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Abstract

The paper studies wage formation in a pairwise matching model with heterogeneous labor. Firms propose wage offers unilaterally but workers can opt to search for alternative contacts besides pairwise meetings. Due to this option, the Diamond Paradox does not emerge but workers earn a non-trivial share of the transferable rent. Since the search option is disproportionately valuable for high skilled workers, the equilibrium wage function is increasing and convex in worker's skills. A mean preserving spread in the skill distribution induces greater wage dispersion. In the short-run, an increase in the average skill level compresses wages and may drive some low skilled workers out of the market. In the long-run, however, higher skill levels stimulate labor demand, counterbalancing the negative short-run effects. The long-run demand effect and the non-linear wage function can help to explain the empirically observed increase in the both supply and price of skilled labor.

JEL Classification: J31, J30.

Keywords: Pairwise matching, wage dispersion, wage formation, Diamond Paradox.

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1 Introduction

The paper provides a theory of equilibrium wages in a continuous-time search model with random matching and heterogeneous labor. Workers differ in productivity (skills). Employers are identical and value worker's skills in similar manner. As in conventional search and matching models (Mortensen and Pissarides, 1994; Pissarides, 2000), agents meet pairwise and bargain over the terms of trade. Instead of Nash bargaining, however, we develop an alternative price formation method which is an extension of the earlier work by Kultti (2000) and Kultti and Virrankoski (2004). The novelty in the approach is that, in the case of disagreement, trading partners do not have to separate forever but may choose to maintain the existing contact and continue search besides the ongoing meeting. Once either of the parties locates another trading partner, transaction is concluded via auction between the two competing agents. If the new agent is another employer, the two employer candidates bid for the sole worker. If it is the employer who locates another worker candidate, the competing workers lower their wage demands until the less skilled is driven to his reservation utility level. Upon the initial pairwise meetings, employers can propose wage offers unilaterally. It is shown that employers' equilibrium strategy is to offer a wage that is just good enough to prevent the worker from exercising his search option. The wage is unique to each skill level.

Two observations about the trading process are immediate. First, the setting allows for the possibility of competition between alike agents even though the initial matching is pairwise. The model thus resembles the 'urn-ball' process (e.g. Butters, 1977; Hall, 1979) where multiple simultaneous contacts are possible and auctions are a standard price formation practice (e.g. Julien et al., 2000). However, since our construction features at most two bidders at any auction, the introduction of worker heterogeneity is more tractable than in the urn-ball model where the number of different bidders at some auctions can be unlimited. Secondly, the search option generates a sharing rule that secures at least some rents also to the worker, so that the model circumvents the Diamond Paradox[1].

[1] Diamond (1971) demonstrated that in a pairwise matching model where one of the traders can set the price unilaterally leads to an equilibrium where the price setter captures all the

It turns out that the search option is disproportionately valuable for high skilled workers. This is because high skilled workers are better protected against competition. In a situation where there are two workers competing for the same vacancy, the better skilled worker must lower his wage demand only to the level that secures the employer the same rent as would be available when hiring the less skilled worker at his reservation wage. In fact, we show that the better skilled earns his reservation utility plus the whole value of the productivity gap between him and the less skilled colleague. This disparity leads to a non-linear wage structure: More skilled workers earn strictly larger fraction of the transferable rent than less skilled workers.

In models with Nash bargaining it is assumed that the transferable rent is shared according to some exogenous sharing rule. If worker heterogeneity is incorporated into these models, our result suggests that the sharing rule should not be constant throughout the worker types. Otherwise the resulting wage schedule is linear (e.g. Rosen and Wasmer, 2005), as opposed to the convex pattern in our setting.

The equilibrium wage dispersion is decreasing with the discount rate, which captures the severeness of the search frictions in the labor market. The reason behind this result is that when the discount rate is low agents are relatively patient and workers' 'wait-and-search' option is more valuable. Since this option is disproportionately valuable for high skilled workers, also the overall wage dispersion is higher when the discount rate is low. Regarding the effect of labor market tightness on equilibrium wage structure, we show that an increase in the ratio of vacant jobs and the number of unemployed always leads to an increase in lower tail wage differentials. In the upper tail, however, the relationship is ambiguous. When the labor market is sufficiently tight initially, a marginal increase in labor demand tends to widen the upper tail wage differentials. However, when the market is 'slack' *ex ante*, increasing demand actually compresses upper tail wages. This is because a marginal increase in demand dilutes part of the high skilled workers' comparative advantage when vacant jobs are the short side of the market. Our numerical examples indicate, however, that the overall wage dispersion is increasing with labor market tightness.

transferable rent.

We also show that a mean preserving spread in the skill distribution is associated with greater wage dispersion. This is because a mean preserving spread increases the lower tail wage differentials more than it reduces the wage gaps in the upper tail. There is a host of evidence indicating that countries where workers' skills are more polarized tend to have higher wage dispersion, too (e.g. Blackburn et al., 1991, and Devroye and Freeman, 2001).

A rise in the average skill level, in turn, has divergent short-run and long-run effects. In the short-run, an increase in the mean skill level creates a negative externality on all wages due to its positive externality on firms' outside option (cf. Rosen and Wasmer, 2005). The effect is especially dramatic for workers in the lower tail of the skill distribution as their market value may collapse completely, but also the upper tail wages become more compressed. As the market adjusts to a new long-run steady state, the vacancy-unemployment ratio is increased and employers are willing to trade with all workers. Greater market tightness then widens wage differentials and thereby counterbalances the negative externality of improved worker productivity on short-run wages.

A number of empirical studies (cf. Katz and Autor, 1999, for an overview) reveal the puzzling trend that a substantial growth in the relative supply of skilled labor has in most industrialized economies been accompanied by increasing 'skill premium' in wages. Our short-run analysis suggests that an increase in the fraction of high skilled labor should reduce that premium. In the long-run, however, the positive labor demand effect tends to widen wage differentials across skill levels. Moreover, if there is a general upgrade in productivity, non-linear wage structure implies that workers with higher skills gain disproportionately. Our theory thus mitigates the need for strong skill-biased technological change (e.g. Katz et al., 1993, Katz and Autor, 1999, and Krusell et al. 2000) to explain the simultaneous increase in the both supply and price of skilled labor.

Meckl and Zink (2002) point out that the wage differentials by skill groups have actually evolved non-monotonically. The time path of the relative wage has typically been U-shaped in a sense that wage differentials by skills fell during the 1970s, and started to increase only during the 1980s and 1990s. The U-shaped time path suggests that the upgrading of skills may have overrun the demand effect and the pace of technological progress in the 1970s while the pattern would have been reversed during the 1980s and the 1990s. Indeed, according to Katz and Autor (1999), the relative supply of skilled labor rose most in the 1970s.

2 The model

2.1 Basic set up

The labor market is populated by a continuum of unemployed workers and a larger continuum of firms who post open vacancies. Workers differ in productivity (skills) which is measured by a continuous index $x \in [\underline{x}, \overline{x}]$. Workers are distributed over this productivity interval according to the distribution function $F(x)$. Firms are identical, have similar valuation for the labor input and have a unit demand for labor. Irrespective to the skill level, all unemployed have a unit supply of labor and they value their work effort at zero.

Trading takes place in private meetings between unemployed workers and firms. In order to locate a potential employer, workers must commit to a search process. Search effort is costless but time-consuming[2] which creates frictions on the functioning of the market. The frequency at which firms receive applications and unemployed workers locate vacant jobs is governed by an exogenous matching function, $M(u, v)$, that gives the total number of matches at each point of time as a function of two inputs, the number of currently unemployed (u) and the number of vacant jobs (v). As usual, the matching function is assumed to be strictly increasing and strictly concave in both arguments and it exhibits constant returns to scale.

Since time is continuous, Poisson arrival rates can be used to measure the flow probabilities of locating (receiving) a vacant job (an application). An unemployed worker locates an open vacancy at rate α while a firm with an unfilled vacancy receives an application from an unemployed worker at rate β. Pairwise matching requires $\alpha u = \beta v = M(u, v)$.

Labor market tightness, $\theta = v/u$, is the ratio between open vacancies and unemployed workers. The CRS-property of the matching function implies that the meeting rates α and β can be determined as a function of labor market

[2] Having direct search costs would introduce just another friction to the search process.

tightness:

$$\alpha = \frac{M(u,v)}{u} \equiv \theta m(\theta) \text{ and } \beta = \frac{M(u,v)}{v} \equiv m(\theta), \tag{1}$$

where $m(\theta) = M(1/\theta, 1)$. The strict concavity of the matching function M implies that $m(\theta)$ is decreasing and convex in θ. Thus, increasing labor market tightness improves (reduces) the rate at which unemployed workers (firms) locate vacant jobs (receive applications); i.e. $\alpha'(\theta) = \partial[\theta m(\theta)]/\partial\theta > 0$ and $\beta'(\theta) = m'(\theta) < 0$. For notational convenience, we continue to denote the meeting rates by α and β. In a long-run steady state, market tightness - and thereby the meeting rates α and β - are endogenously determined.

Opening a vacancy incurs a flow cost denoted by ϕ, which can be thought to capture all the other factors of production except the worker. Under unrestricted entry, firms open new vacancies until the present value of the expected future profits from a filled vacancy equal the present value of the flow cost, Φ. New unemployed workers are born at a constant and exogenous rate η. Steady state requires that $\alpha u = \eta$.[3] Labor contracts are lifelong relationships so that, after a successful match, both the hired worker and the filled vacancy exit the market forever. A long-run steady state equilibrium (i.e. steady state values for u, v, and θ) is completely determined by the pairwise matching condition, firms' free-entry condition and the exogenous 'birth rate' of new unemployed.

2.2 Trading

The trading process postulated here is an extension of the models by Kultti (2000) and Kultti and Virrankoski (2004), the key difference being the assumption of heterogeneous sellers (=unemployed workers). Upon a meeting with a worker, the employer proposes a wage offer. If trading seems unfavorable for the worker, he does not have to break up the contact completely but he may opt to *wait* and *continue search* besides the ongoing meeting. If the worker decides to wait, he locates another employer at rate α. In that case, two employer candidates raise their wage offers until driven to their reservation utilities. On the other hand, at rate β the employer contacts another unemployed and there

[3] This assumption replaces the exogenous job destruction typically assumed in the Mortensen-Pissarides model.

will be two competing workers. The workers reduce their wage demands until either (or both) of them rather leaves the meeting than further lowers the wage demand. Figure 1 depicts the timing of events.

The 'wait-and-search' option is the only trump card in worker's hands. If he did not have that, the employer could propose a wage offer that would make the worker indifferent between accepting the offer and staying unemployed. With no unemployment benefits, that practice would lead to an equilibrium where all workers earn zero wages while the employer keeps all rents; i.e. we would have the Diamond Paradox. It turns out that the search option guarantees the worker a payoff that is generally greater than his reservation utility, so that the Diamond Paradox does not emerge in our setting.

Employer's equilibrium strategy is to propose a wage offer that is just good enough to prevent the worker from using his search option so that no worker ever opts to wait and continue search. The bold arrows in Figure 1 depict the 'equilibrium path' while the dashed arrows represent the 'off-equilibrium paths'. As in Kultti and Virrankoski (2004), the construction of the equilibrium is based on the following conjecture:

Conjecture 1 *(i) If the employer offers a wage that produces the worker less utility than the value of the waiting option, the worker rather waits than leaves the meeting. (ii) Instead of terminating the meeting immediately, the employer is willing to trade at equilibrium wages with any worker - regardless of the skill level of the worker. (iii) If the unemployed worker decides to wait and continue search, then transaction is concluded once a competing agent appears in either side of the negotiation table; i.e. there will be no 'further waiting'.*

The first part of the Conjecture 1 is rather obvious, since the worst scenario in the wait-and-search option is that the worker is driven to his reservation utility level - which is the utility he gets if he opts to discard the initial contact immediately. The second part will hold by assumption. If it did not hold for some low skilled workers, those workers would not have the search option besides the ongoing meeting and could earn at most zero wages. We assume that this is not possible in a long-run steady state but those low skilled workers would be driven out of the market. It turns out that it suffices to assume that the

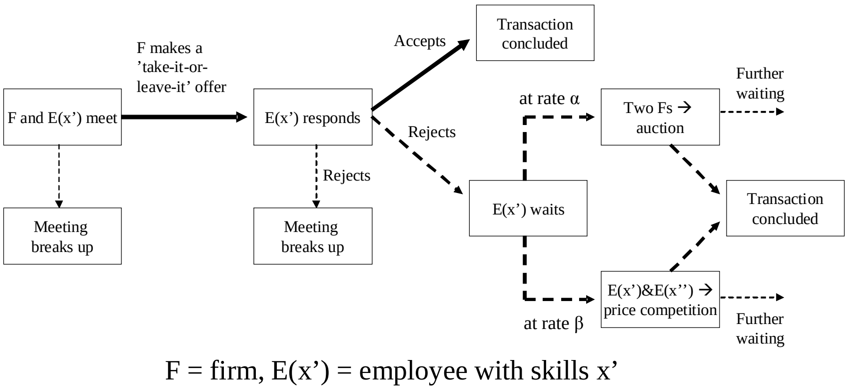

Figure 1: Timing of events

present value of the future output flow generated by the least able worker, $\underline{X}$, at least equals with the present value of the costs, Φ. The third part states that even though the workers would have unlimited possibilities to wait and search, transactions will be concluded immediately after either another employer has been located or another worker candidate has shown up. This will also be shown to hold in equilibrium. Hence, according to Conjecture 1, the 'thin' dashed lines in Figure 1 describe irrelevant off-equilibrium paths.

2.3 The Bellman equations

Both workers and firms are risk neutral and discount their future earnings with a common discount rate r. Assume that the equilibrium wage available for the worker possessing skills x' is $w(x')$. Given the equilibrium wages, the discounted value of an open vacancy is denoted by V_0 while the discounted value of a job filled with a worker with skills x' is denoted by $V(x')$. The discounted value of being unemployed is denoted by $U_0(x')$ and of being employed by $U(x')$.

Under linear preferences, $U(x')$ yields

$$U(x') = \int_{\tau}^{\infty} e^{-(t-\tau)r} w(x')dt = \frac{w(x')}{r}, \ \forall x' \in [\underline{x}, \overline{x}] \,. \tag{2}$$

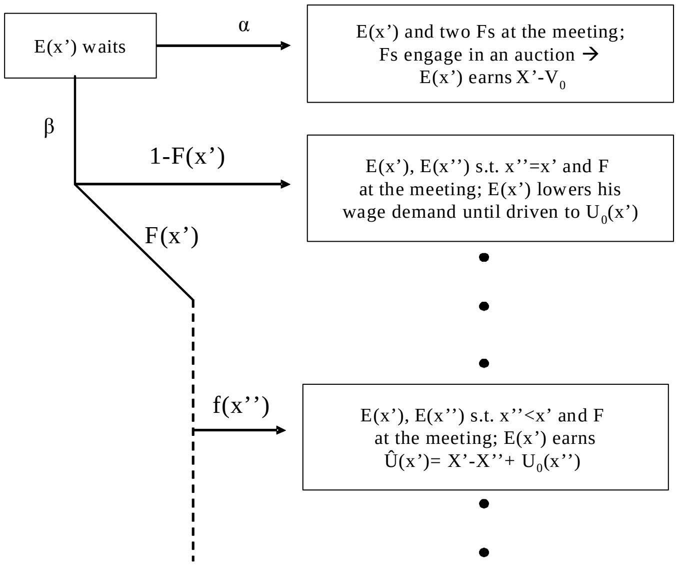

Figure 2: Wait-and-search option

The discounted value of unemployment $U_0(x')$, in turn, can be expressed as

$$\begin{aligned} rU_0(x') &= \alpha \left(U(x') - U_0(x') \right), \ \Leftrightarrow \\ U_0(x') &= \frac{\alpha}{\alpha + r} U(x'). \end{aligned} \tag{3}$$

Similarly, the present value of a filled job reads as

$$V(x') = \frac{x' - w(x')}{r}, \ \forall x' \in [\underline{x}, \overline{x}], \tag{4}$$

while the expected value of a newly opened vacancy yields

$$V_0 = \frac{\beta}{\beta + r} \int_{\underline{x}}^{\overline{x}} V(x) dF(x). \tag{5}$$

For convenience, we define $X' \equiv x'/r$. In a long-run steady state under unrestricted entry, $V_0 = \Phi \equiv \phi/r$.

2.4 The wage function

Consider a meeting between an employer and a worker with skills x'. Figure 2 illustrates the prospects of the worker if he decides to reject the employer's offer and start searching for alternative contacts. If the worker manages to locate another employer, which happens at rate α, two competing firms engage in an auction for the right to hire the worker; i.e. firms raise their bids until the utility from hiring equals their expected reservation utility, V_0. However, if another worker happens to show up (which occurs at rate β), the resulting wage will depend on whether the newcomer is at least equally skilled or less skilled than the incumbent worker.

Assume first that the appearing rival is labeled with a skill level $x'' < x'$. Then the less skilled competitor lowers his wage demand until he gets $U_0(x'')$. The better skilled incumbent knows that his rival's lowest acceptable wage will provide the employer with a discounted value equal to

$$V^C(x'') = x'' - U_0(x''),$$

where $X'' = x''/r$ and the upper index c stands for competition between workers. If the incumbent worker wants to trade, he needs to propose a wage demand that produces the employer at least the same utility; i.e. $V^C(x') = V^C(x'')$. Denote the incumbent worker's utility from such a contract by $\hat{U}(x' \mid x'')$. Then the highest wage the incumbent worker with skills x' is able to demand must satisfy the following condition:

$$X' - \hat{U}(x' \mid x'') = X'' - U_0(x''),$$

so that

$$\hat{U}(x' \mid x'') = U_0(x'') + X' - X'', \forall x' \geq x''. \tag{6}$$

Yet another conjecture, which will be shown to hold in equilibrium, guarantees that the better skilled worker is willing to trade at a competitive situation:

Conjecture 2 $\hat{U}(x' \mid x'') > U_0(x')$.

On the other hand, with probability $1 - F(x')$ we have $\forall x' \leq x''$ so that the arriving competitor is at least equally skilled as the incumbent worker and the incumbent is driven to his reservation utility level $U_0(x')$.

Summarizing this lengthy description with a single equation, the Bellman equation representing the value of the 'wait-and-search' option faced by a worker with skills x' reads as

$$rh(x') = \alpha\left(X' - V_0 - h(x')\right) + \beta[(1 - F(x'))\,U_0\left(x'\right) + \int\limits_{\underline{x}}^{x'} \hat{U}\left(x' \mid x\right) dF\left(x\right) - h(x')], \tag{7}$$

where the first term on the right-hand side captures the chance of ending up to a situation with two competing employers whereas the second term reflects the expected utility available when another worker happens to show up.

Since the initiative is on the employer's side, the equilibrium wage offer proposed upon a meeting is such that it makes the worker indifferent between accepting the offer and exercising the search option. Therefore the equilibrium wages for any skill level $x' \in [\underline{x}, \overline{x}]$ satisfy

$$U\left(x'\right) = h(x') \Leftrightarrow w(x') = rh(x'). \tag{8}$$

The wage at which the transaction is concluded is unique for every $x' \in [\underline{x}, \overline{x}]$. Uniqueness follows as (8) is linear in $w(x)$ and has a unique solution.

Substituting $w(x)$ for $rh(x)$ in (7) and differentiating with respect to x gives

$$w'\left(x\right) = \frac{\left(\alpha + r\right)\left(\alpha + F\left(x\right)\beta\right)}{\left(\alpha + r\right)^2 + F\left(x\right)\beta\alpha + \beta r} \equiv \psi\left(x\right) < 1, \tag{9}$$

where $\psi\left(x\right)$ is the share of the marginal productivity gain that goes to the worker with skills x. $\psi\left(x\right) < 1$ implies that the worker can never capture all the transferable rent. This is because the employer has the first-mover advantage.

Using this formula, and remembering that in a long-run steady state $V_0 = \Phi \equiv \phi/r$, we can derive the steady state equilibrium wages as a function of worker's skill level x:

Lemma 3 *The equilibrium wage function is given by*

$$w(x') = \int\limits_{\underline{x}}^{x'} \psi(x)dx + w\left(\underline{x}\right), \forall x' \in [\underline{x}, \overline{x}]$$

where

$$w\left(\underline{x}\right) = \frac{\alpha\left(\alpha + r\right)}{\left(\alpha + r\right)^2 + \beta r}\left(\underline{x} - \phi\right)$$

is the wage earned by the least able worker in the market.

Proof. See Appendix A.1. ■

Lemma 4 *Conjectures 1-2 hold in an equilibrium established by the wage schedule given in* (8)*.*

Proof. See Appendix A.2. ■

Since $\psi(x)$ is increasing with $F(x)$ and $F'(x) > 0$, $w''(x) > 0$ so that

Proposition 5 *The wage function is non-linear and convex; i.e. worker's share of the matching surplus is increasing with skills.*

Thus, our 'quasi-competitive' setting produces a non-linear pricing rule, even though the buyer's preferences are homogeneous, there are no informational frictions or active market segmentation. The non-linearity of wages stems from the disparity in the values of the wait-and-search options between different skill levels. This disparity is probably most transparently visible in equation (7) (or in Figure 2): The wait-and-search option is the more valuable the less there are equally or better skilled candidates among unemployed workers. This is because the expected utility available from trading in a situation when there are two competing workers and one employer is the larger the smaller is the probability of having a better skilled competitor.

In models where Nash bargaining is applied, the transferable rent is divided according to an exogenous sharing rule. Our result suggests that if one wants to introduce heterogeneity into these models, the sharing rule should not be constant throughout different types. Otherwise the resulting price function is linear. This is easy to see in the current case. the sharing rule obtained under generalized Nash bargaining can be written as

$$(1-\rho)\left(U(x)-U_0(x)\right)=\rho\left(V(x)-\Phi\right), \tag{10}$$

where ρ $(1-\rho)$ denotes worker's (employer's) exogenous 'bargaining power'. Utilizing equations (2), (3) and (4), equation (10) implies

$$w^{NB}(x)=\frac{\rho\left(\alpha+r\right)}{\rho\alpha+r}\left(x-\phi\right),$$

so that Nash wages are linearly increasing with the worker skill level x.

We also report two comparative static results. First,

Proposition 6 *The equilibrium wage dispersion is a decreasing function of the discount rate r.*

Proof. Follows directly from the observation that $\partial\psi(x)/\partial r < 0$. ■

The reason behind this result is that when r is low agents are relatively patient and workers' wait-and-search option more valuable. Since this option is disproportionately valuable to high skilled workers, also the overall wage dispersion is higher when r is low.

Secondly,

Proposition 7 *(i) The lower tail wage differentials increase along with greater labor market tightness. (ii) If*

$$\alpha'(\theta) + \beta'(\theta) > 0,$$

then a marginal increase in market tightness increases the upper tail wage differentials. Otherwise, the upper tail wages become more compressed.

Proof. See Appendix A.3. ■

Greater labor market tightness means that there are more available jobs per one unemployed worker so that it becomes easier for the unemployed to locate a potential employer; i.e. α increases. This fact generally increases the value of the wait-and-search option because higher α means greater probability of having two employers competing for the same worker. In such an occasion high skilled workers benefit disproportionately and wage differentials tend to increase. This prediction is in line with some earlier models (e.g. Acemoglu, 1997) as well as with empirical evidence (e.g. DiNardo et al., 1996). However, greater market tightness also reduces the rate at which firms receive applications (β) which lowers the probability of having two workers competing for the same vacancy. Since high skilled workers are better protected against competition, increasing market tightness also dilutes their comparative advantage. If this effect is strong enough, the upper tail wages may actually become more compressed. This is the case if a marginal increase in θ worsens the congestion on employers' side more than it increases the contact probability on workers' side, i.e. when vacant jobs are the short side of the market or $|\beta'(\theta)| > \alpha'(\theta)$. Our numerical examples in the Appendix indicate, however, that the overall wage dispersion is likely to be increasing with θ.

3 Distribution of skills, technical change and the wage structure

Consider first a mean preserving spread in the distribution of skills. Since the expected value of a filled vacancy remains unchanged, a mean preserving spread affects steady state wages only by changing workers' relative position in the labor market. We obtain

Proposition 8 *A mean preserving spread in the distribution of skills leads to greater wage dispersion.*

Proof. See Appendix A.4. ■

This result is due to the feature that $\psi(x)$ is increasing but concave in F. In other words, a mean preserving spread in the skill distribution F increases the lower tail wage differentials more than it reduces the wage gaps in the upper tail. As a result, the overall wage dispersion becomes wider. There is a host of evidence indicating that countries where workers' skills are more polarized tend to have higher wage dispersion, too (e.g. Blackburn et al., 1991, and Devroye and Freeman, 2001).

On the other hand, any changes in the mean skill level not only affect workers' relative 'bargaining power' but also firm's incentives to open new vacancies. For example, a rise in the mean skill level increases the expected value of a filled vacancy, encouraging more frequent market entry by firms. Therefore a long-run steady state with higher average worker productivity should feature greater market tightness and lower unemployment.

It is instructive, however, to first consider the short run consequences of an increase in average worker skills, before the labor market converges on a new long-run steady state. In the short-run analysis, we keep market tightness θ fixed. It may then happen that the value of having an unfilled vacancy exceeds the value of hiring a worker with lower skill level than some threshold $\hat{x}$; i.e. for

$x \in [\underline{x}, \hat{x})$ $V_0 > V(x)$. Obviously, these workers do not possess the wait-and-search option because any hesitation would trigger the employer to disregard the candidate immediately. Since there are no other outside options, the Diamond paradox tells us that workers with skills $x \in [\underline{x}, \hat{x})$ can earn at most zero wages while the employers keep all the surplus. Moreover, if $rV_0 > x$, the employer refuses to trade at all. Assuming that $V_0 = V(\hat{x})$, the wages for $x' \in [\hat{x}, \bar{x}]$ are given by $w(x') = \int_{\hat{x}}^{x'} \psi(x)dx$.

Thus, in the short-run, an increase in the average skill level creates a negative externality on wages due to its positive externality on firms' outside option (cf. Rosen and Wasmer, 2005). The effect is especially dramatic for workers in the lower tail of the skill distribution - i.e. for $x \in [\underline{x}, \hat{x})$ - as their market value collapses completely. Since $\psi(x)$ is increasing with $F(x)$, also wages for $x \in [\hat{x}, \bar{x}]$ become more compressed.

As the labor market adjusts to a new long-run steady state, so that the free-entry condition $V_0 = \Phi < \underline{x}/r$ is again satisfied, employers are willing to trade with all workers and the vacancy-unemployment ratio is increased. Greater market tightness is shown to widen wage differentials at least on the lower tail of the skill distribution. Our numerical examples indicate that the same is true also for the overall wage dispersion. In the long-run the positive demand effect thereby counterbalances the negative externality of improved worker productivity in the short-run.

Many empirical studies (cf. Katz and Autor, 1999, for an overview) reveal the puzzling trend that a substantial growth in the relative supply of skilled labor has in most industrialized economies been accompanied by increasing 'skill premium' in wages. Our short-run analysis also suggests that an increase in the fraction of high skilled labor should reduce that premium. In the long-run, however, there are potentially two counterbalancing effects: First, an increase in the average skill level of the labor force stimulates labor demand, which tends to widen wage differentials across skill levels. Secondly, if there is a general upgrade in productivity, say by a factor λ, the non-linearity of wages implies that workers with higher skills gain disproportionately. Thus, even if the distribution of skills would be transformed to weight higher skill levels, wage dispersion may still increase, if the positive demand effect and a possible concurrent upgrade in labor productivity are large enough.

A number of studies (e.g. Katz et al., 1993, Katz and Autor, 1999, and Krusell et al. 2000) indicate that skill-biased technological change has to be the key factor explaining the rise in the skill premium. The results in our model, however, mitigate the need for strong skill-biased technological change to explain the simultaneous increase in the both supply and price of skilled labor. Meckl and Zink (2002) point out that the wage differentials by skill groups have actually evolved non-monotonically. The time path of the relative wage has typically been U-shaped in a sense that wage differentials by skills fell during the 1970s, and started to increase only during the 1980s and 1990s. The U-shaped time path suggests that the upgrading of skills may have overrun the demand effect and the pace of technological progress in the 1970s while the pattern would have been reversed during the 1980s and the 1990s. Indeed, according to Katz and Autor (1999), the relative supply of skilled labor rose most in the 1970s.

4 Concluding remarks

The paper studies wage formation in a pairwise matching model under worker heterogeneity. Upon a private meeting, the employer proposes a wage offer to the worker candidate. If the worker is not satisfied with the offer, he can opt to wait and search for an alternative employer but still maintain the existing contact. At the same time, however, the worker runs a risk that a competing worker approaches the employer. Once a rival agent appears in either side of the negotiation table, an auction is triggered between the competing agents. Employer's equilibrium strategy is to propose a wage offer that just prevents the worker from exercising his wait-and-search option. The resulting wage function is non-linear in a sense that more skilled workers earn strictly larger fraction of the transferable rent than less skilled workers. The wait-and-search option anyhow guarantees at least some rents to all workers, so that the model resolves the Diamond Paradox.

The equilibrium wage dispersion is decreasing with the discount rate. This is because lower discount rate increases the value of workers' wait-and-search option, leading to a disproportionate rise in high skilled workers' wages. The relationship between market tightness and wage differentials is unambiguously positive at lower tail skill levels. In the upper tail, however, the relationship may be reversed if vacant jobs are the short side of the market. The reason behind this ambiguity is that an increase in labor demand dilutes part of the high skilled workers' comparative advantage. Our numerical examples indicate, however, that the overall wage dispersion is increasing with labor market tightness.

It is shown that a mean preserving spread in the skill distribution leads to greater wage dispersion. This prediction is supported by empirical evidence (e.g. Blackburn et al., 1991, and Devroye and Freeman, 2001). An increase in the mean skill level, in turn, has very divergent short-run and long-run effects. In the short-run, a rise in the average worker productivity creates a negative externality on wages due to its positive externality on firms' outside options. This effect is especially dramatic for workers in the lower tail of the skill distribution as their market value may completely collapse. In the long-run, however, higher average productivity stimulates market entry by firms, improving workers' position in the market and widening wage gaps.

The long-run demand effect and the non-linearity of wages can together help to understand the widely recognized 'skill premium puzzle'. As high skilled workers are able to gain disproportionately from a general upgrade in productivity, a strong skill-biased technological change might not be needed to explain the simultaneous increase in the both supply and price of skilled labor.

Appendix

A Proofs

A.1 Lemma 3

Proof. Start with deriving the wage rate for the least skilled worker. For $x' = \underline{x}$ eq. (7) reads as

$$rh(\underline{x}) = \alpha\left(X' - V_0 - h(\underline{x})\right) + \beta(U_0\left(\underline{x}\right) - h(x')).$$

Using (2) and (3) and substituting $w(\underline{x})$ for $rh(\underline{x})$ gives

$$\begin{aligned}\frac{\alpha+\beta+r}{r}w(\underline{x}) &= \alpha(X'-V_0)+\frac{\beta\alpha}{(\alpha+r)\,r}w(\underline{x}) \Leftrightarrow \\ w(\underline{x}) &= \frac{\alpha\,(\alpha+r)}{(\alpha+r)^2+\beta r}\,(\underline{x}-\phi)\,.\end{aligned}$$

By (9) we know that

$$w'(x)=\frac{(\alpha+r)\,(\alpha+F\,(x)\,\beta)}{(\alpha+r)^2+F\,(x)\,\beta\alpha+\beta r}=\psi\,(x)\,.$$

Integrating both sides yields

$$w(x)=\int\psi\,(x)\,dx+c\equiv\Psi(x)+c.$$

Now the constant can be solved using $w(\underline{x})$:

$$c=w(\underline{x})-\Psi(\underline{x}).$$

Hence, the wage rate for any other skill level $x'\in[\underline{x},\overline{x}]$ is given by

$$w(x')=\Psi(x')-\Psi(\underline{x})+w(\underline{x})=\int_{\underline{x}}^{x'}\psi\,(x)\,dx+w(\underline{x}).$$

■

A.2 Lemma 4

Proof. Let us start with showing that Conjecture 2 holds. Assume $x'>x''$. Then

$$\begin{aligned}\hat{U}\,(x'\mid x'') &= U_0\,(x'')+X'-X'' \\ &= \frac{\alpha}{\alpha+r}\int_{\underline{x}}^{x''}\frac{\psi\,(x)}{r}dx+\frac{w(\underline{x})}{r}+X'-X''.\end{aligned}$$

On the other hand

$$U_0\,(x')=\frac{\alpha}{\alpha+r}\int_{\underline{x}}^{x'}\frac{\psi\,(x)}{r}dx+\frac{w(\underline{x})}{r},$$

so that

$$\hat{U}\left(x' \mid x''\right) - U_0\left(x'\right) = \frac{1}{r}[x' - x'' - \frac{\alpha}{\alpha + r}\int_{x''}^{x'} \psi\left(x\right) dx],$$

which is greater than zero because $\psi\left(x\right) < 1$ $\forall x \in [\underline{x}, \overline{x}]$. Hence $\hat{U}\left(x' \mid x''\right) > U_0\left(x'\right)$ and Conjecture 2 holds.

Conjecture 1, part (i): The worker with skills x' prefers waiting if $h(x') \geq U_0(x')$. The value of the wait-and-search option

$$rh(x') = \alpha\left(X' - V_0 - h(x')\right) + \beta[(1 - F(x'))\, U_0\left(x'\right) + \int_{\underline{x}}^{x'} \hat{U}\left(x' \mid x\right) dF\left(x\right) - h(x')]$$

is a 'weighted' average of scenarios $X' - V_0$, $U_0\left(x'\right)$ and $\int_{\underline{x}}^{x'} \hat{U}\left(x' \mid x\right) dF\left(x\right)$. Since by Conjecture 2 $\hat{U}\left(x' \mid x''\right) > U_0\left(x'\right)$ $\forall x' > x''$, $x', x'' \in [\underline{x}, \overline{x}]$, we know that $\int_{\underline{x}}^{x'} \hat{U}\left(x' \mid x\right) dF\left(x\right) > U_0\left(x'\right)$. On the other hand,

$$X' - V_0 = X' - \underline{X} + \left(\underline{X} - V_0\right) > \hat{U}\left(x' \mid \underline{x}\right),$$

because $\left(\underline{X} - V_0\right) > w(\underline{x})$. Therefore $X' - V_0 > U_0\left(x'\right)$, so that earning $U_0\left(x'\right)$ is the worst scenario in worker's wait-and-search option, which directly implies that $h(x') \geq U_0(x')$.

Part (ii): Employers are willing to trade at the equilibrium wage if $\forall x' \in [\underline{x}, \overline{x}]$ $V(x') \geq V_0$. Assume $x' > x''$. $V(x') > V(x'')$ because

$$\begin{aligned} V(x') - V(x'') &= X' - X'' - \left(U(x') - U(x'')\right) \\ &= \frac{1}{r}[x' - x'' - \int_{x''}^{x'} \psi\left(x\right) dx] > 0, \end{aligned}$$

since $\psi\left(x\right) < 1$. Therefore it suffices to show that $V\left(\underline{x}\right) \geq V_0 = \Phi$.

$$V\left(\underline{x}\right) = \frac{\underline{x}}{r} - U\left(\underline{x}\right) = \frac{\underline{x}}{r} - \frac{\alpha\left(\alpha + r\right)}{\left(\alpha + r\right)^2 + \beta r}\left(\frac{\underline{x}}{r} - \Phi\right),$$

which is greater than Φ if $\underline{x} \geq r\Phi = \phi$.

Part (iii): Assume an unemployed worker has received a wage offer and decided to wait and search for alternative contacts. We next go through all the possible scenarios, starting from the situation where the unemployed has located another employer.

2 employers & 1 worker: By Bertrand argument we know that the employers raise their wage offers until driven to their reservation values. If the unemployed did not accept the highest offer but opted to search even more contacts, the best can happen is that another employer is located. A third bidder, however, would not increase the highest available bid. On the other hand, there is a possibility that the competing firms receive applications from other unemployed workers. Since there is no upside potential but only a downside risk, the 'incumbent' worker does not want to wait any further but concludes transaction.

1 employer & 2 workers: The second possibility is that the incumbent worker with skills x' finds himself competing with a rival worker candidate labeled with skills x''. Assume $x' > x''$, so that the newcomer could earn $U_0(x'')$ and the incumbent $\hat{U}(x' \mid x'')$. Consider first the possibility that a third worker candidate with skills x''' is to appear. If $x' > x'' > x'''$, then the toughest competition would still take place between the incumbent worker and the second candidate, and they would earn $\hat{U}(x' \mid x'')$ and $U_0(x'')$ respectively. If $x''' > x''$, the second candidate would 'drop out' and still earn $U_0(x'')$ while the incumbent would earn $\hat{U}(x' \mid x''') < \hat{U}(x' \mid x'')$, if $x''' < x'$, or $U_0(x'')$, if $x''' \geq x'$, so that the incumbent would be strictly worse-off. Hence the possibility of a third worker candidate arriving induces a downside risk. Then we need to show that there is no upside potential either. The better skilled incumbent locates an alternative employer at rate α. Then the reservation value for the first employer is the case where it is left alone with the second worker candidate; i.e. the employer can trade at a wage that produces $X'' - U(x'')$. Therefore the highest available wage rate for the incumbent as a result of the bidding game between the employers satisfies $\tilde{U}(x') = U(x'') + X' - X''$. However, the present value of this offer is

$$\tilde{U}_0(x') = U_0(x'') + \alpha/(\alpha + r)(X' - X'') < \hat{U}(x' \mid x''),$$

so that there is no upside potential available for the incumbent from further search. By the similar reasoning the highest utility the less skilled worker could earn by further waiting is $\tilde{U}(x'') = U(x)' - (X' - X'')$. The present value of this scenario is less than the utility available from immediate trading, i.e.

$\tilde{U}_0(x'') < U_0\,(x'')$. This can be verifies by examining

$$\tilde{U}_0(x'') - U_0\,(x'') = \frac{\alpha}{\alpha + r}\int\limits_{x''}^{x'} \frac{\psi\,(x)}{r}dx - \frac{X' - X''}{r},$$

which is a negative number because $\psi\,(x) < 1$.

We have now verified that workers of any type cannot gain from further search in a situation where there are two competing workers and a single employer. In fact, further waiting and search would make any worker strictly worse off. This observation, in turn, implies that if the single employer did not accept the lowest wage demand, both workers would be better off by leaving the employer. As the single worker rationally expects this, he infers that he is better off by accepting the lowest wage demand.

Points a) and b) together imply that trading will take place once a competitive situation is triggered on either employers' or workers' side. ■

A.3 Proposition 7

Proof. Since $\alpha = \theta m(\theta)$ and $\beta = m(\theta)$, $\psi(x)$ can be written as

$$\psi(x) = \frac{K(\theta)\,(\theta + F\,(x))}{K(\theta)^2 + K(\theta) - (1 - F\,(x))\,\theta},$$

where $K(\theta) = \theta + r/m(\theta) > 0$. Differentiating with respect to θ obtains

$$\begin{aligned} sign(\frac{\partial\psi(x)}{\partial\theta}) &= sign(K(\theta)^2\,(1 + K(\theta) - K'(\theta)(\theta + F(x))) + \\ &\quad + (1 - F\,(x))\,(K(\theta)F(x) - \theta K'(\theta)(\theta + F(x))). \qquad (11) \end{aligned}$$

Letting $F(x) \to 0$ obtains

$$sign(\frac{\partial\psi(x)}{\partial\theta}\mid_{F(x)\to 0}) = sign(K(\theta)(\frac{K(\theta)^2 - \theta^2 K'(\theta)}{K(\theta)} + K(\theta) - \theta K'(\theta))).$$

Since

$$K(\theta)^2 - \theta^2 K'(\theta) = \frac{r\theta(\alpha'(\theta) + r/\theta)}{m(\theta)^2} > 0,$$

and

$$K(\theta) - \theta K'(\theta) = \frac{r\alpha'(\theta)}{m(\theta)^2} > 0,$$

we have

$$\frac{\partial\psi(x)}{\partial\theta}\mid_{F(x)\to 0} > 0.$$

On the other hand, when $F(x) \to 1$,

$$sign(\frac{\partial \psi(x)}{\partial \theta} \mid_{F(x)\to 1}) = sign(K(\theta)^2(1 + K(\theta) - K'(\theta)(\theta + 1))).$$

Since

$$1 + K(\theta) - K'(\theta)(\theta + 1)) = \frac{r(\alpha'(\theta) + \beta'(\theta))}{m(\theta)^2},$$

$$\frac{\partial \psi(x)}{\partial \theta} \mid_{F(x)\to 1} > 0$$

only if $\alpha'(\theta) + \beta'(\theta) > 0$, so that a negative relationship between wage differentials and market tightness is possible only within the highest skill levels.

In order to get an idea what happens to the overall wage dispersion as market tighness is increased, let us work through a few numerical examples. Assume $M(u, v) = u^z v^{1-z}$ with $z = .4$, so that $m(\theta) = \theta^{-.4} = \beta$ and $\alpha = \theta^{1-.4}$. Assume a uniform distribution over unit interval; i.e. $F(x) = x$, where $x \in [0, 1]$ and let $r = .25$. Then the wage differential between the highest and the lowest skill level yields

$$w(1) - w(0) = \int_0^1 \frac{(\theta^{1-.4} + .25)(\theta^{1-.4} + x\theta^{-.4})}{(\theta^{1-.4} + .25)^2 + x\theta^{-.4} \times \theta^{1-.4} + \theta^{-.4} \times .25} dx \equiv \Delta_0^1.$$

The following table gives numerical values for Δ_0^1 with different levels of θ:

$\theta = 0.1$	$\Delta_0^1 = 0.599$
$\theta = 0.3$	$\Delta_0^1 = 0.687$
$\theta = 0.5$	$\Delta_0^1 = 0.737$
$\theta = 1.0$	$\Delta_0^1 = 0.804$
$\theta = 1.5$	$\Delta_0^1 = 0.838$
$\theta = 2.0$	$\Delta_0^1 = 0.859$

■

A.4 Proposition 8

Proof. Denote the 'spread' of a distribution by a parameter δ. For a mean-preserving spread of the distribution $F(x)$ it holds that

$$\int_{\underline{x}}^{x'} F_\delta(x; \delta) dx \geq 0, \ \forall x\prime \in [\underline{x}, \overline{x}].$$

The effect of a mean-preserving spread on the steady state wage dispersion between the extreme skill levels $\underline{x}$ and $\overline{x}$ is given by

$$\int_{\underline{x}}^{x'} \frac{\partial \psi(x)}{\partial F} F_{\delta}(x;\delta)dx,$$

which is positive since $\psi(x)$ is incresing and concave in F. ■

References

Acemoglu, D. (1997): "Matching, Heterogeneity, and the Evolution of Income Distribution," *Journal of Economic Growth*, 2, 61–92.

Blackburn, M. L., D. E. Bloom, and R. B. Freeman (1991): "Changes in Earnings Differentials in the 1980s: Concordance, Convergence, Causes, and Consequence," *The Jerome Levy Economics Institute, Wp No. 61*, (Wp No 61).

Butters, G. R. (1977): "Equilibrium Distributions on Sales and Advertising Prices," *Review of Economic Studies*, 44, 465–491.

Devroye, D., and R. Freeman (2001): "Does Inequality in Skills Explain Inequality of Earnings Across Advanced Countries?," *NBER Wp No 8140.*

Diamond, P. A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–168.

DiNardo, J., N. M. Fortin, and T. Lemieux (2001): "Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach," *Econometrica*, 64, 1001–1044.

Hall, R. E. (1979): "A Theory of the Natural Unemployment Rate and the Duration of Employment," *Journal of Monetary Economics*, 5, 153–169.

Julien, B., J. Kennes, and I. King (2000): "Bidding for Labor," *Review of Economic Dynamics*, 3, 619–649.

KATZ, L. F., AND D. H. AUTOR (1999): "Changes in the Wage Structure and Earnings Inequality," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3 of *Handbook of Labor Economics*, pp. 1464–1555. Elsevier Science B.V.

KATZ, L. F., G. W. LOVEMAN, AND D. G. BLANCHFLOWER (1993): "A Comparison of Changes in the Structure of Wages in Four OECD Countries," *NBER Wp No. 4297*.

KRUSELL, P., L. E. OHANIAN, J.-V. RIOS-RULL, AND G. L. VIOLANTE (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1053.

KULTTI, K. (2000): "About Bargaining Power," *Economic Letters*, 69, 341–344.

KULTTI, K., AND J. VIRRANKOSKI (2004): "Option to Wait in a Search Model," *Spanish Economic Review*, 6(2), 153–158.

MECKL, J., AND S. ZINK (2002): "Human-Capital Investment and the Wage Gap," *Economic Theory*, 19, 853–859.

MORTENSEN, D., AND C. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–415.

PISSARIDES, C. A. (2000): *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA, second edn.

ROSEN, Å., AND E. WASMER (2005): "Higher Education Levels, Firms' Outside Options and the Wage Structure," *CEPR*, Dp No. 3186.