Wage Distribution with a Two-Sided Job Auction*

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Abstract

We derive a wage distribution in a model where homogenous unemployed workers and homogenous vacancies can send or receive wage offers. We solve the mixed strategies for wage offers and for the fractions of vacancies and unemployed engaged in sending or receiving offers. The equilibrium market structure is evolutionarily stable, and the pricing game is utilitywise equivalent to auction where the number of competitors is known. We derive a non-degenerate wage distribution, the shape of which depends on the unemployment-vacancy ratio. For a ratio close to one, there exists a wage density function that is first increasing, in the end decreasing, and u-shaped in the middle.

Key words: wage distribution, job search, auctions

JEL codes: J64, J31, J41, D44

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1 Introduction

Labour markets constitute such a large part of the economy that issues of labour economics can rarely be meaningfully addressed in partial equilibrium models. There are roughly two types of general equilibrium models used in modelling labour markets. One is a search model pioneered by Mortensen and Pissarides (see e.g. Pissarides, 2000), where unemployed and firms are involved in a time consuming activity of looking for each other. In these models meetings are pairwise, and wages are determined using the Nash bargaining solution or some analogous procedure. The central concept is so called matching function which tells how fast the parties find each other. In the other branch of models the meetings of the unemployed and vacancies are governed by an urn-ball matching where, say, the employers are contacted by the workers (see e.g. Montgomery, 1991). The advantage of this approach is that the matching function can be determined endogenously, and that it makes multiple meetings and detailed wage formation possible.

Several empirical findings are hard to come by in theoretical models. One of these is the empirical wage distribution. A typical wage distribution for observationally identical workers is hump-shaped and right-skewed (DiNardo, Fortin and Lemieux, 1996). Wage distributions have been generated by search models with varying results. In Burdett and Mortensen (1998), workers receive wage offers from employers at an exogenous rate, and workers search also on the job. With identical workers and identical firms, the wage offer density function is increasing. It can be declining only if firms or workers are heterogenous in productivity.

There are articles (Mortensen, 2000; Bontemps, Robin and van den Berg, 2000) that generate distributions of wages that more closely resemble the observed ones. To achieve this, one needs to assume heterogeneity of workers or firms and some special features of the matching function. These features are not derivable from the basics of the model but are just assumed. Mortensen (2000) considers an on-the-job search model where workers receive wage offers from firms that can make match-specific investments after the firm and worker have met. The meeting rate is the same for employed and unemployed workers, and it depends positively on the number of vacancies. The firms are heterogenous ex post with respect to the amount of capital. Firms who offer higher wages invest more in match-specific capital, because workers who earn higher wages have a lower probability to quit. The resulting wage offer density can be increasing, decreasing, or hump-shaped. However, for the density to be hump-shaped, it is required that the production function is of Cobb-Douglas type, and that the parameter of the production function and the exogenous reservation wage fall within certain limits. In Bontemps, Robin and van den Berg (2000), employed and unemployed workers have different, exogenous meeting rates. Their model allows firms to have different technologies, and they show that a suitable distribution of employer productivity can lead to a hump-shaped distribution for wages.

In this article we demonstrate that one can, with very simple economic reasoning, generate a significant improvement to the wage distribution when one uses an urn-ball model, even though firms and workers are homogenous. We construct a model where unemployed workers can send wage demands to vacant firms that hire the worker who has demanded the lowest wage. We assume that workers do not know how many other workers happen to contact the particular employer. This means that the workers must use a mixed strategy in equilibrium. Simultaneously, vacant firms use a mixed strategy in sending wage offers to unemployed workers who accept the highest offer. We derive the mixed strategies explicitly. In order to accomplish the ideas of this article one cannot stick to the search models where contacts are bilateral.

We get three kind of results in this article. First, the model has different equilibrium market structures depending on the unemployment-vacancy ratio. If the ratio is close to one, there are three equilibrium market structures of which only one is stable in an evolutionary sense. For a small ratio, there exist two equilibrium market structures of which one is stable; the same holds for a large ratio. Second, we derive a non-degenerate wage distribution, the shape of which depends on the unemployment-vacancy ratio. For a ratio close to one, there exists a wage density function (associated with the evolutionarily stable market structure) that is first increasing, in the end decreasing, and u-shaped in the middle. Thus, we do not get exactly the wage distribution observed empirically but one that still has several desirable features. Most of the literature has focused on one of the non-stable equilibria. If the number of unemployed sufficiently exceeds that of vacancies, the wage density function is decreasing; in the opposite case it is increasing. Third, in utility terms the mixed strategy in wage offers is equivalent to a mechanism where job seekers randomly choose vacancies, and wages are determined in an auction where the job seekers know the number of their competitors. Kultti (1999) shows the equivalence between such auctions and posted prices; we thus know that all the three mechanisms are equivalent in utility terms. Our model also predicts that if the relative supply of labour increases, the average observed wage decreases if the unemployed-vacancy ratio differs sufficiently from one; otherwise it increases.

We describe the general idea of the model in Section 2. Sections 3-6 consider a static model that is sufficient to generate a wage distribution. We solve the wage demands and offers in Section 3. In Section 4 we solve the equilibrium market structure, that is, the fractions of unemployed and vacancies who send offers and who receive them. Section 5 analyses the evolutionary stability of the equilibrium. Section 6 presents the main result of this article, the distribution of realised wages. Section 7 presents the main results of two dynamic versions of the model. In the Appendix we derive most of the results of Sections 3-6 as well as the analyses of the dynamic models. Section 8 concludes.

2 The Model

In the most general setting that we consider, everything is in the model, i.e. it is a true general equilibrium model. The measure of workers is L, and the measure of employers is K. Some of them are matched with each other in productive activities, while others are looking for a partner. Denote the measure of unemployed workers (job seekers) by u and the measure of vacancies by v. Production happens in pairs, therefore

$$L - u = K - v. \tag{1}$$

A matched pair produces output worth of unity each period. A worker who is employed at wage $w \in [0, 1]$ gets the wage each period as long as the employment relationship lasts, and correspondingly the employer gets $1 - w$ each period. Utilities are linear such that a worker's utility is w, and firm's utility is $1 - w$. Unmatched agents get zero utility. Time is discrete and extends to infinity. Let $\delta \in [0, 1]$ be the common discount factor.

We focus on the market in a steady state, and for this we need that the matches dissolve every once and a while. We assume an exogenous separation probability $s \in [0,1]$. Each period a match dissolves with probability s, and the firm and the worker enter the pool of vacancies and unemployed. The separation probability is not just something we need to be able to do steady state analysis, but it is a real feature of real labour markets, and it makes possible to study the duration of unemployment, though not an issue in this article.

One of the crucial features of our analysis is that we determine the equilibrium market structure. Usually it is assumed that unemployed workers contact vacancies or vice versa. We do not know which is the better assumption, and consequently we allow for both possibilities and determine which case emerges in equilibrium. To this end we postulate that there are two submarkets. Fraction $x \in [0,1]$ of unemployed workers and fraction $y \in [0,1]$ of vacancies are in the 'vacancy market' where unemployed workers contact vacancies. Each job seeker who decides to go to that market, chooses randomly one of the yv vacancies and sends an application accompanied with a wage demand. We could say that in the vacancy market, vacancies stay (or wait), and workers move. In the 'job seeker market', each of the $(1-y)v$ vacancies sends a wage offer to one of the $(1-x)u$ unemployed workers. Another crucial feature is that the workers do not know which firms the other workers apply to, nor do they know their wage demands. This is an auction with identical valuations but unknown number of bidders. Each firm that has received at least one application hires the worker who has asked for the lowest wage. Likewise, the vacancies that send offers do not know how much the other vacancies offer and to whom. A job seeker chooses the firm that has offered the highest wage. We solve the equilibrium fractions x and y as functions of u/v, and the distributions of wage offers, wage demands, and realised wages.

We focus on symmetric strategies regarding wage demands and offers and probabilities of going to either market. It is clear that there are no pure strategy equilibria, or equilibria with a mass point for that matter, as to wage demands and offers. The heuristic reason is easy to understand by assuming that there is a pure strategy equilibrium where, say, the unemployed demand wage w. There is a positive probability that a particular vacancy is contacted by more than one job seeker. When this happens, the probability that a job seeker gets the job is at most one half. Making a wage demand slightly less than w is a profitable deviation, as then the deviator gets the job for certain, i.e. there is a discrete increase in his probability of getting the job while the wage remains practically the same.[1]

The possibility of two markets is important theoretically, because it allows us to determine the market structure endogenously. It turns out that whether unemployed contact vacancies, or vice versa, or whether two markets with these features exist simultaneously depends on the ratio of unemployed to vacancies. The distribution of wages depends on who contacts whom, and if it is just assumed that contacts take place in one way or the other, there is a chance that the wrong, i.e. non-equilibrium, modelling decision is made. This then produces an incorrect wage distribution.

The aim of this article is to derive the wage distribution produced by the urn-ball models, where there are possibly two markets, and where wage offers and demands originate from a mixed strategy. For this purpose it is sufficient to study a static model where the only things of importance are the measures of unemployed and vacancies. This model is got by setting the discount factor to zero and the separation rate to unity, so that each match lasts exactly one period. A slightly more general model is got by assuming that the discount factor is strictly positive but the separation rate is still unity. This corresponds to a dynamic model where it is assumed that those who match exit the market and are replaced by identical but unmatched agents. Here this is one possible interpretation, but if one wants to think also this as a special case of the general model, it must be assumed that the agents do not remember with whom they have been matched in the previous periods. We conduct most of the analysis via the static model, but in the appendix we provide the full equilibrium analysis of the two dynamic models, too.

[1] For a formal argument see Kultti and Virrankoski (2003) where in an analogous setting it is shown that there exist only non-atomic mixed strategy equilibria in symmetric strategies. Moreover, it is shown that the support of the mixed strategy must be an interval.

3 Distributions of Wage Demands and Offers

We assume that there are two submarkets; in one market vacancies contact unemployed workers by sending wage offers, whereas in the other market unemployed workers send wage demands to vacancies. We examine these two cases separately and start from the first one.

3.1 Vacancies Send Offers to Unemployed Workers

In this job seeker market each vacancy sends a wage offer to one randomly chosen unemployed worker. The Poisson parameter that governs the arrivals of offers to workers is ϕ, which is the ratio of the number of offer-sending vacancies to the the number of offer-receiving workers. (If all unemployed and vacancies are in this market, then $\phi = v/u$.) Let V_s be the utility of a vacancy that sends an offer w. Vacancies use a mixed strategy with cumulative distribution function $H(w)$ with support $[a, b]$. The utility of a vacancy is

$$\begin{aligned} V_s &= e^{-\phi}(1-w) + \phi e^{-\phi}(1-w)H(w) + ... + \frac{\phi^k e^{-\phi}}{k!}(1-w)(H(w))^k + ... \qquad (2) \\ &= (1-w)e^{-\phi(1-H(w))}. \end{aligned}$$

In the first term on the right-hand side, $e^{-\phi}$ is the probability that the worker to whom the vacancy sends an offer does not get any other offers, and the vacancy gets profit $1-w$. In the second term, $\phi e^{-\phi}$ is the probability that the worker gets one other offer, and the vacancy we look at manages to hire the worker if the other vacancy's offer is lower than w, this happens with probability $H(w)$. The rest of the terms capture the probability that the worker gets offers from exactly k other vacancies, times the probability that they offer less than w. The mixed strategy gives $(1-a)e^{-\phi(1-H(a))} = (1-b)e^{-\phi(1-H(b))}$. That is, a vacancy's utility is the same from offer a as from offer b. The lowest offer a equals zero, because there is a positive probability that the worker does not get any other offers. Using $a = 0$, $H(a) = 0$ and $H(b) = 1$, the upper limit of the wage offers is $b = 1 - e^{-\phi}$. The utility of a vacancy is therefore

$$V_s = e^{-\phi}. \qquad (3)$$

Using $V_s = (1-w)e^{-\phi(1-H(w))} = e^{-\phi}$ we get

$$H(w) = -\frac{1}{\phi}\ln(1-w) \tag{4}$$

with support $w \in \left[0, 1-e^{-\phi}\right]$. The density function is

$$h(w) \equiv H'(w) = \frac{1}{\phi(1-w)}, \tag{5}$$

which is increasing in w.

The expected utility of an unemployed worker in this market is equal to U_r:

$$\begin{aligned}
U_r &= \int_a^b \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} h(w) k \left(H(w)\right)^{k-1} w dw \qquad (6)\\
&= \int_a^b \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \frac{k}{\phi(1-w)} \left(\frac{\ln(1-w)^{-1}}{\phi}\right)^{k-1} w dw \\
&= \int_a^b \sum_{k=1}^{\infty} \frac{\left(\ln(1-w)^{-1}\right)^{k-1} e^{-\phi}}{(k-1)!} \frac{w}{1-w} dw \\
&= \int_a^b \sum_{k=0}^{\infty} \frac{\left(\ln(1-w)^{-1}\right)^{k} e^{-\phi}}{k!} \frac{w}{1-w} dw = \int_a^b e^{-\phi} e^{\ln(1-w)^{-1}} \frac{w}{1-w} dw \\
&= e^{-\phi} \int_a^b \frac{w}{(1-w)^2} dw = e^{-\phi} \int_a^b \left(\frac{1}{(1-w)^2} - \frac{1}{1-w}\right) dw \qquad (7)\\
&= e^{-\phi}\left[\ln(1-b) + \frac{1}{1-b} - \ln(1-a) - \frac{1}{1-a}\right] \\
&= 1 - e^{-\phi} - \phi e^{-\phi}.
\end{aligned}$$

The probability that the job seeker gets k offers is $\frac{\phi^k e^{-\phi}}{k!}$. The probability of getting offer w is $h(w)$, and the probability that the wage offered by vacancy i is the highest is $(H(w))^{k-1}$: all $k-1$ offers must be lower than w. The highest offer can be made by any of the k vacancies. In the second line we use (4) and (5) .

3.2 Unemployed Workers Send Applications to Vacancies

Let U_s be the utility of an unemployed job seeker who sends an application with wage demand w. They use a mixed strategy with cumulative distribution function $F(w)$ with support $[A, B]$. Let θ be the appropriate Poisson parameter that governs the meeting probabilities (If all workers and firms are in this market, then $\theta = u/v$.) We have

$$\begin{aligned} U_s &= e^{-\theta}w + \theta e^{-\theta}(1 - F(w))w + ... + \frac{\theta^k e^{-\theta}}{k!}(1 - F(w))^k w + ... \\ &= we^{-\theta}\left[1 + \sum_{k=1}^{\infty}\frac{\theta^k}{k!}(1 - F(w))^k\right] = we^{-\theta}\sum_{k=0}^{\infty}\frac{\theta^k(1 - F(w))^k}{k!} \\ &= we^{-\theta F(w)}. \end{aligned} \tag{8}$$

In the above, $e^{-\theta}$ is the probability that the vacancy to whom the worker sends an offer does not get an offer from any other worker, thus the worker gets w. In the rest of the terms, $\frac{\theta^k e^{-\theta}}{k!}$ is the probability that the vacancy gets applications from k other workers. In these cases, $(1 - F(w))^k$ is the probability that all these wage demands are higher than w, thus the vacancy rejects these applications and hires our worker.

The utility of a job seeker is the same for all $w \in [A, B]$, especially $Ae^{-\theta F(A)} = Be^{-\theta F(B)}$. Clearly, $B = 1$, because the probability that the job seeker in question is the only applicant is positive. Then $Ae^{-\theta F(A)} = e^{-\theta} \Rightarrow A = e^{-\theta}$, and

$$U_s = e^{-\theta}. \tag{9}$$

Next we solve $F(w)$. We have $U_s = e^{-\theta} = we^{-\theta F(w)}$. Taking logarithms results in $\ln e^{-\theta F(w)} = \ln e^{-\theta} - \ln w \Leftrightarrow \theta F(w) = \theta + \ln w$, and the resulting distribution function is

$$F(w) = 1 + \frac{\ln w}{\theta}, \tag{10}$$

with support $w \in \left[e^{-\theta}, 1\right]$. The density function is

$$f(w) \equiv F'(w) = \frac{1}{\theta w}, \tag{11}$$

which is decreasing in w.

The expected utility of a vacancy in this market is equal to V_r:

$$
\begin{aligned}
V_r &= \int_A^B \sum_{k=0}^{\infty} \frac{\theta^k e^{-\theta}}{k!} f(w) k \left(1-F(w)\right)^{k-1} (1-w) dw \qquad (12) \\
&= \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \left[-\left(1-F(B)\right)^k + \left(1-F(A)\right)^k \right] \\
&\quad - \int_A^B \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} f(w) k \left(1-F(w)\right)^{k-1} w dw \\
&= 1 - e^{-\theta} - \int_A^B \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \frac{1}{\theta w} k \left(\frac{\ln w^{-1}}{\theta}\right)^{k-1} w dw \\
&= 1 - e^{-\theta} - \int_A^B \sum_{k=0}^{\infty} \frac{\left(\ln w^{-1}\right)^k}{k!} e^{-\theta} dw = 1 - e^{-\theta} - \int_A^B \frac{e^{-\theta}}{w} dw \\
&= 1 - e^{-\theta} - e^{-\theta} \left(\ln 1 - \ln e^{-\theta}\right) \\
&= 1 - e^{-\theta} - \theta e^{-\theta}.
\end{aligned}
$$

The probability that the vacancy gets k applications is $\frac{\theta^k e^{-\theta}}{k!}$, and the probability that the wage asked by job seeker i is the lowest is $\left(1-F(w)\right)^{k-1}$: all other $k-1$ demands must be higher. The lowest demand can be made by any of the k job seekers.

4 Equilibrium Market Structure

In an equilibrium where two markets coexist, the utility of a vacancy that sends offers is the same as the utility of a vacancy who receives wage demands from unemployed workers. The same equivalence condition between sending offers and receiving them holds for unemployed workers, too. That is, we have $V_s = V_r$ and $U_s = U_r$, and inserting the utilities derived above yields

$$
\begin{aligned}
e^{-\phi} &= 1 - e^{-\theta} - \theta e^{-\theta}, \qquad (13) \\
e^{-\theta} &= 1 - e^{-\phi} - \phi e^{-\phi}. \qquad (14)
\end{aligned}
$$

Equation (13) is called vacancies' equilibrium condition VE, and equation (14) is called unemployed workers' equilibrium condition UE. When both conditions hold, we have

$\theta e^{-\theta} = \phi e^{-\phi}$, and after substitution we get

$$\phi = \frac{\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\theta}}, \quad (15)$$

$$\theta = \frac{\phi e^{-\phi}}{1 - e^{-\phi} - \phi e^{-\phi}}. \quad (16)$$

Using (15) and (16) in $\theta e^{-\theta} = \phi e^{-\phi}$ results in

$$1 - e^{-\theta} - \theta e^{-\theta} - e^{\frac{-\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\theta}}} = 0, \quad (17)$$

$$1 - e^{-\phi} - \phi e^{-\phi} - e^{\frac{-\phi e^{-\phi}}{1 - e^{-\phi} - \phi e^{-\phi}}} = 0. \quad (18)$$

The solution of (17) and (18) is $\theta = \phi \approx 1.146$ which is denoted by θ_0. This means that if both markets coexist, the Poisson parameter that governs the arrival rates is the same, θ_0, in both markets. Denoting u/v by α, the equilibrium fractions of vacancies and unemployed workers in the two markets satisfy

$$\theta_0 = \frac{\alpha x}{y} = \frac{1 - y}{\alpha (1 - x)}, \quad (19)$$

and after a few steps we have

$$x = \frac{\theta_0 (\alpha \theta_0 - 1)}{\alpha (\theta_0^2 - 1)}, \quad (20)$$

$$y = \frac{\alpha \theta_0 - 1}{\theta_0^2 - 1}. \quad (21)$$

Proposition 1 *The vacancy market and the job-seeker market coexist if* $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$.

Proof. The two markets coexist only if $x \in (0, 1)$ and $y \in (0, 1)$. By (20) and (21) this holds only if $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$. ■

The two markets coexist only if there are roughly equally many vacancies and unemployed workers in the economy. Because $x = \frac{\theta_0}{\alpha} y$ and $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$, we get $\frac{\theta_0}{\alpha} \in (1, 1.313)$, which implies that $x > y$ in equilibrium. If $\alpha \notin \left(\frac{1}{\theta_0}, \theta_0\right)$, search is one-sided. We can directly use a result derived in Kultti, Miettunen, Takalo, and Virrankoski (2004)[2]:

[2] A model by Kultti, Miettunen, Takalo and Virrankoski (2004) considers buyers' and sellers' decisions to wait or search, with auction and bargaining as alternative trading mechanisms. It turns out that the

Proposition 2 *i) If* $\alpha < \frac{1}{\theta_0}$, *then* $x = y = 0$, *ii) if* $\alpha > \theta_0$, *then* $x = y = 1$.

Proof. The proof is lengthy, and it is presented in Kultti, Miettunen, Takalo, and Virrankoski (2004). ■

If u/v is small, all the vacancies send wage offers and none of the unemployed workers send wage demands. If u/v is large, all the unemployed workers send wage demands and none of the vacancies send wage offers. The idea of the proof is the following: Assume that $\alpha > \theta_0$, and that all the vacancies send wage offers and none of the unemployed send wage demands. It can be shown that there exists a deviating coalition of vacancies and unemployed such that all deviators would be better off in a market where unemployed send wage demands and none of the vacancies sends offers. Then, the original market cannot be an equilibrium. On the other hand, if $\alpha > \theta_0$ and all the unemployed workers send wage demands and none of the vacancies sends offers, a deviating coalition does not exist. Unemployed would prefer the new market where vacancies send offers only if the Poisson parameter in the new market is large enough, whereas vacancies prefer the new market only if the Poisson parameter is small enough. It can be shown that if $\alpha > \theta_0$, the required supports for the Poisson parameter do not overlap, thus a deviating coalition cannot exist. If $\alpha < \frac{1}{\theta_0}$, an analogous reasoning applies. If there are a lot of unemployed compared to vacancies, Proposition 2 implies that the wage offer density function and the density function for realised wages (determined later in Section 6) are decreasing, whereas in case of relatively numerous vacancies, the density functions are increasing.

The utilities in the two sided market are given by

Proposition 3 *In the two-sided market,* $U_s = V_s = e^{-\theta_0}$, *and* $U_r = V_r = 1-e^{-\theta_0}-\theta_0 e^{-\theta_0}$.

Proof. By $\theta = \phi = \theta_0$ in the two-sided market. ■

The game where the agents send offers using a mixed strategy turns out to have an interesting equivalence with two other trading mechanisms:

Remark 1 *The mixed strategies in wage offers are utilitywise equivalent (i) to an auction where the bidders know the number of competitors, and (ii) to a price-posting game where*

model with auction is utilitywise the same as the wage offer model presented here; also the fractions of staying and moving agents are the same as given by formulae (15) and (16) above.

one side of the market posts prices, and all agents on the other side choose their trading partners based on prices.

The utilities $V_r = 1 - e^{-\theta} - \theta e^{-\theta}$ and $U_s = e^{-\theta}$ given above are the same as the utilities for a seller and a buyer in the static version of Kultti (1999). In his auction model each buyer contacts one randomly chosen seller (buyers move and sellers wait), just like job seekers contact vacancies in the present model, except that buyers do not send price offers but engage in auction (a Bertrand competition) after it has been revealed how many buyers arrived in a seller's location.

5 Stability of the Equilibrium Market Structure

We can interpret the population shares x and y as strategies of the entering unemployed workers and vacancies, that is, as the probabilities of going to the vacancy market. The probabilities of going to the job seeker market are $1 - x$ and $1 - y$. Proposition 1 above and Lemma 1 below show that the model has three equilibrium market structures if $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$: Either $x = y = 0$, or $x = y = 1$, or $x \in (0, 1)$ and $y \in (0, 1)$. The selection between equilibria is modelled by using an evolutionary argument. Outside equilibrium the agents behave myopically and go to the market where their type fared best in the previous period. The adjustment process is differential, and formalisable by replicator dynamics (see e.g. Lu and McAfee, 1996).[3] To define replicator dynamics let us first establish notation for the unemployed workers' and vacancies' average expected utilities U and V, given population shares x and y: $U = xU_s + (1-x)U_r$ and $V = yV_r + (1-y)V_s$. In the replicator dynamics the population shares are determined by the following differential equations:

$$\frac{dx}{dt} = x(U_s - U) = x(1 - x)(U_s - U_r) \qquad (22)$$

$$\frac{dy}{dt} = y(V_r - V) = y(1 - y)(V_r - V_s). \qquad (23)$$

[3] Althoug this is a static (one-shot) model, we can still use replicator dynamics. Instead of assuming agents who live many periods, we assume consecutive generations of one-period-living agents. Or, in a dynamic model, the reservation values are discounted, and the discount factor approaches zero.

Definition 1 *An equilibrium* (x, y) *is evolutionarily stable if there exists a neighbourhood of* (x, y) *where the replicator dynamics converges to the equilibrium.*

The replicator dynamics can be easily performed graphically. In Figure 1 we have drawn the equilibrium curves VE and UE on where, by equations (22) and (23), $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. Next we determine the positions of these curves in (x, y)-space. First we state

Lemma 1 *Equilibrium curves* VE *and* UE *go through* $(0, 0)$ *and* $(1, 1)$.

Proof. In Appendix 1.1 ■

We have thus shown that $(0, 0)$ and $(1, 1)$ are pure strategy equilibria. A mixed-strategy equilibrium where $x \in (0, 1)$ and $y \in (0, 1)$ is economically meaningful only if it is stable. The uniqueness and stability of a mixed-strategy equilibrium is studied in (x, y) -plane (see Figure 1). Along vacancies' equilibrium VE, we have $e^{-\phi} = 1 - e^{-\theta} - \theta e^{-\theta}$. The respective equilibrium condition for unemployed, UE, is $e^{-\theta} = 1 - e^{-\phi} - \phi e^{-\phi}$. Parameter $\theta = \frac{xu}{yv}$ governs the arrival of workers' applications to vacancies, whereas workers receive vacancies' offers governed by parameter $\phi = \frac{(1 - y)\, v}{(1 - x)\, u}$.

Proposition 4 *The mixed-strategy equilibrium where* $x \in (0, 1)$ *and* $y \in (0, 1)$ *is unique and evolutionarily stable.*

Proof. In Appendix 1.2 ■

Lemma 1 tells us that $x = y = 0$ or $x = y = 1$ are also equilibria, by (30) and (31) we know that VE and UE are increasing, and by (20) and (21) we know that at $x \in (0, 1)$ and $y \in (0, 1)$ they have a unique intersection. If $x = y = 1$, all unemployed workers send applications to vacancies, and none of the vacancies send offers to unemployed workers, and if $x = y = 0$, vice versa. However, we know that those equilibria are necessarily unstable, because the equilibrium where $x \in (0, 1)$ and $y \in (0, 1)$ is stable.

6 Aggregate Distribution of Wages

Vacancies and workers draw their wage offers and demands from distributions $H(w)$ and $F(w)$ which are unobserved. The realised distributions (that are observed) differ from the offers and demands because waiting vacancies hire the worker who has demanded the lowest wage, and waiting workers accept the highest offer. Denote the cumulative distribution of realised wages by $G(w)$ in the vacancy market and by $M(w)$ in the job seeker market. We have

$$\begin{aligned} M(w) &= \frac{\sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} (H(w))^k}{1-e^{-\phi}} \\ &= \frac{e^{-\phi(1-H(w))} - e^{-\phi}}{1-e^{-\phi}} \\ &= \frac{e^{-\phi}\left(\frac{1}{1-w} - 1\right)}{1-e^{-\phi}}. \end{aligned} \tag{24}$$

That is, $M(w)$ is the probability that the highest offer, conditional on the job seeker receiving at least one offer, is equal to or less than w. The denominator $1-e^{-\phi}$ conditions for receiving at least one offer. The density function is

$$m(w) = M'(w) = \frac{e^{-\phi}}{(1-e^{-\phi})(1-w)^2}. \tag{25}$$

In the vacancy market the probability of having w as the lowest wage demand received by a vacancy, conditional on receiving at least one application, is

$$\begin{aligned} G(w) &= \frac{\sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \left[1-(1-F(w))^k\right]}{1-e^{-\theta}} \\ &= \frac{1-e^{-\theta} - e^{-\theta} \sum_{k=1}^{\infty} \frac{\theta^k (1-F(w))^k}{k!}}{1-e^{-\theta}} \\ &= \frac{1-e^{-\theta} - e^{-\theta}\left(e^{\theta(1-F(w))} - 1\right)}{1-e^{-\theta}} \\ &= \frac{1-e^{-\theta F(w)}}{1-e^{-\theta}}, \end{aligned} \tag{26}$$

where the denominator $1-e^{-\theta}$ conditions for receiving at least one application. Using $F(w)=1+\dfrac{\ln w}{\theta}$ we end up with

$$G(w)=\frac{1-\dfrac{e^{-\theta}}{w}}{1-e^{-\theta}}. \tag{27}$$

The density function is

$$g(w)\equiv G'(w)=\frac{e^{-\theta}}{\left(1-e^{-\theta}\right)w^{2}}. \tag{28}$$

The value of $m\left(w\right)$ is the probability that w is the highest offer a waiting job seeker gets; that is, $m\left(w\right)$ is the probability that his realised wage is w. Similarly, the value of $g(w)$ is the probability that w is the lowest wage demand that a waiting vacancy receives. The realised aggregate density function of wages, $r(w)$, is a weighted combination of densities $g(w)$ and $m\left(w\right)$ as follows:

$$r(w)=\begin{cases}\dfrac{m\left(w\right)\left(1-x\right)u}{\left(1-x\right)u+yv} & if \quad w\in\left[0,e^{-\theta}\right], \\ \dfrac{m\left(w\right)\left(1-x\right)u+g(w)yv}{\left(1-x\right)u+yv} & if \quad w\in(e^{-\theta},1-e^{-\phi}], \\ \dfrac{g(w)yv}{\left(1-x\right)u+yv} & if \quad w\in(1-e^{-\phi},1].\end{cases} \tag{29}$$

Job seekers do not send wage demands lower than $e^{-\theta}$: For $w<e^{-\theta}$, only vacancies send offers, and there are $\left(1-x\right)u$ job seekers who receive them. Vacancies do not send offers higher than $1-e^{-\phi}$: for $w>1-e^{-\phi}$, only job seekers send offers to yv vacancies. For middle-range wages, $(1-y)v$ vacancies send offers to $\left(1-x\right)u$ unemployed workers and xu unemployed workers send wage demands to yv vacancies. Using the equilibrium values for x and y, from (20) and (21) , and the solutions for the density functions from (25) and (28), and result $\theta=\phi=\theta_0$, gives

Proposition 5 *The density function of realised wages $r(w)$ satisfies*

$$r(w)=\begin{cases}\dfrac{e^{-\theta_0}\left(\theta_0-\alpha\right)}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)\left(1+\alpha\right)\left(1-w\right)^{2}} & \textit{if } w\in[0,e^{-\theta_0}), \\ \dfrac{e^{-\theta_0}}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)\left(1+\alpha\right)}\left[\dfrac{\theta_0-\alpha}{\left(1-w\right)^{2}}+\dfrac{\alpha\theta_0-1}{w^{2}}\right] & \textit{if } w\in\left[e^{-\theta_0},1-e^{-\theta_0}\right], \\ \dfrac{e^{-\theta_0}\left(\alpha\theta_0-1\right)}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)\left(1+\alpha\right)w^{2}} & \textit{if } w\in(1-e^{-\theta_0},1].\end{cases}$$

where $\alpha\in\left(\dfrac{1}{\theta_0},\theta_0\right)$, and θ_0 is the solution to equation (17).

For $u = v$, Figures 2a and 2b show the density functions of realised wages in the job seeker market and in the vacancy market, respectively. Figure 2c shows the aggregate wage density function. The wage distribution is first increasing, in the end decreasing, and u-shaped in the middle. Thus, we do not get exactly the wage distribution observed empirically but one that still has several desirable features. Figures 3a - 3c show that if the ratio u/v increases, the wage density function becomes skewed to the right because the relative size of the vacancy market grows.

A change in the relative number of unemployed workers affects the expected wage in a non-expected way:

Proposition 6 *The expected wage is non-monotonous in u/v. a) If $0 < \alpha < \frac{1}{\theta_0}$, only the job seeker market exists, and an increase in α decreases the expected wage b) If $\alpha > \theta_0$, only the vacancy market exists, and an increase in α decreases the expected wage c) If $\frac{1}{\theta_0} \leq \alpha \leq \theta_0$, a two-sided market exists, and an increase in α increases the expected wage.*

Proof. In Appendix A1.3. ■

Perhaps surprisingly, the expected realised wage can increase as α increases. This outcome results because in the case where the two markets exist, an increase in α increases the relative size of the vacancy market (where job seekers send wage demands). In the vacancy market, the wage density function has a fat right tail, and as α increases, the aggregate distribution will have more mass on the right, despite of the rise of the peak at $w = e^{-\theta_0}$. The probability that a job seeker is not hired in the vacancy market is $\frac{1 - e^{-\theta_0}}{e^{-\theta_0}}$, whereas the probability that he is not hired in the job seeker market is $e^{-\theta_0}$. Clearly, $\frac{1 - e^{-\theta_0}}{e^{-\theta_0}} > e^{-\theta_0}$. However, the utility of a worker is $U_s = e^{-\theta_0}$ in the vacancy market, and it is $U_r = 1 - e^{-\theta_0} - \theta_0 e^{-\theta_0}$ in the job seeker market, as shown in Proposition 3. That is, if the vacancy market grows, the lower matching probability is exactly compensated by an increase in the expected wage.

7 Dynamic Models

The static version of the model is sufficient for generating a wage distribution. Still, one may be interested in dynamics, especially if there are data on discount factors or separation rates. This in mind we provide the central results of two dynamic models in this section. The wage distribution looks pretty much the same as in the static model. The detailed derivation of the results is relegated to the appendix. The analysis mirrors to the most part the analysis of the static case, but the derivation of the equilibrium mixed strategies for wage demands and offers is more complicated. This is because the upper and lower limits of the support of the strategies are now endogenously determined by the expected life time utilities, while in the static model the outside option, or the expected life time utility, of an agent who rejects an offer is zero.

7.1 A Dynamic Partial Equilibrium Model

Instead of assuming one-period-living agents, we now assume that the agents live infinitely long and discount future at rate δ. The agents send and receive offers each period until they are matched. A matched pair exits the economy and produces an output the total discounted value of which is unity. The matched agents are replaced by identical but yet unmatched agents. Each vacancy and worker who send wage offers use a symmetric mixed strategy. The equilibrium fractions of agents in the vacancy market are the same as in the static model:

$$\begin{aligned} x &= \frac{\theta_0 \left(\alpha\theta_0 - 1\right)}{\alpha\left(\theta_0^2 - 1\right)}, \\ y &= \frac{\alpha\theta_0 - 1}{\theta_0^2 - 1}. \end{aligned}$$

Proposition 7 *There exist* $0 < a < A < b < B < 1$ *such that the aggregate density of realized wages in the two-sided market is*

$$r(w) = \begin{cases} \dfrac{m(w)(1-x)\alpha}{(1-x)\alpha + y} & \text{if } a \le w < A, \\ \dfrac{m(w)(1-x)\alpha + g(w)y}{(1-x)\alpha + y} & \text{if } A \le w \le b, \\ \dfrac{g(w)y\alpha}{(1-x)\alpha + y} & \text{if } b < w \le B,. \end{cases}$$

where

$$m(w) = \frac{(1-\delta)e^{-\theta_0}(1-\delta\theta_0 e^{-\theta_0})}{(1-e^{-\theta_0})\left[(1-w)(1-\delta\theta_0 e^{-\theta_0}) - \delta e^{-\theta}\right]^2},$$

$$g(w)=\frac{\left(1-\delta\theta_0e^{-\theta_0}\right)(1-\delta)e^{-\theta_0}}{(1-e^{-\theta_0})\left[w(1-\delta\theta_0e^{-\theta_0})-\delta e^{-\theta_0}\right]^2},$$
$\alpha\in\left(\frac{1}{\theta_0},\theta_0\right)$ *,and where* θ_0 *satisfies (17),* $\theta_0\approx 1.146$.

The above result is derived in detail in Appendix 2. The boundaries are $a=\frac{\delta\left(1-e^{-\theta_0}-\theta_0e^{-\theta_0}\right)}{1-\delta\theta_0e^{-\theta_0}}$, $b=\frac{1-e^{-\theta_0}-\delta\theta_0e^{-\theta_0}}{1-\delta\theta_0e^{-\theta_0}}$, $A=\frac{e^{-\theta_0}}{1-\delta\theta_0e^{-\theta_0}}$, and $B=\frac{1-\delta+\delta e^{-\theta_0}}{1-\delta\theta_0e^{-\theta_0}}$.

The wage distribution is qualitatively very similar to that in the static model: It is low and increasing at low wages, high and decreasing at high wages, and high and u-shaped a the middle-range wages. An increase in δ makes the wage distribution to concentrate towards the middle of the wage range, because $\frac{da}{d\delta}>0$, $\frac{dA}{d\delta}>0$, $\frac{db}{d\delta}<0$, and $\frac{dB}{d\delta}<0$. That is, when job seekers and vacancies become more patient, the job seekers' reservation wage increases, but their highest wage demand decreases because the vacancies are more willing to wait for low wage demands. Increased patience lowers the highest wage offer made by vacancies but increases the lowest offer, because the job seekers are more willing to wait for good offers.

7.2 A Dynamic General Equilibrium Model

The general equilibrium model described in Section 2 gives the same equilibrium market structure as the two other models. For the aggregate wage distribution we have

Proposition 8 *There exist* $0<a<A<b<B<1$ *such that the aggregate density of realised wages in the two-sided markets*

$$r(w)=\begin{cases}\dfrac{m(w)(1-x)\alpha}{(1-x)\alpha+y} & \text{if } a\leq w<A,\\ \dfrac{m(w)(1-x)\alpha+g(w)y}{(1-x)\alpha+y} & \text{if } A\leq w\leq b,\\ \dfrac{g(w)y\alpha}{(1-x)\alpha+y} & \text{if } b<w\leq B,\end{cases}$$

where

$$m(w)=\frac{\left(1-\delta(1-s)\theta_0e^{-\theta_0}\right)(1-\delta(1-s))e^{-\theta_0}}{(1-e^{-\theta_0})\left[(1-w)(1-\delta(1-s)\theta_0e^{-\theta_0})-\delta(1-s)e^{-\theta_0}\right]^2},$$
$$g(w)=\frac{\left(1-\delta(1-s)\;\theta_0e^{-\;\theta_0}\right)(1-\delta(1-s))e^{-\;\theta_0}}{(1-e^{-\;\theta_0})\left[w(1-\delta(1-s)\;\theta_0e^{-\;\theta_0})-\delta(1-s)e^{-\;\theta_0}\right]},$$
$\alpha\in\left(\frac{1}{\theta_0},\theta_0\right)$ *,and where* θ_0 *satisfies (17),* $\theta_0\approx 1.146$.

The above result is derived in detail in Appendix 3. The boundaries are $a = \frac{\delta(1-s)\left(1-e^{-\theta_0}-\theta_0 e^{-\theta_0}\right)}{1-\delta(1-s)\theta_0 e^{-\theta_0}}$, $b = \frac{1-\delta(1-s)\theta_0 e^{-\theta_0}-e^{-\theta_0}}{1-\delta(1-s)\theta_0 e^{-\theta_0}}$, $A = \frac{e^{-\theta_0}}{1-\delta(1-s)\theta_0 e^{-\theta_0}}$, $B = \frac{1-\delta(1-s)\left(1-e^{-\theta_0}\right)}{1-\delta(1-s)\theta_0 e^{-\theta_0}}$. The wage distribution responses to changes in δ in the same way as in the simpler dynamic model, and it gets more concentrated around the middle of the wage range if matches last longer. In order to have a two-sided market, we must have $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$. This imposes restrictions on the relative magnitude of L and K, the total numbers of workers and firms. Let $L/K \equiv \beta$.

Proposition 9 *In order to have a two-sided market, we must have* $\frac{L}{K} \in \left(\underline{\beta}(s), \overline{\beta}(s)\right)$, *where* $s \in [0,1]$, $\underline{\beta}(1) = 1/\theta_0$, $\overline{\beta}(1) = \theta_0$, $\frac{\partial \underline{\beta}}{\partial s} < 0$, *and* $\frac{\partial \overline{\beta}}{\partial s} > 0$.

Proof. If $\beta < 1$, then $\alpha < \beta$ for all values of s less than one. Therefore, in order to have a two-sided market, the lower bound for β, denoted by $\underline{\beta}$, equals $1/\theta_0$ if $s = 1$. A change in s affects α: $\frac{\partial \alpha}{ds} = \frac{-(L-K)\frac{\partial v}{\partial s}}{v^2}$. An increase in s increases the steady state number of vacancies, therefore we have $\frac{\partial \alpha}{ds} > 0$ if $L < K$. Therefore, a decrease in s increases $\underline{\beta}$. If $\beta > 1$, $\alpha > \beta$ for all values of s less than one. The upper bound for β for having a two-sided market is $\overline{\beta}$. If $s = 1$, $\overline{\beta} = \theta_0$. By an analogous reasoning, a decrease in s decreases $\overline{\beta}$. ■

Proposition 9 says that in the general equilibrium model, in order to have a two-sided market, the boundaries for the relative number of workers and firms are tighter than the boundaries for the relative number of unemployed and vacancies.

8 Conclusion

We derive a wage density function for homogenous firms and homogenous workers. Both vacancies and unemployed workers can send wage demands or offers, which is what we often see happening in real labour markets. We show that the symmetric equilibrium for offers and demands is in mixed strategies, and we solve the equilibrium fractions of vacancies and unemployed workers who are engaged in sending or receiving offers. If the measure of firms is roughly the same as the measure of workers, a two-sided market exists. We show that there then exists a wage density function that is first increasing, in the end decreasing, and in the middle either increasing, decreasing or u-shaped. The wage distribution the model produces is not exactly the one observed empirically, but it is fairly close to that. It is notable that we get this distribution without assuming any kind of heterogeneity among workers or firms. The model has several equilibria, but only the evolutionarily stable one produces the interesting wage distribution. For roughly equal-sized pools of firms and workers, the equilibria that are associated with monotonous distributions are unstable. Our model also predicts that the expected realised wage is non-monotonous in the relative supply of labour. Another interesting result is that the mixed strategies that vacancies and job seekers use when sending offers are utilitywise equivalent to auctions where the agents know the number of their competitors.

We believe that our approach offers plenty of chances for applications and generalisations. The meeting technology we use means that the matching function is well determined with a firm microfoundation. Consequently, one can do rigorous comparative statics as nothing comes from outside of the model. In particular, one can determine the response of duration of unemployment spells when the measure of workers or firms changes, or when the expected life-span of matches changes. The model is well suited to consider the implications of worker/firm heterogeneity on wage distribution.

Our view is that the results of this article nicely illuminate the strengths of the urn-ball model over the search models. In the end, it is clear that whatever one can do using the search models, one can also do using the urn-ball models, and with the latter ones one can do much more, with no need to postulate the black box of a matching function. To give another example, it is relatively straightforward to consider a situation where vacancies post wages that are observed by the unemployed workers who strategically decide which vacancy to contact based on the observed wage offers (see e.g. Kultti 1999; Julien, Kennes and King, 2001). This is practically impossible in the search models. Against this background it is somewhat a mystery to us why search models are still used.

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Appendix

A1 THE STATIC MODEL

A1.1 Proof of Lemma 1

Vacancies' equilibrium VE can be written as

$$1-e^{-\frac{xu}{yv}}-\frac{xu}{yv}e^{-\frac{xu}{yv}}-e^{-\frac{(1-y)\,v}{(1-x)\,u}}=0,$$

and unemployed workers' equilibrium UE is

$$1-e^{-\frac{(1-y)\,v}{(1-x)\,u}}-\frac{(1-y)\,v}{(1-x)\,u}e^{-\frac{(1-y)\,v}{(1-x)\,u}}-e^{-\frac{xu}{yv}}=0.$$

The behaviour of VE and UE near $(0,0)$ is analyzed first.

1. $(x,y)\rightarrow(0,y)$. i) VE becomes $-e^{-\frac{(1-y)\,v}{u}}=0$, which cannot hold for any $y\in[0,1]$. ii) UE becomes $-e^{-\frac{(1-y)\,v}{u}}-\frac{(1-y)\,v}{u}e^{-\frac{(1-y)\,v}{u}}=0$, which cannot hold for any $y\in[0,1]$.

2. $(x,y)\rightarrow(x,0)$. i) VE becomes $1-e^{-\frac{v}{(1-x)\,u}}=0$, which cannot hold for any $x\in[0,1]$. ii) UE becomes $1-e^{-\frac{v}{(1-x)\,u}}-\frac{v}{(1-x)\,u}e^{-\frac{v}{(1-x)\,u}}=0$, which does not hold for any $x\in[0,1]$.

Clearly, neither VE nor UE cannot go through $(0,y)$ or $(x,0)$, so they must go through $(0,0)$. We check that this is possible. Assume that along VE, $x/y\rightarrow a$ as $x\rightarrow 0$ and $y\rightarrow 0$. Then

$$\lim_{x\rightarrow0,y\rightarrow0}\frac{1-y}{1-x}=\lim\left(\frac{1}{1-x}-\frac{1}{\frac{1}{y}-\frac{x}{y}}\right)=1-\lim\frac{1}{\frac{1}{y}-a}=1.$$

Then

$$\lim_{x\rightarrow0,y\rightarrow0}\left(1-e^{-\theta}-\theta e^{-\theta}-e^{-\phi}\right)=1-e^{-a\alpha}-a\alpha e^{-a\alpha}-e^{-1/\alpha},$$

which, by for example letting $\alpha=1$, equals zero if $a=1.285$. For any other $\alpha>0$ one can find $a>0$ that satisfies VE going through $(0,0)$. Assume that along UE, $x/y\rightarrow b$

as $x \to 0$ and $y \to 0$. Then

$$\lim_{x\to 0, y\to 0}\left(1-e^{-\phi}-\phi e^{-\phi}-e^{-\theta}\right)=1-e^{-1/\alpha}-\frac{1}{\alpha}e^{-1/\alpha}-e^{-b\alpha},$$

which equals zero if $\alpha = 1$ and $b = 1.33$. Because $a < b$, VE is above UE near $(0,0)$, which is does not contradict UE being steeper than VE in their intersection at strictly positive x and y.

Next check the curves' positions near $(1,1)$.

1. $(x,y) \to (x,1)$. i) VE becomes $-e^{-\frac{xu}{v}}-\frac{xu}{v}e^{-\frac{xu}{v}}=0$, which cannot hold for any $x \in [0,1]$. ii) UE becomes $-e^{-\frac{xu}{v}}=0$, which cannot hold for any $x \in [0,1]$.
2. $(x,y) \to (1,y)$. i) VE becomes $1-e^{-\frac{u}{yv}}-\frac{u}{yv}e^{-\frac{u}{yv}}=0$, which cannot hold for any $y \in [0,1]$. ii) UE becomes $1-e^{-\frac{u}{yv}}=0$, which cannot hold for any $y \in [0,1]$.

We see that VE and UE must go through $(1,1)$. As x and y approach 1, assume that $\frac{1-y}{1-x} \to c$ along VE and $\frac{1-y}{1-x} \to g$ along UE. Then VE becomes

$$\lim_{x\to 1, y\to 1}\left(1-e^{-\theta}-\theta e^{-\theta}-e^{-\phi}\right)=1-e^{-\alpha}-\alpha e^{-\alpha}-e^{-c/\alpha}=0,$$

which holds for example if $\alpha = 1$ and $c = 1.33$. In the limit UE equals

$$\lim_{x\to 1, y\to 1}\left(1-e^{-\phi}-\phi e^{-\phi}-e^{-\theta_0}\right)=1-e^{-g/\alpha}-\frac{g}{\alpha}e^{-g/\alpha}-e^{-\alpha}=0;$$

with $\alpha = 1$ it holds if $g = 1.285$. Near $(1,1)$, UE lies above VE, which is consistent with UE being steeper than VE in their intersection at strictly positive x and y. ■

A1.2 Proof of Proposition 4

The uniqueness is directly seen from (20) and (21). In order to solve the stability of a mixed-strategy equilibrium we determine the positions of VE and UE. When differentiating VE and UE with respect to x and y, we use the following results: $\frac{\partial\theta}{\partial x}=\frac{\alpha}{y}$, $\frac{\partial\theta}{\partial x}=\frac{-\alpha x}{y^2}$, $\frac{\partial\phi}{\partial x}=\frac{\phi}{1-x}$, and $\frac{\partial\phi}{\partial y}=-\frac{1}{(1-x)\,\alpha}$. Differentiating VE with respect to x and y yields

$$\frac{dy}{dx}\mid_{VE}=\frac{\phi e^{-\phi}\alpha y+\theta e^{-\theta}\alpha^2(1-x)}{e^{-\phi}y+\theta^2 e^{-\theta}\alpha(1-x)}, \tag{30}$$

and along UE,

$$\frac{dy}{dx}\mid_{UE}=\frac{e^{-\theta}\alpha^{2}\left(1-x\right)+\phi^{2}e^{-\phi}\alpha y}{\theta e^{-\theta}\alpha\left(1-x\right)+\phi e^{-\phi}y}. \tag{31}$$

Both equilibrium curves have a positive slope. Waiting firms fare equally well as moving firms only if an increase in the share of moving workers is accompanied with an increase in the share of waiting firms. The same kind of intuition applies for workers' equilibrium condition, too. Next we look whether VE is steeper than UE in equilibrium, or the other way round. Subtracting the right-hand side of (31) from that of (30) yields, after a few steps, that $sign\left(\frac{dy}{dx}\mid_{VE}-\frac{dy}{dx}\mid_{UE}\right)=sign\left(2\theta\phi-\theta^{2}\phi^{2}-1\right)$. In equilibrium $\theta=\phi=\theta_0\approx 1.146$. Function $2z^2-z^4-1$ has a unique maximum of zero at $z=1$, therefore $2\theta_0^2-\theta_0^4-1<0$, which indicates that in equilibrium UE is steeper than VE. In studying the stability of the mixed-strategy equilibrium, we compare the utility from waiting and moving for firms and workers when they are off the equilibrium curve. The difference of utilities of waiting and moving for firms is $V_r-V_s=1-e^{-\theta}-\theta e^{-\theta}-e^{-\phi}$. Suppose that a firm is on VE, and then the fraction of moving workers, x, increases. Then

$$\frac{\partial\left(V_r-V_s\right)}{\partial x}=\frac{\partial\left(1-e^{-\theta}-\theta e^{-\theta}-e^{-\phi}\right)}{\partial x}=\frac{\theta e^{-\theta}\alpha}{y}+\frac{\phi e^{-\phi}}{1-x}>0, \tag{32}$$

which indicates that for a firm, it is now more profitable to wait than move, and therefore the fraction of waiting firms, y, will increase. For workers, $U_r-U_s=1-e^{-\phi}-\phi e^{-\phi}-e^{-\theta}$. If a worker is on UE, and then y increases, the utility difference changes by

$$\frac{\partial\left(U_r-U_s\right)}{\partial x}=\frac{\partial\left(1-e^{-\phi}-\phi e^{-\phi}-e^{-\theta}\right)}{\partial y}=-\frac{\phi e^{-\phi}}{\left(1-x\right)\alpha}-\frac{e^{-\theta}\alpha x}{y^{2}}<0. \tag{33}$$

If the fraction of waiting firms increases, waiting becomes less appealing for workers compared to moving, therefore x will increase. In (x,y) -plane, y decreases above VE and increases below it, and x increase on the left of UE and decreases on the right of it.

A1.3 Proof of Proposition 6 (The expected wage in the static model)

a) If only the job-seeker market exists (this happens if $u/v<1/\theta_0$), the density function for the realized wages is

$$m(w)=\frac{e^{-\phi}}{\left(1-e^{-\phi}\right)\left(1-w\right)^{2}},$$

and the expected wage is

$$w^e = \int_0^{1-e^{-\phi}} \frac{e^{-\phi} w}{\left(1-e^{-\phi}\right)(1-w)^2} dw$$
$$= \frac{1-e^{-\phi}-\phi e^{-\phi}}{1-e^{-\phi}}.$$

Differentiating with respect to ϕ $(= v/u)$ yields

$$\frac{dw^e}{d\phi} = \frac{-e^{-\phi}\left(1-\phi-e^{-\phi}\right)}{\left(1-e^{-\phi}\right)^2} > 0.$$

An increase in u/v thus decreases the expected wage.

b) If only the vacancy market exists (this happens if $u/v > \theta_0$), the density function for the realized wages is

$$g(w) = \frac{e^{-\theta}}{\left(1-e^{-\theta}\right) w^2},$$

and the expected wage is

$$w^e = \int_{e^{-\theta}}^{1} \frac{e^{-\theta} w}{\left(1-e^{-\theta}\right) w^2} dw$$
$$= \frac{\theta e^{-\theta}}{1-e^{-\theta}}.$$

Differentiating with respect to θ $(= u/v)$ yields

$$\frac{dw^e}{d\theta} = \frac{e^{-\theta}\left(1-\theta-e^{-\theta}\right)}{\left(1-e^{-\theta}\right)^2} < 0.$$

An increase in u/v thus decreases the expected wage.

c) If both job-seeker market and vacancy market exist, the expected realized wage equals

$$w^e = \int_0^{e^{-\theta_0}} wr(w)dw + \int_{e^{-\theta_0}}^{1-e^{-\theta_0}} wr(w)dw + \int_{1-e^{-\theta_0}}^{1} wr(w)dw$$
$$= \int_0^{e^{-\theta_0}} \frac{e^{-\theta_0}\left(\theta_0-\alpha\right)}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)(1+\alpha)(1-w)^2} wdw$$
$$+ \int_{e^{-\theta_0}}^{1-e^{-\theta_0}} \frac{e^{-\theta_0}}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)(1+\alpha)} \left[\frac{\theta_0-\alpha}{(1-w)^2} + \frac{\alpha\theta_0-1}{w^2}\right] wdw$$
$$+ \int_{1-e^{-\theta_0}}^{1} \frac{e^{-\theta_0}\left(\alpha\theta_0-1\right)}{\left(1-e^{-\theta_0}\right)\left(\theta_0-1\right)(1+\alpha) w^2} wdw,$$

which results in

$$
\begin{aligned}
w^e &= \frac{(5.3186\times 10^{-5})\,(5731-5000\alpha)}{1+\alpha} \\
&+\left(\frac{(0.91657)\,e^{-\ 1.1462}\,(1.1462-\alpha)}{(1-e^{-\ 1.1462})\,(\ 1.1462-1)\,(1+\alpha)}\right)+\left(\frac{(0.76371)\,e^{-\ 1.1462}\,(\alpha(1.1462)-1)}{(1-e^{-\ 1.1462})\,(\ 1.1462-1)\,(1+\alpha)}\right) \\
&+\frac{(2.438\times 10^{-4})\,(5731\alpha-5000)}{1+\alpha}
\end{aligned}
$$

Differentiating with respect to α yields

$$
\frac{dw^e}{d\alpha}=2.0\times 10^{-9}\frac{4.9995\times 10^{8}+28010\alpha}{(1+\alpha)^2}>0.\blacksquare
$$

A2 A DYNAMIC PARTIAL EQUILIBRIUM MODEL

A2.1 Vacancies send offers to unemployed workers

Let us study a situation where the unemployed workers are like urns and vacancies as balls. Our aim is to determine the mixed strategy of the vacancies in a dynamic model focusing on a steady-state. It turns out that the agents' expected utilities are the same as in a corresponding model where the wages are determined by auction. We need the results of the auction to determine the mixed strategies, and that in mind we first determine the agents' expected utilities under auction. The unemployed workers' and the vacancies' utilities are determined by the following equations

$$
U_r^{auc}=(e^{-\phi}+\phi e^{-\phi})\delta U_r^{auc}+(1-e^{-\phi}-\phi e^{-\phi})(1-\delta V_s^{auc}), \tag{34}
$$

$$
V_s^{auc}=e^{-\phi}(1-\delta U_r^{auc})+(1-e^{-\phi})\delta V_{s.}^{auc} \tag{35}
$$

In (34), $e^{-\phi}$ is the probability that no vacancy comes to the unemployed worker, and $\phi e^{-\phi}$ is probability of just one vacancy arriving, in which case the vacancy makes a take-it-or-leave-it offer. In both these cases, the unemployed worker continues to the next period with his discounted reservation value δU_r^{auc}. If he gets two or more vacancies, the vacancies engage in Bertrand competition for the right to employ the worker. The vacancies, regardless of which of them employs the worker, get their discounted reservation value δV_s^{auc}, and the worker gets $1-\delta V_s^{auc}$. In (35), with probability $e^{-\phi}$ the vacancy is the only one that meets the worker, the vacancy makes take-it-or-leave-it offer and gets one minus the unemployed worker's discounted reservation value. If the vacancy has at least one competitor, it gets it discounted reservation value. From these one gets explicit expressions

$$U_r^{auc} = \frac{1 - e^{-\phi} - \phi e^{-\phi}}{1 - \delta\phi e^{-\phi}}, \tag{36}$$

$$V_s^{auc} = \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \tag{37}$$

These utilities are the same as in Kultti (1999), except for the slightly different way of discounting.

Let us now leave the auction and assume that vacancies use a continuous mixed strategy H with support $[a, b]$. A vacancy's expected utility when he offers wage $w \in [a, b]$ is given by V_s:

$$V_s = \sum_{k=0}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \left[H^k(w)(1-w) + \left(1 - H^k(w)\right) \delta V_s \right], \tag{38}$$

which after some simplification equals

$$V_s = e^{-\phi\left(1-H^k(w)\right)}(1-w) + \left(1 - e^{-\phi\left(1-H^k(w)\right)}\right) \delta V_s. \tag{39}$$

Next we use the fact that any wage in the support of the mixed strategy yields the same utility to the vacancy, in particular, this holds for the lowest and the highest wages

$$V_s = V_s(a) = e^{-\phi}(1-a) + (1 - e^{-\phi})\delta V_s = V_s(b) = 1 - b. \tag{40}$$

From this we can solve for

$$b = 1 - e^{-\phi}(1-a) - (1 - e^{-\phi})\delta V_s = 1 - e^{-\phi} + e^{-\phi}\delta U_r - (1 - e^{-\phi})\delta V_s, \tag{41}$$

where the last equality is based on the fact that the lowest wage in the support of the mixed strategy must equal the workers' discounted outside option δU_r, i.e. it must make them indifferent between accepting it and continuing search. Thus, we get

$$a = \delta U_r. \tag{42}$$

We let $h(w) = H'(w)$ and determine the unemployed workers' expected utility as

$$U_r = \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \int_a^b wkh(w)H^{k-1}(w)dw + e^{-\phi}\delta U_r, \tag{43}$$

which equals

$$U_r = \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \left[/_a^b \, H^k(w)w - \int_a^b H^k(w)dw \right] = \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \left[b - \int_a^b H^k(w)dw \right] \quad (44)$$

which in turn equals, using Fubini's theorem,

$$U_r = b - \int_a^b e^{-\phi(1-H(w))} dw. \quad (45)$$

Inserting this and (40) into (41) yields

$$b = 1 - e^{-\phi} - \frac{\delta e^{-\phi}}{1-\delta} \int_a^b e^{-\phi(1-H(w))} dw. \quad (46)$$

Next we impose that the expected utility of a vacancy if it uses b equals that of a vacancy in auction:

$$V_s = V_s(b) = 1 - b = \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}, \quad (47)$$

which yields

$$b = \frac{1 - e^{-\phi} - \delta\phi e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \quad (48)$$

Setting $V_s = V(a)$ we get

$$a = \frac{\delta \left(1 - e^{-\phi} - \phi e^{-\phi}\right)}{1 - \delta\phi e^{-\phi}}. \quad (49)$$

Using the fact that $V_s = V_s(b)$, solving V_s from (39) and equating it with (47) yields

$$e^{-\phi(1-H(w))} \left[1 - w - \frac{\delta e^{-\phi}}{1 - \delta\phi e^{-\phi}} \right] = \frac{(1-\delta)e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \quad (50)$$

From this we can solve the equilibrium mixed strategy

$$H(w) = 1 - \frac{1}{\phi} \ln \left(1 - w - \frac{\delta e^{-\phi}}{1 - \delta\phi e^{-\phi}} \right) + \frac{1}{\phi} \ln \left(\frac{(1-\delta)e^{-\phi}}{1 - \delta\phi e^{-\phi}} \right). \quad (51)$$

The equilibrium mixed strategy is unobservable while the realised wages that result from it generate an observable wage distribution. We denote the cumulative distribution function for realised wages by $M(w)$ and the corresponding density function is denoted by $m(w)$. Let us determine the probability that wage w is observed.

$$\left(1 - e^{-\phi}\right) m(w) = \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} k h(w) H^{k-1}(w) = \phi e^{-\phi(1-H(w))} h(w). \quad (52)$$

Here, $\left(1-e^{-\phi}\right)$ is the probability that a job seeker gets at least one offer. From k offers he chooses the highest one. From this we get

$$m(w)=\frac{e^{-\phi(1-H(w))}-e^{-\phi}}{1-e^{-\phi}} \tag{53}$$

Inserting (51) above and manipulating a little yields an explicit formula

$$m(w)=\frac{(1-\delta)e^{-\phi}(1-\delta\phi e^{-\phi})}{(1-e^{-\phi})\left[(1-w)(1-\delta\phi e^{-\phi})-\delta e^{-\phi}\right]^2}. \tag{54}$$

A2.2 Unemployed workers send applications to vacancies

In the standard auction model the utilities of vacancies and unemployed workers are

$$V_r^{auc}=(e^{-\theta}+\theta e^{-\theta})\delta V_r^{auc}+(1-e^{-\theta}-\theta e^{-\theta})(1-\delta U_s^{auc}), \tag{55}$$

$$U_s^{auc}=e^{-\theta}(1-\delta V_r^{auc})+(1-e^{-\theta})\delta U_s^{auc}. \tag{56}$$

A vacancy gets its reservation value if does not meet any job seekers or just one. In case of two or more job seekers showing up, the vacancy gets one minus the discounted reservation value of a job seeker. Solving these yields

$$V_r^{auc} = \frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta\theta e^{-\theta}}, \tag{57}$$

$$U_s^{auc} = \frac{e^{-\theta}}{1-\delta\theta e^{-\theta}}. \tag{58}$$

Next assume that unemployed workers use a continuous mixed strategy F with support $[A,B]$ .An unemployed worker's expected utility when he asks wage $w\in[A,B]$ is

$$\begin{aligned}U_s &= \sum_{k=0}^{\infty}\frac{\theta^k e^{-\theta}}{k!}\left([1-F(w)]^k w+\left(1-[1-F(w)]^k\right)\delta U_s\right)\\ &\Leftrightarrow\ U_s=we^{-\theta F(w)}+\left(1-e^{-\theta F(w)}\right)\delta U_s\\ &\Leftrightarrow\ U_s=\frac{we^{-\theta F(w)}}{1-\delta+\delta e^{-\theta F(w)}}.\end{aligned} \tag{59}$$

Any wage in support $[A,B]$ yields the same utility to a worker, especially $U_s(A)=U_s(B)$:

$$U_s(A)=Ae^{-\theta F(A)}+\left(1-e^{-\theta F(A)}\right)\delta U_s=Be^{-\theta F(B)}+\left(1-e^{-\theta F(B)}\right)\delta U_s=U_s(B), \tag{60}$$

and using $F(A)=0$ and $F(B)=1$ we have

$$\begin{aligned} U_s &= A, && (61)\\ A &= Be^{-\theta} + \left(1-e^{-\theta}\right)\delta U_s. && (62) \end{aligned}$$

The highest offer the worker makes must leave the firm its discounted reservation value:

$$B = 1 - \delta V_r, \tag{63}$$

and A can be written as

$$A = e^{-\theta}\left(1-\delta V_r\right) + \left(1-e^{-\theta}\right)\delta U_s. \tag{64}$$

Let $f(w) \equiv F'(w)$ and determine a vacancy's expected utility as

$$\begin{aligned} V_r &= \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \int_A^B (1-w)\, f(w) k \left[1-F(w)\right]^{k-1} dw + e^{-\theta}\delta V_r \qquad (65)\\ &= \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \left(-\left[1-F(B)\right]^k + \left[1-F(A)\right]^k\right)\\ &\quad - \int_A^B \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} f(w) k \left[1-F(w)\right]^{k-1} w dw + e^{-\theta}\delta V_r\\ &= \sum_{k=1}^{\infty} \frac{\theta e^{-\theta}}{k!} - \int_A^B \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{(k-1)!} f(w) \left[1-F(w)\right]^{k-1} w dw + e^{-\theta}\delta V_r\\ &= 1 - e^{-\theta} - \theta \int_A^B f(w) w \sum_{k=0}^{\infty} \frac{\theta^k e^{-\theta}\left[1-F(w)\right]^k}{k!} dw + e^{-\theta}\delta V_r\\ &= 1 - e^{-\theta} - \theta \int_A^B e^{-\theta F(w)} f(w) w dw + e^{-\theta}\delta V_r\\ &= 1 - e^{-\theta} + Be^{-\theta F(B)} - Ae^{-\theta F(A)} - \int_A^B e^{-\theta F(w)} dw + e^{-\theta}\delta V_r\\ &= 1 - e^{-\theta} + Be^{-\theta} - A - \int_A^B e^{-\theta F(w)} dw + e^{-\theta}\delta V_r. \end{aligned}$$

Then substitute $1-\delta V_r$ for B and rearrange to have

$$V_r = 1 - A - \int_A^B e^{-\ \theta F(w)} dw. \tag{66}$$

Using (66) and $A = U_s$in (64) we get

$$A = \left[1 - \delta\left(1 - A - \int_A^B e^{-\theta F(w)} dw\right)\right] e^{-\theta} + \delta\left(1-e^{-\theta}\right) A \tag{67}$$

which implies that

$$A = \frac{\left(1 - \delta + \delta \int_A^B e^{-\theta F(w)} dw\right) e^{-\theta}}{1 - \delta}. \tag{68}$$

The lower bound of the support of wage asks cannot be determined explicitly.

Next we impose that the expected utility of an unemployed worker equals that of an unemployed worker in auction:

$$U_s = U_s^{auc} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}} = A. \tag{69}$$

Utilizing (60) yields

$$U_s = Be^{-\theta} + \left(1 - e^{-\theta}\right) \delta U_s, \tag{70}$$

which gives, along with (69), that

$$U_s = \frac{Be^{-\theta}}{1 - \delta + \delta e^{-\theta}} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}}. \tag{71}$$

Solving for B gives

$$B = \frac{1 - \delta + \delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}. \tag{72}$$

Equating U_s given by (59) and that given by (71) yields

$$\frac{we^{-\theta F(w)}}{1 - \delta + \delta e^{-\theta F(w)}} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}} \tag{73}$$

which implies that

$$e^{-\theta F(w)} = \frac{\dfrac{(1 - \delta)\, e^{-\theta}}{1 - \delta\theta e^{-\theta}}}{w - \dfrac{\delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}} \tag{74}$$

Taking logarithms and arranging results in

$$F(w) = \frac{1}{\theta}\left[\ln\left(w - \frac{\delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}\right) - \ln\left(\frac{(1 - \delta)\, e^{-\theta}}{1 - \delta\theta e^{-\theta}}\right)\right]. \tag{75}$$

We denote again the cumulative distribution function for realised wages by $G(w)$ and the corresponding density function by $g(w)$. We get

$$G(w) = \frac{1 - e^{-\theta F(w)}}{1 - e^{-\theta}} \tag{76}$$

$$= \frac{1}{1 - e^{-\theta}} - \frac{(1 - \delta)e^{-\theta}}{(1 - e^{-\theta})\left[w(1 - \delta\,\theta e^{-\theta}) - \delta e^{-\theta}\right]}. \tag{77}$$

The density function is

$$g(w) = \frac{(1-\delta)\left(1-\delta\ \theta e^{-\ \theta}\right)e^{-\ \theta}}{(1-e^{-\ \theta})\left[w(1-\delta\ \theta e^{-\ \theta})-\delta e^{-\ \theta}\right]^2}. \tag{78}$$

A2.3 The equilibrium market structure

In equilibrium, workers are indifferent between sending applications and receiving offers from firms, and firms are indifferent between making offers and receiving applications from workers. The equilibrium condition for a vacancy, VE, is

$$\frac{e^{-\phi}}{1-\delta\phi e^{-\phi}} = \frac{1-e^{-\ \theta}-\ \theta e^{-\ \theta}}{1-\delta\theta e^{-\ \theta}}, \tag{79}$$

and the respective condition UE for a job seeker is

$$\frac{e^{-\ \theta}}{1-\delta\theta e^{-\ \theta}} = \frac{1-e^{-\phi}-\phi e^{-\phi}}{1-\delta\phi e^{-\phi}}. \tag{80}$$

Proceeding as in the static case, it turns out that in equilibrium $\theta = \phi = \theta_0 \approx 1.146$, and

$$x = \frac{\theta_0\left(\alpha\theta_0-1\right)}{\alpha\left(\theta_0^2-1\right)}, \tag{81}$$

$$y = \frac{\alpha\theta_0-1}{\theta_0^2-1}. \tag{82}$$

as in the static model. Along UE,

$$\frac{dy}{dx}\mid_{UE} = \frac{P\dfrac{d\phi}{dx}-Q\dfrac{d\theta}{dx}}{Q\dfrac{d\theta}{dy}-P\dfrac{d\phi}{dy}}, \tag{83}$$

where $Q \equiv \dfrac{-e^{-\theta}}{(1-\delta\theta e^{-\theta})^2}$ and $P \equiv \dfrac{e^{-\phi}\left[(1-\delta)\phi+\delta\left(1-e^{-\phi}\right)\right]}{(1-\delta\phi e^{-\phi})^2}$. Along VE,

$$\frac{dy}{dx}\mid_{VE} = \frac{R\dfrac{d\theta}{dx}-S\dfrac{d\phi}{dx}}{S\dfrac{d\phi}{dy}-R\dfrac{d\theta}{dy}}, \tag{84}$$

where $R \equiv \dfrac{e^{-\theta}\left[(1-\delta)\theta+\delta\left(1-e^{-\theta}\right)\right]}{(1-\delta\theta e^{-\theta})^2}$ and $S \equiv \dfrac{-e^{-\phi}}{(1-\delta\phi e^{-\phi})^2}$. Curve UE is steeper than VE if $(QS-RP)\left(\dfrac{d\phi}{dx}\dfrac{d\theta}{dy}-\dfrac{d\theta}{dx}\dfrac{d\phi}{dy}\right) > 0$. In equilibrium $\phi = \theta = \theta_0 \approx 1.146$, and the sign of $QS - RP$ turns out to be equal to the sign of $(1-\delta)(1-\theta_0)$, which is negative. Expression $\frac{d\phi}{dx}\frac{d\theta}{dy} - \frac{d\theta}{dx}\frac{d\phi}{dy}$ simplifies to $\frac{y-x}{(1-x)^2 y^2}$, which is negative by $x > y$ in equilibrium. These results yield

Proposition A2.1 *In a dynamic model where the agents who exit are replaced by unmatched clones, the mixed-strategy equilibrium where $x \in (0,1)$ and $y \in (0,1)$ is stable.*

As in the static case, the endpoints of both VE and UE are at $(0,0)$ and at $(1,1)$.

A2.4 Distribution of realised wages

Calculating the distribution of realised wages goes analogously to the corresponding task in the static model:

$$r(w) = \begin{array}{l} \frac{m(w)(1-x)\alpha}{(1-x)\alpha + y} \text{ if } a \leq w < A \\ \frac{m(w)(1-x)\alpha + g(w)y}{(1-x)\alpha + y} \text{ if } A \leq w \leq b \\ \frac{g(w)y\alpha}{(1-x)\alpha + y} \text{ if } b < w \leq B \end{array} \tag{85}$$

where, in equilibrium $\theta = \phi = \theta_0$, and $a = \frac{\delta\left(1 - e^{-\theta_0} - \theta_0 e^{-\theta_0}\right)}{1 - \delta\theta_0 e^{-\theta_0}}$, $b = \frac{1 - e^{-\theta_0} - \delta\theta_0 e^{-\theta_0}}{1 - \delta\theta_0 e^{-\theta_0}}$, $A = \frac{e^{-\theta_0}}{1 - \delta\theta_0 e^{-\theta_0}}$, $B = \frac{1 - \delta + \delta e^{-\theta_0}}{1 - \delta\theta_0 e^{-\theta_0}}$.

A3 A DYNAMIC GENERAL EQUILIBRIUM MODEL

A3.1 Vacancies send offers to unemployed workers

Assume that the total number of workers is L and the total number of employers is K. Some of them are matched with each other in productive activities, while others are looking for a partner. The number of unemployed is denoted by u and the number of vacancies by v. Production happens in pairs, therefore

$$L - u = K - v. \tag{86}$$

In each period, a firm-worker pair separates for exogenous reasons with probability s. In a steady state, the number of new matches equals the number of separations in each period:

$$yv(1 - e^{-\theta}) + (1-x)u(1 - e^{-\phi}) = s(K - v). \tag{87}$$

There are yv waiting firms, each of them gets at least one job application with probability $1 - e^{-\theta}$. Each of the $(1-x)u$ waiting job seekers gets at least one job offer with probability

$1 - e^{-\phi}$. For the moment we focus just on the matching market which is assumed to be in a steady state. The only complication to the standard set-up is the exogenous separation probability s, and the fact that a worker who is employed at wage w gets the wage each period as long as the employment relationship lasts, and correspondingly the employer gets $1 - w$ each period.

A3.1.1 Auction

Let the unemployed be urns and the vacancies balls, and let $\phi = \frac{v}{u}$. First we determine their expected life time utilities when wages are determined in auction where the number of competitors is known. The timing is as follows: We determine the expected life time utility of an unemployed worker and a vacancy at the very beginning of a period. The utility of a worker or an employer that has a partner is evaluated right after that, i.e. within the same period before anything else happens. After that the parties produce and get their shares of the production. After that the pair possibly separates. The utility of an unemployed worker is determined by

$$U_r^{auc} = \left(e^{-\phi} + \phi e^{-\phi}\right) \delta U_r^{auc} + \left(1 - e^{-\phi} - \phi e^{-\phi}\right) W^{auc}(\overline{w}) \tag{88}$$

where $\overline{w}$ is the wage that vacancies offer when there are two or more vacancies competing for a worker. We take it as given for now, and determine the equilibrium value later on. We have also used the fact that when an unemployed worker meets exactly one vacancy the vacancy makes a take-it-or-leave-it offer that leaves no surplus to the worker. The utility of a matched worker with wage $\overline{w}$ is determined by

$$W^{auc}(\overline{w}) = \overline{w} + s\delta U_r^{auc} + (1 - s)\delta W^{auc}(\overline{w}). \tag{89}$$

The expected utility of a vacancy is determined by

$$V_s^{auc} = e^{-\phi} E^{auc}\left(\underline{w}\right) + \left(1 - e^{-\phi}\right) \delta V_s^{auc} \tag{90}$$

where $\underline{w}$ is the wage that a vacancy offers when it gets to make a take-it-or-leave-it offer. The expected utility of an employer who employs at wage $\underline{w}$ is determined by

$$E^{auc}(\underline{w}) = 1 - \underline{w} + s\delta V_s^{auc} + (1 - s)\delta E^{auc}(\underline{w}). \tag{91}$$

From these equations we can determine the expected utilities as the function of the wages

$$U_r^{auc} = \frac{\left(1 - e^{-\phi} - \phi e^{-\phi}\right) \overline{w}}{(1-\delta)\left(1 - \delta(1-s)\left(e^{-\phi} + \phi e^{-\phi}\right)\right)} \tag{92}$$

$$W^{auc}(\overline{w}) = \frac{\left(1 - \delta(e^{-\phi} + \phi e^{-\phi})\right) \overline{w}}{(1-\delta)\left(1 - \delta(1-s)\left(e^{-\phi} + \phi e^{-\phi}\right)\right)} \tag{93}$$

$$V_s^{auc} = \frac{e^{-\phi}(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-s)\left(1 - e^{-\phi}\right)\right)} \tag{94}$$

$$E^{auc}(\underline{w}) = \frac{\left(1 - \delta + \delta e^{-\phi}\right)(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-s)\left(1 - e^{-\phi}\right)\right)} \tag{95}$$

Next we determine the two possible equilibrium wages. The higher wage $\overline{w}$, that comes about when several vacancies compete for a worker, must be such that all the vacancies are indifferent between paying the wage and continuing search for a worker, i.e.

$$1 - \overline{w} + s\delta V_s^{auc} + (1-s)\delta E^{auc}(\overline{w}) = \delta V_s^{auc}. \tag{96}$$

Similarly, the lower wage $\underline{w}$, that comes about when a vacancy meets an unemployed worker alone and gets to make a take-it-or-leave-it offer, is such that the worker is indifferent between accepting the wage and continuing search, i.e.

$$\underline{w} + s\delta U_r^{auc} + (1-s)\delta W^{auc}(\underline{w}) = \delta U_r^{auc}. \tag{97}$$

Using (92)-(95), and replacing $\overline{w}$ by $\underline{w}$ in (89), and $\underline{w}$ by $\overline{w}$ in (91), we can solve

$$\overline{w} = \frac{1 - \delta(1-s)\left(e^{-\phi} + \phi e^{-\phi}\right)}{1 - \delta(1-s)\phi e^{-\phi}} \tag{98}$$

$$\underline{w} = \frac{\delta(1-s)\left(1 - e^{-\phi} - \phi e^{-\phi}\right)}{1 - \delta(1-s)\phi e^{-\phi}} \tag{99}$$

Using these data we can finally solve for the expected utility of an unemployed worker who waits

$$U_r^{auc} = \frac{1 - e^{-\phi} - \phi e^{-\phi}}{(1-\delta)\left(1 - \delta(1-s)\phi e^{-\phi}\right)} \tag{100}$$

and for the expected utility of a vacancy that moves

$$V_s^{auc} = \frac{e^{-\phi}}{(1-\delta)(1 - \delta(1-s)\phi e^{-\phi})} \tag{101}$$

A3.1.2 The mixed strategy

The expected utility of a job seeker when the vacancies use a mixed strategy $H(w)$ with support $[a, b]$ is determined by

$$U_r = e^{-\phi}\delta U_r + \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \int_a^b kh(w)H^{k-1}(w)W(w)dw. \tag{102}$$

If the job seeker receives at least one offer, he chooses the highest among k offers. Expression $W(w)$ is the utility of a worker who is employed at wage w, and it is given by

$$W(w) = w + s\delta U_r + (1-s)\delta W(w). \tag{103}$$

The employment relationship continues until it breaks down because of exogenous reasons, with probability s in each period. Solving $W(w)$ and inserting it back to (102) yields the following formula where the last two terms result from partial integration

$$\begin{aligned} U_r \;=\; & e^{-\phi}\delta U_r + \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \int_a^b H^k(w)(1-\delta+\delta s)^{-1} s\delta U_r + \\ & \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \int_a^b H^k(w)(1-\delta+\delta s)^{-1} w - \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \int_a^b H^k(w)(1-\delta+\delta s)^{-1} dw. \end{aligned} \tag{104}$$

Finally, we can simplify this by doing the summations and by changing the order of the summation and integration in the last sum

$$U_r = (1-\delta)^{-1}(1-\delta(1-s)e^{-\phi})^{-1} \left\{ b - ae^{-\phi} - \int_a^b e^{-\phi(1-H(w))} dw \right\}. \tag{105}$$

The expected utility of a vacancy must be the same regardless of which element it chooses from the support of its mixed strategy. Let us denote the utility of a vacancy that uses a by V_s^{1-a}. It is determined by

$$V_s^{1-a} = e^{-\phi}\left[1 - a + s\delta V_s^{1-a} + (1-s)\delta E(1-a)\right] + (1-e^{-\phi})\delta V_s^{1-a}, \tag{106}$$

where $E(1-a)$ is the utility of a matched firm that pays wage a. Analogously, if a vacancy uses b its utility is determined by

$$V_s^{1-b} = 1 - b + s\delta V_s^{1-b} + (1-s)\delta E(1-b) \tag{107}$$

as offering the highest possible wage means that the wage is always accepted. Finally, if the vacancy offers wage w, its utility is determined by

$$V_s^{1-w} = \sum_{k=0}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \left[\begin{array}{c} H^k(w)\left(1 - w + s\delta V_s^{1-w} + (1-s)\delta E(1-w)\right) \\ + \left(1 - H^k(w)\right)\delta V_s^{1-w} \end{array} \right], \tag{108}$$

where $H^k(w)$ is the probability that all other k offers are less than w. The utilities for a firm from being in an employment relationship at a specific wage are easily determined from equations

$$E(a) = 1 - a + s\delta V_s^{1-a} + (1-s)\delta E(a), \tag{109}$$

$$E(b) = 1 - b + s\delta V_s^{1-b} + (1-s)\delta E(b), \tag{110}$$

$$E(w) = 1 - w + s\delta V_s^{1-w} + (1-s)\delta E(w). \tag{111}$$

Solving from these the expected utilities and inserting in formulae (106)-(108) and in turn forcing them to yield the same expected utility as auction, namely that given by (101) allows us to solve for the endpoints of the support of the mixed strategy as well as the mixed strategy itself

$$a = \frac{\delta(1-s)\left(1 - e^{-\phi} - \phi e^{-\phi}\right)}{1 - \delta(1-s)\phi e^{-\phi}} \tag{112}$$

$$b = \frac{1 - \delta(1-s)\phi e^{-\phi} - e^{-\phi}}{1 - \delta(1-s)\phi e^{-\phi}} \tag{113}$$

$$H(w) = \frac{1}{\phi} \ln\left(\frac{1 - \delta(1-s)}{(1-w)\left[1 - \delta(1-s)\phi e^{-\phi}\right] - \delta(1-s)e^{-\phi}}\right) \tag{114}$$

We denote the cumulative distribution function for realised wages by $M(w)$ and the corresponding density function is denoted by $m(w)$. We get

$$M(w) = \frac{e^{-\phi(1-H(w))} - e^{-\phi}}{1 - e^{-\phi}} \tag{115}$$

$$= \frac{(1 - \delta(1-s))e^{-\phi}}{(1 - e^{-\phi})\left[(1-w)(1 - \delta(1-s)\phi e^{-\phi}) - \delta(1-s)e^{-\phi}\right]} - \frac{e^{-\phi}}{1 - e^{-\phi}}. \tag{116}$$

The density function is

$$m(w) = \frac{\left(1 - \delta(1-s)\phi e^{-\phi}\right)(1 - \delta(1-s))e^{-\phi}}{(1 - e^{-\phi})\left[(1-w)(1 - \delta(1-s)\phi e^{-\phi}) - \delta(1-s)e^{-\phi}\right]^2}. \tag{117}$$

A3.2 Unemployed workers send applications to vacancies

A3.2.1 Auction

Let $\theta = \dfrac{u}{v}$. The utility of a vacancy is determined by

$$V_r^{auc} = \left(e^{-\theta} + \theta e^{-\theta}\right)\delta V_r^{auc} + \left(1 - e^{-\theta} - \theta e^{-\theta}\right)E^{auc}(\underline{w}) \tag{118}$$

where $\underline{w}$ is the wage that an unemployed worker offers when there are two or more workers competing for a vacancy. Note that when a vacancy meets exactly one worker the worker makes a take-it-or-leave-it offer that leaves no surplus to the vacancy. The utility of an employer who employes at wage $\underline{w}$ is $E^{auc}(\underline{w})$, determined by

$$E^{auc}(\underline{w}) = 1 - \underline{w} + s\delta V_r^{auc} + (1-s)\delta E^{auc}(\underline{w}). \tag{119}$$

The expected utility of an unemployed worker is determined by

$$U_s^{auc} = e^{-\theta}W^{auc}(\overline{w}) + \left(1 - e^{-\theta}\right)\delta U_s^{auc} \tag{120}$$

where $\overline{w}$ is the wage that a worker offers when it gets to make a take-it-or-leave-it offer. The utility of a matched worker who is paid wage $\overline{w}$ is $W^{auc}(\overline{w})$,determined by

$$W^{auc}(\overline{w}) = \overline{w} + \delta s U_s^{auc} + \delta(1-s)W^{auc}(\overline{w}). \tag{121}$$

From (118)-(121) we can solve

$$V_r^{auc} = \frac{\left(1 - e^{-\theta} - \theta e^{-\theta}\right)(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-s)\left(e^{-\theta} + \theta e^{-\theta}\right)\right)} \tag{122}$$

$$E^{auc}(\underline{w}) = \frac{\left(1 - \delta(e^{-\theta} + \theta e^{-\theta})\right)(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-s)\left(e^{-\theta} + \theta e^{-\theta}\right)\right)} \tag{123}$$

$$U_s^{auc} = \frac{e^{-\theta}\overline{w}}{(1-\delta)\left(1 - \delta(1-s)\left(1 - e^{-\theta}\right)\right)} \tag{124}$$

$$W^{auc}(\overline{w}) = \frac{\left(1 - \delta + \delta e^{-\theta}\right)(1 - \overline{w})}{(1-\delta)\left(1 - \delta(1-s)\left(1 - e^{-\theta}\right)\right)} \tag{125}$$

The two possible equilibrium wages are :

$\underline{w}$: Several workers compete for a vacancy, all of them are indifferent between working at the wage and continuing search:

$$\underline{w} + s\delta U_s^{auc} + (1-s)\delta W^{auc}(\underline{w}) = \delta U_s^{auc}. \tag{126}$$

$\overline{w}$: A worker is the only applicant and gets to make a take-or-leave-it offer. The firm is indifferent between accepting the wage and continuing search:

$$1-\overline{w}+s\delta V_r^{auc}+(1-s)\,\delta E^{auc}(1-\overline{w})=\delta V_r^{auc} \tag{127}$$

Using (122)-(125), and replacing $\underline{w}$ by $\overline{w}$ in (119), and $\overline{w}$ by $\underline{w}$ in (121) we can solve

$$\overline{w}=\frac{1-\delta\,(1-s)\,(1-e^{-\ \theta})}{1-\delta(1-s)\ \theta e^{-\ \theta}}, \tag{128}$$

$$\underline{w}=\frac{\delta\,(1-s)\,e^{-\ \theta}}{1-\delta(1-s)\ \theta e^{-\ \theta}}. \tag{129}$$

Using (128) and (129) we can solve for the expected utility of a vacancy that waits

$$V_r^{auc}=\frac{1-e^{-\ \theta}-\ \theta e^{-\ \theta}}{(1-\delta)\,(1-\delta(1-s)\ \theta e^{-\ \theta})}, \tag{130}$$

and for the expected utility of a moving unemployed worker

$$U_s^{auc}=\frac{e^{-\ \theta}}{(1-\delta)(1-\delta(1-s)\ \theta e^{-\ \theta})}. \tag{131}$$

A3.2.2 The mixed strategy

The expected utility of a vacancy when the workers use mixed strategy $F(w)$ with support $[A,B]$ is determined by

$$V_r=e^{-\ \theta}\delta V_r+\sum_{k=1}^{\infty}\frac{\theta^k e^{-\ \theta}}{k!}\int_A^B kf(w)\,[1-F(w)]^{k-1}\,E(w)dw. \tag{132}$$

The utility of a filled vacancy is

$$E(w)=1-w+s\delta V_r+(1-s)\delta E(w). \tag{133}$$

Solving $E(w)$ and inserting it back to (132) yields the following formula where the last two terms result from partial integration

$$V_r \;=\; e^{-\ \theta}\delta V_r+\sum_{k=1}^{\infty}\frac{\theta^k e^{-\ \theta}}{k!}\,/_A^B\left[-[1-F(w)]^k\right](1-\delta+\delta s)^{-1}s\delta V_r+ \tag{134}$$

$$\sum_{k=1}^{\infty}\frac{\theta^k e^{-\ \theta}}{k!}\,/_A^B\left[-[1-F(w)]^k\right](1-\delta+\delta s)^{-1}(1-w)- \tag{135}$$

$$\sum_{k=1}^{\infty}\frac{\theta^k e^{-\ \theta}}{k!}\int_A^B[1-F(w)]^k(1-\delta+\delta s)^{-1}dw.$$

Finally, we can simplify this by doing the summations and by changing the order of the summation and integration in the last sum

$$V_r = (1-\delta)^{-1}(1-\delta(1-s)e^{-\theta})^{-1}\left[1 - A - (1-B)\,e^{-\phi} - \int_A^B e^{-\theta F(w)}dw\right]. \quad (136)$$

The expected utility of an unemployed worker must be the same regardless of which element he chooses from the support of his mixed strategy. The utility of a worker that uses A is

$$U_s^A = A + s\delta U_s^A + (1-s)\delta W(A), \quad (137)$$

where $W(A)$ is the worker's utility from working at wage A. Offering the lowest possible wage means that the wage is always accepted. The utility of a worker that uses B is

$$U_s^B = e^{-\theta}\left[B + s\delta U_s^B + (1-s)\delta W(B)\right] + (1-e^{-\theta})\delta U_s^B, \quad (138)$$

as with probability $1-e^{-\theta}$ the worker has competitors and his offer B will not be accepted. Finally, if the worker offers wage w, his utility is determined by

$$U_s^w = \sum_{k=0}^{\infty} \frac{\theta^k e^{-\theta}}{k!}\left[\left[1-F(w)\right]^k (w + s\delta U_s^w + (1-s)\delta W(w)) + \left(1-(1-F(w))^k\right)\delta U_s^w\right]. \quad (139)$$

The utilities of being employed at a specific wage are

$$W(A) = A + s\delta U_s^A + (1-s)\,\delta W(A) \quad (140)$$

$$W(B) = h + s\delta U_s^B + (1-s)\,\delta W(B) \quad (141)$$

$$W(w) = w + s\delta U_s^w + (1-s)\,\delta W(w) \quad (142)$$

Solving from these the expected utilities and inserting in formulae (137)-(139) and in turn forcing them to yield the same expected utility as auction, namely that given by (131) allows us to solve for the endpoints of the support of the mixed strategy as well as the mixed strategy itself

$$A = \frac{e^{-\theta}}{1-\delta(1-s)\;\theta e^{-\theta}}, \quad (143)$$

$$B = \frac{1-\delta(1-s)\left(1-e^{-\theta}\right)}{1-\delta(1-s)\;\theta e^{-\theta}}, \quad (144)$$

$$F(w)=\frac{1}{\theta}\ln\left[\frac{w\left[1-\delta(1-s)\ \theta e^{-\ \theta}\right]-\delta(1-s)e^{-\ \theta}}{e^{-\ \theta}(1-\delta(1-s))}\right]. \tag{145}$$

We denote the cumulative distribution function for the realised wages by $G(w)$ and the corresponding density function is denoted by $g(w)$

$$\begin{aligned} G(w) &= \frac{1-e^{-\ \theta F(w)}}{1-e^{-\ \theta}} && (146)\\ &= \frac{1}{1-e^{-\ \theta}}-\frac{(1-\delta(1-s))e^{-\ \theta}}{(1-e^{-\ \theta})\left[w(1-\delta(1-s)\ \theta e^{-\ \theta})-\delta(1-s)e^{-\ \theta}\right]} && (147)\end{aligned}$$

The density function is

$$g(w)=\frac{\left(1-\delta(1-s)\ \theta e^{-\ \theta}\right)(1-\delta(1-s))e^{-\ \theta}}{(1-e^{-\ \theta})\left[w(1-\delta(1-s)\ \theta e^{-\ \theta})-\delta(1-s)e^{-\ \theta}\right]^2} \tag{148}$$

A3.3 The equilibrium market structure

In equilibrium, workers are indifferent between sending applications and receiving offers from firms, and firms are indifferent between making offers and receiving applications from workers. That is, for a job seeker, the equilibrium condition UE is

$$\frac{e^{-\theta}}{(1-\delta)(1-\delta(1-s)\theta e^{-\theta})}=\frac{1-e^{-\phi}-\phi e^{-\phi}}{(1-\delta)\left(1-\delta(1-s)\phi e^{-\phi}\right)}, \tag{149}$$

and for a vacancy, the equilibrium condition VE is

$$\frac{e^{-\phi}}{(1-\delta)(1-\delta(1-s)\phi e^{-\phi})}=\frac{1-e^{-\theta}-\theta e^{-\theta}}{(1-\delta)\left(1-\delta(1-s)\theta e^{-\theta}\right)}. \tag{150}$$

The left-hand side are utilities from sending wage demands or offers, and the right-hand sides are utilities from receiving them. It turns out that in equilibrium $\theta=\phi\equiv\theta_0\approx 1.146$, and there exists a unique equilibrium for strictly positive x and y:

$$\begin{aligned} x &= \frac{\theta_0\left(\alpha\theta_0-1\right)}{\alpha\left(\theta_0^2-1\right)}, && (151)\\ y &= \frac{\alpha\theta_0-1}{\theta_0^2-1}, && (152)\end{aligned}$$

where $\alpha=\frac{u}{v}$. However, two markets coexist only if $x\in(0,1)$ and $y\in(0,1)$. These hold only if $\alpha\in\left(\frac{1}{\theta_0},\theta_0\right)$.

A3.4 Stability of the equilibrium market structure

Applying the stability analysis of the simpler dynamic model above but using $\delta(1-s)$ as a discount factor instead of δ, gives

Proposition A3.1 *In the general equilibrium dynamic model the mixed-strategy equilibrium where $x \in (0,1)$ and $y \in (0,1)$ is stable.*

Plugging the above solutions for x and y into $m(w)$ and $g(w)$ and using the appropriate ranges for the distributions just like in the two models above, and noting that in an equilibrium where two markets coexist we have $\theta = \phi = \theta_0$, we get the equilibrium distribution for the realised wages. The density function has approximately the same shape as the one produced by the static model.

A3.5 Steady state

In a steady state equilibrium the number of matches equals the number of separations:

$$yv(1-e^{-\theta}) + (1-x)u(1-e^{-\phi}) = s(K-v), \tag{153}$$

where $yv(1-e^{-\theta})$ is the number of matches that form in a market where firms are urns, and $(1-x)u(1-e^{-\phi})$ is the number of matches in the market where workers are urns, and $s(K-v)$ is number of matches that break down per period. From (153) we can solve

$$v = \frac{Ks - (1-x)u(1-e^{-\phi})}{y(1-e^{-\theta}) + s} \tag{154}$$

On the other hand, from equation $K - v = L - u$ it follows that

$$v = K - L + u. \tag{155}$$

From (154) and (155) we can solve, noting that in equilibrium $\theta = \phi = \theta_0$:

$$u = \frac{y(1-e^{-\theta_0})(L-K) + Ls}{y(1-e^{-\theta_0}) + (1-x)(1-e^{-\theta_0}) + s} \tag{156}$$

$$v = \frac{(1-x)(1-e^{-\theta_0})(K-L) + Ks}{y(1-e^{-\theta_0}) + (1-x)(1-e^{-\theta_0}) + s} \tag{157}$$

When we substitute (151) and (152) for x and y in (156) and (157) we get that $u = f(L, K, s, \theta_0)$ and $v = g(L, K, s, \theta_0)$.

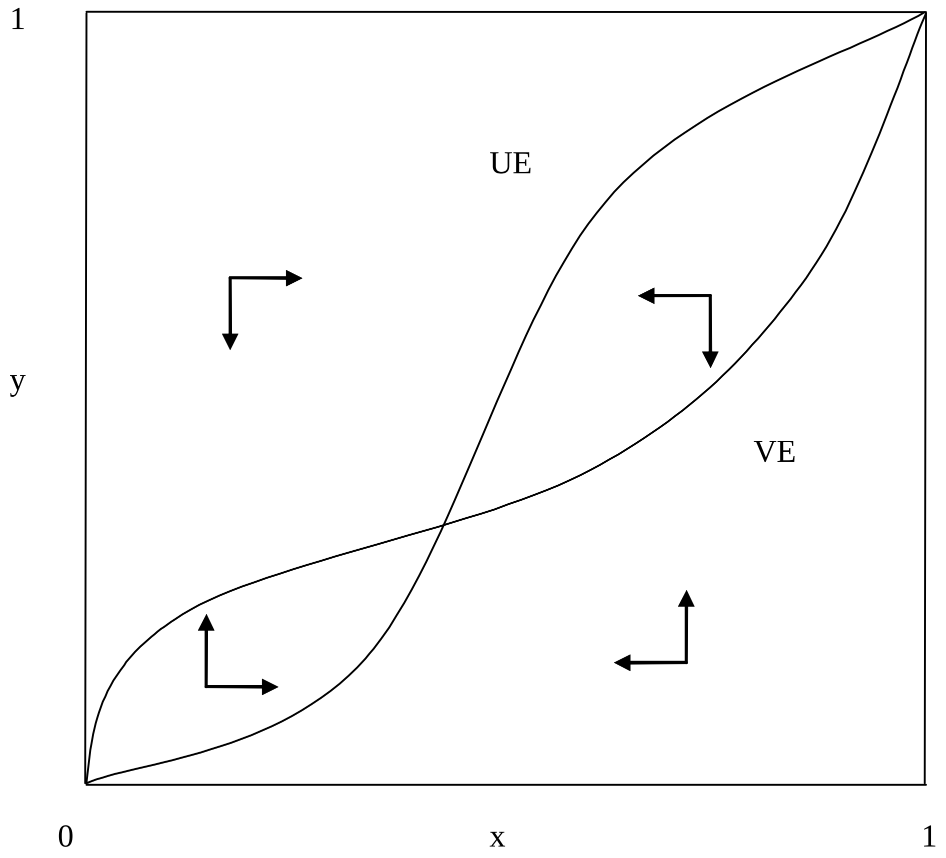

Figure 1: The mixed strategy equilibrium where $x \in (0, 1)$ and $y \in (0, 1)$ is evolutionarily stable.

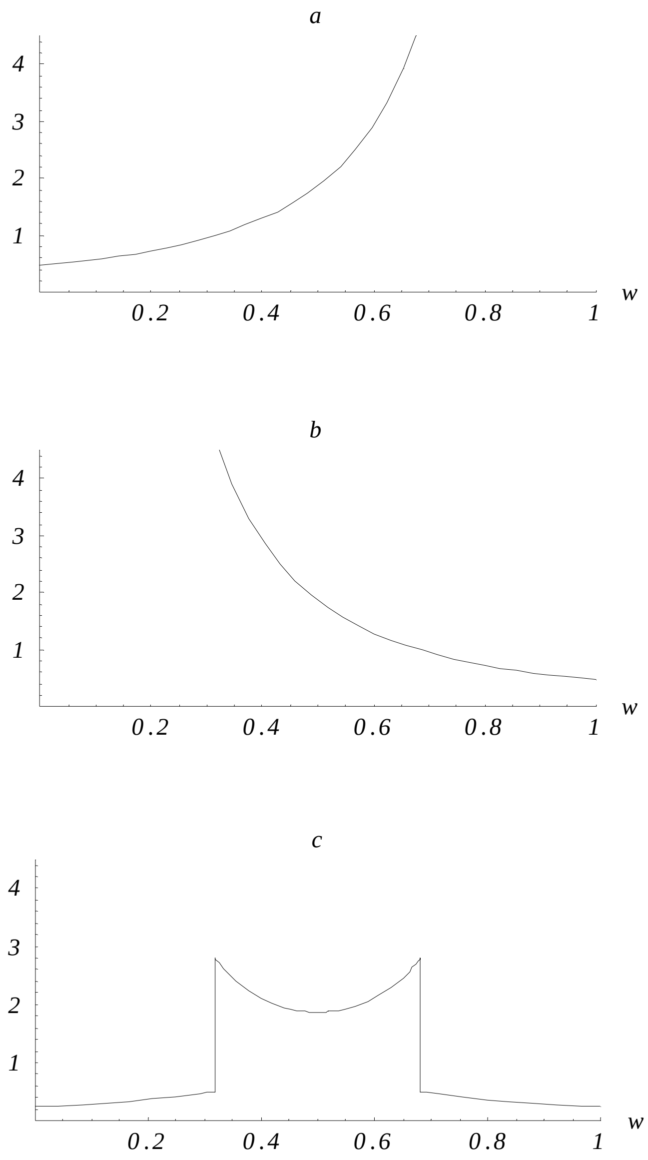

Figure 2: Distributions of realized wages in the static model when $u = v$. a) job seeker market (vacancies send offers), b) vacancy market (job seekers send offers), c) the aggregate distribution

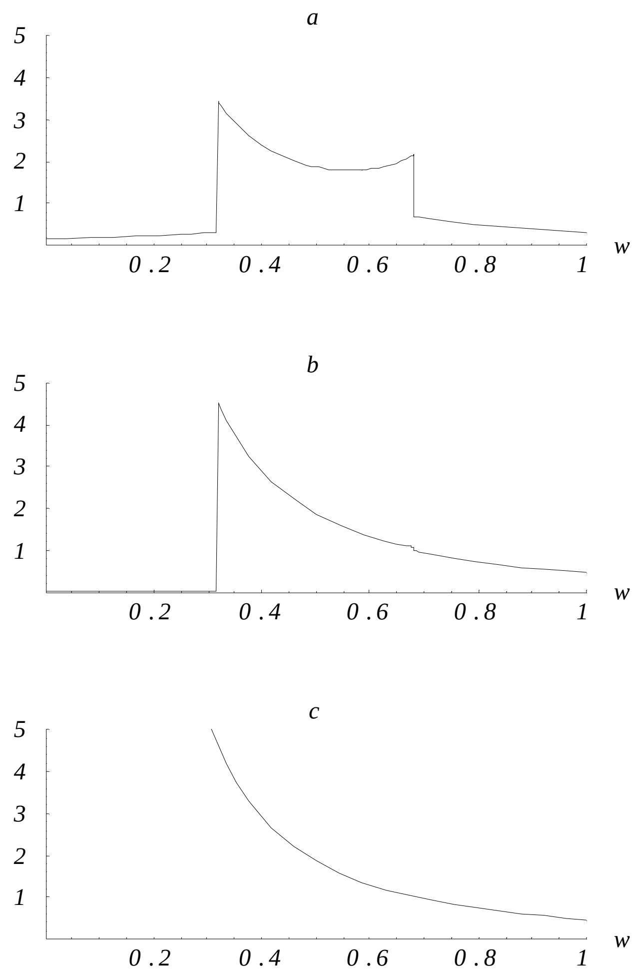

Figure 3: The aggregate distribution of realized wages in the static model, a) $u/v = 1.05$, b) $u/v = 1.14$, c) $u/v = 1.5$