical. It should thus be clear that the results of Proposition 4 would yet be reinforced if the agents' productivity was increasing in their level of specialization (see Bolton and Dewatripont 1994 or Garicano 2000).

Full and Partial outsourcing compared. Exploiting the results of Propositions 3 and 4, we can now compare the relative value of partial and full outsourcing. Importantly, suppliers and principals may have conflicting interests with respect to the type of outsourcing chosen. For instance, partial outsourcing may generate lower profits for the supplier, because it increases the profits of the principals. In a world of free entry, however, the equilibrium outcome would be the one that maximizes the joint profit of the industry. For that reason, we use total industry surplus to determine which type of outsourcing method dominates:

Proposition 5 *For any value of* w, $\alpha \in (0, 1)$, $\Omega \geq 1$, M *and* e_0, *partial outsourcing is always strictly preferred to full outsourcing.*

Proof. See Appendix 5. ■

This result may seem surprising at first glance. Indeed, full outsourcing could be thought as more profitable when congruence is high: instead of creating Ω principal-controlled and one supplier-controlled organizations, full outsourcing allows for one, bigger, firm and hence for more delegation to the agents. However, one must remark that, when congruence is perfect ($\alpha = 1$), full and partial outsourcing are formally equivalent, since all information processing is outsourced in both cases.[18] For α smaller but close to 1, the structure of the supplier-controlled organization is basically the same under full or partial outsourcing. However, the latter option still has the benefit of containing the costs of loss of control, whereas it only has a second-order effect on the mass of information processed by the principals.

Put differently, this proposition demonstrates that securing authority on the management of specific information is valuable. For this reason, the principals' organizations have an *endogenous* comparative advantage at processing this part of the information set. Conversely, the outside supplier has a comparative advantage at processing common information items, because he is less sensitive to wage costs. Thus, in equilibrium, principals should always maintain in-house the treatment of specific information.

Thus, if outsourcing costs are small, the only market equilibrium that survives is one in which each principal focuses on her "core business". She creates an organization that specializes in the treatment of this specific information, and outsources "common-business

[18] Moreover, for $C = 0$, and $w > 0$, total costs are necessarily higher under in-house processing than under either type of outsourcing.

activities” to an outside supplier. Conversely, if outsourcing costs are large, the only equilibrium that survives is the in-house processing one. Note that the rationale for maintaining ”core-business activities” in-house is somewhat different from the arguments developed, for instance, by Domberger (1998). One of his arguments lies in the risk of losing intellectual property rights. However, we show that maintaining ”core business” in-house is beneficial even when such risks are absent. Conversely, an oft-used argument in *favor* of outsourcing is that it increases flexibility. Such a benefit is absent from our setup, since we assume “environmental certainty”. Yet, partial outsourcing remains a profitable option. Finally, Domberger also mentions that external suppliers have greater incentives to improve their processing technology and reduce their costs, because of competitive pressures. Our results justify this presumption: the incentives provided by the market induces the common supplier to provide more efficient services (the structure of its organization can be interpreted as a proxy for technology here), in addition to generating scale economies.

5 Extensions and discussion

5.1 Few types of principals

Proposition 5 shows that ”core business” activities are always retained in-house. What happens if heterogeneity among the principals is less strong? To address this question, assume that lack of congruence only exists between different “types” of principals, and that the number of these types is small compared to the size of the market. That is, there are many *identical* principals of each type, and fewer different types. To solve for the equilibrium market structure, we can proceed in two steps. Denoting by k the number of types in the industry, consider first the k markets in isolation. Since principals have perfectly aligned preferences in each market, by Proposition 2, they fully outsource their activities under the same conditions as before. Second, consider the entire market, now composed of k entities with imperfectly aligned preferences. By Proposition 5, those may also want to outsource industry-wide activities to another, distinct, supplier. Hence, when market size is large but the number of different types limited, the equilibrium structure of the market might be characterized by increasing waves of specialization, and more complex relationships between different organizations. This illustrates the sentence by Adam Smith: “*The division of labor is limited by the extent of the market*” (1880, p18, cited by Domberger 1998, p80).

5.2 Hold-ups and the choice of a supplier

In this and next subsection we shortly discuss how one can introduce the possibility of hold up or ex post bargaining that have been extensively studied in the incomplete contract literature. Recall that, in Section 4, we assumed the principal could only outsource tasks to an *independent* supplier. A natural question that arises is why one of the other principals could not replace this supplier. This should reduce total outsourcing costs by C, and hence be profitable. Quite interestingly, however, this decision might open the way for increased risks of being held up, since the other principal may, in some cases, want to foreclose competitors by not delivering information (like, e.g. in Bolton and Whinston 1993).

To understand why this may happen, one must consider the case in which principals are actual *competitors* on the final market, in which case moral hazard problems may arise. To illustrate this in more detail, assume for a moment that there are exactly two principals, whose preferences are perfectly congruent. Moreover, the value of processed information is equal to V if one principal has access to the information, and equal to $V - \Delta$ if both principals have access to it (because of increased competition).[19] Now, assume that principal 1 acts as a supplier to principal 2. In this case, her profits would be given by:

$$\pi^1_{Supply} = V - \Delta - r\left[d^S_{\text{Full}} + M^S_{\text{Full}}\, e_0\right] - W^S_{\text{Full}} + \left(W_{\text{Org}} + rD_{\text{Org}} - rC - r\left[d^S_{\text{Full}} + M^S_{\text{Full}} e_0\right]\right) \tag{16}$$

where the term between parentheses in (16) is the price principal 2 is ready to pay. If the delivery of information is not contractible, however, she may promise to sell information to principal 2 in the first place, but deviate and hold up information ex post. This strategy generates a profit:

$$\pi^1_{Holdup} = V - rD_{\text{Org}} - W_{\text{Org}}. \tag{17}$$

Comparing (16) and (17), not delivering processed information is profitable when:

$$\Delta \geq 2r\left[D_{\text{Org}} - d^S_{\text{Full}} - M^S_{\text{Full}}\, e_0\right] + 2W_{\text{Org}} - W^S_{\text{Full}} - rC,$$

that is, if competition strongly affects the profitability of the principals' decisions (*i.e.* if Δ is large enough).

By contrast, the risk of hold-up is reduced with an independent supplier. Since the price of information is contracted in advance, the profits of the supplier are increasing in the number of actual buyers.[20] Summing up, if the benefits of the different principals are independent (they are not competing on the final market), and if C is not too large, the most efficient outcome is that only one principal creates an organization, and processes information for the entire market. If instead the profits of the different principals are strongly linked to one another, each principal may reduce moral hazard problems by outsourcing to an independent supplier who does not value information directly.

[19] Note that the value of information has been assumed constant ($\Delta = 0$) until now.

[20] One may argue that principals could make higher price offers, conditional on being the only one to receive the information. However, such (illegal) contracts are more difficult to enforce. Moreover, one must remember that if the supplier delivers information to one principal only, his profits are necessarily negative if $C > 0$ (see Section 4).

5.3 Ex-post price renegotiation

Another typical problem analyzed in the literature is the possibility of ex post renegotiation, which has been shown to generate underinvestment in relationship-specific assets. In Section 4, we assumed that the supplier has full bargaining power. More generally of course, the supplier may, ex post, only manage to extract some fraction β of the surplus generated by outsourcing. Yet, it is easy to check that, when the supplier anticipates ex-post price renegotiation, first order conditions (8) and (9) become:

$$f'\left(e_1^{S*}\right) = -\frac{1/n_1^{S*} + w/\left(r\Omega\beta\right)}{e_0}; \tag{18}$$

$$f'\left(e_l^{S*}\right) = -\frac{1/n_l^{S*} + w/\left(r\Omega\beta\right)}{1/n_{l-1}^{S*} + w/\left(r\Omega\beta\right)}\left(e_{l-1}^{S*} - \frac{f\left(e_{l-1}^{S*}\right)}{f'\left(e_{l-1}^{S*}\right)}\right)^{-1}, \forall l \geq 2. \tag{19}$$

Put differently, the behaviour of the supplier is determined exactly as in Section 4, albeit for the fact that ex-post price bargaining reduces the outside supplier's incentives to reduce delays. Technically, this amounts to changing the parameter Ω into $\tilde{\Omega} \equiv \beta\Omega$. Thus, our results directly extend to ex post price bargaining, with the only difference that market size must be sufficiently large, or β sufficiently close to 1, for outsourcing to arise in equilibrium.

6 Conclusions

We proposed a model of information processing in which the principal must decide whether to retain activities in-house or to outsource them, knowing that this decision also affects the amount of authority she has over agents. Central to the analysis was the question of understanding how hierarchies are shaped, and what determines the comparative advantage of either form of delegation.

In the absence of outsourcing, our results reproduce some of those in the information processing literature: the larger the mass of information or the more overloaded is the principal, the bigger and the more hierarchized her organization will be. Moreover, this organization then represents the entire hierarchy of “processing units” who work for the principal. One additional feature of our model was then to relate the structure of this organizations to external market conditions, and to show that the task performed by each agent depends on his hierarchical position.

Next, the paper showed that ”common business tasks” are sometimes performed better by the market than by such an organization. In that case, outsourcing takes place and the hierarchy of “processing units” encompasses more than the principal's organization. Independent organizations in the upstream market process information for the principal. However, the organization of the principal remains in operation, focusing on what becomes its ”core business activities". When the principal has a specific interest in processing a particular piece of information, her incentives to process it quickly are stronger than the outside supplier's, and conversely for "common business" information items. For this reason, under outsourcing the upstream market is made of independently managed organization(s) that provide "common business" services for the whole industry, whereas the downstream market consists of organizations, run by the principals, that focus on "specific information".

The next question we addressed is when outsourcing arises in equilibrium. Our results have shown that the value of outsourcing increases when the size of the market is large, when the preferences of the principals become sufficiently congruent, and when wages increase compared to the marginal cost of delay. That is, if there are sufficiently many principals who need similar information to make their decisions and/or if the urgency of decision-making matters less.[21] To express these results differently, we could say that "globalization" (see also Hummels et al., 1998) and/or economic slowdowns favor outsourcing: "globalization" increases market size, and economic downturns reduce profitability, thereby making the principals less impatient. However, outsourcing has contrasted effects on the upstream and downstream markets. Since they process a smaller mass of information by themselves, downstream firms (that of the principals) will tend to become "flatter" and less staffed, which could reduce employment. However, setup costs are then likely to also be reduced, which could trigger additional entry in the market. Instead, the opposite trend takes place in the upstream market: we also showed that different principals have an incentive to outsource their tasks to a common supplier. Thus, outsourcing and "globalization" induce concentration in the upstream market. As a consequence, outside suppliers must hire additional workers and create more hierarchized organizations. In the absence of entry in the downstream market, this process leads to a reduction in total employment in the industry, which generates a reduction in total costs and shortens delays.

This paper extended the analysis of information processing to incorporate strategic organizational decisions generally disregarded by the information processing literature. However, our analysis of course ignored several issues that have been recently addressed by the research on the theory of firm. For instance, the paper abstracted form the lack of congruence between the agents and the principal (Friebel and Raith 2000, Dessein 2002, Dewatripont and Tirole 2003), imperfect synergies (Vayanos 1998), heterogeneity of skills (Prat 1997) or general equilibrium considerations (Marin and Verdier 2002). Yet, raising the issue of the span of authority as a representation for the boundaries of the firm allowed us to exploit the insights of the information processing literature. This provided us with novel microfoundations for our analysis of the comparative advantages of managing information in organizations versus markets.

[21] Using a different approach, Grossman and Helpman (2003) and Marin and Verdier (2002) provide very interesting general equilibrium analyses of the interactions between market conditions, corporate, and/or market structure.

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Appendix

Appendix 1: Two preliminary lemmas

Proof of Lemma 1

Proof. We demonstrate this lemma for layer 1, and for the case in which $n_1 = 2$. By recursion, the proof holds for all layers $l \geq 2$ and any $n_l \geq 3$.
Denote the two agents in the layer by a and b. e_a and e_b are their effort levels, and m_a and m_b are the size of their respective information sets. The cost of processing information is then:

$$\begin{aligned} TC &= r\left[2\lambda + \max\left\{e_a m_a, e_b m_b\right\} + e_0\left(f\left(e_a\right) m_a + f\left(e_b\right) m_b + \max\left\{0,\, M - m_a - m_b\right\}\right)\right] \\ &\quad + w\left(e_a m_a + e_b m_b\right). \end{aligned}$$

Therefore, if $m_a + m_b < M$ and $e_a m_a > e_b m_b$, we have:

$$\frac{\partial TC}{\partial e_a} = \left(r\left[1 + e_0 f'\left(e_a\right)\right] + w\right) m_a.$$

If $\partial TC/\partial e_a > 0$, $\forall e_a$, agent a (and, by extension, also agent b) should exert 0 effort. In the complementary case where $\partial TC/\partial e_a < 0$ for $e_a \to 0$, this FOC implies:

$$f'\left(e_a^*\right) = -\left(1 + w/r\right)/e_0.$$

Since $f\left(0\right) = 1$ and $f\left(e\right)$ is decreasing and convex in e, this also implies that:

$$f\left(e_a^*\right) < 1 - e_a^* \tfrac{1 + w/r}{e_0}. \tag{20}$$

Using (20) to compute $\partial TC/\partial m_a$ shows that the latter derivative is strictly negative, which implies that $m_a + m_b < M$ is suboptimal. In other words, it is always optimal to set $m_b = M - m_a$.

Next, and still for the case $e_a m_a > e_b m_b$, we compute the optimal level of effort for agent b:

$$\frac{\partial TC}{\partial e_b} = \left(r e_0 f'\left(e_b\right) + w\right) m_b \Rightarrow f'\left(e_b^*\right) = -\tfrac{w/r}{e_0},$$

which implies that $e_b^* > e_a^*$. Using the properties of the function $f\left(e\right)$, the latter implies that:

$$f\left(e_a^*\right) - f\left(e_b^*\right) > \left(e_b^* - e_a^*\right) \tfrac{w/r}{e_0},$$

which in turn yields:

$$\begin{aligned} \frac{\partial TC}{\partial m_a} &= r\left[e_a^* + e_0\left(f\left(e_a^*\right) - f\left(e_b^*\right)\right)\right] + w\left(e_a^* - e_b^*\right) \\ &> r\left[e_a^* + e_0\left(e_b^* - e_a^*\right) \tfrac{w/r}{e_0}\right] + w\left(e_a^* - e_b^*\right) = e_a^* > 0, \end{aligned}$$

and hence that $e_b m_b < e_a m_a$ cannot be cost minimizing. Hence, we must also have:

$$e_b m_b = e_a m_a. \tag{21}$$

Finally, it remains to show that $m_a = m_b$ is optimal. We demonstrate this by contraction. Assume $m_a > m_b$, in which case e_a is strictly less than e_b, by (21). Therefore, total costs can be reduced by decreasing m_a, subject to $e_a m_a$ constant. ■

Lemma 2 *The effort levels that minimize a function of the type:*

$$F\left(\{e_l\},\{n_l\},L;\gamma_0,\delta,\tilde{\lambda}\right)=\gamma_0\prod_{j=1}^{L}f\left(e_j\right)+\sum_{k=1}^{L}\left[e_k\left(\frac{1}{n_k}+\delta\right)\prod_{j=k+1}^{L}f\left(e_j\right)+\tilde{\lambda}n_k\right]$$

are given by:

$$\begin{cases} f'\left(e_1^*\right)=-\dfrac{1/n_1+\delta}{\gamma_0}, \\ f'\left(e_l^*\right)=-\dfrac{1/n_l+\delta}{1/n_{l-1}+\delta}\left(e_{l-1}^*-\dfrac{f\left(e_{l-1}^*\right)}{f'\left(e_{l-1}^*\right)}\right)^{-1},\forall l\geq 2 \end{cases} \tag{22}$$

Proof. Differentiating $F(\cdot)$ with respect to e_l to derive the first order condition, we find:

$$\begin{aligned} \frac{1}{n_l}+\delta &= \frac{f'(e_l)}{f(e_l)}\left[\gamma_0\prod_{j=1}^{l}f\left(e_j\right)+\sum_{k=1}^{l-1}\left[e_k\left(\frac{1}{n_k}+\delta\right)\prod_{j=k+1}^{l}f\left(e_j\right)\right]\right], \text{ or:} \\ f'\left(e_l^*\right) &= \frac{-\left(1/n_l+\delta\right)f\left(e_l^*\right)}{\gamma_0\prod_{k=1}^{l}f\left(e_k^*\right)+\sum_{k=1}^{l-1}e_k^*\left(1/n_k+\delta\right)\prod_{k=j+1}^{l}f\left(e_k^*\right)}. \end{aligned} \tag{23}$$

Applying (23) to layer 1 yields the first line in (22). To derive the optimal level of effort in the other layers, we must compare the value of (23) for two adjacent layers. First, we decompose (23) into:

$$f'\left(e_l^*\right)=\frac{-\left(1/n_l+\delta\right)}{e_{l-1}^*\left(1/n_{l-1}+\delta\right)+\left[\gamma_0\prod_{k=1}^{l-1}f\left(e_k^*\right)+\sum_{k=1}^{l-2}e_k^*\left(1/n_k+\delta\right)\prod_{k=j+1}^{l-1}f\left(e_k^*\right)\right]}. \tag{24}$$

Second, for layer $l-1$ (23) yields:

$$f'\left(e_{l-1}^*\right)=\frac{-\left(1/n_{l-1}+\delta\right)f\left(e_{l-1}^*\right)}{\gamma_0\prod_{k=1}^{l-1}f\left(e_k^*\right)+\sum_{k=1}^{l-2}e_k^*\left(1/n_k+\delta\right)\prod_{k=j+1}^{l-1}f\left(e_k^*\right)}. \tag{25}$$

Noting that the term between brackets on the denominator of (24) equals the denominator in (25) then shows that this optimization problem is recursive, and yields the second line in (22). ■

Appendix 2: Proof of proposition 1

In any given organization, the optimal levels of effort are the ones that minimize (1). That objective function can however be re-written into:

$$\begin{aligned} TC_{\text{Org}}\left(\{e_l\},\{n_l\},L\right) &= rM\left[e_0\prod_{j=1}^{L}f\left(e_j\right)+\sum_{l}e_l\left(\frac{1}{n_l}+\frac{w}{r}\right)\prod_{k=l+1}^{L}f\left(e_k\right)+\frac{\lambda n_l}{M}\right] \\ &= rM\,F\left(\{e_l\},\{n_l\},L;e_0,w/r,\lambda/M\right). \end{aligned}$$

Put differently, the principal faces the same problem as in Lemma 2, with $\gamma_0=e_0$, $\delta=w/r$ and $\tilde{\lambda}=\lambda/M$. Applying the results of Lemma 2 then yields (4) and (5) in the main text. Next, $\partial e_l^*/\partial n_l>0$ and $\partial e_l^*/\partial e_0>0$ are then directly proven by computing the derivative of these equations.

Next, the optimal value of n_l directly results from the first order condition:

$$\frac{\partial D}{\partial n_l}=-e_l^*\frac{M_l}{\left(n_l^*\right)^2}+\lambda=0\Rightarrow n_l^*=\sqrt{e_l^*M_l/\lambda}.$$

To show that the optimal number of layers is increasing in M and in e_0, consider an organization which, given M, λ, $f(\cdot)$ and e_0 should be structured into L^* layers, and in which the principal receives a batch of mass M_{Org}. Now, consider the (suboptimal) choice of instructing some additional agents, in number $\bar{n}$, to process this batch another round, before she starts processing remaining items. Total costs increase by:

$$\begin{aligned}\Delta &= \lambda\bar{n} + \left[\frac{\bar{e}}{\bar{n}} + (f(\bar{e}) - 1)\, e_0 + w\bar{e}\right] M_{\text{Org}}, \\ &\quad \text{with: } \bar{e} \text{ } s.t. \text{ } f'(\bar{e}) = -(1/\bar{n} + w)/e_0.\end{aligned}$$

Since $f'' > 0$, this implies that $f(\bar{e}) < 1 - \frac{\bar{e}}{e_0}(1/\bar{n} + w)$, and hence that $\partial\Delta/\partial M_{\text{Org}} < 0$. Therefore, a sufficiently large increase in M (which must increase M_{Org} in the L^*-layer organization) implies that Δ becomes negative: the optimal number of layers must be increasing in M. Similarly, $\partial\Delta/\partial e_0 < 0$, and the optimal number of layers is thus also increasing in e_0.

Finally, we turn to processing delays. By revealed preferences, we have:

$$\begin{aligned} D^*_{\text{Org}}(w_1) + \tfrac{w_2}{r}\omega^*_{\text{Org}}(w_1) &\geq D^*_{\text{Org}}(w_2) + \tfrac{w_2}{r}\omega^*_{\text{Org}}(w_2) \quad \text{and} \\ D^*_{\text{Org}}(w_1) + \tfrac{w_1}{r}\omega^*_{\text{Org}}(w_1) &\leq D^*_{\text{Org}}(w_2) + \tfrac{w_1}{r}\omega^*_{\text{Org}}(w_2), \end{aligned}$$

where $D^*_{\text{Org}}(w)$ is the optimal level of delay given the wage level w and $\left(w\,\omega^*_{\text{Org}}(w)\right)$ is the optimal total wage cost given that wage level. Thus, for $w_1 < w_2$, we have $\omega^*_{\text{Org}}(w_1) \geq \omega^*_{\text{Org}}(w_2)$, and hence $D^*_{\text{Org}}(w_2) \geq D^*_{\text{Org}}(w_1)$. ■

Appendix 3: Proof of Proposition 3

If the supplier has $\Omega \to \infty$ customers, by (11), the optimal level of effort in layer 1 is 0. Thus layer 1 must be suppressed. Hence, the optimal level of effort in layer 2 is defined by (11). Thus, layer 2 must also be suppressed. By recursion, it is optimal for the supplier to set $L = 0$, which implies that $M^S_{\text{Full}} = M$. However, the principal can obtain the same mass of information at lower cost by simply not outsourcing. This shows that, for any $C \geq 0$, there is no equilibrium in which a same supplier has $\Omega \to \infty$ customers.

To prove point *i*, remember that $w \to 0$ implies that the principal sets up an organization that is "fully efficient." Yet, under full outsourcing, a decrease in α decreases effort levels (and increases processing delays), whereas it has no effect under in-house processing. By continuity, full outsourcing must be strictly dominated by in-house processing for low wages and congruence α.

Conversely, for α sufficiently close to 1 and some given –and finite– value of Ω, the cost borne by the loss of authority is small. Thus, processing costs are more sensitive to an increase in w under in-house processing compared to full outsourcing. By continuity, for C small enough, there must exist values of w such that full outsourcing strictly dominates in-house processing for high enough values of α, which proves point *ii*.

For w arbitrarily large, by (11), the outside supplier also selects effort levels that are arbitrarily close to zero, in which case the delay reduction achieved through outsourcing is too small to compensate for any outsourcing cost $C > 0$, which proves point *iii*. ■

6.1 Appendix 4: Proof of Proposition 4

Consider first the case of in-house processing, in which Ω independent organizations process information. Total costs of the industry are then:

$$\Omega TC_{\text{Org}} = \Omega r \left[\underbrace{M \sum_{l=1}^{L^*} e_l^* \frac{\prod_{j=l+1}^{L^*} f(e_j^*)}{n_l^*}}_{Z_1} + \underbrace{M e_0 \prod_{j=1}^{L^*} f\left(e_j^*\right)}_{Z_2} + \underbrace{\lambda \sum_{l=1}^{L^*} n_l^*}_{Z_3} + \underbrace{\frac{w}{r} M \sum_{l=1}^{L^*} e_l^* \prod_{j=l+1}^{L^*} f\left(e_j^*\right)}_{Z_4} \right].$$

Now, consider the following *suboptimal* case of partial outsourcing: each principal outsources the common part α of the information set M. However, assume that, in this process, she maintains the structure of her organization unchanged. Next, assume that the common supplier also creates an organization with the same structure. Total sectorial costs then become:

$$\Omega r \left[C + \underbrace{\max\{\alpha, 1-\alpha\} M \sum_{l=1}^{L^*} e_l^* \frac{\prod_{j=l+1}^{L^*} f(e_j^*)}{n_l^*}}_{Z_1^{\text{Partial}}} + \underbrace{M e_0 \prod_{j=1}^{L^*} f\left(e_j^*\right)}_{Z_2^{\text{Partial}}} + \underbrace{\lambda \sum_{l=1}^{L^*} n_l^*}_{Z_3^{\text{Partial}}} + \right.$$

$$\left. \underbrace{+\frac{w}{r}(1-\alpha) M \sum_{l=1}^{L^*} e_l^* \prod_{j=l+1}^{L^*} f\left(e_j^*\right)}_{Z_4^{\text{Partial}}} \right] + \underbrace{w \alpha M \sum_{l=1}^{L^*} e_l^* \prod_{j=l+1}^{L^*} f\left(e_j^*\right)}_{Z_5^{\text{Partial}}}.$$

Hence:

1. By outsourcing the common fraction α of their information set, principals manage to reduce processing delay to a fraction of its initial level, *i.e.* $Z_1^{\text{Partial}} = \max\{\alpha, 1-\alpha\} Z_1 < Z_1$.
2. Since organizational structure is unchanged, the total size of the batch reaching the principal must also remain unchanged: $Z_2 = Z_2^{\text{Partial}}$
3. Since all organizations have the same structure as under in-house processing, hiring costs are the same for the supplier and for the principals: $Z_3^{\text{Partial}} = Z_3$.
4. Finally, outsourcing allows each principal to reduce wage costs in her organization to a fraction $(1-\alpha)$ of its initial level. Conversely, only one common supplier has to bear the remaining fraction α of that initial wage cost. Thus, total *sectorial* wage costs are reduced to $\Omega Z_4^{\text{Partial}} + Z_5^{\text{Partial}} < \Omega Z_4$. Total wage cost is thus reduced by $(\Omega - 1)\alpha Z_4$.

This shows that, for $\alpha \in (0,1)$, there must exist a range $(0, C^{\max}]$ such that, for any C in that interval, partial outsourcing dominates in-house processing.

Clearly, for $\alpha = 0$, partial outsourcing and in-house processing are formally equivalent, and thus yield identical profits. ■

Appendix 5: Proof of Proposition 5

We follow the same procedure as for the proof of Proposition 4: consider the optimal supplier's organization under full outsourcing. Total sectorial costs in that case are given by:

$$\Omega r \left[C + \underbrace{\left[\Omega\left(1-\alpha\right)+\alpha\right] M \sum_{l=1}^{L^{S^*}} e_l^{S^*} \frac{\prod_{j=l+1}^{L^{S^*}} f\left(e_j^{S^*}\right)}{n_l^{S^*}}}_{\bar{Z}_1} + \underbrace{e_0 \, M \prod_{j=1}^{L^{S^*}} f\left(e_j^{S^*}\right)}_{\bar{Z}_2} + \underbrace{\lambda \sum_{l=1}^{L^{S^*}} n_l^{S^*}}_{\bar{Z}_3} \right] + ...$$

$$... + \underbrace{\left[\Omega\left(1-\alpha\right)+\alpha\right] M \, w \sum_{l=1}^{L^{S^*}} e_l^{S^*} \prod_{j=l+1}^{L^{S^*}} f\left(e_j^{S^*}\right)}_{\bar{Z}_4}.$$

Following the same reasoning as in the proof of Proposition 4, suboptimal partial outsourcing reduces $\bar{Z}_1$ while $\bar{Z}_2$, $\bar{Z}_3$ and $\bar{Z}_4$ are unchanged. Hence, partial outsourcing must necessarily increase total sectorial profits, and full outsourcing is dominated. Clearly, for $\alpha = 1$, full and partial outsourcing are formally equivalent, and yield identical profits. ■